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Old age or dependence. Which social insurance?

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Abstract

We consider a society where individuals differ according to their productivity and their risk of mortality and dependency. We show that according to the most reasonable estimates of correlations among these three characteristics, if one had to choose between a public pension system and a long-term care social insurance, the latter should be chosen by a utilitarian social planner. With a Rawlsian planner, the balance between the two schemes does depend on the comparison between the probabilities of the worst off individual and the probabilities of the rest of society.

JEL: H2, H5.

Keywords: long term care, pension, mortality risk, optimal taxation, liquidity constraints.

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1 Introduction

Assume that a government has to choose between a public pension system and a long term care social (LTC) insurance. Admittedly this is an odd dilemma that could be explained by limited public funds. We show in this short paper that priority should be given to the LTC scheme. This finding is a bit paradoxical given that in a number of countries we observe generous pensions systems and quasi-inexistent public support to dependence in the old age.

To achieve this result, we consider a two-period economy with three states of nature: the first period (people work and save), the second period (people retire) with a healthy state, and the second period with dependency (Cremer et al. (2010), Cremer and Pestieau (2014)). Society comprises a number of individuals who differ in their productivity and their probability of survival and dependence. We have strong evidence concerning the correlation between those probabilities. According to the recent waves of Survey of Health, Ageing and Retirement in Europe (SHARE), we indeed observe a positive correlation between income and longevity, and a negative correlation between income and dependence, conditional or not upon survival. Given these stylized facts, and assuming the existence of saving and LTC private insurance, it is intuitive that a redistributive LTC scheme brings more utility to the poor than a redistributive social security. One dollar devoted to a LTC public benefits them more than a dollar spent on public pensions. This is true as long as individuals, particularly those with low income, have a positive saving and a positive LTC insurance, or alternatively they can have negative saving and/or private LTC insurance (the absence of liquidity constraints). When these assumptions do not hold, we show that the superiority of LTC social insurance over public pensions still holds under some plausible conditions.

When the objective of the government is Rawlsian, then the desirability of public LTC scheme depends on the comparison between the probabilities of the worst-off individual and the average probabilities. However, the superiority of LTC benefits over social security is more limited than in the utilitarian case.

2 The Model

Consider a two-period model, where individuals work and save in the first period and retire in the second. In the second period people face different risks of mortality and dependence. There are I types of individuals. The proportion of type i ($i = 0, 1, \dots, I$) individuals is denoted by n_i , with the total number of individuals born in the first period being normalized to unity: $\sum_{i=0}^I n_i = 1$. Each individual of type i is characterized by three characteristics: (i) w_i (labor productivity in the first period), (ii) π_i (the probability to be alive in the second period), and (iii) p_i (the probability of becoming dependent in the second period). From the Survey of Health, Ageing and Retirement in Europe (SHARE), we know that at least for the European countries covered the following relations hold:

- longevity (π_i) increases with income.
- conditional upon survival, the probability of dependency (p_i) decreases with income.
- the probability of dependency ($\pi_i p_i$) decreases with income.

Consistent with these facts, we assume $cov(w_i, \pi_i) > 0$, $cov(w_i, p_i) < 0$ and $cov(w_i, \pi_i p_i) < 0$.¹

Let c_i denote individual i 's first period consumption, $\ell_i \in [0, \bar{\ell}]$ denote labor supply, d_i represent the second period consumption if (s)he is healthy, and m_i denote care expenditures in case of dependency. An individual's expected lifetime utility is given by

$$U_i = u(c_i - v(\ell_i)) + \pi_i(1 - p_i)u(d_i) + \pi_i p_i H(m_i).$$

In the following we denote $x_i = c_i - v(\ell_i)$. We assume $u' > 0$, $u'' < 0$, $v' > 0$, $v'' > 0$, $H' > 0$, $H'' < 0$, $v'(0) = 0$ and $v'(\bar{\ell}) > \max_{i=1, \dots, I} w_i$. We also assume $H'(y) > u'(y)$ for all $y > 0$, to reflect costly needs for dependency.

Private saving is invested in a perfect annuity market with a zero interest rate. From saving s_i , type i has a return s_i/π_i . There is also a private insurance market against dependency. From the insurance purchase P_i , type i receives $P_i/(\pi_i p_i)$, where the return is inversely proportional to the individual's risk of dependency.

¹Our stylized model in the following yields that life-time incomes are increasing in wage w_i .

