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The impacts of pollution control policies on the pollution in small open economies*

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Abstract

We examine the effects of tax policies on pollution in a small open economy. There are two pollution causes: consumption activities of households and production activities of firms. In this setup, we examine how tax policies affect pollution and the small open economy. Considering that pollution control policies are undertaken only for temporary periods in countries, we mainly focus on duration of governments' pollution control policies; that is, permanent and temporary policies. The main finding is that the government in the small open economy need to tackle pollution problems from a long-term perspective in order to decrease the level of pollution.

Keywords: Pollution, Pollution control policies, Temporary and permanent policies, Small open economy

JEL Classification Code: F41, H21, Q52

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1 Introduction

Since economic activity itself creates environmental pollution, the control and regulation of pollution is a significant and difficult issue in almost all countries. Even if developed countries have superior technologies to combat pollution to some extent, pollution which come from different pollutants has been serious problems for even these countries. Needless to say, other countries experience much severer impacts of pollution. If governments address pollution problem seriously, the control and regulation of pollution impedes the economic growth and production activities. For example, when the regulation of industrial pollution reduces the level of production, export countries might lose opportunities to acquire foreign assets. Because the acquisition of foreign assets enables export countries to achieve rapid growth of their economies, the enforcement of pollution control policy is a very serious problem. As a result, many countries are forced to struggle with the trade-off between the regulation of pollution and economic development.

For instance, the activities of multinational oil companies in the Niger Delta region of Nigeria have caused substantial land, water, and air pollution. However, the regulation of pollution has not been a priority issue in Nigeria, because oil exploration and production enable Nigeria to maintain its rapid growth. Similarly, China would not have achieved its dramatic economic growth over the preceding decades if it had regulated pollution from the initial stage of its development.

Since some governments focus on improving levels of economic activities, they are more likely to be short-sighted. This means that pollution control policies are not determined from a longer-term perspective but rather from a shorter-term one. Therefore, in some countries, pollution control policies are undertaken only for a temporary period. From a theoretical point of view, after the temporary policies are repealed, the pollution level returns to its original level; hence, at least, the temporary enforcement of pollution control policies does not seem to lead to any serious problem in the long run.

However, we can find some examples that show the repeal of pollution control policies has a considerably negative impact on the environment.[1] We here highlight two cases.[2] First, a high-occupancy vehicle (HOV) policy is known to reduce air pollution, carbon emissions, traffic congestion on roads, and the need for parking space because HOVs generally refer to vehicles carrying two or more persons on a single journey. Because the major thoroughfare in Jakarta, Jalan Sudirman, suffered from heavy traffic congestion, the government introduced the HOV policy ("three-in-one" policy) in 1992, which required all private cars to carry at least three passengers during peak hours. However, in March 2016, the Indonesian government abruptly canceled the HOV policy. Thereafter, in Jakarta, travel delays became 46% worse during the morning rush hour and 87% worse during the evening rush hour. Furthermore, Hanna et al. (2017) demonstrated that the pollutants per daily trip from motor vehicles dramatically increased due to heavy traffic congestion after the HOV restrictions were eliminated. Next, we introduce the case of Australia. The Australian government imposed a carbon tax through the Clean Energy Act in 2011. Although the legislation achieved a reduction in the country's carbon emissions, it was repealed in 2014 to lower costs for Australian businesses and to ease the cost of living pressure on households. As a result, carbon emissions and electricity demand in Australia have jumped dramatically. Specifically, carbon dioxide emissions from electricity increased by 6.4 million tonnes in 2015. Furthermore, according to the Department of the Environment and Energy of the Australian Government, the emissions level continued to increase after the repeal of the carbon tax, and might exceed the original level before the introduction of the emissions control policy.[3]

What would occur for such short-run pollution control policies? To tackle this question, we examine how pollution control policies, represented by tax policies, affect the pollution in

[1] This may also apply to the United States in near future. Since January 2017, the current US administration has sought to upend the environmental protection policies enforced by the previous administration, because excessive regulations are harming US industry and thus, reducing regulation will be good for the economic activities. At the moment, we cannot correctly evaluate the negative impact on the environment by the repeal of the environmental protection policy; however, as for the health problem of US residents, Cutler and Dominici (2018) have already concluded that the policy rollbacks will lead to an additional 80,000 U.S. deaths per year and respiratory problems in more than 1 million people, as a result of damage that would be difficult for a future administration to undo.

[2] We are grateful for the useful comments of Minoru Nakada about these cases.

[3] For example, see Figure 3 in http://www.environment.gov.au/system/files/resources/9437fe27-64f4-4d16-b3f1-4e03c2f7b0d7/files/aust-emissions-projects-chart-data-2016.xlsx

a small open economy. The reason that we focus on the small open economy is that while the small open economy relies too much on less stringent pollution controls, the pollution emission in the economy has a larger contribution to the world pollution emission year by year, implying that the small open economy needs to take responsibility for own large pollution emission. Thus, we construct a dynamic general equilibrium model of the small open economy with pollution, and incorporate pollution control policies into this model.

Our economy in this study has the following three characteristics. First, we suppose the pollution evolved over time; that is, economic activities, such as consumption and production, generate flows of pollution, and thus, the pollution accumulates over time. In addition, a certain part of this stock is broken down at every time owing to the assimilative capacity of the environment.

Second, we consider two pollution causes that generate the flow of pollution. The first cause is the production activities of firms. Owing to increases in demand for food, shelter, and housing, more commodities are produced. Therefore, firms need to dispose of more waste, which contains toxic chemicals and pollutants, leading to water, air, and soil pollution. For instance, many industries drain the waste in the fresh water that goes into rivers, canals, and later the sea. Furthermore, to produce huge amounts of energy through fossil fuels like coal and oil, and nuclear fission and fusion, thermal power stations produce substantial amount of ash in the atmosphere, and nuclear plants dispose of left-over radioactive materials that contain harmful and toxic chemicals.

The other pollution cause we consider is the consumption activities of households. As for soil pollution, households often use highly toxic fertilizers and pesticides to remove insects and bacteria from their crops. In addition, each household produces tonnes of garbage each year, such as aluminum and plastic produced in modern technology. This garbage becomes part of landfills and causes soil pollution. With regard to water pollution, households produce chemical waste water and sewage, and release it into rivers. Finally, considering air pollution, transportation has assumed an important role in our lives and cars, trains, shipping vessels, and airplanes all burn a lot of fossil fuels to operate.

Third, in this study, a government plays an important role in decreasing the level of pollution. The reason is that our model includes the pollution control policies that directly reduce economic activities as the pollution causes. As discussed previously, we are interested in the duration of policy implementation. In summary, some countries want to avoid pollution control policies that impede economic activities, so that they have strong incentives to conduct pollution control policies not from a long-term perspective, but only for a certain period. Therefore, by comparing the effects of temporary policies with permanent ones, we consider the dynamic movement of pollution. Then, we find that the pollution level decreases during the pollution control policy, and that after the policy is repealed, the pollution level always exceeds the original level in our small open economy. Importantly, as in the above-mentioned cases in Australia and Indonesia, our numerical examples show a serious problem that countries may face after the repeal of pollution control policy.

Our study is related to existing investigations in dynamic models with pollution as well as international macrodynamic models. As for dynamic models with pollution, many studies incorporate pollution into macrodynamic models. Earlier studies, such as Lopez (1994) and Selden and Song (1995), construct dynamic models with pollution generated by production but not by consumption. On the other hand, John and Pecchenino (1994), John et al. (1995), and McConnell (1997) develop dynamic models with pollution generated by consumption rather than production. The latter authors mainly examine an inverted U-shaped curve of pollution intensity over time. In addition, many researchers focus on the relationship between economic growth and pollution, and examine whether sustainable endogenous growth is achieved or not (e.g., Huang and Cai, 1994; Bovenberg and Smulders, 1995; Michel and Rotillon, 1995; Smulders and Gradus, 1996; Chevé, 2000; Gupta and Barman, 2009; Greiner, 2005). The purposes of these studies are not the same as ours. In other words, they do not focus on the effects of temporary implementation of pollution control policies.

The structure of our model is closely related to international macrodynamic models, such as those of Sen and Turnovsky (1990), Turnovsky (1997), Schubert and Turnovsky (2002), and Nakamoto and Futagami (2016), in that the models in these studies focus on temporary policies in small open economies. These studies find that when policies temporarily change under the assumption of perfect foresight, the small open economy does not return to its original steady state after the policy implementation is completed. This insight on public policies is completely different from the results of dynamic macroeconomic models of closed economies. The temporary implementation of public policies in closed economies does not affect the qualitative impact on the long-run economy, that is, the long-run equilibrium coincides with the original steady state. However, unlike our study, these studies do not include pollutants in their dynamic models, and furthermore, their attention is not to examine the effects of pollution control policies.

Finally, our motivation is like that of Rondeau and Bulte (2007) and Nakamoto and Futagami (2016) in that these studies cast some doubt on the usefulness of environmental protection policies. Rondeau and Bulte (2007) and Nakamoto and Futagami (2016) focus on renewable natural resources, such as forestry, fish, and wildlife stocks, while the current study is interested in pollution accumulation generated by two different pollutants. In addition, Rondeau and Bulte (2007) make use of a single-country partial equilibrium model with the interaction between habitat and open-access resource. Nakamoto and Futagami (2016) and the present study make use of dynamic models in a small open economy.

The remainder of this paper is organized as follows. In the next section, we present our set-up. Section 3 shows the dynamic system under our set-up. Section 4 shows the uniqueness of the steady-state equilibrium with the saddle-path stability. Section 5 examines the effects of pollution abatement policies on the economy. Section 6 gives numerical examples. Section 7 summarizes our findings.

2 Baseline Model

We consider a small open economy in which the world interest rate, r, is constant. The population is constant and normalized to unity. Denoting the time index by t, we express the production function, which satisfies constant returns to scale with respect to capital ($\hat{k}_t$) and labor (l_t), as follows:

$$y_t = F(\hat{k}_t, l_t) = l_t f(k_t),$$

where y_t is output and $k_t \equiv \hat{k}_t / l_t$ denotes capital intensity. The intensive form of the production function, $f(k_t)$, is monotonically increasing and strictly concave in k_t, and satisfies the Inada conditions. Taking account of competitive factors and final goods markets, the real rent r and real wage rate w_t are determined by

$$r = f'(k_t), \quad w_t = f(k_t) - k_t f'(k_t), \tag{1a}$$

respectively. Since the real rent is constant in the small open economy, from (1a), the capital intensity is constant and hence, the wage rate is fixed as well ($k_t = \bar{k}$ and $w_t = \bar{w}$). As a result, the production function is simply given by

$$y_t = l_t f(\bar{k}). \tag{1b}$$

2.1 Pollution

Following the existing papers with the stock of pollution (e.g., Huang and Cai, 1994; Chevé, 2000), we suppose that the net flow of pollution caused by consumption and output adds to the stock of pollution, and the ratio θ of the pollution stock is broken down owing to the assimilative capacity of the environment. Then, the accumulation equation of pollution is given by

$$\dot{p}_t = \alpha_c G(c_t) + \alpha_y N(l_t f(\bar{k})) - \theta p_t, \tag{2}$$

where p_t is the level of the pollution stock, c_t is the level of consumption, and θ is the natural decay rate of the pollution stock. The functions $G(c_t)$ and $N(l_t f(\bar{k}))$ represent the flow of pollution. We assume that

$$G'(c_t) > 0,\ G''(c_t) > 0,\ N'(l_t f(\bar{k})) > 0,\ N''(l_t f(\bar{k})) > 0, \tag{3a}$$

$$\lim_{c_t \to 0} G'(c_t) = 0,\ \lim_{c_t \to \infty} G'(c_t) = \infty,\ \lim_{l_t \to 0} N'(l_t f(\bar{k})) = 0,\ \lim_{l_t \to 1} N'(l_t f(\bar{k})) = N'(f(\bar{k})). \tag{3b}$$

Assumption $(3a)$ means that the flow of pollution is monotonically increasing and strictly convex. In other words, the more active the economy becomes, the higher the flow level of pollution is. In what follows, assumption $(3b)$ corresponds to the Inada conditions. Furthermore, α_c and α_y in (2) are non-negative parameters that specify each pollution flow. For instance, when $\alpha_c > 0$ and $\alpha_y = 0$, we consider the case in which the pollution flow is generated by consumption, not output.

