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Kohei Okada

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Graduate School of Economics
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

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Kohei Okada†

Abstract

Employing an overlapping-generations model of R&D-based growth with endogenous education decision-making and government's education policy, we examine how government's education policy and human capital accumulation influence R&D activity. We show that an increase in government's public education expenditure has an inverted U-shaped effect on the growth rate at the steady state. We examine how increased public education expenditure affects welfare and show that an increase in the public education expenditure has an inverted U-shaped effect on the steady state level of welfare.

Keywords: Education expenditure, Human capital accumulation, R&D

JEL Classification: H52, I25, J24, O30

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†Graduate School of Economics, Osaka University. Address: 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: koheiokada257@gmail.com

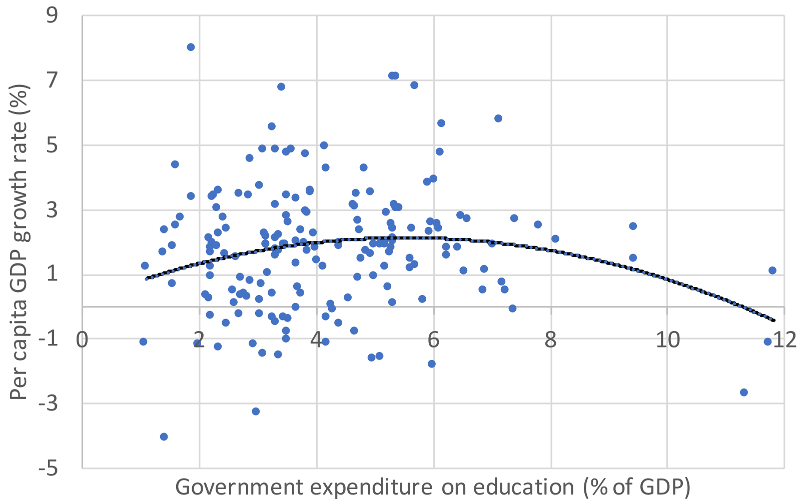

Figure 1: The hump-shaped relationship between per capita GDP growth rate and government expenditure on education. Source: World Bank. Cross-country data for the period from 1975 to 2005.

1 Introduction

Many countries have achieved economic development and a concomitant increase in the number of people who receive education. Throughout this process, governments have adopted various educational policies. At the same time, R&D activity has played a significant role in economic growth. Our analysis focusses on how government's education policy affects R&D activity and economic growth. From Figure 1, we find that there is a hump-shaped relationship between per capita GDP growth rate and government expenditure on education[1]. This indicates that an increase in government expenditure on education tends to increase the economic growth rate when the level of government expenditure on education is low. However, as government expenditure on education increases, it tends to reduce the economic growth rate.

To explain this phenomenon, we construct a simple overlapping-generations model of R&D-based growth, with endogenous education decision-making and education expenditure. We assume that

[1]The World Development Indicators (World Bank, 2019) are used to calculate government expenditure on education and per capita output growth rates. We use data on 180 countries for the period from 1975 to 2005 and estimate simple regressions, in which the average per capita output growth rate (Growth) is the dependent variable and is a function of government expenditure on education (Education1) and the value of its square (Education2). The following equation generates simple estimation results using ordinary least squares:

$$Growth = 0.162(0.249) + 0.714(2.783)Education1 - 0.065(-2.846)Education2,$$

where the figures in parentheses are the values of the t-statistics. The equation above suggests that there is a hump-shaped correlation between government expenditure on education and the per capita GDP growth rate.

each individual life comprises three periods. In the first period, individuals receive public education provided by government. In the second period, individuals invest in their own education, supply efficient units of labor, pay income tax, consume differentiated goods, and save any remaining income. In the final period, individuals retire and consume differentiated goods. In this model, there is a production sector and an R&D sector. In the production sector, a single firm produces a differentiated good by using effective labor. Similarly, in the R&D sector, R&D firms use the effective labor to invent new differentiated goods. The equilibrium dynamics of this economy are characterized by the level of human capital. We show that increasing the government's public education expenditure has an inverted U-shaped effect on the economic growth rate at the steady state. We also examine how increased public education expenditure affects welfare and show that increasing the government's public education expenditure has an inverted U-shaped effect on the welfare level at the steady state.

