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Dynamic analysis of demographic change and human capital accumulation in an R&D-based growth model*

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Abstract

Employing an overlapping-generations model of R&D-based growth with endogenous fertility, mortality, and education choice, we examine how demographic changes and human capital accumulation influence R&D activity. We show that multiple steady states can exist in this economy. One steady state has a high level of human capital and the other has a low level. In the steady state with high (low) level of human capital, there is a high (low) level of R&D activity, a low (high) fertility rate, and a high (low) old-age survival rate. In addition, we show that the government can steer an economy away from a poverty trap trajectory by investing in public health. We also show that an improvement in the government's public health policy has an inverted U-shaped effect on the growth rate at the steady state.

Keywords: Demographic change, Human capital accumulation, R&D

JEL Classification: I25, J10, O10, O30

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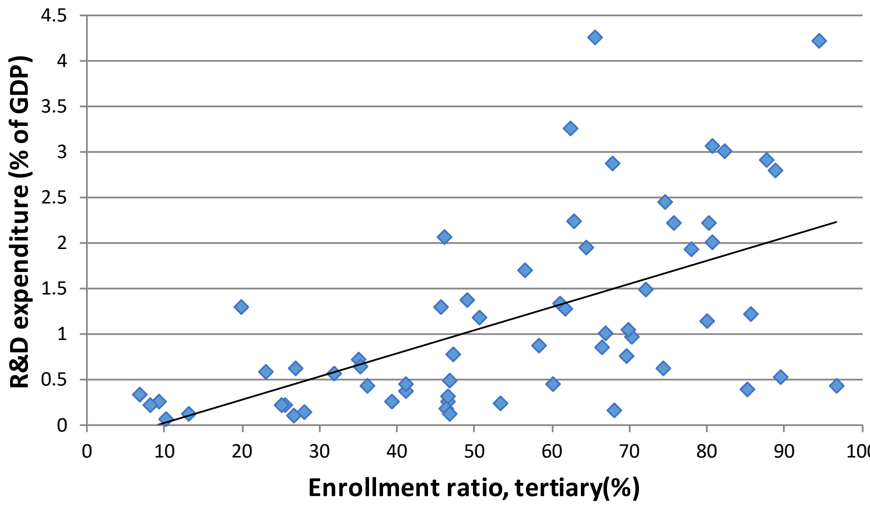

Figure 1: R&D expenditure and enrollment rate of tertiary education. Source: World Bank. Cross-country data for 2015. Simple ordinary least squares estimation shows that there is statistically significant positive correlation between the enrollment ratio and the level of R&D expenditure per unit of GDP.

1 Introduction

In most developed countries, people receive higher levels of education than do people in developing countries. Moreover, most developed countries face low fertility rates and population aging. On the contrary, developing countries face high fertility rates and high mortality rates. In addition, most developed countries more actively undertake R&D and realize more rapid technological change than developing countries. Figure 1 shows the relationship between the enrollment ratio and the level of R&D expenditure per unit of GDP. According to Figure 1, we find positive correlation between the enrollment rate and R&D activity. Figure 2 shows the relationship between the fertility rate and the level of R&D expenditure per unit of GDP. We find negative correlation between the fertility rate and R&D activity. Meanwhile, Figure 3 shows the relationship between the old-age dependency ratio and the level of R&D expenditure per unit of GDP. As Figure 3 shows, there is positive correlation between the old-age survival rate and R&D activity. This remarkable empirical evidence suggests that there are mutual relationships among demographic changes, education, and R&D activity. Therefore, this study examines how demographic changes and human capital accumulation influence R&D activity. We show that education and the human capital accumulation play an important role in economic development in an R&D-based growth model. We clarify the historical conditions that

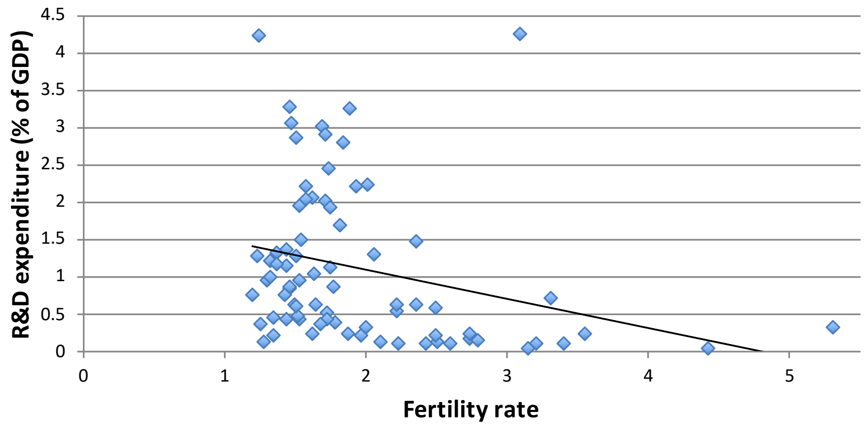

Figure 2: R&D expenditure and fertility rate. Source: World Bank. Cross-country data for 2015. Simple ordinary least squares estimation indicates that there is statistically significant negative correlation between the fertility rate and the level of R&D expenditure per unit of GDP.

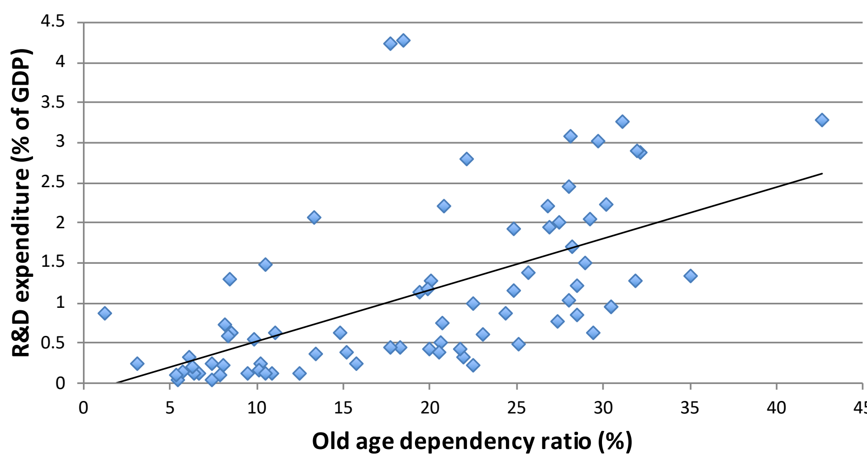

Figure 3: R&D expenditure and old-age dependency ratio. Source: World Bank. Cross-country data for 2015. Simple ordinary least squares estimation shows that there is statistically significant positive correlation between the old-age dependency ratio and the level of R&D expenditure per unit of GDP.

determine whether a country becomes a developed country or a developing country.

We construct a simple overlapping-generations model of R&D-based growth with endogenous fertility, mortality, and education choice. Each individual lives for three periods. In the first period of their lives, individuals do not make any decisions. In the second period, individuals raise children and invest in their education, supply efficient units of labor, pay income tax, consume differentiated goods, and save any remaining income. In the final period, individuals retire and consume differentiated goods. Old individuals live through old age with an endogenous survival rate, which is determined by the government's public health policy. In this model, there is a production sector and an R&D sector. In the production sector, a single firm produces a differentiated good by using effective labor. Similarly, in the R&D sector, R&D firms use the effective labor to invent new differentiated goods. The equilibrium dynamics of this economy are characterized by the level of human capital. We show that multiple steady states can exist in this economy. One of them has a high level of human capital and the other has a low level. In the steady state with the high (low) level of human capital, there is a high (low) level of R&D activity, a low (high) fertility rate, and a high (low) old-age survival rate. In addition, we examine how government policy affects this economy. We show that the government can steer an economy away from a poverty trap trajectory by investing in public health. We also show that an improvement in the government's public health policy has an inverted U-shaped effect on the growth rate at the steady state. We examine how the improved public health policy affects welfare and show that the government may face a trade-off between improving the welfare levels of the current generation and those of future generations.

This study is related to research on demographic change, human capital accumulation, and R&D-based growth. In particular, this study is related to research in the unified growth literature, such as Galor and Weil (2000), Galor and Moav (2002), and Galor (2011). They explain the transition from the Malthusian regime to the sustained growth regime by considering the interrelationship of demographic change, human capital accumulation, and technological change. These studies argue that demographic change, human capital accumulation, and technological change play an important role in shifting from the Malthusian regime to the sustained growth regime. In addition, they argue that the difference between developed and developing countries is their timing in taking off from the Malthusian regime. Specifically, developing countries do not stay in the Malthusian regime and automatically move to the sustained growth regime. However, many developing countries face sustained challenges of low economic growth, high fertility rate, and high mortality rate. Therefore, developing countries may be caught in a poverty trap, thereby hindering the transition from the Malthusian regime to the sustained growth regime. Hence, many studies allow the existence of multiple steady states with the poverty trap, including Blackburn and Cipriani (2002), Moav (2005), Kimura and Yasui (2007), Chen (2010), and Chakraborty and Chakraborty (2018), taking endogenous fertility and education choice into account. They argue that individuals' education decisions play an important role in economic development. However, they do not consider R&D activities. In contrast to this research, we consider the relationship between demographic change, education, and R&D activity simultaneously. This study is also related to Chakraborty (2004), Hashimoto and Tabata (2005), and Agénor (2015). whose works endogenize the old-age survival rate by focusing on public health policy and exploring the dynamic relationship between mortality and economic growth. However, they do not consider fertility, education, and R&D simultaneously. In contrast to this research, we consider these factors and can explain the facts shown in Figures 2 and 3. Furthermore, this research is related to Chu et al. (2013), Strulik et al. (2013), and Hashimoto and Tabata (2016), who investigate the relationship between demographic change, education, and R&D activity simultaneously. However, Chu et al. (2013) and Strulik et al. (2013) do not consider mortality. Hashimoto and Tabata (2016) consider mortality but regard it as exogenous; moreover, they do not refer to a multiplicity of steady states. In contrast to this research, we consider endogenous mortality and show the existence of multiple steady states. Lastly, this study is related to Futagami and Konishi (2017), who explore the dynamic relationship between endogenous fertility, mortality, and R&D. In addition to their research, we study how education and human capital accumulation affect the economy.

