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Dynamic Analysis of Education, Automation, and Economic Growth

Kohei Okada

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Graduate School of Economics
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

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Kohei Okada†

Abstract

Ever since the onset of the Industrial Revolution, automation has had significant impacts on economic growth, labor, the education decision-making of individuals, and education policy. In this study, we aim to examine the complex relationship between education, automation, and economic growth. We employ an overlapping-generations model with endogenous education decision-making and automation. Our findings show that an economy converges to a steady state where automation occurs and per capita output is high if productivity is high. On the other hand, we show that an economy converges to a steady state where automation does not occur and per capita output is low if productivity is low. In addition, we examine how education subsidy policy affects the economy when productivity is low. If the efficiency of education is high, the government can steer an economy away from a steady state without automation by investing more resources in education. If the efficiency of education is low, there can exist multiple steady states where automation occurs in one but not in the other.

Keywords: Education, Automation, Economic growth

JEL Classification Numbers: E22, J24, O10, O30

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†Graduate School of Economics, Osaka University. Address: 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: sge005ok@student.econ.osaka-u.ac.jp

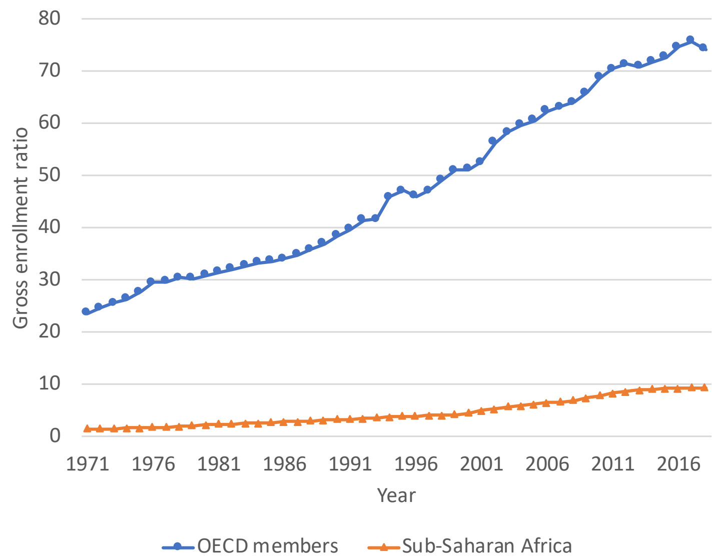

Figure 1: Trends in gross educational enrollment ratio in developed and developing countries. Source: World Bank.

1 Introduction

In the developed centuries since the onset of the Industrial Revolution, the rise of automation has allowed machines to take over tasks which previously were performed by human workers. This transformation dramatically reduced the costs associated with production, increasing productivity and freeing up human labor. On the other hands, developing countries cannot introduces machines in their production well. It makes productions of developing countries low and hinder their economic growth. Figure 1 shows the trends in gross educational enrollment ratio in both developed and developing countries. In the developed countries, the gross enrollment ratio of the school-aged population has steadily increased over the last four decades. This in turn means that the number of skilled workers has increased and the number of unskilled workers has decreased in these countries. In addition, Figure 1 shows a significant and growing gap in the gross educational enrollment ratio between developed and developing countries. Therefore, there are more unskilled workers in developing countries than in developed countries. Firms in developing countries can employ these unskilled workers by paying low wage. Therefore, these firms can have these unskilled workers engage in manual labor at low cost in spite of not introducing machine. However, it is inefficient that workers engage in manual job which can be easily produced by machines. This phenomena hinders economic growth in developing countries.

On the other hands, automation is often also perceived as a threat to employment of workers across many industries and occupations. For example, a recent study by Frey and Osborne (2017) examined 702 occupations and found that almost half of them could be automated within the next two decades.

In particular, unskilled occupations, typically performed by workers with low levels of education, are at the greatest risk of being replaced by machines. Given the rising importance of automation in economies throughout the world, we have developed a simple macro model that incorporates automation and education decision-making to explore their effects on macroeconomy. In addition, we discuss how education policy affects economic growth and automation.

We construct a simple small open overlapping-generations model with endogenous education decision-making and automation. In the model, each individual's life consists of three periods (childhood, adulthood, and old age). In the first period of their lives, individuals do not make any decisions. In the second period, individuals invest in children's education, supply labor, pay lump-sum taxes, consume final goods, and save any remaining income. In the final period, individuals retire and consume final goods.

In this model, there is a final goods sector and an intermediate goods sector. In the final goods sector, firms produce final goods by combining intermediate goods. In the intermediate goods sector, firms produce intermediate goods by using labor or capital (machine). Each intermediate goods firm chooses the form of input (labor or capital) that minimizes its costs. With our model, we show that an economy converges to a steady state where automation occurs and per capita output is high if productivity of intermediate goods is high. On the other hand, we show that an economy converges to a steady state where automation does not occur and per capita output is low if productivity of intermediate goods is low.

In addition, we examine how education policy affects the economy when productivity of intermediate goods is low. If the efficiency of education is high, the government can steer an economy away from the steady state without automation by investing more resources in education. If the efficiency of education is low, there can exist multiple steady states where automation occurs at one of them but not in the other.

This study relates to and draws on the existing literature regarding automation, education and economic growth. An early study by Zeira (1998) develops a growth model with capital-labor substitution, which forms the basis for analyzing automation in subsequent studies. Recent studies by Acemoglu and Restrepo (2018c) and by Hémous and Olsen (2014) generalize the model developed by Zeira (1998). They examine the relationship between automation and economic growth in dynamic general equilibrium models and explore the effects of automation on the labor market and income inequality. Chu et al. (2018) introduce automation into a Schumpeterian growth model to explore the effects of R&D and automation subsidies on economic growth. However, these existing studies on automation do not discuss endogenous education decision-making and education policy.

Prettner and Strulik (2019) analyze the effects of R&D-driven automation on economic growth, education, and inequality when high-skilled workers are complements to machines and low-skilled workers are substitutes to machines. Their study analytically analyzes only balanced growth path and examine the transitional dynamics only in a numerical approach. The present study analyzes the transitional dynamics of the economy and also the dynamic relationships between education, automation, and economic growth. Additionally, Acemoglu and Restrepo (2018a,b) examine how automation affects wage inequality between skilled and unskilled workers. However, they discuss this issue in the context of a static partial equilibrium model, and do not incorporate education decision-making.

The present study also relates to the existing literature on intergenerational mobility. Maoz and Moav (1999) and Fan and Zhang (2013) employ overlapping-generations models with education decision-making to discuss intergenerational mobility and economic development. However, they do not take automation factors into account, whereas our study examines intergenerational mobility and automation simultaneously. We therefore are able to discuss the relationship between wage gaps, automation, and economic development.

The rest of this paper is organized as follows: Section 2 explains the basic structure of the model; Section 3 analyzes equilibrium and dynamics; Section 4 examines how education policy affects the economy; and Section 5 contains the conclusion.

2 The model

2.1 Individuals

Time is discrete and denoted by $t = 0, 1, 2, \cdots$. Each individual's life consists of three periods (childhood, adulthood, and old age). In the first period (childhood), individuals do not make any decisions and they are raised by their parents. In the second period (adulthood), individuals invest in education for their children, supply 1 unit of labor, consume final goods, pay lump-sum taxes, and save any remaining income. In the final period (old age), individuals retire and consume final goods. Members of the cohort born in period $t-1$ become active workers in period t. Thus, we call this cohort generation t. The number of population is constant and normalized at one.

There are two kinds of workers, high-skilled workers and low-skilled workers. Let us define λ_t as the number of high-skilled workers. The superscript $j \in \{H, L\}$ represents the type of workers' skill and $j = H$ ($j = L$) means high (low) skill. High-skilled workers can engage in production of all intermediate goods; low-skilled workers cannot engage in production of all intermediate goods. As discussed later, the individual's type of skill is determined by his / her parents' education investment level and government's education expenditure level. Individuals whose type is j derive their utility from the level of investment for their children e_t^j, their own consumption during adulthood $c_{1,t}^j$, and their own consumption during old age $c_{2,t+1}^j$.

The lifetime utility of individuals in generation t is expressed as

$$u_t^j = \gamma \log e_t^j + \log c_{1,t}^j + \beta \log c_{2,t+1}^j, \quad j \in \{H, L\}. \tag{1}$$

where the positive parameter γ denotes the weights of the children's educational outcome and $\beta \in (0,1]$ denotes the discount factor. In an individual's second period of life (adulthood), they are endowed with 1 unit of time, which is devoted to working in the labor market inelastically. Individuals divide their disposable income $w_t^j - \tau$ between consumption, investment in each child e_t^j, and saving s_t^j for their old age. Here, w_t^j and τ are the wage rate of an individual whose skill is j and the lump-sum tax, respectively. R represents the gross interest rate and R is constant over time because this economy is supposed to be small open. Thus, the budget constraints for individuals in generation t are expressed as follows:

$$c_{1,t}^j = w_t^j - \tau - d_t^j n_t^j - s_t^j, \tag{2}$$

$$c_{2,t+1}^j = R s_t^j. \tag{3}$$

The j type worker maximizes (1) subject to (2) and (3). From the utility maximization problem, we obtain the following solutions:

$$e_t^j = \frac{\gamma}{1+\beta+\gamma}(w_t^j - \tau) \tag{4}$$

$$s_t^j = \frac{\beta}{1+\beta+\gamma}(w_t^j - \tau). \tag{5}$$

2.2 Final goods sector

The final goods sector is perfectly competitive. The final goods y_t are produced by combining the intermediate goods of a unit measure of $i \in [0,1]$. The production function of final goods is given by

$$\log y_t = \int_0^1 \log B x_t(i) di, \tag{6}$$

where $x_t(i)$ is the input of intermediate goods i and B denotes the productivity of intermediate goods. Let us denote the demand for intermediate goods i as $x_t(i)$. From profit maximization condition, $x_t(i)$ is given by

