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Abstract

We examine two firms' strategic choices of capital structure in the presence of negative bankruptcy spillovers. The low-profitability firm (denoted by firm L) that bankrupts earlier affects the high-profitability firm (denoted by firm H). Against negative bankruptcy spillovers, firm H takes either of the two contrasting responses: decreasing leverage to prepare for operations in the worse cash flow scenario after firm L's bankruptcy or increasing leverage to bankrupt simultaneously with firm L. Firm H takes the simultaneous bankruptcy strategy when the tax benefits of increased debt dominate the cash flows from operations after firm L's bankruptcy. With more negative bankruptcy spillovers, a smaller profitability difference, and lower volatility, firm H is more likely to choose the simultaneous bankruptcy strategy. The simultaneous bankruptcy equilibrium shows a novel mechanism of bankruptcy cascades through firms' strategic choices of capital structure with negative bankruptcy spillovers. This mechanism can potentially explain the empirical findings of bankruptcy contagion and herding behavior for corporate financial policies.

JEL Classifications Code: G13; G32; G33.

Keywords: capital structure; bankruptcy spillovers; contagion; herding; real options.

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1 Introduction

A firm's bankruptcy can bring about negative spillovers to other firms not only through financial networks (e.g., Jorion and Zhang (2007) and Jorion and Zhang (2009)) but also through real networks such as supply chains (e.g., Hertzel, Li, Officer, and Rodgers (2008) and Kolay, Lemmon, and Tashjian (2016)) and strategic alliances (e.g., Boone and Ivanov (2012)). In the worst cases, negative bankruptcy spillovers lead to bankruptcy cascades over firms that are linked financially and/or operationally to each other. Although a number of papers examine default contagion from the financial market viewpoints (e.g., credit risks and derivatives in Duffie, Eckner, Horel, and Saita (2009) and Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015), financial network stability in Allen and Gale (2000) and Eisenberg and Noe (2001)), few papers examine this problem from a perspective of corporate financial policy.

This study points to a new perspective for bankruptcy cascades; we show that firms' strategic capital structure choices can cause bankruptcy contagion. To do this, we extend Leland (1994) to a setup of two firms with negative bankruptcy spillovers. The negative bankruptcy spillovers are modeled by the assumption that cash flows of a surviving firm decrease after the other firm's bankruptcy. We derive the payoff dominant equilibrium of the game in which the firms strategically choose initial capital structure to maximize their firm values. The equilibrium results are summarized below.

The low-profitability firm (denoted by firm L) does not change the capital structure and bankruptcy timing from those of the nonstrategic case because the bankruptcy spillover effects do not affect firm L, which bankrupts earlier than firm H does. On the other hand, the high-profitability firm (denoted by firm H) can either decrease or increase debt from the nonstrategic level. In the former case, firm H decreases debt ex ante by anticipating operations in the worse cash flow scenario after firm L's bankruptcy (called the sequential bankruptcy equilibrium). In the latter case, firm H increases debt ex ante to go bankrupt simultaneously with firm L (called the simultaneous bankruptcy equilibrium). In the simultaneous bankruptcy equilibrium, firm H can maximize the tax benefits of debt until bankruptcy through the strategic increase of debt. In other words, firm H intentionally abandons operations after firm L's bankruptcy and goes bankrupt simultaneously with firm L.

Firm H's choice depends on the tradeoff between cash flows from operations after firm L's bankruptcy and the tax benefits of increased debt. When the tax benefits dominate the cash flows, firm H chooses the high leverage to bankrupt with firm L. This simultaneous bankruptcy equilibrium is more likely to occur for more negative bankruptcy spillovers, a smaller profitability difference, lower market volatility, and higher market growth rate. Compared to sequential bankruptcy, simultaneous bankruptcy reduces social welfare, which is defined as the sum of two firm values and the government value, because firm H's high leverage and hastened bankruptcy reduce the tax revenues of the government.

The simultaneous bankruptcy equilibrium shows a novel mechanism of bankruptcy cascades through the firms' strategic choices of capital structure in the presence of negative bankruptcy spillovers. Indeed, this new mechanism suggests that financially and/or operationally connected firms (e.g., firms with cross-holdings, firms in a supply chain, etc.) have ex-ante herding incentives to increase debt in order to go bankrupt together, which leads to social loss (i.e., systemic risk).

Although the empirical relevance of this mechanism has not been studied, this mechanism can potentially account for the empirical findings of default clustering. Emery and Cantor (2005) show that all affiliates within the same corporate family often go bankrupt together. Many papers, including Das, Duffie, Kapadia, and Saita (2007), Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015), and Azizpour, Giesecke, and Schwenkler (2018), show that default contagion can occur even for firms that do not belong to the same corporate family. Our results may also have implications regarding financial contagion and systemic risk. Acharya and Yorulmazer (2007) show the possibility that banks get exposed to similar risks ex ante in order to fail together. The bank herding incentive stems from a "too-many-to-fail" problem—the regulator finds it optimal ex post to bail out failed banks when the number of failed banks is too large. We complement their results by showing that, even in the absence of the regulator's bailout, banks cross-holding interbank loans have incentives to increase debt and systemic risk so that they fail together.

From a perspective of corporate financial policy, our results could potentially account for the empirical findings of Leary and Roberts (2014). They show that firms' financial policies, such as issues of new equity and debt, are partly driven by responses to their peers. Although the learning or competition effects often explain the herding behavior for corporate financial policies, we add an alternative contagion explanation—a firm can adjust its capital structure to that of another firm, whose bankruptcy negatively affect its operations, and aim to go bankrupt together.

Moreover, we extend the baseline model to several cases. In a setup of more than two firms, a firm's strategic increase of debt can amplify the negative externalities, which leads to another firm's strategic increase of debt. This negative feedback loop can cause bankruptcy cascades over multiple firms. With more negative bankruptcy spillovers, a larger cash flow scale of firm H over that of firm L, and lower merger cost and volatility, firm H is more likely to bail out firm L. With more negative bankruptcy spillovers and a lower debt renegotiation cost and volatility, firm H is more likely to renegotiate debt. Although bailout and debt renegotiation opportunities can prevent firm H from choosing the simultaneous bankruptcy equilibrium with high leverage and increase social welfare, they can prevent firm H from choosing the sequential bankruptcy equilibrium with low leverage and decrease social welfare.

Lambrecht (2001), Morellec and Zhdanov (2008), Nishihara and Shibata (2010), and Matveyev and Zhdanov (2019) are most relevant to our study. These papers extend Leland (1994) to duopoly market models. Lambrecht (2001) examines the effects of debt on entry and exit decisions, but he does not consider the strategic capital structure choice. Morellec and Zhdanov (2008) examine the firms' strategic capital structure choices when anticipating a takeover opportunity in the future. Nishihara and Shibata (2010) and Matveyev and Zhdanov (2019) examine the strategic investment timing and financing decisions, as well as bankruptcy timing. All of these papers focus on the competitive intra-industry effects—a firm's cash flows increase after another firm's bankruptcy–and they find a firm's incentive to decrease leverage and to win industry competition. In contrast, we focus on the negative bankruptcy spillover effects of bankruptcy—a firm's cash flows decrease after another firm's bankruptcy—and we find a firm's incentive to increase leverage and to go bankrupt together. Then, this paper complements the previous literature by showing that negative bankruptcy spillovers can distort the capital structure in an opposite direction from the competition effects.

The remainder of this paper is organized as follows. Section 2 introduces the model setup. After Section 3.1 derives the benchmark solutions with no bankruptcy spillover, Section 3.2 shows the equilibrium solutions with bankruptcy spillovers. In Section 3.3, we explore the solutions in numerical examples. After Section 4 explains the extended models, Section 5 concludes the paper.

2 Model Setup

2.1 Firms until bankruptcy

We extend the standard setup (e.g., Leland (1994), Goldstein, Ju, and Leland (2001), and Lambrecht and Myers (2008)) to the following model consisting of two firms. Consider two firms (denoted by firms H and L) that face the common shock $X(t)$. We do not necessarily consider a duopoly market, but the two firms may have customer-supplier relationships or belong to the same conglomerate. Firm $i \in \{H, L\}$ receives continuous streams of earnings before interest and taxes (EBIT) $a_i X(t) - b_i$, where a_i and b_i are positive constants satisfying $a_H/b_H \geq a_L/b_L$. This inequality means that firm H has higher profitability than firm L does. As in the standard literature, the state variable $X(t)$ follows a geometric Brownian motion

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t)\ (t > 0), \quad X(0) = x,$$

where $B(t)$ denotes the standard Brownian motion defined in a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$ and $\mu, \sigma(> 0)$ and $x(> 0)$ are constants. Assume that the initial value, $X(0) = x$, is sufficiently high to exclude a firm's bankruptcy at the initial time. For convergence, we assume that $r > \mu$, where a positive constant r denotes the risk-free interest rate.

Following the standard literature, at time 0, firm $i \in \{H, L\}$ issues consol debt with coupon C_i to maximize its firm value. Shareholders of firm i continue to receive cash flows $(1-\tau)(a_i X(t) - b_i - C_i)$ until one of the firms bankrupts, where $\tau \in (0, 1)$ denotes a corporate tax rate. Throughout this paper, we assume that managers operate the firm on behalf of shareholders' interests, and hence we do not distinguish between shareholders and managers.

2.2 Bankruptcy and its negative spillovers

Shareholders of firm $i \in \{H, L\}$ can stop paying coupon C_i and declare default at any time in their own interests. They do not consider the debt value in place. At the default time t, as in Lambrecht and Myers (2008) and Nishihara and Shibata (2019), firm i is liquidated by the value $U_i(X(t))$, where the function $U_i(\cdot)$ is assumed to be affine in numerical examples.[1] We assume liquidation bankruptcy because we prefer to focus on a

[1]For simplicity, we assume that the functional form $U_i(X(t))$ is unchanged by firm j's bankruptcy. In reality, compared to operational cash flows, scrap value is less affected by another firm's bankruptcy. The main results

simple case in which a bankrupt firm stops operations and causes negative spillovers to the other firm in operations.[2] We presume that debt is risky, i.e., the principal of debt C_i/r is higher than the liquidation value $U_i(X(t))$.[3] Then, by the absolute priority rule, debt holders receive the liquidation value $U_i(X(t))$, while shareholders receive nothing. For simplicity, as in Lambrecht and Myers (2008) and Nishihara and Shibata (2019), we assume no opportunity of debt renegotiation in the baseline model. We will also examine a setup with debt renegotiation in Section 4.3.

After firm i's bankruptcy, the other firm $j(\neq i)$'s EBIT falls to $\delta_j a_j X(t) - \epsilon_j b_j$ if it continues operations. The EBIT contraction parameters $\delta_j \in (0,1)$ and $\epsilon_j(\geq 1)$ stand for the negative bankruptcy spillovers, where a lower δ_j and higher ϵ_j mean stronger spillover effects. The negative externalities can arise from real and financial linkages between the two firms. For instance, a firm may be forced to purchase more costly parts from other firms when its major supply chain partner goes bankrupt. If firms cross-hold shares, a firm loses dividends after the other firm's bankruptcy. When cross-holding debts with infinite maturity, a firm loses coupons after the other firm's bankruptcy. Although the loss of coupons is not counted in EBIT, it decreases cash flows. Hence, the cross-holding model becomes almost the same model. In many cases, the negative effect will not be permanent. For example, consider firms that provide trade credit or short-term debt to each other. Default by a firm causes bad debt loss to the other firm, but the loss is temporary rather than permanent. The main results and implications in this paper will vary only quantitatively but not qualitatively, even if we model such temporary effects. Thus, for simplicity, we model the permanent and negative effects of the other firm's bankruptcy on EBIT.

in this paper will remain unchanged if the bankruptcy spillover effects on the liquidation value are smaller than that on the cash flows.

[2]The assumption of liquidation bankruptcy is not essential, and we may assume reorganization bankruptcy. The main results in this paper will remain unchanged as far as a firm's reorganization bankruptcy reduces the other firm's earnings. Several papers, such as Morellec, Valta, and Zhdanov (2015), Shibata and Nishihara (2015), and Antill and Grenadier (2019) examine reorganization bankruptcy with debt renegotiation.

[3]Indeed, in all numerical examples, the firms issue risky debt rather than riskless debt to maximize their firm values.

2.3 Capital structure

All agents have complete information about all parameter values of both firms.[4] At time 0, the equity, debt, and firm values of firm $i \in \{H, L\}$ are priced under the rational expectation of its own bankruptcy time, the opponent firm $j(\neq i)$'s bankruptcy time, and the bankruptcy spillover effects. Following the standard literature (e.g., Leland (1994) and Goldstein, Ju, and Leland (2001)), at time 0, firm i chooses the optimal capital structure (i.e., coupon C_i) to maximize its firm value by the tradeoff between the tax benefits and bankruptcy costs of debt. For model tractability, we do not consider the dynamic adjustment of capital structure, but Section 4.3 discusses the effects of the leverage adjustment on the model results.

The model's key feature is that the firms make the capital structure decisions from strategic considerations of each other's capital structure and bankruptcy timing, as well as the negative bankruptcy spillovers. Morellec and Zhdanov (2008), Nishihara and Shibata (2010), and Matveyev and Zhdanov (2019) investigate the strategic decisions of capital structure in a duopoly market, while Miao (2005) examines capital structure in a perfectly competitive market. These papers examine the competition effects of bankruptcy—a firm's bankruptcy increases the other firm's cash flows. In contrast, we examine contagion effects—a firm's bankruptcy decreases the other firm's cash flows. By this difference, we will show that a firm can take the capital structure which is quite different from that found in previous literature.