2.2 Households

The welfare level of the household is given by

$$U_0 \equiv \int_0^\infty \{u(c_t) - v(p_t) + \Gamma(L_t)\} e^{-\rho t} dt, \tag{4}$$

where $\rho(>0)$ is the rate of time preference and $L_t(=1-l_t)$ is the leisure time. We assume that $u'(c_t) > 0$, $u''(c_t) < 0$, $v'(p_t) > 0$, $v''(p_t) > 0$, $\Gamma'(L_t) > 0$, and $\Gamma''(L_t) < 0$. We further assume each of the following Inada conditions $\lim_{c_t \to 0} u'(c_t) = \infty$, $\lim_{c_t \to \infty} u'(c_t) = 0$, $\lim_{p_t \to 0} v'(p_t) = 0$, $\lim_{p_t \to \infty} v'(p_t) = \infty$, $\lim_{L_t \to 0} \Gamma'(L_t) = \infty$, and $\lim_{L_t \to 1} \Gamma'(L_t) = \Gamma'(1)$.

The accumulation of the foreign assets holding, b_t, evolves as

$$\dot{b}_t = rb_t + (1-\tau^y)l_t f(\bar{k}) - (1+\tau^c)c_t + z_t, \tag{5}$$

where z_t is the lump-sum transfer from the government. We suppose that τ^y is the rate of income tax and τ^c is the rate of consumption tax. τ^j ($j = y$ and c) are constant over time. These rates satisfy $0 \leq \tau^y < 1$ and $\tau^c \geq 0$. The government keeps the following balanced budget over time:

$$\tau^c c_t + \tau^y l_t f(\bar{k}) = z_t. \tag{6}$$

We here make use of consumption and income taxes as the methods of pollution control by the government. Then, one may wonder the use of these tax policies, because taxing pollution/emission directly is more efficient. However, because policy makers place a great deal of weight on economic activities in own countries, it is difficult to impose green taxes called as pollution or emission taxes upon polluting firms directly in even a large economy like the United State (e.g., see the environmental policy of the Donald Trump administration and the Kyoto Protocol without the United States ratification). Because many developed countries continue to evade their responsibility to provide adequate financial resources, people and governments in other countries will not accept green taxes actively. Next, when the green taxes are imposed on polluting firms in less developed countries, its policy may be directly linked to the survival problem of residents because people may lose their job and may be out of work for a long time without adequate support by governments. On the contrary, imposing the green taxes on polluting firms in developed countries may change life-style, but it does not lead to the serious risk of death under social security system. As a result, imposing green taxes is more difficult in less developed countries than developed ones, implying that it may be needed to consider alternative ways of controlling pollution.

Based on the fact that the consumption and income taxes are not imposed on polluting firms directly and the primary purpose of their tax policies is not to reduce pollution, it is easier for people and governments in many countries to accept these taxes to control certain pollutants. For example, when the pollution flow is generated by consumption, we adopt the rate of consumption tax τ^c as the pollution prevention policy, but not the rate of income tax τ^y. In other words, increasing the rate of consumption tax directly has a negative impact on the demand for the commodities of consumption, which means that the pollution cause will be indirectly reduced. On the other hand, when the pollution flow is caused by output, we adopt the rate of income tax τ^y as the pollution prevention policy, but not the rate of consumption tax τ^c. An increase in the rate of income tax directly has a negative impact on working, which implies that it will lead to a lower level of pollution indirectly.

3 Dynamic system of small open economy

The representative household maximizes its lifelong utility (4) subject to the evolution of pollution (2) and the budget constraint (5). We set the current-value Hamiltonian:

$$\mathcal{H} = u(c_t) - v(p_t) + \Gamma(L_t) - \mu_t\big[(2)\big] + \lambda_t\big[(5)\big], \tag{7}$$

where μ_t and λ_t show the shadow values for equations (2) and (5), respectively.

The first-order conditions of the maximization problem are

$$H_{c_t}: \; u'(c_t) - \mu_t\alpha_c G'(c_t) = \lambda_t(1+\tau^c), \tag{8a}$$

$$H_{l_t}: \; \lambda_t(1-\tau^y)f(\bar{k}) = \Gamma'(L_t) + \mu_t\alpha_y N'(l_t f(\bar{k}))f(\bar{k}), \tag{8b}$$

$$H_{p_t}: \; -v'(p_t) + \mu_t\theta = \dot{\mu}_t - \rho\mu_t, \tag{8c}$$

$$H_{b_t}: \; \lambda_t r = -\dot{\lambda}_t + \rho\lambda_t. \tag{8d}$$

The transversality conditions are given by

$$\lim_{t\to\infty}\mu_t p_t e^{-\rho t} = 0, \quad \lim_{t\to\infty}\lambda_t b_t e^{-\rho t} = 0. \tag{8e}$$

In what follows, since the rate of time preference ρ and the interest rate r are both fixed, we require $r = \rho$ for our system to have a finite interior steady-state value for the shadow value of foreign assets. Therefore, assuming that $r = \rho$, from equation ($8d$), we obtain the constant level of shadow value λ_t:

$$\lambda_t = \bar{\lambda}, \tag{9}$$

where $\bar{\lambda}$ should be determined endogenously.

Substituting (9) into (8a) and totally differentiating it, we can obtain the following:

$$c_t = c(\mu_t, \bar{\lambda}, \tau^c), \tag{10}$$

where

$$\frac{\partial c_t}{\partial \mu_t} = -\frac{\alpha_c G'(c_t)}{\theta_c}(<0), \quad \frac{\partial c_t}{\partial \bar{\lambda}} = -\frac{1+\tau^c}{\theta_c}(<0), \quad \frac{\partial c_t}{\partial \tau^c} = -\frac{\bar{\lambda}}{\theta_c}(<0),$$
$$\theta_c = \mu_t \alpha_c G''(c_t) - u''(c_t)(>0).$$

Next, substituting (9) into (8b), we totally differentiate (8b) and obtain

$$l_t = l(\mu_t, \bar{\lambda}, \tau^y), \tag{11}$$

where

$$\frac{\partial l_t}{\partial \mu_t} = -\frac{\alpha_y N'(l_t f(\bar{k}))f(\bar{k})}{\theta_l}(<0), \quad \frac{\partial l_t}{\partial \bar{\lambda}} = \frac{(1-\tau^y)f(\bar{k})}{\theta_l}(>0), \quad \frac{\partial l_t}{\partial \tau^y} = -\frac{\bar{\lambda} f(\bar{k})}{\theta_l}(<0),$$
$$\theta_l = -\Gamma''(L_t) + \alpha_y \mu_t N''(l_t f(\bar{k}))f(\bar{k})^2(>0).$$

Finally, substituting (10) and (11) into the evolution of the pollution stock (2), we can rewrite (2) as follows:

$$\dot{p}_t = \alpha_c G(c(\mu_t, \bar{\lambda}, \tau^c)) + \alpha_y N(l(\mu_t, \bar{\lambda}, \tau^y)f(\bar{k})) - \theta p_t, \tag{12a}$$

and the dynamic equation of the shadow price of the pollution stock in (8c) is

$$\dot{\mu}_t = \mu_t(\rho + \theta) - v'(p_t). \tag{12b}$$

4 Steady-state equilibrium and stability

We denote the steady-state levels of variables by asterisk. Then, we observe that the steady-state levels of the pollution stock and shadow price of the pollution stock are determined by $\dot{p}_t = 0$ in (12a) and $\dot{\mu}_t = 0$ in (12b) as follows:

$$\dot{\mu}_t = 0: \quad \mu^*(\rho + \theta) = v'(p^*). \tag{13a}$$

$$\dot{p}_t = 0: \quad \alpha_c G(c(\mu^*, \bar{\lambda}^*, \tau^c)) + \alpha_y N(l(\mu^*, \bar{\lambda}^*, \tau^y)) = \theta p^*. \tag{13b}$$

Using Figure 1, we now confirm the uniqueness of the steady-state equilibrium and the saddle-path stability. First, we depict the $\dot{\mu}_t = 0$ locus. From (13a), we observe that

$$\left.\frac{\partial \mu^*}{\partial p^*}\right|_{\dot{\mu}_t=0} = \frac{v''(p^*)}{\rho+\theta}(>0). \tag{14a}$$

Furthermore, we observe that when the level of the shadow price of the pollution stock μ^* approaches zero, the level of the pollution stock approaches zero as well. When μ^* goes to infinity, the level of the pollution stock becomes infinite. Therefore, as observed in Figure 1, the $\dot{\mu}_t = 0$ locus is an upward-sloping curve that goes through the origin.

We next examine the $\dot{p}_t = 0$ locus defined by (13b). We show the slope of the $\dot{p}_t = 0$ locus given $\bar{\lambda}$, τ^c and τ^y:

$$\left.\frac{\partial \mu^*}{\partial p^*}\right|_{\dot{p}_t=0} = \frac{\theta}{\alpha_c G'(c^*)\frac{\partial c^*}{\partial \mu^*} + \alpha_y N'(l^* f(\bar{k})) f(\bar{k}) \frac{\partial l^*}{\partial \mu^*}}(<0). \tag{14b}$$

Furthermore, when the shadow value of the pollution stock μ^* goes to zero, we observe that $\lim_{\mu^* \to 0} \tilde{c} = (u')^{-1}(\bar{\lambda}(1+\tau^c))(>0)$ from (8a) and that $\lim_{\mu^* \to 0} \tilde{L} = (\Gamma')^{-1}(\bar{\lambda}(1-\tau^y)f(\bar{k}))$ from (8b), because $l_t < 1$; hence, $\tilde{l} = 1 - (\Gamma')^{-1}(\bar{\lambda}(1-\tau^y)f(\bar{k}))(>0)$. In summary, the stock of pollution takes a finite level to satisfy $\lim_{\mu^* \to 0} p^* = \left(\alpha_c G(\tilde{c}) + \alpha_y N(\tilde{l} f(\bar{k}))\right)/\theta$. On the other hand, as the level of shadow value of the pollution stock μ^* approaches infinity, the level of the pollution stock goes to zero. This result is because the levels of consumption and labor supply are zero, as easily observed from (8a) and (8b). As a result, the $\dot{p}_t = 0$ locus is downward sloping, as in Figure 1, where the value of the intercept at the horizontal axis ($p-$axis) is finite. Therefore, since the $\dot{\mu}_t = 0$ locus crosses the $\dot{p}_t = 0$ once, the steady-state equilibrium is uniquely determined, as shown in Figure 1.

We now examine the stability around the steady state. As for the $\dot{\mu}_t = 0$ locus, the right-hand side of this locus holds that $\dot{\mu}_t = \mu_t(\rho+\theta) - v'(p_t)(<0)$ under a fixed level of shadow value of the pollution stock μ_t, because of $v''(p_t) > 0$. Alternatively, the opposite inequality holds in its left-hand side. Next, the upper side of the $\dot{p}_t = 0$ locus shows that $\dot{p}_t = \alpha_c G(c(\mu_t, \bar{\lambda}, \tau^c)) + \alpha_y N(l(\mu_t, \bar{\lambda}, \tau^y)) - \theta \bar{p}(<0)$ given the level of the pollution stock $\bar{p}$. This is because $\partial G/\partial \mu_t < 0$ and $\partial N/\partial l_t < 0$. Below the $\dot{p}_t = 0$ locus, we observe that $\dot{p}_t = \alpha_c G(c(\mu_t, \bar{\lambda}, \tau^c)) + \alpha_y N(l(\mu_t, \bar{\lambda}, \tau^y)) - \theta \bar{p}(>0)$. Therefore, as confirmed in Figure 1, the steady-state equilibrium becomes saddle-point stable. The stable root β^s and the unstable one β^u of our system in $(12a)$ and $(12b)$ are given by:[4]

$$\beta^s = \frac{\rho - \sqrt{\rho^2 - 4(-\theta(\rho+\theta) + v''(p_t)\frac{\partial \dot{p}_t}{\partial \mu_t})}}{2}(<0), \quad \beta^u = \frac{\rho + \sqrt{\rho^2 - 4(-\theta(\rho+\theta) + v''(p_t)\frac{\partial \dot{p}_t}{\partial \mu_t})}}{2}(>0), \tag{15}$$

where

$$\frac{\partial \dot{p}_t}{\partial \mu_t} = \alpha_c G'(c^*)\frac{\partial c_t}{\partial \mu_t} + \alpha_y N'(l^* f(\bar{k}))\frac{\partial l_t}{\partial \mu_t}(<0).$$

The abovementioned results are summarized as follows.