The government imposes (i) linear income taxes ($\tau \geq 0$), (ii) flat-rate pension ($r \geq 0$), and (iii) uniform long-term care benefit ($q \geq 0$). Individuals choose labor supply (ℓ_i), savings (s_i), and private insurance (P_i) while taking the government's scheme as given:

$$U_i = u((1 - \tau)w_i\ell_i - s_i - P_i - v(\ell_i)) + \pi_i(1 - p_i)u(s_i/\pi_i + r) + \pi_i p_i H(s_i/\pi_i + P_i/(\pi_i p_i) + q + r). \quad (1)$$

In the following, we assume:

Assumption 1 $x_i \equiv c_i - v(\ell_i) > 0$ holds for all i at the optimum of (1).

Namely, individuals would not transfer all his/her first-period incomes to the second period through P_i and s_i . Here we assume that w_i 's are sufficiently high, and/or the expected cost for dependency is sufficiently low. The case where there is an individual with $w_i = 0$ will be discussed in Section 4.

The FOCs with respect to ℓ_i , s_i and P_i are:

$$u'(x_i) ((1 - \tau)w_i - v'(\ell_i)) = 0, \quad (2)$$

$$-u'(x_i) + (1 - p_i)u'(d_i) + p_i H'(m_i) \leq 0, \quad (3)$$

$$-u'(x_i) + H'(m_i) \leq 0. \quad (4)$$

Let the solution values be ℓ_i^* , s_i^* and P_i^* respectively. The first condition is written with an equal sign, implying for an interior solution for labor. The two other solutions are not necessarily interior, implying that some individuals may be constrained to have a non-negative level of saving or of LTC insurance. Formally: $s_i^* \geq 0$; $I_i^* \geq 0$. In case of interior solutions, we have

$$u'(c_i) = u'(d_i) = H'(m_i).$$

Concerning those solutions, we distinguish two cases in which they are interior:

- Given the parameters of the model, all the solutions are interior. This will be the case when both q and r are small, or alternatively when the tax rate is low for some reason (political decision or tax distortions).
- Liquidity constraints are assumed away, implying that individuals can have negative saving or insurance premium.

We now turn to the optimal level of public benefits chosen by a government that is utilitarian or Rawlsian.

3 Utilitarian Case

The problem of the utilitarian government is to maximize the following Lagrangian:

$$\begin{aligned} \mathcal{L} = & \sum n_i \{ u((1-\tau)w_i \ell_i^* - s_i^* - I_i^* - v(\ell_i^*)) \\ & + \pi_i(1-p_i)u(s_i^*/\pi_i + r) + \pi_i p_i H(s_i^*/\pi_i + I_i^*/(\pi_i p_i) + q + r) \} \\ & + \mu \sum n_i (\tau w_i \ell_i^* - \pi_i r - \pi_i p_i q), \end{aligned} \quad (5)$$

For simplicity, the stars with respect to x_i , d_i , m_i and ℓ_i are dropped in the remainder of the paper. The FOCs on q and r are as follows:

$$\frac{\partial \mathcal{L}}{\partial q} = \sum n_i \pi_i p_i H'(m_i) - \mu \bar{\pi} p, \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial r} = \sum n_i \pi_i \{ (1-p_i)u'(d_i) + p_i H'(m_i) \} - \mu \bar{\pi}, \quad (7)$$

where the bar denote the population average of the respective parameter.

We adopt the view point of tax reform wherein we consider that the tax is given, not necessarily optimal, and we look at the welfare incidence of increasing q at the expense of r while keeping a balanced budget. This is given by:

$$\begin{aligned} \frac{\partial \mathcal{L}^c}{\partial q} & \equiv \frac{\partial \mathcal{L}}{\partial q} + \frac{\partial \mathcal{L}}{\partial r} \frac{\partial r}{\partial q} \Big|_{d\tau=0} \\ & = \left(1 - \frac{\bar{\pi} p}{\bar{\pi}} \right) \sum n_i \pi_i p_i H'(m_i) - \frac{\bar{\pi} p}{\bar{\pi}} \sum n_i \pi_i (1-p_i) u'(d_i), \end{aligned}$$

or

$$\frac{\partial \mathcal{L}^c}{\partial q} = \left(1 - \frac{\bar{\pi} p}{\bar{\pi}} \right) \bar{\pi} p \left(cov \left(H'(m_i), \frac{\pi_i p_i}{\bar{\pi} p} \right) - cov \left(u'(d_i), \frac{\pi_i (1-p_i)}{\bar{\pi} (1-p)} \right) + \Delta \right), \quad (8)$$

where $\Delta \equiv \sum n_i \{ H'(m_i) - u'(d_i) \}$.

