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Abstract

This paper theoretically examines how import tariffs by a developed country (the North) and a developing country (the South) affect innovation, foreign direct investment (FDI), wages, and welfare using a North–South quality ladder model. We show that a Northern import tariff raises the relative wage of Northern labor to Southern labor, but impedes innovation and FDI. Because of the decrease in innovation and increased prices, this may worsen Northern welfare. By contrast, a Southern import tariff raises the relative wage of Southern labor to Northern labor and promotes innovation and FDI. As a result, it can improve Southern welfare. These results imply that the North has a weaker incentive than the South to impose an import tariff, and this is consistent with actual experience.

Keywords: foreign direct investment, innovation, intellectual property rights protection

JEL classification: F43, O33, O34

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1 Introduction

Since the start of the Trump administration, the US government has largely adopted a protectionist policy concerning some aspects of international trade. As evidence, Fajgelbaum et al. (2020, p. 9) reported that in 2018, the US had increased average import tariffs on about $247 billion worth of Chinese products from 3.0% to 15.5%. They also showed that, in the same year, China had increased the average tariff on about $93 billion worth of US exports from 8.4% to 18.9% in retaliation. Regarding an explanation for the initial US action, according to a presidential memorandum signed on March 22, 2018, the increased tariffs by the USA targeting China were at least partly in response to allegedly insufficient protection by China of the intellectual property of US companies. The question then naturally arises as to what effect increased tariffs by a developed country such as the USA (hereafter, the North) and a developing country such as China (hereafter, the South) have on innovation and international technology transfer, given that intellectual property rights (IPR) are not well protected in the South.

In this paper, we theoretically investigate the effects of import tariffs on innovation, foreign direct investment (FDI) from the North to the South, wages, and welfare using a North–South growth model based on the quality ladder-type product-cycle model first developed by Grossman and Helpman (1991). In our model, a higher-quality product invented in the North replaces the current product if innovation occurs through successful R&D. The Northern inventor can then choose to produce in the North or shift production to the South through FDI to employ its lower-wage labor. However, some of these goods produced in the South are subject to imitation because of its imperfect protection of IPR. Unlike Grossman and Helpman (1991), we also assume that each government imposes an ad valorem tariff on imported goods.

We provide the following three main results. First, a unilateral increase in the Northern tariff impedes innovation and FDI, although it also raises the relative wage of Northern to Southern labor. In the Northern market, the higher Northern tariff raises the duty-inclusive price of the goods produced in the South. For Northern firms, this reduces the competition pressure from Southern firms, so that they can enjoy higher profits through raising the price in the North. On the other hand, the Northern tariff also reduces the Northern demand for the goods produced by FDI firms in the South through raising the duty-inclusive price. As a result, the higher Northern tariff decreases the profits of the FDI firms. Through both these effects, an increased Northern tariff depresses FDI from the North to the South. In addition, it raises the Northern relative wage because more firms choose to produce in the North than in the South. The higher Northern wage also leads to a higher cost of R&D in the North, and thus, an increased Northern tariff impedes innovation.

Second, contrary to the effects of a Northern tariff, a unilateral increase in the Southern tariff raises the Southern relative wage and promotes innovation and FDI. When the Southern tariff increases, Northern firms need to lower the pre-tariff price of the goods sold in the South to gain the Southern market. This reduces the profits of Northern firms, such that a higher Southern tariff promotes FDI in the South. The increased production transfer from the North to the South reduces the Northern relative wage and thereby the cost of R&D in the North. Accordingly, the higher Southern tariff promotes innovation.

These two results imply that simultaneous tariff increases by the North and South decrease innovation and FDI if the impact of the Northern tariff increase surpasses that of the Southern tariff. We show that this is the case when the population is relatively large in the North and small in the South. Conversely, if the relative population size of the North compared with the South is sufficiently small, the symmetric tariff rate between the two countries that maximizes innovation and FDI is not zero, but strictly positive.

Third, we also show the effects of a unilateral tariff increase on the country's own welfare as follows. A unilateral tariff increase by the South improves the welfare of the South if the initial tariff rate is lower than a certain positive level. This is because an increase in the Southern tariff promotes innovation and increases cheaper imitated goods through increasing production transfer to the South. This result implies that the welfare-maximizing tariff rate for the South is strictly positive. By contrast, a unilateral tariff increase by the North tends to worsen the welfare of the North if the Southern tariff is lower and the imitation rate in the South is higher. In fact, a tariff increase in the North exerts a positive effect on the North's welfare because it shifts production from the South to the North and raises the Northern wage. However, if the Southern tariff is low and imitation is active in the South, production in the South tends to be small because fewer Northern firms choose to undertake FDI. As a result, a lower Southern tariff and active imitation weakens the positive effect of a Northern tariff increase on the Northern wage through the production shift to the North. Thus, the total welfare effects of the Northern tariff tend to be negative in this case because the negative effects from the decreased innovation and increased prices tend to dominate the positive effect because of the increased income of the North.

The effects of trade cost have already been examined in a number of studies using quality ladder-type growth models.[1] Seminal studies include Dinopoulos and Segerstrom (1999a, 1999b) and Dinopoulos and Syropoulos (1997). However, both of these studies assumed two structurally identical countries,

[1] Using expanding variety as opposed to quality ladder-type growth models, Baldwin and Robert-Nicoud (2008), Dinopoulos and Unel (2011), Gustafsson and Segerstrom (2010), Naito (2017), and Sampson (2016) also investigate the effects of trade costs.

that is, a North–North setting. By contrast, we focus on the trade policy in two asymmetric countries: a North that is innovative and a South that is the recipient of production transfer. Therefore, the analyses are complementary. In addition, as summarized above, our results show that a tariff increase by a country has differing effects depending on whether the country is the North or the South, which is not possible in a North–North setting. Dinopoulos and Segerstrom (2007) constructed a North–South model where a Southern firm can produce some state-of-the-art good through imitation, and considered how decreasing trade cost affects both innovation and imitation. However, they assumed the trade cost to be symmetric between both countries and thus, did not analyze either the case of asymmetric trade costs or the effect of a unilateral tariff increase by just one country. By contrast, our model allows unequal tariff rates between the North and the South so that we can readily investigate the effect of a unilateral tariff increase. Grieben and Şener (2009) also examined the effects of Northern and Southern unilateral tariff reduction in a North–South product-cycle model. However, they assumed imitation by Southern firms to be the only channel for technology diffusion from the North to the South and did not consider FDI, much like Dinopoulos and Segerstrom (2007).

Unlike these studies, we incorporate production transfer by Northern firms into the model. As pointed out by Keller (2004), FDI is one of the major channels for technology diffusion across countries. Moreover, with so-called "tariff-jumping FDI", tariffs are likely to affect the incentive for FDI and thus labor demand.[2] Our model captures this tariff effect not considered in Dinopoulos and Segerstrom (2007) and Grieben and Şener (2009). In addition, unlike either of these previous studies, we examine the welfare effects of a unilateral tariff increase.[3] To date, many theoretical studies, including Kennan and Riezman (1988) and Syropoulos (2002), have concluded that the optimal tariff for a large country tends to be high. However, as discussed by Naito (2019), in practice, we frequently observe the opposite: economically larger countries tend to set lower tariffs. As discussed, our model shows that the Northern tariff worsens the welfare of the North under a certain condition, while the Southern tariff improves the welfare of the South if the initial tariff rate is set sufficiently low. Therefore, the results of the welfare analysis in this paper are consistent with the actual tendencies shown between country size and the optimal tariff.

The remainder of the paper is structured as follows. Section 2 develops the North–South quality

[2] For example, Belderbos and Sleuwaegen (1998), Chen and Moore (2010), and Ghodsi (2020) empirically showed that tariffs imposed by a host country exert a significantly positive effect on FDI to that country, which is consistent with the tariff-jumping motive.

[3] A number of studies have examined the welfare effects of changes in unilateral tariffs in a two-country general equilibrium model, e.g., Gros (1987), Opp (2010), Felbermayr, Jung, and Larch (2013). However, these employ static as opposed to growth models.

ladder model with tariff and Section 3 derives the market equilibrium path. Section 4 presents the comparative statics and Section 5 provides the welfare analysis. Section 6 concludes.

2 The model

We introduce FDI into the two-country quality ladder model developed by Grossman and Helpman (1991). Consider an economy consisting of two countries, the North and the South, denoted N and S, respectively. We assume perfect capital mobility between the countries so that their interest rates equalize. With a fixed number (measure) of identical households, let $L_i(0)$ denote the number of households in country $i \in \{N, S\}$. A member of each household supplies one unit of labor inelastically at each time point. Assume that the member size of each household is unity at the initial time and grows at a constant rate $g_L(\geq 0)$, such that the quantity of labor supplied in country i at time t is given by $L_i(0)e^{g_L t} \equiv L_i(t)$. We select Southern labor as the numeraire and normalize the Southern wage to be unity at every time point.

There is a continuum of goods, indexed by $\omega \in [0, 1]$, produced in the North or the South. One unit of good output requires one unit of labor input. Each good is classified by a countable infinite number of "generations" $j = 0, 1, 2, \cdots$. We normalize the generation number at the initial time to be zero for all goods. If innovation occurs in industry ω, a one-step new generation of good ω is developed. Therefore, generation j of good ω can be produced after the jth innovation in industry ω. As described in Section 2.3, innovation occurs because of successful R&D efforts by a Northern firm. Different generations of a good possess different "qualities". The quality of generation j of good ω is $q(j, \omega) = \lambda^j$, where the rate of quality increase between any two consecutive generations, $\lambda\ (> 1)$, is identical for all goods.

We assume that the government of each country imposes an ad valorem tariff on imported goods. The tariff rate of country i is $\tau_i\ (\geq 0)$, which is common to all imported goods. The government transfers all of the tariff revenues to the households of its country as a lump sum and runs a balanced budget at each time point.

2.1 Households

Each household in country i maximizes the following lifetime utility:

$$U_i = \int_0^\infty e^{-(\rho - g_L)t} \log u_i(t) dt,$$

where ρ $(> g_L)$ is a common subjective discount rate and $\log u_i(t)$ represents instantaneous utility at time t. We specify the instantaneous utility function as:

$$\log u_i(t) = \int_0^1 \log \left[\sum_j q(j,\omega) d_i(j,\omega,t) \right] d\omega, \tag{1}$$

where $d_i(j,\omega,t)$ denotes the per capita consumption of good ω of generation j at time t. The intertemporal budget constraint of each household in country i is given by

$$\int_0^\infty e^{-\int_0^t r(s)ds + g_L t} E_i(t) dt = A_i(0) + \int_0^\infty e^{-\int_0^t r(s)ds + g_L t} w_i(t) dt + \int_0^\infty e^{-\int_0^t r(s)ds + g_L t} T_i(t) dt, \tag{2}$$

where $r(t)$ is the interest rate, $E_i(t)$ and $A_i(0)$ denote consumption expenditure per capita and initial asset holdings per capita, respectively, $w_i(t)$ and $T_i(t)$ denote wages and a lump-sum transfer per capita by the government, respectively.

We solve this utility maximization problem in two stages. First, for each product, a household chooses a single generation $J(\omega,t)$ that carries the lowest quality-adjusted price $p(j,\omega,t)/q(j,\omega)$. This implies the following static demand function:

$$d_i(j,\omega,t) = \begin{cases} E_i(t)/p(j,\omega,t) & \text{for } j = J(\omega,t), \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Second, the household chooses a time pattern of expenditure to maximize its lifetime utility. Such intertemporal utility maximization requires that

$$\frac{\dot{E}_i(t)}{E_i(t)} = r(t) - \rho. \tag{4}$$

2.2 Production

The firm that developed the current latest generation of good ω (hereafter, the "leader" firm in industry ω) can produce it monopolistically under IPR protection if the firm chooses to operate in the North. A leader firm can become a "multinational" firm by shifting production to the South. Following Lai (1998), Glass and Wu (2007), and Tanaka and Iwaisako (2014), we assume that a Northern firm can transfer production to the South instantaneously without cost. The multinational firm enjoys a lower labor cost for production, but faces the risk of imitation because the South does not sufficiently enforce IPR protection. If good ω is imitated at time t, the leader firm in industry ω cannot earn profits for time t because perfect competition with copied goods prevails in the industry at that time. For simplicity, we assume that whether a good is under imitation is determined independently at each time point. More specifically, if a leader firm chooses to produce a good in the South, Southern firms imitate that good at some constant probability $m \in [0, 1)$ at any time point.[4] Therefore, under the law of large numbers, $m \times 100$ percent of all multinational firms are imitated at any time point. We interpret this imitation probability m as the degree of IPR protection; higher m implies weaker IPR enforcement in the South.