Proposition 1 *The steady-state equilibrium is uniquely determined and satisfies the saddle-path stability.*

Denoting an element of an eigenvector corresponding to the stable root β^s as Δ^s, we obtain the linearly approximated system of $(12a)$ and $(12b)$ along the stable path:

$$p_t = p^* + (p_0 - p^*)e^{\beta^s t}, \tag{16a}$$

$$\mu_t = \mu^* + \Delta^s(p_0 - p^*)e^{\beta^s t}, \tag{16b}$$

where Δ^s is given by

$$\Delta^s = \frac{v''(p^*)}{\rho + \theta - \beta^s}(>0). \tag{16c}$$

To determine the fixed level of shadow value (9), we make use of the accumulation equation of the foreign asset holding. Approximating (5) linearly around the steady state, we can derive the following equation:

$$\dot{b}_t = r(b_t - b_c^*) + \Omega\Delta^s(p_0 - p^*)e^{\beta^s t}, \tag{17a}$$

where

$$\Omega = f(\bar{k})\frac{\partial l_t}{\partial \mu_t} - \frac{\partial c_t}{\partial \mu_t}. \tag{17b}$$

[4]Our linearized system is given by

$$\begin{bmatrix} \dot{\mu}_t \\ \dot{p}_t \end{bmatrix} = \begin{bmatrix} \rho + \theta & -v''(p^*) \\ \partial \dot{p}_t / \partial \mu_t & -\theta \end{bmatrix} \begin{bmatrix} \mu_t - \mu^* \\ p_t - p^* \end{bmatrix}.$$

This has the trace and the determinant as follows:

$$\mathrm{Tr} = \rho(>0), \quad \mathrm{Det} = -\theta(\rho+\theta) + v''(p^*)\frac{\partial \dot{p}_t}{\partial \mu_t}(<0).$$

Solving the dynamic equation ($17a$), we obtain:[5]

$$b_t - b^* = \Theta^s(p_0 - p^*)e^{\beta^s t}, \tag{17c}$$

where Θ^s is defined by

$$\Theta^s = \frac{\Omega\Delta^s}{\beta^s - r} = -\frac{v''(p^*)}{(\rho - \beta_s)(\rho + \theta - \beta_s)}\left(f(\bar{k})\frac{\partial l_t}{\partial \mu_t} - \frac{\partial c_t}{\partial \mu_t}\right)$$

Setting $t = 0$ in ($17c$), we obtain

$$b^* - b_0 = \Theta^s(p^* - p_0). \tag{17d}$$

Finally, in order to express the steady-state equilibrium, we take account of the economy starting at time T_j and corresponding to a policy set (τ_j^c, τ_j^y). Moreover, the viable steady state is associated with the initially given levels of the pollution stock and the foreign assets p_{T_j} and b_{T_j} at time T_j. Following ($13a$), ($13b$), ($17d$), and the following $\dot{b}_t = 0$ equation, we show a viable steady state j with (τ_j^c, τ_j^y) as follows:

$$\mu_j^*(\rho + \theta) = v'(p_j^*). \tag{18a}$$

$$\alpha_c G(c(\mu_j^*, \bar{\lambda}_j^*, \tau_j^c)) + \alpha_y N(l(\mu_j^*, \bar{\lambda}_j^*, \tau_j^y)) = \theta p_j^*, \tag{18b}$$

$$r b_j^* = c(\mu_j^*, \bar{\lambda}_j^*, \tau_j^c) - l(\mu_j^*, \bar{\lambda}_j^*, \tau_j^y) f(\bar{k}). \tag{18c}$$

$$b_j^* - b_0 = \Theta_j^s(p_j^* - p_0). \tag{18d}$$

5 The effects of pollution control policies

In this section, we analyze how the pollution control policy affects the level of the pollution stock as well as the other economic variables. We must mention the following two points.

[5]Solving ($17a$) yields:

$$b_t = b^* + \left(b_0 - b^* - \frac{\Omega\Delta^s(p_0 - p^*)}{\beta^s - r}\right)e^{rt} + \Omega\Lambda^s\frac{p_0 - p^*}{\beta^s - r}e^{\beta^s t}.$$

Then, we obtain ($17c$). The parentheses on the right-hand side must be zero owing to the intertemporal solvency condition; that is,

$$b_0 - b^* = \frac{\Omega\Lambda^s(p_0 - p^*)}{\beta^s - \rho}.$$

First, when investigating the long-run effects of policies on the pollution stock, we deal with two pollution causes separately; that is, assuming that $\alpha_c > 0$ and $\alpha_y = 0$ ($\alpha_c = 0$ and $\alpha_y > 0$), we examine the policy effect in the case in which the pollution flow is generated by consumption (output). In what follows, as for the pollution control policies, we suppose that when the pollution flow is generated by consumption, the pollution control policy is to increase the rate of consumption tax. This assumption is because increasing the rate of consumption tax directly decreases the demand for the consumption commodities, which means that the level of pollution cause will be reduced. This dampens an increase in the level of the pollution stock. Next, we suppose that when the pollution flow is generated by output, the pollution control policy is to raise the rate of income tax. This assumption is because an increase in the rate of income tax directly restrains the household from working, and hence, it will lead to a lower level of pollution flow. This dampens an increase in the level of the pollution stock.

We now consider the difference of temporary policies in closed and small open economies. In closed economy models, a temporary policy change affects the transitional path over time, and after the temporary change is removed, the economy gradually returns to the original steady state. Therefore, we conclude that the temporary policy change does not have any qualitative impacts on the long-run economy. On the other hand, in small open economy models, when the policy is temporarily conducted, the position of the long-run steady state does not coincide with the original one.

To consider these policies, we suppose that the economy at the initial time $T_j = T_0$ is in the steady state in which $\tau_j^c = \tau_0^c$ and $\tau_j^y = \tau_0^y$. Then, from $(18a) - (18d)$, we denote the original steady-state equilibrium at time T_0 as follows:[6]

$$p_0 = p_0^* = P(\bar{\lambda}_0, \tau_0^c, \tau_0^y),\ P_{\bar{\lambda}} < (>) 0 \text{ if } \alpha_c > 0 \text{ and } \alpha_y = 0,\ (\alpha_c = 0 \text{ and } \alpha_y > 0),\ P_{\tau^c} < 0,\ P_{\tau^y} < 0, \tag{19a}$$

$$\mu_0 = \mu_0^* = M(\bar{\lambda}_0, \tau_0^c, \tau_0^y),\ M_{\bar{\lambda}} < (>) 0 \text{ if } \alpha_c > 0 \text{ and } \alpha_y = 0\ (\alpha_c = 0 \text{ and } \alpha_y > 0),\ M_{\tau^c} < 0,\ M_{\tau^y} < 0, \tag{19b}$$

$$b_0 = b_0^* = B(\bar{\lambda}_0, \tau_0^c, \tau_0^y), \tag{19c}$$

$$\bar{\lambda}_0 = \bar{\lambda}_0^* = \Lambda(\tau_0^c, \tau_0^y),\quad \Lambda_{\tau^c} < 0 \ \text{ if } \alpha_c > 0 \text{ and } \alpha_y = 0,\quad \Lambda_{\tau^y} > 0 \ \text{ if } \alpha_c = 0 \text{ and } \alpha_y > 0. \tag{19d}$$

[6] See Appendix A.

Suppose that the government announces the temporary changes of the policy instruments from the original levels (τ_0^c, τ_0^y) to a new level (τ_1^c, τ_1^y) at the initial period. In this case, the unanticipated change leads to an initial jump of the shadow value of foreign assets. After the initial change of policy instruments at time 0, some duration, $t \in [0, T)$, has passed, and thereafter, the policy instruments return to the original level after time T. Importantly, because of the assumption of perfect foresight, the household can initially anticipate the policy return at time T. This assumption means that the level of the shadow value of foreign assets does not change over time, including time T, except for the initial jump at time 0. In other words, the level of this shadow value continues to be fixed so as to sustain the intertemporal solvency condition. Finally, since only this effect remains in the long run, the temporary policy changes have long-run impacts on the level of the pollution stock.

5.1 Consumption as the pollution cause

Assuming that the pollution flow is generated only by consumption (i.e., $\alpha_c > 0$ and $\alpha_y = 0$), we now examine the effects of changes in pollution control policies on the pollution stock in the long run. Conducting a comparative static analysis, we obtain the following proposition.

Proposition 2 *Assume that $\alpha_c > 0$ and $\alpha_y = 0$. Then, a permanent increase in the rate of consumption tax decreases the steady-state level of the pollution stock, but a temporary increase leads to a more severe level of the pollution stock in the steady state.*

Proof. See Appendices B and C. ■

We now consider the dynamic behavior of the economy shown in Proposition 2. When $\alpha_c > 0$ and $\alpha_y = 0$, Figure 2 illustrates the effects of an increase in the rate of consumption tax, where E_0 is the original steady state. Note that PER (or TEM) in this figure shows the cases in which the pollution control policies change permanently (or temporarily).

First, we observe that the enforcement of increasing the rate of consumption tax does not change the $\dot{\mu}_t = 0$ locus. On the other hand, the $\dot{p}_t = 0$ locus is influenced by a change in the rate of consumption tax.

When the pollution flow is caused by consumption alone (i.e., $\alpha_c > 0$ and $\alpha_y = 0$), from (18*b*), the permanent increase in the rate of consumption tax always shifts the $\dot{p}_t = 0$ locus downward given τ^y and μ^*:

$$\left.\frac{\partial p_j^*}{\partial \tau_j^c}\right|_{\dot{p}_t=0,\ \mu_j^*=\text{constant}} = \underbrace{\frac{\alpha_c G'(c_j^*)}{\theta}}_{(+)} \Bigg(\underbrace{\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\Lambda_{\tau^c}}_{(\#1)} + \underbrace{\frac{\partial c_j^*}{\partial \tau_j^c}}_{(\#2)} \Bigg)(<0). \tag{20}$$

This derivative with respect to the $\dot{p}_t = 0$ locus consists of two parts. The term (#1) shows the indirect impact that is caused by a change in the shadow value of foreign assets $\bar{\lambda}$, and the term (#2) indicates the direct impact. The indirect impact takes a positive value, while the direct impact takes a negative value; however, the direct impact dominates the indirect one as follows:[7]

$$\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\Lambda_{\tau^c} + \frac{\partial c_j^*}{\partial \tau_j^c} = \underbrace{\frac{1}{B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}}}_{(-)} \underbrace{\left(-\frac{f(\bar{k})}{r}\frac{\partial c_j^*}{\partial \tau^c}\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*} \right)}_{(+)}(<0), \tag{21}$$

where we make use of $\partial c_j^*/\partial \tau^c < 0$ in (10) and $\partial l_j^*/\partial \bar{\lambda}_j^* > 0$ in (11).

Figure 2(a) shows that when the initial economy is at the point E_0, the permanent increase in the rate of consumption tax initially leads to the downward jump of the shadow value of the pollution stock at the point I. Thereafter, the level of the pollution stock monotonically decreases toward the new steady-state equilibrium E_1.

In what follows, from ($17c$) and ($18d$), we characterize changes in foreign assets over time. We here note that Θ_j^s has a negative sign. Figure 2(b) depicts a negative relationship between foreign assets and the pollution stock. When the rate of consumption tax permanently increases, the level of the pollution stock monotonically decreases, as shown in Figure 2(a). Thus, the level of foreign assets monotonically increases from E_0 to E_1, as shown in Figure 2(b). The reason is that increasing the rate of consumption tax reduces the level of consumption, which increases the level of foreign assets.

We now turn to the case in which the rate of consumption tax temporarily increases. In this case, the key element to understand the dynamic behavior of the economy is the

[7] This can be shown as follows:

$$\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\Lambda_{\tau^c} + \frac{\partial c_j^*}{\partial \tau_j^c} = \frac{\partial c_j^*}{\partial \bar{\lambda}}\left(\frac{-B_{\tau^c} + \Theta_j^s P_{\tau^c}}{B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}}\right) + \frac{\partial c_j^*}{\partial \tau^c} = \frac{1}{B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}}\left[\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\left(-B_{\tau^c} + \Theta_j^s P_{\tau^c}\right) + \frac{\partial c_j^*}{\partial \tau^c}\left(B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}\right)\right],$$

where we use

$$\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\Theta_j^s P_{\tau^c} - \frac{\partial c_j^*}{\partial \tau^c}\Theta_j^s P_{\bar{\lambda}} = 0, \quad \text{and} \quad -\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}B_{\tau^c} + \frac{\partial c_j^*}{\partial \tau^c}B_{\bar{\lambda}} = -\frac{f(\bar{k})}{r}\frac{\partial c_j^*}{\partial \tau^c}\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}.$$

This proves (21).

dynamic behavior of the shadow value of foreign assets $\bar{\lambda}$. Because of the temporary change in the pollution control policy, the level of the shadow value of foreign assets initially jumps. Thereafter, even if the rate of consumption tax returns to the original level after the period of implementation $t = [0, T]$, the level of the shadow value of foreign assets does not go back to its original level. This outcome is because the household can initially anticipate the policy change at time T; that is, the level of the shadow value of foreign assets stays there permanently in order to sustain the intertemporal solvency condition. Therefore, the indirect effect (#1) in (20) remains in the long run. This result implies that the temporary increase in the rate of consumption tax leads to the upward shift of the $\dot{p}_t = 0$ locus in the long run:

$$\left.\frac{\partial p_j^*}{\partial \tau_j^c}\right|_{\dot{p}_t=0,\ \mu_j^*=\text{constant}} = \underbrace{\frac{\alpha_c G'(c_j^*)}{\theta}}_{(+)} \underbrace{\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}}_{(+)} \Lambda_{\tau^c}(>0). \tag{22}$$

As shown in Figure 2(a), the upward shift of the $\dot{p}_t = 0$ locus in the long run leads to a higher level of the pollution stock in the long run.