This rest of this paper is organized as follows. Section 2 shows the basic structure of the model, Section 3 analyzes equilibrium and dynamics. Section 4 analyses government policy. Finally, the conclusion is described in Section 5.

2 The model

2.1 Individuals

Time is discrete and denoted by $t = 0, 1, 2, \cdots$. Each individual lives for three periods (childhood, adulthood, and old age). In the first period (childhood), individuals do not make any decisions, and they are raised by their parents. In the second period (adulthood), individuals raise their children and invest in their education, supply efficient units of labor, pay income tax, consume differentiated goods, and save any remaining income. In the final period (old age), individuals retire and consume differentiated goods. An individual dies at the beginning of his or her old age with probability $1 - \lambda_t \in [0, 1]$ and lives through old age with probability $\lambda_t \in [0, 1]$. Members of the cohort born in period $t - 1$ become active workers in period t. Thus, we call this cohort generation t and use N_t to represent the number of adults who are alive in period t. Let n_t denote the number of children for each adult. Therefore, the relationship between the sizes of the adult populations during any two consecutive periods can be expressed as $N_{t+1} = n_t N_t$. Individuals derive their utility from the number of children n_t, their children's level of human capital h_{t+1}, their own consumption during adulthood $C_{1,t}$, and their own consumption during old age $C_{2,t+1}$. The lifetime utility of individuals in generation t is expressed as

$$u_t = \eta \log n_t + \log C_{1,t} + \beta\lambda_t(\gamma \log h_{t+1} + \log C_{2,t+1}), \tag{1}$$

where the positive parameters η and γ denote the weights of the number of their children and the children's level of human capital, respectively. $\beta \in (0, 1]$ denotes the discount factor. Following Mariani et al. (2010), we assume that intergenerational altruism is eventually magnified by a higher λ_t, because the success of children has a greater effect on those parents who will live long enough to witness it. We specify the subutility function $C_{k,t}$ for $k \in \{1, 2\}$ as

$$C_{k,t} \equiv \left[\int_0^{A_t} c_{k,t}(i)^\alpha di\right]^{\frac{1}{\alpha}}, \tag{2}$$

where $c_{k,t}(i)$ represents the consumption of differentiated good $i \in [0, A_t]$. A_t denotes the variety of differentiated goods or the level of technological knowledge in this economy, which grows through

R&D. In individuals' second period of life (adulthood), they are endowed with one unit of time, which is devoted to working l_t in the labor market, raising n_t identical children, and educating each child e_t. We assume that a fixed amount of time z is required to bear and raise a child. Individuals divide their disposable income $(1-\tau)w_t h_t l_t$ between consumption and saving s_t for their old age[1]. Following Yaari (1965), we assume that insurance companies are risk neutral and that the private annuities market is competitive. Insurance companies promise individuals a payment $(R_t/\lambda_t)s_t$, in exchange for which the estate s_t accrues to the companies, where λ_t is the average probability of surviving and R_{t+1} represents the gross interest rate. In the absence of a bequest motive, individuals are willing to invest their assets in such insurance. Here, w_t and $\tau \in [0,1)$ are the wage rate for efficient units of labor and the income tax rate, respectively. Thus, the budget and time constraints for individuals in generation t are expressed as follows:

$$E_{1,t} = (1-\tau)w_t h_t l_t - s_t, \tag{3}$$

$$E_{2,t+1} = \frac{R_{t+1}}{\lambda_t} s_t, \tag{4}$$

$$E_{1,t} = \int_0^{A_t} p_t(i) c_{1,t}(i) di, \tag{5}$$

$$E_{2,t+1} = \int_0^{A_{t+1}} p_{t+1}(i) c_{2,t+1}(i) di, \tag{6}$$

$$l_t + z n_t + e_t n_t = 1, \tag{7}$$

where $E_{1,t}$, $E_{2,t+1}$, and $p(i)$ denote the expenditure of an individual in adulthood, the expenditure of an individual in old age, and the price of good i, respectively[2].

We assume that the human capital production function is given by the following expression:

$$h_{t+1} = \phi(e_t h_t)^{\sigma_e} \bar{h}_t^{\sigma_h}, \quad \phi > 0, \quad \sigma_e, \sigma_h \in (0.1]. \tag{8}$$

ϕ, σ_e, and σ_h are parameters. h_t reflects externalities from the human capital stock of parents and $\bar{h}_t$ reflects externalities from the human capital stock of the society. In equilibrium, $\bar{h}_t = h_t$ holds,

[1] In this model, we do not take account of the goods cost of child rearing. If we were to consider this, the dynamics of per capita human capital h_t and the variety of differentiated goods A_t would depend on each other and the model would be quite complicated. Therefore, we do not take account of the goods cost of child rearing to simplify the analysis. However, this simplification is quite common in the literature, for example, Galor and Moav (2002).

[2] In our specifications, we ignore the utility cost of labor effort. However, our results still hold even if we take account of the utility cost of labor effort and the substitution among leisure and time spent doing other things.

because all individuals are homogeneous in this economy. Individuals maximize their utility given these externalities[3].

We next consider the individual's utility maximization. By maximizing the subutility function (2) subject to the budget constraint (5), we obtain the demand for differentiated good i as follows:

$$c_{k,t}(i) = \frac{p_t(i)^{-\epsilon}}{P_t^{1-\epsilon}} E_{k,t}, \tag{9}$$

where $\epsilon \equiv \frac{1}{1-\alpha}$ and P_t is the price index defined by $P_t \equiv \left[\int_0^{A_t} p_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$. This demand function implies that indirect utility becomes a linear function of expenditure as follows:

$$C_{k,t} = \frac{E_{k,t}}{P_t}. \tag{10}$$

Let us denote the total demand for good i as $x_t(i)$. $x_t(i)$ is given by

$$\begin{aligned} x_t(i) &= c_{1,t}(i)N_t + c_{2,t}(i)\lambda_{t-1}N_{t-1}, \\ &= \frac{p_t(i)^{-\epsilon}}{P_t^{1-\epsilon}}(E_{1,t}N_t + E_{2,t}\lambda_{t-1}N_{t-1}). \end{aligned} \tag{11}$$

By maximizing (1) subject to (3), (4), (7), (8), and (10), we obtain the following solution:

$$n_t = \frac{\eta - \beta\gamma\sigma_e\lambda_t}{(1+\eta+\beta\lambda_t)z}, \tag{12}$$

$$e_t = \frac{\beta\gamma\sigma_e z\lambda_t}{\eta - \beta\gamma\sigma_e\lambda_t}, \tag{13}$$

$$s_t = \frac{\beta(1-\tau)w_t h_t\lambda_t}{1+\eta+\beta\lambda_t}, \tag{14}$$

$$l_t = \frac{1+\beta\lambda_t}{1+\eta+\beta\lambda_t}. \tag{15}$$

According to (12) and (13), the fertility rate decreases with the old-age survival rate λ_t (i.e., $\frac{\partial n_t}{\partial \lambda_t}$), whereas investment in education for each child increases with λ_t (i.e., $\frac{\partial e_t}{\partial \lambda_t}$). An increase in the old-age survival rate stimulates demand for consumption relative to demand for both quantity and quality of children, because this increase induces individuals to anticipate the need to consume

[3] Yakita (2010) and Hashimoto and Tabata (2016) discuss these assumptions of human capital production function in detail.

goods over a longer period of time. In response to this change, individuals shift their time from child rearing to work. Therefore, the fertility rate decreases and working time and savings increase with the old-age survival rate. On the other hand, an increase in the old-age survival rate has three effects on investment in education for each child. First, it motivates parents to shift their time from education to work (i.e., the first positive effect). Second, if the old-age survival rate increases, parents invest more in education for each child because of the inter-generational externality of education (i.e., the second positive effect). Third, the decrease in the fertility rate that results from the rise in the old-age survival rate decreases the opportunity costs that parents incur from providing their children with an education, which motivates parents to invest more in education for each child (i.e., the negative effect). In equilibrium, the first positive effect and the negative effect offset one another. Consequently, investment in education for each child increases with the old-age survival rate because of the second positive effect.

We make the following assumption, which is a sufficient condition for ensuring $n_t > 0$.

Assumption 1

$$\eta > \gamma$$

This assumption implies that having a family must be more important than investing in the education of children[4].

