Discussion Papers In Economics And Business

Educational funds and economic growth:

private versus public funds

Kohei Okada

Discussion Paper 19-07

May 2019

Graduate School of Economics
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Educational funds and economic growth: private versus public funds[*]

Kohei Okada[†]

Abstract

This study constructs an overlapping generations model in order to examine the relationship between education and R&D activities. Using this model, we analyze the effect of individuals' borrowing of educational funds on economic growth. We first consider the benchmark case in which individuals borrow educational funds at a market interest rate. We next consider that individuals borrow educational funds from a government at a constant rate of interest. If the policy interest rate is too low, the number of skilled workers increases, but economic growth is not achieved in the long run, because an increase in the demand for educational funds owing to the low policy interest rate crowds out funds for R&D investment and hinders economic growth. In addition, this study examines welfare and intergenerational inequality. When the government lowers the policy interest rate, the current generation's welfare level increases. However, future generations' welfare levels will decrease. This study shows that the government faces a trade-off between the current generation's welfare and future generations' welfare.

Keywords: Education policy, Occupational choice, Intergenerational inequality, R&D

JEL Classification Numbers: I22, I24, H52, O10

[*]I would like to thank Koichi Futagami, Tetsuo Ono, Tatsuro Iwaisako, Takeo Hori, Masashi Tanaka, Miho Sunaga, and Takaaki Morimoto for their helpful comments and suggestions. In addition, I thank participants of the 2017 Japanese Economic Association Spring Meeting held at Ritsumeikan University, and the 49th Economic Theory and Policy Workshop held at Tokyo Institute of Technology for their helpful comments and suggestions. I acknowledge financial support from the Grant-in-Aid for JSPS Fellows of the Japan Society for the Promotion of Science No.JP18J15399

[†]Graduate school of economics, Osaka unversity. Address: 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. Phone: +81-6-6850-5232. E-mail: sge005ok@student.econ.osaka-u.ac.jp

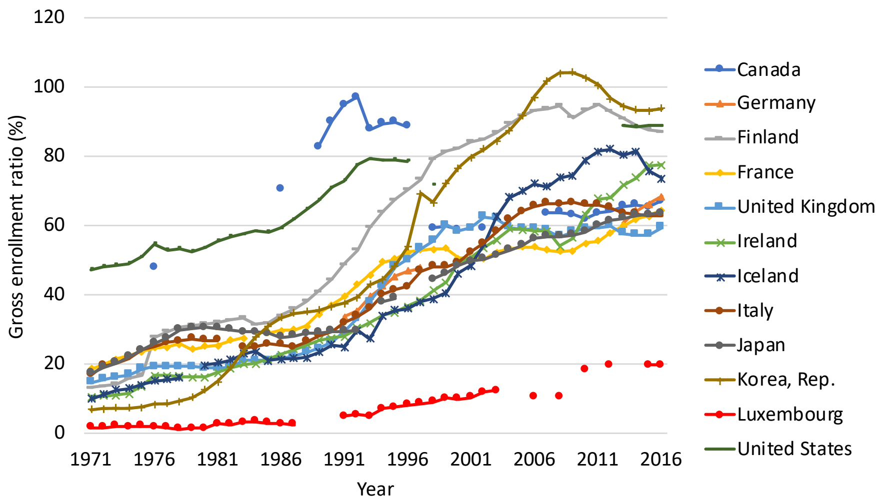

Figure 1: Gross enrollment ratio trends of Canada, Germany, Finland, France, the United Kingdom, Ireland, Iceland, Italy, Japan, Korea, Luxembourg, and the United States

1 Introduction

Economic development has been achieved in many countries, bringing an increase in the number of people who receive higher education. In this process, governments have adopted various educational policies. Figure 1 shows that the gross enrollment ratios of Canada, Germany, Finland, France, the United Kingdom, Ireland, Iceland, Italy, Japan, Korea, Luxembourg, and the United States have increased. Figure 1 confirms that people receiving higher education in developed countries have increased[1]. For example, in the United States, the gross enrollment ratio, which was about 47% in 1971, has been around 90% since 2010.

On the other hand, let us focus on the R&D activity because it is quite important factor for economic growth. We can divide twelve countries mentioned above into two types. First type is Type A countries whose R&D expenditure tends to increase and R&D would be undertaken actively. Type A countries include Germany, France, United Kingdom, Italy, Japan, and United States. Second type is Type B countries whose R&D expenditure tends to decrease and R&D would not be undertaken actively. Type B countries include Canada, Finland, Ireland, Iceland, and Luxembourg. Figure 2 shows the trends of average R&D expenditure of Type A countries and Type B countries. From Figure 2, we can find that average R&D expenditure of Type A countries tends to increase. For example, in Korea, the R&D expenditure, which was about 2.26% of GDP in 1996,

[1]Autor et al. (1995) empirically show this trend.

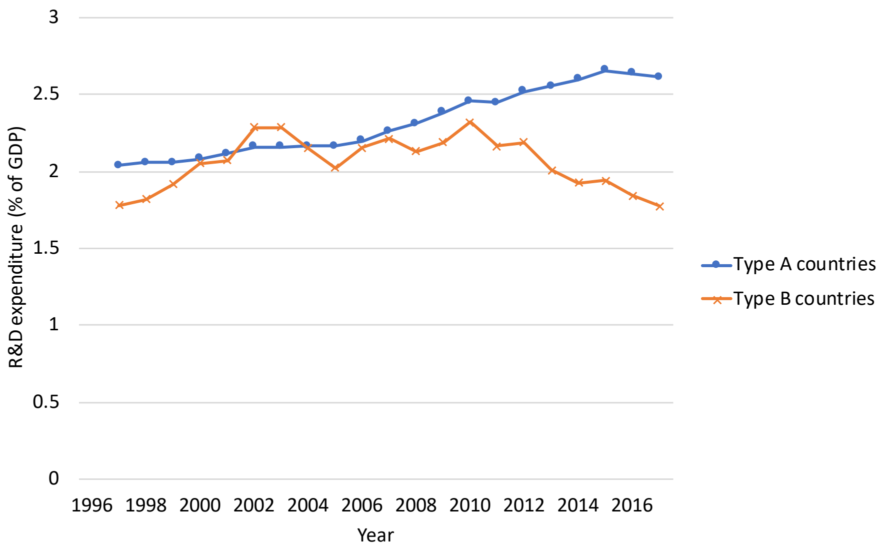

Figure 2: Trends of average R&D expenditure of Type A and Type B countries

has been around 4.24% since 2016. On the other hand, we can find that average R&D expenditure of Type B countries tends to decrease. For example, in Canada, the R&D expenditure, which was about 2.02% of GDP in 2001, has been around 1.61% since 2016.

In order to do so, this study focuses on the effect of borrowing educational funds on R&D activity. In developed countries, many people who receive education borrow funds as scholarships to pay for their education expenses. According to "Education at a glance 2014" (2014) published by the OECD, many students receive scholarships in OECD countries. In particular, in Europe and the United States, almost 80% of students receives scholarships. In addition, such scholarship projects are led by their governments, and significant financial resources are needed to promote the scholarship projects. Thus, governments raise financial resources. For example, in Japan, most students who receive scholarships do so by borrowing in the form of a student loan from the Japan Student Services Organization (JASSO)[2] The total rental amount of the Japan Student Services Organization was 5 billion dollars in 2003, which increased to 18.3 billion dollars in 2016. The Japan Student Services Organization's budget for financial year 2017 was 1.95 billion dollars, of which 6.9 billion dollars was covered by repayment from borrowers. The Japanese government covers a part of the remaining budget by raising funds, such as government loans and Fiscal Loan

[2]The Japan Student Services Organization (JASSO) is an independent administrative corporation established under the Ministry of Education, Culture, Sports, Science and Technology. JASSO mainly administers scholarship programs in Japan.

Funds, to the value of 13.8 billion dollars. The remainder is financed through private borrowing by the JASSO of 3.37 billion dollars. Therefore, the Japanese government procures significant funds for the scholarship project. In the United States, as in Japan, agencies called Affiliate Computing Services provide student loans; the total loan amount increased from about 45.5 billion dollars in 2003 to 100 billion dollars in 2011. In this study, we analyze macroeconomic trends taking into account scholarship projects operated by a government. This study examines how this expansion of demand for funds affects supplies of funds for other sectors.

We construct an overlapping generations model in which economic growth is induced by expanding the variety of goods by R&D activities. In addition, we introduce a mechanism for an individual to borrow educational funds. If an individual receives education, he or she becomes a skilled worker. Skilled workers earn more income than do unskilled workers who do not receive education. However, skilled workers must borrow educational funds to pay educational costs. In this study, we analyze the following two cases. The first case is (1) a private funds regime in which individuals borrow educational funds at a market interest rate in the financial market. The second case is (2) a public funds regime in which individuals borrow educational funds from a government at a constant interest rate. We show a condition by which the economy can grow over time in the private funds regime and also in the public funds regime. In the private funds regime, the economy can grow over time if the skill premium and R&D productivity are sufficiently high. In the public funds regime, if the policy interest rate is too low, expansion of demand for educational funds crowds out funds for R&D activities. Therefore, overly fast expansion of skilled workers can interrupt economic growth.