3 Model Solutions

3.1 Nonstrategic solutions

As a benchmark, we examine two firms with no bankruptcy spillover (i.e., $\delta_i = \epsilon_i = 1$ for $i = \{H, L\}$). Hereafter, this benchmark case is called the nonstrategic case. Suppose that firm i issues debt with coupon $C(\geq 0)$ and will bankrupt at the threshold $y(> 0)$. As in the standard literature (e.g., Leland (1994) and Goldstein, Ju, and Leland (2001)),

[4] It might be difficult for firms to precisely estimate the EBIT contraction parameters. In Section 4.4, we discuss a case with incomplete information about bankruptcy spillovers.

we can calculate the following equity, debt, and firm values:

$$
\begin{aligned}
E_i^N(x,y,C) &= \mathbb{E}[\int_0^T \mathrm{e}^{-rt}(1-\tau)(a_iX(t)-b_i-C)\mathrm{d}t] \\
&= (1-\tau)\left\{\frac{a_ix}{r-\mu}-\frac{b_i+C}{r}+\left(\frac{x}{x\wedge y}\right)^{\gamma}\left(-\frac{a_i(x\wedge y)}{r-\mu}+\frac{b_i+C}{r}\right)\right\} \qquad (1)\\
D_i^N(x,y,C) &= \mathbb{E}[\int_0^T \mathrm{e}^{-rt}C\mathrm{d}t+\mathrm{e}^{-rT}U_i(X(T))] \\
&= \frac{C}{r}-\left(\frac{x}{x\wedge y}\right)^{\gamma}\left(\frac{C}{r}-U_i(x\wedge y)\right), \qquad (2)\\
F_i^N(x,y,C) &= E_i^N(x,y,C)+D_i^N(x,y,C) \\
&= \frac{(1-\tau)a_ix}{r-\mu}-\frac{(1-\tau)b_i}{r}+\frac{\tau C}{r}-\left(\frac{x}{x\wedge y}\right)^{\gamma}\left(\frac{(1-\tau)a_i(x\wedge y)}{r-\mu}+\frac{\tau C-(1-\tau)b_i}{r}-U_i(x\wedge y)\right), \qquad (3)
\end{aligned}
$$

where T denotes the stopping time (i.e., the bankruptcy time) $T=\inf\{t\geq 0 \mid X(t)\leq y\}$. The notation $x\wedge y$ represents the minimum of x and y, and γ denotes the negative characteristic root $\gamma = 0.5-\mu/\sigma^2-\sqrt{(0.5-\mu/\sigma^2)^2+2r/\sigma^2}$. Throughout the paper, the superscript N stands for the nonstrategic case. In (1), the first two terms represent the equity value of the permanently operating firm, and the extra term represents the value of the option to default at the threshold y. In (2), C/r represents the riskless debt value, and the extra term represents the expected loss of bankruptcy. Recall that on bankruptcy, debt holders receive the liquidation value, i.e., $U_i(x\wedge y)$, although they lose the value of future coupon streams, i.e., C/r. The firm value (3) can be decomposed of the three parts, i.e., the unlevered firm value, tax benefits of debt, and bankruptcy costs of debt. The first two terms represent the unlevered firm value, while the third and forth terms represent the tax benefits and bankruptcy costs of debt, respectively.

For debt in place, (i.e., coupon C), firm i's shareholders optimize the bankruptcy threshold y to maximize the equity value (1). Hence, we can derive the bankruptcy threshold as the following function of C:

$$x_i^N(C) = \arg\max_{y\geq 0} E_i^N(x,y,C) = \frac{\gamma(r-\mu)(b_i+C)}{(\gamma-1)ra_i}. \qquad (4)$$

By substituting $y=x_i^N(C)$ into (1)–(3), we have the equity, debt, and firm values under the rational expectation of the bankruptcy time. Firm i chooses its coupon C to maximize the firm value. Then, we have the optimal coupon

$$C_i^N = \arg\max_{C\geq 0} F_i^N(x,C), \qquad (5)$$

where we denote $F_i^N(x,C) = F_i^N(x,x_i^N(C),C)$. We can numerically verify that in (5), $F_i^N(x,C)$ is unimodal with respect to C and has a unique maximizer C_i^N. For simplicity, under the optimal capital structure, we denote the bankruptcy threshold by $x_i^N = x_i^N(C_i^N)$. We also denote the equity, debt, and firm values by $E_i^N(x) = E_i^N(x, x_i^N, C_i^N), D_i^N(x) = D_i^N(x, x_i^N, C_i^N)$, and $F_i^N(x) = F_i^N(x, x_i^N, C_i^N)$, respectively.

In numerical examples, we always have

$$\frac{\partial F_i^N(x,C)}{\partial C} \begin{cases} > 0 & (C \in [0, C_i^N)) \\ = 0 & (C = C_i^N) \\ < 0 & (C \in (C_i^N, \bar{C}_i)) \\ = 0 & (C > \bar{C}_i), \end{cases} \tag{6}$$

where $\bar{C}_i = (\gamma - 1)ra_i x/\gamma(r-\mu) - b_i$. Note that $\bar{C}_i$ is the solution to $x_i^N(\bar{C}_i) = x$, which means that for $C \geq \bar{C}_i$, firm i bankrupts at time 0, leading to $F_i^N(x) = U_i(x)$. In numerical examples, we can easily compute the unique maximizer C_i^N because of the unimodality (6). We can also numerically verify that $C_H^N > C_L^N$ (cf. Figure 1) and

$$x_H^N < x_L^N, \tag{7}$$

which implies that compared to firm L, firm H issues more debt but defaults later because of its higher profitability. In theoretical analysis in Section 3.2, we assume (6) and (7).

3.2 Baseline solutions

In this subsection, we examine the baseline case with negative bankruptcy spillovers (i.e., $\delta_i < 1, \epsilon_i \geq 1$ for $i = \{H, L\}$). Under assumption (7), firm L, which bankrupts earlier than firm H bankrupts, has no incentive to deviate from the nonstrategic coupon C_L^N, which generates the firm value $F_L^N(x)$. On the other hand, firm H, which bankrupts later than firm L bankrupts, cannot gain the firm value $F_H^N(x)$ by the nonstrategic coupon C_H^N because firm H's EBIT falls after firm L's bankruptcy. Thus, firm H can potentially deviate from C_H^N to maximize its firm value. Intuitively, firm H can decrease its coupon below C_H^N to anticipate the worse cash flow scenario, where EBIT decreases after firm L's bankruptcy. In Section 3.2.2, we will show that this straightforward result arises under certain parameter sets. However, we will show that firm H counterintuitively increases its coupon beyond C_H^N under certain parameter sets.

3.2.1 Solutions for exogenous capital structure

In this subsection, we suppose that firms H and L have issued debt with coupons C_H and C_L, respectively, at time 0. Then, we derive the firm $i \in \{H, L\}$'s bankruptcy threshold (denoted by $x_i(C_i, C_j)$) as well as the equity, debt, and firm values (denoted by $E_i(x, C_i, C_j)$, $D_i(x, C_i, C_j)$, and $F_i(x, C_i, C_j)$) as the functions of C_H and C_L. We define

$$K_1(C_L) = \frac{\delta_H a_H C_L}{a_L} + \frac{\delta_H a_H b_L}{a_L} - \epsilon_H b_H, \tag{8}$$

$$K_2(C_L) = \frac{a_H C_L}{a_L} + \frac{a_H b_L}{a_L} - b_H, \tag{9}$$

$$K_3(C_L) = \frac{a_H C_L}{\delta_L a_L} + \frac{\epsilon_L a_H b_L}{\delta_L a_L} - b_H, \tag{10}$$

and also define $E_i^R(x, y, C)$ and $x_i^R(C)$ by replacing a_i and b_i with $\delta_i a_i$ and $\epsilon_i b_i$, respectively, in (1) and (4). The superscript R stands for the revised equity value and threshold due to EBIT contraction. By (8)–(10), we can readily show that $K_1(C_L) < K_2(C_L) < K_3(C_L)$, $x_H^R(K_1(C_L)) = x_H^N(K_2(C_L)) = x_L^N(C_L)$, and $x_H^N(K_3(C_L)) = x_L^R(C_L)$. In other words, $C_H = K_1(C_L)$ is the coupon such that firm H's revised threshold agrees with firm L's nonstrategic threshold. Similarly, $C_H = K_2(C_L)$ is the coupon such that firm H's nonstrategic threshold agrees with firm L's nonstrategic threshold, whereas $C_H = K_3(C_L)$ is the coupon such that firm H's nonstrategic threshold agrees with firm L's revised threshold. The following proposition shows the firms' policies and values for exogenously given (C_H, C_L). For the proof, refer to Appendix A.

Proposition 1 *Suppose that firms H and L issue debt with coupon C_H and C_L, respectively. Suppose that the initial state variable x is higher than $x_H^N(C_H)$ and $x_L^N(C_H)$. Firm i's bankruptcy threshold $x_i(C_i, C_j)$, equity value $E_i(x, C_i, C_j)$, debt value $D_i(x, C_i, C_j)$, firm value $F_i(x, C_i, C_j)$ are given as follows.*

Sequential bankruptcy case I: $C_H \in [0, K_1(C_L))$.

$$\begin{aligned}
x_H(C_H, C_L) &= x_H^R(C_H) < x_L(C_L, C_H) = x_L^N(C_L),\\
E_H(x, C_H, C_L) &= E_H^N(x, x_L^N(C_L), C_H) + \left(\frac{x}{x_L^N(C_L)}\right)^{\gamma} E_H^R(x_L^N(C_L), x_H^R(C_H), C_H),\\
E_L(x, C_L, C_H) &= E_L^N(x, x_L^N(C_L), C_L),\\
D_H(x, C_H, C_L) &= D_H^N(x, x_H^R(C_H), C_H),\\
D_L(x, C_L, C_H) &= D_L^N(x, x_L^N(C_L), C_L),\\
F_H(x, C_H, C_L) &= E_H^N(x, x_L^N(C_L), C_H) + D_H^N(x, x_H^R(C_H), C_H) + \left(\frac{x}{x_L^N(C_L)}\right)^{\gamma} E_H^R(x_L^N(C_L), x_H^R(C_H), C_H),\\
F_L(x, C_L, C_H) &= F_L^N(x, x_L^N(C_L), C_L).
\end{aligned}$$

Simultaneous bankruptcy case: $C_H \in [K_1(C_L), K_3(C_L)]$.

$$\begin{aligned}
x_H(C_H, C_L) = x_L(C_L, C_H) &= x_H^N(C_H) \vee x_L^N(C_L),\\
E_H(x, C_H, C_L) &= E_H^N(x, x_H^N(C_H) \vee x_L^N(C_L), C_H),\\
E_L(x, C_L, C_H) &= E_L^N(x, x_H^N(C_H) \vee x_L^N(C_L), C_L),\\
D_H(x, C_H, C_L) &= D_H^N(x, x_H^N(C_H) \vee x_L^N(C_L), C_H),\\
D_L(x, C_L, C_H) &= D_L^N(x, x_H^N(C_H) \vee x_L^N(C_L), C_L),\\
F_H(x, C_H, C_L) &= F_H^N(x, x_H^N(C_H) \vee x_L^N(C_L), C_H),\\
F_L(x, C_L, C_H) &= F_L^N(x, x_H^N(C_H) \vee x_L^N(C_L), C_L),
\end{aligned}$$

where $x_H^N(C_H) \vee x_L^N(C_L)$ *denotes the maximum of* $x_H^N(C_H)$ *and* $x_L^N(C_L)$.

Sequential bankruptcy case II: $C_H > K_3(C_L)$.

$$\begin{aligned}
x_H(C_H, C_L) &= x_H^N(C_H) > x_L(C_L, C_H) = x_L^R(C_L),\\
E_H(x, C_H, C_L) &= E_H^N(x, x_H^N(C_L), C_H),\\
E_L(x, C_L, C_H) &= E_L^N(x, x_H^N(C_H), C_L) + \left(\frac{x}{x_H^N(C_H)}\right)^\gamma E_L^R(x_H^N(C_H), x_L^R(C_L), C_L),\\
D_H(x, C_H, C_L) &= D_H^N(x, x_H^N(C_H), C_H),\\
D_L(x, C_L, C_H) &= D_L^N(x, x_L^R(C_L), C_L),\\
F_H(x, C_H, C_L) &= F_H^N(x, x_H^N(C_H), C_H),\\
F_L(x, C_L, C_H) &= E_L^N(x, x_H^N(C_H), C_L) + D_L^N(x, x_L^R(C_L), C_L) + \left(\frac{x}{x_H^N(C_H)}\right)^\gamma E_L^R(x_H^N(C_H), x_L^R(C_L), C_L).
\end{aligned}$$

For $C_H < K_1(C_L)$, firm H's revised threshold $x_H^R(C_H)$ is lower than firm L's threshold $x_L^N(C_L)$. Then, firm H bankrupts later than firm L bankrupts because of low levels of debt. For $C_H > K_3(C_L)$, firm L's revised threshold $x_L^R(C_L)$ is lower than firm H's threshold $x_H^N(C_H)$. Then, firm H bankrupts earlier than firm L because of high levels of debt. We call these two cases the sequential bankruptcy cases I and II, respectively. In these cases, the equity and debt values of the firm that bankrupts first are the same as those of the nonstrategic case (say, $E_L(x, C_L, C_H) = E_L^N(x, x_L^N(C_L), C_L)$ and $D_L(x, C_L, C_H) = D_L^N(x, x_L^N(C_L), C_L)$ in the sequential bankruptcy cases I). The negative bankruptcy spillover effects affect he equity and debt values of the firm that bankrupts later (say, $E_H(x, C_L, C_H)$ and $D_H(x, C_L, C_H)$ in the sequential bankruptcy cases I).