$$x_t(i) = \frac{y_t}{p_t(i)}. \tag{7}$$

2.3 Intermediate goods sector

There are intermediate goods indicated by $i \in [0,1]$. Each intermediate good can be produced by capital or by human labor, depending on whether it has been technologically automated or not. Each intermediate goods firm decides whether to automate or not. Intermediate goods in the range $[0, I]$ are technologically automated and can be produced by either labor or capital (machine). Intermediate goods in the range $(I, 1]$ are not technologically automated and must be produced by labor. Intermediate goods in the range $(I, S]$ are not technologically automated and must be produced with low- and/or high-skilled labor. Intermediate goods in the range $(S, 1]$ are not technologically automated and can be performed only by high-skilled workers (high-skilled labor). The production function of intermediate goods is given by

$$x_t(i) = \begin{cases} h_t(i) + l_t(i) + k_t(i) & \text{if} \quad i \in [0, I] \\ h_t(i) + l_t(i) & \text{if} \quad i \in (I, S] \\ h_t(i) & \text{if} \quad i \in (S, 1] \end{cases}, \tag{8}$$

where $k_t(i)$, $l_t(i)$, and $h_t(i)$ denote the input of capital, low-skilled labor, and high-skilled labor, respectively, to produce intermediate good i. We assume that capital depreciates perfectly during one period. Let us consider the profit maximization problem of the intermediate goods firm i. First, the firm determines which inputs to use for production. Then, the free entry condition determines the price of intermediate goods i. For $i \in [0, I]$ the cost minimization problem of intermediate goods firm i is as follows:

$$\min_{h_t(i), l_t(i), k_t(i)} \quad w_t^H h_t(i) + w_t^L l_t(i) + R k_t(i),$$
$$\text{s.t.} \quad x_t(i) = h_t(i) + l_t(i) + k_t(i).$$

From this cost minimization problem, we obtain

$$x_t(i) = \begin{cases} k_t(i) & \text{if} \quad R = \min\left\{w_t^H, w_t^L, R\right\} \\ l_t(i) & \text{if} \quad w_t^L = \min\left\{w_t^H, w_t^L, R\right\} \\ h_t(i) & \text{if} \quad w_t^H = \min\left\{w_t^H, w_t^L, R\right\} \end{cases}. \tag{9}$$

(9) implies that the intermediate goods firm i chooses the cheapest input to produce good i. From the free entry condition, the price of good i is given by

$$p_t(i) = \begin{cases} R & \text{if} \quad R = \min\left\{w_t^H, w_t^L, R\right\} \\ w_t^L & \text{if} \quad w_t^L = \min\left\{w_t^H, w_t^L, R\right\} \\ w_t^H & \text{if} \quad w_t^H = \min\left\{w_t^H, w_t^L, R\right\} \end{cases}. \tag{10}$$

Similarly, for $i \in (I, S]$, the cost minimization problem of intermediate goods firm i is as follows:

$$\begin{aligned} \min_{h_t(i), l_t(i)} \quad & w_t^H h_t(i) + w_t^L l_t(i), \\ \text{s.t.} \quad & x_t(i) = h_t(i) + l_t(i). \end{aligned}$$

From this cost minimization problem, we obtain

$$x_t(i) = \begin{cases} l_t(i) & \text{if} \quad w_t^H > w_t^L \\ h_t(i) & \text{if} \quad w_t^H \leq w_t^L \end{cases}. \tag{11}$$

From the free entry condition, the price of good i is given by

$$p_t(i) = \begin{cases} w_t^L & \text{if} \quad w_t^H > w_t^L \\ w_t^H & \text{if} \quad w_t^H \leq w_t^L \end{cases}. \tag{12}$$

For $i \in (S, A]$, the intermediate goods firm only uses high-skilled workers. From the free entry condition, the price of good i is given by

$$p_t(i) = w_t^H. \tag{13}$$

2.4 Government

A government collects lump-sum taxes from individuals and invests it in education. We denote per capita government's expenditure for education as G_t. This expenditure is financed by the government's balanced budget. Noting that the number of population is normalized at one, the government's budget constraint is

$$\tau = G_t. \tag{14}$$

Let q_t^j denote the probability of a j type worker's child becoming a skilled worker. We assume that q_t^j depends on j type worker's total expenditure for education E_t^j and the efficiency of education ϕ. E_t^j is given by

$$E_t^j = e_t^j + G_t. \tag{15}$$

We assume that $q_t^j = q(E_t^j;\phi) \in [0,1]$, $\frac{\partial q(E_t^j;\phi)}{\partial E_t^j} > 0$, $\frac{\partial^2 q(E_t^j;\phi)}{\partial (E_t^j)^2} < 0$, $\lim_{E_t^j \to 0} q(E_t^j;\phi) = \underline{q} \geq 0$, $\lim_{E_t^j \to \infty} q(E_t^j;\phi) = \bar{q} \leq 1$, and $\frac{\partial q(E_t^j;\phi)}{\partial \phi} > 0$. Combining (4), (14), and (15), we obtain

$$E_t^j = \frac{\gamma}{1+\beta+\gamma} w_t^j + \frac{1+\beta}{1+\beta+\gamma}\tau. \tag{16}$$

We find that E_t^j depends on w_t^j, τ, and ϕ. Therefore, we can regard q_t^j as a function of w_t^j, τ, and ϕ and denote it as $q_t = q(w_t^j;\tau,\phi)$. From (16), we find that $\frac{\partial q(w_t^j;\tau,\phi)}{\partial w_t^j} > 0$, $\frac{\partial^2 q(w_t^j;\tau,\phi)}{\partial (w_t^j)^2} < 0$ and $\frac{\partial q(w_t^j;\tau,\phi)}{\partial \tau} > 0$. This probability of children becoming skilled workers allows our model to address intergenerational mobility together with differential investment in education. If $w_t^j < \tau$, the individual's disposable income becomes negative. Therefore, we assume that the government can implement the education policy only if $w_t^j > \tau$ holds.

3 Equilibrium

3.1 No automation equilibrium

If $R > w_t^L$ holds, no production of intermediate goods are automated in the range $[0, I]$ because the marginal cost of low-skilled labor (i.e., w_t^L) is lower than the marginal cost of capital (i.e., R). We call this equilibrium "*no automation equilibrium.*" From (7), (8), and (9), the demand for capital to produce each intermediate good is

$$k_t(i) = 0. \tag{17}$$

From (7), (8), (9), (11), and (12), the demand for low-skilled labor to produce each intermediate good is

$$l_t(i) = \begin{cases} \frac{y_t}{w_t^L} & \text{if} \quad i \in [0, S] \\ 0 & \text{otherwise} \end{cases}. \tag{18}$$

From (7), (8), (9), (11), (12), and (13), the demand for high-skilled labor to produce each intermediate good is

$$h_t(i) = \begin{cases} \frac{y_t}{w_t^H} & \text{if} \quad i \in (S, 1] \\ 0 & \text{otherwise} \end{cases}. \tag{19}$$

In this no automation equilibrium, no intermediate goods firm uses capital, and the intermediate firm whose index i is in the rate $[0, S]$ uses low-skilled labor. The low-skilled labor market-clearing condition is

$$(1 - \lambda_t) = \int_0^S l_t(i) di. \tag{20}$$

The left-hand side of (20) represents the supply of low-skilled labor and the right-hand side of (20) represents the demand of low-skilled labor. The high-skilled labor market equilibrium condition is

$$\lambda_t = \int_S^1 h_t(i) di. \tag{21}$$

The left-hand side of (21) represents the supply of high-skilled labor and the right-hand side of (21) represents the demand of high-skilled labor. As shown in Appendix A, $w_t^H > w_t^L$ holds in this equilibrium (see Appendix A). Therefore, the intermediate goods firm whose index i is in the rate $(I, S]$ uses low-skilled labor in this equilibrium.

Let us define ξ_t as $\xi_t \equiv \frac{\lambda_t}{1-\lambda_t}$. As shown in Appendix B, the wage rate of low-skilled labor is given by

$$w_t^L = B \left(\frac{S\xi_t}{1 - S} \right)^{1-S} \equiv w_N^L(\xi_t), \tag{22}$$

Let us define $w_N^L(\xi_t)$ as the right-hand side of (22). Similarly, the wage rate of high-skilled labor is given by

$$w_t^H = B \left(\frac{1 - S}{S\xi_t} \right)^S \equiv w_N^H(\xi_t). \tag{23}$$

Let us define $w_N^H(\xi_t)$ as the right-hand side of (23). From (22), we can rewrite $R > w_t^L$ as follows:

$$\xi_t < \frac{1 - S}{S} \left(\frac{R}{B} \right)^{\frac{1}{1-S}} \equiv \hat{\xi}_N. \tag{24}$$

Let us define $\hat{\xi}_N$ as the right-hand side of (24).