This study is related to studies on human capital accumulation and R&D-based growth, for instance, Grossmann (2007), Chu et al. (2016), Hashimoto and Tabata (2016), and Okada (2020). Grossmann (2007) examines how education expenditure affects R&D-based growth. However, he does not find a hump-shaped relationship between growth rate and education expenditure. Chu et al. (2016) find a hump-shaped relationship between growth rate and cultural preference for education. Hashimoto and Tabata (2016) find a hump-shaped relationship between growth rate and life expectancy. Okada (2020) find a hump-shaped relationship between growth rate and government's public health expenditure. However, Chu et al. (2016) and Okada (2020) do not consider education expenditure. Hashimoto and Tabata (2016) examine how education expenditure affects the growth rate. However, they do not find a hump-shaped relationship between growth rate and education expenditure.

The rest of this paper is organized as follows. Section 2 shows the basic structure of the model, Section 3 analyzes equilibrium and dynamics. Section 4 analyzes education policy. Finally, the conclusion is given in Section 5.

2 The model

2.1 Individuals

Time is discrete and denoted by $t = 0, 1, 2, \cdots$. Each individual lives for three periods (childhood, adulthood, and old age). In the first period (childhood), individuals do not make any decisions and receive public education. In the second period (adulthood), individuals invest in their own education, supply efficient units of labor, pay income tax, consume differentiated goods, and save any remaining income. In the final period (old age), individuals retire and consume differentiated goods. Members of the cohort born in period $t-1$ become active workers in period t. Thus, we call this cohort generation t. The population size is constant and normalized to 1. Individuals derive their utility from their own consumption during adulthood $C_{1,t}$, and their own consumption during old age $C_{2,t+1}$. The lifetime utility of individuals in generation t is expressed as

$$u_t = \log C_{1,t} + \beta \log C_{2,t+1}, \tag{1}$$

where the positive γ denotes the weights of the children's level of human capital. $\beta \in (0, 1]$ denotes the discount factor. We specify the subutility function $C_{k,t}$ for $k \in \{1, 2\}$ as

$$C_{k,t} \equiv \left[\int_0^{A_t} c_{k,t}(i)^{\alpha} di \right]^{\frac{1}{\alpha}}, \tag{2}$$

where $c_{k,t}(i)$ represents the consumption of differentiated good $i \in [0, A_t]$. A_t denotes the variety of differentiated goods or the level of technological knowledge in this economy, which grows through R&D. In individuals' second period of life (adulthood), they are endowed with one unit of time, which is devoted to working l_t in the labor market and investing in their own education e_t. Individuals divide their disposable income $(1-\tau)w_t h_t l_t$ between consumption and saving s_t for their old age. Here, w_t and $\tau \in [0, 1)$ are the wage rate for efficient units of labor, and the income tax rate, respectively. Thus, the budget and time constraints for individuals in generation t are expressed as follows:

$$E_{1,t} = (1-\tau)w_t h_t l_t - s_t, \tag{3}$$

$$E_{2,t+1} = R_{t+1}s_t, \tag{4}$$

$$E_{1,t} = \int_0^{A_t} p_t(i)c_{1,t}(i)di, \tag{5}$$

$$E_{2,t+1} = \int_0^{A_{t+1}} p_{t+1}(i)c_{2,t+1}(i)di, \tag{6}$$

$$l_t + e_t = 1, \tag{7}$$

where $E_{1,t}$, $E_{2,t+1}$, and $p(i)$ denote the expenditure of an individual in adulthood, the expenditure of an individual in old age, and the price of good i, respectively.

We assume that the human capital production function is given by the following expression:

$$h_{t+1} = \phi e_{t+1}^{\sigma} l_{E,t}^{1-\sigma}, \quad \phi > 0, \quad \sigma \in (0.1]. \tag{8}$$

where ϕ and σ are parameters, and. $l_{E,t}$ reflects the effective labor of teachers who teach in public school.