The intermediate case (i.e., $C_H \in [K_1(C_L), K_3(C_L)]$) is most interesting. For $C_H \in [K_1(C_L), K_2(C_L)]$, firm H does not bankrupt earlier than firm L because $x_H^N(C_H)$ is not higher than $x_L^N(C_L)$, but firm H does not postpone bankruptcy after firm L's bankruptcy because its revised threshold $x_H^R(C_H)$ is not lower than $x_L^N(C_L)$. For $C_H \in (K_2(C_L), K_3(C_L)]$, firm L does not bankrupt earlier than firm H because $x_L^N(C_L)$ is lower than $x_H^N(C_H)$, but firm L does not postpone bankruptcy after firm H's bankruptcy because its revised threshold $x_L^R(C_L)$ is not lower than $x_H^N(C_H)$. Thus, firms H and L bankrupt simultaneously at the bankruptcy threshold $x_H^N(C_H) \vee x_L^N(C_L)$. We call this case the simultaneous bankruptcy case. In this case, the equity and debt values agree with those of the nonstrategic case with the bankruptcy threshold $x_H^N(C_H) \vee x_L^N(C_L)$.

With no bankruptcy spillover (i.e., $\delta_i = \epsilon_i = 1$), we have $K_1(C_L) = K_2(C_L) = K_3(C_L)$, which means that simultaneous bankruptcy occurs only for $C_H = K_1(C_L) = K_2(C_L) = K_3(C_L)$. However, with negative bankruptcy spillovers, a firm's bankruptcy threshold is revised upward after the other firm's bankruptcy. As the spillover effects are stronger (i.e., a lower δ_i and higher ϵ_i), the simultaneous bankruptcy region becomes larger.

3.2.2 Solutions for endogenous capital structure

In this subsection, we consider the two firms' game, where firm $i \in \{H, L\}$ optimizes C_i to maximize its firm value $F_i(x, C_i, C_j)$ at time 0. We define

$$C_H^* = \arg \max_{C \in [0, K_1(C_L^N)]} F_H(x, C, C_L^N), \tag{11}$$

$$C_H^{**} = K_2(C_L^N), \tag{12}$$

where $C_H^{**} > C_H^N$ follows from assumption (7). Definition (11) means that for firm L's nonstrategic coupon C_L^N, C_H^* is firm H's optimal coupon in the sequential bankruptcy case I. By definition (12), C_H^{**} is the coupon for which firm H's nonstrategic threshold is equal to firm L's nonstrategic threshold. As we will see in Appendix B, C_H^{**} is firm H's optimal coupon in the simultaneous bankruptcy case (cf. Figure 1). The following proposition derives the Nash equilibria of the game. For the proof, refer to Appendix B.

Proposition 2 *Suppose that the initial state variable x is higher than x_L^N. The payoff dominant equilibrium is given as follows. Firm L chooses coupon C_L^N. If $F_H(x, C_H^*, C_L^N) > F_H^N(x, C_H^{**})$ holds, firm H chooses coupon C_H^*. If $F_H(x, C_H^*, C_L^N) = F_H^N(x, C_H^{**})$ holds, firm H chooses either coupon C_H^* or C_H^{**}. If $F_H(x, C_H^*, C_L^N) < F_H^N(x, C_H^{**})$ holds, firm H chooses coupon C_H^{**}. If any other Nash equilibrium exists, its strategy profile (C_H, C_L) satisfies either*

$$C_L \in (C_L^N, \bar{C}_L), \ C_H = K_2(C_L)$$

or

$$C_L \geq \bar{C_L},\ C_H \geq \bar{C_H}.$$

In the payoff dominant equilibrium, firm L chooses the nonstrategic coupon C_L^N and gains the nonstrategic firm value $F_L^N(x)$. Intuitively, firm L, which bankrupts earlier than firm H bankrupts, does not suffer from negative bankruptcy spillovers. Then, firm L has no incentive to change the nonstrategic coupon because it cannot increase the firm value beyond the nonstrategic value. Shareholders of firm L declare default at the nonstrategic threshold x_L^N, where debt holders receive all the liquidation value.

On the other hand, firm H's optimal coupon depends on the parameter values. First, we explain the case of $F_H(x, C_H^*, C_L^N) > F_H^N(x, C_H^{**})$. In this case, firm H chooses the low coupon $C_H^*(< K_1(C_L^N))$, which leads to sequential bankruptcy.[5] We call this outcome the sequential bankruptcy equilibrium. In other words, firm H chooses a modest level of leverage to prepare for operations in the worse cash flow scenario after firm L's bankruptcy. After firm L's bankruptcy, firm H continues to operate until the state variable $X(t)$ falls to the revised threshold $x_H^R(C_H^*)$. At $x_H^R(C_H^*)$, shareholders of firm H declare default, and debt holders receive all the liquidation value. In this equilibrium, the expected firm value is equal to $F_H(x, C_H^*, C_L^N)$ at time 0. This sequential bankruptcy equilibrium corresponds to the straightforward response to negative bankruptcy spillovers, as explained in the beginning of Section 3.2.

Next, we turn to the case of $F_H(x, C_H^*, C_L^N) < F_H^N(x, C_H^{**})$. In this case, firm H chooses the high coupon $C_H^{**}(= K_2(C_L^N))$, which leads to simultaneous bankruptcy. We call this outcome the simultaneous bankruptcy equilibrium. Note that $x_H^N(C_H^{**}) = x_L^N$ holds. In other words, firm H increase debt to equalize its nonstrategic bankruptcy threshold with that of firm L. This strategic increase of debt is explained as follows. For $C_H \in [K_1(C_L^N), C_H^{**}]$, firm H's bankruptcy threshold remains at x_L^N, which is independent of C_H, as is the bankruptcy cost. Then, firm H maximizes the tax benefits of debt by increasing its coupon to the maximum level C_H^{**} (cf. $F_H(x, C, C_L^N)$ increasing in $C_H \in [K_1(C_L^N), C_H^{**}]$ in Figure 1). If firm H increase its coupon C_H beyond C_H^{**}, the bankruptcy threshold changes to $x_H^N(C_H)$, which decreases the firm value via the increased bankruptcy cost (cf. $F_H(x, C, C_L^N)$ decreasing in $C_H \geq C_H^{**}$ in Figure 1). Thus, firm H intentionally chooses the coupon C_H^{**} to bankrupt simultaneously with firm L at x_L^N. In

[5]We can easily show that C_H^* is strictly lower than $K_1(C_L^N)$ when the sequential bankruptcy equilibrium is chosen. For details, see Appendix B.

this equilibrium, the expected firm value is equal to $F_H(x, C_H^{**}, C_L^N) = F_H^N(x, C_H^{**})$ at time 0. This simultaneous bankruptcy equilibrium corresponds to the counterintuitive response to negative bankruptcy spillovers, as explained in the beginning of Section 3.2. Indeed, firm H increases debt beyond the nonstrategic level rather than decreasing debt in the presence of negative bankruptcy spillovers.

In another equilibrium, if it exists, firms H and L issue debt with coupon $C_H = K_2(C_L)(> C_H^{**})$ and $C_L(> C_L^N)$, respectively, or issue debt to bankrupt immediately. In any case, they bankrupt simultaneously. In other words, both firms issue more debt and simultaneously bankrupt earlier than in the payoff dominant equilibrium. This equilibrium can be regarded as a worse equilibrium than the payoff dominant simultaneous bankruptcy equilibrium.

3.2.3 Empirical implications

In this subsection, we explain the empirical implications from Proposition 2. It determines firm H's capital structure choice of whether firm H's cash flows after firm L's bankruptcy are sufficiently high compared to the tax benefits of increased debt. Notably, when the cash flows are insufficient, firm H aims to cause simultaneous bankruptcy. More negative bankruptcy spillovers (i.e., a lower δ_i and higher ϵ_i) and more tax benefits (i.e., a higher τ) increase firm H's incentive to choose the simultaneous bankruptcy equilibrium. To our knowledge, no paper shows that firms' strategic capital structure choices lead to bankruptcy cascades. Our results provide a novel mechanism of bankruptcy contagion—a firm has an ex-ante herding incentive to increase debt and to go bankrupt simultaneously with another firm on which it financially and/or operationally depends.

Obviously, the simultaneous bankruptcy equilibrium is likely to arise for affiliates in the same corporate family. Emery and Cantor (2005) show the empirical evidence of bankruptcy cascades over affiliates belonging to the same corporate family or conglomerate. Similarly, we predict that the simultaneous bankruptcy equilibrium applies for firms belonging to the same supply chain network (cf. Hertzel, Li, Officer, and Rodgers (2008) and Kolay, Lemmon, and Tashjian (2016)). For instance, in the Japanese automotive industry, a large automaker such as Toyota has developed a strong supply chain network with specific auto parts suppliers called "keiretsu" companies. Keiretsu companies depend greatly on the large automaker in terms of sales and profits, and hence, these companies are unlikely to generate sufficient earnings if the large automaker goes into financial distress. Then, we predict that keiretsu companies can adjust capital structure to align their bankruptcy timing with that of their leader.

Trade-credit linkages between firms in the same supply chain network are akin to interbank loan linkages between banks. Indeed, a bank's bankruptcy can lead to negative spillovers to other banks through interbank loan linkages (cf. Allen and Gale (2000) and Eisenberg and Noe (2001)).[6] Schepens (2016) shows the empirical evidence of bank capital structure driven by the tax benefits of debt, although the tax benefits are often overlooked in the banking literature. In recent years, Hugonnier and Morellec (2017) and Sundaresan and Wang (2015) have examined the bank capital structure by extending the setup of Leland (1994). The main difference from the nonfinancial corporate model of this paper is to include deposits in addition to equity and debt, but we have similar results even in the banking models where the tax benefits of debt are present. Thus, we argue that banks can adjust capital structure ex ante in order to go bankrupt together, which causes systemic banking crises. Acharya and Yorulmazer (2007) also show the possibility that banks take the same risk ex ante to fail together. In their model, the driver of the herding behavior is a too-many-to-fail problem—the regulator finds it ex post optimal to bail out failed banks when the number of failed banks is too large. Banks try to fail simultaneously in order to be bailed out by the regulator. We complement their arguments by showing that only in the presence of negative bankruptcy spillovers (e.g., interbank loan linkages) do banks have incentives to increase debt and to go bankrupt together.

Many papers, including Das, Duffie, Kapadia, and Saita (2007), Benzoni, Collin-Dufresne, Goldstein, and Helwege (2015), and Azizpour, Giesecke, and Schwenkler (2018), show the empirical evidence that default contagion is not limited to firms with counterparty risk. In some cases, the mechanism of the simultaneous bankruptcy equilibrium may be the story behind these findings, although the empirical relevance has yet to be studied. In addition, the mechanism can potentially explain the peer effects in corporate financial policies. For instance, Leary and Roberts (2014) show that firms' financial policies (e.g., leverage, equity and debt issuance, etc.) are affected by their peer firms' financial policies. The learning or competition effects often explain the herding behavior, our results add an alternative explanation—a firm that anticipates negative externalities from bankruptcy of its peer firm in the same industry (cf. Lang and Stulz (1992)) can adjust its capital structure in response to its peer firm to align the bankruptcy timing.

Lastly, empirical evidence shows that there is substantial inter- and intra-variation in leverage (e.g., MacKay and Phillips (2005)). Our model can also explain a wide variation

[6]As explained in Section 2.2, we have the same result even in a setup with temporary spillover effects.

in leverage through the difference between the sequential and simultaneous bankruptcy equilibria (cf. the multiple equilibria for $F_H(x, C_H^*, C_L^N) = F_H^N(x, C_H^{**})$ in Proposition 2). Miao (2005) and Morellec and Zhdanov (2008) explain the dispersion in leverage through the competitive intra-industry effects. They show that in oligopoly markets, a firm can decrease leverage to survive longer or to win competition. The sequential bankruptcy equilibrium, where firm H decreases leverage in preparation of operations in the future worse cash flow scenario, is similar to the results of Miao (2005) and Morellec and Zhdanov (2008). However, the simultaneous bankruptcy equilibrium is opposite from the previous results.[7] In fact, unlike in the competitive intra-industry model, firm H increases leverage to go bankrupt simultaneously with firm L. Thus, this paper complements the previous literature by showing the opposite capital structure stemming from the contagion effect.

3.3 Numerical analysis

3.3.1 The baseline results

We numerically examine the payoff dominant equilibrium in Proposition 2 in full details. We omit depicting the other type of equilibria (i.e., worse versions of the simultaneous bankruptcy equilibrium) because their properties are similar to those of the payoff dominant simultaneous bankruptcy equilibrium. The baseline parameter values are set in Table 1, where the values of r, μ, σ, and τ are standard in dynamic corporate finance literature and reflect a typical S&P firm (e.g., Morellec (2001) and Arnold (2014)). The liquidation value function is set by $U_i(x) = 0.5(a_i x/(r-\mu) - b_i/r)$. The bankruptcy spillover effects are represented by $\delta_i = 0.94 (< 1)$ $(i = H, L)$. We set the baseline values of a_i, b_i, and δ_i $(i = H, L)$ to make firm H's values in the two equilibria (i.e., $F_H(x, C_H^*, C_L^N)$ and $F_H^N(x, C_H^{**})$) almost equal because we want to examine how the two equilibria switch due to the parameter values.