3.2 Partial automation equilibrium

If $w_t^L = R$ holds, some production of intermediate goods are automated in the range $[0, I]$. In this equilibrium, there exists a threshold value $\hat{I}_t$ such that intermediate goods in the range $[0, \hat{I}_t]$ are automated and intermediate goods in the range $(\hat{I}, I]$ are not automated. $\hat{I}_t$ is determined to satisfy $w_t^L = R$. We call this equilibrium "*partial automation equilibrium.*" From (7), (9), and (10), the demand for capital to produce each intermediate good is

$$k_t(i) = \begin{cases} \frac{y_t}{R} & \text{if} \quad i \in [0, \hat{I}_t] \\ 0 & \text{otherwise} \end{cases}. \tag{25}$$

From (7), (9), (10), (11), and (12), the demand for low-skilled labor to produce each intermediate good is

$$l_t(i) = \begin{cases} \frac{y_t}{w_t^L} & \text{if} \quad i \in (\hat{I}_t, S] \\ 0 & \text{otherwise} \end{cases}. \tag{26}$$

The demand for high-skilled labor to produce intermediate good is expressed as (19). Note that this economy is small open. Let us define the supply of capital from the international asset market as K_t. The capital market-clearing condition is

$$K_t = \int_0^{\hat{I}_t} k_t(i) di. \tag{27}$$

The right-hand side of (27) is the capital demand of intermediate goods firms. In this partial automation equilibrium, the intermediate firms whose index i is in the rate $[0, \hat{I}_t]$ use capital. The low-skilled labor market-clearing condition is

$$(1 - \lambda_t) = \int_{\hat{I}_t}^{S} l_t(i) di. \tag{28}$$

The left-hand side of (28) represents the supply of low-skilled labor and the right-hand side of (28) represents the demand of low-skilled labor. In this equilibrium, the intermediate firms whose index i is in the rate $(\hat{I}_t, S]$ use low-skilled labor because $w_t^H > w_t^L$ holds (see Appendix A). The high-skilled labor market-clearing condition is (21) also in this partial automation equilibrium. The wage rate of low-skilled labor is given by

$$w_t^L = R. \tag{29}$$

As shown in Appendix C, the wage rate of high-skilled labor is given by

$$w_t^H = B\left(\frac{B}{R}\right)^{\frac{S}{1-S}} \equiv w_P^H. \tag{30}$$

Let us define w_P^H as the right-hand side of (30). As shown in Appendix C, we obtain $\hat{I}_t$ as follows:

$$\hat{I}_t = S - \frac{1-S}{\xi_t}\left(\frac{R}{B}\right)^{\frac{1}{1-S}}. \tag{31}$$

3.3 Full automation equilibrium

If $w_t^L > R$ holds, all production of intermediate goods are automated in the range $[0, I]$ because the marginal cost of low-skilled labor (i.e., w_t^L) is higher than the marginal cost of capital (i.e., R). We call this equilibrium "*full automation equilibrium.*" From (7), (9), and (10), the demand for capital to produce each intermediate good is

$$k_t(i) = \begin{cases} \frac{y_t}{R} & \text{if} \quad i \in [0, I] \\ 0 & \text{otherwise} \end{cases}. \tag{32}$$

From (7), (9), (10), (11), and (12), the demand for low-skilled labor to produce each intermediate good is

$$l_t(i) = \begin{cases} \frac{y_t}{w_t^L} & \text{if} \quad i \in (I, S] \\ 0 & \text{otherwise} \end{cases}. \tag{33}$$

The demand for high-skilled labor to produce intermediate goods is expressed as (19). The capital market-clearing condition is

$$K_t = \int_0^I k_t(i)di. \tag{34}$$

The right-hand side of (34) is the capital demand of intermediate goods firms. In this full-automation equilibrium, the intermediate firms whose index i is in the rate $[0, I]$ use capital. The low-skilled labor market-clearing condition is

$$(1-\lambda_t) = \int_I^S l_t(i)di. \tag{35}$$

The left-hand side of (35) represents the supply of low-skilled labor and the right-hand side of (35) represents the demand of low-skilled labor. In this equilibrium, the intermediate firms whose index i is in the rate $(I, S]$ use low-skilled labor because $w_t^H \geq w_t^L$ holds (this is discussed later). The high-skilled labor market-clearing condition is expressed as (21) also in this full automation equilibrium. As shown in Appendix D, the wage rate of low-skilled labor is given by

$$w_t^L = B\left(\frac{B}{R}\right)^{\frac{I}{1-I}}\left[\frac{(S-I)\xi_t}{1-S}\right]^{\frac{1-S}{1-I}} \equiv w_F^L(\xi_t). \tag{36}$$

Let us define $w_F^L(\xi_t)$ as the right-hand side of (36). Similarly, the wage rate of high-skilled labor is given by

$$w_t^H = B\left(\frac{B}{R}\right)^{\frac{I}{1-I}}\left[\frac{1-S}{(S-I)\xi_t}\right]^{\frac{S-I}{1-I}} \equiv w_F^H(\xi_t). \tag{37}$$

Let us define $w_F^H(\xi_t)$ as the right-hand side of (37). From (37), we can rewrite $w_t^L > R$ as follows:

$$\xi_t > \frac{1-S}{S-I}\left(\frac{R}{B}\right)^{\frac{1}{1-S}} \equiv \hat{\xi}_P. \tag{38}$$

Let us define $\hat{\xi}_P$ as the right-hand side of (38). We make the following assumption to clarify whether $\hat{\xi}_P$ is larger than $\hat{\xi}_N$:

Assumption 1

$$B > R.$$

Under Assumption 1, $\hat{\xi}_P > \hat{\xi}_N$ holds from (24) and (38).

Let us discuss that $w_t^H \geq w_t^L$ holds in the full automation equilibrium. High-skilled workers can engage in production of intermediate goods in the range $(0, S]$. In other words, high-skilled workers can work as low-skilled workers. Let us consider the case in which $w_t^H < w_t^L$ holds, all high-skilled workers work as low-skilled workers. However, the production of intermediate goods $i \in (S, 1]$ which need high-skilled worker exist. Therefore, w_t^H must diverge to positive infinity. Thus, there is no equilibrium. Hence, $w_t^H \geq w_t^L$ holds in equilibrium and $w_t^H = w_t^L$ is the corner solution of the wage rate. From (36) and (37), $\frac{1-S}{(S-I)\xi_t} = 1$ holds when $w_t^H = w_t^L$ holds. Let us define $\bar{\xi}_F$ as $\bar{\xi}_F \equiv \frac{1-S}{S-I}$.

From (37) and $\frac{1-S}{(S-I)\xi_t} = 1$, if $\xi_t \geq \bar{\xi}_F$, the wage rate is given by

$$w_t^H = w_t^L = B\left(\frac{B}{R}\right)^{\frac{I}{1-I}} \equiv \bar{w}. \tag{39}$$

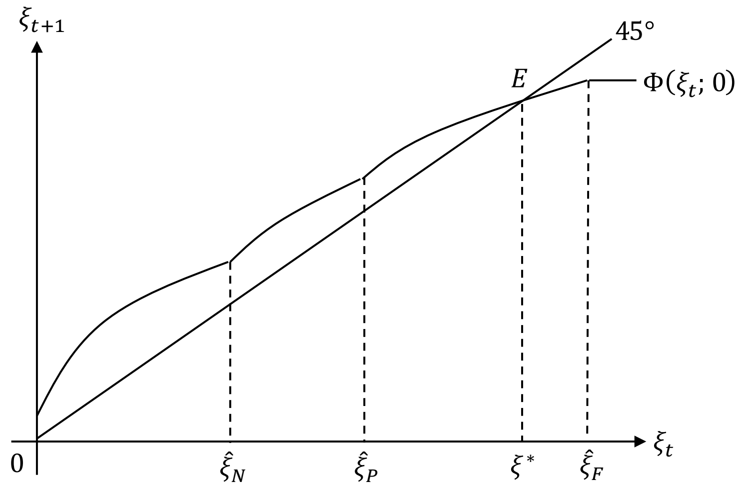

Figure 2: Dynamics of ξ_t when $\tau = 0$ and $B > \bar{B}(0)$

Let us define $\bar{w}$ as the right-hand side of (36).

We finally characterize the output level of final goods. We obtain the following Lemma 1.

Lemma 1 *y_t increases with ξ_t and does not depend on τ.*

Proof see Appendix E.

From Lemma 1, we can regard y_t as a function of ξ_t and denote $y_t = y(\xi_t)$.

3.4 Dynamics

The dynamics of this economy is characterized by the proportion of high-skilled individuals to low-skilled individuals ξ_t. At time $t+1$, the number of skilled workers whose parents are skilled is $q(w_t^H; \tau, \phi)\lambda_t$ and the number of skilled workers whose parents are unskilled is $q(w_t^L; \tau, \phi)(1-\lambda_t)$. Therefore, the total number of skilled workers at time $t+1$ is

$$\lambda_{t+1} = q(w_t^H; \tau, \phi)\lambda_t + q(w_t^L; \tau, \phi)(1-\lambda_t). \tag{40}$$

From (22), (23), (29), (30), (36), (37), (39), and (40), and $\xi_t \equiv \frac{\lambda_t}{1-\lambda_t}$, the dynamics of ξ_t is given by

$$\xi_{t+1} = \Phi(\xi_t; \tau) \equiv \begin{cases} \Phi_N(\xi_t; \tau) & \text{if} \quad 0 \le \xi_t < \hat{\xi}_N \\ \Phi_P(\xi_t; \tau) & \text{if} \quad \hat{\xi}_N \le \xi_t \le \hat{\xi}_P \\ \Phi_F(\xi_t; \tau) & \text{if} \quad \hat{\xi}_P < \xi_t \le \hat{\xi}_F \\ \Phi_{\bar{F}}(\tau) & \text{if} \quad \hat{\xi}_F < \xi_t \end{cases}, \tag{41}$$

where

$$\Phi_N(\xi_t;\tau) \equiv \frac{q\big(w_N^H(\xi_t);\tau,\phi\big)\xi_t + q\big(w_N^L(\xi_t);\tau,\phi\big)}{[1-q\big(w_N^H(\xi_t);\tau,\phi\big)]\xi_t + 1 - q\big(w_N^L(\xi_t);\tau,\phi\big)}, \tag{42}$$

$$\Phi_P(\xi_t;\tau) \equiv \frac{q\big(w_P^H;\tau,\phi\big)\xi_t + q\big(R;\tau,\phi\big)}{[1-q\big(w_P^H;\tau,\phi\big)]\xi_t + 1 - q\big(R;\tau,\phi\big)}, \tag{43}$$

$$\Phi_F(\xi_t;\tau) \equiv \frac{q\big(w_F^H(\xi_t);\tau,\phi\big)\xi_t + q\big(w_F^L(\xi_t);\tau,\phi\big)}{[1-q\big(w_F^H(\xi_t);\tau,\phi\big)]\xi_t + 1 - q\big(w_F^L(\xi_t);\tau,\phi\big)}, \tag{44}$$

$$\Phi_{\bar{F}}(\tau) \equiv \frac{q(\bar{w};\tau,\phi)}{1-q(\bar{w};\tau,\phi)}. \tag{45}$$

From (22), the wage rate of low-skilled workers increases if B increases in no automation equilibrium. On the other hands, the interest rate is constant because this economy is small open. Therefore, if B becomes higher, it tends to generate the automation. Let us define $\bar{B}(\tau)$ as the threshold level of productivity of intermediate good B. When the productivity of intermediate goods B is higher than $\bar{B}(\tau)$, the automation occurs in the steady state. We obtain the following Lemma 2.