Next, we consider how the price and quantity supplied of each good are determined. Each good is produced by either (i) the leader firm in the North monopolistically; (ii) the multinational firm in the South monopolistically; or (iii) imitators in the South under perfect competition.[5] From the demand function (3), a leader firm can maximize profits by selling at the upper limit of the price such that rival firms that could produce an old generation in the same product line cannot operate. For a leader firm, the optimal price in the Northern and Southern markets can be different because of tariffs.

2.2.1 Northern firms

First, we consider what level of price a leader firm producing in the North charges in the Northern market. We assume that any firm can freely produce generations older than the currently latest in each product line because of expired patents.[6] In this case, leader firms do not undertake R&D because they cannot take more than a one-step quality lead over the nearest follower firms in the same product line. Thus, the potential strongest rivals for a Northern leader firm are the follower firms that can produce the current second to newest generation of the same good. A follower firm could cut the (pre-tariff) price down to its marginal cost, which would be $w_N(t)$ if produced in the North and $w_S(t) = 1$ if produced in the South. This implies that the lowest possible duty-inclusive price of a follower's good imported from the South to the North is $1 + \tau_N$. Therefore, the optimal price for a Northern leader firm in the Northern market is $\lambda \min\{w_N(t), 1 + \tau_N\}$ because it needs to set the lowest quality-adjusted price to sell the good.

In this paper, we focus on the case where the tariff rate in the North is low enough to satisfy $1+\tau_N \leq w_N(t)$ (see footnote 5). Under this assumption, a Northern leader firm prices its good at $p_{NN}(t) = \lambda(1 + \tau_N)$ in the North. The demand for a Northern leader's good by Northern consumers is

$$x_{NN}(t) = \frac{E_N(t)L_N(t)}{\lambda(1 + \tau_N)}. \tag{5}$$

[4] Grossman and Lai (2004) adopt a similar assumption.

[5] We focus on the case where a leader firm does not divide the location of production between the North and the South. In equilibrium, this is satisfied if and only if $\tau_N < [w_N(t) - 1]/\lambda$. The proof is provided in Appendix C available from the authors upon request.

[6] Even without this assumption, leader firms do not undertake R&D, and our results do not change at all if $w_N(t) < (1-m)\lambda$ and there is no advantage over follower firms in the R&D process. The proof is provided in Appendix C, which is available from the authors upon request.

The Northern leader's profits from the sale in the North are given by

$$\pi_{NN}(t) = [\lambda(1+\tau_N) - w_N(t)]\,\frac{E_N(t)L_N(t)}{\lambda(1+\tau_N)}. \tag{6}$$

In a similar way, we derive the price a leader firm producing in the North charges in the Southern market. As we focus on the case where the Northern wage is not lower than the Southern wage, a follower firm could set a lower (duty-inclusive) price in the Southern market when it produced in the South than when it produced in the North. To set the lowest quality-adjusted price in the Southern market, a Northern leader firm needs to choose a *duty-inclusive* price not higher than λ because the marginal cost of the follower firm that produced in the South would be equal to $w_S(t) = 1$. Therefore, the optimal *pre-tariff* price that a Northern leader firm charges Southern consumers is $p_{NS}(t) = \lambda/(1+\tau_S)$. As the duty-inclusive price is λ, the demand for a Northern leader's good by Southern consumers is

$$x_{NS}(t) = \frac{E_S(t)L_S(t)}{\lambda}. \tag{7}$$

The Northern leader's profits from the sale in the South are given by[7]

$$\pi_{NS}(t) = \left[\frac{\lambda}{1+\tau_S} - w_N(t)\right]\frac{E_S(t)L_S(t)}{\lambda}. \tag{8}$$

Adding (6) and (8), we obtain the total profits of a Northern leader firm as follows:

$$\pi_N(t) = \left[1 - \frac{w_N(t)}{\lambda(1+\tau_N)}\right]E_N(t)L_N(t) + \left[\frac{1}{1+\tau_S} - \frac{w_N(t)}{\lambda}\right]E_S(t)L_S(t). \tag{9}$$

2.2.2 Multinationals

Next, we consider what level of price a multinational firm charges Northern consumers. For the same reason as in the decision by a Northern leader firm, a multinational firm needs to choose a *duty-inclusive* price not higher than $\lambda \min\{w_N(t), 1+\tau_N\}$ in the North to set the lowest quality-adjusted price. Under the assumption that $1+\tau_N \leq w_N(t)$, the optimal *pre-tariff* price in the North is $p_{FN}(t) = \lambda$ for a multinational firm. Because the duty-inclusive price is $\lambda(1+\tau_N)$, the demand for an unimitated multinational's good by Northern consumers is

$$x_{FN}(t) = \frac{E_N(t)L_N(t)}{\lambda(1+\tau_N)}. \tag{10}$$

The unimitated multinational's profits from sale in the North are given by

$$\pi_{FN}(t) = (\lambda - 1)\,\frac{E_N(t)L_N(t)}{\lambda(1+\tau_N)}. \tag{11}$$

[7] If $1+\tau_S > \lambda/w_N(t)$, a Northern leader firm could not earn positive profits by selling the good in the South. In this case, no Northern leader firm would supply the good in the South, that is, $x_{NS}(t) = 0$. However, we rule out this case.

We derive the price a multinational firm charges Southern consumers in a similar way. Just as we considered the pricing of a Northern leader firm, a follower firm could set a lower (duty-inclusive) price in the South when it produces in the South. To set the lowest quality-adjusted price, a multinational firm needs to choose a price not higher than λ in the South because the marginal cost would be $w_S(t) = 1$ if a follower firm produced in the South. Therefore, the optimal price in the Southern market is $p_{FS}(t) = \lambda$ for a multinational firm. The demand for an unimitated multinational's good by Southern consumers is

$$x_{FS}(t) = \frac{E_S(t)L_S(t)}{\lambda}. \tag{12}$$

The unimitated multinational's profits from sale in the South are given by

$$\pi_{FS}(t) = (\lambda - 1)\,\frac{E_S(t)L_S(t)}{\lambda}. \tag{13}$$

Consequently, from (11) and (13), the total profits of a multinational firm are

$$\pi_F(t) = \left(1 - \frac{1}{\lambda}\right)\left[\frac{E_N(t)L_N(t)}{1+\tau_N} + E_S(t)L_S(t)\right]. \tag{14}$$

2.2.3 Imitated goods

If a good is imitated at time t, any firm in the South can produce and export the latest-generation good at that time. In the Southern market, the price of the good falls to $w_S(t) = 1$, which is equal to the marginal cost of imitators. The demand for an imitated good by Southern consumers then becomes

$$x_{MS}(t) = E_S(t)L_S(t). \tag{15}$$

In the Northern market, an imitated good is imported from the South and sold at $1 + \tau_N$ after tariff.[8] Therefore, the demand for an imitated good by Northern consumers is

$$x_{MN}(t) = \frac{E_N(t)L_N(t)}{1+\tau_N}. \tag{16}$$

2.3 R&D and FDI

Following Grossman and Helpman (1991), we assume an R&D process as follows: if a firm devotes $a_N X(t)\tilde{I}$ units of Northern labor for a time interval of length dt to research good ω, it succeeds in developing the next generation of good ω with probability $\tilde{I}dt$, where a_N is a parameter and $X(t)$ represents the difficulty of R&D. As in Dinopoulos and Segerstrom (1999a), Dinopoulos and Thompson

[8] Under the assumption that $1 + \tau_N \leq w_N(t)$, a leader firm whose good is imitated cannot earn profits, irrespective of whether it is produced in the North or the South.

(2000), Şener and Zhao (2009), and others, we assume that the growth rate of $X(t)$ is equal to the growth rate of the total labor supply, g_L, so that the model is free from the scale effect.[9] For a finite size of R&D activities in equilibrium, the expected gain from R&D must not exceed the cost of R&D. Thus, letting $v_N(t)$ denote the stock market value of a Northern leader firm, we have:

$$v_N(t) \leq w_N(t) a_N X(t) \quad \text{with equality if } \; I(t) > 0, \tag{17}$$

where $I(t)$ denotes the innovation rate at time t, which is assumed to be the same in every industry in the symmetric equilibrium.

Once a Northern firm succeeds in inventing a new-generation good, it can become a multinational firm by shifting production to the South without cost. Therefore, as long as both Northern leaders and multinational firms exist in equilibrium, the market values of a Northern leader and a multinational firm must be equal; that is, the following equality must hold at each time point:

$$v_N(t) = v_F(t), \tag{18}$$

where $v_F(t)$ denotes the stock market value of a multinational firm.

Next, we consider the no-arbitrage conditions between the stocks of a leader firm and the risk-free asset. Assuming that the shareholders of a firm hold a well-diversified portfolio, the expected return from the stocks of a leader firm must be equal to the return from the risk-free asset. The shareholders of a Northern leader firm then earn dividends $\pi_N(t)dt$ and capital gains $\dot{v}_N(t)dt$ over a time interval of length dt. At the same time, the Northern leader firm loses its monopolistic rent through the development of a new generation of the same good by another firm at the innovation rate $I(t)$ over the time interval. Thus, the shareholders are faced with a capital loss of $v_N(t)$ with probability $I(t)dt$. These imply that the no-arbitrage condition with respect to the stocks of a Northern leader firm is[10]

$$r(t)v_N(t) = \pi_N(t) + \dot{v}_N(t) - I(t)v_N(t). \tag{19}$$

A multinational firm earns profits $\pi_F(t)$ if its good is not produced by imitators at time t. This event occurs with probability $1 - m$ at each time point. Meanwhile, because of imitation, the multinational firm cannot earn any profits at time t with probability m. Thus, the expected dividends that shareholders of a multinational firm obtain at time t is $(1 - m)\pi_F(t)$. In addition, over a time interval of length dt,

[9] Our model can thus be interpreted as an extension of "first-generation" fully endogenous growth models such as Grossman and Helpman (1991) because it becomes one if we assume $X(t) = 1$ and $g_L = 0$ for all t.