The reason that the temporary pollution control policy increases the level of the pollution stock in the long run is that the direction of initial jump of the shadow value of foreign assets is downward, as observed by $\Lambda_{\tau^c} < 0$ of (19d). To understand its consequence, it is helpful to make use of (8b) under $\alpha_y = 0$:

$$\bar{\lambda}_j^*(1-\tau^y)f(\bar{k}) = \Gamma'(1-\bar{l}_j^*). \tag{23}$$

As easily confirmed in the above equation (23), the constant level of the shadow value of foreign assets leads to the fixed level of labor supply $\bar{l}_j^*$. Then, the initial change of labor supply determines the direction of jump of the shadow value of foreign assets. An increase in the rate of consumption tax has a negative impact on the level of consumption at the beginning. Since the household tries to maintain its welfare level, it decreases the level of labor supply and increases the level of leisure time. Therefore, from (23), a lower level of marginal utility of leisure leads to the downward jump of the shadow value of foreign assets at the initial time. Importantly, after the rate of consumption tax returns to the original rate, the level of consumption increases to the original level; however, the lower level of the shadow value of foreign assets remains. As a result, from (10), the lower level of the shadow value of foreign assets leads to a higher level of consumption than its original level. Finally, noting that only this effect is kept in the long run, we find that the level of the pollution stock increases.

In Figure 2, we confirm the dynamic behavior of the economy in which the initial economy starts from the original steady state E_0. When the government raises the rate of consumption tax at the initial time, the $\dot{p}_t = 0$ locus moves downward to TEM: $\dot{p}_t = 0$ $(t < T)$, and the shadow value of the pollution stock jumps downward toward the point I'. In duration $[0, T)$, the level of the pollution stock monotonically decreases toward I'' along the unstable path. When the economy reaches the point I'', the level of the shadow value of the pollution stock begins to increase when the economy is still on the unstable path. When the government returns to the original policy at time T, the economy just reaches the point I'''. Importantly, even if the government returns the rate of consumption tax to its original rate, the $\dot{p}_t = 0$ locus does not go back to its original position, because the level of the shadow value of foreign assets does not return to its original level. As a result, after this policy reversion, the economy moves along the stable path to reach the new steady-state equilibrium E_2. At the new steady state, the level of the pollution stock is greater than its original level.

Making use of (17c) and (17d), we next consider the dynamic behavior of foreign assets. From Figure 2(b) we observe that in the period $[0, T)$, the level of foreign assets monotonically increases from E_0 toward Q, based on the fact that $p_0 > p_T$. After the economy follows the stable saddle path to reach the steady-state equilibrium E_2, the level of foreign assets monotonically decreases from Q to E_2.

5.2 Output as the pollution cause

In this subsection, we consider that the flow of pollution is caused by output alone (i.e., $\alpha_c = 0$ and $\alpha_y > 0$). The results of comparative static analysis can be summarized as follows.

Proposition 3 *Assume that $\alpha_c = 0$ and $\alpha_y > 0$. Then, a permanent increase in the rate of income tax decreases the steady-state level of the pollution stock, but a temporary increase leads to a more severe level of the pollution stock.*

Proof. See Appendices B and C. ■

Figure 3 shows the effects of an increase in the rate of income tax on the whole economy, where $\alpha_c = 0$ and $\alpha_y > 0$. An increase in the rate of income tax affects the $\dot{p}_t = 0$ locus, but not the $\dot{\mu}_t = 0$ locus. The movement of $\dot{p}_t = 0$ and $\dot{\mu}_t = 0$ loci is essentially the same as that in the case of the consumption tax rate. From (18b), when the rate of income tax permanently increases, the $\dot{p}_t = 0$ locus moves downward given τ^c and μ_j^*:

$$\left.\frac{\partial p_j^*}{\partial \tau_j^y}\right|_{\dot{p}_t=0,\quad \mu_j^*=\text{constant}} = \underbrace{\frac{\alpha_y N'(l_j^* f(\bar{k}))f(\bar{k})}{\theta}}_{(+)} \left(\underbrace{\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\Lambda_{\tau^y}}_{(\#3)} + \underbrace{\frac{\partial l_j^*}{\partial \tau^y}}_{(\#4)} \right)(< 0). \tag{24}$$

The term (#3) shows the indirect impact on the pollution stock through a change in the shadow value of foreign assets and the term (#4) expresses the direct impact. As for (#3) and (#4), we observe that the direct impact dominates the indirect one, as follows:[8]

$$\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\Lambda_{\tau^y} + \frac{\partial l_j^*}{\partial \tau^y} = \underbrace{\frac{1}{B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}}}_{(-)} \Bigg(\underbrace{\frac{1}{r}\frac{\partial l_j^*}{\partial \tau^y}\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}}_{(+)} \Bigg)(< 0). \tag{25}$$

where $\partial c_j^*/\partial \bar{\lambda}_j^* < 0$ (see (10)) and $\partial l_j^*/\partial \tau^y < 0$ (see (11)).

We now consider the case in which the rate of income tax increases permanently in Figure 3(a). Suppose that the economy starts at the initial steady-state equilibrium E_0. Because of the increase in the rate of income tax, the shadow value of the pollution stock initially jumps downward toward the point I. Thereafter, the level of the pollution stock monotonically decreases until the new steady-state equilibrium E_1. The dynamic behavior of the pollution stock and its shadow value are qualitatively the same as that in the case of the permanent increase in the rate of consumption tax in Figure 2(a). Figure 3(b) shows that the relationship between the foreign assets and the pollution stock is positive, which is opposite to the relationship depicted in Figure 2(b). This result is because an increase in the level of output generates a higher level of pollution flow and a larger household income. This results in a positive relationship between the pollution stock and foreign assets. Therefore, Figure

[8] We can arrange for the terms (#3) and (#4), as follows:

$$\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\Lambda_{\tau^y} + \frac{\partial l_j^*}{\partial \tau^y} = \frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\left(\frac{-B_{\tau^y} + \Theta_j^s P_{\tau^y}}{B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}}\right) + \frac{\partial l_j^*}{\partial \tau^y} = \frac{1}{B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}}\left[\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\left(-B_{\tau^y} + \Theta_j^s P_{\tau^y}\right) + \frac{\partial l_j^*}{\partial \tau^y}\left(B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}\right)\right]$$

where we confirm the following:

$$\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\Theta_j^s P_{\tau^y} - \frac{\partial l_j^*}{\partial \tau^y}\Theta_j^s P_{\bar{\lambda}} = 0, \quad \text{and} \quad -\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}B_{\tau^y} + \frac{\partial l_j^*}{\partial \tau^y}B_{\bar{\lambda}} = \frac{1}{r}\frac{\partial l_j^*}{\partial \tau^y}\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}.$$

Then, we obtain (25).

3(b) shows that when the rate of income tax permanently increases, the level of foreign assets monotonically decreases toward the new steady-state equilibrium E_2.

We now consider the impact of a temporary increase in the rate of income tax. When this rate increases, the level of the shadow value of foreign assets jumps at the same time. Hereafter, the level of the shadow value of foreign assets remains there a long time, even if the rate of income tax returns to its original rate. Since only this effect remains in the long run, the $\dot{p}_t = 0$ locus does not return to the original position. In detail, the $\dot{p}_t = 0$ locus in the long run locates over the original position, as confirmed below:

$$\left.\frac{\partial p_j^*}{\partial \tau^y}\right|_{\dot{p}_t=0,\ \mu_j^*=\text{constant}} = \underbrace{\frac{\alpha_y N'(l_j^* f(\bar{k})) f(\bar{k})}{\theta}}_{(+)} \underbrace{\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}}_{(+)} \Lambda_{\tau^y} (> 0). \tag{26}$$

The long-run position is shown by TEM : $\dot{p}_t = 0 (T < t)$ in Figure 3(a). From this figure, we confirm that the upward shift of the $\dot{p} = 0$ locus in the long run generates a more severe pollution stock in the new steady-state equilibrium E_2. This result is the same as that in the case of a temporary increase in the rate of consumption tax.

On the other hand, the direction of initial jump of the shadow value of foreign assets is the opposite to that in the case of a consumption tax increase. In other words, the shadow value of foreign assets jumps upward, because $\Lambda_{\tau^y} > 0$ (see (19d)). The upward direction of the shadow value of foreign assets is determined by the following equation (8a) under $\alpha_c = 0$:

$$\bar{\lambda}_j^*(1 + \tau^c) = u'(\bar{c}_j^*). \tag{27}$$

Note that the level of consumption is fixed. Increasing the rate of income tax leads to a lower level of income at the beginning. As a result, such a lower level of income generates a downward jump of consumption at the initial period, and for a long time thereafter, the level of consumption does not change. The downward shift of consumption leads to an upward jump of the shadow value of foreign assets from (27). Noting that this effect remains in the long run, we find that a greater level of the shadow value of foreign assets leads to a greater level of labor supply, as confirmed in (11). As a result, the long-run level of the pollution stock increases.

In Figure 3, we observe the dynamic behavior of the entire economy by the temporary increase in the rate of income tax. The transition processes of the pollution stock and its shadow value in Figure 3(a) are qualitatively the same as that in Figure 2(a). In summary, after the shadow value of the pollution stock jumps from E_0 to I', the level of the pollution stock monotonically decreases in the period $(0, T]$ from I' to I''' through the point I'', and thereafter, when the rate of income tax returns to its original level, the level of the pollution stock increases from the point I''' to the new steady-state equilibrium E_2.

Looking at Figure 3(b), we need to mention the dynamic movement of foreign assets, because its movement differs from that in Figure 2(b). In detail, the level of foreign assets monotonically decreases from E_0 to Q during the policy enforcement, and after the rate of income tax goes back to the original one, the level of foreign assets increases toward E_2. Finally, the level of foreign assets in the new steady-state equilibrium E_2 is greater than the original level.

6 Numerical Examples

As stated in introduction, some countries experienced severe pollution after the repeal of temporary pollution control policies. These interesting cases are theoretically captured by our main findings in Propositions 2 and 3; that is, the temporary enforcement of policy leads to a higher level of pollution in the long run. Given our theoretical findings, the numerical analyses in our model play a pivotal role in giving richer policy implications. Specifically, when countries become environmentally friendly from a short-term perspective, the numerical analyses will show how severe the pollution problem in the countries becomes after the repeal of the pollution control policy, which is the theoretically unsolved issue. Then, the purposes of this section of the paper are as follows. First, we confirm the results obtained in Propositions 2 and 3 by conducting numerical analyses. Second, under a longer duration of policy enforcement and a higher rate of consumption or income tax, we examine the quantitative impacts on the pollution stock as well as other economic variables of the whole economy in the long run. To identify the effects of the pollution control policies, we focus on the following two cases, $(\alpha_c, \alpha_y) = (1, 0)$ and $(0, 1)$, throughout this section.

We here specify the production function, the utility functions, and the pollution functions. The production of the single homogeneous good is given by the production process $f(\bar{k}) = A\bar{k}^{\alpha}$, where $0 < \alpha < 1$ and $A > 0$, so that the return to capital is $r = A\alpha\bar{k}^{\alpha-1}$ and the constant level of capital stock is $\bar{k} = \left(\frac{A\alpha}{r}\right)^{\frac{1}{1-\alpha}}$. Next, the utility functions are given by $u(c_t) = \frac{c_t^{1-\delta}}{1-\delta}$, $v(p_t) = \frac{p_t^{1+\epsilon}}{1+\epsilon}$ and $\Gamma(L_t) = \frac{L_t^{1-\gamma}}{1-\gamma}$, where $\delta > 0$, $\epsilon > 0$, and $\gamma > 0$. As for the pollution functions, we assume that $G(c_t) = \frac{c_t^{1+g}}{1+g}$ and $N(l_t f(\bar{k})) = \frac{(l_t f(\bar{k}))^{1+n}}{1+n}$ in which g and n are positive parameters.