Lemma 2 *Let us define $\bar{B}(\tau)$ as the threshold level of productivity of intermediate good B. There exists $\bar{B}(\tau)$ and $\bar{B}(\tau)$ is a decreasing function of τ.*

Proof see Appendix F.

For simplicity, we assume following the assumption:

Assumption 2

$$\frac{\partial^2 \Phi_N(\xi_t,\tau)}{\partial \xi_t^2} < 0 \quad \textit{and} \quad \frac{\partial^2 \Phi_F(\xi_t,\tau)}{\partial \xi_t^2} < 0.$$

Under Assumption 2, we obtain the following Proposition 1.

Proposition 1 *Suppose that $\tau = 0$. If $B > \bar{B}(0)$ holds, there exists a unique steady state where automation occurs and economy converges to this steady state.*

Proof see Appendix G.

Let us define ξ^* as the steady state value of ξ_t. Figure 2 shows the dynamics of ξ_t when $B > \bar{B}(0)$[1]. Let us consider the case in which $\hat{\xi}_P < \xi^* \leq \hat{\xi}_F$ holds. There exists a unique steady state where automation occurs and economy converges to this steady state. Figure 3 shows the relationship between ξ_t and the number of automated productions of intermediate goods. Let us consider the case where $\hat{\xi}_P < \xi^* \leq \hat{\xi}_F$ holds.

[1] A numerical example reveals that there is a parameter configuration such that there exists a unique steady state where automation occurs and Assumption 2 holds. For example, if we specify $q(e_t^j) = \frac{\rho+\mu e_t^j}{\theta+e_t^j}$ and assume that $\beta = (0.98)^{25}, \gamma = 0.14, \mu = 1, \rho = 1, \theta = 8, z = 0.088, B = 10, I = 0.3, S = 0.4$, and $R = 1.3$.

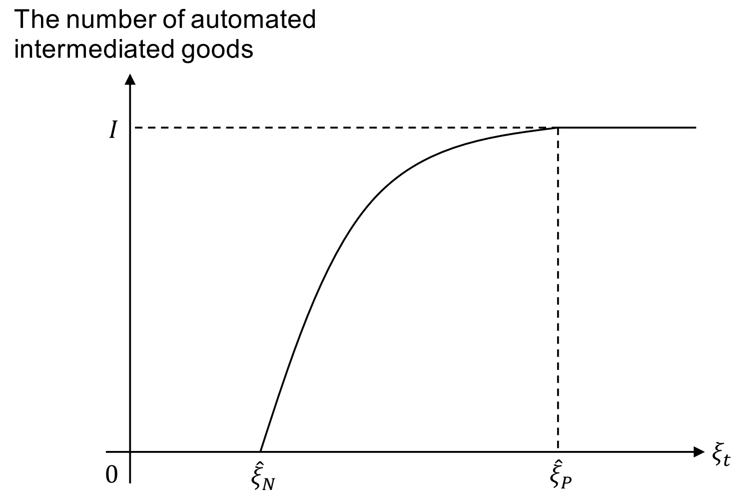

Figure 3: Transition of the number of automated intermediate goods

Suppose that the initial proportion of high-skilled individuals to low-skilled individuals ξ_0 is smaller than $\hat{\xi}_N$. While the economy is less developed and $\xi_t < \hat{\xi}_N$ holds, automation does not occur because of the low wage rate of low-skilled workers. As the economy develops, some intermediate goods are gradually automated. Finally, ξ_t become higher than $\hat{\xi}_P$ and all productions of intermediate goods whose index is in the range $[0, I]$ are automated. Figure 4 shows the relationship between ξ_t and the number of intermediated goods produced by low-skilled workers. While the economy is less developed and $\xi_t < \hat{\xi}_N$ holds, low-skilled workers engage in the production of intermediate goods in the range $[0, S]$ because no automation occurs. As the economy develops, some low-skilled worker's jobs are gradually substituted by machine. Finally, ξ_t become higher than $\hat{\xi}_P$ and all productions of intermediate goods whose index is in the range $[0, I]$ are substituted by machine. From Lemma 1, during this development process, the per capita output y_t becomes higher. This mechanism explains the launching of the Industrial Revolution. Before the Industrial Revolution, $\xi_t < \hat{\xi}_N$ holds. However, the Industrial Revolution is launched and automation occurs when ξ_t becomes higher than $\hat{\xi}_N$. Then, some low-skilled workers' jobs are substituted by machines and the per capita output become higher.

We obtain the following Proposition 2.

Proposition 2 *Suppose that $\tau = 0$ and $S < 1 - q(R, 0)$. If $B \leq \bar{B}(0)$ holds, there exists a unique steady state where automation does not occur and economy converges to this steady state.*

Proof see Appendix H.

Figure 5 shows the dynamics of ξ_t when $B \leq \bar{B}(0)$. In this case, $\xi^* \leq \hat{\xi}_N$ holds. There exists a unique steady state where automation does not occur and economy converge to this steady state. Finally, we discuss how the technological progress of the automation technology affects the economic development. We regards the

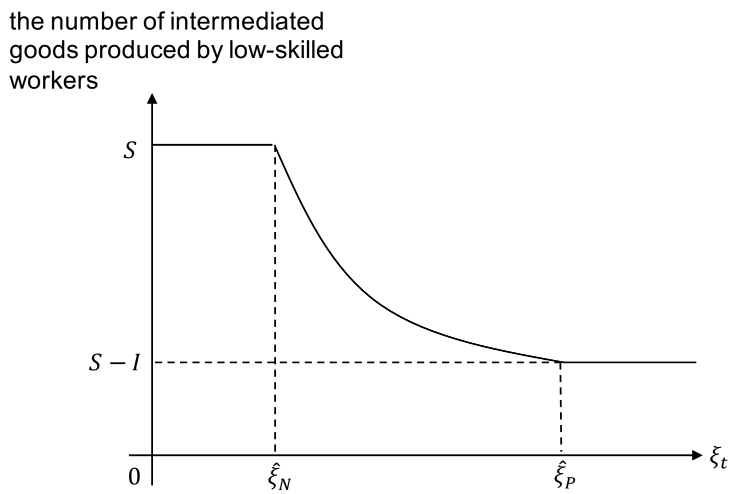

Figure 4: Transition of the number of intermediated goods produced by low-skilled workers

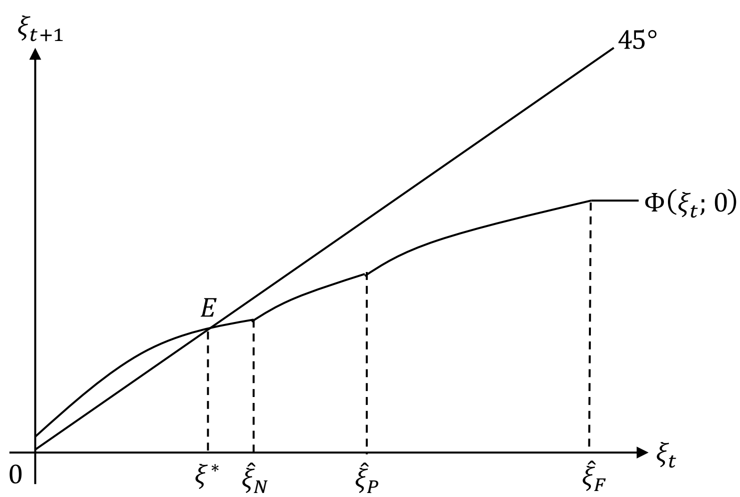

Figure 5: Dynamics of ξ_t when $\tau = 0$ and $B \leq \bar{B}(0)$

increase of I as the technological progress of the automation technology. From (22), (23), (25), (42), and (43), I does not affect $\hat{\xi}_N$, $\Phi_N(\xi_t;\tau)$ and $\Phi_P(\xi;\tau)$. Therefore, if I changes, the steady state value ξ^* does not change and the automation never occurs.

4 Education policy

In this section, we consider how the government's education policy affects this economy. The government can set the level of lump-sum tax in this model. Therefore, we focus on how changes in the tax affect this economy. We obtain following Lemma 3.

Lemma 3 $\frac{\partial \Phi(\xi_t;\tau)}{\partial \tau} > 0$ *holds.*

Proof see Appendix I.

If the level of lump-sum tax is higher than the income, the individual's disposable income becomes negative. Note that $w_N^L(\xi_t)$ is the lowest wage rate in equilibrium. Therefore, we assume that the government can implement the education policy only if $w_N^L(\xi_t) > \tau$ holds. From (22), we obtain

$$\begin{aligned} & w_N^L(\xi_t) > \tau \\ \to \quad & \xi_t > \frac{1-S}{S}\left(\frac{\tau}{B}\right)^{\frac{1}{1-S}} \equiv \bar{\xi}. \end{aligned} \tag{46}$$

Let us define $\bar{\xi}$ as the right-hand side of (46). From (46), the government can implement the education policy only if $\xi_t > \bar{\xi}$. Hereafter, we consider the case that $\bar{\xi} < \hat{\xi}_N$ holds[2]. Let us define $\tilde{\tau}$ as the value which satisfies $B = \bar{B}(\tilde{\tau})$. We obtain the following Proposition 3.

Proposition 3 *Suppose that* $\bar{B}(R) < B < \bar{B}(0)$ *and* $\tilde{\tau} < \tau < R$ *hold. If* $\bar{\xi} < \frac{q\left(w_N^L(\bar{\xi});\tau,\phi\right)}{1-q\left(w_N^H(\bar{\xi});\tau,\phi\right)}$ *holds, the economy converges to a steady state where automation occurs.*

Proof see Appendix J.