Furthermore, this study conducts welfare analysis and examines intergenerational inequality. If the government lowers the policy interest rate, it has two effects on the economy. One is a positive effect on the welfare level. A reduction in the policy interest rate increases the number of skilled workers. It increases individuals' disposable income. The other is a negative effect on welfare. If the increase in the demand for educational funds reduces the supply of funds for R&D, it decreases the future generations' income. Therefore, it has a negative effect on the welfare level of future generations. When the government lowers the policy interest rate, the current generation's welfare is increased by the positive effect. However, future generations' welfare is reduced by the negative effect. This study shows that the government faces a trade-off between the current generation's welfare and future generations' welfare.

This study is related to much research on economic growth and education. Boldrin and Montes (2005) analyze the relationship between education funds and economic growth. However, they focus only on accumulation of physical capital and human capital, and do not take account of R&D activities. On the other hand, the present study considers that the demand for funds for paying educational expenses crowds out the supply of funds to the R&D sector and that economic growth may not be achieved over the long run. Dalgaard and Kreiner (2001) and Strulik (2005) analyze the relationship between R&D investment and human capital accumulation. However, these studies do not consider financial markets for education expenses. Acemoglu (1998) and Galor and Moav (2000) focus on an increase in the number of workers with higher education along with technological progress after World War II. they argue that this increase is caused by skill-biased technological change. However, these studies do not analyze the relationship between financial markets for educational funds and economic growth. Grossmann (2007) analyzes the impact of R&D subsidy policy and educational policy on R&D and economic growth. His research and the present study both analyze the effect of the government's educational policy on R&D and economic growth. However, Grossmann (2007) does not analyze the government's scholarship policy such that individuals borrow the educational funds from a government. Morimoto (2017) analyzes the relationship between R&D subsidy policy and occupational choice. He finds the existence of an optimal R&D subsidy rate. In addition, he analyzes the relationship between R&D subsidy policy and intergenerational inequality. However, he focuses only on the R&D subsidy policy.

The rest of this paper is organized as follows. Section 2 shows the basic structure of the model, Section 3 analyzes the equilibrium and dynamics. Section 4 examines welfare and intergenerational inequality. The conclusion is described in Section 5.

2 The model

2.1 Individuals

Time is discrete and denoted by $t = 0, 1, 2, \cdots$. Each individual lives three periods (youth, adulthood, and old age). The population size of each generation is normalized to one. In the first period of their lives (youth), individuals decide whether to receive education or not. If they receive education, they have to borrow funds to pay the cost of education. However, they can work as skilled workers when they reach adulthood. If they do not receive education, they do not need to borrow. However, they become unskilled workers. In the second period of their lives (adulthood), individuals work and obtain wage income according to their skill. Individuals who borrow funds in youth have to repay it from their income in this period. In the public funds regime, all individuals pay a lump-sum tax. Individuals save the remaining income. We assume for simplicity that individuals do not consume in the first and second periods. In the final period of their lives (old age), individuals obtain a subsidy from the government and consume their total wealth. We also assume that each individual does not have any bequest motives for their children.

When young, if individuals do not receive education, they become unskilled workers and supply one unit of labor. On the other and, skilled workers who receive education can supply $h(>1)$ unit of labor. If an individual wants to obtain education, he or she must pay the educational cost depending on his or her ability θ. Each individual's ability is randomly given by the uniform distribution defined over $[0,1]$; that is, the distribution function of ability is $F(\theta)=\theta$ and the density function is $f(\theta)=1$. Furthermore, we assume that an individual whose ability is θ has to pay $\frac{\gamma}{\theta}$ as the educational cost. This assumption means that the lower ability an individual has, the more educational cost he or she has to pay.

Each individual chooses to become a skilled worker if and only if his or her disposable income as a skilled worker is higher than that as an unskilled worker. Therefore, if the following condition is satisfied, an individual whose ability is θ chooses to become a skilled worker.

$$\begin{aligned} & w_t h - R\frac{\gamma}{\theta} \geq w_t \times 1, \\ \rightarrow \quad & \theta \geq \frac{R\gamma}{w_t(h-1)} \equiv \hat{\theta}_{t-1}, \end{aligned} \tag{1}$$

where R and $\hat{\theta}_{t-1}$ are the gross rate of interest of educational loan and the threshold of ability, respectively. Individuals whose ability is above (under) this threshold become skilled (unskilled) workers.

We consider two cases in the following analyses. One is that individuals borrow from an asset market (private funds). The other is that they borrow from the government (public funds).

Therefore, R is defined as

$$R \equiv \begin{cases} R_t & \text{if individuals borrow from the asset market (private funds),} \\ R_B & \text{if individuals borrow from the government (public funds),} \end{cases}$$

where R_t and R_B are the gross rate of market interest and the gross rate of policy interest set by the government, respectively.

2.2 Final goods sector

Final goods Y_t are allocated to consumption, production of intermediate goods, R&D activities, and investment in education. The final goods are treated as a numeraire. The final goods are produced by competitive firms using the following production function:

$$Y_t = H_t^{1-\alpha} \int_0^{A_t} x_t(j)^\alpha dj, \tag{2}$$

where H_t is human capital devoted to production and $x_t(j)$ is intermediate goods $j \in [1, A_t]$. A_t denotes the variety of intermediate goods. In this economy, human capital is used only in the final goods sector. Human capital is given by the sum of unskilled workers' labor supply and skilled workers' labor supply, that is,

$$H_t = \int_0^{\hat{\theta}_{t-1}} 1 d\theta + \int_{\hat{\theta}_{t-1}}^1 hd = h - (h-1)\hat{\theta}_{t-1}. \tag{3}$$

Given the wage rate w_t and the price of intermediate goods $p_t(j)$, the profit-maximizing conditions with respect to H_t and $x_t(j)$ are

$$p_t(j) = \alpha H_t^{1-\alpha} x_t(j)^{\alpha-1}, \tag{4}$$

$$w_t = (1-\alpha) H_t^{-\alpha} \int_0^{A_t} x_t(j)^\alpha dj. \tag{5}$$

2.3 Intermediate goods sector

There is a number of differentiated intermediate goods. A single firm produces each intermediate good $j \in [0, A_t]$ and can supply a differentiated good monopolistically. The monopoly is protected by perfect patent protection. Each monopolistic firm produces one unit of an intermediate good by using one unit of final goods. The producer of intermediate good j maximizes the following profit,

$$\pi_{xt}(j) = p_t(j)x_t(j) - x_t(j),$$

subject to the inverse demand function of the final goods sector (4). From the profit-maximization condition, the price of intermediate goods j is

$$p_t(j) = \frac{1}{\alpha}. \tag{6}$$

Hence, all intermediate goods have the same price. Therefore, the firm-specific index j in the differentiated goods sector can be dropped. By substituting this into demand function (4), we obtain the output level of intermediate goods:

$$x_t = \alpha^{\frac{2}{1-\alpha}} H_t. \tag{7}$$

Thus, the profit of each intermediate good firm is given by

$$\pi_{xt} = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} H_t. \tag{8}$$

2.4 R&D sector

In this economy, R&D firms also use the final goods to invent new intermediate goods. After invention, the firms sell a blueprint of a new intermediate good to an entrepreneur in the intermediate goods sector. R&D firms use η unit of the final goods to invent one unit of new intermediate goods. We suppose that there are no spillover effects from the accumulated intermediate goods that have been invented. Free entry into R&D races leads to the following zero-profit condition:

$$v_t \leq \eta \qquad \text{with equality if} \quad \Delta A_t > 0. \tag{9}$$

The value of v_t is equal to the present value of future profits given by

$$v_{t-1} = \sum_{T=t}^{\infty} \frac{\pi_{xT}}{\Pi_{\tau=t}^{T} R_\tau}.$$

After some manipulations, we can obtain the following no-arbitrage condition.

$$R_t = \frac{v_t + \pi_{xt}}{v_{t-1}}. \tag{10}$$

Each individual saves at the gross rate of market interest R_t determined by (10). Moreover, if an individual borrows funds from the asset market, he or she faces this market interest rate.