2.2 Production

There are differentiated goods indicated by $i \in [0, A_t]$. A single firm produces each good. Each firm supplies a differentiated good monopolistically and sets its price. The monopoly is protected through perfect patent protection. Each monopolistic firm produces one unit of good by using one unit of effective labor. The producer of good i maximizes the following profit:

$$\pi_t(i) = p_t(i)x_t(i) - w_t x_t(i), \tag{16}$$

[4]This assumption ensures the existence of a consistent solution; see Strulik (2004).

subject to the total demand function for good i (11). From the profit maximization condition, the price of good i is

$$p_t(i) = \frac{1}{\alpha} w_t \equiv p_t. \tag{17}$$

Hence, all goods have the same price. Thus, the firm-specific index i in the differentiated goods sector can be dropped. By substituting (17) into the demand function (11), we obtain the output level of the differentiated good:

$$x_t = \frac{p_t^{-\epsilon}}{\int_0^{A_t} p_t^{1-\epsilon} dj} (E_{1,t} N_t + E_{2,t} \lambda_{t-1} N_{t-1}) = \frac{E_{1,t} N_t + E_{2,t} \lambda_{t-1} N_{t-1}}{A_t p_t}. \tag{18}$$

The total expenditure is treated as a numeraire ($E_{1,t} N_t + E_{2,t} \lambda_{t-1} N_{t-1} = 1$). Therefore, we can rewrite (18) by using (17) as follows:

$$x_t = \frac{1}{A_t p_t} = \frac{\alpha}{w_t A_t}. \tag{19}$$

The profit of each differentiated good firm is given by

$$\pi_t = \left(\frac{1}{\alpha} - 1 \right) w_t x_t. \tag{20}$$

2.3 R&D

R&D firms use the effective labor to invent new differentiated goods. After invention, the firms sell a blueprint of a new good to an entrepreneur. Development of $A_{t+1} - A_t$ new blueprints requires $l_{R,t}$ units of effective labor input. Let us define ΔA_t as $\Delta A_t \equiv A_{t+1} - A_t$. Thus, given research productivity of ψ_t, output is expressed as follows:

$$\Delta A_t = \psi_t A_t l_{R,t}, \tag{21}$$

A_t implies the spillover from general knowledge accumulated by past innovation. To remove the strong scale effect, we follow Laincz and Peretto (2006) and Chu et al. (2013). We assume that ψ_t is decreasing in the scale of the economy $h_t N_t$. We specify productivity as $\psi_t \equiv \frac{\psi l_{R,t}^{\nu-1}}{(h_t N_t)^{\nu}}$ where $\psi > 0$

and the parameter $\nu \in (0,1)$ inversely measures the negative duplication externality discussed in Jones (1995) and Jones and Williams (2000). In accordance with Jones (1995), the research productivity is given for each firm. $g_t \equiv \frac{\Delta A_t}{A_t}$ is the growth rate of product variety. Given $l_{R,t}$, the growth rate of product variety features decreasing returns to scale in $l_{R,t}$. The R&D firms' profits π_t^R are given by

$$\begin{aligned}\pi_t^R &= v_t \Delta A_t - w_t l_{R,t}, \\ &= \left(v_t - \frac{w_t}{\psi_t A_t} \right) \Delta A_t, \qquad (22)\end{aligned}$$

where v_t is the price of a blueprint of a newly invented good. Because an entrepreneur pays the price of the blueprint of the new good to the R&D firm, this price corresponds to the price of equity that is sold to the household. Free entry into R&D races leads to the following zero-profit condition:

$$v_t \leq \frac{w_t}{\psi_t A_t} \qquad \text{with equality if} \quad \Delta A_t > 0. \qquad (23)$$

As shown in Appendix A, R&D is always undertaken and (23) holds with equality. The value of v_t equals the present value of future profits as follows:

$$v_t = \sum_{T=t+1}^{\infty} \frac{\pi_T}{\Pi_{\nu=t+1}^{T} R_\nu}.$$

After some manipulations, we obtain the following no-arbitrage condition.

$$R_{t+1} = \frac{v_{t+1} + \pi_{t+1}}{v_t}. \qquad (24)$$

Each individual saves at the gross rate of interest R_{t+1} determined by (24).

2.4 Government

The government collects income tax from individuals and invests it in the public health service G_t. We assume that the public health service is produced by effective labor $l_{G,t}$ and, for simplicity, that the production function of the public health service is linear, as follows:

$$G_t = \delta l_{G,t} \tag{25}$$

where $\delta > 0$ is a constant parameter[5]. The old-age survival rate λ_t is improved by the per capita public health service $b_t \equiv \frac{G_t}{N_t}$. The old-age survival rate λ_t is given by

$$\lambda_t = f(b_t), \tag{26}$$

which satisfies $f'(b_t) \geq 0$, $f''(b_t) < 0$, $f(0) \geq 0$, and $\lim_{b_t \to \infty} f(b_t) = \mu \leq 1$. Public health expenditure is financed by the government's balanced budget. Therefore, the government's budget constraint is

$$w_t l_{G,t} = \tau w_t h_t l_t N_t. \tag{27}$$

3 Equilibrium

3.1 Market equilibrium

We first describe the equilibrium condition of the asset market. Individuals' savings must be directed to purchase either newly issued stocks for R&D or existing stocks of operating firms owned by the preceding generation. Therefore, the asset market equilibrium condition is

$$s_t N_t = w_t l_{R,t} + v_t A_t. \tag{28}$$

The left-hand side of (28) is the saving volume of the adult individuals. The first term on the right-hand side is the total investment in newly issued stocks, which is equal to the total cost of R&D activities. The second term is the purchase of existing stocks. Note that the second term

[5]We consider that nurses and doctors provide the public health service. Effective labor $l_{G,t}$ includes their human capital and we can express their raw labor as $\frac{l_{G,t}}{h_t}$; see (29). Therefore, we consider the skills of nurses and doctors in this simple liner functional specification. In addition, the public health service is produced without goods. If we take account of goods input in the production of public health service in addition to the labor input, then λ_t depends on both h_t and A_t and the dynamics become quite complicated. This simplification allows us to undertake a simple analysis. Futagami and Konishi (2017) analyze the situation in which the old-age survival rate depends on the variety of goods in R&D-based growth model without human capital.

implicitly assumes that ex-dividend stocks are traded between adult and old generations in each period. In other words, the old generation always receives all dividends before selling the stocks.

We next describe the labor market equilibrium condition. Effective labor is used for production of differentiated goods, R&D, and public health services. The labor market-clearing condition is

$$h_t l_t N_t = l_{R,t} + A_t x_t + l_{G,t}, \tag{29}$$

where $l_{R,t}$, $A_t x_t$, and $l_{G,t}$ denote the quantities of effective labor engaging in R&D, production activities, and improving public health, respectively.

From (15), (25), and (27), we obtain

$$b_t = \frac{(1+\beta\lambda_t)\delta\tau h_t}{1+\eta+\beta\lambda_t} \equiv b(\lambda_t; h_t, \tau). \tag{30}$$

Note that $b(0; h_t, \tau) = \frac{\tau\delta h_t}{1+\eta} \geq 0$, $\lim_{\lambda_t \to \infty} b(\lambda_t; h_t, \tau) = \delta\tau h_t \leq \infty$, $\frac{\partial b(\lambda_t; h_t, \tau)}{\partial \lambda_t} \geq 0$, $\frac{\partial^2 b(\lambda_t; h_t, \tau)}{\partial \lambda_t^2} \leq 0$. Let us define $f^{-1}(\lambda_t)$ as the inverse function of (26). This inverse function satisfies the following characteristics: $\frac{\partial f^{-1}(\lambda_t)}{\partial \lambda_t} \geq 0$, $\frac{\partial^2 f^{-1}(\lambda_t)}{\partial \lambda_t^2} > 0$, and $f^{-1}(0) \leq 0$. λ_t and b_t are determined by the following equation:

$$f^{-1}(\lambda_t) = b(\lambda_t; h_t, \tau). \tag{31}$$

We define λ_t^* and b_t^* as the solution of (31) with respect to λ_t and $b_t^* \equiv b(\lambda_t^*, h_t, \tau)$, respectively. In addition, we can regard λ_t^* as a function of h_t and τ, and denote $\lambda_t^* = \lambda(h_t; \tau)$. Figure 4 shows the relationship between $f^{-1}(\lambda_t)$ and $b(\lambda_t, h_t, \tau)$. Figure 4 shows the determination of (λ_t^*, b_t^*). As shown in Figure 4, (λ_t^*, b_t^*) is uniquely determined. From (30), we find that $\frac{\partial b(\lambda_t; h_t, \tau)}{\partial h_t} \geq 0$. Therefore, if h_t increases, the line of $b(\lambda_t; h_t, \tau)$ shifts upward in Figure 5. We find that $\frac{\partial \lambda_t^*}{\partial h_t} > 0$ from Figure 5 and obtain the following Proposition 1.