When $B < \bar{B}(0)$ holds, there exists a unique steady state without automation if the government does not implement the education policy. Figure 6 shows the dynamics of ξ_t when $\tilde{\tau} < \tau < R$ and $\bar{\xi} < \frac{q\left(w_N^L(\bar{\xi});\tau,\phi\right)}{1-q\left(w_N^H(\bar{\xi});\tau,\phi\right)}$ hold. From Lemma 3, $\Phi(\xi_t;\tau)$ shifts upward in $[\bar{\xi},\infty)$ in Figure 6. Suppose that the government implements education policy at time 0. If the government does not implement any education policies, economy converges to a steady state where automation does not occur and the proportion of high-skilled individuals to low-skilled individuals is low. However, if the government implements education policy, economy converges to a steady state where the automation occurs and the proportion of high-skilled individuals to low-skilled individuals

[2] From (24) and (46), $\bar{\xi} < \hat{\xi}_N$ holds if and only if $\tau < R$ holds.

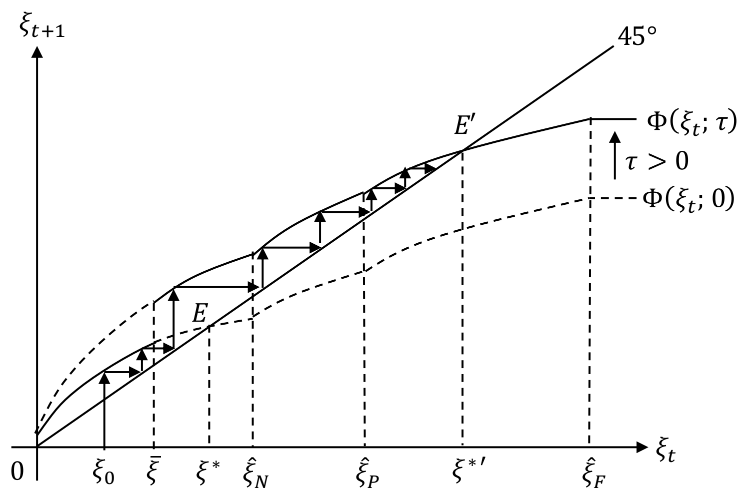

Figure 6: Dynamics of ξ_t when $\tau > 0$ and $\bar{\xi} < \xi^*$

is high. From Lemma 1, the per capita output of steady state with automation is higher than that of a steady state without automation. From $\frac{\partial q\left(w_N^L(\bar{\xi});\tau,\phi\right)}{\partial \phi} > 0$, $\bar{\xi} < \frac{q\left(w_N^L(\bar{\xi});\tau,\phi\right)}{1-q\left(w_N^H(\bar{\xi});\tau,\phi\right)}$ holds if ϕ is sufficiently high. Therefore, this equilibrium occurs when the efficiency of education is sufficiently high.

Proposition 4 *Suppose that $\bar{B}(R) < B < \bar{B}(0)$ and $\tilde{\tau} < \tau < R$ hold. If $\bar{\xi} \geq \frac{q\left(w_N^L(\bar{\xi});\tau,\phi\right)}{1-q\left(w_N^H(\bar{\xi});\tau,\phi\right)}$ holds, there exist multiple steady states. The automation occurs at one of them but not the other.*

Proof see Appendix J.

Figure 7 shows the case that $\tilde{\tau} < \tau < R$ and $\bar{\xi} \geq \frac{q\left(w_N^L(\bar{\xi});\tau,\phi\right)}{1-q\left(w_N^H(\bar{\xi});\tau,\phi\right)}$ hold. In this case, there exist multiple steady states[3]. The automation occurs at one of them and not at the other. Suppose that the government implements education policy at time 0. If the initial proportion of high-skilled individuals to low-skilled individuals ξ_0 is higher than $\bar{\xi}$, the economy converges to a steady state where the automation occurs and per capita output is high. However, if the initial proportion of high-skilled individuals to low-skilled individuals ξ_0 is lower than $\bar{\xi}$, the economy converges to the steady state where the automation does not occur and per capita output is high. From Lemma 1, the per capita output of the steady state with automation is higher than a steady state without automation. From $\frac{\partial q\left(w_N^L(\bar{\xi});\tau,\phi\right)}{\partial \phi} > 0$, $\bar{\xi} < \frac{q\left(w_N^L(\bar{\xi});\tau,\phi\right)}{1-q\left(w_N^H(\bar{\xi});\tau,\phi\right)}$ holds if ϕ is sufficiently low. Therefore, this equilibrium occurs when the efficiency of education is sufficiently low.

[3]Our model setting of individual's behavior follows Fan and Zhang (2013). As discussed in Fan and Zhang (2013), if we assume that the government imposes income tax, multiple steady states can also exist. However, imposing income tax in this model makes analysis very complicated because government expenditure for education G_t depends on λ_t. For analytical simplicity, we assume that the government impose lump-sum tax.

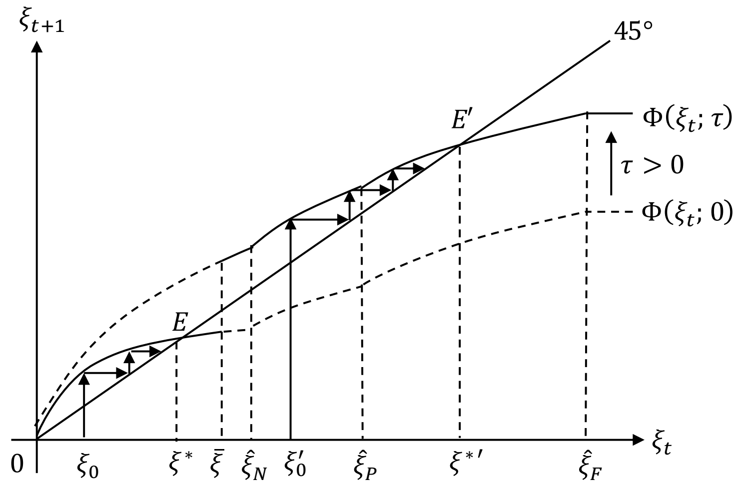

Figure 7: Dynamics of ξ_t when $\tau > 0$ and $\bar{\xi} \geq \xi^*$

5 Conclusion

We constructed a simple small open overlapping-generations model with endogenous education decision-making and automation. We examined the relationship between education, automation, and economic growth. We showed that the economy converge to a steady state where automation occurs and per capita output is high if productivity of intermediate goods is high. On the other hand, we showed that the economy converge to a steady state where automation does not occur and per capita output is low if productivity of intermediate goods is low. In addition, we examined how education policy affects the economy when productivity of intermediate goods is low. If the efficiency of education is high, the government can steer an economy away from a steady state without automation by investing in education. If the efficiency of education is low, there exist multiple steady states. Automation occurs at one of them but not the other.

Appendix

Appendix A: Proof of that $w_t^H > w_t^L$ holds in no automation equilibrium and partial automation equilibrium.

We first examine no automation equilibrium. Let us guess that $w_t^H > w_t^L$ holds in no automation equilibrium. Then, we can obtain (22) and (23). From (22) and (23), we obtain

$$\begin{aligned} \frac{w_t^H}{w_t^L} &= \frac{1-S}{S\xi_t}, \\ &> \frac{1-S}{S\hat{\xi}_N}, \\ &= \left(\frac{B}{R}\right)^{\frac{1}{1-S}} > 1. \end{aligned} \tag{A1}$$

Note that $\xi_t < \hat{\xi}_N$ holds in no automation equilibrium. The last inequality holds doe to Assumption 1. Therefore, we can verify that $w_t^H > w_t^L$ holds in no automation equilibrium.

We next examine partial automation equilibrium. Let us guess that $w_t^H > w_t^L$ holds in no automation equilibrium. Then, we can obtain (28) and (29). From (28) and (29), we obtain

$$\frac{w_t^H}{w_t^L} = \left(\frac{B}{R}\right)^{\frac{1}{1-S}} > 1. \tag{A2}$$

Therefore, we can verify that $w_t^H > w_t^L$ holds in partial automation equilibrium.

Appendix B: Derivation of (22) and (23)

From (18) and (20), we obtain,

$$\frac{y_t}{w_t^L} = \frac{1-\lambda_t}{S}. \tag{A3}$$

From (19) and (21), we obtain

$$\frac{y_t}{w_t^H} = \frac{\lambda_t}{1-S}. \tag{A4}$$

Noting that $w_t^H > w_t^L$ and $R > w_t^L$ holds in no automation equilibrium. Considering (8), (9), and (11), we can rewrite (6) as follows:

$$\log y_t = \int_0^S \log Bl_t(i)di + \int_S^1 \log Bh_t(i)di. \tag{A5}$$

Substituting (18) and (19) into (A5), we obtain

$$\log y_t = \int_0^S \log B\frac{y_t}{w_t^L}di + \int_S^1 \log B\frac{y_t}{w_t^H}di. \tag{A6}$$

Substituting (A3) and (A4) into (A6) and rearranging them, we obtain

$$y_t = B\left(\frac{1-\lambda_t}{S}\right)^S\left(\frac{\lambda_t}{1-S}\right)^{1-S}. \tag{A7}$$

Combining (A3), (A7), and $\xi_t \equiv \frac{\lambda_t}{1-\lambda_t}$ and rearranging them, we obtain (22) as follows:

$$w_t^L = B\left(\frac{S\xi_t}{1-S}\right)^{1-S}.$$

Combining (A4), (A7), and $\xi_t \equiv \frac{\lambda_t}{1-\lambda_t}$ and rearranging, we obtain (23) as follows:

$$w_t^H = B\left(\frac{1-S}{S\xi_t}\right)^S.$$

Appendix C: Derivation of (30) and (31)

From (25) and (27), we obtain

$$\frac{y_t}{R} = \frac{K_t}{\hat{I}_t}. \tag{A8}$$

From (26) and (28), we obtain

$$\frac{y_t}{w_t^L} = \frac{1-\lambda_t}{S-\hat{I}_t}. \tag{A9}$$

From (19) and (21), we obtain

$$\frac{y_t}{w_t^H} = \frac{\lambda_t}{1-S}. \tag{A10}$$

Noting that $w_t^H > w_t^L = R$ holds in partial automation equilibrium. Considering (8), (9), and (11), we can rewrite (6) as follows:

$$\log y_t = \int_0^{\hat{I}_t} \log Bk_t(i)di + \int_{\hat{I}_t}^{S} \log Bl_t(i)di + \int_S^1 \log Bh_t(i)di. \tag{A11}$$