We next consider the individual's utility maximization. By maximizing the subutility function (2) subject to the budget constraint (5), we obtain the demand for differentiated good i as follows:

$$c_{k,t}(i) = \frac{p_t(i)^{-\epsilon}}{P_t^{1-\epsilon}} E_{k,t}, \tag{9}$$

where $\epsilon \equiv \frac{1}{1-\alpha}$ and P_t is the price index defined by $P_t \equiv \left[\int_0^{A_t} p_t(j)^{1-\epsilon}dj\right]^{\frac{1}{1-\epsilon}}$. This demand function implies that indirect utility becomes a linear function of expenditure as follows:

$$C_{k,t} = \frac{E_{k,t}}{P_t}. \tag{10}$$

Let us denote the total demand for good i as $x_t(i)$. $x_t(i)$ is given by

$$\begin{aligned} x_t(i) &= c_{1,t}(i)N_t + c_{2,t}(i)\lambda_{t-1}N_{t-1}, \\ &= \frac{p_t(i)^{-\epsilon}}{P_t^{1-\epsilon}}(E_{1,t}N_t + E_{2,t}\lambda_{t-1}N_{t-1}). \end{aligned} \tag{11}$$

By maximizing (1) subject to (3), (4), (7), (8), and (10), we obtain the following solution:

$$e_t = \frac{\sigma}{1+\sigma}, \tag{12}$$

$$l_t = \frac{1}{1+\sigma}, \tag{13}$$

$$s_t = \frac{\beta(1-\tau)w_t h_t}{(1+\beta)(1+\sigma)}. \tag{14}$$

2.2 Production

There are differentiated goods indicated by $i \in [0, A_t]$. A single firm produces each good. Each firm supplies a differentiated good monopolistically and sets its price. The monopoly is protected by perfect patent protection. Each monopolistic firm produces one unit of good by using one unit of effective labor. The producer of good i maximizes the following profit:

$$\pi_t(i) = p_t(i)x_t(i) - w_t x_t(i), \tag{15}$$

subject to the total demand function for good i (11). From the profit maximization condition, the price of good i is

$$p_t(i) = \frac{1}{\alpha}w_t \equiv p_t. \tag{16}$$

Hence, all goods have the same price. Thus, the firm-specific index i in the differentiated goods sector can be dropped. By substituting (16) into the demand function (11), we obtain the output level of the differentiated good:

$$x_t = \frac{p_t^{-\epsilon}}{\int_0^{A_t} p_t^{1-\epsilon}dj}(E_{1,t}N_t + E_{2,t}\lambda_{t-1}N_{t-1}) = \frac{E_{1,t}N_t + E_{2,t}\lambda_{t-1}N_{t-1}}{A_t p_t}. \tag{17}$$

The total expenditure is treated as a numeraire ($E_{1,t}N_t + E_{2,t}\lambda_{t-1}N_{t-1} = 1$). Therefore, we can rewrite (17) by using (16) as follows:

$$x_t = \frac{1}{A_t p_t} = \frac{\alpha}{w_t A_t}. \tag{18}$$

The profit of each differentiated good firm is given by

$$\pi_t = \left(\frac{1}{\alpha} - 1\right) w_t x_t. \tag{19}$$

2.3 R&D

R&D firms use the effective labor to invent new differentiated goods. After invention, the firms sell a blueprint of a new good to an entrepreneur who produces the differentiated good. Development of $A_{t+1} - A_t$ new blueprints requires $l_{R,t}$ units of effective labor input. Let us define ΔA_t as $\Delta A_t \equiv A_{t+1} - A_t$. Thus, given research productivity of ψ_t, output is expressed as follows:

$$\Delta A_t = \psi_t A_t l_{R,t}, \tag{20}$$

where A_t implies the spillover from general knowledge accumulated by past innovations. We specify productivity as $\psi_t \equiv \psi l_{R,t}^{\nu-1}$ where $\psi > 0$ and the parameter $\nu \in (0,1)$ inversely measures the negative duplication externality discussed in Jones (1995) and Jones and Williams (2000). In accordance with Jones (1995), the research productivity is given for each firm. $g_t \equiv \frac{\Delta A_t}{A_t}$ is the growth rate of product variety. Given $l_{R,t}$, the growth rate of product variety features decreasing returns to scale in $l_{R,t}$. The R&D firms' profits π_t^R are given by

$$\begin{aligned} \pi_t^R &= v_t \Delta A_t - w_t l_{R,t}, \\ &= \left(v_t - \frac{w_t}{\psi_t A_t}\right) \Delta A_t, \end{aligned} \tag{21}$$

where v_t is the price of a blueprint of a newly invented good. Because an entrepreneur pays the price of the blueprint of the new good to the R&D firm, this price corresponds to the price of the equity that is sold to the household. Free entry into R&D leads to the following zero-profit condition:

$$v_t \le \frac{w_t}{\psi_t A_t} \qquad \text{with equality if} \quad \Delta A_t > 0. \tag{22}$$