Based on these functions, we make use of the following parameters:

Production and pollution parameters: $\alpha = 0.35, \quad A = 0.5, \quad g = n = 1, \quad \theta = 0.05.$

Taste parameters: $\delta = 0.5, \ \epsilon = 0.5, \quad \gamma = 0.5,$

Price and tax rates: $r = (\rho =)0.08, \quad \tau_0^c = \tau_0^y = 0,$

Initial values of foreign assets: $b_0 = 0.35.$

Some parameters used in our simulations are standard ones: $\alpha = 0.35$ and $r = 0.08$. Moreover, setting $A = 0.5$ yields $\bar{k} = 3.33$. As for the pollution parameters, because the parameters g and n, represented by the elasticities of pollution, are freely chosen, we set $g = n = 1$. Furthermore, the choice of θ is 0.05, which means that the 5% of the pollution stock is broken down at every time. To grasp the intuition obtained in the numerical simulation easily, we assume the same values of elasticity parameters in the utility function with consumption and leisure $\delta = \gamma = 0.5$. This assumption is because these elasticities play a critical role in changing the levels of pollution causes (consumption and labor supply).

Next, noting that there are two stock variables, we provide the initially given value of foreign assets by $b_0 = 0.35$; however, we do not provide the initial level of the pollution stock. The reason is that the initial economy is at the steady state, that is, the initial economy must satisfy the equations $(18a) - (18d)$. Considering that $b_0 = b_0^*(= 0.35)$, we notice that the value of the left-hand side in $(18d)$ is zero. If the initial level of the pollution stock is freely given, two values of $\bar{\lambda}_0^*$ and μ_0^* at the initial economy must be determined by three equations $(18a) - (18c)$ given $b_0^* = b_0$ and $p_0^* = p_0$. Therefore, the number of equations is more than the number of variables. Thus, by setting $(p_0^*, \mu_0^*, \bar{\lambda}_0^*)$ to satisfy three equations $(18a) - (18c)$, we determine the initially unique values of $(p_0^*, \mu_0^*, \bar{\lambda}_0^*)$. Then, Table 1 shows the initial economy in our simulation.[9]

[9] We omit $\bar{\lambda}_0^*$ in Table 1 because $\bar{\lambda}_0^*$ is uniquely given by the level of consumption or labor supply using $(8a)$ and $(8b)$ as follows:

$$\lambda_0^* = u'(c_0^*), \quad \text{when } \alpha_c = 0, \quad \text{and} \quad \alpha_y = 1.$$

Finally, we set the initial rates of tax by $\tau_0^c = \tau_0^y = 0$, while the tax rates of consumption and income, τ_1^c and τ_1^y, respectively, are set between 4% and 16%. Furthermore, when the tax rate increases temporarily, we assume that the implementation period T is 2, 4, or 6.

6.1 Consumption as the pollution cause

We now assume that the pollution flow is generated by consumption ($\alpha_c = 1$ and $\alpha_y = 0$). Then, Table 2(a) shows the quantitative effects of the permanent increase in the rate of consumption tax and Table 2(b) shows those of the temporary increase at $T = 2$, 4, or 6. When the initial economy starts from $p_0 = 0.3468$, as in Table 1, we observe from Table 2(a) that the long-run levels of pollution are lower than the original level, showing that when the policy is permanently enforced, the steady-state level of the pollution stock is less than its original level. For instance, when the rate of consumption tax is 4%, the level of consumption decreases by 1.6% in the long run, and hence, the long-run level of the pollution stock decrease by around 5%. This relationship supports the result of Proposition 2. In addition, we find from Table 2(a) that the greater the rate of consumption tax, the lower the long-run level of the pollution stock. For instance, when the rate of consumption tax is set at 12%, the level of the pollution stock decreases by about 10%, which is nearly double the percentage change under a 4% consumption tax rate.

We turn to the effects of the temporary increase in the rate of consumption tax, which is shown in Table 2(b). Then, the findings in the numerical simulation are summarized as follows. First, we confirm that the long-run levels of the pollution stock are greater than the original level, as stated in Proposition 2. For example, when the rate of consumption tax is temporarily set at 4% where $T = 2$, the long-run level of the pollution stock increases by around 1.7%. Furthermore, at the cancellation time of the policy ($t = T = 2$), Table 2(b) shows that the level of the pollution stock, p_T decreases by around 0.8%. This result corresponds to point I''' of Figure 2(a).

Second, Table 2(b) shows that the larger the rate of consumption tax, the greater the long-run level of the pollution stock. For example, when the rate of consumption tax is set

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$$\lambda_0^* = \frac{\Gamma'(1 - l_0^*)}{f(\bar{k})}, \quad \text{when } \alpha_c = 1, \quad \text{and} \quad \alpha_y = 0.$$

where $\tau_0^c = \tau_0^y = 0$ at the initial economy.

at 12% under $T = 2$, the long-run level of the pollution stock increases by around 4.6% compared with its original level, which is more than double its percentage change under a 4% consumption tax rate.

Third, we observe from Table 2(b) that a longer period of policy implementation leads to a lower level of pollution stock at the cancellation time of policy and a higher level of the pollution stock in the long run. For instance, let us consider that the period of policy implementation increases threefold, that is, from $T = 2$ to $T = 6$. Then, we observe that the change from $T = 2$ to $T = 6$ further decreases the level of the pollution stock by 0.08% at the cancellation time of the policy and increases the long-run level of the pollution stock by 0.14%.

As for this relationship between the level of the pollution stock and the implementation duration of the policy, Figure 4 may aid our understanding further. We assume that the implementation duration of the temporary policy is given by $T = 2, 4$, and 6, and that the rate of consumption tax is set at the range in which $\tau^c \in [0.04, 0.16]$. Then, we notice that TEM(2) and TEM(2)L mean that the implementation duration of policy, T is 2. Moreover, TEM(2) indicates the stage of the economy just when the policy has been repealed (i.e., $t = T = 2$), and TEM(2)L shows the long-run economy (i.e., $t = \infty$). Furthermore, note that the broken straight-horizontal line at $p_0 = 0.3468$ means the initial value of the pollution stock. We observe from this figure that the longer the implementation period, the larger the effects of a temporary increase. As for the quantitative impacts, Figure 4 shows that the steady-state levels of the pollution stock are almost the same irrespective of the implementation period under our setting.[10]

Finally, the characterization of foreign assets in Figure 2(b) can be observed in Tables 2(a) and 2(b). From Table 2(a), we observe that the long-run level of foreign assets is higher than its original level, which corresponds to the relationship between points E_0 and E_1 in Figure 2(b). As for the quantitative impacts, for example, when the rate of consumption tax is set at 4%, the long-run level of foreign assets increases by around 2.7%. Furthermore, Table 2(a) shows that a higher rate of consumption tax leads to a greater level of foreign assets. Specifically, setting the rate of consumption tax as 12% makes the steady-state level

[10]When the implementation period is very short (e.g., $T = 0.1$), the steady-state levels of pollution are very close to the original level; however, we omit this case, because we want to focus on the interesting cases.

of foreign assets increase by around 6%, which is more than double the percentage change of foreign assets under a 4% consumption tax rate.

Figure 2(b) shows that the level of foreign assets at time T is greater than the original level (see the point Q) but the long-run level of foreign assets is lower than its original level (see the point E_2). Looking at Table 2(b), for instance, we find that when the rate of consumption tax is given at 4% under $T = 2$, the long-run level of foreign assets, b_2^*, is given by $0.3499(< b_0 = 0.35)$ and the level of foreign assets at the cancellation time of policy, b_T, is given by $0.354(> b_0)$.[11]

6.2 Output as the pollution cause

Assuming that $\alpha_c = 0$ and $\alpha_y = 1$, we consider the case in which the pollution cause is output. Table 3(a) shows the quantitative effects of the permanent increase in the rate of income tax. For example, when the rate of income tax is set at 4%, the long-run level of the pollution stock decreases by around 7%, as a result of the decrease in the level of labor supply by about 2.8%. Looking at the case at $\tau^y = 0.12$, surprisingly, we find that the long-run level of the pollution stock dramatically decreases by about 20%, where the level of labor supply reduces by 9.5%.

In what follows, we pay attention to the effect of the temporary increase in the rate of income tax, which is shown in Table 3(b). The points that we mention are the same as those in the last subsection. First, Table 3(b) shows that the long-run levels of the pollution stock are larger than the original level, which supports our finding in Proposition 3. For instance, when the rate of income tax is set at 4% where $T = 2$, the long-run level of the pollution stock increases by 4.3%. In addition, such pollution control policy decreases the level of the pollution stock by below 1% at the cancellation period ($T = 2$). This result corresponds to the point I''' in Figure 3(a). As for the quantitative effects, we can argue from Table 3(b) that the temporary increase in the rate of income tax does not decrease the level of the pollution stock largely, and furthermore, leads to non-negligible serious contamination after the repeal of the temporary enforcement of tax policy.

[11] Table 2(b) shows that when $\tau^c = 0.04$ and 0.08, $b_2^* = 0.35$ under $T = 6$, which means that the long-run levels of foreign assets are lower than the original level; however, because the quantitative effects are very small, we give $b_2^* = 0.35$ as a result of rounding off the value to four decimal places.

Second, Table 3(b) shows that a higher rate of income tax leads to a greater level of pollution stock in the long run. For example, setting $\tau^y = 0.12$ at $T = 2$, we confirm from Table 3(b) that the long-run level of the pollution stock increases by around 10%, which is more than double the percentage change under a 4% income tax rate.

Third, Table 3(b) and Figure 5 show that the longer the period of the implementation period, the greater the long-run level of the pollution stock. For instance, let us assume that the rate of income tax is set as 4%. Then, we find that when the period of implementation lengthens from $T = 2$ to $T = 6$, the long-run level of the pollution stock increases from $p_2^* = 0.3149$ to 0.3305, which further increases by around 5%.

Finally, we confirm the levels of foreign assets by increasing the rate of income tax. When the rate of income tax is permanently set as 4%, the level of foreign assets decreases by around 5.7 %, shown as point E_1 in Figure 3(b). Turning to the temporary increase of the rate of income tax, Table 3(b) shows that the levels of foreign assets are lower than the initially given level at the cancellation time of the policy, while its long-run levels are greater. This characterization of the dynamic behavior of foreign assets shows points Q and E_2 of Figure 3(b). For example, when the rate of income tax is given by 4% under $T = 2$, the long-run level of foreign assets decreases by 3.8%, and the long-run level of foreign assets does not increase significantly below 0.1%.

7 Conclusion

In this study, we have constructed a dynamic model of a small open economy with two pollution causes. Assuming that the pollution flow is generated by consumption or output and that the pollutants accumulate in the economy, we examine the impacts of pollution control policies on the pollution stock. When the pollution cause is consumption, we pick up the consumption tax rate as a pollution control policy, because increasing the rate of consumption tax has a negative impact on the household's consumption directly, which will reduce the level of pollution cause. Alternatively, when the pollution cause is output, we suppose that the income tax rate is a pollution control policy. We make this assumption because an increase in the rate of income tax directly has a negative impact on working, and it will lead to a lower level of pollution flow. Then, our main finding is that when the pollution control policy is permanently conducted, the level of the pollution stock decreases in the long run, and when the pollution control policy is temporarily conducted, it increases in the long run. This result is independent of the pollution causes. Our numerical examples support this finding.

The numerical simulation shows that the temporary enforcement of pollution control policy may have little merit in small open economies. When the implementation duration of the pollution control policy is longer, and/or when the pollution control policy is stricter, we observe from our simulation that the temporary enforcement of pollution control policy leads to a greater amount of pollution stock. In summary, when small open economies become environmentally friendly from a short-term perspective, they experience more severe pollution after the repeal of the pollution control policy.

According to our findings, governments must tackle pollution problems from a long-term perspective, not a short-term perspective. This argument corresponds to the case of Japan at the initial stage of development. Shimaoka et al. (2016, Chapter 5) mention that when industrial pollution was widespread in Japan at the initial stage of development, the Japanese government regarded stringent industrial pollution policies as indispensable means for economic development. In fact, during the economic boom in Japan from 1960 to 1980, water quality made some remarkable improvements, owing to significant reduction of serious pollution by heavy metals stemming from regulations on industrial wastewater.[12]

[12]According to the Ministry of the Environment in Japan, the rates of non-conformity for health-related pollutants (cadmium, lead, and creatinine) based on old environmental quality standards has significantly decreased. Specifically, in 1971, the rate of non-conformity was indicated at around 1.4% for lead, around 1.1% for creatinine, and around 0.7% for cadmium; however, in 1982, the rates of non-conformity for these pollutants were all below 0.1%.

Appendix A.