Proposition 1 *An increase of per capita capital h_t increases the old-age survival rate λ_t^*.*

If per capita human capital h_t increases, the income of individuals increases. This income increase raises the government's income, because the government imposes income tax. Therefore, the government can invest more in public health and the old-age survival rate rises. We obtain the following

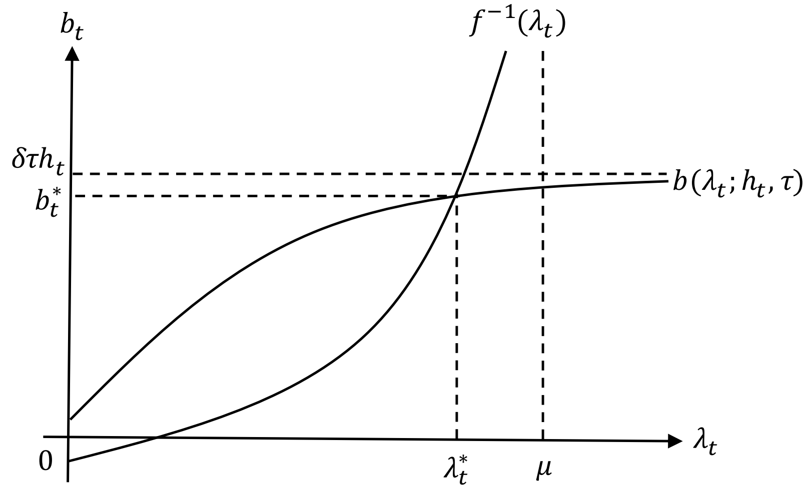

Figure 4: Determination of (λ_t^*, G_t^*)

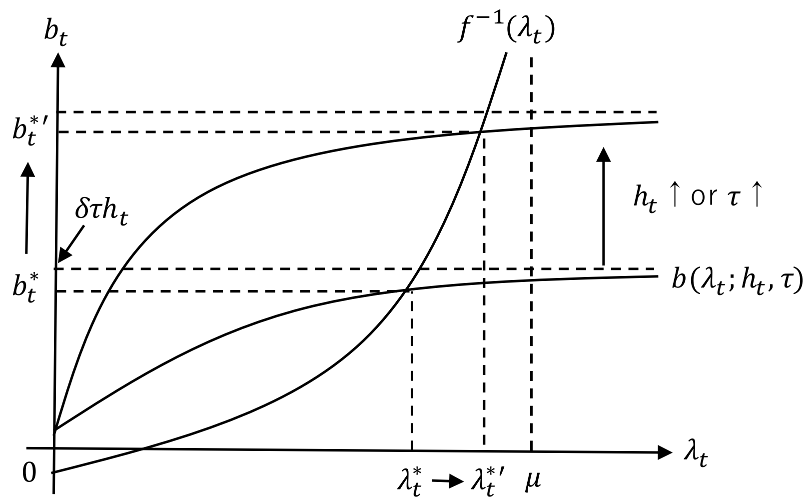

Figure 5: The effect of H_t and τ on (λ_t^*, G_t^*)

proposition.

Proposition 2 *An increase of per capita human capital h_t decreases the fertility rate n_t.*

Proof. From (12) and $\frac{\partial \lambda_t^*}{\partial h_t} > 0$, we find $\frac{\partial n_t}{\partial h_t} < 0$.
From Proposition 1, if per capita human capital h_t increases, the old-age survival rate λ_t^* rises. If the old survival rate λ_t^* rises, individuals put more weight on old-age consumption. Therefore, individuals increase their efficient units of labor to save more. With increased efficient units of labor, individuals have to decrease the number of children owing to time constraints. Therefore, if per capita human capital h_t increases, the fertility rate n_t decreases.

From (14), (21), (23), and (28), the growth rate of product variety g_t is determined by the following equation:

$$g_t + 1 = \frac{\beta \psi^{\frac{1}{\nu}} (1-\tau)\lambda_t}{1+\eta+\beta\lambda_t} g_t^{-\frac{1-\nu}{\nu}}. \tag{32}$$

This defines the relationship between g_t and λ_t. We obtain the following proposition.

Proposition 3 *An increase of per capita human capital h_t increases the growth rate of product variety g_t.*

Proof. see Appendix B.
If per capita human capital h_t increases, the old-age survival rate increases. This increases savings and thus, investment in R&D increases. Consequently, the growth rate of product variety g_t increases.

From (30), we find that $\frac{\partial b(\lambda_t; h_t, \tau)}{\partial \tau} \geq 0$. Therefore, if τ increases, the line of $b(\lambda_t; h_t, \tau)$ shifts upward in Figure 5. We find that $\frac{\partial \lambda_t^*}{\partial \tau} > 0$ from Figure 5 and obtain the following Lemma 1.

Lemma 1 *An increase of the tax rate τ increases the old-age survival rate λ_t^*.*

If the government raises the tax rate, the government can collect more tax revenue and invest more in public health. Therefore, the old-age survival rate λ_t^* rises.

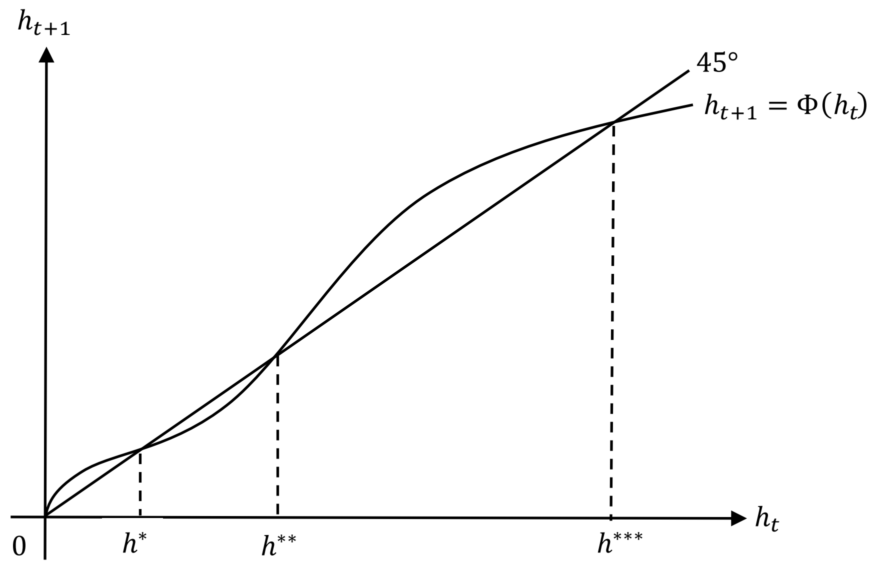

Figure 6: Dynamics of h_t

3.2 Dynamics

The dynamics of this economy are characterized by per capita human capital h_t. From (8), (13), and $\lambda_t^* = \lambda(h_t;\tau)$, we obtain

$$h_{t+1} = \underbrace{\phi \left[\frac{\beta\gamma\sigma_e z \lambda(h_t;\tau)}{\eta - \beta\gamma\sigma_e \lambda(h_t;\tau)} \right]^{\sigma_e} h_t^{\sigma_e+\sigma_h}}_{\equiv \Phi(h_t)}. \tag{33}$$

Let us define the right-hand side of (33) as $\Phi(h_t)$. Figure 6 shows the phase diagram of h_t when the economy has multiple steady states[6]. In this case, the economy has three steady states. The first one is stable and its per capita human capital is h^*. The second one is the unstable steady state. We denote the per capita human capital of this steady state as h^{**}. The third one is stable and its per capita human capital is high. We denote the per capita human capital of this steady state as h^{***}. If we specify $f(b_t) = \frac{\rho+\mu b_t}{\theta+b_t}$, we obtain the following proposition[7].

Proposition 4 *Suppose $\sigma_h \in (1-2\sigma_e, 1-\sigma_e)$, and ρ is sufficiently low. There exists ϕ such that*

[6]A numerical example reveals that there is a parameter configuration such that Assumption 1 is satisfied and multiple steady states exist. For example, if we specify $f(G_t) = \frac{\rho+\mu b_t}{\theta+b_t}$ and assume that $\beta = (0.98)^{25}$, $\delta = 1$, $\gamma = 0.14$, $\eta = 0.21$, $\theta = 5$, $\mu = 1$, $\rho = 1$, $\sigma_e = 0.635$, $\sigma_h = 0.165$, $\tau = 0.1$, $\phi = 34.02$, and $z = 0.088$, then multiple steady states exist. The justifications for this parameter setting are described in Section 4. If ϕ is in $[30.6, 35.6]$, multiple steady states exist.

[7]The functional form $f(b_t) = \frac{\rho+\mu b_t}{\theta+b_t}$ satisfies $f'(b_t) \geq 0$, $f''(b_t) < 0$, $f(0) \geq 0$, and $\lim_{b_t\to\infty} f(b_t) = \mu \leq 1$.

the economy has multiple steady states. When the economy has three steady states, if the initial per capita human capital is low ($h_0 < h^{**}$), per capita human capital h_t converges to h^*. However, if the initial per capita human capital is high ($h_0 > h^{**}$), per capita human capital h_t converges to h^{***}.

Proof. see Appendix C

If the initial per capita human capital is sufficiently high (i.e., $h_0 > h^{**}$), the old-age survival rate is high because of Proposition 5. Then, investment in education for each child is also high. Therefore, per capita human capital converges to steady state h^{***}. However, if the initial per capita human capital is sufficiently low (i.e., $h_0 < h^{**}$), the old-age survival rate is low. Then, the investment in education for each child is also low. Therefore, per capita human capital converges to steady state h^*. This complementarity results in the existence of multiple steady states.

From Propositions 1, 2, 3, and, 4, we obtain the following results. In the steady state with a high level of human capital h^{***}, the level of R&D activity is high, the fertility rate is low, and the old-age survival rate is high. However, in the steady state with a low level of human capital h^*, the level of R&D activity is low, the fertility rate is high, and the old-age survival rate is low. These results correspond to the empirical results, as shown in Figures 1, 2, and 3.

4 Government policy

In this section, we consider how government policy affects this economy. We concentrate on the case in which multiple steady states exist in this economy. The government can set the income tax rate in this model. Therefore, we focus on how changes of the tax rate affect this economy. First, we consider the effects on economic growth. Let us denote g^{***} as the growth rate of product variety of the steady state with a high level of human capital h^{***} and λ^{***} as the old-age survival rate of the steady state with h^{***}. We obtain the following proposition[8].