As shown in Appendix A, R&D is always undertaken and (22) holds with equality. The value of v_t equals the present value of future profits as follows:

$$v_t = \sum_{T=t+1}^{\infty} \frac{\pi_T}{\Pi_{\nu=t+1}^{T} R_\nu}.$$

After some manipulations, we obtain the following no-arbitrage condition:

$$R_{t+1} = \frac{v_{t+1} + \pi_{t+1}}{v_t}. \tag{23}$$

Each individual saves at the gross rate of interest R_{t+1} determined by (23).

2.4 Government

The government collects income tax from individuals and invests it in public education. We assume that the government employs teachers and they supply public education services. Let us define $l_{G,t}$ as the effective labor that is employed as teachers. Public education expenditure is financed by the government's balanced budget. Therefore, the government's budget constraint is

$$w_t l_{E,t} = \tau w_t h_t l_t. \tag{24}$$

3 Equilibrium

3.1 Market equilibrium

We first describe the equilibrium condition of the asset market. Individuals' savings must be directed to purchase either the newly issued stocks for R&D or the existing stocks of the firms operating that have been owned by the preceding generation. Therefore, the asset market equilibrium condition is

$$s_t = w_t l_{R,t} + v_t A_t. \tag{25}$$

The left-hand side of (25) is the savings of the adult individuals. The first term on the right-hand side is the total investment in newly issued stocks, which is equal to the total cost of R&D activities. The second term is the purchase of existing stocks. Note that the second term implicitly assumes that ex-dividend stocks are traded between adult and old generations in each period. In other words, the old generation always receives all dividends before selling the stocks.

We next describe the labor market equilibrium condition. Effective labor is used for production of differentiated goods, R&D, and public education services. The labor market-clearing condition is

$$h_t l_t = l_{R,t} + A_t x_t + l_{E,t}, \tag{26}$$

where $l_{R,t}$, $A_t x_t$, and $l_{E,t}$ denote the quantities of effective labor engaging in R&D, production activities, and the supply of public education services, respectively.

From (14), (20), (22), and (25), the growth rate of product variety g_t is determined by the following equation:

$$g_t + 1 = \frac{\beta \psi^{\frac{1}{\nu}} (1-\tau) h_t}{(1+\beta)(1+\sigma)} g_t^{-\frac{1-\nu}{\nu}}. \tag{27}$$

3.2 Dynamics

The dynamics of this economy is characterized by human capital h_t, and is given by

$$h_{t+1} = \frac{\sigma^\sigma \phi}{1+\sigma} (\tau h_t)^{1-\sigma} \equiv \Phi(h_t; \tau). \tag{28}$$

Let us define the right-hand side of (28) as $\Phi(h_t;\tau)$. Figure 2 shows the phase diagram of h_t. we obtain the following proposition.