Figure 1 shows the firm value functions $F_H(x, C, C_L^N), F_H^N(x, C)$, and $F_L^N(x, C)$ with respect to coupon C. In the figure, we see that $F_i^N(x, C)$ $(i = H, L)$ are unimodal and satisfy assumption (6). Their maximums are attained at $C_H^N = 0.958$ and $C_L^N = 0.748$, and the bankruptcy thresholds are $x_H^N = 0.679$ and $x_L^N = 0.874$, which satisfy assumption (7). This implies that with no spillover effect, firm H issues more debt but defaults later because of its higher profitability than firm L. As shown in Proposition 2, in the payoff

[7] Static models, such as Brander and Lewis (1986) and Fulghieri and Nagarajan (1996), argue that higher leverage can lead to competitive advantages in oligopoly markets, although they do not examine the contagion effects.

Table 1: Baseline parameter values.

r	μ	σ	τ	a_H/b_H	a_L/b_L	δ_i	ϵ_i	x
0.06	0.01	0.2	0.15	1/0.4	1/1	0.94	1	2

Table 2: Baseline results.

x_H	$C_H(x)$	$LV_H(x)$	$CS_H(x)$	$E_H(x)$	$D_H(x)$	$F_H(x)$	$SW(x)$
0.874	1.348	0.589	0.0179	12.096	17.306	29.402	54.734
x_L	$C_L(x)$	$LV_L(x)$	$CS_L(x)$	$E_L(x)$	$D_L(x)$	$F_L(x)$	
0.874	0.748	0.426	0.0234	12.096	8.967	21.063	

dominant equilibrium, firm L's financing and bankruptcy policies as well as the firm value are unchanged with bankruptcy spillovers.

On the other hand, firm H's value $F_H(x, C, C_L^N)$ is different from the nonstrategic firm value $F_H^N(x, C)$. See the top panel of Figure 1. We have $K_1(C_L^N) = 1.243, C_H^{**} = K_2(C_L^N) = 1.349$, and $K_3(C_L^N) = 1.46$. For $C < 1.243$, We have the sequential bankruptcy case I in Proposition 1 for $C < 1.243$, the simultaneous bankruptcy case for $C \in [1.243, 1.46]$, and the sequential bankruptcy case II for $C > 1.46$. We have $C_H^* = \arg\max_{C\in[0,1.243]} F_H(x, C, C_L^N) = 0.931(< C_H^N = 0.958)$ as the optimal coupon in the sequential bankruptcy case. As we explained after Proposition 2, for $C_H \in [1.243, 1.349]$, firm H's bankruptcy threshold stays at $x_L^N = 0.874$, and hence, $F_H(x, C, C_L^N)$ monotonically increases in this region because the tax benefits of debt increase. For $C_H \geq 1.349$, $F_H(x, C, C_L^N)$ agrees with $F_H^N(x, C)$ and monotonically decreases. Then, we have $C_H^{**} = 1.349$ as the optimal coupon in the simultaneous bankruptcy case.

The simultaneous bankruptcy equilibrium arises because the firm value $F_H^N(x, C_H^{**}) = F_H(x, C_H^{**}, C_L^N) = 29.402$ is higher than $F_H(x, C_H^*, C_L^N) = 29.394$. In equilibrium, firm H issues debt with the high coupon $C_H^{**} = 1.349(> C_H^N = 0.958)$ and goes bankrupt at $x_L^N = 0.874$ simultaneously with firm L. Table 2 summarizes the baseline values in equilibrium. If firm H did not take into account strategic considerations, firm H would issue debt with the nonstrategic coupon $C_H^N = 0.958$ and operate in the worse cash flow scenario after firm L's bankruptcy until the bankruptcy threshold $x_H^R(C_H^N) = 0.722$ is reached. However, in equilibrium, firm H, that anticipates the negative spillover effects, strategically increases debt to bankrupt simultaneously with firm L. In other words, firm H's strategic increase of debt causes bankruptcy cascades in this numerical example.

3.3.2 The impacts of bankruptcy spillovers

Now, we explore the comparative statics with respect to the key parameters. We focus on firm H's results in the payoff dominant equilibrium because firm L's results are the same as those of the nonstrategic case. Figure 2 shows firm H's bankruptcy threshold x_H, coupon C_H, leverage $LV_H (= D_H(x)/F_H(x))$, credit spread $CS_H (= C_H/D_H(x) - r)$, equity value $E_H(x)$, debt value $D_H(x)$, firm value $F_H(x)$, and social welfare $SW(x)$ with varying levels of the spillover effect parameter δ_H. Recall that a lower δ_H means more negative bankruptcy spillover effects. The other parameter values are set in Table 1. The social welfare $SW(x)$ is defined by the sum of the two firm values and government value through the corporate tax revenue.[8]

In Figure 2, the values other than $F_H(x)$ jump at $\delta_H = 0.9417$. This is because firm H chooses C_H^{**} (i.e., the simultaneous bankruptcy equilibrium) for $\delta_H \leq 0.9417$ and C_H^* (i.e., the sequential bankruptcy equilibrium) for $\delta_H > 0.9417$. A lower δ_H decreases cash flows after firm L's bankruptcy, and hence, it increases firm H's incentive to choose C_H^{**} and to bankrupt simultaneously with firm L. It is straightforward that x_H, C_H, LV_H, and CS_H are higher in the simultaneous bankruptcy case than in the sequential bankruptcy case. In the panels of C_H and LV_H, a higher δ_H, which means weaker externalities, can decrease C_H and LV_H by shifting from the simultaneous bankruptcy equilibrium to the sequential bankruptcy equilibrium, although within the sequential bankruptcy region, a higher δ_H straightforwardly increases C_H and LV_H. In other words, firm H's capital structure is nonmonotonic with respect to the bankruptcy spillover effects.

In the panels of $E_H(x)$ and $D_H(x)$ in Figure 2, we see that $E_H(x)$ ($D_H(x)$) in the simultaneous bankruptcy equilibrium is lower (higher) than those in the sequential bankruptcy equilibrium. This is explained by the higher coupon in the simultaneous bankruptcy equilibrium. In fact, the higher coupon decreases residual payoffs to shareholders and accelerates bankruptcy, and hence, it decreases $E_H(x)$. The higher coupon increases coupon payments although it speeds up bankruptcy. The former effect dominates the latter effect, and hence, the higher coupon increases $D_H(x)$. Then, a higher δ_H can increase $E_H(x)$ and decrease $D_H(x)$ by shifting from the simultaneous bankruptcy equilibrium to the sequential bankruptcy equilibrium, although within the sequential bankruptcy region, a higher δ_H decreases $E_H(x)$ and increases $D_H(x)$. As in the capital structure, the equity and debt values are nonmonotonic with respect to the bankruptcy spillover effects.

[8] The main results are unchanged even if we include the consumer surplus which decreases by a firm's bankruptcy.

Lastly, the panel of $SW(x)$ shows that $SW(x)$ jumps upward at $\delta_H = 0.9417$. In the simultaneous bankruptcy equilibrium, $SW(x)$ is lower than that of the sequential bankruptcy equilibrium because firm H's high coupon C_H^{**} and bankruptcy threshold x_L^N reduce the tax revenues of the government. This suggests that firm H's strategic choice of simultaneous bankruptcy reduces the social welfare.

3.3.3 The impacts of profitability

Figure 3 shows the equilibrium results with varying levels of firm H's operating cost b_H. The other parameter values are set in Table 1, where firm L's operating cost b_L is set at 1. The values other than $F_H(x)$ jump at $b_H = 0.391$. This is because firm H chooses C_H^* (i.e., the sequential bankruptcy equilibrium) for $b_H < 0.391$ and C_H^{**} (i.e., the simultaneous bankruptcy equilibrium) for $b_H \geq 0.391$. A higher b_H decreases firm H's advantage over firm L in terms of profitability, and hence, it decreases cash flows by operations after firm L's bankruptcy. Then, for a higher b_H, firm H is more likely to choose C_H^{**} to bankrupt simultaneously with firm L.

Notably, Figure 3 shows that C_H, LV_H, and $D_H(x)$ are nonmonotonic with respect to b_H. This is explained by the shift from the simultaneous bankruptcy equilibrium to the sequential bankruptcy equilibrium. In fact, C_H, LV_H, and $D_H(x)$ jump upward at $b_H = 0.391$ by shifting from the simultaneous bankruptcy equilibrium to the sequential bankruptcy equilibrium, although they monotonically decrease in each equilibrium region. Most of the structural models (e.g., Leland (1994)) based on the tradeoff theory predict a positive relation between leverage and profitability (i.e., C_H, LV_H, and $D_H(x)$ decreasing in b_H), although empirical studies (e.g., Titman and Wessels (1988) and Frank and Goyal (2015)) show a negative relation. Our results for C_H, LV_H, and $D_H(x)$ can potentially explain the negative relation between leverage and profitability by shifting from the simultaneous bankruptcy equilibrium to the sequential bankruptcy equilibrium. Indeed, our model suggests that higher profitability leads a firm to take lower leverage to prepare for operating in a worse cash flow scenario in the future.

In the panel of $E_H(x)$, $E_H(x)$ monotonically decreases in b_H, and $E_H(x)$ jumps downward at $b_H = 0.391$ by shifting from the the simultaneous bankruptcy equilibrium to the sequential bankruptcy equilibrium. In the panel of $SW(x)$, we can also see a downward jump of $SW(x)$ at $b_H = 0.391$. This is because firm H's high coupon C_H^{**} and bankruptcy threshold x_L^N reduce the tax revenues of the government.

3.3.4 The impacts of cash flow volatility

Figure 4 shows the equilibrium results with varying levels with varying levels of the market volatility σ. The other parameter values are set in Table 1. The values other than $F_H(x)$ jump at $\sigma = 0.213$. Indeed, firm H chooses C_H^{**} (i.e., the simultaneous bankruptcy equilibrium) for $\sigma \leq 0.213$ and C_H^* (i.e., the sequential bankruptcy equilibrium) for $\sigma > 0.213$. A lower σ increases firm L's default distance, and hence, it increases the tax benefit effects until firm L's bankruptcy and decreases the future cash flow effects after firm L's bankruptcy. Then, a lower σ increases firm H's incentive to choose C_H^{**} and to enjoy the tax benefits until firm L's bankruptcy rather than choosing C_H^* to prepare operations after firm L's bankruptcy. Although we omit depicting a figure, a higher market growth rate μ, that increases firm L's default distance, also increases firm H's incentive to choose the simultaneous bankruptcy equilibrium. These results are counterintuitive and lead to new empirical predictions that all else being equal, bankruptcy cascades are more likely to hit a firm with a lower volatility and higher growth rate of cash flows.

The panel of LV_H shows leverage monotonically decreasing in σ, which is consistent with the standard result (e.g., Titman and Wessels (1988) and Leland (1994)). Leverage jumps downward at $\sigma = 0.213$ by shifting from the simultaneous bankruptcy equilibrium to the sequential bankruptcy equilibrium. More notably, the panel of CS_H presents the nonmonotonic shape because of the capital structure change at $\sigma = 0.213$. In fact, CS_H jumps downward at $\sigma = 0.213$ by shifting from the simultaneous bankruptcy equilibrium to the sequential bankruptcy equilibrium, although it monotonically increase in each equilibrium region. Most of the structural models (e.g., Leland (1994)) shows a positive relation between credit spread and volatility because the leverage decreasing with higher volatility does not fully offset the bankruptcy risk increasing with higher volatility. The same logic accounts for CS_H increasing with higher σ for each equilibrium region. However, in our model, at $\sigma = 0.213$, the firm changes the capital structure policies from the high-risk policy that leads to simultaneous bankruptcy to the low-risk policy that leads to sequential bankruptcy. Then, unlike in the previous results, CS_H can decrease in σ.

The panels of $E_H(x)$ and $D_H(x)$ show that a higher σ leads to a wealth transfer from debt holders to shareholders. This result is consistent with the standard result (e.g., Leland (1994)). Note that the shift from the simultaneous bankruptcy equilibrium to the sequential bankruptcy equilibrium at $\sigma = 0.213$ amplifies the wealth transfer. In the panel of $SW(x)$, we can also find an upward jump of $SW(x)$ at $\sigma = 0.213$. This is because firm H's high coupon C_H^{**} and bankruptcy threshold x_L^N reduce the tax revenues of the government.

To summarize, the numerical examples show that a small difference in parameter values can greatly change firm H's capital structure by switching between the simultaneous bankruptcy equilibrium and the sequential bankruptcy equilibrium. A switch between the two equilibria also causes several comparative static results different from the previous results of the standard structural model, leading to new empirical predictions. The novel result of the simultaneous bankruptcy equilibrium is more likely to occur for more negative bankruptcy spillovers, a smaller profitability difference, lower market volatility, and higher market growth rate.