Proposition 5 *If the tax rate τ increases, the level of human capital of the unstable steady state h^{**} decreases and that of the steady state with a low level of human capital h^* and a high level of human capital h^{***} increases.*

[8]Propositions 5 and 6 do not need to specify the functional form of $f(b_t)$.

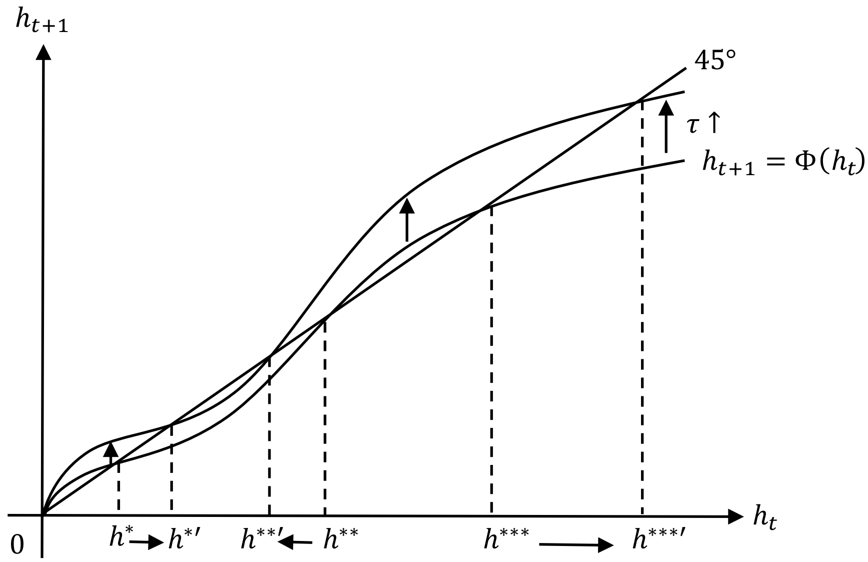

Figure 7: Effect of change of the tax rate on development

Proof. see Appendix D

Let us consider an economy whose initial per capita human capital h_0 satisfies $h^* < h_0 < h^{**}$. If the government does not implement any policy, this economy can converge to the steady state with low per capita human capital h^*. However, if the government sets a sufficiently high tax rate and implements a health-improving policy, investment in education for each child increases. Then, human capital accumulation is stimulated and this economy converges to the steady state with high per capita human capital h^{***}. In addition, if the government sets a higher tax rate, the steady-state level of human capital h^* and h^{***} increases.

We next obtain the following proposition.

Proposition 6 *Suppose* $\frac{\partial^2 \lambda^{***}}{\partial \tau^2} < 0$*, in which an increase of tax rate* τ *has an inverted U-shaped effect on the steady-state growth rate* g^{***}*.*

Proof. see Appendix E

As shown in Appendix E, if the tax rate τ increases, the steady state old-age survival rate λ^{***} increases. Therefore, individuals save more and R&D investment increases. Then, the growth rate increases. However, if the tax rate τ increases, the disposable income decreases. Therefore, individuals' saving volume decreases, R&D investment decreases, and the growth rate decreases. In addition, if $\frac{\partial^2 \lambda^{***}}{\partial \tau^2} < 0$ holds, the first positive effect on the growth rate decreases when τ increases.

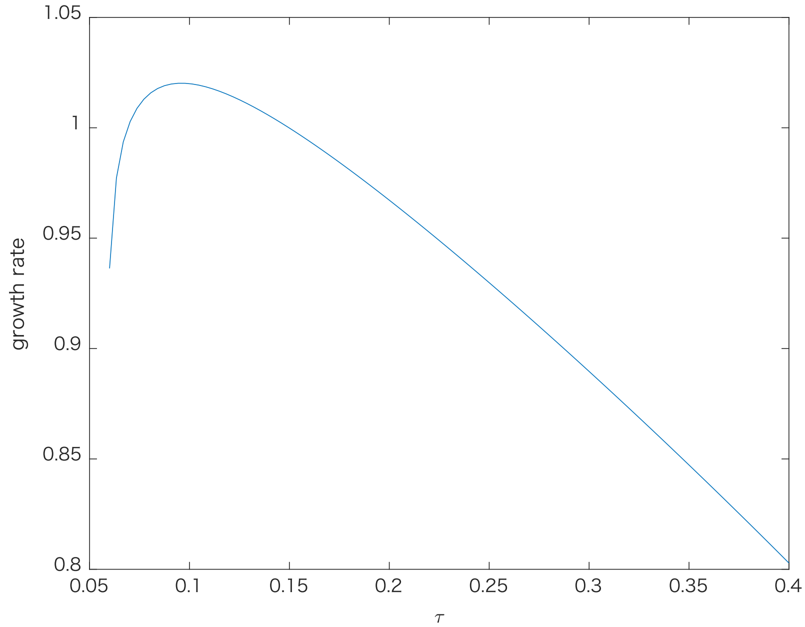

Figure 8: The inverted U-shaped effect of the tax rate on the steady-state growth rate. We specify $f(G_t) = \frac{\rho+\mu b_t}{\theta+b_t}$ and assume that $\beta = (0.98)^{25}$, $\delta = 1$, $\gamma = 0.14$, $\eta = 0.21$, $\theta = 5$, $\mu = 1$, $\rho = 1$, $\sigma_e = 0.635$, $\sigma_h = 0.165$, $\phi = 34.02$, and $z = 0.088$.

Hence, an increase of tax rate τ has an inverted U-shaped effect on the steady-state growth rate g^{***}. Figure 8 shows a numerical example of the inverted U-shaped effect of the tax rate on the steady-state growth rate. Considering $\frac{\partial \lambda^{***}}{\partial \tau} > 0$ and Proposition 5, there is an inverted U-shaped effect between λ^{***} and the steady-state growth rate g^{***}. Our simple regression supports this inverted U-shaped relationship[9]. More formal empirical evidence in support of this pattern can be found in An and Jeon (2006). Their regression using panel data from OECD countries over the period 1960–2000 also reflects an inverted U-shaped relationship between the old-age dependency ratio and economic growth.

Next, we calibrate the model to examine how the change of tax rate affects welfare. There are nine structural parameters $\{\alpha, \beta, \delta, \gamma, \eta, \sigma_e, \sigma_h, \phi, \psi, \nu, z\}$. We specify $f(G_t) = \frac{\rho+\mu b_t}{\theta+b_t}$ and normalize

[9] The World Development Indicators (World Bank 2019) are used to calculate the old-age dependency ratio and the per capita output growth rate. We use data on 157 countries for the period from 1991 to 2016 and estimate simple regressions, in which the average per capita output growth rate (Growth) as the dependent variable is a function of the average old-age dependency ratio (Old1) and the value of its square (Old2). The following equation provides simple estimation results using ordinary least squares:

$$Growth = -0.896(-1.134) + 0.535(4.032)Old1 - 0.017(-3.780)Old2,$$

where the figures in parentheses are the values of the t-statistics. The equation above suggests that there is an inverted U-shaped correlation between the old-age dependency ratio and the per capita GDP growth rate. Hashimoto and Tabata (2016) show a similar result.