3 Equilibrium

In this section, we consider the private funds regime and the public funds regime. Regardless of the characteristics of these two regimes, from (2) and (7), the following holds;

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} A_t H_t. \tag{11}$$

From (5) and (7), the wage rate w_t is

$$w_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t. \tag{12}$$

3.1 (1) Private funds regime

We first consider the private funds regime in which individuals who need educational funds borrow from the asset market. The interest rate is determined by the no-arbitrage condition (10). By using (1), (3), (8), (9), and (12), the no-arbitrage condition (10) becomes

$$R_t = \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h] A_t}{\alpha\gamma + \eta A_t} \equiv R_t^M. \tag{13}$$

Here, note that $\frac{\partial R_t^M}{\partial A_t} = \frac{[\eta+(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\alpha\gamma}{(\alpha\gamma+\eta A_t)^2} > 0$. From (1) and (13), the threshold of ability $\hat{\theta}_{t-1}$ becomes

$$\hat{\theta}_{t-1} = \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma+\eta A_t)}.$$

This shows that when A_t decreases, $\hat{\theta}_{t-1}$ increases. Because $\theta \in [0,1]$, this determines the lower bound of A_t. The lower bound $\underline{A}$ becomes

$$\underline{A} = \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}]\gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\eta(h-1)}. \tag{14}$$

When $A_t < \underline{A}$, $\hat{\theta}_{t-1} = 1$. Thus, $\hat{\theta}_{t-1}$ becomes

$$\hat{\theta}_{t-1} = \begin{cases} \frac{[\eta+(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma+\eta A_t)} & \text{if} \quad A_t \geq \underline{A}, \\ 1 & \text{if} \quad A_t < \underline{A}. \end{cases} \tag{15}$$

When $A_t < \underline{A}$, all individuals become unskilled workers. Therefore, no educational loans are made. Consequently, the asset market equilibrium condition is given by

$$\int_0^1 w_t d\theta = v_t A_t + v_t \Delta A_t. \tag{16}$$

The left-hand side of (16) is unskilled workers' saving. The right-hand side of (16) is investment in assets. By using (9) and (12), we can rewrite (16) as follows:

$$\underbrace{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}A_t}_{\equiv S_1^M(A_t)} = \underbrace{\eta A_{t+1}}_{\equiv D_1^M(A_{t+1})}. \tag{17}$$

Let us denote the left-hand side of (17) as $S_1^M(A_t)$ and the right-hand side of (17) as $D_1^M(A_{t+1})$.

When $A_t \geq \underline{A}$, some individuals receive education and become skilled workers. The individuals who receive education access the asset market to obtain educational funds. Unlike the public funds regime, individuals' demand for educational funds directly affects the asset market. Therefore, the asset market equilibrium condition is given by

$$\int_0^{\hat{\theta}_{t-1}} w_t d\theta + \int_{\hat{\theta}_{t-1}}^1 (w_t h - R_t^M \frac{\gamma}{\theta}) d\theta = \int_{\hat{\theta}_t}^1 \frac{\gamma}{\theta} d\theta + v_t A_t + v_t \Delta A_t. \tag{18}$$

The left-hand side of (18) is the sum of unskilled workers' saving and skilled workers' saving. The right-hand side of (18) is the sum of demand for education funds and investment in assets. By using (9), (12), (13), and (14), we can rewrite (18) as follows:

$$\underbrace{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h A_t - \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h]\gamma A_t}{\alpha\gamma + \eta A_t} \left\{ \log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma + \eta A_t)}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h]\gamma} + 1 \right\}}_{\equiv S_2^M(A_t)}$$
$$= \underbrace{\gamma \log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma + \eta A_{t+1})}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h]\gamma} + \eta A_{t+1}}_{\equiv D_2^M(A_{t+1})}. \tag{19}$$

Let us define the left-hand side of (19) as $S_2^M(A_t)$ and the right-hand side of (19) as $D_2^M(A_{t+1})$. Now, the following lemma holds.

Lemma 1 $\frac{\partial S_2^M(A_t)}{\partial A_t} > 0$, $\frac{\partial^2 S_2^M(A_t)}{\partial A_t^2} > 0$, $\frac{\partial D_2^M(A_{t+1})}{\partial A_{t+1}} > 0$, *and* $\frac{\partial^2 D_2^M(A_{t+1})}{\partial A_{t+1}^2} < 0$ *hold.*

Proof. See Appendix A.

We next assume the following with respect to the R&D productivity parameter.

Assumption 1 *R&D productivity parameter* η *satisfies the following condition*

$$\eta < (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}. \tag{A1}$$

Assumption 1 means that R&D productivity is sufficiently high. Under Assumption 1, $S_2^M(\underline{A}) > D_2^M(\underline{A})$ holds. Because two cases exist, we can draw phase diagrams that describe the dynamics. One is that (i) $S_2^M(A_t)$ and $D_2^M(A_{t+1})$ have no intersection point. The other is that (ii) $S_2^M(A_t)$ and $D_2^M(A_{t+1})$ have two intersection points. Figure 3 represents the dynamics of case (i). In this case, the economy continues to grow in the long run. From (15), the threshold of ability $\hat{\theta}_{t-1}$ decreases and the enrollment rate continues to rise over time. Figure 4 represents the dynamics of case (ii). In this case, if the initial technology level A_0 is too low ($A_0 < A_1^*$), the economy converges to A_1^*

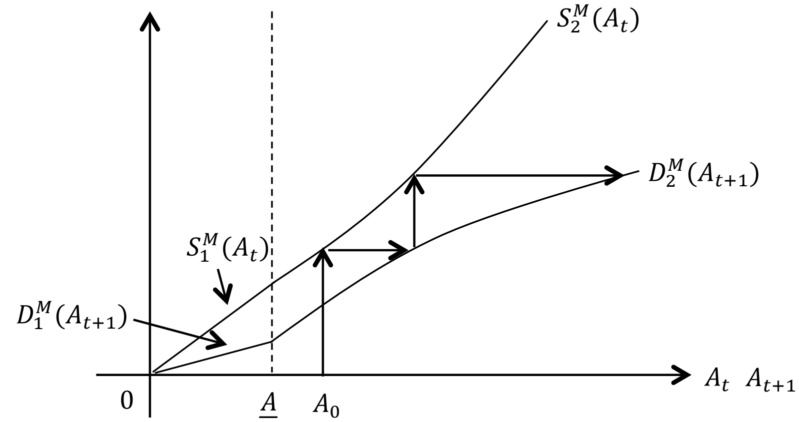

Figure 3: Phase diagram of case (i)

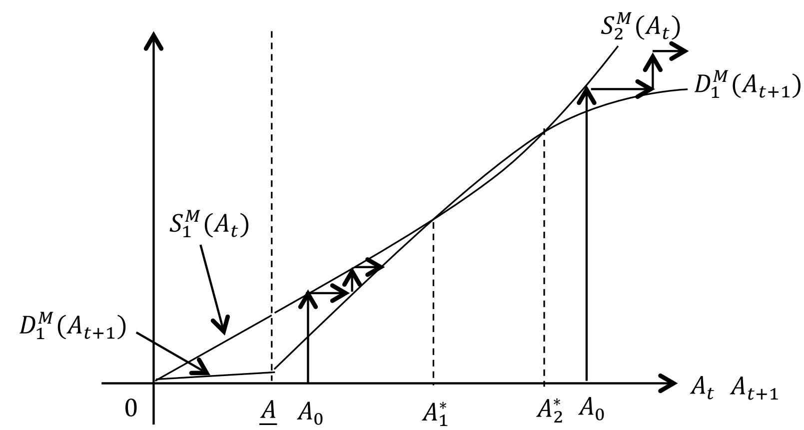

Figure 4: Phase diagram of case (ii)

and cannot continue to grow in the long run. From (15), the threshold of ability $\hat{\theta}_{t-1}$ and the enrollment rate become constant in the long run. On the other hand, if the initial technology level A_0 is sufficiently high ($A_0 > A_2^*$), the economy continues to grow in the long run and the enrollment rate rises. If the initial technology level A_0 is at a middle level ($A_1^* < A_0 < A_2^*$), the economy does not grow from the initial level because R&D does not take place[3].

We investigate whether case (i) or case (ii) occurs. For this purpose, let us define $\psi(A)$ as

[3]Following previous researches, we assume that the technology level dose not diminish and A_t dose not decrease. When $A_1^* < A_0 < A_2^*$, we can find that $S_2^M(A_0) < D_2^M(A_0)$ from Figure 4. Considering that A_t dose not decrease (i.e. $A_1 = A_0$) and (19) is derived from $v_t = \eta$, (9) has to hold with inequality (i.e. $v_0 < \eta$) to satisfy the asset market equilibrium condition ($S_2^M(A_0) = D_2^M(A_1)$).

$\psi(A) \equiv D_2^M(A) - S_2^M(A)$, that is,

$$\begin{aligned}
\psi(A) &\equiv D_2^M(A) - S_2^M(A) \\
&= \gamma \log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma} + \gamma \log(\alpha\gamma + \eta A) + \eta A - (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}hA \\
&\quad + \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma A}{\alpha\gamma + \eta A}\left\{\log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma} + \gamma \log(\alpha\gamma + \eta A) + 1\right\}
\end{aligned}$$

Differentiating $\psi(A)$ with respect to A, we obtain

$$\begin{aligned}
\psi'(A) = &\frac{\gamma\eta}{\alpha\gamma + \eta A} + \eta - (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h + \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\alpha\gamma^2}{(\alpha\gamma + \eta A)^2} \\
&\times \left\{\log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma + \eta A)}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma} + 1\right\} + \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma\eta A}{(\alpha\gamma + \eta A)^2}. \qquad (20)
\end{aligned}$$

The value of A that maximizes $\psi(A)$ is determined from equation $\phi'(A) = 0$. Let us denote the maximizer as $\hat{A}$. Substituting $\hat{A}$ into $\psi(A)$, we examine whether $\psi(\hat{A})$ takes a positive value or not. However, because it is difficult to explore the condition analytically, we conduct a numerical analysis. By setting $\alpha = 0.2$ and $\gamma = 1$, we can draw Figure 5 to show how the pair of parameters (h, η) determines the two cases.