In this section, for simplicity, we abbreviate the arguments in each function as $G_c \equiv G'(c^*)$ and $N_l \equiv N'(l^* f(\bar{k}))$, so as not to confuse readers. First, totally differentiating $(18a)$, we obtain

$$p_j^* = p(\mu_j^*). \tag{A.1}$$

Making use of $(18a)$, we obtain

$$\frac{\partial p_j^*}{\partial \mu_j^*} = \frac{\rho + \theta}{v''}(> 0).$$

Substituting $(A.1)$ into $(18b)$ and totally differentiating it, we show that

$$\mu_j^* = M(\bar{\lambda}_j^*, \tau^c, \tau^y), \tag{A.2}$$

where

$$M_{\bar{\lambda}} \equiv \frac{\partial \mu_j^*}{\partial \bar{\lambda}_j^*} = \frac{\alpha_c G_c \frac{\partial c_j^*}{\partial \bar{\lambda}_j^*} + \alpha_y N_l f(\bar{k}) \frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}}{\theta_\mu},$$

$$M_{\tau^c} \equiv \frac{\partial \mu_j^*}{\partial \tau_j^c} = \frac{\alpha_c G_c \frac{\partial c_j^*}{\partial \tau^c}}{\theta_\mu}(< 0),$$

$$M_{\tau^y} \equiv \frac{\partial \mu_j^*}{\partial \tau_j^y} = \frac{\alpha_y N_l f(\bar{k}) \frac{\partial l_j^*}{\partial \tau^y}}{\theta_\mu}(< 0)$$

Note that we make use of the following:

$$\theta_\mu = \frac{\theta(\rho + \theta)}{v''(p_j^*)} - \alpha_c G_c \frac{\partial c_j^*}{\partial \mu_j^*} - \alpha_y N_l f(\bar{k}) \frac{\partial l_j^*}{\partial \mu_j^*}(> 0).$$

Therefore, substituting $(A.2)$ into $(A.1)$ yields

$$p_j^* = P(\bar{\lambda}_j^*, \tau_j^c, \tau_j^y), \tag{A.3}$$

where

$$P_{\bar{\lambda}} \equiv \frac{\partial p_j^*}{\partial \bar{\lambda}_j^*} = \frac{\rho + \theta}{v''(p_j^*)} M_{\bar{\lambda}},$$

$$P_{\tau^c} \equiv \frac{\partial p_j^*}{\partial \tau_j^c} = \frac{\rho + \theta}{v''(p_j^*)} M_{\tau^c}(< 0),$$

$$P_{\tau^y} \equiv \frac{\partial p_j^*}{\partial \tau_j^y} = \frac{\rho + \theta}{v''(p_j^*)} M_{\tau^y}(< 0),$$

Substituting $(A.2)$ into (10) and (11) and further substituting these into $(18c)$, we obtain

$$rb_j^* + l(M(\bar{\lambda}_j^*, \tau_j^c, \tau_j^y), \bar{\lambda}_j^*, \tau^y) f(\bar{k}) = c(M(\bar{\lambda}_j^*, \tau_j^c, \tau_j^y), \bar{\lambda}_j^*, \tau_j^c). \tag{A.4}$$

Totally differentiating $(A.4)$, we obtain the following:

$$b_j^* = B(\bar{\lambda}_j^*, \tau_j^c, \tau_j^y), \tag{A.5}$$

where derivatives with respect to each argument are

$$B_{\bar{\lambda}} \equiv \frac{\partial b_j^*}{\partial \bar{\lambda}_j^*} = \frac{1}{r}\left[-f(\bar{k})\left(\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*} + \frac{\partial l_j^*}{\partial \mu_j^*}M_{\bar{\lambda}}\right) + \frac{\partial c_j^*}{\partial \bar{\lambda}_j^*} + \frac{\partial c_j^*}{\partial \mu_j^*}M_{\bar{\lambda}}\right],$$

$$B_{\tau^c} \equiv \frac{\partial b_j^*}{\partial \tau_j^c} = \frac{1}{r}\left[-f(\bar{k})\frac{\partial l_j^*}{\partial \mu_j^*}M_{\tau^c} + \frac{\partial c_j^*}{\partial \mu_j^*}M_{\tau^c} + \frac{\partial c_j^*}{\partial \tau_j^c}\right],$$

$$B_{\tau^y} \equiv \frac{\partial b_j^*}{\partial \tau_j^y} = \frac{1}{r}\left[-f(\bar{k})\left(\frac{\partial l_j^*}{\partial \mu_j^*}M_{\tau^y} + \frac{\partial l_j^*}{\partial \tau^y}\right) + \frac{\partial c_j^*}{\partial \mu_j^*}M_{\tau^y}\right].$$

Finally, when substituting $(A.3)$ and $(A.5)$ into $(18d)$, we derive the following:

$$\bar{\lambda}_j^* = \Lambda(\tau_j^c, \tau_j^y), \tag{A.6}$$

where

$$\Lambda_{\tau^c} \equiv \frac{\partial \bar{\lambda}_j^*}{\partial \tau_j^c} = \frac{-B_{\tau^c} + \Theta_j^s P_{\tau^c}}{B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}}(< 0), \tag{A.7}$$

$$\Lambda_{\tau^y} \equiv \frac{\partial \bar{\lambda}_j^*}{\partial \tau_j^y} = \frac{-B_{\tau^y} + \Theta_j^s P_{\tau^y}}{B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}}(> 0). \tag{A.8}$$

We now examine the signs of the denominator and numerator in $(A.7)$ and $(A.8)$ under each pollution cause, where we omit variables in each function for simplicity.

First, we calculate $B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}}$ as follows:

$$B_{\bar{\lambda}} - \Theta_j^s P_{\bar{\lambda}} = \frac{1}{r\theta_\mu}\Bigg\{\left(-f\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*} + \frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\right)\Bigg(\theta\frac{\partial p_j^*}{\partial \mu_j^*}\underbrace{-\alpha_c G_c\frac{\partial c_j^*}{\partial \mu_j^*} - \alpha_y N_l f\frac{\partial l_j^*}{\partial \mu_j^*}}_{(\#A1)}\Bigg)$$
$$+\Bigg(\underbrace{1}_{(\#A1)} - \frac{\partial p_j^*}{\partial \mu_j^*}\frac{v''\rho}{(\rho-\beta_s)(\rho+\theta-\beta_s)}\Bigg)\left(\frac{\partial c_j^*}{\partial \mu_j^*} - f\frac{\partial l_j^*}{\partial \mu_j^*}\right)\left(\alpha_c G_c\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*} + \alpha_y N_l f\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\right)\Bigg\}, \tag{A.9}$$

in which $(\#A1)$ is given by

$$(\#A1) = \left(-f\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*} + \frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\right)\left(-\alpha_c G_c\frac{\partial c_j^*}{\partial \mu_j^*} - \alpha_y N_l f\frac{\partial l_j^*}{\partial \mu_j^*}\right) + \left(\frac{\partial c_j^*}{\partial \mu_j^*} - f\frac{\partial l_j^*}{\partial \mu_j^*}\right)\left(\alpha_c G_c\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*} + \alpha_y N_l f\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\right),$$
$$= (\alpha_y N_l f + \alpha_c G_c f)\left(\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\frac{\partial c_j^*}{\partial \mu_j^*} - \frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\frac{\partial l_j^*}{\partial \mu_j^*}\right)(< 0).$$

Therefore, $(A.9)$ can be rewritten as

$$(A.9) = \underbrace{\frac{1}{r\theta_\mu}}_{(+)} \Bigg\{ \underbrace{\left(-f\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*} + \frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\right)\theta\frac{\partial p_j^*}{\partial \mu_j^*} + f(\alpha_y N_l + \alpha_c G_c)\left(\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\frac{\partial c_j^*}{\partial \mu_j^*} - \frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\frac{\partial l_j^*}{\partial \mu_j^*}\right)}_{(-)}$$

$$\underbrace{-\frac{\partial p_j^*}{\partial \mu_j^*}\frac{v''\rho}{(\rho-\beta_s)(\rho+\theta-\beta_s)}\left(\frac{\partial c_j^*}{\partial \mu_j^*} - f\frac{\partial l_j^*}{\partial \mu_j^*}\right)\left(\alpha_c G_c\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*} + \alpha_y N_l f\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\right)}_{(\#A2)}\Bigg\}. \tag{A.10}$$

As for $(\#A2)$, because $\frac{\partial c_j^*}{\partial \mu_j^*} < 0$, $\frac{\partial l_j^*}{\partial \mu_j^*} < 0$, $\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*} < 0$ and $\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*} > 0$, we cannot specify the sign of $\left(\frac{\partial c_j^*}{\partial \mu_j^*} - f\frac{\partial l_j^*}{\partial \mu_j^*}\right)$ as well as $\left(\alpha_c G_c\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*} + \alpha_y N_l f\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}\right)$ in $(\#A2)$.

On the other hand, assuming that $\alpha_c > 0$ and $\alpha_y = 0$, we show that

$$(\#A2) = -\frac{\alpha_c G_c v''\rho}{(\rho-\beta_s)(\rho+\theta-\beta_s)}\frac{\partial p_j^*}{\partial \mu_j^*}\frac{\partial c_j^*}{\partial \mu_j^*}\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}(<0), \tag{A.11}$$

which means that the sign of $(A.9)$ is negative. Next, when assuming that $\alpha_c = 0$ and $\alpha_y > 0$, we confirm the negative sign of $(\#A2)$,

$$(\#A2) = \frac{\alpha_y N_l f^2 v''\rho}{(\rho-\beta_s)(\rho+\theta-\beta_s)}\frac{\partial p_j^*}{\partial \mu_j^*}\frac{\partial l_j^*}{\partial \mu_j^*}\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}(<0), \tag{A.12}$$

implying that $(A.9)$ has a negative sign. As a result, we find that $(A.9)$ has a negative sign if $\alpha_c > 0$ and $\alpha_y = 0$ or $\alpha_c = 0$ and $\alpha_y > 0$.

The pollution cause is consumption:
We assume that $\alpha_c > 0$ and $\alpha_y = 0$, so that $\frac{\partial l_j^*}{\partial \mu_j^*} = 0$. Hence, the denominator of $(A.7)$ has a negative sign.

Based on the negative sign of the denominator of $(A.7)$, our focus is to show the positive sign of the numerator in $(A.7)$.

$$\begin{aligned} &-B_{\tau^c} + \Theta_j^s P_{\tau^c} = -\frac{1}{r}\left(\frac{\partial c_j^*}{\partial \mu_j^*}M_{\tau^c} + \frac{\partial c_j^*}{\partial \tau^c}\right) + \Theta_j^s\frac{\partial p_j^*}{\partial \mu_j^*}M_{\tau^c}, \\ &= -\frac{1}{r\theta_c\theta_\mu}\frac{\partial c_j^*}{\partial \tau^c}\left[-(\alpha_c G_c)^2\left(1 - \frac{\rho(\rho+\theta)}{(\rho-\beta_s)(\rho+\theta-\beta_s)}\right) + \theta_c\left(\theta\frac{\partial p_j^*}{\partial \mu_j^*} - \alpha_c G_c\frac{\partial c_j^*}{\partial \mu_j^*}\right)\right], \\ &= \underbrace{-\frac{1}{r\theta_c\theta_\mu}\frac{\partial c_j^*}{\partial \tau^c}}_{(+)}\underbrace{\left(\frac{\rho(\rho+\theta)(\alpha_c G_c)^2}{(\rho-\beta_s)(\rho+\theta-\beta_s)} + \theta\theta_c\frac{\partial p_j^*}{\partial \mu_j^*}\right)}_{(+)} > 0. \end{aligned} \tag{A.13}$$

Finally, we show that $\Lambda_{\tau^c} < 0$.

The pollution cause is output:

We now assume that $\alpha_c = 0$ and $\alpha_y > 0$, and hence, $\frac{\partial c_j^*}{\partial \mu_j^*} = 0$. Noting that the denominator of $(A.8)$ has a negative sign, we show the negative sign of the numerator in $(A.8)$.

$$
\begin{aligned}
&- B_{\tau^y} + \Theta_j^s P_{\tau^y} = -\frac{1}{r}\left[-f(\bar{k})\left(\frac{\partial l_j^*}{\partial \mu_j^*}M_{\tau^y} + \frac{\partial l_j^*}{\partial \tau^y}\right)\right] + \Theta_j^s \frac{\partial p_j^*}{\partial \mu_j^*}M_{\tau^y}.\\
&= \frac{f}{r\theta_\mu \theta_l}\frac{\partial l_j^*}{\partial \tau^y}\left[(\alpha_y N_l f)^2\left(1 - \frac{\rho(\rho+\theta)}{(\rho-\beta_s)(\rho+\theta-\beta_s)}\right) + \theta_l\left(\theta\frac{\partial p_j}{\mu_j^*} - \alpha_y N_l f\frac{\partial l_j^*}{\partial \mu_j^*}\right)\right],\\
&= \underbrace{\frac{f}{r\theta_\mu \theta_l}\frac{\partial l_j^*}{\partial \tau^y}}_{(-)}\Bigg[\underbrace{(\alpha_y N_l f)^2\frac{\rho(\rho+\theta)}{(\rho-\beta_s)(\rho+\theta-\beta_s)} + \theta\theta_l\frac{\partial p_j^*}{\partial \mu_j^*}}_{(+)}\Bigg] < 0. \qquad \text{(A.14)}
\end{aligned}
$$

Hence, we show that $\Lambda_{\tau^y} > 0$.