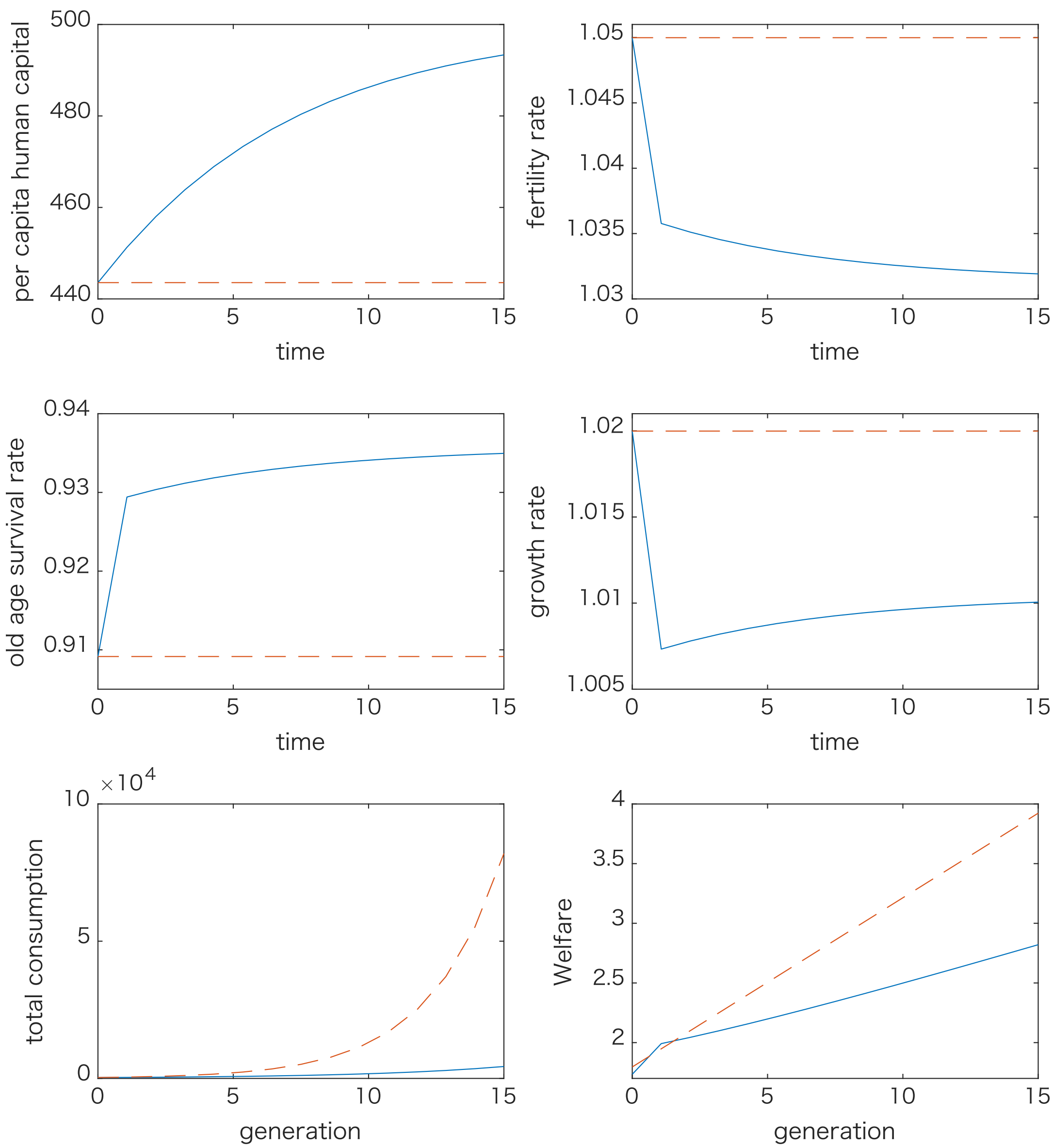

Figure 9: Response to change of tax rate. The dotted lines denote the steady-state value and the solid lines denote the tax change response. The t-th generation's total consumption is the sum of its consumption level for adult age and that for old age.

δ, μ, and ρ to 1. Following Futagami and Konishi (2017), we set the minimum value of λ_t to 0.2 (i.e., $\theta = 5$). Considering the range of the values estimated by Norrbin (1993) and Basu (1996), we set α at 1/1.05. We regard one period in this overlapping-generations economy as 25 years and set $\beta = (0.98)^{25}$. Following Strulik et al. (2013), we set $\gamma = 0.14$ and $z = 0.088$. We choose η to make the fertility rate 2.1/2 in the steady state with h^{***} (i.e., $\eta = 0.21$). Following de la Croix and Doepke (2003), we set $\sigma_e = 0.635$. Following Cardak (2004), we set $\sigma_e + \sigma_h = 0.8$ (i.e., $\sigma_h = 0.165$). The average old-age dependency ratio of OECD countries from 1991 to 2016 is 0.2076. Therefore, we choose ϕ to make the old-age dependency ratio 0.2067 in the steady state with h^{***} (i.e., $\phi = 34.02$)[10]. Following Jones and Williams (2000), we set $\nu = 0.5$. We choose the values of the R&D productivity parameter ψ to make the annual growth rate of output 2% in the steady state with h^{***} (i.e., $\psi = 2.69$). We set the initial population size $N_0 = 1$ and the initial economy size $A_0 = 1$.

Suppose that the economy is in the steady state with h^{***} at time 0 and the government raises the tax rate $\tau = 0.1$ to $\tau = 0.13$ at time 1. Figure 9 shows the levels of variables at the initial steady state and those at the transition pass after the tax change. From Figure 9, we find that if the government raises the tax rate, the welfare level of individuals of the first generation (i.e., people who become adults at time 1) increases from the steady-state level. However, after the second generation (i.e., people who become adults after time 2), the welfare levels decrease from the steady-state levels of individuals. Therefore, the government faces a trade-off between improving the welfare levels of the current versus future generations. Let us consider the reason for this conflict. From Lemma 1, the old-age survival rate increases if the government raises the tax rate. This increases the expected utility of adults of the t-th generation. However, from (12), the fertility rate decreases if the old-age survival rate increases. If the fertility rate decreases, the utility level of adults of the t-th generation decreases. Therefore, if this negative effect is sufficiently small, the welfare level of adults of the t-th generation increases. On the contrary, if the tax rate τ increases, the disposable income decrease. Therefore, individuals' savings decreases and R&D investment decreases. If R&D

[10]The old-age dependency ratio is the ratio of people older than 64 years to the working-age population of those aged 15–64 years. Noting that one period in this overlapping generations economy as 25 years, we calculate the old-age dependency ratio of the steady state with h^{***} by following equation:

$$\text{old age dependency ratio} = \frac{\frac{75-64}{25}\lambda^{***}}{\frac{25-15}{25}(n^{***})^2 + n^{***} + \frac{64-50}{25}\lambda^{***}},$$

where n^{***} and λ^{***} denote the fertility rate and old-age survival rate of the steady state with h^{***}, respectively.

investment decreases, the growth rate also decreases. Hence, individuals' total consumption levels increase more slowly over time owing to the low growth rate. Therefore, the welfare level gradually becomes lower from the steady-state level after the tax increases. Hence, if the government raises the tax rate, the welfare level of the current generation increases and the welfare levels of future generations decrease. The government tends to decide the tax rate by placing more weight on the current generation (i.e., adult agents) than on other generations, because children do not have voting rights and some old agents are dead at time t. However, such a decision tends to harm social welfare, because the welfare levels of future generations are sacrificed for the welfare gain of the current generation.

5 Conclusion

We constructed an overlapping-generations model of R&D-based growth with endogenous fertility, mortality, and education choice. We showed that multiple steady states can exist in this economy. One of them has a high level of human capital and the other has a low level. In the steady state with the high (low) level of human capital, there is a high (low) level of R&D activity, a low (high) fertility rate, and a high (low) old-age survival rate. In addition, we showed that the government can steer the economy away from a poverty trap trajectory by investing in public health. We also showed that an improvement in the government's public health policy has an inverted U-shaped effect on the growth rate at the steady state. We examined how the improved public health policy affects welfare and showed that the government faces a trade-off between improving the welfare levels of the current generation and future generations. In this study, we did not take account of the goods cost of child rearing and, for simplification, assumed that the public health service is produced without goods. However, these factors are worth analyzing in this framework. The expansion of the cost of child rearing and the provision of public health service is a promising direction for future research.

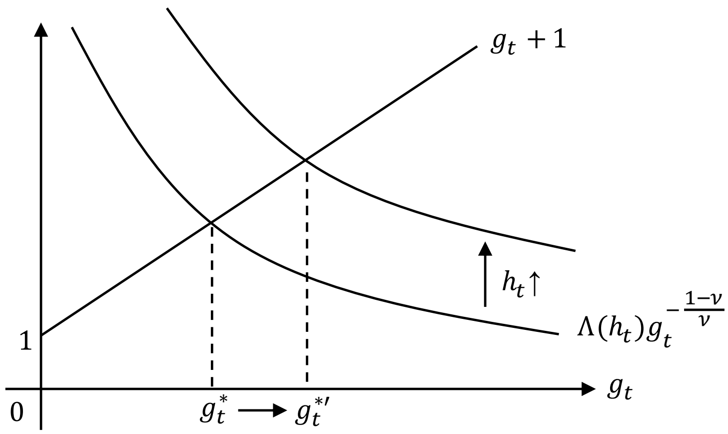

Figure 10: Determination of g_t and the effect of H_t on g_t

Appendix

Appendix A: Proof of that R&D is always undertaken

We show that R&D is always undertaken in this model. Suppose that R&D is not undertaken and $\Delta A_t = 0$. Then, (23) holds with strict inequality as follows:

$$v_t < \frac{w_t}{\psi_t A_t}. \tag{A1}$$

From (22), we have

$$g_t = \frac{\Delta A_t}{A_t} = \psi \left(\frac{l_{R,t}}{h_t N_t} \right)^{\nu}. \tag{A2}$$

From (A2), if $\Delta A_t = 0$, $l_{R,t} = 0$. By using (15), (19), (21), (27), and (29), we obtain the wage rate w_t as follows:

$$w_t = \frac{\alpha}{\left[\frac{(1+\beta\lambda_t)(1-\tau)}{1+\eta+\beta\lambda_t} - \left(\frac{g_t}{\psi} \right)^{\frac{1}{\nu}} \right] H_t}, \tag{A3}$$

where $H_t \equiv h_t N_t$ is aggregate human capital. From (A3), when $\Delta A_t = 0$ (i.e., $g_t = 0$), $w_t = \frac{\alpha(1+\eta+\beta\lambda_t^*)H_t}{(1+\beta+\beta\lambda_t^*)(1-\tau)}$. In addition, when $l_{R,t} = 0$, ψ_t approaches infinity. Therefore, the right-hand side of (A1) converges to 0 when $\Delta A_t = 0$. Meanwhile, when $\Delta A_t = 0$ (i.e., $l_{R,t} = 0$), we obtain the following equation from (28),

$$v_t = \frac{s_t N_t}{A_t} = \frac{\beta(1-\tau) w_t \lambda_t^* H_t}{(1+\eta+\beta\lambda_t^*)A_t} > 0. \tag{A4}$$

These results contradict (A1). Therefore, $\Delta A_t > 0$ always holds in this economy.