4 Extensions

4.1 More than two firms

We can easily extend the baseline model consisting of two firms to the following setup with n firms. Firm $i \in \{1, 2, , \dots, n\}$ receives EBIT $a_i X(t) - b_i$, where $a_1/b_1 < a_2/b_2 < \cdots < a_n/b_n$, while no firm is bankrupt, whereas firm i's EBIT decreases to $\delta(k) a_i X(t) - \epsilon(k) b_i$, where the EBIT contraction parameter $\delta(j) \in (0, 1)$ ($\epsilon(j) \geq 1$) decreases (increases) with the number of bankrupt firms, k. For simplicity, we assume that the negative bankruptcy spillover effects are symmetric to all firms.

As in Section 3.2, we can derive the Nash equilibria for the game of n firms. Most notably, the simultaneous bankruptcy equilibrium can arise as follows. The least-profitability firm 1 chooses the nonstrategic coupon and bankruptcy threshold. Firm 2 increases debt beyond the nonstrategic level to bankrupt simultaneously with firm 1 because it anticipates the bankruptcy spillover effects $\delta(1)$ and $\epsilon(1)$. Firm 3 increases debt beyond the nonstrategic level to bankrupt simultaneously with firms 1 and 2 because it anticipates the effects $\delta(2)$ and $\epsilon(2)$ amplified by firm 2's bankruptcy. This procedure repeats until firm n chooses the high coupon to bankrupt simultaneously with firms $1, 2, \dots, n-1$, due to the amplified effects $\delta(n-1)$ and $\epsilon(n-1)$. A firm's strategy to increase debt and to go bankrupt simultaneously with lower-profitability firms amplifies negative externalities, leading to another higher-profitability firm's strategic debt increase and simultaneous bankruptcy. Through this negative feedback loop channel, bankruptcy cascades spread over multiple firms.

4.2 Bailout

In the presence of negative externalities, firm H can potentially bail out firm L. For instance, Mistrulli (2011) observes that a bank bails out another bank affiliated with the same conglomerate. Yang, Birge, and Parker (2015) document bailout examples through supply chain networks. Now, we examine a model in which firm H can merge firm L when firm L goes bankrupt at x_L^N.[9] Suppose that the merger requires the sunk cost $MC(> 0)$. After the merger, firm H receives EBIT $(a_H+a_L)X(t)-b_H-b_L$ and pays coupon $C_H+C_L^N$ until bankruptcy. As in (4), we can derive the merged firm H's bankruptcy threshold as

$$x_H^M(C_H, C_L^N) = \frac{\gamma(r-\mu)(b_H + b_L + C_H + C_L^N)}{(\gamma-1)r(a_H + a_L)}, \tag{13}$$

where the superscript M stands for the merged firm.

At time 0 prior to merger, firm H's equity and debt values can be derived as

$$\begin{aligned} E_H^M(x, C_H, C_L^N) &= E_H^N(x, x_L^N, C_H) + \left(\frac{x}{x_L^N}\right)^\gamma (E_H^N(x_L^N, x_H^M(C_H, C_L^N), C_H) \\ &\quad + E_L^N(x_L^N, x_H^M(C_H, C_L^N), C_L^N) - MC), \qquad (14) \\ D_H^M(x, C_H, C_L^N) &= D_H^N(x, x_H^M(C_H, C_L^N), C_H). \qquad (15) \end{aligned}$$

In (14), the first term represents the value before merger and the second term represents the value after merger. Debt value (15) implies that original debt holders of firm H continues to receive coupon C_H after merger until bankruptcy. Firm H chooses C_H to maximize the firm value (i.e., the sum of (14) and (15)) in the bailout case. Firm H decides whether to bail out firm L by comparing the firm values with and without bailout. Although it is costly for firm H to bail out firm L, firm H can bail out to avoid the negative bankruptcy spillover effects.

Clearly, more negative spillover effects and a lower merger cost increases firm H's incentive to bail out firm L. When firm H's EBIT scale is larger than that of firm L (i.e., $a_H/a_L >> 1$ and $b_H/b_L >> 1$), firm H is more likely to bail out firm L. This is because firm H can avoid relatively large spillover effects with relatively small distortion (i.e., $x_H^M(C_H, C_L^N)$ is close to $x_H^N(C_H)$). For instance, our model predicts that a large automaker, such as Toyota, is likely to bail out a smaller keiretsu supplier if its bankruptcy greatly damages the automaker's production line via the supply chain channel.

[9] For robustness check, we also examined an alternative model in which firm H can provide subsidies for firm L to delay firm L's bankruptcy. In fact, some firms provide more favorable pricing for financially distressed customers (e.g., see Yang, Birge, and Parker (2015)). The results are the same as those of the merger model presented in this subsection.

We now explore the impacts of cash flow volatility σ. Figure 5 shows firm H's bankruptcy threshold x_H^M, coupon C_H^M, leverage LV_H^M, credit spread CS_H^M, equity value $E_H^M(x)$, debt value $D_H^M(x)$, firm value $F_H^M(x)$, and social welfare $SW^M(x)$ with varying levels of σ. For comparison, we also depict the results in the baseline model with no bailout by dashed lines. For the baseline parameter values in Table 1, no bailout occurs even if we set $MC = 0$. To see the possibility of bailout changing with σ, we quadruple the firm H's EBIT scale (i.e., $a_H = 4$ and $b_H = 1.6$) in Table 1.

In Figure 5, firm H bails out firm L for $\sigma \leq 0.238$, whereas firm H does not bail out firm L (i.e., the solid and dashed lines align) and sequential bankruptcy occurs for $\sigma > 0.238$. For a lower σ, firm H is more likely to bail out firm L partially because bailout is a more effective measure to resolve the simultaneous bankruptcy equilibrium rather than the sequential bankruptcy equilibrium. In fact, the simultaneous bankruptcy equilibrium, which occurs in the case with no bailout for $\sigma \leq 0.213$, is fully replaced by bailout. We can see from Figure 5 that for $\sigma \leq 0.213$, bailout leads to lower x_H^M, C_H^M, LV_H^M, and CS_H^M than those for the case with no bailout. In other words, firm H reduces leverage to anticipate its operations after bailout of firm L. The panel of $SW^M(x)$ shows that for $\sigma \leq 0.213$, bailout, which delays bankruptcy, increases social welfare.

On the other hand, for $0.213 < \sigma \leq 0.238$, the sequential bankruptcy equilibrium in the baseline case is also replaced by bailout. In this region, bailout leads to higher x_H^M, C_H^M, LV_H^M, and CS_H^M than those for the case with no bailout. This is because firm H does not prepare for operations in the worse cash flow scenario after firm L's bankruptcy but rather anticipates bailout of firm L. The panel of $SW^M(x)$ shows that, for $0.213 < \sigma \leq 0.238$, bailout, which accelerates bankruptcy, decreases social welfare through the channel of tax revenues of the government. These results are counterintuitive and contrary to the results for $\sigma \leq 0.213$.

4.3 Debt renegotiation

Although the baseline model focuses on non-renegotiable debt, firm L's bankruptcy may give a debt restructuring opportunity to firm H. In reality, debt holders sometimes accept debt renegotiation apart from the absolute priority rule to avoid bankruptcy costs when the business environment deteriorates through an inevitable incident.[10] In this subsection, we consider a model in which firm H can renegotiate debt at firm L's bankruptcy time.

[10] This type of debt renegotiation is more likely to occur in a case with temporary losses rather than permanent losses from another firm's bankruptcy.

Assume that debt renegotiation requires sunk cost $DC(>0)$. At threshold x_L^N, the firm can renew the coupon to maximize the firm value. In debt renegotiation, shareholders gain the portion η of the surplus, while debt holders gain the portion $1-\eta$ of the surplus, where $\eta \in [0,1]$ stands for the bargaining power of shareholders.

The renewed coupon is $C_H^D = \arg\max_{C \geq 0} F_H^R(x_L^N, x_H^R(C), C)$, and the renewed bankruptcy threshold is $x_H^R(C_H^D)$. Recall that $F_H^R(\cdot,\cdot,\cdot)$ and $x_H^R(\cdot)$ denote the revised firm value and bankruptcy threshold, i.e., (3) and (4) with $\delta_i a_i$ and $\epsilon_i b_i$ rather than a_i and b_i. The superscript D stands for the debt renegotiation case. If the debt renegotiation surplus is positive, i.e., $F_H^R(x_L^N, x_H^R(C_H^D), C_H^D) - F_H(x_L^N, x_H^R(C_H), C_H) - DC > 0$, firm H prefers to restructure debt. In this case, firm H's equity and debt values at time 0 can be derived as

$$
\begin{aligned}
E_H^D(x, C_H, C_L^N) &= E_H^N(x, x_H^R(C_H), C_H) + \left(\frac{x}{x_L^N}\right)^{\gamma} \eta (F_H^R(x_L^N, x_H^R(C_H^D), C_H^D) \\
&\quad - F_H^N(x_L^N, x_H^R(C_H), C_H) - DC), \qquad (16) \\
D_H^D(x, C_H, C_L^N) &= D_H^N(x, x_H^R(C_H), C_H) + \left(\frac{x}{x_L^N}\right)^{\gamma} (1-\eta)(F_H^R(x_L^N, x_H^R(C_H^D), C_H^D) \\
&\quad - F_H^N(x_L^N, x_H^R(C_H), C_H) - DC). \qquad (17)
\end{aligned}
$$

In (16), the first and second terms represent the equity value with no debt renegotiation and the shareholders' gain from debt renegotiation, respectively. Similarly, in (17), the first and second terms represent the debt value with no debt renegotiation and the debt holders' gain from debt renegotiation, respectively. Firm H chooses C_H to maximize the firm value (i.e., the sum of (16) and (17)) in the debt renegotiation case.

Clearly, more negative spillover effects and a lower debt renegotiation cost, which increase the debt renegotiation surplus, increases firm H's incentive to choose debt renegotiation. We now explore the impacts of cash flow volatility σ. Figure 6 shows firm H's bankruptcy threshold x_H^D, coupon C_H^D, leverage LV_H^D, credit spread CS_H^D, equity value $E_H^D(x)$, debt value $D_H^D(x)$, firm value $F_H^D(x)$, and social welfare $SW^D(x)$ in the case allowing debt renegotiation with varying levels of σ. For comparison, we also depict the results in the baseline model with no debt renegotiation by dashed lines. To see the possibility of debt renegotiation changing with σ, we set $DC = 3$, and the other parameter values are set in Table 1.

In Figure 6, firm H renegotiate debt for $\sigma \leq 0.298$, whereas firm H does not renegotiate debt (i.e., the solid and dashed lines align) for $\sigma > 0.298$. In both cases, firm H operates after firm L's bankruptcy. For a lower σ, firm H is more likely to renegotiate debt partially because debt renegotiation is a more effective measure to resolve the simultaneous bankruptcy equilibrium rather than the sequential bankruptcy equilibrium. In fact, the simultaneous bankruptcy equilibrium, which occurs in the case with no debt renegotiation for $\sigma \leq 0.213$, is fully replaced by debt renegotiation. The panel of x_H^D shows that debt renegotiation greatly decreases x_H^D below x_H. In the panel of C_H^D, we find that for $\sigma \leq 0.298$, C_H^D is equal to C_H^{**}. This is because firm H, that anticipates future debt renegotiation at firm L's bankruptcy threshold x_L^N, chooses the initial coupon C_H^{**} to maximize the tax benefits until debt renegotiation.

The panels of LV_H^D and CS_H^D show that LV_H^D and CS_H^D are lower than LV_H and CS_H for $\sigma \leq 0.213$, while LV_H^D and CS_H^D are higher than LV_H and CS_H for $0.213 < \sigma \leq 0.298$. Contrary to the region $\sigma \leq 0.213$, in the region $0.213 < \sigma \leq 0.298$, firm H, which anticipates debt renegotiation in the future, issues more debt at time 0 than it does in the sequential bankruptcy equilibrium in the baseline model. The panel of $SW^D(x)$ shows that debt renegotiation increases social welfare for $\sigma \leq 0.213$. In this region, the shift from the simultaneous bankruptcy equilibrium to the debt renegotiation equilibrium delays firm H's bankruptcy and increases both values of firm H and the government. On the other hand, the panel of $SW^D(x)$ shows that debt renegotiation decreases social welfare for $0.213 < \sigma \leq 0.298$, yet it delays firm H's bankruptcy and increases firm H's value. This result is counterintuitive but is explained as follows. In this region, with debt renegotiation, firm H enjoy the tax benefits of debt by setting the high coupon $C_H^D = C_H^{**}$ at time 0. However the high coupon decreases tax revenues of the government until firm L's bankruptcy. In other words, debt renegotiation, which increases firm H's tax benefits of debt, causes a wealth transfer from the government to firm H, decreasing total social welfare via the decrease in the government value.

The results in the debt renegotiation case also yield implications about capital structure adjustment and debt maturity. For simplicity, this paper assumes that firms issue debt with infinite maturity at time 0. In the real world, firms can use short-term debt rather than long-term debt and adjust capital structure. The leverage adjustment with short-term debt plays the same role as debt renegotiation. As in the debt renegotiation case, firm H is more likely to issue short-term debt and adjust leverage for more negative spillover effects, lower debt issuance and adjustment costs, and a lower cash flow volatility. The dynamic leverage adjustment can prevent firm H from choosing the simultaneous bankruptcy equilibrium with high leverage and increase social welfare, but it can prevent firm H from choosing the sequential bankruptcy equilibrium with low leverage and decrease social welfare.