Appendix B.

We now examine the effects of a temporary change in each tax rate (τ_j^c and τ_j^y) on the long-run level of pollution. As stated in the Section 5, the government announces changes of the policy instruments from the original level τ_0^i to a new level τ_1^i until time T where $i = c$ and y. After time T, the government reverts permanently to its original level τ_0^i. Since the household can initially anticipate the policy change at time T, the new information arrives only at the initial time, which means that the fixed level of the shadow value $\bar{\lambda}$ jumps to the new steady state at the initial economy and remains there permanently.

When a temporary change in tax rate arises, we divide the transitional dynamics into two separate ones, called Periods 1 and 2, as follows.

Period 1: $0 \le t < T$

During Period 1 (i.e., the period of policy implementation τ_1^i), the economy moves along the unstable transitional path:

$$p_t = p_1^* + D_1 e^{\beta_1^s t} + D_2 e^{\beta_1^u t}, \tag{B.1a}$$

$$\mu_t = \mu_1^* + D_1 \Delta_1^s e^{\beta_1^s t} + D_2 \Delta_1^u e^{\beta_1^u t}, \tag{B.1b}$$

$$b_t = b_1^* + \Theta_1^s D_1 e^{\beta_1^s t} + \Theta_1^u D_2 e^{\beta_1^u t}, \tag{B.1c}$$

where Θ_1^u is defined by

$$\Theta_1^u = \frac{v''(p_1^*)}{(\rho - \beta_1^u)(\rho + \theta - \beta_1^u)} \left(f(\bar{k}) \frac{\partial l_1^*}{\partial \mu_1^*} - \frac{\partial c_1^*}{\partial \mu_1^*} \right).$$

Note that β_1^s and β_1^u represent the stable and unstable roots, respectively, under $(\tau_1^c, \tau_1^y, p_0, b_0)$.

Given the initial levels of foreign assets and the pollution stock $(b_0,\ p_0)$, the steady-state levels, written by p_1^*, μ_1^*, and b_1^*, are determined under the new level of tax rate τ_1^i ($i = c$ or y) where we make use of $(A.6)$:

$$p_1^* = P(\bar{\lambda}_1, \tau_1^c, \tau_1^y) = P(\Lambda(\tau_1^c, \tau_1^y, b_0, p_0), \tau_1^c, \tau_1^y), \tag{B.2a}$$

$$\mu_1^* = M(\bar{\lambda}_1, \tau_1^c, \tau_1^y) = M(\Lambda(\tau_1^c, \tau_1^y, b_0, p_0), \tau_1^c, \tau_1^y), \tag{B.2b}$$

$$b_1^* = B(\bar{\lambda}_1, \tau_1^c, \tau_1^y) = B(\Lambda(\tau_1^c, \tau_1^y, b_0, p_0), \tau_1^c, \tau_1^y) \tag{B.2c}$$

$$\bar{\lambda}_1 = \Lambda(\tau_1^c, \tau_1^y, b_0, p_0) \tag{B.2d}$$

In this period, we notice that $\bar{\lambda}_0 \neq \bar{\lambda}_1$, because it jumps at the initial period.

Period 2: $T \geq t$

During Period 2 (i.e., the period of policy return τ_0^i), the economy follows the stable path as follows:

$$p_t = p_2^* + D_1' e^{\beta_2^s t}, \tag{B.3a}$$

$$\mu_t = \mu_2^* + D_1' \Delta_s e^{\beta_2^s t}, \tag{B.3b}$$

$$b_t = b_2^* + \Theta_2^s D_1' e^{\beta_2^s t}, \tag{B.3c}$$

The steady-state levels of p_2^*, a_2^*, and b_2^* are determined by

$$p_2^* = P(\bar{\lambda}_2^*, \tau_0^c, \tau_0^y) = P(\Lambda(\tau_0^c, \tau_0^y, b_T, p_T), \tau_0^c, \tau_0^y), \tag{B.4a}$$

$$\mu_2^* = M(\bar{\lambda}_2^*, \tau_0^c, \tau_0^y) = M(\Lambda(\tau_0^c, \tau_0^y, b_T, p_T), \tau_0^c, \tau_0^y) \tag{B.4b}$$

$$b_2^* = B(\bar{\lambda}_2^*, \tau_0^c, \tau_0^y) = B(\Lambda(\tau_0^c, \tau_0^y, b_T, p_T), \tau_0^c, \tau_0^y), \tag{B.4c}$$

$$\bar{\lambda}_2^* = \Lambda(\tau_0^c, \tau_0^y, b_T, p_T). \tag{B.4d}$$

Notice that the level of shadow value does not change $\bar{\lambda}_1 = \bar{\lambda}_2$, because the household anticipates the removal of the tax rate under the assumption of perfect foresight.

We denote the policy changes by $d\tau^i \equiv \tau_1^i - \tau_0^i$ $(i = c,\ \ y)$. The approximation of the steady-state changes of pollution with the differentials yields

$$\begin{aligned} p_2^* - p_1^* &= P(\bar{\lambda}_2^*, \tau_0^c, \tau_0^y) - P(\bar{\lambda}_1^*, \tau_1^c, \tau_1^y), \\ &= -P_{\tau^c} d\tau^c - P_{\tau^y} d\tau^y, , \qquad \text{(B.5a)} \\ p_1^* - p_0^* &= P(\bar{\lambda}_1^*, \tau_1^c, \tau_1^y) - P(\bar{\lambda}_0^*, \tau_0^c, \tau_0^y), \\ &= P_{\bar{\lambda}} \left(\Lambda_{\tau^c} d\tau^c + \Lambda_{\tau^y} d\tau^y\right) + P_{\tau^c} d\tau^c + P_{\tau^y} d\tau^y. \qquad \text{(B.5b)} \end{aligned}$$

Notice that the equality $\bar{\lambda}_1 = \bar{\lambda}_2$ holds.

From $(B.5a)$ and $(B.5b)$, we confirm the effects of a temporary change in each tax rate on the pollution as follows:

$$p_2^* - p_0^* = P_{\bar{\lambda}} \left(\Lambda_{\tau^c} d\tau^c + \Lambda_{\tau^y} d\tau^y\right) \tag{B.6}$$

This shows the effect of the temporary change in the environmental policies on the level of the pollution stock.

Appendix C.

We now examine the effects of a permanent increase in the pollution abatement policy on the pollution stock in the long run, where for simplicity we omit the arguments in each function as before.

First, assuming that $\alpha_c > 0$ and $\alpha_y = 0$, we consider the case in which the pollution flow is generated by consumption. Using $(B.5b)$, we show how the permanent increase in the rate of consumption tax affects the level of the pollution stock as follows:

$$\begin{aligned}
\frac{dp_j^*}{d\tau^c} &= P_{\bar{\lambda}}\Lambda_{\tau^c} + P_{\tau^c}, \\
&= \frac{\rho+\theta}{v''\left(B_{\bar{\lambda}} - \frac{\Theta_j^s(\rho+\theta)M_{\bar{\lambda}}}{v''}\right)}\left[M_{\bar{\lambda}}\left(-B_{\tau^c} + \frac{\Theta_j^s(\rho+\theta)M_{\tau^c}}{v''}\right) + M_{\tau^c}\left(B_{\bar{\lambda}} - \frac{\Theta_j^s(\rho+\theta)M_{\bar{\lambda}}}{v''}\right)\right], \\
&= \frac{\rho+\theta}{v''\left(B_{\bar{\lambda}} - \frac{\Theta_j^s(\rho+\theta)M_{\bar{\lambda}}}{v''}\right)}\left(-M_{\bar{\lambda}}B_{\tau^c} + M_{\tau^c}B_{\bar{\lambda}}\right), \\
&= \underbrace{\frac{\rho+\theta}{v''\left(B_{\bar{\lambda}} - \frac{\Theta_j^s(\rho+\theta)M_{\bar{\lambda}}}{v''}\right)}}_{(-)}\underbrace{\left[\left(-\frac{f(\bar{k})}{r}\right)\frac{\partial l_j^*}{\partial \bar{\lambda}_j^*}M_{\tau^c}\right]}_{(+)}(< 0).
\end{aligned} \tag{C.1}$$

Hence, $dp_j^*/d\tau^c < 0$.

In what follows, we assume that $\alpha_c = 0$ and $\alpha_y > 0$, so that the pollution flow is caused by output. Similarly, when using $(B.5b)$, we show how the permanent increase in the rate of income tax affects the level of the pollution stock:

$$\begin{aligned}
\frac{dp_j^*}{d\tau^y} &= P_{\bar{\lambda}}\Lambda_{\tau^y} + P_{\tau^y}, \\
&= \frac{\rho+\theta}{v''\left(B_{\bar{\lambda}} - \frac{\Theta_j^s(\rho+\theta)M_{\bar{\lambda}}}{v''}\right)}\left[M_{\bar{\lambda}}\left(-B_{\tau^y} + \frac{\Theta_j^s(\rho+\theta)M_{\tau^y}}{v''}\right) + M_{\tau^y}\left(B_{\bar{\lambda}} - \frac{\Theta_j^s(\rho+\theta)M_{\bar{\lambda}}}{v''}\right)\right], \\
&= \frac{\rho+\theta}{v''\left(B_{\bar{\lambda}} - \frac{\Theta_j^s(\rho+\theta)M_{\bar{\lambda}}}{v''}\right)}(-M_{\bar{\lambda}}B_{\tau^y} + M_{\tau^y}B_{\bar{\lambda}}), \\
&= \underbrace{\frac{\rho+\theta}{v''\left(B_{\bar{\lambda}} - \frac{\Theta_j^s(\rho+\theta)M_{\bar{\lambda}}}{v''}\right)}}_{(-)}\underbrace{\left[\frac{M_{\tau^y}}{r}\frac{\partial c_j^*}{\partial \bar{\lambda}_j^*}\right]}_{(+)}(< 0)
\end{aligned} \tag{C.2}$$

Therefore, we find that $dp_j^*/d\tau^y < 0$.