[10] If a Northern leader firm transfers production to the South and becomes a multinational firm, it can obtain the value $v_F(t) - v_N(t)$. However, from (18), this is zero in equilibrium.

the shareholders obtain capital gains $\dot{v}_F(t)dt$, and are faced with a capital loss of $v_F(t)$ with probability $I(t)dt$ given the loss of monopolistic rent through the development of a new generation of the same good by another firm. Thus, the no-arbitrage condition between the stocks of a multinational firm and the risk-free asset is

$$r(t)v_F(t) = (1-m)\pi_F(t) + \dot{v}_F(t) - I(t)v_F(t). \tag{20}$$

2.4 Labor markets

In the North, labor is devoted to production and R&D activities. Letting $n_N(t) \in (0,1)$ represent the number (measure) of industries in which the Northern leader firms produce state-of-the-art goods, the aggregate labor demand for production in the North is given by $n_N(t)(x_{NN}(t) + x_{NS}(t))$. The aggregate labor demand for R&D activities is given by $a_N X(t) I(t)$ because firms undertaking R&D target all industries. From (5) and (7), the labor market-clearing condition in the North is

$$n_N(t)\left[\frac{E_N(t)L_N(t)}{\lambda(1+\tau_N)} + \frac{E_S(t)L_S(t)}{\lambda}\right] + a_N X(t)I(t) = L_N(t). \tag{21}$$

In the South, multinational firms and imitators demand labor for production. We define $n_S(t) \equiv 1 - n_N(t)$, which is the number of industries where the goods are produced in the South. The goods produced in the South are imitated at probability m at each time point. From the law of large numbers, the goods are produced by the multinational firms monopolistically in $(1-m)n_S(t)$ industries and produced by Southern imitators in $mn_S(t)$ industries. The aggregate labor demand of multinationals is $(1-m)n_S(t)(x_{FN}(t) + x_{FS}(t))$ and that of Southern imitators is $mn_S(t)(x_{MN}(t) + x_{MS}(t))$. From (10), (12), (15), and (16), the labor market-clearing condition in the South becomes

$$(1-m)n_S(t)\left[\frac{E_N(t)L_N(t)}{\lambda(1+\tau_N)} + \frac{E_S(t)L_S(t)}{\lambda}\right] + mn_S(t)\left[\frac{E_N(t)L_N(t)}{1+\tau_N} + E_S(t)L_S(t)\right] = L_S(t). \tag{22}$$

2.5 Government budget constraints

The Northern government imposes the tariff on imports from the South by the multinational firms and the Southern imitators. In the Northern market, the sales of the good supplied by a multinational firm are $p_{FN}(t)x_{FN}(t) = E_N(t)L_N(t)/(1+\tau_N)$ and the sales of the good supplied by the Southern imitators are $x_{MN}(t) = E_N(t)L_N(t)/(1+\tau_N)$. Thus, the Northern tariff revenue is given by $n_S(t)\tau_N E_N(t)L_N(t)/(1+\tau_N)$. As the Northern government transfers all tariff revenue to Northern households, it determines the lump-sum transfer per capita $T_N(t)$ to satisfy the following budget constraint at each time point:

$$T_N(t)L_N(t) = n_S(t)\tau_N \frac{E_N(t)L_N(t)}{1+\tau_N}.$$

The Southern government imposes the tariff on imports from the North by the Northern leader firms. The sales of a Northern leader firm in the Southern market are $p_{NS}(t)x_{NS}(t) = E_S(t)L_S(t)/(1+\tau_S)$, which implies that the Southern tariff revenue is given by $n_N(t)\tau_S E_S(t)L_S(t)/(1+\tau_S)$. Therefore, the Southern government determines $T_S(t)$ to satisfy the following budget constraint:

$$T_S(t)L_S(t) = n_N(t)\tau_S \frac{E_S(t)L_S(t)}{1+\tau_S}. \tag{23}$$

3 The equilibrium

In this section, we discuss the market equilibrium. To simplify notation, we define world aggregate expenditure as $E(t) \equiv E_N(t)L_N(t) + E_S(t)L_S(t)$ and the share of Northern aggregate expenditure to world aggregate expenditure $\phi \equiv E_N(t)L_N(t)/E(t)$. On the equilibrium path, ϕ becomes constant over time because Northern expenditure $E_N(t)$ and Southern expenditure $E_S(t)$ always grow at the same rate given the Euler equation (4). By using $E(t)$ and ϕ, the labor market equilibrium conditions (21) and (22) are rewritten as

$$n_N(t)\left(\frac{\phi}{1+\tau_N} + 1 - \phi\right)\frac{E(t)}{\lambda} + a_N X(t)I(t) = L_N(t), \tag{24}$$

$$(1-m+m\lambda)n_S(t)\left(\frac{\phi}{1+\tau_N} + 1 - \phi\right)\frac{E(t)}{\lambda} = L_S(t). \tag{25}$$

In this model, there is no state variable, except population size and R&D difficulty, whose growth rates are exogenous and constant. Consequently, as shown in Appendix A, this model does not have a transitional process and the economy jumps to the steady state immediately at the initial time. In the steady state, $E_i(t)$, $I(t)$, $n_N(t)$, $n_S(t)$, and $w_N(t)$ are constant over time. We therefore omit the time index of these variables hereafter. As E_i is constant, the interest rate $r(t)$ is also constant and equal to ρ all the time from (4).

For analytical tractability, we focus on the case where the Southern households initially have no assets. Then, from (2) and (23), the budget constraint of a Southern household is

$$E_S = 1 + n_N E_S \frac{\tau_S}{1+\tau_S}, \tag{26}$$

where the left-hand side (LHS) is expenditure per capita, the first term on the right-hand side (RHS) is wage income, and the second term on the RHS is the per capita lump-sum transfer from the Southern government's tariff revenue. Because $E_S = (1-\phi)E(t)/L_S(t)$ by the definition of $E(t)$ and ϕ, the budget constraint (26) can be rewritten as $1-\phi=(1+\tau_S)L_S(t)/[(1+\tau_S n_S)E(t)]$. Substituting (25) into this equation to eliminate $E(t)$, we obtain ϕ as a decreasing function of n_S as follows:

$$\frac{\phi}{1-\phi}=(1+\tau_N)\left[\frac{\lambda}{1-m+m\lambda}\frac{1+\tau_S n_S}{(1+\tau_S)n_S}-1\right]. \tag{27}$$

This equation implies that an increase in n_S has two effects on the share of Southern aggregate expenditure to world aggregate expenditure, $1-\phi$, through the budget constraint of a Southern household. First, as shown in (25), an increase in n_S raises the Southern wage compared with the world aggregate expenditure, $1/E(t)$, because it expands the demand for Southern labor. The increased wage income has a positive effect on $1-\phi$. Second, an increase in n_S reduces the transfer payment to Southern households because it decreases the tariff revenue of the Southern government. The decreased transfer payment has a negative effect on $1-\phi$. Nevertheless, the first effect necessarily dominates the second, so that ϕ is decreasing with n_S, as shown in (27).

Next, to analyze the equilibrium, we describe two key equations with respect to I and n_S. We derive the first equation from the labor market-clearing conditions in the two countries. Combining (24) and (25), we have the following equation:

$$I=\frac{L_N(t)}{a_N X(t)}-\left(\frac{1}{n_S}-1\right)\frac{1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}. \tag{28}$$

As $X(t)$ grows at the same rate as $L_N(t)$ and $L_S(t)$, the relation between n_S and I satisfying (28) is depicted as an upward sloping curve in Figure 1. We refer to this as the *LC curve* given the labor constraint. It shows the combinations of n_S and I that are consistent with equilibrium in the labor markets of the two countries.

The LC curve is upward sloping because the innovation rate I satisfying the labor constraints increases with n_S for two reasons. First, as n_S increases, the number of industries producing in the North, n_N, contracts, and thus, labor demand for production decreases in the North. Second, an increase in n_S reduces the quantity of labor demanded by the Northern leader firm, $[\phi/(1+\tau_N)+1-\phi](E(t)/\lambda)$. This second effect results from the increase in the Southern wage compared with the world aggregate expenditure, $1/E(t)$, in the Southern labor market, as shown in (25). A higher Southern wage increases the marginal cost of a follower firm when produced in the South, so that the Northern leader firms raise the relative price of their goods to the world aggregate expenditure and thereby decrease production. Because the abovementioned two effects reduce the labor inputs for production in the North, the Northern labor resources available for R&D increase. This is why an innovation rate I consistent with the equilibrium in the labor markets increases with n_S.

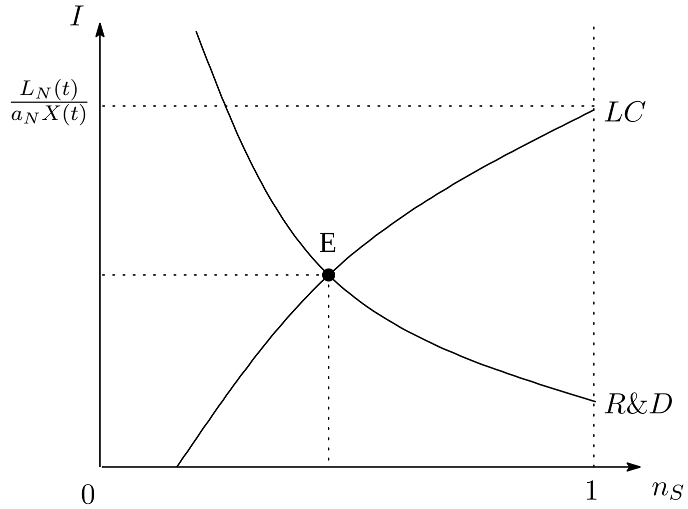

Figure 1: The equilibrium

The second equation with respect to n_S and I is derived from the free-entry condition in R&D. Because $v_F(t) = w_N a_N X(t)$ and $\dot{v}_F(t)/v_F(t) = \dot{X}(t)/X(t)$ from (17) and (18), substituting (14) and (20) into the former yields

$$\frac{(1-m)\left(1-\frac{1}{\lambda}\right)\left(\frac{\phi}{1+\tau_N}+1-\phi\right)E(t)}{\rho+I-\frac{\dot{X}(t)}{X(t)}} = w_N a_N X(t). \tag{29}$$

The LHS of (29) represents the expected gain from R&D that is equal to the present value of expected profits discounted by the interest rate, the hazard rate of monopolistic rent due to another firm's innovation, and the capital gain. The RHS of (29) then represents the cost of R&D.

The Northern wage (compared with the Southern wage), which affects the cost of R&D, is determined by the condition on FDI. When production is carried out in both countries continuously, $\dot{v}_N(t)/v_N(t) = \dot{v}_F(t)/v_F(t)$ must be satisfied because (18) holds at each time point. Therefore, from (19) and (20), the profits of a Northern leader firm and a multinational firm must satisfy $\pi_N(t) = (1-m)\pi_F(t)$ in equilibrium. Substituting (9), (14), and (27) into this equation, we obtain the Northern wage as a decreasing function of n_S:

$$w_N = \tau_N(1-m)(\lambda-1) + \frac{1+\tau_N(1-n_S)}{1+\tau_S n_S}(1-m+m\lambda). \tag{30}$$

The reason why the Northern wage is decreasing with n_S can be explained as follows. For both a Northern leader firm and a multinational firm, profits per unit consumption expenditure in the local market (the North for a Northern leader firm and the South for a multinational firm) are higher than those in the other market because the firms can differentiate their prices in the two markets.[11] As shown in (27), an increase in n_S raises the share of Southern aggregate expenditure to world aggregate expenditure, $1-\phi$, and therefore increases the profits of a multinational firm compared with a Northern leader firm. Because this increases the incentive for a production transfer to the South, to restore equilibrium, the Northern wage must decline so that the incentive for production in the North can increase.

Combining (25) and (29), we have the following equation:

$$\frac{\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\frac{L_S(t)}{X(t)}}{\rho+I-\frac{\dot{X}(t)}{X(t)}}=n_S w_N a_N. \tag{31}$$

From (30), the RHS of (31) is increasing with n_S and tends to zero as $n_S \to 0$.[12] Therefore, the relation between n_S and I satisfying (30) and (31) is negative and asymptotes to the vertical axis, as depicted in Figure 1. We refer to this as the *R&D curve*, which shows the combinations of n_S and I that are consistent with an incentive to carry out R&D.

The R&D curve is downward sloping because the innovation rate I consistent with an incentive for R&D decreases with n_S. In the Southern labor market, an increase in n_S pushes up the Southern wage when compared with world aggregate expenditure, $1/E(t)$. This decreases the demand for multinational firms' goods through increasing their prices compared with aggregate expenditure. As a result, it decreases the profits of multinational firms and thereby the expected gain by R&D. This effect appears as n_S on the RHS of (31). Meanwhile, an increase in n_S also reduces the cost of R&D through lowering the Northern wage w_N, as shown by (30). However, the decrease in the expected gain necessarily dominates the decrease in the cost. Therefore, as n_S increases, the innovation rate I must be lower in terms of the incentive for R&D.

The intersection of the LC and R&D curves provides the equilibrium values of n_S and I. As depicted in Figure 1, the LC and R&D curves intersect only once if the R&D curve lies below the LC curve around the upper limit of n_S. Then, there exists a unique interior equilibrium such that n_S and I are positive. Depending on the equilibrium value of n_S, (25), (27), and (30) determine the equilibrium values of $E(t)$, ϕ, and w_N, respectively.

[11] Equations (6), (8), (11), and (13) show that the profits per unit consumption expenditure of a Northern leader firm are $1-w_N(t)/[\lambda(1+\tau_N)]$ in the North and $1/(1+\tau_S)-w_N(t)/\lambda$ in the South, while those of a multinational firm are $(1-1/\lambda)/(1+\tau_N)$ in the North and $1-1/\lambda$ in the South.