4.4 Incomplete information

So far, we have assumed that firms H and L have complete information about all parameter values. In reality, it may be difficult to know the bankruptcy spillover effects prior to bankruptcy. Then, we examined a model in which the firms know the spillover parameters $\delta_i \in (0, 1)$ and $\epsilon_i \geq 1$ as some distributions. Unlike in the baseline model, firm H chooses C to maximize the expectation of $F_H(x, C, C_L^N)$ with respect to δ_H and ϵ_H in (11), yet C_H^{**} in the simultaneous bankruptcy case remains unchanged. We computed the results for uniformly distributed δ_H and ϵ_H. We verified that the effects of incomplete information are very small and that the main results in the baseline models robustly hold true. Hence, we omitted depicting a figure.

4.5 Bidimensional state process

Although the baseline model assumes that firms H and L face the common shock $X(t)$, it is more realistic to assume the bidimensional geometric Brownian motion with a positive correlation for two firms' cash flows. For the bidimensional state process, we cannot obtain analytical forms for the equity, debt, and firm values. Hence, it is harder to compute the equilibrium results. In particular, unlike in the baseline model, the order of bankruptcy for a given strategy profile (C_H, C_L) is not known at time 0 but depends on the sample path of the state process $X(t)$. Then, not only firm H but also firm L can anticipate the negative bankruptcy spillover effects. This leads to the possibility that, through strategic considerations, both firms can either decrease debt for preparation of the worse cash flow scenario or increase debt to bankrupt simultaneously. We believe that, with a sufficiently high correlation, the results remain quite similar to the baseline results; two types of equilibria (i.e., high- and low-leverage equilibria) arise depending on the parameter values.

In the bidimensional setup, one may consider a model in which a firm can optimize the correlation between the two firms' EBIT in addition to capital structure. In fact, a firm could change the correlation by investing in different projects. The choice of correlation is examined in the banking model in Acharya and Yorulmazer (2007). They show that banks can increase the interbank correlation in cash flows by choosing similar sets of loans. In their model, banks herd in order to fail and to be bailed out together in the presence of a too-many-to-fail problem. The same herding behavior can arise in our setup, even though we do not consider a bailout opportunity. Indeed, a firm has an incentive to increase the correlation coefficient to one in order to go bankrupt at exactly the same time with the other firm and to avoid the negative bankruptcy spillover effects.

5 Conclusion

This paper develops the optimal capital structure model of two firms with negative externalities of bankruptcy. We examine the firms' capital structure and bankruptcy timing choices in equilibrium. The equilibrium results are summarized below.

The low-profitability firm L's capital structure and bankruptcy timing remain unchanged from those of the nonstrategic case. On the other hand, the high-profitability firm H takes either of the two contrasting capital structures: lower leverage than the nonstrategic leverage to prepare for operations after firm L's bankruptcy (i.e., the sequential bankruptcy equilibrium) or higher leverage than the nonstrategic leverage to bankrupt simultaneously with firm L (i.e., the simultaneous bankruptcy equilibrium). The capital structure choice is determined by the tradeoff between the cash flows from operations after firm L's bankruptcy and the tax benefits of increased debt. With more negative bankruptcy spillovers, a smaller profitability difference, and lower volatility, firm H is more likely to choose the simultaneous bankruptcy equilibrium. The model can yield nonstandard comparative static results by switching between the sequential and simultaneous equilibria.

Most notably, the simultaneous bankruptcy equilibrium shows a novel mechanism of bankruptcy cascades—firms with negative bankruptcy spillovers, such as firms with cross-holdings and firms in the same supplier chain, can intentionally increase leverage ex ante to go bankrupt together. Although the empirical relevance of this mechanism has yet to be studied, this mechanism can potentially account for the empirical findings of default clustering, financial contagion, and herding behavior for corporate financial policies.

A Proof of Proposition 1

Recall that we define $x_i^R(C)$ by replacing a_i and b_i with $\delta_i a_i$ and $\epsilon_i b_i$, respectively, in (4). Then, $x_i^N(C) < x_i^R(C)$ follows from $\delta_i < 1$ and $\epsilon_i \geq 1$.

Sequential bankruptcy case I: $C_H \in [0, K_1(C_L))$.

By (4) and (8), we have

$$
\begin{aligned}
x_H^R(C_H) &= \frac{\gamma(r-\mu)(\epsilon_H b_H + C_H)}{(\gamma-1)r\delta_H a_H} \\
&< \frac{\gamma(r-\mu)(\epsilon_H b_H + K_1(C_L))}{(\gamma-1)r\delta_H a_H} \\
&= x_L^N(C_L).
\end{aligned} \tag{18}
$$

By (18), we have the bankruptcy thresholds $x_H(C_H, C_L) = x_H^R(C_H)$ and $x_L(C_L, C_H) = x_L^N(C_L)$, as well as the inequality $x_H(C_H, C_L) < x_L(C_L, C_H)$. Then, the equity and debt values of firm L are the same as those in the nonstrategic case. We can calculate the equity and debt values of firm H as follows:

$$
\begin{aligned}
E_H(x, C_H, C_L) &= \mathbb{E}[\int_0^{T_1} \mathrm{e}^{-rt}(1-\tau)(a_H X(t) - b_H - C_H)\mathrm{d}t + \int_{T_1}^{T_2} \mathrm{e}^{-rt}(1-\tau)(\delta_H a_H X(t) - b_H - C_H)\mathrm{d}t] \\
&= E_H^N(x, x_L^N(C_L), C_H) + \left(\frac{x}{x_L^N(C_L)}\right)^\gamma E_H^R(x_L^N(C_L), x_H^R(C_H), C_H), \\
D_H(x, C_H, C_L) &= \mathbb{E}[\int_0^{T_2} \mathrm{e}^{-rt} C_H \mathrm{d}t + \mathrm{e}^{-rT_2} U_H(X(T_2))] \\
&= D_H^N(x, x_H^R(C_H), C_H),
\end{aligned}
$$

where we define firm L's bankruptcy time $T_1 = \inf\{t \geq 0 \mid X(t) \leq x_L^N(C_L)\}$ and firm H's bankruptcy time $T_2 = \inf\{t \geq 0 \mid X(t) \leq x_H^R(C_H)\}$.

Simultaneous bankruptcy case: $C_H \in [K_1(C_L), K_3(C_L)]$.

In the same manner as derivation of (18), we can show that $x_H^R(C_H) \geq x_L^N(C_L)$. By (4) and (8), we can show that

$$
\begin{aligned}
x_H(C_H) &= \frac{\gamma(r-\mu)(b_H + C_H)}{(\gamma-1)r a_H} \\
&\leq \frac{\gamma(r-\mu)(b_H + K_3(C_L))}{(\gamma-1)r a_H} \\
&= x_L^R(C_L).
\end{aligned} \tag{19}
$$

The two inequalities imply that firms H and L have the same bankruptcy threshold $x_H^N(C_H) \vee x_L^N(C_L)$. Then, we have the equity value $E_i(x, C_i, C_j) = E_i^N(x, x_H^N(C_H) \vee x_L^N(C_L), C_i)$ and debt value $D_i(x, C_i, C_j) = D_i^N(x, x_H^N(C_H) \vee x_L^N(C_L), C_i)$ for $i = \{H, L\}$.

Sequential bankruptcy case II: $C_H > K_3(C_L)$.

As in derivation of (19), we can show that $x_H^N(C_H) > x_L^R(C_L)$. This leads to $x_H(C_H, C_L) = x_H^N(C_H)$ and $x_L(C_L, C_H) = x_L^R(C_L)$. By exchanging the roles of firms H and L in the sequential bankruptcy case I, we can derive the expressions of the equity and debt values in this case.

B Proof of Proposition 2

First, we prove that the strategy profile

$$(C_H^*, C_L^N) \qquad (F_H(x, C_H^*, C_L^N) > F_H^N(x, C_H^{**})) \tag{20}$$

$$(C_H^*, C_L^N) \text{ or } (C_H^{**}, C_L^N) \qquad (F_H(x, C_H^*, C_L^N) = F_H^N(x, C_H^{**})) \tag{21}$$

$$(C_H^{**}, C_L^N) \qquad (F_H(x, C_H^*, C_L^N) < F_H^N(x, C_H^{**})) \tag{22}$$

is a Nash equilibrium of the game, and we will later show that it is the payoff dominant equilibrium. Consider firm H's response C_H to firm L's coupon C_L^N. By Proposition 1, we have firm H's value

$$F_H(x, C_H, C_L^N) = \begin{cases} E_H^N(x, x_L^N(C_L), C_H) + D_H^N(x, x_H^R(C_H), C_H) & \\ + \left(\frac{x}{x_L^N(C_L)}\right)^\gamma E_H^R(x_L^N(C_L), x_H^R(C_H), C_H), & (C_H \in [0, K_1(C_L^N))), \\ F_H^N(x, x_L^N, C_H) & (C_H \in [K_1(C_L^N), C_H^{**}]), \\ F_H^N(x, C_H) & (C_H > C_H^{**}), \end{cases} \tag{23}$$

Note that this function is continuous with respect to C_H. By (3), $F_H^N(x, x_L^N, C_H)$ monotonically increases in $C_H \in [K_1(C_L^N), C_H^{**}]$. By the unimodality (6), $F_H^N(x, C_H)$ monotonically decreases in $C_H > C_H^N$. By (4) and assumption (7), we have

$$\begin{aligned} C_H^N &= \frac{(\gamma-1) r a_H x_H^N}{\gamma(r-\mu)} - b_H \\ &< \frac{(\gamma-1) r a_H x_L^N}{\gamma(r-\mu)} - b_H \\ &= K_2(C_L^N) = C_H^{**}. \end{aligned}$$

Hence, we have

$$C_H^{**} = \arg \max_{C_H \geq K_1(C_L^N)} F_H(x, C_H, C_L^N) \tag{24}$$

(cf. Figure 1). By (11) and (24), firm H's best response to firm L's coupon C_L^N is C_H^* for $F_H(x, C_H^*, C_L^N) > F_H^N(x, C_H^{**})$, C_H^* or C_H^{**} for $F_H(x, C_H^*, C_L^N) = F_H^N(x, C_H^{**})$, and C_H^{**} for $F_H(x, C_H^*, C_L^N) < F_H^N(x, C_H^{**})$.

Next, we consider firm L's best response to firm H's coupon C_H^* or C_H^{**}. We have $x_L^N = x_H^N(C_H^{**}) > x_H^N(C_H^*)$. Hence, firm L gains $F_L^N(x, C_L^N)$ by choosing coupon C_L^N. By $\delta_L < 1$ and $\epsilon_L \geq 1$, we have $F_L^N(x, C_L) \geq F_L(x, C_L, C_H)$ for any C_L and C_H. Then, by the unimodality (6), we have

$$F_L^N(x, C_L^N) > F_L^N(x, C_L) \geq \max\{F_L(x, C_L, C_H^*), F_L(x, C_L, C_H^{**})\} \tag{25}$$

for any $C_L \neq C_L^N$. Inequality (25) implies that firm L's best response to firm H's coupon C_H^* or C_H^{**} is C_L^N. We have completed the proof that the strategy profile (20)–(22) is a Nash equilibrium of the game.

Now, we examine other equiilibria. Suppose that another Nash equilibrium (C_H, C_L) exists. If $C_L = C_L^N$ holds, the above argument shows that (C_H, C_L) agrees with the strategy profile (20)–(22). Suppose that $C_L < C_L^N$. By (4), we immediately have

$$x_L^N(C_L) < x_L^N. \tag{26}$$

By the unimodality (6), we have $F_L^N(x, C_L^N) > F_L^N(x, C_L)$. By the optimality of C_L, we have $F_L(x, C_L, C_H) \geq F_L(x, C_L^N, C_H)$. Then, we have

$$F_L^N(x, C_L^N) > F_L^N(x, C_L) \geq F_L(x, C_L, C_H) \geq F_L(x, C_L^N, C_H),$$

which leads to

$$x_H^N(C_H) > x_L^N. \tag{27}$$

Note that if (27) does not hold, firm L deviates from C_L to C_L^N and increases the firm value from $F_L(x, C_L^N, C_H)$ to $F_L^N(x, C_L^N)$. By (26) and (27), we have $x_H^N(C_H) > x_L^N(C_L)$, which means that firm H's value becomes $F_H^N(x, C_H)$. By (6), (7), and (27), we have $C_H > C_H^N$ and $\partial F_H^N(x, C_H)/\partial C < 0$. Then, we can take $C'_H(< C_H)$ satisfying $x_H^N(C'_H) > x_L^N(C_L)$ and $F_H^N(x, C'_H) > F_H^N(x, C_H)$. Thus, C_H is not firm H's best response, which leads to contradiction.

Next, suppose that $C_L \in (C_L^N, \bar{C}_L)$. By assumption (6), we have $\partial F_L^N(x, C_L)/\partial C < 0$. If $x_H^N(C_H) < x_L^N(C_L)$ holds, firm L can increase the firm value by decreasing the coupon below C_L, which contradicts with the optimality of C_L. Hence, we have $x_H^N(C_H) \geq x_L^N(C_L)$, which implies that firm H's value becomes $F_H^N(x, C_H)$. We have $C_H > C_H^N$ by $C_L > C_L^N, x_H^N(C_H) \geq x_L^N(C_L)$, and assumption (7). Then, by the unimodality (6), we have $\partial F_H^N(x, C_H)/\partial C < 0$. If $x_H^N(C_H) > x_L^N(C_L)$ holds, firm H can increase the firm value by decreasing the coupon below C_H, which contradicts with the optimality of C_H. Hence, we have $x_H^N(C_H) = x_L^N(C_L)$, i.e., $C_H = K_2(C_L)$. In fact, this strategy profile (C_H, C_L) becomes a Nash equilibrium, if any firm cannot increase the firm value by decreasing the coupon and choosing the sequential bankruptcy.