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Table 1: The original steady state where $\tau^c = 0$ and $\tau^y = 0$

	b_0	p_0	c_0	l_0	μ_0
$\alpha_c = 1$ and $\alpha_y = 0$	0.35	0.3468	0.1862	0.2076	4.5301
$\alpha_c = 0$ and $\alpha_y = 1$	0.35	0.3017	0.2016	0.2278	4.2251

Table 2(a): Steady state E_1 where $\alpha_c = 1$ and $\alpha_y = 0$

τ^c	b_1^*	p_1^*	c_1^*	l_1^*	μ_1^*
0.04	0.3593	0.3292	0.1832	0.2026	4.4137
0.08	0.365	0.3183	0.1795	0.1972	4.3395
0.12	0.3709	0.31067	0.1759	0.1919	4.26

Table 2(b): Steady state E_2 where $\alpha_c = 1$ and $\alpha_y = 0$ at $T = 2$, 4, or 6

	τ^c	b_2^*	p_2^*	c_2^*	l_2^*	μ_2^*	b_T	p_T
$T = 2$	0.04	0.3499	0.3526	0.1863	0.2026	4.5555	0.354	0.346
	0.08	0.3497	0.3579	0.1864	0.1972	4.5796	0.3573	0.3459
	0.12	0.3495	0.3627	0.1864	0.1919	4.6022	0.3604	0.3451
$T = 4$	0.04	0.3499	0.3529	0.1862	0.2026	4.5572	0.3553	0.3447
	0.08	0.3499	0.3583	0.1862	0.1972	4.584	0.3595	0.3431
	0.12	0.3491	0.3634	0.1863	0.1919	4.6079	0.3629	0.3409
$T = 6$	0.04	0.35	0.3531	0.1862	0.2026	4.5586	0.3564	0.3431
	0.08	0.35	0.3586	0.1862	0.1972	4.5866	0.3612	0.3404
	0.12	0.3499	0.364	0.1862	0.1919	4.6135	0.366	0.3376

Table 3(a): Steady state E_1 where $\alpha_c = 0$ and $\alpha_y = 1$

τ^y	b_1^*	p_1^*	c_1^*	l_1^*	μ_1^*
0.04	0.3302	0.2792	0.1952	0.2214	4.0648
0.08	0.3144	0.2623	0.1881	0.2137	3.9395
0.12	0.2967	0.2445	0.1808	0.206	3.8037

Table 3(b): Steady state E_2 where $\alpha_c = 0$ and $\alpha_y = 1$ at $T = 2$, 4, or 6

	τ^y	b_2^*	p_2^*	c_2^*	l_2^*	μ_2^*	b_T	p_T
$T = 2$	0.04	0.3502	0.3149	0.1952	0.233	4.2905	0.3367	0.2998
	0.08	0.3503	0.3259	0.1881	0.2398	4.3448	0.3256	0.2978
	0.12	0.3507	0.3376	0.1808	0.2471	4.4005	0.3143	0.2953
$T = 4$	0.04	0.3502	0.3216	0.1952	0.2313	4.3352	0.3276	0.2953
	0.08	0.3501	0.3382	0.1881	0.2367	4.4241	0.3095	0.2898
	0.12	0.3501	0.3551	0.1808	0.2428	4.5093	0.2913	0.2838
$T = 6$	0.04	0.3501	0.3305	0.1952	0.229	4.3944	0.3173	0.2913
	0.08	0.3501	0.3551	0.1881	0.2326	4.5302	0.2915	0.2826
	0.12	0.35	0.3797	0.1808	0.2372	4.6559	0.2665	0.2732

Figure 1: Phase diagram

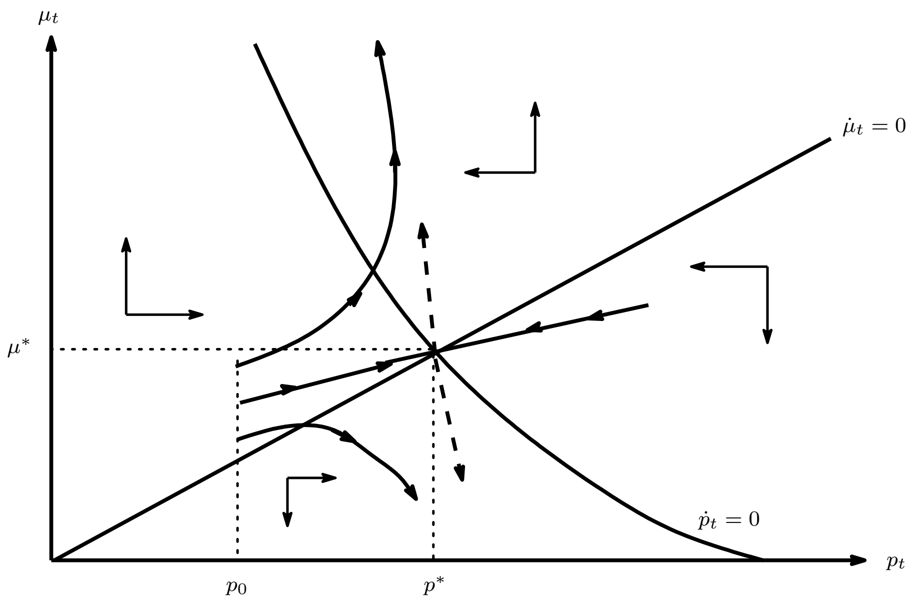

Figure 2: The pollution cause is consumption

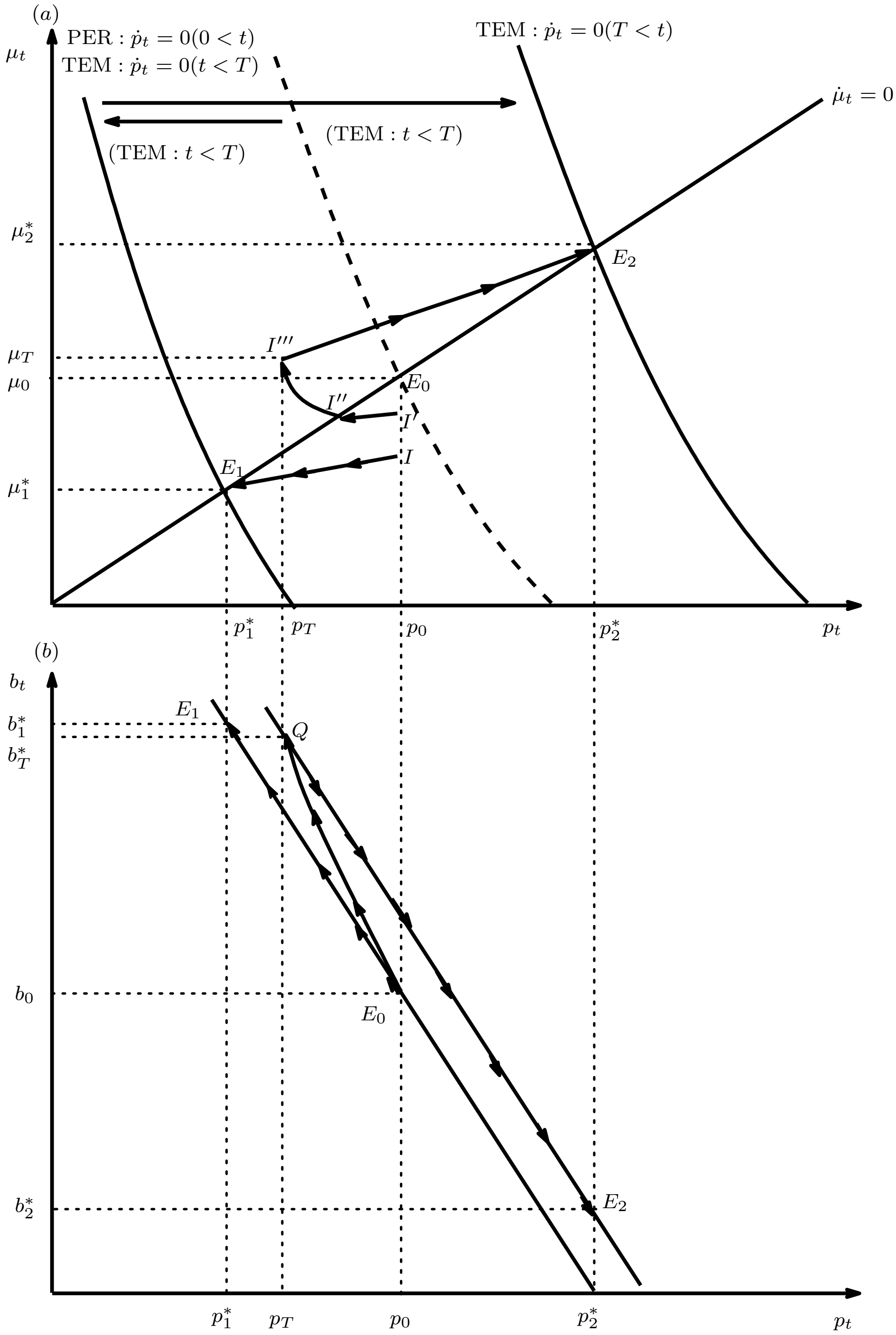

Figure 3: The pollution cause is output

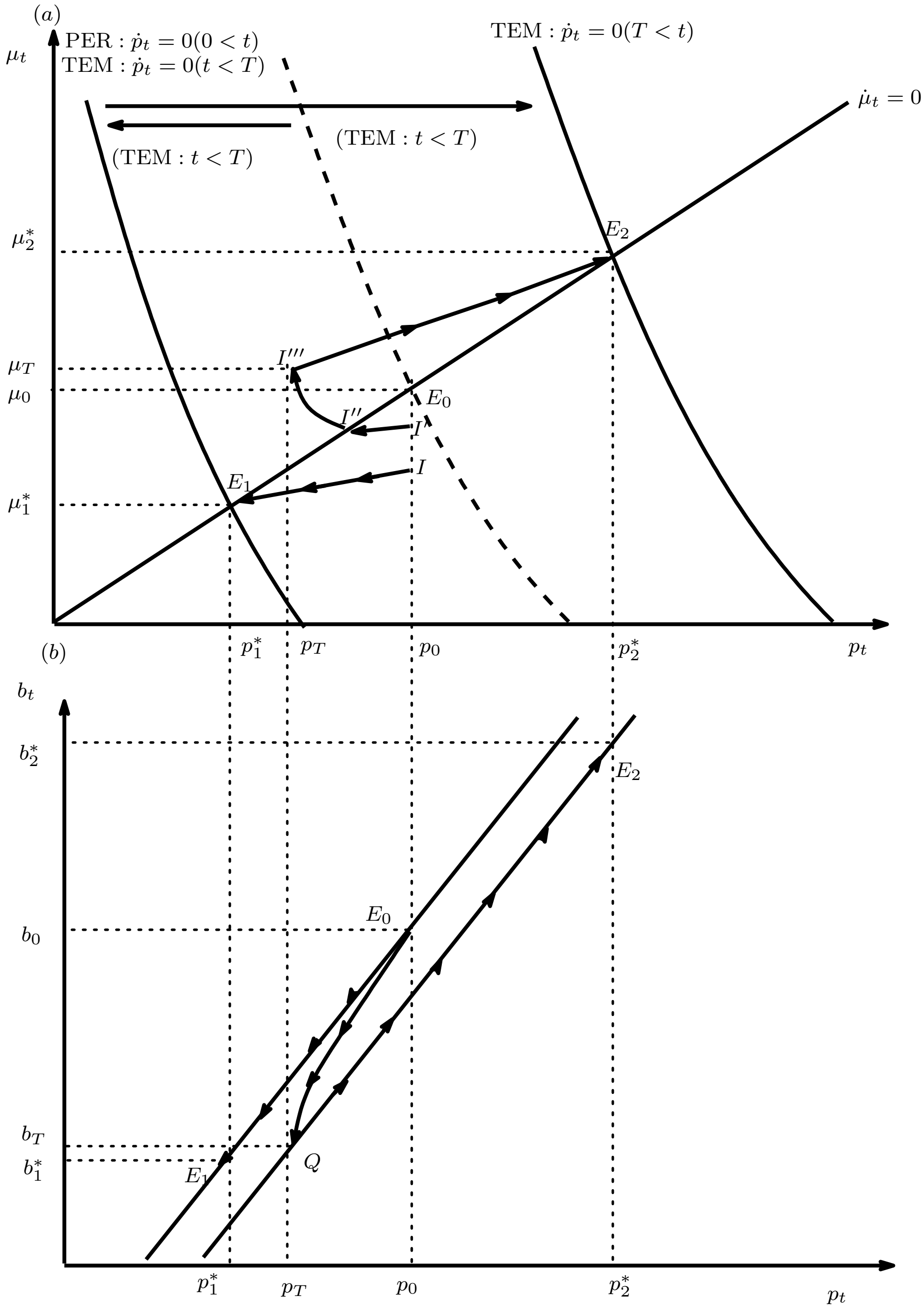

Fig 4: Pollution cause is consumption

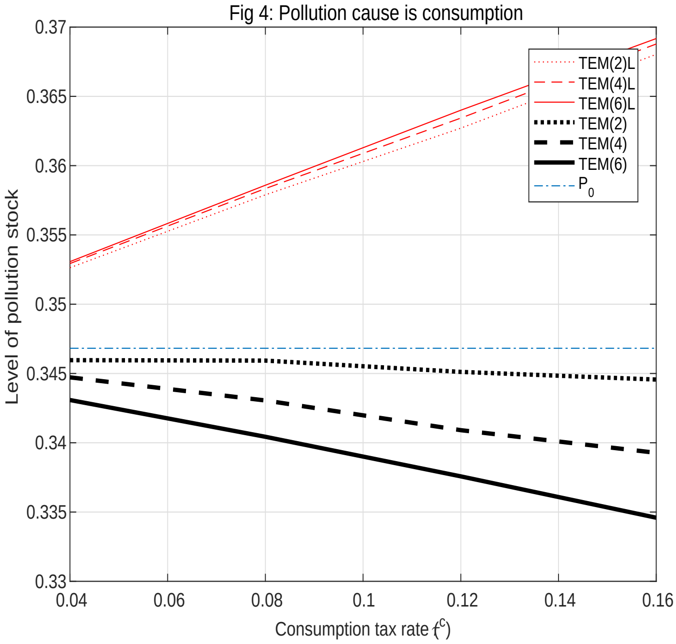

Figure 5: Pollution cause is output