[12] For the proof that the RHS is increasing with n_S, see (53) in Appendix B. A detailed derivation is also provided in Appendix C, which is available from the authors upon request.

4 Comparative statics

In this section, we conduct comparative statics using Figure 1.

4.1 A unilateral tariff increase

We first consider the effects of a unilateral tariff increase by the North. Given n_S, an increase in τ_N raises w_N, as shown in (30). This is because the Northern tariff affects the incentive for FDI. For Northern leader firms, an increase in τ_N reduces competition pressure in the Northern market because it raises the lowest price that the follower firms in the South could charge after the tariff. This enables the Northern leader firms to raise the price in the Northern market, which increases their profits. Meanwhile, the higher Northern tariff pushes up the duty-inclusive price of goods produced by multinational firms. Accordingly, it reduces the demand for the goods and consequently, the profits of multinational firms. Because the increased profits of a Northern leader firm and the decreased profits of a multinational firm decrease the incentive for FDI, the relative wage of Northern to Southern labor must become higher to restore equilibrium.

Given that the higher Northern wage leads to a higher cost of R&D, an increase in τ_N negatively affects the incentive for R&D and the innovation rate I for a given n_S from the R&D equilibrium condition (31). This means that an increase in τ_N shifts the R&D curve downward. However, the LC curve (28) does not change because the Northern leader firm, the multinational firm, and an imitator change the labor input proportionately in response to the tariff increase, as shown in (24) and (25). As a result, an increased Northern tariff lowers both n_S and I, as in Figure 2. In addition, from (30), it raises the relative wage of Northern labor to Southern labor through both the direct effect discussed above and an indirect effect through the decrease in n_S. These results are summarized as follows.

Proposition 1. *A unilateral tariff increase by the North reduces innovation and FDI from the North to the South, although it also raises the relative wage of Northern to Southern labor.*

Next, we analyze the effects of a unilateral tariff increase by the South. As the Southern tariff increases, the Northern leader firms need to lower their pre-tariff export prices to the South, p_{NS}. Given n_S, this reduces the Northern leaders' profits, and consequently, increases the incentive for FDI. As shown in (30), the Northern wage w_N must decrease to restore equilibrium as τ_S increases. Because the decreased Northern wage reduces the cost of R&D, a higher τ_S positively affects the incentive for R&D and innovation rate I for a given n_S from the R&D equilibrium condition (31). Therefore, the R&D curve shifts upward. However, the LC curve does not change because an increase in τ_S does not affect

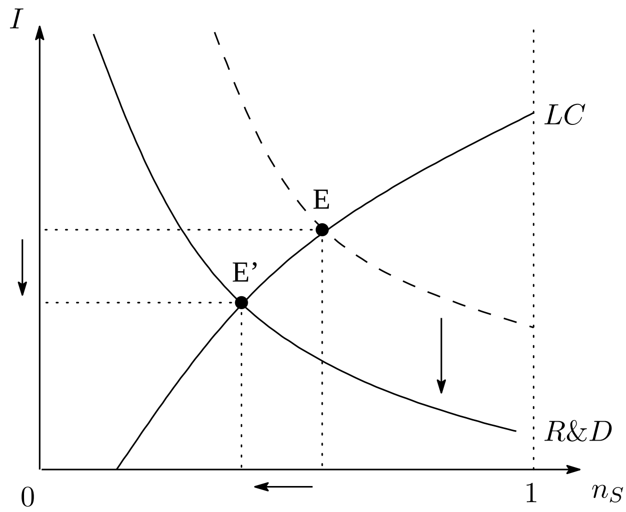

Figure 2: The effect of a unilateral tariff increase by the North

the labor input for production. As a result, a tariff increase by the South increases both n_S and I, as in Figure 3. In addition, equation (30) shows that it reduces w_N through both the direct effect and the indirect effect through the increase in n_S.

Proposition 2. *A unilateral tariff increase by the South promotes innovation and FDI from the North to the South. Moreover, it raises the relative wage of Southern to Northern labor.*

With a North–South innovation-imitation model not including FDI, Grieben and Şener (2009) concluded that a unilateral reduction of the Southern (Northern) import tariff has no effect on innovation in their basic model, but decreases (increases) innovation rate in their extended model with a perfectly competitive low-tech sector in the South. Our results for Propositions 1 and 2 on innovation are similar to the results of their extended model, but different to those of their basic model.

4.2 Tariff increases by both countries

In the former section, we concluded that a tariff increase by the South promotes innovation and FDI. Nevertheless, this does not necessarily imply that such a policy change is favorable. This is because a tariff increase by the South may result in a retaliatory tariff by the North, which has a negative effect on innovation and FDI. Next, we discuss the effects of simultaneous tariff increases by the North and South. In Section 4.2, we assume that the initial tariff rate in the North is not higher than that in the South; that

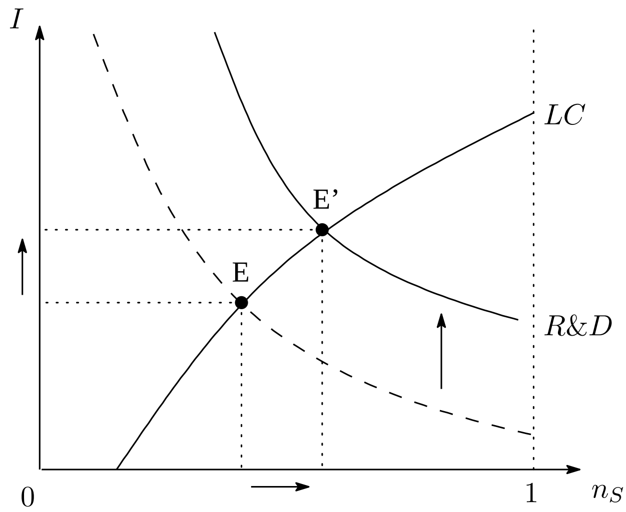

Figure 3: The effect of a unilateral tariff increase by the South

is, $\tau_N = \tau$ and $\tau_S = \tau + \bar{\tau}$ where $\tau \geq 0$ and $\bar{\tau} \geq 0$.[13] Under this setting, we analyze the effects of an increase in τ.

We interpret the effects of tariff increases by both countries as the combined effects of unilateral tariff increases by the North and South. Similarly to a unilateral tariff increase, neither the Northern nor Southern tariffs affect the LC curve from (28). Meanwhile, (31) shows that the tariff increases by both countries affect the R&D curve only through the change in w_N. If an increase in τ raises w_N for a given value of n_S, it moves the R&D curve downward and vice versa. Partially differentiating (30) with respect to τ, we have

$$\begin{aligned} \left.\frac{\partial w_N}{\partial \tau}\right|_{n_S=\text{given}} = & \frac{\lambda - 2n_S[(1-m+m\lambda) - (1-m)(\lambda-1)\tau_S]}{(1+\tau_S n_S)^2} \\ & + \frac{(1-m)(\lambda-1)\tau_S^2 n_S^2 + (\tau_S - \tau_N)(1-m+m\lambda)n_S(1-n_S)}{(1+\tau_S n_S)^2}. \end{aligned} \tag{32}$$

This equation shows that given n_S, an increase in τ raises w_N if (i) $\tau_S \geq (1-m+m\lambda)/[(1-m)(\lambda-1)]$; or (ii) $\tau_S < (1-m+m\lambda)/[(1-m)(\lambda-1)]$ and $n_S < \lambda/\{2[(1-m+m\lambda)-(1-m)(\lambda-1)\tau_S]\} \equiv \hat{n}_S$. Consequently, if the R&D and LC curves intersect at a value of n_S lower than $\hat{n}_S$, an increase in τ moves the R&D curve downward around the intersection of the two curves, and the equilibrium to the lower left along the LC curve. This implies, in that case, that simultaneous tariff increases by both countries reduce the equilibrium values of I and n_S. This is the case if the following condition is satisfied.

[13] In fact, according to the World Bank Open Data (https://data.worldbank.org/), the weighted mean tariff rate applied in 2017 was 4.28% in "low & middle income" countries, which was higher than the 2.02% in "high income" countries.

Proposition 3. *Suppose that the initial tariff rate in the North is not higher than that in the South:* $\tau_N \leq \tau_S$*. Then, simultaneous tariff increases by the North and South to the same degree reduce innovation and FDI if* $L_N(t)/[a_N X(t)] + \rho - \dot{X}(t)/X(t) > L_S(t)/[(1 - m + m\lambda)a_N X(t)]$.

Proof. See Appendix B. □

Proposition 3 implies that the tariff increases by both countries tend to be detrimental to innovation and FDI if the share of population size is relatively large in the North and small in the South. In such a case, the negative effects of an increase in the Northern tariff tend to dominate the positive effects of an increase in the Southern tariff. However, the negative effects on innovation and FDI are quantitatively smaller than for a unilateral tariff increase by the North. This is because the increase in the Southern tariff works towards alleviating the negative effects.

Using the proof of Proposition 3, we also have the following result.

Corollary. *Suppose that the initial tariff rate is zero in the North and South:* $\tau_N = \tau_S = 0$*. Then, simultaneous tariff increases by the North and South to the same degree reduce innovation and FDI if and only if* $L_N(t)/[a_N X(t)] + \rho - \dot{X}(t)/X(t) > L_S(t)/[(1 - m + m\lambda)a_N X(t)]$.

Proof. See Appendix B. □

Unlike Proposition 3, the condition in this corollary is necessary and sufficient for the result, as it implies a symmetric tariff rate between the two countries that maximizes innovation and FDI is not zero, but strictly positive if the condition is not satisfied. Furthermore, the condition is more restrictive when m is small through strong IPR protection in the South. This is because a smaller m results in a larger equilibrium value of n_S, as discussed in Section 4.3. As n_S is large, the simultaneous tariff increases by both countries tend to reduce w_N from (32) and move the R&D curve upward around the equilibrium, which increases innovation and FDI.

4.3 A change in IPR protection in the South

Next, we analyze how the imitation rate m influences innovation and FDI to consider the effects of strengthening IPR protection in the South.

Equation (28) shows that a decrease in m rotates the LC curve counterclockwise around the point $(1, L_N(t)/a_N X(t))$ on the n_S-I plane. Given n_S, a lower imitation rate decreases the number of imitated goods with larger production volumes, and thus decreases labor demand in the South. The smaller labor demand lowers the wage of the South compared with the world aggregate expenditure, $1/E(t)$, in the

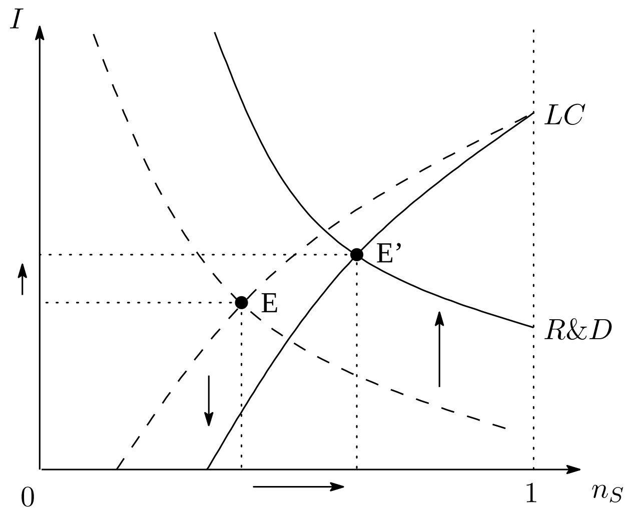

Figure 4: The effect of a decrease in the imitation rate

Southern labor market. Because this pushes down the marginal cost of follower firms if they produce in the South, the Northern leader firms set a lower price and expand production, which increases labor demand in the North. As a result, for a given value of n_S, a lower imitation rate decreases the quantity of labor available for R&D in the North and negatively affects the innovation rate I.