Proposition 1 *The economy has a unique steady state.*

Proof: see Appendix B

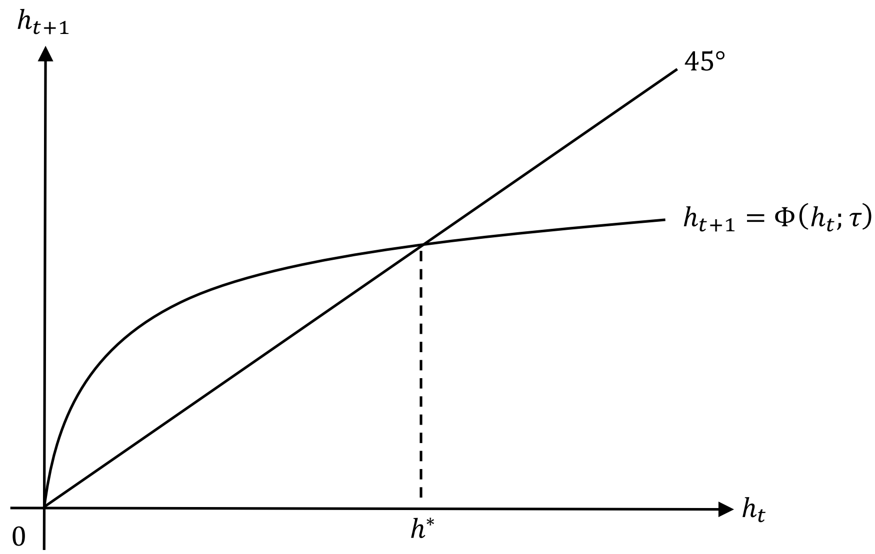

Figure 2: Dynamics of h_t

4 Education policy

In this section, we consider how government policy affects this economy. The government can set the income tax rate in this model. Therefore, we focus on how changes of the tax rate affect this economy. First, we consider the effects on economic growth. Let us denote g^* as the growth rate of product variety of the steady state. We obtain the following proposition.

Proposition 2 *An increase of tax rate τ has an inverted U-shaped effect on the steady state growth rate g^* and it is maximized at $\tau = 1 - \sigma$.*

Proof: see Appendix C

Let us define h^* as the level of human capital in the steady state. If τ increases, $\Phi(h_t, \tau)$ shifts upward in Figure 2 and h^* increases. Therefore, individuals save more and R&D investment increases. Then, the growth rate increases (the positive effect). However, if the tax rate τ increases, disposal income decreases. Individuals' savings consequently decrease, R&D investment decreases, and thus the growth rate decreases (the negative effect). In addition, we can find that $\frac{\partial^2 h^*}{\partial \tau^2} < 0$ holds. Therefore, the positive effect on the growth rate decreases when τ increases. Hence, an increase of tax rate τ has an inverted U-shaped effect on the steady-state growth rate g^*. Figure 3 shows a numerical example of the inverted U-shaped effect of the tax rate on the steady-state growth rate. We obtain the following proposition.

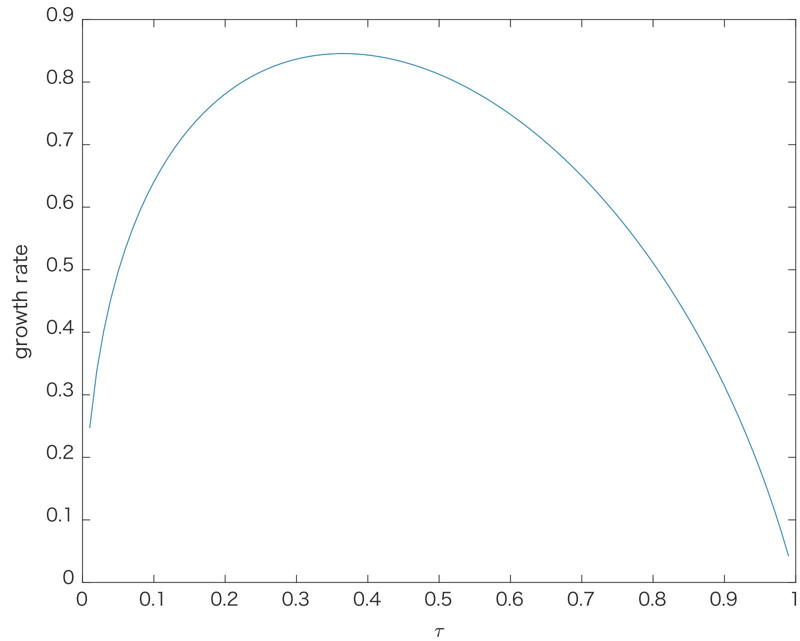

Figure 3: The inverted U-shaped effect of the tax rate on the steady-state growth rate. We assume that $\beta = (0.98)^{25}$, $\nu = 0.5$, $\sigma = 0.635$, $\phi = 1$, and $\psi = 4.9347$.

Proposition 3 *The steady state level of welfare is maximized at $\tau = 1 - \sigma$.*

Proof: see Appendix D

The government can maximize not only the steady state growth rate but also the steady state level of welfare by setting $\tau = 1 - \sigma$.[2]

5 Conclusion

We constructed an overlapping-generations model of R&D-based growth with endogenous education choice. We showed that increasing the government's public education expenditure has an inverted U-shaped effect on the growth rate at the steady state. We examined how increased public education spending affects welfare, and showed that increasing the government's public education expenditure has an inverted U-shaped effect on the steady state level of welfare.