Figure 5 shows that case (i) occurs if h and η are sufficiently low. If the skill premium h is high, an individual's incentive to become a skilled worker is high. Then, an individual's higher incentive increases the number of skilled workers, which increases demand for educational funds. The larger demand of educational funds crowds out investments for R&D. The supply of funds for R&D activities decreases, and then, long-run economic growth is not attained. On the other hand, if R&D productivity is high (η is low), long-run economic growth can be attained, because R&D firms invent actively. We can show the following proposition

Proposition 1 *Under Assumption 1, if h and η are sufficiently low, case (i) occurs. Then, the economy continues to grow in the long run and the enrollment rate rises over time.*

Proof. See Appendix B.

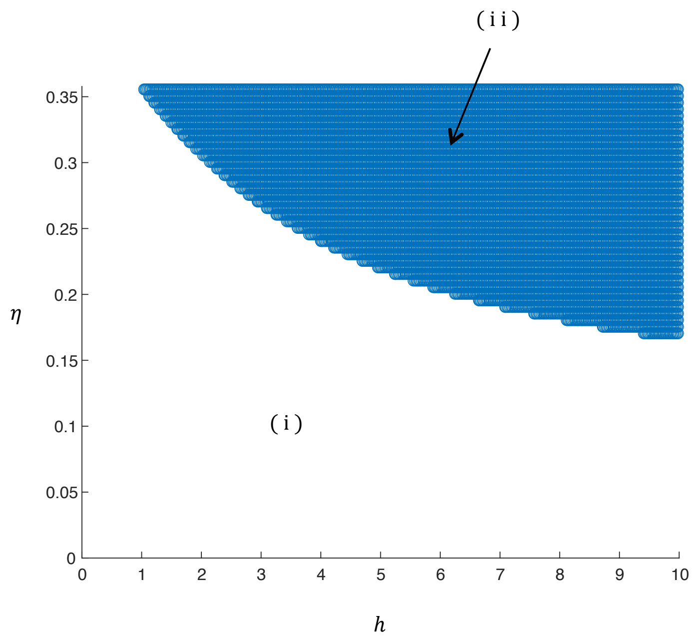

Figure 5: Parameter region of (h, η) for case (i) or (ii)

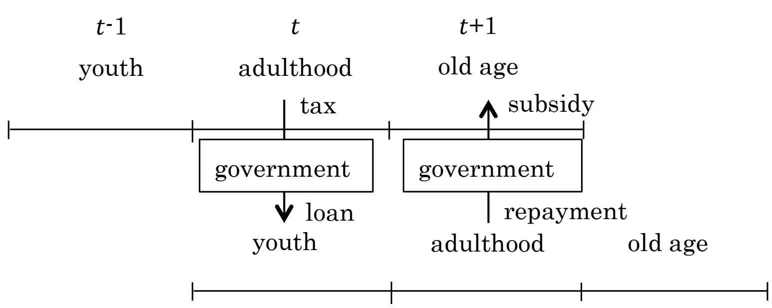

Figure 6: Government policy for each generation

3.2 (2) Public funds regime

We consider the public funds regime in which each individual borrows from the government. In this case, the government lends the funds to individuals who need to borrow at policy interest rate R_B. We assume that individuals cannot invest the funds borrowed from the government in the asset market instead of paying education fees. The government collects a lump-sum tax T_t from each adult and uses the tax revenue to lend funds to young individuals who want to receive education. Thus, the following holds:

$$\int_0^1 T_t d\theta = \int_{\hat{\theta}_t}^1 \frac{\gamma}{\theta} d\theta. \tag{21}$$

Furthermore, the government receives $R_B \frac{\gamma}{\theta}$ repayment from individuals who borrowed funds in youth and return the revenue to each old as subsidy G_t. Thus, the following relationship holds:

$$\int_{\hat{\theta}_{t-1}}^1 R_B \frac{\gamma}{\theta} d\theta = \int_0^1 G_t d\theta. \tag{22}$$

We next consider the asset market. We first derive the threshold of ability between skilled workers and unskilled workers in the equilibrium. From (1) and (12), the threshold of ability $\hat{\theta}_{t-1}$ becomes

$$\hat{\theta}_{t-1} = \frac{R_B \gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)A_t}.$$

This shows that when A_t decreases, $\hat{\theta}_{t-1}$ increases. Because $\theta \in [0,1]$, this determines the lower bound of A_t as follows:

$$\underline{A} = \frac{R_B \gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}. \tag{23}$$

Thus, $\hat{\theta}_{t-1}$ becomes

$$\hat{\theta}_{t-1} = \begin{cases} \frac{R_B \gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)A_t} & \text{if} \quad A_t \geq \underline{A}, \\ 1 & \text{if} \quad A_t < \underline{A}. \end{cases} \tag{24}$$

From (1), (3), (8), (9), (10), and (12), the gross market interest rate R_t is given by

$$R_t = 1 + \frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h}{\eta} - \frac{R_B \alpha\gamma}{\eta A_t} \equiv R_t^G. \tag{25}$$

To analyze a situation in which individuals borrow educational funds from the government, we assume that if the government changes the policy interest rate at time t, the government sets a lower policy interest rate than the market interest rate determined by (25)[4].

When $A_t < \underline{A}$, all individuals become unskilled workers. Therefore, there is no borrowing of educational funds. In this case, the asset market equilibrium condition is also given by (17).

On the other hand, when $A_t \geq \underline{A}$, some individuals receive education and become skilled workers. Therefore, the government's lending policy is undertaken. The asset market equilibrium condition is given by

$$\int_0^{\hat{\theta}_{t-1}} (w_t - T_t) d\theta + \int_{\hat{\theta}_{t-1}}^1 \left(w_t h - R_B \frac{\gamma}{\theta} - T_t \right) d\theta = v_t A_t + v_t \Delta A_t. \tag{26}$$

The left-hand side of (26) is the sum of unskilled workers' saving and skilled workers' saving. The right-hand side of (26) is investment in assets. By using (9), (12), (21), and (24), we can rewrite (26) as follows:

$$\underbrace{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h A_t - R_B \gamma \left[\log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)A_t}{R_B \gamma} + 1 \right]}_{\equiv S_2^G(A_t)}$$
$$= \underbrace{\gamma \log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)A_{t+1}}{R_B \gamma} + \eta A_{t+1}}_{\equiv D_2^G(A_{t+1})}. \tag{27}$$

Let us define the left-hand side of (27) as $S_2^G(A_t)$ and the right-hand side of (27) as $D_2^G(A_{t+1})$. We can show that the following lemma holds, as for $S_2^G(A_t)$ and $D_2^G(A_{t+1})$.

Lemma 2 $\frac{\partial S_2^G(A_t)}{\partial A_t} > 0$, $\frac{\partial^2 S_2^G(A_t)}{\partial A_t^2} > 0$, $\frac{\partial D_2^G(A_{t+1})}{\partial A_{t+1}} > 0$, *and* $\frac{\partial^2 D_2^G(A_{t+1})}{\partial A_{t+1}^2} < 0$ *hold.*

Proof. See Appendix C.

---

[4]The government sets $R_B < R_t^G$. From this relationship and (25), we obtain $R_B < \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h] A_t}{\alpha\gamma + \eta A_t} = R_t^M$. This result means that the government sets a lower policy interest rate than the market interest rate of the private funds regime.

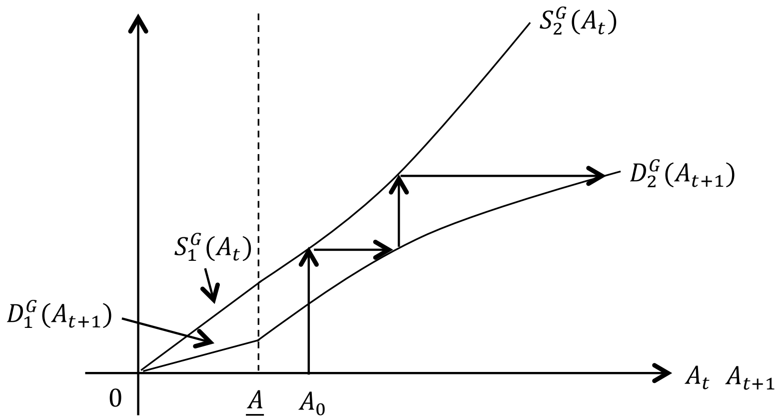

Figure 7: Phase diagram of case (iii)

We make Assumption 1 also in the public funds regime. Under Assumption 1, $S_2^G(\underline{A}) > D_2^G(\underline{A})$ holds. Because two cases can exist, we can draw phase diagrams that describe the dynamics. One is that (iii) $S_2^G(A_t)$ and $D_2^G(A_{t+1})$ have no intersection point. The other is that (iv) $S_2^G(A_t)$ and $D_2^G(A_{t+1})$ have two intersection points. Figure 7 represents the dynamics of case (iii). In this case, the economy continues to grow in the long run. From (24), the threshold of ability $\hat{\theta}_{t-1}$ decreases and the enrollment rate also continues to rise over time. Figure 8 represents the dynamics of case (iv). In this case, if the initial technology level A_0 is too low ($A_0 < A_1^{**}$), the economy converges to A_1^{**} and cannot continue to grow in the long run. From (24), the threshold of ability $\hat{\theta}_{t-1}$ and the enrollment rate become constant in the long run. On the other hand, if the initial technology level A_0 is too high ($A_0 > A_2^{**}$), the economy continues to grow in the long run and the enrollment rate rises. If the initial technology level A_0 is at a middle level ($A_1^{**} < A_0 < A_2^{**}$), the economy does not grow from the initial level because R&D dose not taken place ((9) holds with inequality).