Suppose that $C_L \geq \bar{C}_L$, i.e., $x \leq x_L^N(C_L)$. If $x_H^N(C_H) < x$ holds, firm L can increase the firm value by decreasing the coupon below $\bar{C}_L$, which contradicts with the optimality of C_L. Hence, we have $x_H^N(C_H) \geq x$, i.e., $C_H \geq \bar{C}_H$. This strategy profile (C_H, C_L) becomes a Nash equilibrium, if any firm cannot increase the firm value by decreasing the coupon and choosing sequential bankruptcy.

Lastly, we show that the strategy profile (20)–(22) is the payoff dominant equilibrium. Consider another Nash equilibrium (C_H, C_L). As we proved above, $C_L > C_L^N$ and $C_H > C_H^{**} > C_H^N$ hold, and firm i's value is $F_i^N(x, C_i)$ for $i \in \{H, L\}$. By the unimodality (6), we have $F_L^N(x, C_L^N) > F_L^N(x, C_L)$ and $F_H^N(x, C_H^{**}) > F_H^N(x, C_H)$. The proof is complete.

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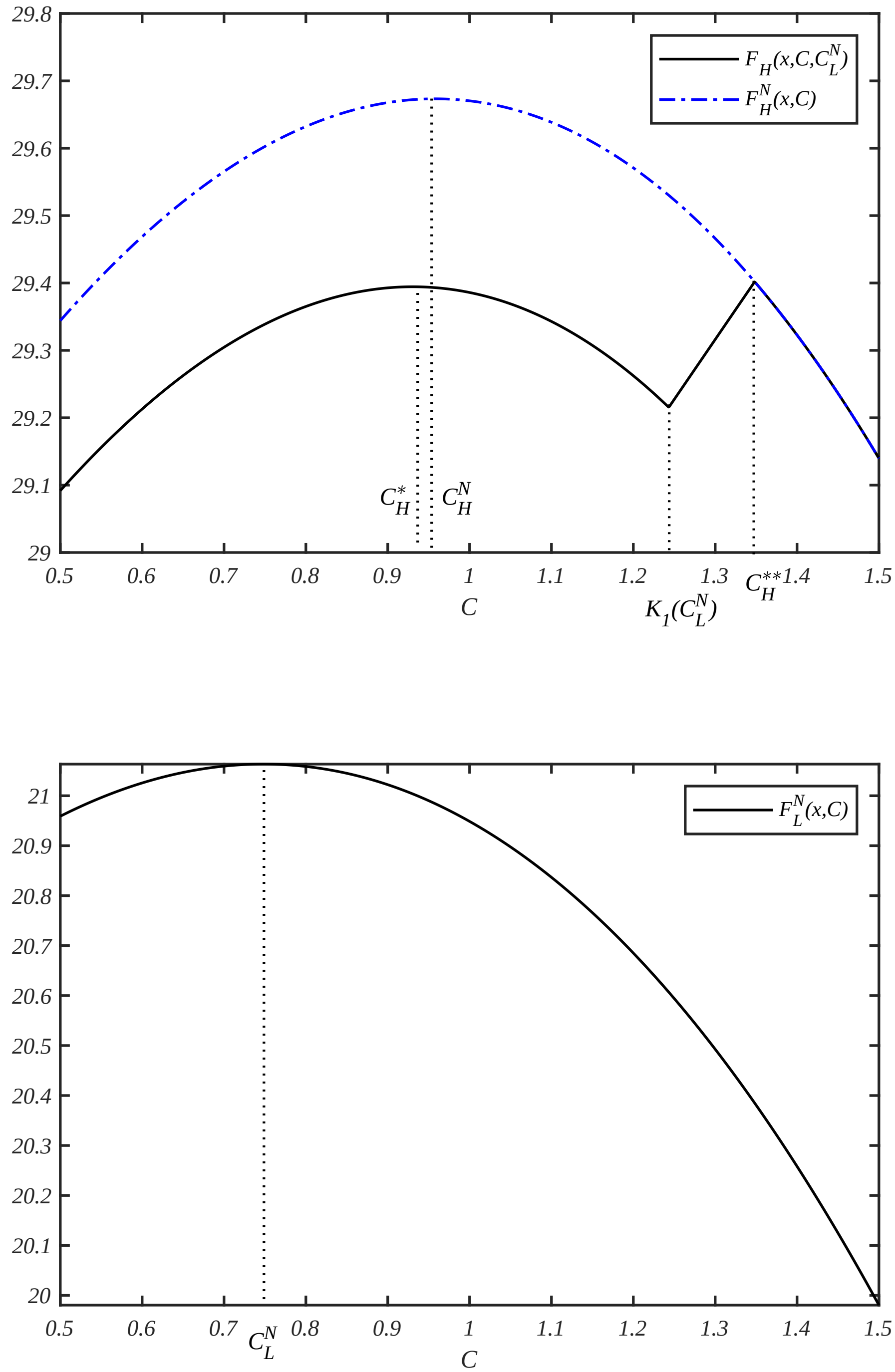

Figure 1: Firms H and L's values. This figure shows $F_H(x, C, C_L^N)$, $F_H^N(x, C)$, and $F_L^N(x, C)$ as functions of coupon C. The parameter values are set in Table 1.

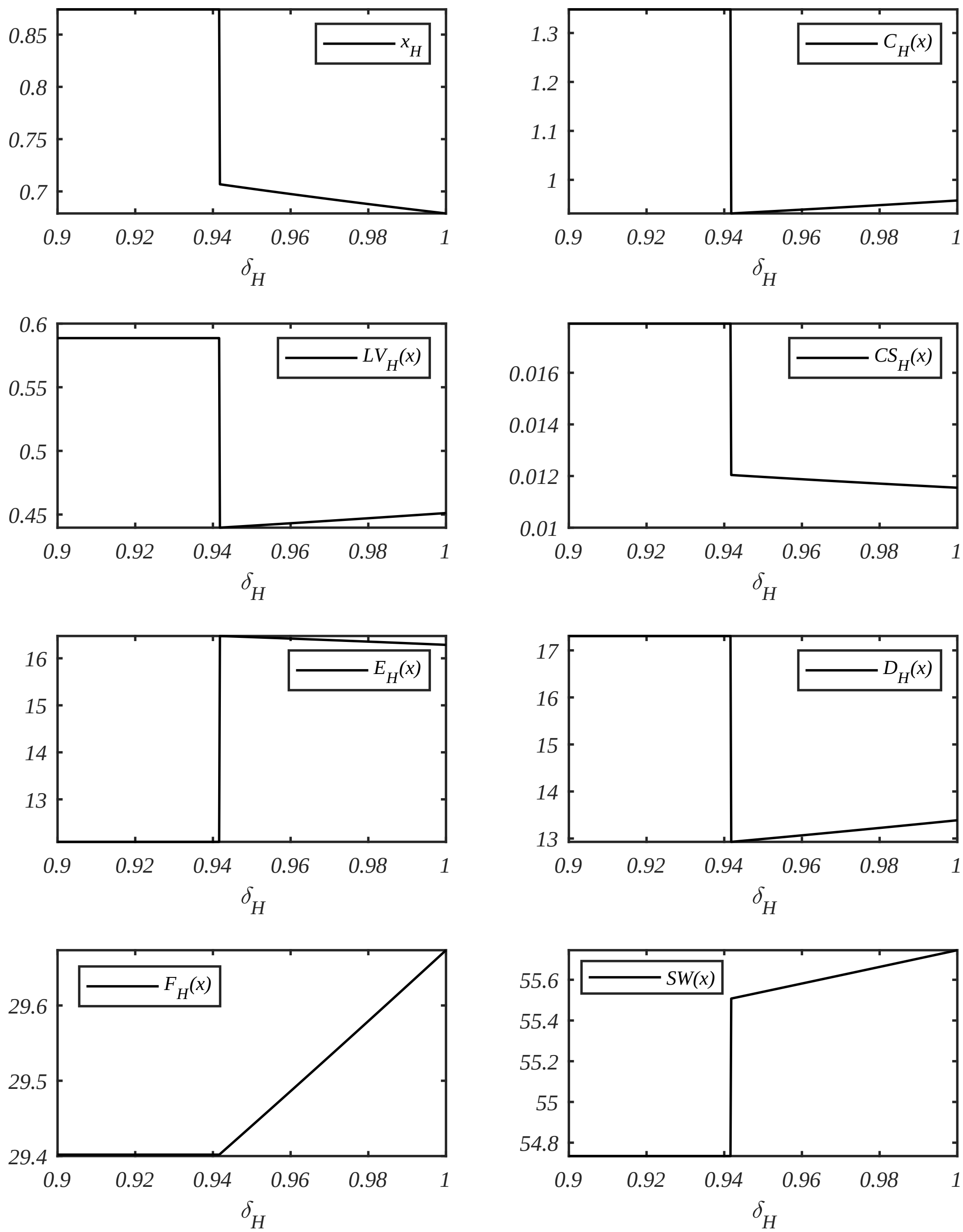

Figure 2: Comparative statics with respect to the spillover parameter δ_H. This figure shows firm H's bankruptcy threshold x_H, coupon C_H, leverage LV_H, credit spread CS_H, equity value $E_H(x)$, debt value $D_H(x)$, firm value $F_H(x)$, and social welfare $SW(x)$. The other parameter values are set in Table 1. Simultaneous bankruptcy occurs for $\delta_H \leq 0.9417$, while sequential bankruptcy occurs for $\delta_H > 0.9417$.

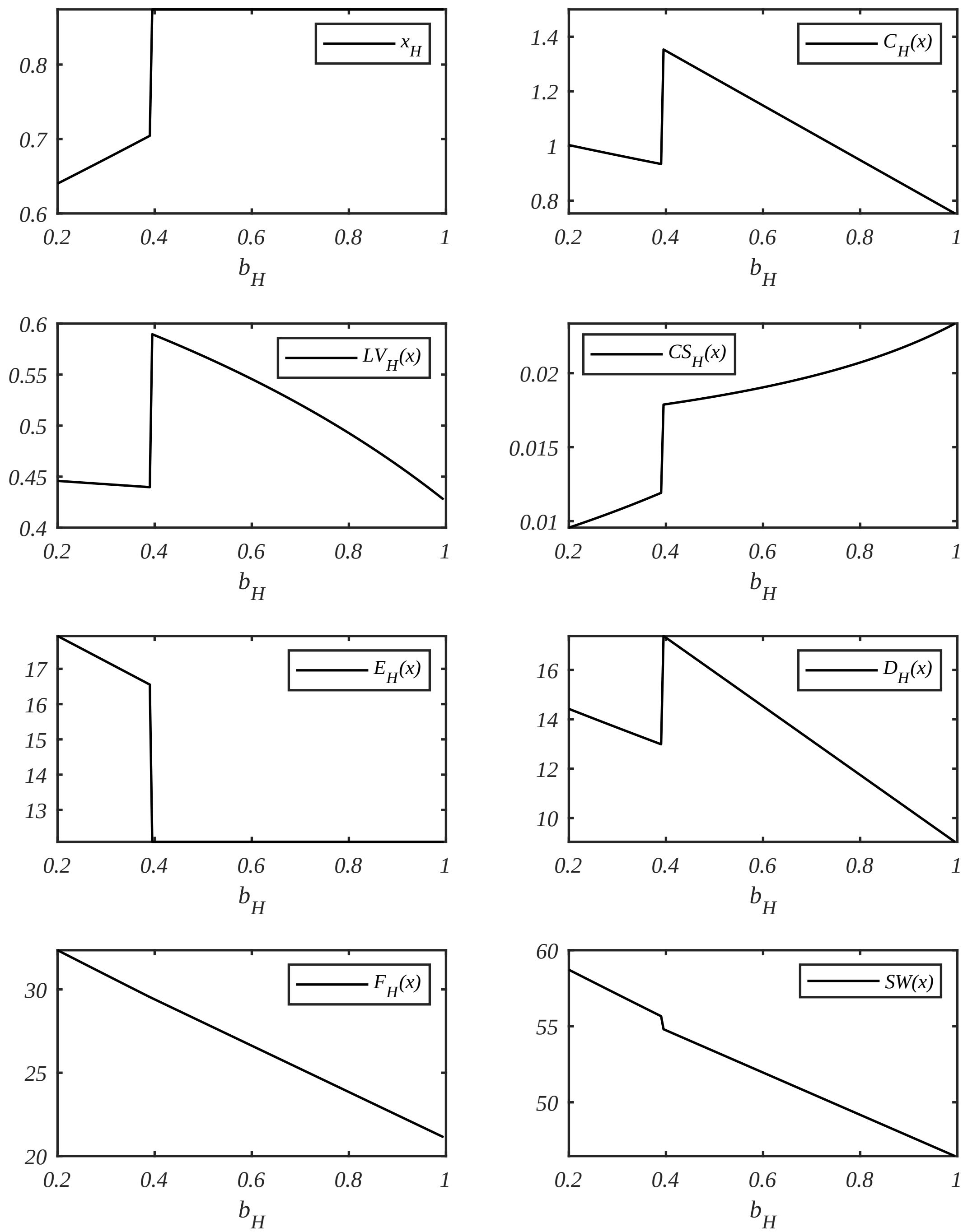

Figure 3: Comparative statics with respect to firm H's operating cost b_H. This figure shows firm H's bankruptcy threshold x_H, coupon C_H, leverage LV_H, credit spread CS_H, equity value $E_H(x)$, debt value $D_H(x)$, firm value $F_H(x)$, and social welfare $SW(x)$. The other parameter values are set in Table 1. Sequential bankruptcy occurs for $b_H < 0.391$, while simultaneous bankruptcy occurs for $b_H \geq 0.391$.