Regarding the second-period consumption (m_i and d_i), the following property holds (regardless of the liquidity constraints). The proof is given in the Appendix:

Lemma 1 $\partial m_i / \partial w_i \geq 0$ and $\partial d_i / \partial w_i \geq 0$ with strict inequality when $s_i^* > 0$.

We assume in the following that wage differentials represent the dominant source of heterogeneity of m_i and d_i . Namely, we assume that w varies sufficiently across individuals.

If π is constant or has a low variance, then, from our stylized facts concerning the correlations between πp and $\pi(1-p)$ on the one hand and w on the other hand, we have $\text{cov}(H'(m_i), \pi_i p_i / \bar{\pi p}) > 0$ and $\text{cov}(u'(d_i), \pi_i(1-p_i) / \overline{\pi(1-p)}) < 0$. The LTC social insurance realizes targeted expenditures but the public pension favors the productive individuals who also live longer. Notice that we assume that the tax distortions are independent of the type of insurances. Therefore, as long as $\Delta = 0$ ($H'(m_i) - u'(d_i) = 0$ for all i), it is always desirable to increase q at the expense of r ; in other words, $\partial \mathcal{L}^c / \partial q > 0$ in the case of interior solutions. $\Delta = 0$ holds if both s_i^* and I_i^* are positive for all individuals. For low values of both q and r (and τ), this condition is fulfilled.

To get $\frac{\partial \mathcal{L}^c}{\partial q} \leq 0$, we have to assume that Δ is negative and large enough. For an illustrative purpose, suppose that $H(y) = u(y - L)$ where $L > 0$ stands for the resources needed to compensate for the dependency.

Lemma 2 *Suppose that $H(y) = u(y - L)$. If $q \leq L$, then $\Delta \geq 0$.*

Only if $q > L$ (the government fully compensates resources the need for the dependency), we have $H'(m_i) = u'(s_i^* / \pi_i + q + r - L) < u'(s_i^* / \pi_i + r)$,² so that $\Delta < 0$. However, note that $q > L$ is a necessary but not sufficient condition for $\frac{\partial \mathcal{L}^c}{\partial q} \leq 0$. This leads us to our first theorem.

Theorem 1 *It is always desirable to have a balanced budget increase in LTC benefits at the expense of social security benefits, as long as the liquidity constraint is not binding for all individuals. In case part of the population is subject to a liquidity constraint, this dominance of LTC over social security still holds as long as LTC benefits are not too high relative to pension benefits.*

²From (3) and (4), $-u'(x_i) + H'(m_i) < -u'(x_i) + (1 - p_i)u'(d_i) + p_i H'(m_i) \leq 0$, so that $I_i^* = 0$ for all i .

Combining (6) and the revenue-side optimization, we obtain:

$$\begin{aligned}
\frac{\partial \mathcal{L}^c}{\partial \tau} &= \frac{\partial \mathcal{L}}{\partial \tau} + \frac{\partial \mathcal{L}}{\partial q} \frac{\partial q}{\partial \tau} \Big|_{dr=0} \\
&= \sum n_i u'_i(x_i)(-w_i \ell_i) + \sum n_i \pi_i p_i H'(m_i) \frac{\bar{y} + \tau \frac{\partial \bar{y}}{\partial \tau}}{\pi p} \quad (9) \\
&= -cov(u'(x_i), y_i) + cov(H'(m_i), \frac{\pi_i p_i}{\pi p}) \bar{y} - \Gamma \bar{y} + \mu \tau \frac{\partial \bar{y}}{\partial \tau},
\end{aligned}$$

where $y_i \equiv w_i \ell_i$ and $\Gamma \equiv \sum n_i \{u'(x_i) - H'(m_i)\} \geq 0$. This implies:

$$\tau^* = \frac{-cov\left(u'(x_i), \frac{y_i}{\bar{y}}\right) + cov\left(H'(m_i), \frac{\pi_i p_i}{\pi p}\right) - \Gamma}{-\mu \frac{\partial \bar{y}}{\partial \tau} \cdot 1/\bar{y}} > 0. \quad (10)$$

The denominator is the conventional efficiency term. It is positive. The first term of the numerator is the traditional equity term $-cov(u'(x_i), y_i) > 0$. These two terms correspond to the conventional optimal tax formula (e.g., Sheshinski (1972) and Hellwig (1986)). This redistributive impact of the conventional first term of the numerator is reinforced by the second term, which is positive and reflects the redistributive impact of the LTC benefit. Note that if instead of using the tax proceeds for LTC they were used for pensions, the second term of the numerator would be negative, reflecting the fact that pensions tend to benefit the high-income individuals. The last term of the numerator represents the cost of the binding liquidity constraints (4).