Proposition 2 *Under Assumption 1, given h and η, a unique policy interest rate $\hat{R}_B > 0$ exists satisfying the following:*

(1) *When $R_B > \hat{R}_B$ (the policy interest rate is high), case (iii) is obtained. Then, the economy continues to grow in the long run and the enrollment rate rises.*

(2) *When $R_B < \hat{R}_B$ (the policy interest rate is low), case (iv) is obtained. If the level of initial technology A_0 is sufficiently low, economic growth and the rise of the enrollment rate finally stops in the long run.*

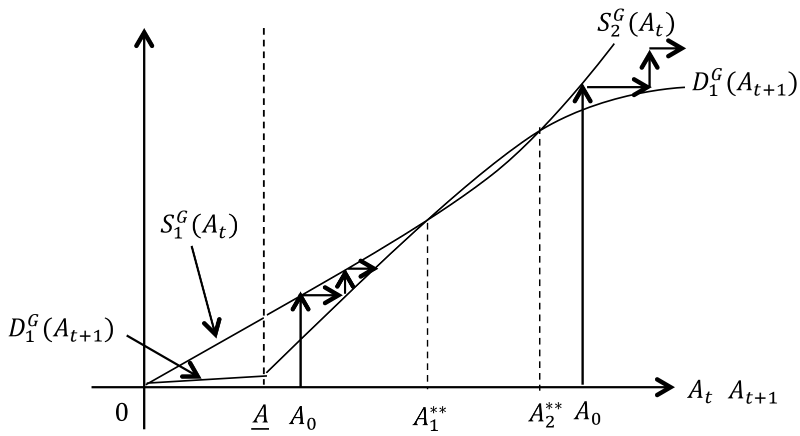

Figure 8: Phase diagram of case (iv)

Proof. See Appendix D.

If the level of technology A_t rises, the wage rate w_t also rises. The increase of the wage rate w_t decreases the threshold of ability $\hat{\theta}_{t-1}$. Therefore, the number of skilled workers increases. On the other hand, if the wage income increases, individuals save more. This saving finances R&D activities. If the policy interest rate R_B is too low, expansion of demand for educational funds crowds out funds for R&D activities more. Therefore, overly fast expansion of skilled workers interrupts economic growth.

Proposition 3 *Under Assumption 1, if the skill premium of skilled workers h is high and/or R&D productivity is low (η is high), then, $\hat{R}_B$ is high. Therefore, the government has to set the policy interest rate sufficiently high to achieve the long-run economic growth.*

Proof. See Appendix E.

If the skill premium h is high, an individual's incentive to become a skilled worker is high. Then, the individual's higher incentive increases the number of skilled workers. The increase of skilled workers increases demand for educational funds, which crowds out investment in R&D activities. Therefore, if the government does not set a higher policy interest rate, the supply of funds for R&D firms decreases and economic growth is not sustained. On the other hand, if R&D productivity is high (η is low), the economy can grow in the long run regardless of the lower policy interest rate, because R&D firms invent actively.

Last, we compare the private and public funds regimes. Suppose the government adopts an education policy by which it lends funds to individuals who receive education at time t and $A_t < A_1^{**}$. From Proposition 1, if h and η are sufficiently low, case (i) in which the economy continues to grow occurs in the private funds regime. Under this h and η, there is a policy interest rate threshold $\hat{R}_B$ that distinguishes between case (iii) in which the economy continues to grow in the long run and case (iv) in which the economy cannot grow in the long run if $A_t < A_1^{**}$ from Proposition 2. If the government sets a higher interest rate, R_B, than $\hat{R}_B$ at time t, case (iii) occurs, in which the economy continues to grow in the long run. On the other hand, if the government sets a lower interest rate, R_B, than $\hat{R}_B$, case (iv) occurs, in which the economy cannot grow in the long run if $A_t < A_1^{**}$. Therefore, long-run economic growth may be hindered by the government educational policy that sets too low a rate of policy interest.

4 Welfare and intergenerational inequality

In this section, we consider welfare and intergenerational inequality. In this economy, all individuals obtain utility from consumption when old. Therefore, we define the welfare levels of this economy at time t as the sum of consumption of old individuals at time t. The welfare levels of the private funds regime is

$$U_t^M = \begin{cases} R_{t+1}^M (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t & \text{if} \quad A_t < \underline{A}, \\ R_{t+1}^M S_2^M(A_t) & \text{if} \quad A_t \geq \underline{A} \end{cases} \tag{28}$$

The welfare levels of the public funds regime is[5]

$$U_t^G = \begin{cases} R_{t+1}^G (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t & \text{if} \quad A_t < \underline{A}, \\ R_{t+1}^G S_2^G(A_t) - (R_{t+1}^G - R_B)\int_{\hat{\theta}_t}^{1} \frac{\gamma}{\theta} d\theta & \text{if} \quad A_t \geq \underline{A}. \end{cases} \tag{29}$$

Suppose the government changes the policy interest rate at time 0. Figures 9 and 10 are the results of the numerical calculation under the following parameter setting: $h = 1.5$, $\alpha = 0.2$, $\gamma = 2$, and $\eta = 0.3055$. Figure 9 shows the effect of changes in the policy interest rate R_B at time 0 on

[5]The detailed calculation method of the welfare levels is described in Appendix F.

the welfare levels of the current generation[6]. According to Figure 9, the lower policy interest rate increases the welfare levels of the current generation. Figure 10 shows how changes of policy interest rate R_B at time 0 affect the welfare levels of each individual before the 10th generation. As shown in Figure 10, the government can raise the welfare levels of the 0th to 4th generations by setting the policy interest rate lower. However, when the government lowers the policy interest rate, the welfare level of individuals after the 5th generation declines. By the results of the numerical calculation, $\hat{R}_B = 0.8$. If the government sets R_B lower than 0.8, case (iv) is obtained. Then, economic growth finally stops in the long run.

Decreases in the policy interest rate have two effects on welfare levels. The first effect is positive on all generations' welfare levels. Suppose that the government lowers the policy interest rate at time 0. From (1), the enrollment rate of young individuals $(1 - \hat{\theta}_t)$ increases after time 0. The rise of the enrollment rate increases the number of skilled workers who earn more income than do unskilled workers. Therefore, the welfare level, which is the sum of consumption of all individuals in the same generation, increases. In addition, from (25), the decrease in the policy interest rate increases the market interest rate. When the market interest rate rises, individuals' savings receipts increase. Therefore, their consumption levels in old age increase and their welfare levels can be improved.

On the other hand, the second effect is negative. Suppose that the government lowers the rate policy interest at time 0. Then, the enrollment rate is raised and the demand for educational funds increases by the first effect. The expansion of demand for educational funds crowds out funds for R&D activities. When the level of R&D activities decreases, the rate of economic growth declines and the future technical level falls. Then, the wage rate and the market interest rate decline in the future from (12) and (25). Therefore, future generations' income and consumption decline. This outcome reduces the future generations' welfare levels. If individuals of the current generation earn lower income, they invest less. Then, R&D becomes more inactive and the technology level of the next generation decreases. The lower technology level reduces the next generation's income. Therefore, the impact of the negative effect gradually enlarges through the generations.

[6] The reason the old people's welfare level is not shown in Figure 9 is that the welfare level of the old at time 0 is not affected by the policy change. The enrollment rate of the old $(1 - \hat{\theta}_{-2})$ and the market interest rate faced by the old R_0 is not affected by the policy interest rate change at time 0. Therefore, the welfare level of the old at time 0 is not affected by the policy change.

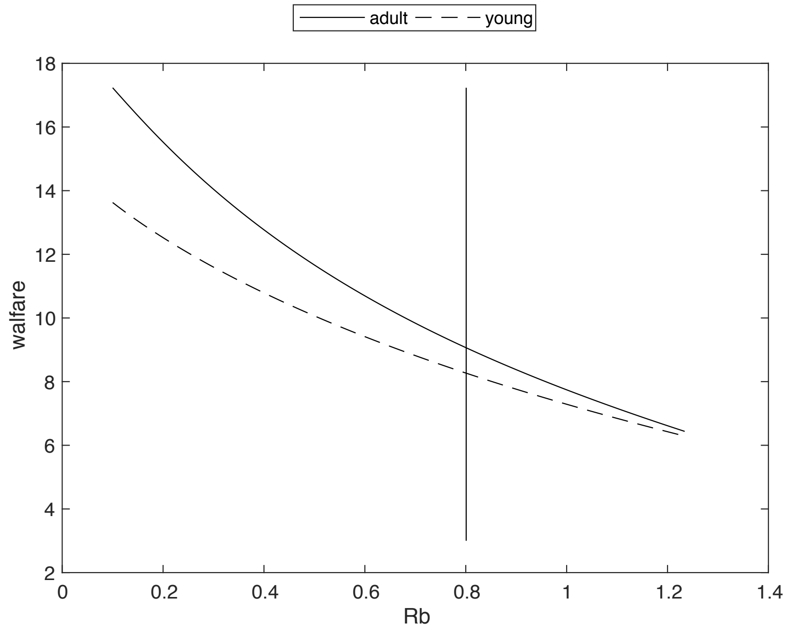

Figure 9: Relationship between policy interest rate and current generation welfare: The vertical line shows the welfare levels of young generation or adult generation at time 0. The horizontal line shows the policy interest rate.