Equations (30) and (31) imply that a decrease in m shifts the R&D curve upward for two reasons. First, a lower imitation rate increases the return from successful R&D. This is because it increases the expected profits of a multinational firm through (i) raising the probability of earning profits and (ii) reducing the marginal cost of production (the wage of the South) compared with the world aggregate expenditure $1/E(t)$. Second, a lower imitation rate has a negative effect on the Northern wage w_N and thereby the cost of R&D. This is because it stimulates production transfer to the South through increasing the expected profits of a multinational firm. Partially differentiating (30) with respect to m verifies this effect:

$$\left.\frac{\partial w_N}{\partial m}\right|_{n_S=\text{given}} = (\lambda - 1)\left[-\tau_N + \frac{1+\tau_N(1-n_S)}{1+\tau_S n_S}\right] \geq 0, \tag{33}$$

where the inequality holds because $\tau_N \leq 1/(1+\tau_S n_S)$ must be satisfied to ensure nonnegative profits of Northern leader firms in the Southern market, as shown in Appendix B. Through the abovementioned effects, a lower m positively affects the incentive for R&D and the innovation rate I for a given value of n_S. In equation (31), the first effect is represented by a decrease in m on the LHS, while the second effect is represented by a decrease in w_N on the RHS, both of which show that a lower m must increase I, given n_S.

Figure 4 depicts the effects of a decrease in m on the equilibrium. The intersection of the LC and R&D curves moves from point E to point E'. Figure 4 shows that a lower m unambiguously increases the equilibrium value of n_S. Because (30) shows that w_N is decreasing with n_S, a decreased m lowers w_N through both the direct effect represented by (33) and the indirect effect by the increase in n_S. Meanwhile, Figure 4 does not indicate whether a lower m increases the equilibrium value of I because it includes both the negative effect through tightening the labor constraint in the North and the positive effect through improving the incentive for R&D. On this point, we conclude as follows using total differentiation.

Proposition 4. *A decrease in the imitation rate through strengthening IPR protection in the South (i) promotes innovation, (ii) increases FDI, and (iii) decreases the relative wage of Northern to Southern labor.*

Proof. See Appendix B. □

Note that Proposition 4 holds regardless of whether the tariff in both countries is zero or positive, and implies that tariffs do not qualitatively change the effects of strengthening IPR protection in the South on innovation and FDI. In addition, the result for Proposition 4 is consistent with those for Gustafsson and Segerstrom (2011), Lai (1998), and Tanaka and Iwaisako (2014) using North–South innovation–FDI models with exogenous imitation not including tariffs.

5 Welfare analysis

In this section, we examine how unilateral tariff increases by the North and South impact their own welfare. To this end, we first derive the instantaneous utility of a household in each country. From (1), the instantaneous utility can be decomposed into two parts, utility from quality and utility from quantity, as follows:

$$\log u_i(t) = \int_0^1 \log \lambda^{J(\omega,t)} d\omega + \int_0^1 \log d_i(\omega,t) d\omega, \tag{34}$$

where $J(\omega,t)$ is the generation number of the state-of-the-art quality of good ω at time t, and $d_i(\omega,t)$ denotes the demand for that good. Hereafter, we let $\log Q(t)$ and $\log D_i$ denote the first and second terms of (34), respectively. The growth rate of $Q(t)$ is given by

$$\frac{d \log Q(t)}{dt} = (\log \lambda) I. \tag{35}$$

Meanwhile, substituting (3) and the prices into $\log D_i$ yields the utility from quantity as follows:

$$
\begin{aligned}
\log D_N &= \int_0^1 \log d_N(\omega, t) d\omega \\
&= n_N \log \frac{E_N}{\lambda(1+\tau_N)} + (1-m) n_S \log \frac{E_N}{\lambda(1+\tau_N)} + m n_S \log \frac{E_N}{1+\tau_N} \\
&= \log E_N - \log(1+\tau_N) - (1 - m n_S) \log \lambda, \qquad (36)
\end{aligned}
$$

$$
\begin{aligned}
\log D_S &= \int_0^1 \log d_S(\omega, t) d\omega \\
&= n_N \log \frac{E_S}{\lambda} + (1-m) n_S \log \frac{E_S}{\lambda} + m n_S \log E_S \\
&= \log E_S - (1 - m n_S) \log \lambda. \qquad (37)
\end{aligned}
$$

By rewriting (22), the Northern expenditure in (36) is expressed as

$$
E_N = (1+\tau_N) \frac{L_S(t)}{L_N(t)} \left(\frac{\lambda}{1-m+m\lambda} \frac{1}{n_S} - E_S \right). \qquad (38)
$$

The Southern expenditure in (37) and (38) is derived from (26) as follows:

$$
E_S = \frac{1}{1 - n_N \frac{\tau_S}{1+\tau_S}} = \frac{1+\tau_S}{1+\tau_S n_S}. \qquad (39)
$$

Substituting (35) into (34), we have the lifetime utility of a household in country i as

$$
\begin{aligned}
U_i &= \int_0^\infty e^{-(\rho - g_L)t} \log u_i(t) dt \\
&= \int_0^\infty e^{-(\rho - g_L)t} \left[\log Q(0) + (\log \lambda) I t + \log D_i \right] dt \\
&= \frac{1}{\rho - g_L} \left[\log Q(0) + \frac{(\log \lambda) I}{\rho - g_L} + \log D_i \right],
\end{aligned}
$$

where $\log D_i$ is given by (36) - (39). From this lifetime utility, we next derive the welfare change from a tariff increase in each country.

5.1 Welfare effect of a Northern tariff increase

By differentiating the Northern household's lifetime utility with respect to τ_N, we obtain the Northern welfare change from a marginal increase in its tariff rate as follows:

$$
\begin{aligned}
\frac{\partial U_N}{\partial \tau_N} = \frac{1}{\rho - g_L} \Bigg[& \underbrace{\frac{\log \lambda}{\rho - g_L} \frac{\partial I}{\partial \tau_N}}_{\substack{\text{innovation-impeding effect} \\ (-)}} + \underbrace{\frac{1}{E_N} \frac{\partial E_N}{\partial \tau_N}}_{\substack{\text{income effect} \\ (+)}} \underbrace{- \frac{1}{1+\tau_N}}_{\substack{\text{price-raising effect} \\ (-)}} \\
& + \underbrace{(\log \lambda) m \frac{\partial n_S}{\partial \tau_N}}_{\substack{\text{competition-weakening effect} \\ (-)}} \Bigg]. \qquad (40)
\end{aligned}
$$

Equation (40) shows that the total welfare effect of a Northern tariff increase (an increase in τ_N) can be decomposed into the following four parts. First, the increased Northern tariff impedes innovation, as shown in Proposition 1, and thus reduces welfare. We refer to this welfare effect as the *innovation-impeding effect*. Second, the increased Northern tariff increases Northern income as we later demonstrate. We refer to this welfare effect as the *income effect*. Third, the Northern tariff increase raises prices in the North and thus reduces welfare. We refer to this welfare effect as the *price-raising effect*. Finally, the Northern tariff increase impedes FDI and consequently decreases the number of goods produced by the Southern imitators mn_S. The price of the imitated goods is lower than that of the goods produced by Northern and multinational firms, so that a decrease in the number of the imitated goods reduces welfare. We refer to this welfare effect as the *competition-weakening effect*.

The sum of the income effect and price-raising effect is equal to $\frac{1}{\rho-g_L}\frac{1+\tau_N}{E_N}\frac{\partial\frac{E_N}{1+\tau_N}}{\partial\tau_N}$. From (38) and (39), differentiating $E_N/(1+\tau_N)$ with respect to τ_N yields

$$\frac{\partial\frac{E_N}{1+\tau_N}}{\partial\tau_N}=\frac{L_S(t)}{L_N(t)}\left[-\frac{\lambda}{1-m+m\lambda}+(E_S n_S)^2\frac{\tau_S}{1+\tau_S}\right]\frac{1}{n_S{}^2}\frac{\partial n_S}{\partial\tau_N}>0, \tag{41}$$

where the inequality holds because $\lambda/(1-m+m\lambda)>1$, $E_S n_S=(1+\tau_S)n_S/(1+\tau_S n_S)<1$, and $\partial n_S/\partial\tau_N<0$ from Proposition 1. As the price-raising effect is necessarily negative, this means that the income effect is necessarily positive. This is because the increased Northern tariff raises Northern tariff revenue, the Northern wage, as shown in Proposition 1, and the value of the holding stocks proportionate to the Northern wage from (17) and (18).

Although both positive and negative welfare effects exist, the total welfare effect is negative under a certain parameter condition because the sum of the innovation-impeding and price-raising effects surpasses the positive income effect. This result is summarized as follows.

Proposition 5. *If* $\frac{\log\lambda}{\rho-g_L}\left[\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}\right]>\frac{\lambda}{1-m+m\lambda}(1+\tau_S)$, *a unilateral tariff increase by the North worsens the North's welfare.*

Proof. See Appendix B. □

Note that the condition in Proposition 5 does not depend on the value of τ_N as it implies that the welfare-maximizing tariff rate for the North is zero if the condition is satisfied.

From the condition in Proposition 5, the welfare effect tends to be negative when Northern labor is larger, the imitation rate in the South is higher, and the Southern tariff rate is lower. A tariff increase by the North has a positive effect on the North's welfare because it shifts production from the South to the North and raises the Northern wage. However, this positive welfare effect is weak under large Northern labor, a high imitation rate in the South, and a low Southern tariff rate because the number of industries producing in the South is relatively smaller in these cases. That is why the total welfare effects of the Northern tariff tend to be negative.

5.2 Welfare effect of a Southern tariff increase

Next, by differentiating the Southern household's lifetime utility with respect to τ_S, we obtain the Southern welfare change from a marginal increase in its tariff rate as follows:

$$\frac{\partial U_S}{\partial \tau_S} = \frac{1}{\rho - g_L}\Bigg[\underbrace{\frac{\log\lambda}{\rho - g_L}\frac{\partial I}{\partial \tau_S}}_{\substack{\text{innovation-enhancing effect} \\ (+)}} + \underbrace{\frac{1}{E_S}\frac{\partial E_S}{\partial \tau_S}}_{\substack{\text{income effect} \\ (+) \text{ or } (-)}} + \underbrace{(\log\lambda)m\frac{\partial n_S}{\partial \tau_S}}_{\substack{\text{competition-strengthening effect} \\ (+)}} \Bigg]. \quad (42)$$

Equation (42) shows that the total welfare effect of a Southern tariff increase (an increase in τ_S) can be decomposed into the following three parts. First, the Southern tariff increase enhances innovation, as shown in Proposition 2, and thus raises welfare. We refer to this welfare effect as the *innovation-enhancing effect*. Second, the Southern tariff increase may increase or decrease Southern expenditure because it affects the transfer payment from the tariff revenue of the Southern government. We refer to this effect as the *income effect*. From equation (39), if the elasticity of n_N with respect to τ_S is lower than $1/(1+\tau_S)$, a higher Southern tariff increases tariff revenue, and thus has a positive effect on Southern income and welfare.[14] Finally, the Southern tariff increase promotes FDI and increases the number of the goods produced by Southern imitators mn_S. The increase in imitated goods improves welfare because they are cheaper than the goods produced by Northern leaders and multinationals. We refer to this welfare effect as the *competition-strengthening effect*.

As shown in Appendix B, under a certain parameter condition, the positive welfare effects surpass the negative part of the income effect, which is the negative effect on the tariff revenue from the decrease in Northern firms. Accordingly, the total welfare effect of an increase in the Southern tariff is positive in that case, as in the following proposition.

Proposition 6. *If* $\frac{\log\lambda}{\rho-g_L}\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} + (\log\lambda)m + \left[\frac{\log\lambda}{\rho-g_L}\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} - 1\right]\tau_S > 0$, *a unilateral tariff increase by the South improves the South's welfare.*

Proof. See Appendix B. □

[14] Differentiating (39) with respect to τ_S, we have

$$\frac{\partial E_S}{\partial \tau_S} = \frac{(E_S)^2 n_N}{1+\tau_S}\left[\frac{1}{1+\tau_S} - \left(-\frac{\tau_S}{n_N}\frac{\partial n_N}{\partial \tau_S}\right)\right].$$

Therefore, $\partial E_S/\partial \tau_S > 0$ if $-(\tau_S/n_N)(\partial n_N/\partial \tau_S) < 1/(1+\tau_S)$.