Substituting (19), (25), and (26) into (A11), we obtain

$$\log y_t = \int_0^{\hat{I}_t} \log B\frac{y_t}{R}di + \int_{\hat{I}_t}^{S} \log B\frac{y_t}{w_t^L}di + \int_S^1 \log B\frac{y_t}{w_t^H}di. \tag{A12}$$

Substituting (A8), (A9), and (A10) into (A12) and rearranging them, we obtain

$$y_t = B\left(\frac{K_t}{\hat{I}_t}\right)^{\hat{I}_t}\left(\frac{1-\lambda_t}{S-\hat{I}_t}\right)^{S-\hat{I}_t}\left(\frac{\lambda_t}{1-S}\right)^{1-S}. \tag{A13}$$

Combining (A8) and (A13) rearranging them, we obtain

$$K_t = \hat{I}_t\left(\frac{B}{R}\right)^{\frac{1}{1-\hat{I}_t}}\left(\frac{1-\lambda_t}{S-\hat{I}_t}\right)^{\frac{S-\hat{I}_t}{1-I}}\left(\frac{\lambda_t}{1-S}\right)^{\frac{1-S}{1-\hat{I}_t}}. \tag{A14}$$

Substituting (A14) into (A13), we obtain

$$y_t = B\left(\frac{B}{R}\right)^{\frac{\hat{I}_t}{1-\hat{I}_t}}\left(\frac{1-\lambda_t}{S-\hat{I}_t}\right)^{\frac{S-\hat{I}_t}{1-\hat{I}_t}}\left(\frac{\lambda_t}{1-S}\right)^{\frac{1-S}{1-\hat{I}_t}}. \tag{A15}$$

Combining (29), (A9), and (A15) and rearranging them, we obtain (31), as follows:

$$\hat{I}_t = S - \frac{1-S}{\xi_t}\left(\frac{R}{B}\right)^{\frac{1}{1-S}}.$$

Combining (A10), (A15), and (31), and rearranging them, we obtain (30), as follows:

$$w_t^H = B\left(\frac{B}{R}\right)^{\frac{S}{1-S}}.$$

Appendix D: Derivation of (36) and (37)

From (32) and (34), we obtain

$$\frac{y_t}{R} = \frac{K_t}{I}. \tag{A16}$$

From (33) and (35), we obtain

$$\frac{y_t}{w_t^L} = \frac{1-\lambda_t}{S-I}. \tag{A17}$$

From (19) and (21), we obtain

$$\frac{y_t}{w_t^H} = \frac{\lambda_t}{1-S}. \tag{A18}$$

Noting that $w_t^H > w_t^L > R$ holds in full automation equilibrium. Considering (8), (9), and (11), we can rewrite (6) as follows:

$$\log y_t = \int_0^I \log Bk_t(i)di + \int_I^S \log Bl_t(i)di + \int_S^1 \log Bh_t(i)di. \tag{A19}$$

Substituting (19), (32), and (33) into (A19), we obtain

$$\log y_t = \int_0^I \log B\frac{y_t}{R}di + \int_I^S \log B\frac{y_t}{w_t^L}di + \int_S^1 \log B\frac{y_t}{w_t^H}di. \tag{A20}$$

Substituting (A16), (A17), and (A18) into (A20) and rearranging them, we obtain

$$y_t = B\left(\frac{K_t}{I}\right)^I \left(\frac{1-\lambda_t}{S-I}\right)^{S-I} \left(\frac{\lambda_t}{1-S}\right)^{1-S}. \tag{A21}$$

Combining (A21) and (A16) and rearranging them, we obtain

$$K_t = I\left(\frac{B}{R}\right)^{\frac{1}{1-I}} \left(\frac{1-\lambda_t}{S-I}\right)^{\frac{S-I}{1-I}} \left(\frac{\lambda_t}{1-S}\right)^{\frac{1-S}{1-I}}. \tag{A22}$$

Substituting (A22) into (A21), we obtain

$$y_t = B\left(\frac{B}{R}\right)^{\frac{I}{1-I}} \left(\frac{1-\lambda_t}{S-I}\right)^{\frac{S-I}{1-I}} \left(\frac{\lambda_t}{1-S}\right)^{\frac{1-S}{1-I}}. \tag{A23}$$

Combining (A17), (A23), and $\xi_t \equiv \frac{\lambda_t}{1-\lambda_t}$ and rearranging them, we obtain (36) as follows:

$$w_t^L = B\left(\frac{B}{R}\right)^{\frac{I}{1-I}} \left[\frac{(S-I)\xi_t}{1-S}\right]^{\frac{1-S}{1-I}}.$$

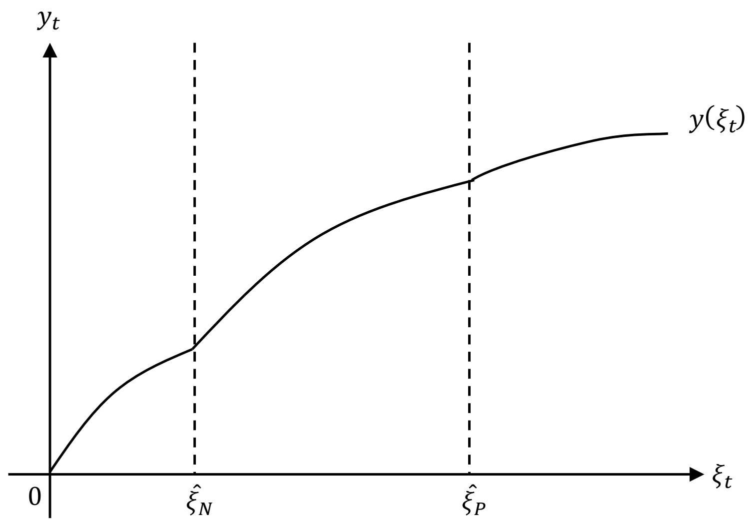

Figure 8: The relationship between ξ_t and y_t

Combining (A18), (A23), and $\xi_t \equiv \frac{\lambda_t}{1-\lambda_t}$, and rearranging, obtain (36) as follows:

$$w_t^H = B\left(\frac{B}{R}\right)^{\frac{I}{1-I}}\left[\frac{1-S}{(S-I)\xi_t}\right]^{\frac{S-I}{1-I}}.$$

Appendix E: Proof of Lemma1

We first examine the no automation equilibrium. From (A7), we obtain

$$y_t = \frac{\Psi_N \xi_t^{1-S}}{1+\xi_t} \equiv y_N(\xi_t), \tag{A24}$$

where $\Psi_N \equiv \frac{B}{1-S}\left(\frac{1-S}{S}\right)^S$. Let us define $y_N(\xi_t)$ as the right-hand side of (A24). From (A24), we can show that

$$\frac{\partial y_N(\xi_t)}{\partial \xi_t} = \Psi_N \xi_t^{-S} \frac{1-S-S\xi_t}{(1+\xi_t)^2} > 0. \tag{A25}$$

From (A24), we find that $y_N(\xi_t)$ does not depend on τ.

We next examine the partial automation equilibrium. From (31) and (A15), we can show that

$$y_t = \frac{\Psi_P \xi_t}{1+\xi_t} \equiv y_P(\xi_t), \tag{A26}$$

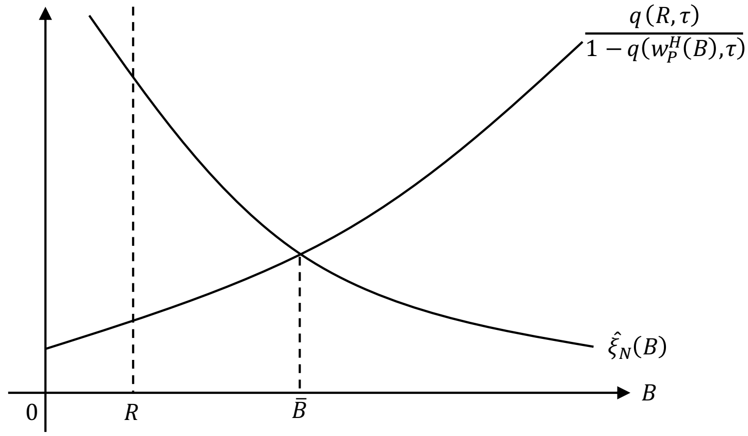

Figure 9: Determination of $\tilde{B}$

where $\Psi_P \equiv \frac{B}{1-S}\left(\frac{B}{R}\right)^{\frac{S}{1-S}}$. Let us define $y_P(\xi_t)$ as the right-hand side of (A26). From (A26), we obtain

$$\frac{\partial y_P(\xi_t)}{\partial \xi_t} = \frac{\Psi_P}{(1+\xi_t)^2} > 0. \tag{A27}$$

From (A26), we find that $y_P(\xi_t)$ does not depend on τ.

We finally examine the full automation equilibrium. From (A23), we obtain

$$y_t = \frac{\Psi_F \xi_t^{\frac{1-S}{1-I}}}{1+\xi_t} \equiv y_F(\xi_t), \tag{A28}$$

where $\Psi_F \equiv \frac{B}{1-S}\left(\frac{B}{R}\right)^{\frac{I}{1-I}}\left(\frac{1-S}{S-I}\right)^{\frac{S-I}{1-I}}$. Let us define $y_F(\xi_t)$ as the right-hand side of (A28). From (A28), we obtain

$$\frac{\partial y_F(\xi_t)}{\partial \xi_t} = \Psi_F \xi_t^{-\frac{S-I}{1-I}} \frac{1-S-(S-I)\xi_t}{(1-I)(1+\xi_t)^2} > 0. \tag{A29}$$

From (A28), we find that $y_F(\xi_t)$ does not depend on τ. Figure 8 shows the graphs of $y_N(\xi_t)$, $y_P(\xi_t)$, and $y_F(\xi_t)$. From Figure 8, we find that y_t increases with ξ_t.