The lower policy interest rate leads to the higher welfare levels of the current generation, as shown in Figure 9, because the positive effect is greater than the negative effect. As shown in Figure10, when the government sets a low policy interest rate, the welfare levels of the 0th to 4th generation increase, because the positive effect is greater than the negative effect. However, the welfare levels of individuals after the 5th generation decline, because the negative effect is greater than the positive effect. The government faces a trade-off between improving the welfare of the current generation and increasing the welfare of future generations.

5 Conclusion

We introduced borrowing educational funds by individuals into an endogenous growth model of a variety-expansion type R&D. Using this model, we analyzed the effect of individual's borrowing of educational funds on economic growth. When individuals access the asset market directly and borrow educational funds at a market interest rate (private funds regime), if the skilled worker's skill premium and R&D firms' productivity are low, economic growth cannot be achieved in the long run. When individuals borrow educational funds from the government with a fixed interest

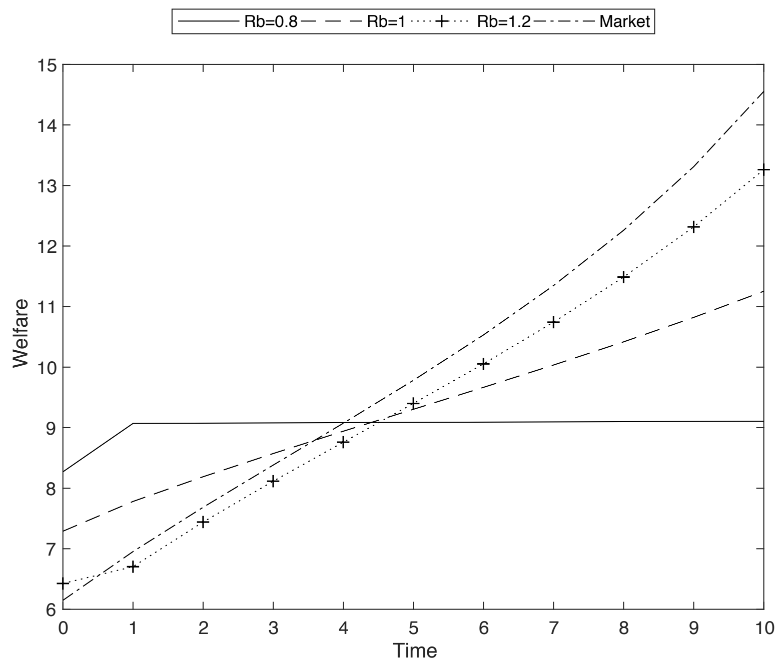

Figure 10: Relationship between policy interest rate and intergenerational inequality: The vertical line shows the welfare levels of young generation or adult generation at time 0. The horizontal line shows time.

rate set by the government (public funds regime), if the policy interest rate is too low, expansion of demand for educational funds crowd out funds for R&D activities. Therefore, overly fast expansion of skilled workers interrupts economic growth. In addition, we suggest that even though long-run economic growth is attained when the individual borrows from the financial market, the economy cannot grow in the long run because of the government's excessive education policy. In addition, we examined welfare and intergenerational inequality. If the government lowers the policy interest rate on educational fund lending, it has both a positive effect and a negative effect on the economy. When the government lowers the policy interest rate of educational fund lending, the current generation's welfare increases by the positive effect. However, future generations' welfare will decrease by the indirect negative effect. This study showed that the government faces a trade-off between the current generation's welfare and future generations' welfare.

Appendix

Appendix A : Proof of Lemma 1

From (14), we obtain

$$\alpha\gamma + \eta\underline{A} = \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}. \tag{30}$$

We also obtain

$$\begin{aligned}
\frac{\partial S_2^M(A_t)}{\partial A_t} &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h - \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\alpha\gamma^2}{(\alpha\gamma+\eta A_t)^2}\left\{\log\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma+\eta A_t)}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma} + 1\right\} \\
&\qquad - \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma A_t}{(\alpha\gamma+\eta A_t} \times \frac{\eta}{\alpha\gamma+\eta A_t} \\
&= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h - \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\alpha\gamma^2}{(\alpha\gamma+\eta A_t)^2} \\
&\qquad \times \left\{\log\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma+\eta A_t)}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma} + \alpha\gamma + \eta A_t\right\}
\end{aligned} \tag{31}$$

and $\frac{\partial^2 S_2^M(A_t)}{\partial A_t^2}$ is calculated as follows

$$\begin{aligned}
\frac{\partial^2 S_2^M(A_t)}{\partial A_t^2} &= \frac{2[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma\eta}{(\alpha\gamma+\eta A_t)^3}\left\{\alpha\gamma\log\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma+\eta A_t)}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma} + \alpha\gamma + \eta A_t\right\} \\
&\quad - \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma}{(\alpha\gamma+\eta A_t)^2}\left(\frac{\alpha\gamma\eta}{\alpha\gamma+\eta A_t} + \eta\right) \\
&= \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma\eta}{(\alpha\gamma+\eta A_t)^3}\left\{2\alpha\gamma\log\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma+\eta A_t)}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma} + \eta A_t\right\}.
\end{aligned} \tag{32}$$

From this equation, if $2\alpha\gamma\log\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma+\eta A_t)}{[\eta+(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma} \geq 0$, $\frac{\partial^2 S_2(A_t)}{\partial A_t^2} > 0$ holds. Considering $A_t \geq \underline{A} = \frac{[\eta+(1-\alpha)\alpha^{\frac{1-\alpha}{1+\alpha}}h]\gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\eta(h-1)}$, after some manipulations, we obtain the following inequality,

$$2\alpha\gamma\log\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma+\eta A_t)}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma} \geq 0$$

Therefore, when $A_t \geq \underline{A}$, $\frac{\partial^2 S_2^M(A_t)}{\partial A_t^2} > 0$ holds. We examine the sign of $\frac{\partial S_2^M(A_t)}{\partial A_t}$; when $A_t = \underline{A}$, the following holds

$$\left.\frac{\partial^2 S_2^M(A_t)}{\partial A_t^2}\right|_{A_t=\underline{A}} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h - \frac{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h]\alpha\gamma^2}{(\alpha\gamma + \eta\underline{A})^2} \times \left\{\log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma + \eta\underline{A})}{[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h]\gamma} + \alpha\gamma + \eta\underline{A}\right\}.$$

From (30), we obtain

$$\begin{aligned}\left.\frac{\partial S_2^M(A_t)}{\partial A_t}\right|_{A_t=\underline{A}} &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h - \frac{\gamma[(1-\alpha)\alpha^{1+\alpha} 1 - \alpha h][(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)]^2}{\{\gamma[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h]\}^2} \\ &\quad \times \left\{\underbrace{\alpha\gamma \log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)\gamma[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h]}{\gamma[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h](1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}}_{=0} + \frac{\gamma[\eta + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} h]}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}\right\} \\ &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} > 0. \end{aligned} \tag{33}$$

Considering $\frac{\partial^2 S_2^M(A_t)}{\partial A_t^2} > 0$, $\frac{\partial S_2^M(A_t)}{\partial A_t} > 0$ holds when $A_t \geq \underline{A}$.