Proposition 6 shows that a tariff increase by the South improves the South's welfare if $\frac{\log\lambda}{\rho-g_L}\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} \geq 1$. Even if $\frac{\log\lambda}{\rho-g_L}\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} < 1$, it improves the South's welfare as long as the Southern tariff is so small as to satisfy $\tau_S < \left[\frac{\log\lambda}{\rho-g_L}\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} + (\log\lambda)m\right]\left[1 - \frac{\log\lambda}{\rho-g_L}\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}\right]^{-1} (> 0)$. That is, the condition in Proposition 6 is necessarily satisfied if τ_S is zero and this implies that the welfare-maximizing tariff rate for the South is strictly positive. Thus, the result shows that, in contrast to the North, the South has a stronger incentive to raise tariffs.

The results of the welfare analysis in this section have implications for the optimal tariff literature. Many theoretical studies have concluded that larger countries tend to set higher tariffs, but the opposite is actually observed.[15] As discussed, our model shows that the optimal tariff for the North may be zero, whereas that for the South is necessarily positive. The difference is due to the dissimilar effects of tariffs in the two countries on innovation, FDI, and prices. In contrast to extant theoretical studies, our welfare analysis can then explain the observation that larger countries tend to set lower tariffs.

6 Concluding remarks

Using a North–South quality ladder model, this paper investigated how import tariffs affect innovation, FDI, and welfare under imperfect IPR protection in a developing country. The conclusion is that a unilateral tariff increase by a developed country reduces innovation and FDI, while that by a developing country promotes innovation and FDI. In addition, because of the decrease in innovation, a unilateral tariff increase by the developed country tends to worsen its welfare if the country is large, the tariff rate of the developing country is low, and the protection of IPR in the developing country is weak. By contrast, a unilateral tariff increase by the developing country improves its welfare if the initial tariff rate is sufficiently low, which implies that the optimal tariff rate for the developing country is strictly positive. The effects of simultaneous tariff increases by both countries then depend on the relative country size and the degree of IPR protection in the developing country: they tend to reduce innovation and FDI when the relative size of the developed country to the developing country is large and the protection of IPR in the developing country is weak. In addition, we showed that strengthening IPR protection in a developing country promotes innovation and FDI even when tariffs are imposed, which is the same as in the case of free trade.

Possible directions of further research include extensions to address the following issues. First, we

[15] The exception is Naito (2019), who obtained a result consistent with the actual tendency using a two-country growth model. However, the engine of growth in that model is not R&D, but rather capital accumulation by both countries, nor is it a North–South model.

assumed that the process of FDI is costless for analytical tractability. This could be justified if production startup costs are small or at least similar between the two countries. However, it would be useful to examine whether our results continue to hold, even with a cost of FDI. Second, we assumed harmonization in the patent system, such as breadth of coverage, between the developed and developing countries. To investigate the effects of tariffs when the breadth of patent coverage differs between the two countries would also be interesting.[16] Third, we did not consider the differences between skilled and unskilled labor. In reality, the R&D sector is likely to require more skilled labor than the production sector. To examine how a tariff increase affects innovation, FDI, and wage gap in such a setup would also be important. Moreover, we could explore how increases in skilled and unskilled labor affect innovation and skill premium, as in Chu, Cozzi, and Furukawa (2015), or endogenize skill acquisition, as in Dinopoulos and Segerstrom (1999a) and Cozzi and Impullitti (2010). Fourth, we ruled out R&D activities by the developing country in inventing a new product. In fact, R&D spending has recently increased in a few emerging countries such as China. International trade between a developed country and a "developing" country that undertakes both imitation and R&D may then be an intermediate case between North–South trade and North–North trade. It may be interesting to investigate how tariffs affect innovation and technology transfer in this case. As all of these extensions are worth examining, but beyond the scope of this paper, we defer them to future research.

Appendix A Dynamics of the model

In this appendix, we show that this model does not have transitional dynamics.

Rewriting the Southern labor market-clearing condition (25) and applying $n_S(t) \equiv 1 - n_N(t)$, we obtain

$$[1 - n_N(t)] \left(\frac{\phi}{1 + \tau_N} + 1 - \phi \right) \frac{E(t)}{\lambda} = \frac{L_S(t)}{1 - m + m\lambda}. \tag{43}$$

Note that ϕ must be constant on the equilibrium path because $E_N(t)$ and $E_S(t)$ grow at the same rate from (4). Adding both sides of this equation to those of (24) and rewriting, we have the following equilibrium innovation rate:

$$I(t) = \frac{L_N(t)}{a_N X(t)} + \frac{L_S(t)}{(1 - m + m\lambda) a_N X(t)} - \frac{1}{a_N \lambda} \left(\frac{\phi}{1 + \tau_N} + 1 - \phi \right) \frac{E(t)}{X(t)}. \tag{44}$$

[16] Iwaisako, Tanaka, and Futagami (2010) considered this situation and examined how extending patent breadth in the South affects innovation and FDI with no tariff.

For both a Northern leader firm and a multinational firm to exist at each time point, (18) must be satisfied over time. Differentiating both sides of (18) with respect to t yields $\dot{v}_N(t) = \dot{v}_F(t)$. Substituting this and (18) into (19) and (20), we have $\pi_N(t) = (1-m)\pi_F(t)$. Therefore, from (9), (14), and the definitions of ϕ and $E(t)$, we obtain

$$\left[1 - \frac{w_N(t)}{\lambda(1+\tau_N)}\right]\phi E(t) + \left[\frac{1}{1+\tau_S} - \frac{w_N(t)}{\lambda}\right](1-\phi)E(t) = (1-m)\left(1-\frac{1}{\lambda}\right)\left[\frac{\phi E(t)}{1+\tau_N} + (1-\phi)E(t)\right].$$

Rewriting this equation, we have the Northern wage (compared with the Southern wage) as follows.

$$w_N(t) = \frac{\lambda\left(\phi + \frac{1-\phi}{1+\tau_S}\right)}{\frac{\phi}{1+\tau_N} + (1-\phi)} - (1-m)(\lambda - 1)$$

Hence, the Northern wage must be constant over time on the equilibrium path because ϕ is constant. As the Northern wage w_N is constant, (17) and (18) imply that the market values of a Northern leader firm and a multinational firm, $v_N(t)$ and $v_F(t)$, grow at the same rate as $X(t)$. Accordingly, from (9), (17), and (19), the equilibrium interest rate is expressed as follows.

$$r(t) = \frac{1}{w_N a_N}\left\{\left[1 - \frac{w_N}{\lambda(1+\tau_N)}\right]\phi + \left(\frac{1}{1+\tau_S} - \frac{w_N}{\lambda}\right)(1-\phi)\right\}\frac{E(t)}{X(t)} + \frac{\dot{X}(t)}{X(t)} - I(t)$$

Substituting (44) into this equation, we have

$$r(t) = \frac{1}{w_N a_N}\left(\phi + \frac{1-\phi}{1+\tau_S}\right)\frac{E(t)}{X(t)} + \frac{\dot{X}(t)}{X(t)} - \frac{L_N(t)}{a_N X(t)} - \frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}. \tag{45}$$

Next, we compute the equation of motion with respect to $E(t)/X(t)$. Taking the logarithm of $E(t)/X(t)$ and differentiating it with respect to t yields

$$\begin{aligned}
\frac{[E(t)\dot{/}X(t)]}{E(t)/X(t)} &= \frac{\dot{E}(t)}{E(t)} - \frac{\dot{X}(t)}{X(t)} \\
&= \frac{\dot{E}_N(t)}{E_N(t)}\frac{E_N(t)L_N(t)}{E(t)} + \frac{\dot{L}_N(t)}{L_N(t)}\frac{E_N(t)L_N(t)}{E(t)} \\
&\quad + \frac{\dot{E}_S(t)}{E_S(t)}\frac{E_S(t)L_S(t)}{E_(t)} + \frac{\dot{L}_S(t)}{L_S(t)}\frac{E_S(t)L_S(t)}{E(t)} - \frac{\dot{X}(t)}{X(t)} \\
&= [r(t) - \rho + g_L]\left(\frac{E_N(t)L_N(t)}{E(t)} + \frac{E_S(t)L_S(t)}{E(t)}\right) - \frac{\dot{X}(t)}{X(t)} \\
&= r(t) - \rho + g_L - \frac{\dot{X}(t)}{X(t)},
\end{aligned} \tag{46}$$

where the second and fourth equalities use the definition of $E(t)$ and the third equality uses the Euler equation (4). Substituting (45) into (46), we obtain

$$\frac{[E(t)\dot{/}X(t)]}{E(t)/X(t)} = \frac{1}{w_N a_N}\left(\phi + \frac{1-\phi}{1+\tau_S}\right)\frac{E(t)}{X(t)} - \frac{L_N(t)}{a_N X(t)} - \frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} - (\rho - g_L). \tag{47}$$

As $\dot{X}(t)/X(t) = g_L$, $L_N(t)/X(t)$ and $L_S(t)/X(t)$ are constant. Thus, (47) has a unique interior steady state that is unstable. In the equilibrium, $E(t)/X(t)$ must jump to this steady-state value at the initial time point and then become constant because $E(t)/X(t)$ is jumpable. Otherwise, either (43) or (44) would be violated at a certain finite time point. This result implies that $r(t) = \rho$ for all t from (46). Then, $E_i(t)$ must be constant over time from the Euler equation (4). Also, (25) and (44) show that $n_S(t)$, $n_N(t)$, and $I(t)$ must be constant over time because $E(t)$, $X(t)$, $L_N(t)$, and $L_S(t)$ grow at the same rate on the equilibrium path.

Therefore, we conclude that the equilibrium path of this model does not have a transitional process and immediately jumps to the steady state.

Appendix B Proofs

Proof of Proposition 3

For the proof, it is sufficient to show that the LC curve is above the R&D curve at $\hat{n}_S$ because the former is upward sloping and the latter is downward sloping. From (28), the LC curve is given by

$$I = \frac{L_N(t)}{a_N X(t)} - \left(\frac{1}{n_S} - 1\right) \frac{1}{1-m+m\lambda} \frac{L_S(t)}{a_N X(t)} \equiv f(n_S).$$

Substituting (30) into (31), we have the R&D curve as follows:

$$\begin{aligned} I &= \frac{(1-m)(\lambda-1)}{1-m+m\lambda} \frac{L_S(t)}{a_N X(t)} \frac{1}{n_S w_N} - \left[\rho - \frac{\dot{X}(t)}{X(t)}\right] \\ &= \frac{(1-m)(\lambda-1)}{1-m+m\lambda} \frac{L_S(t)}{a_N X(t)} \frac{1+\tau_S n_S}{n_S\{\tau_N \lambda - \tau_N \left[(1-m+m\lambda) - (1-m)(\lambda-1)\tau_S\right] n_S + (1-m+m\lambda)\}} \\ &\qquad - \left[\rho - \frac{\dot{X}(t)}{X(t)}\right] \\ &\equiv h(n_S). \end{aligned}$$

Therefore, the following inequality holds from the definition of $\hat{n}_S$:

$$
\begin{aligned}
&f(\hat{n}_S) - h(\hat{n}_S)\\
&= \frac{L_N(t)}{a_N X(t)} - \left(\frac{1}{\hat{n}_S} - 1\right)\frac{1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\\
&\quad - \frac{(1-m)(\lambda-1)}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\frac{1+\tau_S \hat{n}_S}{\hat{n}_S[(\tau_N\lambda/2)+(1-m+m\lambda)]} + \rho - \frac{\dot{X}(t)}{X(t)}\\
&= \frac{L_N(t)}{a_N X(t)} + \rho - \frac{\dot{X}(t)}{X(t)}\\
&\quad - \frac{1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\frac{(\lambda/\hat{n}_S)[(\tau_N/2)+1] - [(\tau_N\lambda/2)+(1-m+m\lambda)] + (1-m)(\lambda-1)\tau_S}{(\tau_N\lambda/2)+(1-m+m\lambda)}\\
&= \frac{L_N(t)}{a_N X(t)} + \rho - \frac{\dot{X}(t)}{X(t)}\\
&\quad - \frac{1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\frac{(\tau_N\lambda/2)+(1-m+m\lambda)-(1-m)(\lambda-1)(\tau_N+\tau_N\tau_S+\tau_S)}{(\tau_N\lambda/2)+(1-m+m\lambda)}\\
&\geq \frac{L_N(t)}{a_N X(t)} + \rho - \frac{\dot{X}(t)}{X(t)} - \frac{1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}.
\end{aligned}
\tag{48}
$$

This inequality shows that, if $L_N(t)/[a_N X(t)] + \rho - \dot{X}(t)/X(t) > L_S(t)/[(1-m+m\lambda)a_N X(t)]$, then $f(\hat{n}_S) > h(\hat{n}_S)$ and thus the LC curve is above the R&D curve at $\hat{n}_S$.