Appendix B: Proof of Proposition 3

We check the effect of h_t on g_t. From (32) and $\lambda_t^* = \lambda(h_t;\tau)$, we obtain

$$g_t + 1 = \underbrace{\frac{\beta\psi^{\frac{1}{\nu}}(1-\tau)\lambda(h_t;\tau)}{1+\eta+\beta\lambda(h_t;\tau)}}_{\equiv \Lambda(h_t)} g_t^{-\frac{1-\nu}{\nu}}. \tag{A5}$$

Let us define $\Lambda(h_t)$ as $\Lambda(h_t) \equiv \frac{\beta\psi^{\frac{1}{\nu}}(1-\tau)\lambda(h_t;\tau)}{1+\eta+\beta\lambda(h_t;\tau)}$. Figure 10 shows the relationship between the left-hand and right-hand sides of (A5). From Figure 10, we find that (A5) has a unique solution of g_t and we denote this solution as g_t. We obtain

$$\frac{\partial\lambda(h_t;\tau)}{\partial h_t} = \frac{\beta(1+\eta)\psi^{\frac{1}{\nu}}(1-\tau)}{(1+\eta+\beta\lambda_t^*)^2}\frac{\partial\lambda(h_t;\tau)}{\partial h_t} > 0. \tag{A6}$$

Therefore, when h_t increases, the line of $\Lambda(h_t;\tau) g_t^{-\frac{1-\nu}{\nu}}$ shifts upward and g_t increases, as shown in Figure 10.

Appendix C: Proof of Proposition 4

Suppose $f(b_t) = \frac{\rho+\mu b_t}{\theta+b_t}$ and from this equation, we obtain

$$f^{-1}(\lambda_t) = \frac{\theta\lambda_t - \rho}{\mu - \lambda_t}. \tag{A7}$$

Substituting (A7) into (31) and rearranging them, we obtain

$$h_t = \frac{(1+\eta+\beta\lambda_t^*)(\theta\lambda_t^* - \rho)}{(1+\beta\lambda_t^*)(\mu-\lambda_t^*)\delta\tau} \tag{A8}$$

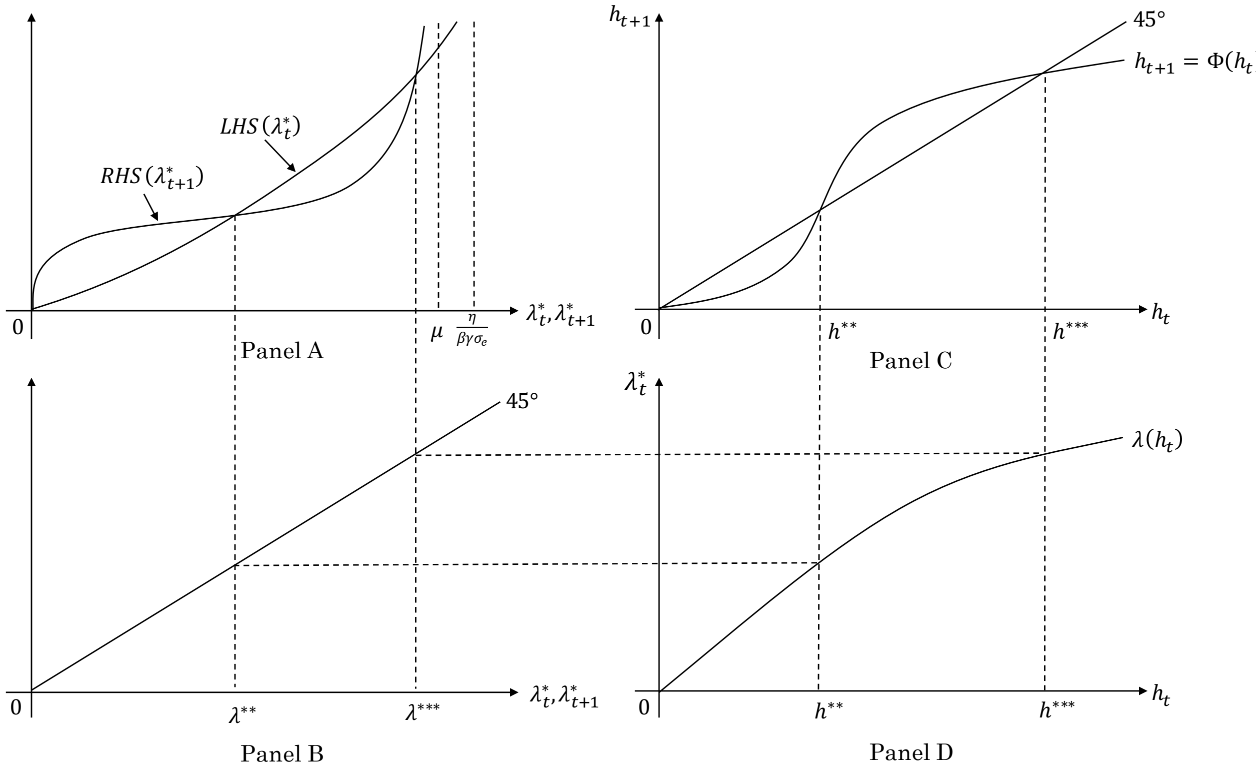

Figure 11: Derivation of the phase diagram of h_t

If h_t is given, λ_t^* is determined to satisfy (A8). From $\lambda_t^* = \lambda(h_t; \tau)$, (33), and (A8), we obtain the following inequalities

$$h_{t+1} \geq h_t,$$

$$\rightarrow \quad \underbrace{\frac{\beta\gamma\sigma_e \phi^{\frac{1}{\sigma_e}} z \lambda_t^*}{\eta - \beta\gamma\sigma_e\lambda_t^*}}_{\equiv LHS(\lambda_t^*)} \geq \underbrace{\left[\frac{(1+\eta+\beta\lambda_t^*)(\theta\lambda_t^* - \rho)}{(1+\beta\lambda_t^*)(\mu - \lambda_t^*)\delta\tau}\right]^{\frac{1-\sigma_e-\sigma_h}{\sigma_e}}}_{\equiv RHS(\lambda_t^*)}. \tag{A9}$$

Let us define the left-hand side of (A9) as $LHS(\lambda_t^*)$ and the right-hand side of (A9) as $RHS(\lambda_t^*)$. From (A9), we find that $LHS(0) = 0$, $RHS(\frac{\theta}{\rho}) = 0$, $\lim_{\lambda_t^* \to \frac{\eta}{\beta\gamma\sigma_e}} LHS(\lambda_t^*) = \infty$, and $\lim_{\lambda_t^* \to \mu} RHS(\lambda_t^*) = \infty$. Therefore, $\lim_{\lambda_t^* \to \mu} RHS(\lambda_t^*) > \lim_{\lambda_t^* \to \mu} LHS(\lambda_t^*)$, because $\mu \leq 1$ and $\eta > \gamma$. From (A9), we obtain the following:

$$\frac{\partial LHS(\lambda_t^*)}{\partial \lambda_t^*} = \frac{\beta\gamma\eta\sigma_e\phi^{\frac{1}{\sigma_e}}z}{(\eta-\beta\gamma\sigma_e\lambda_t^*)^2} \geq 0, \tag{A10}$$

$$\frac{\partial RHS(\lambda_t^*)}{\partial \lambda_t^*} = \frac{\theta[(1+\beta\lambda_t^*)^2\mu+\eta(\mu+\beta\lambda_t^{*2})]-\{1+(1-\mu)\eta+[2(1+\eta)+\beta\lambda_t^*]\lambda_t^*\}\rho}{\sigma_e(1+\eta+\beta\lambda_t^*)^2}$$
$$\times\,\delta(1-\sigma_e-\sigma_h)\tau(\theta\lambda_t^*-\rho)^{\frac{1-2\sigma_e-\sigma_h}{\sigma_e}}\left[\frac{1+\eta+\beta\lambda_t^*}{(1+\beta\lambda_t^*)(\mu-\lambda_t^*)\delta\tau}\right]^{\frac{1-\sigma_h}{\sigma_e}}. \tag{A11}$$

First, let us consider the case in which $\rho=0$ as the benchmark. When $\rho=0$, we obtain the following equation from (A11)

$$\left.\frac{\partial RHS(\lambda_t^*)}{\partial \lambda_t^*}\right|_{\rho=0} = \frac{[\mu(1+\beta\lambda_t^*)^2+\eta(\mu+\beta\lambda_t^{*2})]\delta(1-\sigma_e-\sigma_h)\tau}{\theta\sigma_e(1+\eta+\beta\lambda_t^*)^2}$$
$$\times\,(\lambda_t^*)^{\frac{1-2\sigma_e-\sigma_h}{\sigma_e}}\left[\frac{(1+\eta+\beta\lambda_t^*)\theta}{(1+\beta\lambda_t^*)(\mu-\lambda_t^*)\delta\tau}\right]^{\frac{1-\sigma_h}{\sigma_e}}. \tag{A11'}$$