We obtain $\frac{\partial D_2^M(A_{t+1})}{\partial A_{t+1}}$ as follows:

$$\frac{\partial D_2^M(A_{t+1})}{\partial A_{t+1}} = \frac{\gamma\eta}{\alpha\gamma + \eta A_{t+1}} + \eta > 0 \tag{34}$$

and also obtain $\frac{\partial^2 D_2^M(A_{t+1})}{\partial A_{t+1}^2}$ as follows:

$$\frac{\partial^2 D_2^M(A_{t+1})}{\partial A_{t+1}^2} = -\frac{\gamma\eta^2}{(\alpha\gamma + \eta A_{t+1})^2} < 0. \tag{35}$$

Appendix B : Proof of Proposition 1

From Assumption 1, $D_2^M(\underline{A}) < S_2^M(\underline{A})$ holds. Therefore, $\psi(\underline{A}) \equiv D_2^M(\underline{A}) - S_2^M(\underline{A}) < 0$ holds. In addition, From Lemma 1, $\psi(A)$ is a concave function with respect to A. Therefore, $\psi'(\underline{A}) < 0$ is a

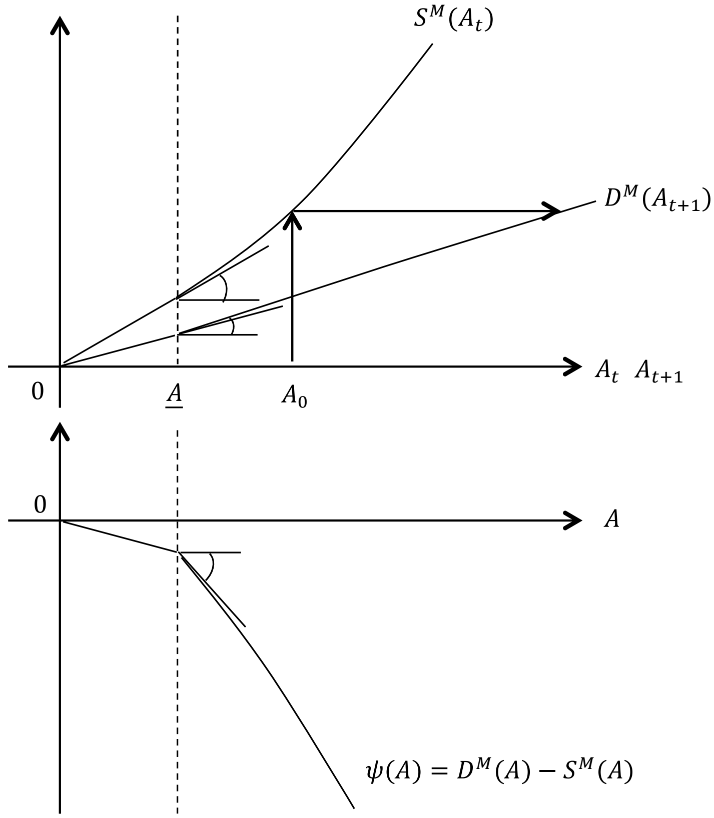

Figure 11

sufficient condition for case (i) to occur. From (20), we obtain

$$\begin{aligned}\psi'(\underline{A}) &= \frac{\gamma\eta}{\alpha\gamma+\eta\underline{A}} + \eta - (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h + \frac{[\eta+(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\alpha\gamma^2}{(\alpha\gamma+\eta\underline{A})^2} \\ &\quad\times\left\{\log\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(\alpha\gamma+\eta\underline{A})}{[\eta+(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma}+1\right\} + \frac{[\eta+(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h]\gamma\eta\underline{A}}{(\alpha\gamma+\eta\underline{A})^2} \\ &= \left[2\eta+(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1+\alpha)\right]\left[\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}{\eta+(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h} - \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h-\eta}{2\eta+(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1+\alpha)}\right]\end{aligned} \tag{36}$$

We define the following functions to examine the sign of $\psi'(A)$.

$$\Lambda(\eta,h) \equiv \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}{\eta+(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h}, \tag{37}$$

$$\Omega(\eta,h) \equiv \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h-\eta}{2\eta+(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1+\alpha)}. \tag{38}$$

We can obtain the following:

$$\frac{\partial\Omega(\eta,h)}{\partial\eta} = -\frac{[2\eta+(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1+\alpha)]+2[(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h-\eta]}{[2\eta+(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1+\alpha)]^2} < 0 \tag{39}$$

$$\frac{\partial^2\Omega(\eta,h)}{\partial\eta^2} = \frac{2}{[2\eta+(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1+\alpha)]^4} > 0 \tag{40}$$

$$\begin{aligned}\Lambda(0,h) &= \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1+\alpha)} \\ &= \frac{h}{h-1+\alpha}\end{aligned} \tag{41}$$

$$\begin{aligned}\Omega(0,h) &= \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}h} \\ &= \frac{h-1}{\alpha h}.\end{aligned} \tag{42}$$

We define $\hat{h}$ as h when equality $\Omega(0,h) = \Lambda(0,h)$ holds.

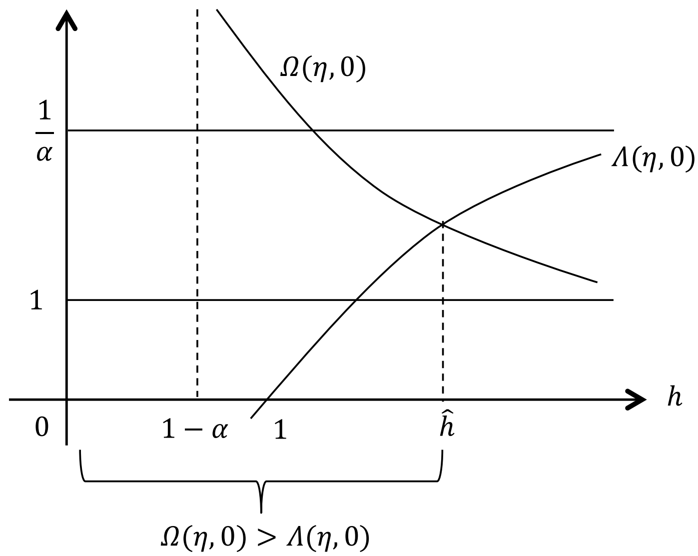

Figure 12

According to Figure 12, when $h < \hat{h}$, $\Omega(0) > \Lambda(0)$ holds. Now, we draw the relationship between (37) and (38) in Figure 13.

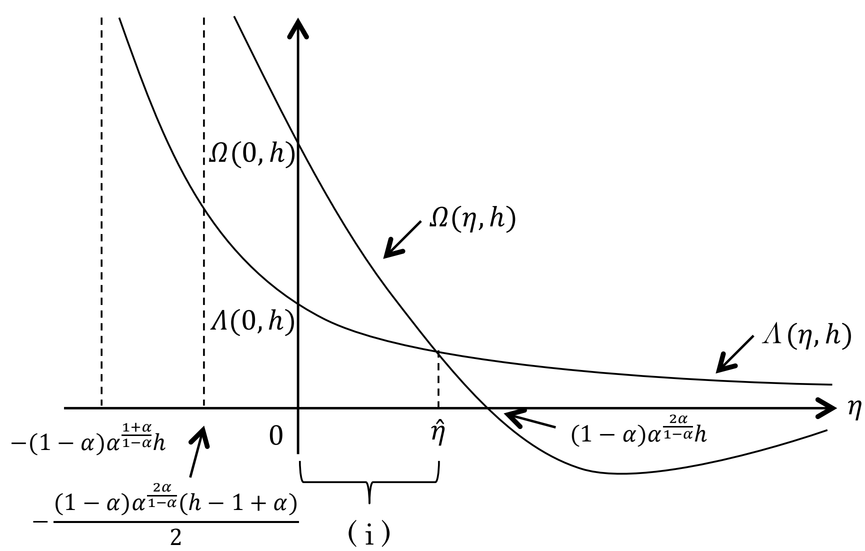

Figure 13

We define $\hat{\eta}$ such that $\Omega(\eta,h) = \Lambda(\eta,h)$ holds. According to Figure 13, when $\eta \leq \hat{\eta}$, case (i) occurs. Therefore, when $h \leq \hat{h}$ and $\eta \leq \hat{\eta}$ holds, case (i) occurs.

Appendix C : Proof of Lemma 2

$\frac{\partial S_2^G(A_t)}{\partial A_t}$ is given by

$$\frac{\partial S_2^G(A_t)}{\partial A_t} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h - \frac{R_B\gamma}{A_t}. \tag{43}$$

When $A_t \geq \underline{A}$, we show the following:

$$\begin{aligned}\frac{\partial S_2^G(A_t)}{\partial A_t} &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h - \frac{R_B\gamma}{A_t}\\ &> (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1) - \frac{R_B\gamma}{A_t}\\ &= \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}{A_t}\left(A_t - \frac{R_B\gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}\right)\\ &= \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}{A_t}(A_t - \underline{A}) \geq 0.\end{aligned} \tag{44}$$

$\frac{\partial^2 S_2^G(A_t)}{\partial A_t^2}$ is calculated as

$$\frac{\partial^2 S_2^G(A_t)}{\partial A_t^2} = \frac{R_B\gamma}{A_t^2} > 0. \tag{45}$$

We obtain $\frac{\partial D_2^G(A_{t+1})}{\partial A_{t+1}}$ as follows:

$$\frac{\partial D_2^G(A_{t+1})}{\partial A_{t+1}} = \frac{\gamma}{A_{t+1}} + \eta > 0. \tag{46}$$

Thus, $\frac{\partial^2 D_2^G(A_{t+1})}{\partial A_{t+1}^2}$ becomes

$$\frac{\partial^2 D_2^G(A_{t+1})}{\partial A_{t+1}^2} = -\frac{\gamma}{A_{t+1}^2} < 0. \tag{47}$$

Appendix D : Proof of Proposition 2

We investigate conditions in which case (iii) occurs. Let us define $\psi(A)$ as $\psi(A) \equiv D_2^G(A) - S_2^G(A)$, that is,

$$\begin{aligned}\psi(A) &\equiv D_2^G(A) - S_2^G(A) \\ &= (1+R_B)\gamma \log A - [(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h - \eta]A + R_B\gamma \\ &\qquad\qquad - (1+R_B)\gamma \log \frac{R_B\gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}. \end{aligned} \tag{48}$$