Appendix F: Proof of Lemma 2

From Figure 2 and (41), we find that $\Phi_N(\xi_t;\tau)$ does not have intersection with 45-degree line if $\hat{\xi}_N < \Phi_N(\hat{\xi}_N;\tau)$ holds. We can rewrite $\hat{\xi}_N < \Phi_N(\hat{\xi}_N;\tau)$ as $\hat{\xi}_N < \Phi_P(\hat{\xi}_N,\tau)$ because $\Phi_N(\xi_t,\tau)$ and $\Phi_P(\xi_t,\tau)$ are

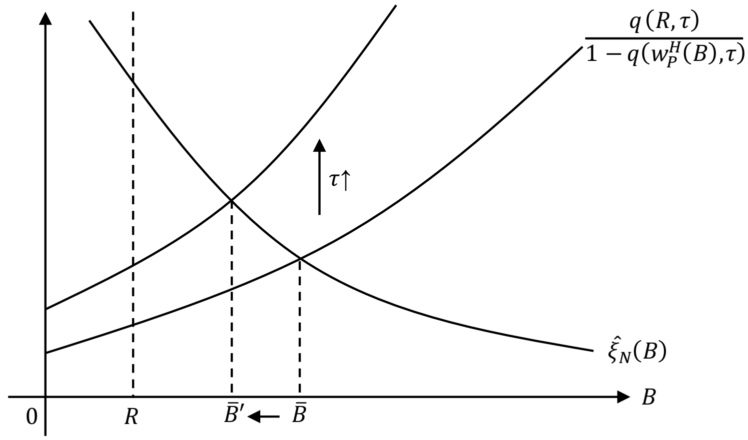

Figure 10: The effect of the change of τ on $\tilde{B}$

continuous at $\xi_t = \hat{\xi}_N$. Therefore, we can rewrite $\hat{\xi}_N < \Phi_N(\hat{\xi}_N;\tau)$ as the following inequality by using (43),

$$\hat{\xi}_N < \frac{q(w_P^H;\tau,\phi)\hat{\xi}_N + q(R;\tau,\phi)}{[1-q(w_P^H;\tau,\phi)]\hat{\xi}_N + 1 - q(R;\tau,\phi)}. \tag{A30}$$

We can solve (A30) with respect to $\hat{\xi}_N$ as follows:

$$\hat{\xi}_N < \frac{q(R;\tau,\phi)}{1-q(w_P^H;\tau,\phi)}. \tag{A31}$$

Therefore, $\hat{\xi}_N < \Phi_N(\hat{\xi}_N;\tau)$ holds if (A31) holds. From (24), we find that $\hat{\xi}_N$ depends on B and $\hat{\xi}_N$ decreases with B. Therefore, we denote $\hat{\xi}_N$ as $\hat{\xi}_N(B)$. From (30), we find that w_P^H depends on B and w_P^H increases with B. Therefore, we denote w_P^H as $w_P^H(B)$. Hence, $\frac{q(R;\tau,\phi)}{1-q(w_P^H(B);\tau,\phi)}$ increases with B. We can draw the relationship between $\hat{\xi}_N(B)$ and $\frac{q(R;\tau,\phi)}{1-q(w_P^H(B);\tau,\phi)}$ in Figure 9. From Figure 9, there is a unique intersection between $\hat{\xi}_N(B)$ and $w_P^H(B)$ and we denote the value of this intersection as $\bar{B}(\tau)$. Therefore, $\Phi_N(\xi_t;\tau)$ does not have intersection with 45-degree line if $B > \bar{B}(\tau)$.

Figure 10 shows the effect of the change of τ on $\tilde{B}(\tau)$. If τ increases, $\frac{q(R;\tau,\phi)}{1-q(w_P^H(B);\tau,\phi)}$ shifts upward and $\hat{\xi}_N(B)$ does not change. From Figure 10, $\bar{B}(\tau)$ decreases if τ increases.

Appendix G: Proof of Proposition 1

We first show that there exists a unique steady state. From (42), we obtain

$$\frac{\partial \Phi_N(\xi_t;\tau)}{\partial \xi_t} = \frac{(1+\xi_t)\left[\xi_t \frac{\partial q\left(w_N^H(\xi_t);\tau,\phi\right)}{\partial \xi_t} + \frac{\partial q\left(w_N^L(\xi_t);\tau,\phi\right)}{\partial \xi_t}\right] + q\left(w_N^H(\xi_t);\tau,\phi\right) - q\left(w_N^L(\xi_t);\tau,\phi\right)}{\{[1-q\left(w_N^H(\xi_t);\tau,\phi\right)]\xi_t + 1 - q\left(w_N^L(\xi_t);\tau,\phi\right)\}^2}. \tag{A32}$$

We obtain the following Lemma 4.

Lemma 4 *The following inequality holds:*

$$\xi_t \frac{\partial q\big(w_N^H(\xi_t);\tau,\phi\big)}{\partial \xi_t} + \frac{\partial q\big(w_N^L(\xi_t);\tau,\phi\big)}{\partial \xi_t} > 0.$$

Proof. We obtain

$$\xi_t \frac{\partial q\big(w_N^H(\xi_t);\tau,\phi\big)}{\partial \xi_t} + \frac{\partial q\big(w_N^L(\xi_t);\tau,\phi\big)}{\partial \xi_t} = \xi_t \frac{\partial q\big(w_N^H(\xi_t);\tau,\phi\big)}{\partial w_N^H(\xi_t)} \frac{\partial w_N^H(\xi_t)}{\partial \xi_t} + \frac{\partial q\big(w_N^L(\xi_t);\tau,\phi\big)}{\partial w_N^L(\xi_t)} \frac{\partial w_N^L(\xi_t)}{\partial \xi_t}. \quad \text{(A33)}$$

From (22) and (23), we obtain

$$\frac{\partial w_N^L(\xi_t)}{\partial \xi_t} = BS \left(\frac{1-S}{S\xi_t}\right)^S, \quad \text{(A34)}$$

$$\frac{\partial w_N^H(\xi_t)}{\partial \xi_t} = -\frac{BS}{\xi_t} \left(\frac{1-S}{S\xi_t}\right)^S. \quad \text{(A35)}$$

Substituting (A34) and (A35) in to the right-hand side of (A33) and rearranging them, we obtain

$$\xi_t \frac{\partial q\big(w_N^H(\xi_t);\tau,\phi\big)}{\partial \xi_t} + \frac{\partial q\big(w_N^L(\xi_t);\tau,\phi\big)}{\partial \xi_t} = \left[\frac{\partial q\big(w_N^L(\xi_t);\tau,\phi\big)}{\partial w_N^L(\xi_t)} - \frac{\partial q\big(w_N^H(\xi_t);\tau,\phi\big)}{\partial w_N^H(\xi_t)}\right] BS \left(\frac{1-S}{S\xi_t}\right)^S > 0 \quad \text{(A36)}$$

From $\frac{\partial^2 q(w_t^j;\tau,\phi)}{\partial (w_t^j)^2} < 0$ and $w_N^H(\xi_t) > w_N^L(\xi_t)$, the right-hand side is larger than 0. Hence, we obtain Lemma 4.

From (A32), Lemma 4, and $q\big((w_N^L(\xi_t);\tau,\phi\big) < q\big((w_N^H(\xi_t);\tau,\phi\big)$, we obtain

$$\frac{\partial \Phi_N(\xi_t;\tau)}{\partial \xi_t} > 0. \quad \text{(A37)}$$

If we assume that $\frac{\partial^2 \Phi_N(\xi_t;\tau)}{\partial \xi_t^2} < 0$, $\Phi_N(\xi_t;\tau)$ is a concave function. From (22) and (23), $\lim_{\xi_t \to 0} w_N^L(\xi_t) = 0$ and $\lim_{\xi_t \to 0} w_N^H(\xi_t) = \infty$ holds. Therefore, $\lim_{\xi_t \to 0} \Phi_N(\xi) = \frac{q}{1-\underline{q}} > 0$.

From (43) and $q'(w_t^j) > 0$, we obtain

$$\frac{\partial \Phi_P(\xi_t;\tau)}{\partial \xi_t} = \frac{q(w_P^H;\tau,\phi) - q(R;\tau,\phi)}{\{[1-q(w_p^R)]\xi_t + 1 - q(R;\tau,\phi)\}} > 0. \quad \text{(A38)}$$

From (A38), we obtain $\frac{\partial^2 \Phi_P(\xi_t;\tau)}{\partial \xi_t^2} < 0$. Therefore, we can find that $\Phi_P(\xi_t;\tau)$ is a concave function. In addition, $\lim_{\xi_t \to 0} \Phi_P(\xi_t;\tau) = \frac{q(R;\tau,\phi)}{1-q(R;\tau,\phi)} > 0$.

From (44), we obtain

$$\frac{\partial \Phi_F(\xi_t;\tau)}{\partial \xi_t} = \frac{(1+\xi_t)\left[\xi_t \frac{\partial q\left(w_F^H(\xi_t);\tau,\phi\right)}{\partial \xi_t} + \frac{\partial q\left(w_F^L(\xi_t);\tau,\phi\right)}{\partial \xi_t}\right] + q\left(w_F^H(\xi_t);\tau,\phi\right) - q\left(w_F^L(\xi_t);\tau,\phi\right)}{\{[1-q\left(w_F^H(\xi_t);\tau,\phi\right)]\xi_t + 1 - q\left(w_F^L(\xi_t);\tau,\phi\right)\}^2}. \tag{A39}$$

We obtain the following Lemma 5.