Proof of Corollary

As discussed in the main text, the condition under which the simultaneous tariff increases reduce innovation and FDI is equivalent to the condition to satisfy $\partial w_N/\partial\tau\big|_{n_S=\text{given}} > 0$. Equation (32) implies that under $\tau_S = \tau_N = 0$, $\partial w_N/\partial\tau\big|_{n_S=\text{given}} > 0$ if and only if $n_S < \lambda/[2(1-m+m\lambda)] = \hat{n}_S$. Because (48) holds with equality if $\tau_S = \tau_N = 0$, the LC curve is above the R&D curve at $\hat{n}_S$ and the equilibrium value of n_S is lower than $\hat{n}_S$ if and only if $L_N(t)/[a_N X(t)]+\rho-\dot{X}(t)/X(t) > L_S(t)/[(1-m+m\lambda)a_N X(t)]$.

Proof of Proposition 4

The proofs on (ii) and (iii) are provided in the text and Figure 4. In this appendix, we show (i).

For the proof, we first show that parameters need to satisfy $1/(1+\tau_S n_S) \geq \tau_N$. In the equilibrium, the profit a Northern leader firm can obtain from a sale in the South must be nonnegative. Therefore, from (8) and (30), the following condition must be satisfied:

$$\begin{aligned}\frac{\lambda}{1+\tau_S}-w_N &= \frac{\lambda}{1+\tau_S}-\tau_N(1-m)(\lambda-1)-\frac{1+\tau_N(1-n_S)}{1+\tau_S n_S}(1-m+m\lambda)\\ &= \frac{(1-m)(\lambda-1)(1+\tau_S)-\tau_S\lambda(1-n_S)}{(1+\tau_S)(1+\tau_S n_S)}-\tau_N(1-m)(\lambda-1)-\frac{\tau_N(1-n_S)}{1+\tau_S n_S}(1-m+m\lambda)\\ &= (1-m)(\lambda-1)\left(\frac{1}{1+\tau_S n_S}-\tau_N\right)-\frac{1-n_S}{1+\tau_S n_S}\left[\frac{\tau_S\lambda}{1+\tau_S}+\tau_N(1-m+m\lambda)\right]\\ &\geq 0.\end{aligned}$$

For this condition to be satisfied, we need to assume

$$\frac{1}{1+\tau_S n_S}\geq\tau_N. \tag{49}$$

Next, combining (28) and (31) yields

$$\begin{aligned}&\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\\ &\quad = w_N\left\{\left[\rho-\frac{\dot{X}(t)}{X(t)}+\frac{L_N(t)}{a_N X(t)}+\frac{1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\right]n_S-\frac{1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\right\}.\end{aligned}$$

Totally differentiating this equation, we have

$$\begin{aligned}&\left\{\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\frac{1}{w_N}\frac{\partial w_N}{\partial n_S}+\left[\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}+\frac{1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}w_N\right]\frac{1}{n_S}\right\}dn_S\\ &\quad +\left\{\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\frac{1}{w_N}\frac{\partial w_N}{\partial m}+\frac{\lambda-1}{(1-m+m\lambda)^2}\frac{L_S(t)}{a_N X(t)}\left[w_N(1-n_S)+\lambda\right]\right\}dm=0. \quad (50)\end{aligned}$$

From (30), we obtain

$$\frac{\partial w_N}{\partial m}=(\lambda-1)\left[-\tau_N+\frac{1+\tau_N(1-n_S)}{1+\tau_S n_S}\right]\geq 0, \tag{51}$$

$$\frac{\partial w_N}{\partial n_S}=-\frac{\tau_N+\tau_S(1+\tau_N)}{(1+\tau_S n_S)^2}(1-m+m\lambda)\leq 0, \tag{52}$$

$$\frac{\partial w_N}{\partial n_S}n_S+w_N=\frac{1+\tau_N(1-n_S)-\tau_N n_S(1+\tau_S n_S)}{(1+\tau_S n_S)^2}(1-m+m\lambda)+\tau_N(1-m)(\lambda-1)>0, \tag{53}$$

where the first and the third inequalities use (49). Therefore, (50), (51), and (53) imply that

$$\frac{dn_S}{dm}=-\frac{(1-m)(\lambda-1)\frac{1}{w_N}\frac{\partial w_N}{\partial m}+\frac{\lambda-1}{1-m+m\lambda}\left[w_N(1-n_S)+\lambda\right]}{(1-m)(\lambda-1)\frac{1}{n_S w_N}\left(\frac{\partial w_N}{\partial n_S}n_S+w_N\right)+\frac{w_N}{n_S}}<0.$$

Differentiating (28) with respect to m, we obtain

$$\begin{aligned}\frac{dI}{dm} &= \frac{1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\frac{1}{(n_S)^2}\left[\frac{dn_S}{dm}+n_S(1-n_S)\frac{\lambda-1}{1-m+m\lambda}\right] \\ &= -\frac{\frac{\lambda-1}{1-m+m\lambda}\frac{L_S(t)}{a_N X(t)}\frac{1}{(n_S)^2}}{(1-m)(\lambda-1)\frac{1}{n_S w_N}\left(\frac{\partial w_N}{\partial n_S}n_S+w_N\right)+\frac{w_N}{n_S}} \\ &\quad \times\left[\frac{1-m}{w_N}\frac{\partial w_N}{\partial m}+1+\frac{(1-m)(\lambda-1)}{1-m+m\lambda}n_S-n_S(1-n_S)\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\frac{1}{w_N}\frac{\partial w_N}{\partial n_S}\right]<0,\end{aligned}$$

where the last inequality uses (51) - (53). Thus, a decrease in m through strengthening IPR protection in the South increases the innovation rate.

Proof of Proposition 5

From (28), (38), (39), and (41), we have

$$\frac{\partial I}{\partial \tau_N} = \frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}\frac{1}{{n_S}^2}\frac{\partial n_S}{\partial \tau_N}, \tag{54}$$

$$\begin{aligned}\frac{1+\tau_N}{E_N}\frac{\partial \frac{E_N}{1+\tau_N}}{\partial \tau_N} &= -\frac{\frac{\lambda}{1-m+m\lambda}-\frac{\tau_S}{1+\tau_S}(E_S n_S)^2}{\frac{\lambda}{1-m+m\lambda}-E_S n_S}n_S\frac{1}{{n_S}^2}\frac{\partial n_S}{\partial \tau_N} \\ &= -\frac{\frac{\lambda}{1-m+m\lambda}-\left(1-\frac{1}{1+\tau_S n_S}\right)(E_S n_S)}{\frac{\lambda}{1-m+m\lambda}-E_S n_S}n_S\frac{1}{{n_S}^2}\frac{\partial n_S}{\partial \tau_N} \\ &= -A(n_S)\frac{1}{{n_S}^2}\frac{\partial n_S}{\partial \tau_N},\end{aligned} \tag{55}$$

where

$$A(n_S) \equiv n_S + n_S\frac{\frac{1}{1+\tau_S n_S}(E_S n_S)}{\frac{\lambda}{1-m+m\lambda}-E_S n_S}. \tag{56}$$

We can show that the second term of (56) is an increasing function of n_S as follows:

$$\begin{aligned}\frac{\partial}{\partial n_S}\log\left[n_S\frac{\frac{1}{1+\tau_S n_S}(E_S n_S)}{\frac{\lambda}{1-m+m\lambda}-E_S n_S}\right] &= \frac{1}{n_S}-\frac{\tau_S}{1+\tau_S n_S}+\frac{1}{E_S n_S}\frac{\partial(E_S n_S)}{\partial n_S}+\frac{1}{\frac{\lambda}{1-m+m\lambda}-E_S n_S}\frac{\partial(E_S n_S)}{\partial n_S} \\ &= \frac{1}{n_S(1+\tau_S n_S)}+\frac{\frac{\lambda}{1-m+m\lambda}}{E_S n_S\left(\frac{\lambda}{1-m+m\lambda}-E_S n_S\right)}\frac{\partial(E_S n_S)}{\partial n_S} \\ &> 0,\end{aligned}$$

where the inequality holds because $\lambda/(1-m+m\lambda) > 1 > E_S n_S$ and $\partial(E_S n_S)/\partial n_S > 0$ from (39). Therefore, $A(\cdot)$ is an increasing function of n_S. Because $A(n_S)$ depends on τ_N only through n_S and

E_S, and $\partial n_S/\partial\tau_N < 0$ from Proposition 1, $A(\cdot)$ is a decreasing function of τ_N. From (28), (30) and (31), n_S equals

$$n_S{}^* \equiv \frac{\lambda}{1-m+m\lambda}\left[\frac{\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}}{\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}}+1-\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\tau_S\right]^{-1} \tag{57}$$

at $\tau_N = 0$, so that $n_S \leq n_S{}^*$ and $A(n_S) \leq A(n_S{}^*)$ for any $\tau_N \geq 0$.

Substituting (54) and (55) into (40) yields the Northern welfare change due to the marginal increase in its tariff as follows:

$$\begin{aligned}\frac{\partial U_N}{\partial\tau_N} &= \frac{1}{\rho-g_L}\left[\frac{\log\lambda}{\rho-g_L}\frac{\partial I}{\partial\tau_N}+\frac{1}{E_N}\frac{\partial E_N}{\partial\tau_N}-\frac{1}{1+\tau_N}+(\log\lambda)m\frac{\partial n_S}{\partial\tau_N}\right]\\ &= \frac{1}{\rho-g_L}\frac{1}{n_S{}^2}\frac{\partial n_S}{\partial\tau_N}F(n_S),\end{aligned}$$

where

$$F(n_S) \equiv \frac{\log\lambda}{\rho-g_L}\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} - A(n_S) + m(\log\lambda)n_S^2. \tag{58}$$