[2] Barro (1990) shows that there is an inverted-U shaped relationship between growth rate and government expenditure for production. Our results are consistent with his. In contrast to Barro (1990), we focus on education policy and R&D-based growth.

Appendix

Appendix A: Proof that R&D is always undertaken

We show that R&D is always undertaken in this model. Suppose that R&D is not undertaken and $\Delta A_t = 0$. Then, (22) holds with strict inequality as follows:

$$v_t < \frac{w_t}{\psi_t A_t}. \tag{A1}$$

From (20), we have

$$g_t = \frac{\Delta A_t}{A_t} = \psi \left(\frac{l_{R,t}}{h_t N_t} \right)^{\nu}. \tag{A2}$$

From (A2), if $\Delta A_t = 0$, $l_{R,t} = 0$. By using (13), (18), (22), (24), and (26), we obtain the wage rate w_t as follows:

$$w_t = \frac{\alpha}{\frac{1-\tau}{1+\sigma} h_t - \left(\frac{g_t}{\psi} \right)^{\frac{1}{\nu}}}, \tag{A3}$$

From (A3), when $\Delta A_t = 0$ (i.e., $g_t = 0$), $w_t = \frac{\alpha(1+\sigma)}{(1-\tau)h_t}$. In addition, when $l_{R,t} = 0$, ψ_t approaches infinity. Therefore, the right-hand side of (A1) converges to 0 when $\Delta A_t = 0$. Meanwhile, when $\Delta A_t = 0$ (i.e., $l_{R,t} = 0$), we obtain the following equation from (14) and (28),

$$v_t = \frac{s_t}{A_t} = \frac{\beta(1-\tau) w_t h_t}{(1+\beta)(1+\sigma) A_t} > 0. \tag{A4}$$

These results contradict (A1). Therefore, $\Delta A_t > 0$ always holds in this economy.

Appendix B: Proof of Proposition 1

From (28), we obtain $\frac{\partial \Phi(h_t)}{\partial h_t} = \frac{\sigma^{\sigma}(1-\sigma)\phi\tau^{1-\sigma}}{1+\sigma} h_t^{-\sigma} > 0$, $\lim_{h_t \to 0} \frac{\partial \Phi(h_t)}{\partial h_t} = \infty$, $\lim_{h_t \to \infty} \frac{\partial \Phi(h_t)}{\partial h_t} = 0$, $\frac{\partial^2 \Phi(h_t)}{\partial h_t^2} = -\frac{\sigma^{\sigma+1}(1-\sigma)\phi}{\tau}^{1-\sigma} h_t^{-\sigma-1} < 0$. Therefore, the economy has a unique steady state as shown in Figure 2.

Appendix C: Proof of Proposition 2

From (28), we obtain

$$h^* = \left(\frac{\sigma^\sigma \phi}{1+\sigma}\right)^{\frac{1}{\sigma}} \tau^{\frac{1-\sigma}{\sigma}}. \tag{A5}$$

From (27), we obtain

$$g^* + 1 = \Omega(1-\tau)\tau^{\frac{1-\sigma}{\sigma}}(g^*)^{-\frac{1-\nu}{\nu}}, \tag{A6}$$

where $\Omega \equiv \frac{\beta\psi^{\frac{1}{\nu}}}{(1+\beta)(1+\sigma)}\left(\frac{\sigma^\sigma\phi}{1+\sigma}\right)^{\frac{1}{\sigma}}$. Totally differentiating (A6) with respect to g^* and τ, we obtain

$$\frac{dg^*}{d\tau} = \frac{\sigma\Omega\tau^{\frac{1-2\sigma}{\sigma}}(g^*)^{-\frac{1-\nu}{\nu}}(1-\sigma-\tau)}{1+\frac{(1-\nu)(1-\tau)\Omega}{\nu}(g^*)^{-\frac{1}{\nu}}} \tag{A7}$$

From (A7), if $\tau < 1-\sigma$ ($\tau > 1-\sigma$), $\frac{dg^*}{d\tau} > 0$ $\left(\frac{dg^*}{d\tau} < 0\right)$.