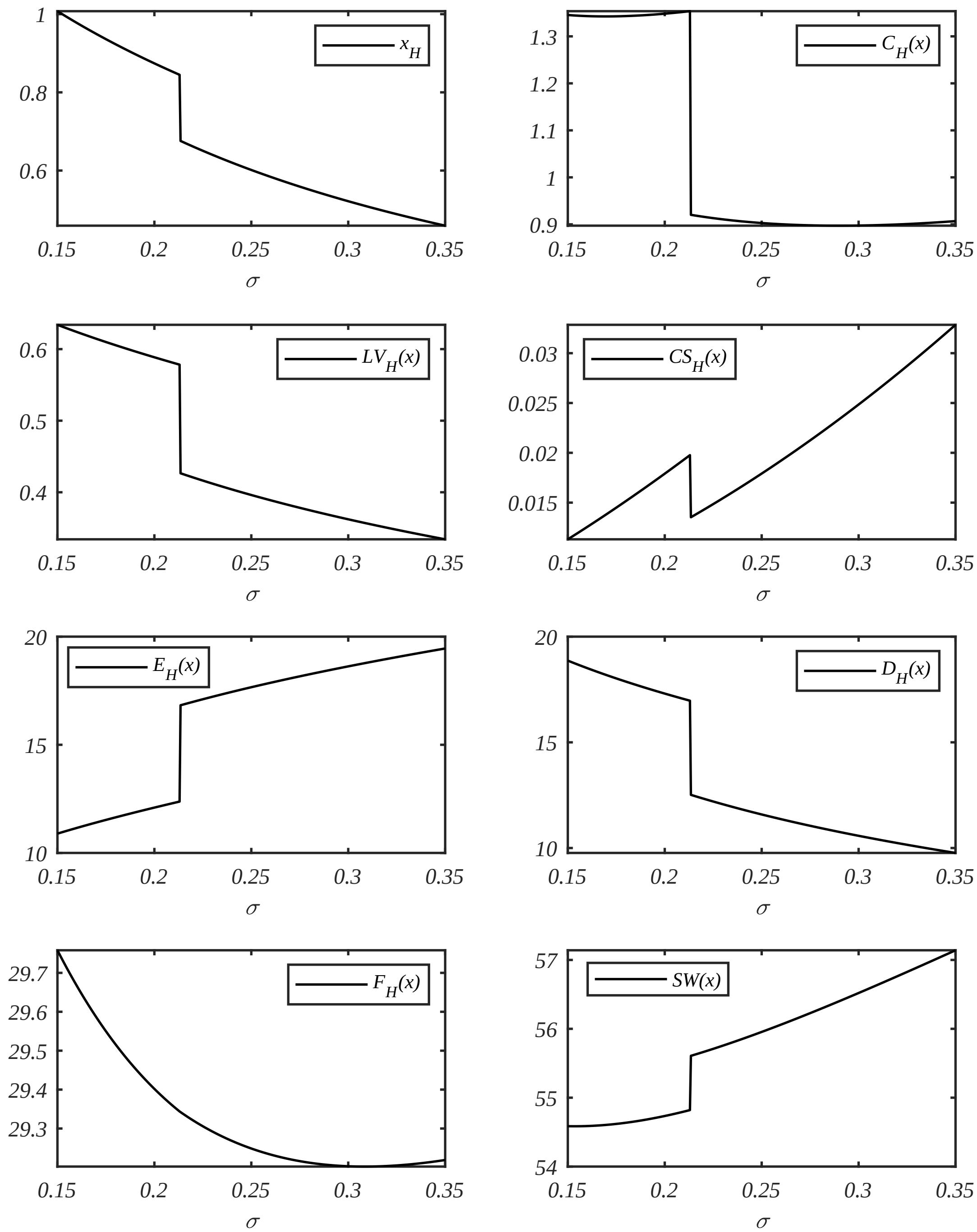

Figure 4: Comparative statics with respect to market volatility σ. This figure shows firm H's bankruptcy threshold x_H, coupon C_H, leverage LV_H, credit spread CS_H, equity value $E_H(x)$, debt value $D_H(x)$, firm value $F_H(x)$, and social welfare $SW(x)$. The other parameter values are set in Table 1. Simultaneous bankruptcy occurs for $\sigma \leq 0.213$, while sequential bankruptcy occurs for $\sigma > 0.213$.

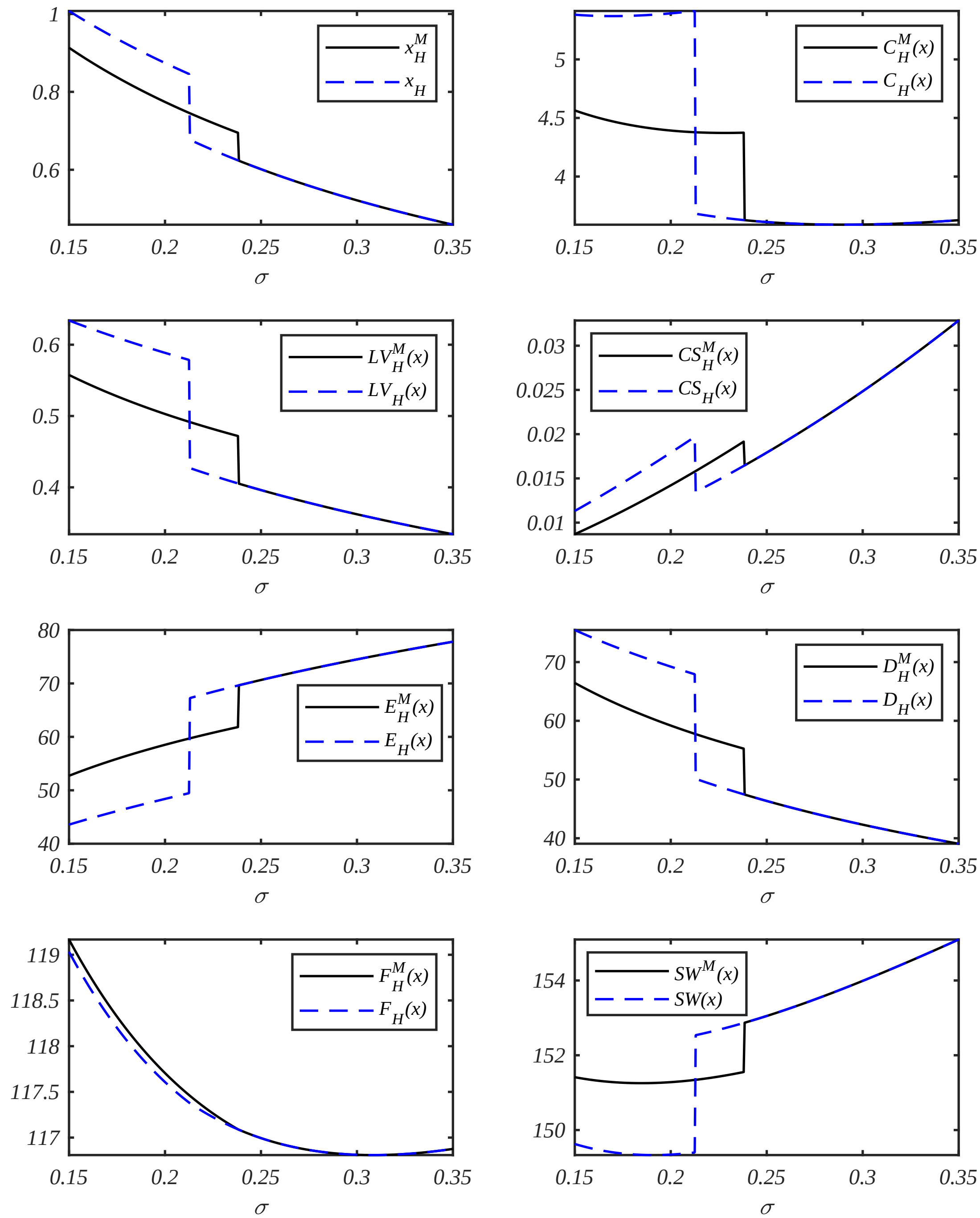

Figure 5: Bailout. Comparative statics with respect to cash flow volatility σ. This figure shows firm H's bankruptcy threshold x_H^M, coupon C_H^M, leverage LV_H^M, credit spread CS_H^M, equity value $E_H^M(x)$, debt value $D_H^M(x)$, firm value $F_H^M(x)$, and social welfare $SW^M(x)$ in the case with bailout. The parameter values other than $a_H/b_H = 4/1.6$ and $MC = 1$ are set in Table 1. Firm H bails out firm L for $\sigma \leq 0.238$, while firm H does not bail out firm L for $\sigma > 0.238$.

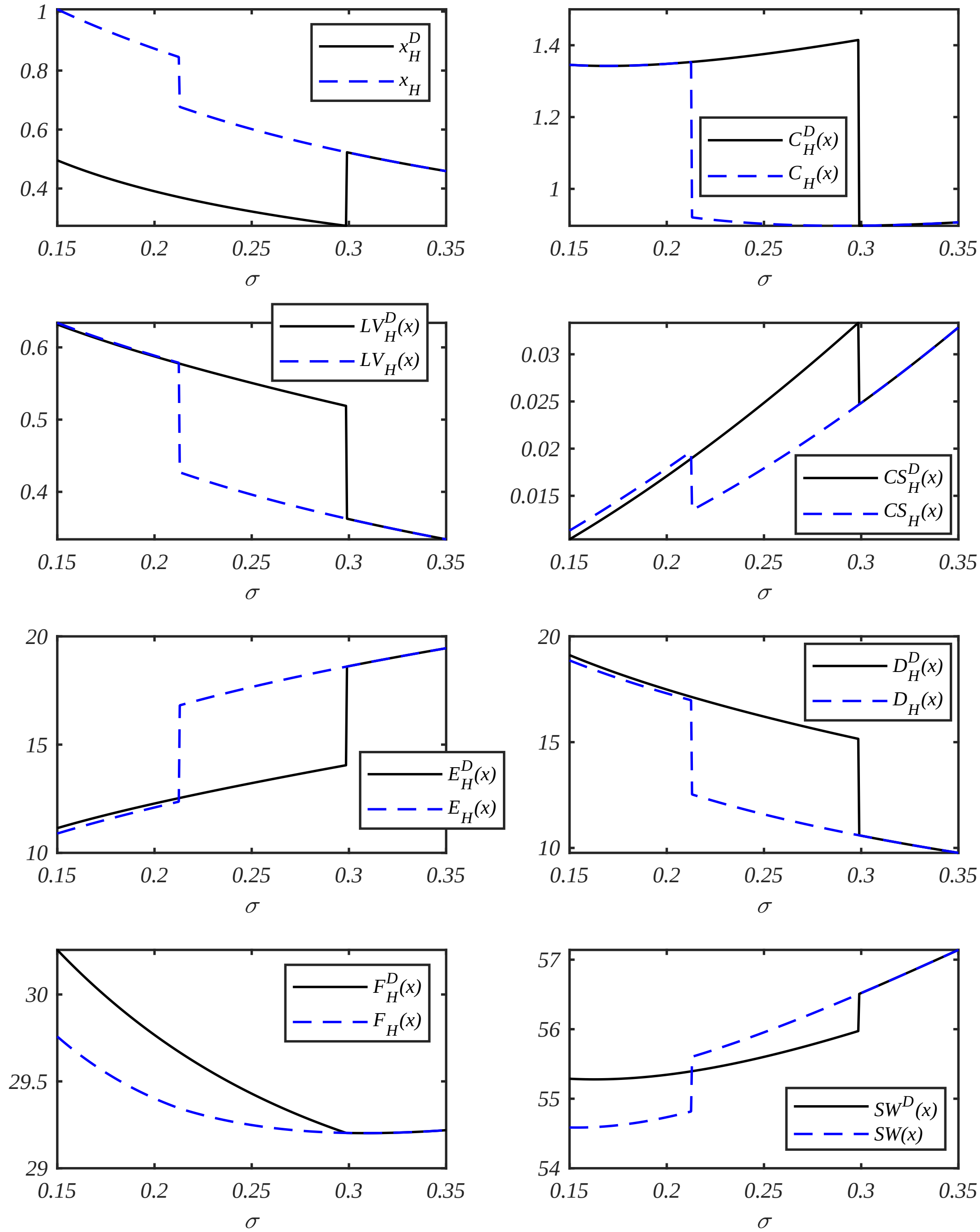

Figure 6: Debt renegotiation. Comparative statics with respect to cash flow volatility σ. This figure shows firm H's bankruptcy threshold x_H^D, coupon C_H^D, leverage LV_H^D, credit spread CS_H^D, equity value $E_H^D(x)$, debt value $D_H^D(x)$, firm value $F_H^D(x)$, and social welfare $SW^D(x)$ in the case with debt renegotiation. The parameter values other than $DC = 3$ are set in Table 1. Firm H renegotiates debt for $\sigma \leq 0.229$, while firm H does not renegotiate debt for $\sigma > 0.229$.