Differentiating $\psi(A)$ leads to

$$\psi'(A) = \frac{(1+R_B)\gamma}{A} - [(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h - \eta].$$

This derivative equals zero when $A = \frac{(1+R_B)\gamma}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h - \eta} \equiv \tilde{A}$. It is obvious that $\psi(A)$ takes a maximum value when $A = \tilde{A}$. The maximum value of $\psi(A)$ is

$$\psi(\tilde{A}) = \gamma \left\{ (1+R_B) \log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(1+R_B)}{[(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h - \eta]R_B} - 1 \right\}. \tag{49}$$

We next examine the sign of $\psi(\tilde{A})$. If the following inequality holds, then $\psi(\tilde{A})$ takes a positive value

$$(1+R_B) \log \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(1+R_B)}{[(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h - \eta]R_B} - 1 \geq 0.$$

This can be rewritten as

$$\underbrace{\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(1+R_B)}{[(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h - \eta]R_B}}_{\equiv \phi(R_B;h,\eta)} \geq e^{\frac{1}{1+R_B}}. \tag{50}$$

We define the left-hand side of (50) as $\phi(R_B; h, \eta)$. It is obvious that $\phi(R_B; h, \eta)$ and $e^{\frac{1}{1+R_B}}$ are decreasing functions of R_B. Moreover, we show that $\lim_{R_B \to 0} \phi(R_B; h, \eta) = \infty$, $\lim_{R_B \to \infty} \phi(R_B; h, \eta) = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} h - \eta}$, $\lim_{R_B \to -1} e^{\frac{1}{1+R_B}} = \infty$ and $\lim_{R_B \to \infty} e^{\frac{1}{1+R_B}} = 1$. The following inequality holds un-

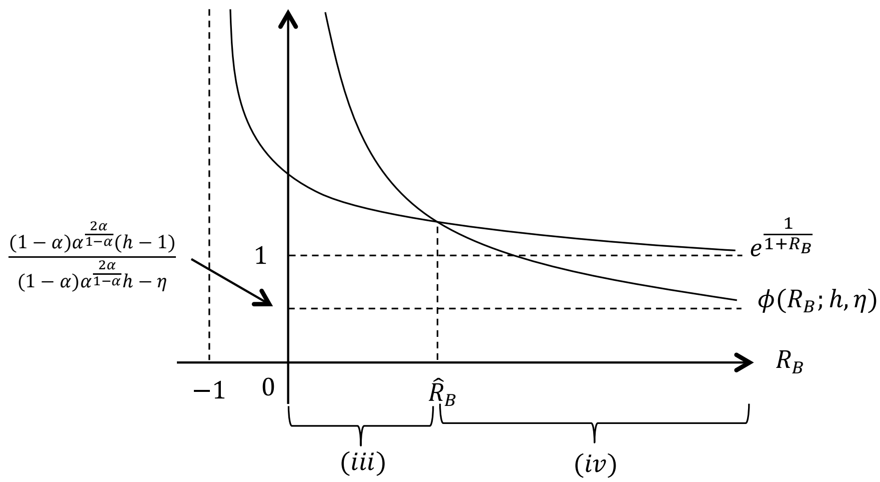

Figure 14: Determination of $\hat{R}_B$

der (A1):

$$\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)}{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h-\eta} < 1. \tag{51}$$

From the preceding arguments, Figure 14 shows the graph of the left-hand side and that of the right-hand side of (50). We define $\hat{R}_B$ as the policy interest rate at which (50) holds with equality. When $R_B \leq \hat{R}_B$, case (iii) is obtained.

Appendix E : Proof of Proposition 3

We investigate the effects of the change of h and η on $\hat{R}_B$. We obtain

$$\frac{\partial \phi(R_B; h, \eta)}{\partial h} = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(1+R_B)[(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}-\eta]}{[(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h-\eta]^2 R_B} \geq 0 \tag{52}$$

and

$$\frac{\partial \phi(R_B; h, \eta)}{\partial \eta} = \frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}(h-1)(1+R_B)}{[(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}h-\eta]^2 R_B} \geq 0. \tag{53}$$

Therefore, if h or η increases, $\phi(R_B; h, \eta)$ shifts, as described in Figure 15. From Figure 15, we find that $\hat{R}_B$ rises if h or η increase.

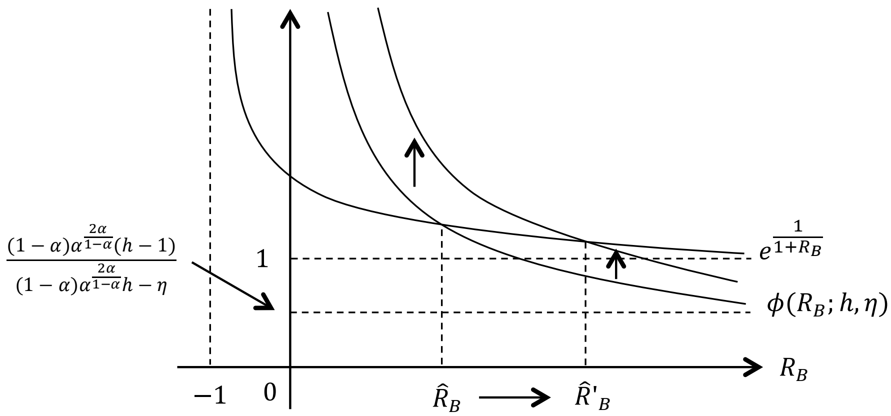

Figure 15

Appendix F : Welfare

We consider the welfare of this economy. All individuals obtain utility from consumption when old. Therefore, we define the welfare levels of this economy at time t as the sum of consumption of old individuals at time t.

(1) Private funds regime

First, we consider the private funds regime. We define U_t^M as the welfare levels of this economy. When $A_t < \underline{A}$, all individuals become unskilled workers. Therefore, aggregate consumption of all individuals is given by

$$\begin{aligned} U_t^M &= \int_0^1 R_{t+1}^M w_t d\theta \\ &= R_{t+1}^M (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t \end{aligned} \tag{54}$$

Moreover, when $A_t \geq \underline{A}$, aggregate consumption of all individuals is given by

$$\begin{aligned} U_t^M &= \int_0^{\hat{\theta}_{t-1}} R_{t+1}^M w_t d\theta + \int_{\hat{\theta}_{t-1}}^1 R_{t+1}^M \left(w_t h - R_t^M \frac{\gamma}{\theta} \right) d\theta \\ &= R_{t+1}^M S_2^M (A_t) \end{aligned} \tag{55}$$

(2) Public funds regime

Next, we consider the public funds regime. We define U_t^G as the welfare levels of this economy. When $A_t < \underline{A}$, all individuals become unskilled workers. Therefore, aggregate consumption of all individuals is given by

$$\begin{aligned} U_t^G &= \int_0^1 R_{t+1}^G w_t d\theta \\ &= R_{t+1}^G (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t. \end{aligned} \tag{56}$$

Moreover, when $A_t \geq \underline{A}$, aggregate consumption of all individuals is given by

$$\begin{aligned} U_t^G &= \int_0^{\hat{\theta}_{t-1}} \left[R_{t+1}^G (w_t - T_t) + G_{t+1}\right] d\theta + \int_{\hat{\theta}_{t-1}}^1 \left[R_{t+1}^G \left(w_t h - R_{t+1}^G \frac{\gamma}{\theta} - T_t\right) + G_{t+1}\right] d\theta \\ &= R_{t+1}^G S_2^G(A_t) - (R_{t+1}^G - R_B) \int_{\hat{\theta}_t}^1 \frac{\gamma}{\theta} d\theta. \end{aligned} \tag{57}$$

Appendix G : Data

- Figure 1 and Figure 2 is based on data released by World bank.

References

[1] Acemoglu, Daron. "Why do new technologies complement skills? Directed technical change and wage inequality." Quarterly Journal of Economics (1998): 1055–1089.

[2] Boldrin, Michele, and Ana Montes. "The intergenerational state education and pensions." The Review of Economic Studies 72.3 (2005): 651–664.

[3] Dalgaard, Carl-Johan, and Claus Thustrup Kreiner. "Is declining productivity inevitable?" Journal of Economic Growth 6.3 (2001): 187–203.

[4] Galor, Oded, and Omer Moav. "Ability-biased technological transition, wage inequality, and economic growth." Quarterly Journal of Economics (2000): 469–497.

[5] Grossmann, Volker. “How to promote R&D-based growth? Public education expenditure on scientists and engineers versus R&D subsidies.” Journal of Macroeconomics 29.4 (2007): 891–911.

[6] Morimoto, Takaaki. “Occupational choice and entrepreneurship: effects of R&D subsidies on economic growth.” Journal of Economics (2017): 1–25.

[7] OECD “Education at a glance 2014” (2014).

[8] Rivera-Batiz, Luis A., and Paul M. Romer. “International trade with endogenous technological change.” European Economic Review 35.4 (1991): 971–1001.

[9] Strulik, Holger. “The role of human capital and population growth in R&D- based models of economic growth.” Review of International Economics 13.1 (2005): 129–145.

[10] Japan Student Services Organization “Nihongakuseishienkiko Gaiyo 2016” (2016).