Next, we derive a sufficient condition that the first term of $F(n_S)$ surpasses the second term $A(n_S)$, which ensures $\partial U_N/\partial\tau_N < 0$. To this end, we rewrite $A(n_S)$ as follows:

$$\begin{aligned}A(n_S) &= \left[1+\frac{\frac{1}{1+\tau_S n_S}(E_S n_S)}{\frac{\lambda}{1-m+m\lambda}-E_S n_S}\right]n_S\\ &= \left[\frac{\frac{\lambda}{1-m+m\lambda}-\frac{\tau_S n_S}{1+\tau_S n_S}(E_S n_S)}{\frac{\lambda}{1-m+m\lambda}-E_S n_S}\right]n_S\\ &= \left[\frac{\frac{\lambda}{1-m+m\lambda}-\frac{\tau_S n_S}{1+\tau_S n_S}\frac{1+\tau_S}{1+\tau_S n_S}n_S}{\frac{\lambda}{1-m+m\lambda}-\frac{1+\tau_S}{1+\tau_S n_S}n_S}\right]n_S\\ &= \left[\frac{\frac{\lambda}{1-m+m\lambda}(1+\tau_S n_S)^2-\tau_S(1+\tau_S)n_S{}^2}{\frac{\lambda}{1-m+m\lambda}(1+\tau_S n_S)^2-(1+\tau_S)(1+\tau_S n_S)n_S}\right]n_S,\end{aligned}$$

where the third equality uses (39). For any $\tau_N \geq 0$,

$$\begin{aligned}A(n_S) &\leq A(n_S{}^*)\\ &= \frac{\frac{\lambda}{1-m+m\lambda}(1+\tau_S n_S{}^*)^2 n_S{}^* - \tau_S(1+\tau_S)n_S{}^{*3}}{\frac{\lambda}{1-m+m\lambda}(1+\tau_S n_S{}^*)^2-(1+\tau_S)(1+\tau_S n_S{}^*)n_S{}^*}\\ &\leq \frac{\frac{\lambda}{1-m+m\lambda}(1+\tau_S n_S{}^*)^2 n_S{}^*}{\frac{\lambda}{1-m+m\lambda}(1+\tau_S n_S{}^*)^2-(1+\tau_S)(1+\tau_S n_S{}^*)n_S{}^*}\\ &= \frac{\frac{\lambda}{1-m+m\lambda}(1+\tau_S n_S{}^*)n_S{}^*}{\frac{\lambda}{1-m+m\lambda}(1+\tau_S n_S{}^*)-(1+\tau_S)n_S{}^*},\end{aligned} \tag{59}$$

where ${n_S}^*$ is given by (57). From (57), the denominator of the RHS of (59) is rewritten as follows:

$$\begin{aligned}
& \frac{\lambda}{1-m+m\lambda}(1+\tau_S {n_S}^*) - (1+\tau_S){n_S}^* \\
= \;& \frac{\lambda}{1-m+m\lambda}\left\{1+\tau_S \frac{\lambda}{1-m+m\lambda}\frac{\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}}{\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}+\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}\left[1-\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\tau_S\right]}\right\} \\
& -(1+\tau_S)\frac{\lambda}{1-m+m\lambda}\frac{\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}}{\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}+\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}\left[1-\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\tau_S\right]} \\
= \;& \frac{\lambda}{1-m+m\lambda}\frac{\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}}{\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}+\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}\left[1-\frac{(1-m)(\lambda-1)}{1-m+m\lambda}\tau_S\right]} \\
= \;& {n_S}^*\left[\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}\right]\left[\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}\right]^{-1}.
\end{aligned}$$

Substituting this into the RHS of (59), we obtain

$$\begin{aligned}
A(n_S) \;&\leq\; \frac{\frac{\lambda}{1-m+m\lambda}(1+\tau_S {n_S}^*){n_S}^*}{{n_S}^*\left[\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}\right]\left[\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}\right]^{-1}} \\
&=\; \frac{\lambda}{1-m+m\lambda}(1+\tau_S {n_S}^*)\frac{\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}}{\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}} \\
&\leq\; \frac{\lambda}{1-m+m\lambda}(1+\tau_S)\frac{\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}}{\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}}.
\end{aligned}$$

By applying this to (58), we have

$$\begin{aligned}
F(n_S) \;&\geq\; \frac{\log\lambda}{\rho-g_L}\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} - \frac{\lambda}{1-m+m\lambda}(1+\tau_S)\frac{\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}}{\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}} + m(\log\lambda)n_S^2 \\
&=\; \frac{\frac{L_S(t)}{(1-m+m\lambda)a_N X(t)}}{\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}}\left\{\frac{\log\lambda}{\rho-g_L}\left[\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}\right]-\frac{\lambda}{1-m+m\lambda}(1+\tau_S)\right\} + m(\log\lambda)n_S^2.
\end{aligned}$$

Therefore, if $\frac{\log\lambda}{\rho-g_L}\left[\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}\right] > \frac{\lambda}{1-m+m\lambda}(1+\tau_S)$, the sum of the innovation-impeding and price-raising effects dominates the income effect, and $F(n_S) > 0$. Because $\partial n_S/\partial\tau_N < 0$ from Proposition 1, we conclude that $\partial U_N/\partial\tau_N < 0$ for any $\tau_N \geq 0$ if $\frac{\log\lambda}{\rho-g_L}\left[\frac{L_N(t)}{a_N X(t)}+\rho-\frac{\dot{X}(t)}{X(t)}\right] > \frac{\lambda}{1-m+m\lambda}(1+\tau_S)$.

Proof of Proposition 6

Differentiating (28) and (39) with respect to τ_S, we derive the effect of the marginal increase in Southern tariff as follows:

$$\frac{\partial I}{\partial \tau_S} = \frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} \frac{1}{n_S{}^2} \frac{\partial n_S}{\partial \tau_S},$$

$$\frac{\partial E_S}{\partial \tau_S} = \frac{(E_S)^2 n_N}{1+\tau_S} \left(\frac{1}{1+\tau_S} - \frac{\tau_S}{n_N} \frac{\partial n_S}{\partial \tau_S} \right).$$

Substituting these into (42) and rewriting, we obtain

$$\begin{aligned}
\frac{\partial U_S}{\partial \tau_S} &= \frac{1}{\rho - g_L} \left[\frac{B}{n_S{}^2} \frac{\partial n_S}{\partial \tau_S} + \frac{E_S n_N}{1+\tau_S} \left(\frac{1}{1+\tau_S} - \frac{\tau_S}{n_N} \frac{\partial n_S}{\partial \tau_S} \right) + (\log \lambda) m \frac{\partial n_S}{\partial \tau_S} \right] \\
&= \frac{1}{\rho - g_L} \left\{ \left[\frac{B}{n_S{}^2} - \frac{\tau_S}{1+\tau_S n_S} + (\log \lambda) m \right] \frac{\partial n_S}{\partial \tau_S} + \frac{E_S n_N}{(1+\tau_S)^2} \right\} \\
&= \frac{1}{\rho - g_L} \left\{ \frac{B + B\tau_S n_S + [(\log \lambda) m - \tau_S] n_S{}^2 + (\log \lambda) m \tau_S n_S{}^3}{n_S{}^2 (1+\tau_S n_S)} \frac{\partial n_S}{\partial \tau_S} + \frac{E_S n_N}{(1+\tau_S)^2} \right\},
\end{aligned} \quad (60)$$

where $B \equiv \frac{\log \lambda}{\rho - g_L} \frac{L_S(t)}{(1-m+m\lambda)a_N X(t)} > 0$ and the second equality uses (39). If the coefficient of $\partial n_S / \partial \tau_S$ in (60) is positive, $\partial U_S / \partial \tau_S > 0$ because $\partial n_S / \partial \tau_S > 0$ from Proposition 2. A sufficient condition for this is that the quadratic part $G(n_S) \equiv B + B\tau_S n_S + [(\log \lambda) m - \tau_S] n_S{}^2$ is positive. (i) If $(\log \lambda) m - \tau_S \geq 0$, then $G(n_S) > 0$ for all $n_S \in (0,1)$. (ii) If $(\log \lambda) m - \tau_S < 0$, the quadratic function $G(\cdot)$ is concave. Therefore, if $G(1) > 0$, then $G(n_S)$ is positive for all $n_S \in (0,1)$ because $G(0) > 0$. The condition for $G(1) > 0$ is rewritten as $B + (\log \lambda) m + (B-1)\tau_S > 0$.

In conclusion, $\partial U_S / \partial \tau_S > 0$ if $B + (\log \lambda) m + (B-1)\tau_S > 0$.

References

[1] Baldwin, R. E., Robert-Nicoud, F., 2008. Trade and growth with heterogeneous firms. *Journal of International Economics*, 74(1), 21–34.

[2] Belderbos, R., Sleuwaegen, L., 1998. Tariff jumping DFI and export substitution: Japanese electronics firms in Europe. *International Journal of Industrial Organization*, 16(5), 601–638.

[3] Chen, M. X., Moore, M. O., 2010. Location decision of heterogeneous multinational firms. *Journal of International Economics*, 80(2), 188–199.

[4] Chu, A. C., Cozzi, G., Furukawa, Y., 2015. Effects of economic development in China on skill-biased technical change in the US. *Review of Economic Dynamics*, 18(2), 227–242.

[5] Cozzi, G., Impullitti, G., 2010. Government spending composition, technical change and wage inequality. *Journal of the European Economic Association*, 8(6), 1325–1358.

[6] Dinopoulos, E., Segerstrom, P., 1999a. A Schumpeterian model of protection and relative wages. *American Economic Review*, 89(3), 450–472.

[7] Dinopoulos, E., Segerstrom, P., 1999b. The dynamic effects of contingent tariffs. *Journal of International Economics*, 47(1), 191–222.

[8] Dinopoulos, E., Segerstrom, P., 2007. North–South trade and economic growth. CEPR Discussion Paper No. 5887.

[9] Dinopoulos, E., Syropoulos, C., 1997. Tariffs and Schumpeterian growth. *Journal of International Economics*, 42(3–4), 425–452.

[10] Dinopoulos, E., Thompson, P., 2000. Endogenous growth in a cross-section of countries. *Journal of International Economics*, 51(2), 335–362.

[11] Dinopoulos, E., Unel, B., 2011. Quality heterogeneity and global economic growth. *European Economic Review*, 55(5), 595–612.

[12] Fajgelbaum, P. D., Goldberg, P. K., Kennedy, P. J., Khandelwal, A. K., 2020. The return to protectionism. *Quarterly Journal of Economics*, 135(1), 1–55.

[13] Felbermayr, G., Jung, B., Larch, M., 2013. Optimal tariffs, retaliation, and the welfare loss from tariff wars in the Melitz model. *Journal of International Economics*, 89(1), 13–25.

[14] Ghodsi, M., 2020. How do technical barriers to trade affect foreign direct investment? Tariff jumping versus regulation haven hypotheses. *Structural Change and Economic Dynamics*, 52, 269–278.

[15] Glass, A. J., Wu, X., 2007. Intellectual property rights and quality improvement. *Journal of Development Economics*, 82(2), 393–415.

[16] Grieben W-H., Şener, F., 2009. Globalization, rent protection institutions, and going alone in freeing trade. *European Economic Review*, 53(8), 1042–1065.

[17] Gros, D., 1987. A note on the optimal tariff, retaliation and the welfare loss from tariff wars in a framework with intra-industry trade. *Journal of International Economics*, 23(3–4), 357–367.

[18] Grossman, G. M., Helpman, E., 1991. *Innovation and Growth in the Global Economy*. MIT Press, Cambridge, MA.

[19] Grossman, G. M., Lai, E. L.-C., 2004. International protection of intellectual property. *American Economic Review*, 94(5), 1635–1653.

[20] Gustafsson, P., Segerstrom, P., 2010. Trade liberalization and productivity growth. *Review of International Economics*, 18(2), 207–228.

[21] Gustafsson, P., Segerstrom, P. S., 2011. North–South trade with multinational firms and increasing product variety. *International Economic Review*, 52(4), 1123–1155.

[22] Iwaisako, T., Tanaka, H., Futagami, K., 2011. A welfare analysis of global patent protection in a model with endogenous innovation and foreign direct investment. *European Economic Review*, 55(8), 1137–1151.

[23] Keller, W., 2004. International technology diffusion. *Journal of Economic Literature*, 42(3), 752–782.

[24] Kennan, J., Riezman, R., 1988. Do big countries win tariff wars? *International Economic Review*, 29(1), 81–85.

[25] Lai, E. L-C., 1998. International intellectual property rights protection and the rate of product innovation. *Journal of Development Economics*, 55(1), 133–153.

[26] Naito, T., 2017. Growth and welfare effects of unilateral trade liberalization with heterogeneous firms and asymmetric countries. *Journal of International Economics*, 109, 167–173.

[27] Naito, T., 2019. A larger country sets a lower optimal tariff. *Review of International Economics*, 27(2), 643–665.

[28] Opp, M. M., 2010. Tariff wars in the Ricardian Model with a continuum of goods. *Journal of International Economics*, 80(2), 212–225.

[29] Sampson, T., 2016. Dynamic selection: An idea flows theory of entry, trade, and growth. *Quarterly Journal of Economics*, 131(1), 315–380.

[30] Şener, F., Zhao, L., 2009. Globalization, R&D and the iPod Cycle. *Journal of International Economics*, 77(1), 101–108.

[31] Syropoulos, C., 2002. Optimum tariffs and retaliation revisited: How country size matters. *Review of Economic Studies*, 69(3), 707–727.

[32] Tanaka, H., Iwaisako, T., 2014. Intellectual property rights and foreign direct investment: A welfare analysis. *European Economic Review*, 67, 107–124.