Below we have a further discussion for the case of $H(y) = u(y - L)$. Lemma 2 implies the following:

Theorem 2 *Suppose that $H(y) = u(y - L)$. Let (τ^*, q^*, r^*) be the utilitarian social optimum. If $q^* \leq L$, then $r^* = 0$.*

Evaluated at $q \leq L$ and $r = 0$, taking account of the government budget balance, the total effect of the tax increase for the increase of q is given by (9). Whether there exists the optimum at $q^* \leq L$ depends on the sign of (9) at $q \leq L$. The qualitative features are as follows. Other things being equal, (9) is lower (and q^* is lower) when the distribution of income and risk of dependency are more equal, or when the tax distortions are high.³

³When $q \leq L$ and $r = 0$, we can show that $\Gamma = 0$. Dividing (9) by $\bar{y} u'(x_i)$, it is

Between q and r , the priority is given to q until $q = L$. $r^* > 0$ might happen only when $q^* > L$ (individuals are overly insured under dependency), which we do not observe in reality.

4 Rawlsian Case

Suppose that there is an individual 0 whose $w_0 = 0$. For this individual, we have $s_0^* = 0$ and $P_0^* = 0$. Suppose that the government's social objective is to maximize the second-period utility of individual 0:

$$\mathcal{L} = \pi_0(1 - p_0)u(r) + \pi_0 p_0 H(q + r) + \mu \sum n_i(\tau w_i \ell_i^* - \pi_i r - \pi_i p_i q), \quad (11)$$

Since individual 0 does not pay the payroll tax, the optimal tax rate under this social objective is the peak of the Laffer curve: $\tau^* = \frac{\bar{y}}{-\partial \bar{y} / \partial \tau}$. The issue here is how to allocate the tax revenue between r and q .

$$\frac{\partial \mathcal{L}}{\partial q} = \pi_0 p_0 H'(r + q) - \mu \bar{\pi} \bar{p}, \quad (12)$$

$$\frac{\partial \mathcal{L}}{\partial r} = \pi_0 \{(1 - p_0)u'(r) + p_0 H'(r + q)\} - \mu \bar{\pi}. \quad (13)$$

From these FOCs, we have:

$$\frac{\partial \mathcal{L}^c}{\partial q} = \left(1 - \frac{\bar{\pi} \bar{p}}{\bar{\pi}}\right) \pi_0 p_0 H'(r + q) - \frac{\bar{\pi} \bar{p}}{\bar{\pi}} \pi_0 (1 - p_0) u'(r) \quad (14)$$

In other words, a compensated increase of LTC benefits, q , is desirable if and only if:

$$H'(r + q) > u'(r) \Phi, \quad (15)$$

where $\Phi = \frac{1 - p_0}{p_0} \frac{\bar{\pi} \bar{p} / \bar{\pi}}{1 - \bar{\pi} \bar{p} / \bar{\pi}} < 1$.

The parameter Φ depends on the average probability of dependency ($\bar{\pi} \bar{p} / \bar{\pi}$) and the probability of dependency of the poorest individuals (p_0). Note that in case of $\pi_i = \bar{\pi}$ for all i , $\Phi < 1$ since $p_0 > \bar{p}$.

increasing in $-\text{cov}(u'(x_i) / \overline{u'(x_i)}, y_i / \bar{y})$ and decreasing in $\partial \bar{y} / \partial \tau \cdot 1 / \bar{y}$. Noting that $\mu = \text{cov}(u'(x_i), \pi_i p_i / \bar{\pi} \bar{p}) + u'(x_i)$, evaluated at the left side of the Laffer curve ($\bar{y} + \tau \partial \bar{y} / \partial \tau > 0$), the value is increasing in $\text{cov}(u'(x_i) / \overline{u'(x_i)}, \pi_i p_i / \bar{\pi} \bar{p})$.

Clearly in the Rawlsian case, the superiority of LTC benefits over social security is more limited than in the utilitarian case.

Theorem 3 *In the Rawlsian case, a balanced budget increase in LTC benefits is desirable as long as these are not too high relative to pension benefits and if the dependence probability of the poorest is higher than that of the average population.*

5 Conclusion

This paper has studied the design of both a social LTC insurance and a public pension system. Both benefits were uniform as well as the payroll tax rate. Under the realistic assumption of a positive correlation between income and the survival probability and of a negative correlation of the dependency probability and income for the skilled and a lower probability of turning dependent, we show that a utilitarian government should give priority to the LTC scheme relative to the pension program. When the government adopts a Rawlsian criterion, both programs are needed and the relative advantage of one over the other will depend on the comparison between the average probabilities of survival and dependency and those of the worst-off individuals. In this paper, we use linear instruments. In a companion paper (Nishimura and Pestieau (2016)) we instead use non-linear instruments but with just two types of individuals. In the optimal non-linear scheme, our stylized facts determine the features of the optimal tax policies on the saving and the LTC schemes.