Appendix D: Proof of Proposition 3

From (3) and (14), we obtain

$$E_{1,t} = \frac{(1-\tau)w_t h_t}{(1+\beta)(1+\sigma)}. \tag{A8}$$

From (4) and (14), we obtain

$$E_{2,t} = R_{t+1}\frac{\beta(1-\tau)w_t h_t}{(1+\beta)(1+\sigma)}. \tag{A9}$$

By using (16), we rewrite $P_t \equiv \left[\int_0^{A_t} p_t(j)^{1-\epsilon}dj\right]^{\frac{1}{1-\epsilon}}$ as follows:

$$P_t = \frac{A_t^{\frac{1}{1-\epsilon}} w_t}{\alpha}. \tag{A10}$$

Combining (18) and (19), we obtain

$$\pi_t = \frac{1-\alpha}{A_t}. \tag{A11}$$

From (20) and $\psi_t \equiv \psi l_{R,t}^{\nu-1}$, we obtain

$$\psi_t = \psi^{\frac{1}{\nu}} g_t^{\frac{\nu-1}{\nu}}. \tag{A12}$$

Note that $h_{t+1} = h_t = h^*$ and $g_{t+1} = g_t = g^*$ hold in the steady state. Then, from (22), (A3), (A11) and (A12), we can rewrite (23) as follows:

$$\begin{aligned} R_{t+1} &= \frac{w_{t+1}\psi_t A_t}{w_t \psi_{t+1} A_{t+1}} + \frac{(1-\alpha)\psi_t A_t}{A_{t+1} w_t}, \\ \rightarrow \quad R^* &= (1+g^*)\left[1 + \frac{(1-\alpha)\psi^*}{w^*}\right], \end{aligned} \tag{A13}$$

where R^*, ψ^* and w^* denote steady state values of R_t, ψ_t and w_t, respectively. Combining (1), (10), (A5), (A8), (9), and (A13), we obtain the welfare level of t th generation u_t^* as follows:

$$u_t^* = \Lambda + (1+\beta)\log(1-\tau)\tau^{\frac{1-\sigma}{\sigma}} + \frac{\beta}{\alpha}\log(1+g^*) + \beta\log\left[1 + \frac{(1-\alpha)\psi^*}{w^*}\right] + \frac{\alpha(1+\beta)}{1-\alpha}\log A_t, \tag{A14}$$

where $\Lambda \equiv (1+\beta)\log\frac{\alpha}{(1+\beta)(1+\sigma)}\left(\frac{\sigma^\sigma \phi}{1+\sigma}\right)^{\frac{1}{\sigma}} + \beta\log\beta$. We can easily confirm that the second term on the right-hand side of (A14) is maximized at $\tau = 1-\sigma$. From Proposition 2, the third term on the right-hand side of (A14) is also maximized at $\tau = 1-\sigma$. Let us define $Z(\tau)$ as $Z(\tau) \equiv \frac{(1-\alpha)\psi^*}{w^*}$. Noting (A3), (A5), and (A12), we rewrite $Z(\tau)$ as follows:

$$Z(\tau) = \frac{1-\alpha}{\alpha\beta}(g^*)^{\frac{\nu-1}{\nu}}\left[(1+\beta)(1-\tau)\tau^{\frac{1-\sigma}{\sigma}}\Omega - (g^*)^{\frac{1}{\nu}}\right]. \tag{A15}$$

Differentiating $Z(\tau)$ with respect to τ, We obtain

$$\frac{\partial Z(\tau)}{\partial \tau} = \frac{(1-\alpha)(1+\beta)(1-\nu)(1-\tau)\left(1-\tau^{\frac{1-\sigma}{\sigma}}\right)\sigma\tau^{\frac{1-2\sigma}{\sigma}}\Omega^2(g^*)^{\frac{\nu-2}{\nu}}(1-\sigma-\tau)}{\alpha\beta^2\nu\left[1 + \frac{(1-\nu)(1-\tau)\Omega}{\nu}(g^*)^{-\frac{1}{\nu}}\right]}. \tag{A16}$$

From (A16), we can find that $Z(\tau)$ is maximized at $\tau = 1 - \sigma$. Therefore, the fourth term on the right-hand side of (A14) is maximized at $\tau = 1 - \sigma$. Given A_t, the welfare level of t th generation u_t^* is maximized at $\tau = 1 - \sigma$. From Proposition 2, for any time t, A_t is maximized at $\tau = 1 - \sigma$ in the steady state. Therefore, the steady state level of welfare is maximized at $\tau = 1 - \sigma$.

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