From (A10), $\lim_{\lambda_t\to 0}\frac{\partial LHS(\lambda_t^*)}{\partial\lambda_t^*}=\frac{\beta\gamma\sigma_e\phi^{\frac{1}{\sigma_e}}}{\eta}<\infty$. Suppose that $\sigma_h\in(1-2\sigma_e,1-\sigma_e)$. Then, we find that $\left.\frac{\partial RHS(\lambda_t^*)}{\partial\lambda_t^*}\right|_{\rho=0}>0$, $\lim_{\lambda_t^*\to 0}\left.\frac{\partial RHS(\lambda_t^*)}{\partial\lambda_t^*}\right|_{\rho=0}=\infty$ and $\lim_{\lambda_t^*\to\mu}\left.\frac{\partial RHS(\lambda_t^*)}{\partial\lambda_t^*}\right|_{\rho=0}=\infty$ from (A11'). If ϕ increases, $\frac{\partial LHS(\lambda_t^*)}{\partial\lambda_t^*}$ increases. Therefore, there exists ϕ such that $LHS(\lambda_t^*)$ and $RHS(\lambda_t^*)|_{\rho=0}$ have their intersections at $\lambda_t^*=0,\lambda^{**},\lambda^{***}$, as drawn in Panel A of Figure 11. If $LHS(\lambda_t^*)$ and $RHS(\lambda_t^*)|_{\rho=0}$ have three intersections, there exist three steady states with respect to h_t, as shown in Panel C of Figure 11. Next, let us consider the case in which $\rho>0$. From (A9), we find that $RHS(\lambda_t^*)$ shifts left, as shown in Panel A of Figure 12, if ρ increases. From (A11), we find that $\frac{\partial RHS(\lambda_t^*)}{\partial\lambda_t^*}>0$ if ρ is not so high. We also find that $\lim_{\lambda_t^*\to\frac{\rho}{\theta}}\frac{\partial RHS(\lambda_t^*)}{\partial\lambda_t^*}=\infty$ and $\lim_{\lambda_t^*\to\mu}\frac{\partial RHS(\lambda_t^*)}{\partial\lambda_t^*}=\infty$ from (A11). From Figure 12, if $\rho>0$ and ρ is not so high, $LHS(\lambda_t^*)$ and $RHS(\lambda_t^*)$ have three intersections and those values are positive ($\lambda^{***}>\lambda^{**}>\lambda^*>0$). Then, if ρ is not so high, three steady states exist with respect to h_t and the values of these steady states are positive ($h^{***}>h^{**}>h^*>0$), as shown in Panel C of Figure 12.

In addition, if $h_{t+1}\geq h_t$ holds, $LHS(\lambda_t^*)\geq RHS(\lambda_t^*)$ holds from (A8). Therefore, we obtain Panel C of Figure 12, which is equivalent to Figure 6.

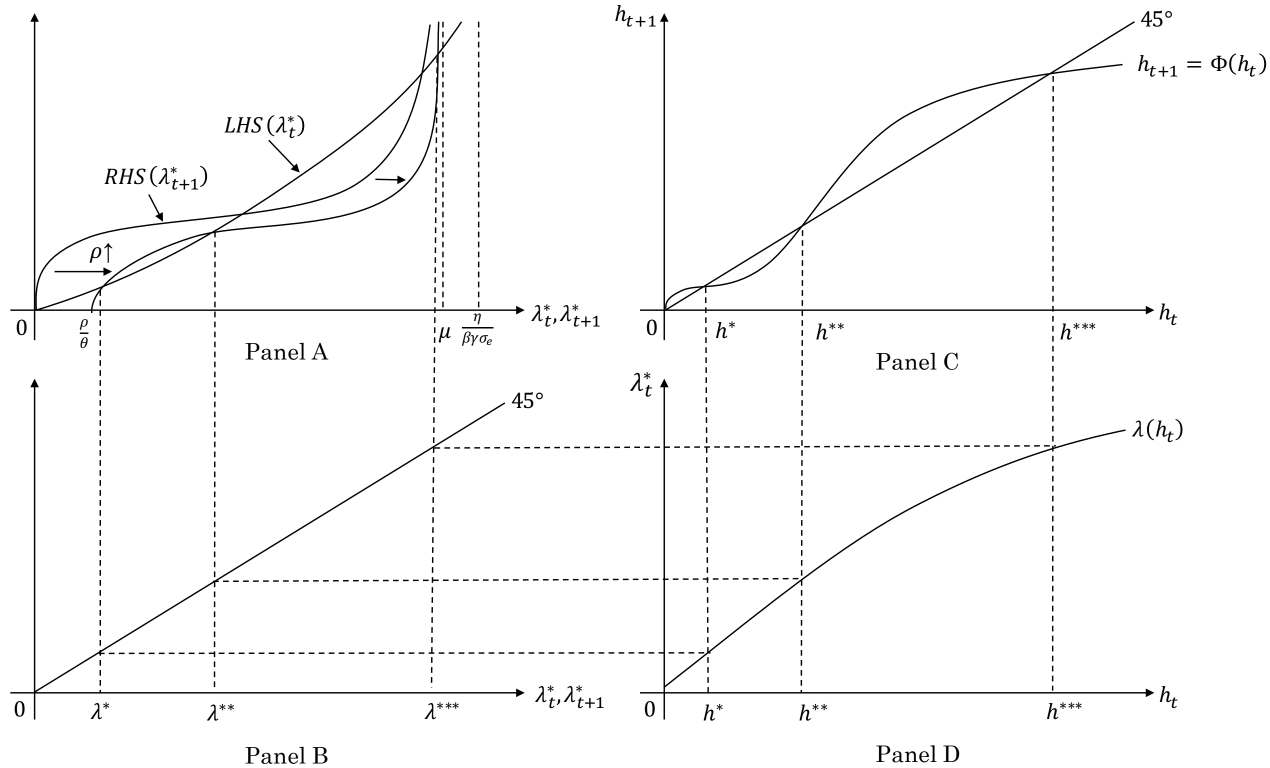

Figure 12: Derivation of Figure 4

Appendix D: Proof of Proposition 5

From Lemma 1, λ_t^* is a function of τ such that $\lambda_t^* = \lambda(h_t;\tau)$ and $\frac{\partial\lambda(h_t;\tau)}{\partial\tau} > 0$. From (33), we obtain

$$\frac{\partial\Phi(h_t)}{\partial\tau} = \phi\left\{\frac{\beta\gamma\sigma_e z\frac{\partial\lambda(h_t;\tau)}{\partial\tau}}{[\eta-\beta\gamma\sigma_e\lambda(h_t;\tau)]^2}\right\}^{\sigma_e} h_t^{\sigma_e+\sigma_h} > 0. \tag{A12}$$

From (A12), we find that the line of $\Phi(h_t)$ shifts upward in Figure 7 if τ increases. Then, h^{**} decreases and h^* and h^{***} increase when τ increases.

Appendix E: Proof of Proposition 6

From Lemma 1, λ_t^* is a function of τ such that $\lambda_t^* = \lambda(h_t;\tau)$ and $\frac{\partial\lambda(h_t;\tau)}{\partial\tau} > 0$. If τ increases, the line of $\Phi(h_t)$ shifts upward in Figure 7 and h^{***} increases. From Proposition 1, $\frac{\partial\lambda(h_t;\tau)}{\partial h_t} > 0$. Therefore,

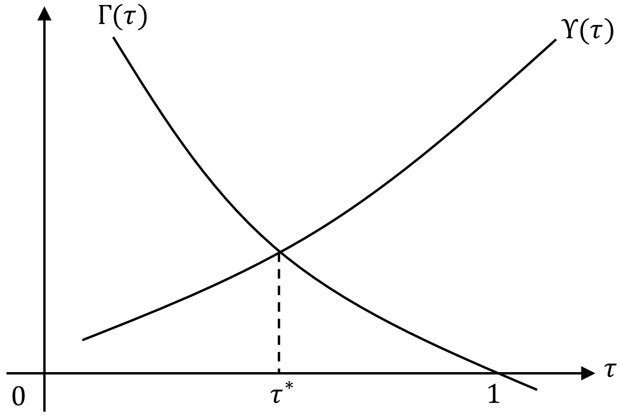

Figure 13: Determination of τ^*

$\frac{\partial \lambda^{***}}{\partial \tau} > 0$ holds. Totally differentiating (32) with respect to g_t and τ, we obtain

$$\frac{dg^{***}}{d\tau} = \frac{\beta \psi^{\frac{1}{\nu}} \nu g^{***} \left[(1+\eta)(1-\tau)\frac{\partial \lambda^{***}}{\partial \tau} - (1+\eta+\beta\lambda^{***})\lambda^{***}\right]}{(1+\eta+\beta\lambda^{***})\left[(1+\eta+\beta\lambda^{***}]\nu(g^{***})^{\frac{1}{\nu}} + \beta\psi^{\frac{1}{\nu}}(1-\nu)(1-\tau)\lambda^{***}\right]}. \tag{A13}$$

Let us define $\Gamma(\tau)$ as $\Gamma(\tau) \equiv (1+\eta)(1-\tau)\frac{\partial \lambda^{***}}{\partial \tau}$ and $\Upsilon(\tau)$ as $\Upsilon(\tau) \equiv (1+\eta+\beta\lambda^{***})\lambda^{***}$. If $\Gamma(\tau)$ is larger than $\Upsilon(\tau)$, $\frac{dg^{***}}{d\tau} > 0$ holds. Suppose $\frac{\partial^2 \lambda^{***}}{\partial \tau^2} < 0$, we can obtain

$$\frac{\partial \Gamma(\tau)}{\partial \tau} = -(1+\eta)\left[\frac{\partial \lambda^{***}}{\partial \tau} - (1-\tau)\frac{\partial^2 \lambda^{***}}{\partial \tau^2}\right] < 0, \tag{A14}$$

$$\frac{\partial \Upsilon(\tau)}{\partial \tau} = (1+\eta+2\beta\lambda^{***})\frac{\partial \lambda^{***}}{\partial \tau} > 0. \tag{A15}$$

From (A14) and (A15), we find the relationship between $\Gamma(\tau)$ and $\Upsilon(\tau)$ in Figure 13. From Figure 13, there is a unique intersection between $\Gamma(\tau)$ and $\Upsilon(\tau)$ and we denote the value of this intersection as τ^*. From (A13), if $\tau < \tau^*(\tau > \tau^*)$, $\frac{dg^{***}}{d\tau} > 0 \left(\frac{dg^{***}}{d\tau} < 0\right)$.

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