Appendix

Proof of Lemma 1: Let $f(\tau, w_i) \equiv (1 - \tau)w_i\ell_i^* - v(\ell_i^*)$. From the Envelope Theorem, $\partial f/\partial w_i = (1 - \tau)\ell_i^* > 0$. When $s_i^* > 0$ and $P_i^* > 0$, (3) and (4) imply $u'(f(\tau, w_i) - s_i^* - P_i^*) = H'(m_i) = u'(s_i^*/\pi_i + r)$, so $x_i = \frac{s_i^*}{\pi_i} + r = \frac{f(\tau, w_i) - P_i^* - r}{1 + \pi_i} + r$. Differentiating $-u' \left(\frac{f(\tau, w_i) - P_i^* - r}{1 + \pi_i} + r \right) + H' \left(\frac{f(\tau, w_i) - P_i^* - r}{1 + \pi_i} + \frac{P_i^*}{\pi_i p_i} + q + r \right) = 0$, we obtain:

$$\frac{\partial P_i^*}{\partial w_i} = \frac{(u''(x_i) - H''(m_i))/(1 + \pi_i) \cdot \partial f/\partial w_i}{(u''(x_i) - H''(m_i))/(1 + \pi_i) + H''(m_i)/(\pi_i p_i)}. \quad (16)$$

The denominator of (16) is negative due to the second-order condition with respect to P_i^* . Then $\frac{\partial x_i}{\partial w_i} = \frac{1}{1 + \pi_i} \left(\frac{\partial f}{\partial w_i} - \frac{\partial P_i^*}{\partial w_i} \right) = \frac{H''(m_i)/(\pi_i p_i) \partial f / \partial w_i}{u''(x_i) - H''(m_i) + H''(m_i)(1 + \pi_i)/(\pi_i p_i)} > 0$. Since $H''(m_i) \frac{\partial m_i}{\partial w_i} = u''(d_i) \frac{\partial d_i}{\partial w_i} = u''(x_i) \frac{\partial x_i}{\partial w_i}$, we have $\frac{\partial m_i}{\partial w_i} > 0$ and $\frac{\partial d_i}{\partial w_i} > 0$.

When the individual optimum faces the liquidity constraint for P_i^* , then (3) and (4) are characterized by $-u'(f(\tau, w_i) - s_i^*) + (1 - p_i)u'(s_i^*/\pi_i + r) + p_i H'(s_i^*/\pi_i + q + r) \leq 0$ and $P_i^* = 0$. When $s_i^* > 0$, differentiating the former equation, we obtain $\frac{\partial s_i^*}{\partial w_i} =$

$$\frac{u''(x_i)}{u''(x_i) + (1 - p_i)u''(d_i)/\pi_i + p_i H''(m_i)/\pi_i} \frac{\partial f}{\partial w_i} > 0. \text{ When } s_i^* = 0 \text{ and } P_i^* = 0, \partial m_i / \partial w_i = 0.$$

When $s_i^* = 0$ and $P_i^* > 0$, then differentiating $-u'(f(\tau, w_i) - P_i^*) + H' \left(\frac{P_i^*}{\pi_i p_i} + q + r \right) = 0$, $\frac{\partial x_i}{\partial w_i} = \frac{\partial f}{\partial w_i} - \frac{\partial P_i^*}{\partial w_i} = \frac{H''(m_i)/(\pi_i p_i)}{u''(x_i) + H''(m_i)/(\pi_i p_i)} \frac{\partial f}{\partial w_i} > 0$. Since $H''(m_i) \frac{\partial m_i}{\partial w_i} = u''(x_i) \frac{\partial x_i}{\partial w_i}$, we have $\frac{\partial m_i}{\partial w_i} > 0$. *Q.E.D.*

Proof of Lemma 2: Suppose that $q \leq L$. If (4) holds in equality, then (3) implies that $-u'(x_i) + (1 - p_i)u'(d_i) + p_i H'(m_i) = (1 - p_i)(u'(d_i) - H'(m_i)) \leq 0$ for all i . When $q \leq L$ and $P_i^* = 0$, $u'(d_i) = u'(s_i^*/\pi_i + r) \leq H'(s_i^*/\pi_i + r + q) = H'(m_i)$ for all i . Therefore, $u'(d_i) \leq H'(m_i)$ for all i when $q \leq L$. We therefore have $\Delta \geq 0$. *Q.E.D.*

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