Lemma 5 *The following equality holds:*

$$\xi_t \frac{\partial q\left(w_F^H(\xi_t);\tau,\phi\right)}{\partial \xi_t} + \frac{\partial q\left(w_F^L(\xi_t);\tau,\phi\right)}{\partial \xi_t} > 0.$$

Proof. We obtain

$$\xi_t \frac{\partial q\left(w_F^H(\xi_t);\tau,\phi\right)}{\partial \xi_t} + \frac{\partial q\left(w_F^L(\xi_t);\tau,\phi\right)}{\partial \xi_t} = \xi_t \frac{\partial q\left(w_F^H(\xi_t);\tau,\phi\right)}{\partial w_F^H(\xi_t)} \frac{\partial w_F^H(\xi_t)}{\partial \xi_t} + \frac{\partial q\left(w_F^L(\xi_t);\tau,\phi\right)}{\partial w_F^L(\xi_t)} \frac{\partial w_F^L(\xi_t)}{\partial \xi_t}. \tag{A40}$$

From (36) and (37), we obtain

$$\frac{\partial w_F^L(\xi_t)}{\partial \xi_t} = B\left(\frac{B}{R}\right)^{\frac{I}{1-I}} \frac{S-I}{1-I}\left[\frac{1-S}{(S-I)\xi_t}\right]^{\frac{S-I}{1-I}}, \tag{A41}$$

$$\frac{\partial w_F^H(\xi_t)}{\partial \xi_t} = -B\left(\frac{B}{R}\right)^{\frac{I}{1-I}} \frac{S-I}{(1-I)\xi_t}\left[\frac{1-S}{(S-I)\xi_t}\right]^{\frac{S-I}{1-I}}. \tag{A42}$$

Substituting (A41) and (A42) in to the right-hand side of (A40) and rearranging them, we obtain

$$\begin{aligned}\xi_t &\frac{\partial q\left(w_F^H(\xi_t);\tau,\phi\right)}{\partial \xi_t} + \frac{\partial q\left(w_F^L(\xi_t);\tau,\phi\right)}{\partial \xi_t} \\ &= \left[\frac{\partial q\left(w_F^L(\xi_t);\tau,\phi\right)}{\partial w_F^L(\xi_t)} - \frac{\partial q\left(w_F^H(\xi_t);\tau,\phi\right)}{\partial w_F^H(\xi_t)}\right] B\left(\frac{B}{R}\right)^{\frac{I}{1-I}} \frac{S-I}{1-I}\left[\frac{1-S}{(S-I)\xi_t}\right]^{\frac{S-I}{1-I}} > 0.\end{aligned} \tag{A43}$$

From $\frac{\partial^2 q(w_t^j;\tau,\phi)}{\partial (w_t^j)^2} < 0$ and $w_F^H(\xi_t) > w_F^L(\xi_t)$, the right-hand side is larger than 0. Hence, we obtain Lemma 5.

From (A39), Lemma 5, and $q\big((w_F^L(\xi_t)\big) < q\big((w_F^H(\xi_t);\tau,\phi\big)$, we obtain

$$\frac{\partial \Phi_F(\xi_t;\tau)}{\partial \xi_t} > 0. \tag{A44}$$

If we assume that $\frac{\partial^2 \Phi_F(\xi_t;\tau)}{\partial \xi_t^2} < 0$, $\Phi_F(\xi_t;\tau)$ is a concave function. From (36) and (37), $\lim_{\xi_t \to 0} w_F^L(\xi_t) = 0$ and $\lim_{\xi_t \to 0} w_F^H(\xi_t) = \infty$ holds. Therefore, $\lim_{\xi_t \to 0} \Phi_F(\xi_t;\tau) = \frac{q}{1-\underline{q}} > 0$. $\Phi_N(\xi_t;\tau)$ and $\Phi_P(\xi_t;\tau)$ are continuous at $\hat{\xi}_N$. $\Phi_P(\xi_t;\tau)$ and $\Phi_F(\xi_t;\tau)$ are continuous at $\hat{\xi}_P$. $\Phi_F(\xi_t;\tau)$ and $\Phi_{\bar{F}}(\tau)$ are continuous at $\hat{\xi}_F$. Therefore, $\Phi(\xi_t;\tau)$ is a continuous function. As shown above, $\Phi_N(\xi_t)$, $\Phi_P(\xi_t)$, and $\Phi_F(\xi_t)$ are piecewise concave and have positive intersections with vertical axis. Of course, properties described above all hold when $\tau = 0$. Hence, there exists a unique steady state when $\tau = 0$. As shown in Appendix F, the economy converge to a steady state where automation occurs if $B > \bar{B}(0)$.

Appendix H: Proof of Proposition 2

As shown in Appendix F, all economies converge to a steady state where automation does not occur if $B \leq \bar{B}(0)$. However, there exists the lower bound of B because of Assumption 1. Therefore, we have to check the condition that $R < \bar{B}(0)$ holds. From Figure 9, $R < \bar{B}(0)$ holds if $\frac{q(R,0)}{1-q\left(w_P^H(R),0\right)} < \hat{\xi}(R)$ holds. This condition is rewritten by $\frac{q(R,0)}{1-q(R,0)} < \frac{1-S}{S}$ by using (24) and (30). Therefore, $R < \bar{B}(0)$ holds if $S < 1-q(R,0)$ holds.

Appendix I: Proof of Lemma 3

From (42), we obtain

$$\frac{\partial \Phi_N(\xi_t;\tau)}{\partial \tau} = \frac{(1+\xi_t)\left[\frac{\partial q\left(w_N^H(\xi_t);\tau,\phi\right)}{\partial \tau}\xi_t + \frac{\partial q\left(w_N^L(\xi_t);\tau,\phi\right)}{\partial \tau}\right]}{\{[1-q\left(w_N^H(\xi_t);\tau,\phi\right)]\xi_t + 1 - q\left(w_N^L(\xi_t);\tau,\phi\right)\}^2} > 0. \tag{A45}$$

From (43), we obtain

$$\frac{\partial \Phi_P(\xi_t;\tau)}{\partial \tau} = \frac{(1+\xi_t)\left[\frac{\partial q(w_P^H;\tau,\phi)}{\partial \tau}\xi_t + \frac{\partial q(R;\tau,\phi)}{\partial \tau}\right]}{\{[1-q(w_P^H;\tau,\phi)]\xi_t + 1 - q(R;\tau,\phi)\}^2} > 0. \tag{A46}$$

From (44), we obtain

$$\frac{\partial \Phi_F(\xi_t;\tau)}{\partial \tau} = \frac{(1+\xi_t)\left[\frac{\partial q\left(w_F^H(\xi_t);\tau,\phi\right)}{\partial \tau}\xi_t + \frac{\partial q\left(w_F^L(\xi_t);\tau,\phi\right)}{\partial \tau}\right]}{\{[1-q\left(w_F^H(\xi_t);\tau,\phi\right)]\xi_t + 1 - q\left(w_F^L(\xi_t);\tau,\phi\right)\}^2} > 0. \tag{A47}$$

From (45), we obtain

$$\frac{\partial \Phi_{\bar{F}}(\tau)}{\partial \tau} = \frac{\frac{\partial q(\bar{w};\tau,\phi)}{\partial \tau}}{[1-q(\bar{w};\tau,\phi)]^2} > 0. \tag{A48}$$

From (A45) to (A48), $\frac{\partial \Phi(\xi_t;\tau)}{\partial \tau} > 0$ holds.

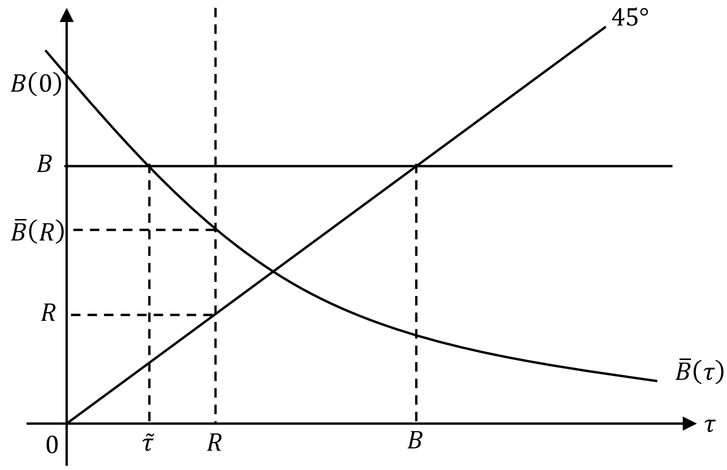

Figure 11: Determination of $\tilde{\tau}$

Appendix J: Proof of Proposition 3 and Proposition 4

As shown in Appendix F, $\hat{\xi}_N < \Phi_N(\hat{\xi}_N; \tau, \phi)$ holds if $B > \bar{B}(\tau)$ holds. Now, the government must set τ to satisfy $\tau < R$. As shown in Figure 11, B must be higher than $\bar{B}(R)$ to make $\tilde{\tau}$ strictly lower than R. We consider the condition that $\bar{\xi} < \xi^*$. From Figure 6 and (41), we find that $\bar{\xi} < \xi^*$ holds if $\bar{\xi} < \Phi_N(\bar{\xi})$ holds. From (42), we obtain

$$\begin{aligned} &\bar{\xi} < \Phi_N(\bar{\xi}; \tau), \\ \rightarrow \quad &\bar{\xi} < \frac{q\big(w_N^H(\bar{\xi}); \tau, \phi\big)\bar{\xi} + q\big(w_N^L(\bar{\xi}); \tau, \phi\big)}{\big[1 - q\big(w_N^H(\bar{\xi}); \tau, \phi\big)\big]\,\bar{\xi} + 1 - q\big(w_N^L(\bar{\xi}); \tau, \phi\big)}. \end{aligned} \tag{A49}$$

We can solve (A49) with respect to $\bar{\xi}$ and $\bar{\xi} < \Phi_N(\bar{\xi}; \tau, \phi)$ holds if $\bar{\xi} < \frac{q\left(w_N^H(\bar{\xi});\tau,\phi\right)}{1-q\left(w_N^L(\bar{\xi});\tau,\phi\right)}$ holds. Therefore, $\bar{\xi} < \xi^*$ holds if $\bar{\xi} < \frac{q\left(w_N^H(\bar{\xi});\tau,\phi\right)}{1-q\left(w_N^L(\bar{\xi});\tau,\phi\right)}$ holds. Therefore, we obtain Proposition 3. On the other hand, $\bar{\xi} \geq \xi^*$ holds if $\bar{\xi} \geq \frac{q\left(w_N^H(\bar{\xi});\tau,\phi\right)}{1-q\left(w_N^L(\bar{\xi});\tau,\phi\right)}$ holds. Then, we obtain Proposition 4.

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