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Abstract

In this study, we analyze how an improvement in child mortality affects fertility, child labor, and investments education investments. We consider an overlapping generations model, in which skilled and unskilled workers coexist. Improvement in child mortality has different effects on skilled and unskilled workers. We study three alternative policies of increasing the proportion of skilled workers in the economy: improvement in child mortality, a ban on child labor, and child education. The ban on child labor means that the government enforces a law that prohibits a household from supplying child labor. The model reveals that improvements in child mortality and a ban on child labor can decrease the proportion of skilled workers and the average income in the economy. However, the child education policy, which supports both skilled and unskilled workers' investments in the education of their children by building schools, increases the proportion of skilled workers and the average income in the economy.

Keywords: Child mortality, Child labor, Fertility, Education, Health, Overlapping generations model

JEL Classification: D1, I1, I2, J1, O1

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1. Introduction

Many countries have experienced significant improvements in child mortality over the past several decades (UNIGME, 2019). However, extant empirical studies indicate that the least developed countries, such as the sub-Saharan African countries, still have high child mortality. UNIGME (2019) states it is impossible for these developing countries to reduce under-five mortality by 2030 as per the SDG targets. Moreover, many theoretical and empirical studies have elucidated that these countries show features such as high fertility, low investments in education, high mortality, abundant child labor, and low income.

In this study, we clarify how an improvement in child mortality affects the behaviors of heterogeneous individuals and demographics by focusing on fertility, child labor, and investments in education. While existing studies (e.g., Dessy 2000; Hazan and Berdugo 2002; Kalemli-Ozcan 2002, 2003, 2008; Strulik 2004; Azarnert 2006; Fioroni 2010) also analyze this issue in developing countries, they do not consider the heterogeneity of individuals. Since this study considers the heterogeneity among individuals, that is, the presence of both skilled and unskilled individuals, an improvement in child mortality has different effects on these different individuals. This study thus reveals that an improvement in child mortality causes skilled workers to decrease fertility and the child labor supply and increase investments in education. Conversely, a reduction in child mortality makes unskilled individuals increase both fertility and the child labor supply. These opposite behaviors affect demographics and can decrease the proportion of skilled workers in the economy. Hence, if there is a significant improvement in child mortality that increases the proportion of unskilled individuals, our model can explain why poor countries still have high fertility and abundant child labor, especially in sub-Saharan African countries, although these countries have experienced progress in terms of child mortality.

Specifically, we consider the heterogeneity of individuals according to Fan and Zhang (2013). Skilled workers can freely choose investments in education for their children, while unskilled workers are less productive in educating their children, that is, they face the upper bound for the investments in education to their children. We also incorporate child mortality into the proposed model, similarly to Chakraborty (2004). Here, the mortality in adulthood and old age improves through public health expenditure by the government. Several studies (e.g., Kalemli-Ozcan 2002, 2003, 2008; Strulik 2004; Sato and Yamamoto 2005; Azarnert 2006; Fironi 2010; Strulik and Weisdorf 2014) incorporate child mortality into their models. For instance, Kalemli-Ozacan (2002, 2003, 2008) introduces child mortality, which is constant over time, and analyzes how its uncertainty affects economic growth. Sato and Yamamoto (2005) assume that the survival of children is deterministic and analyze how an improvement in child mortality affects the demographic transition. Strulik (2004) assumes that child mortality depends on the per capita income. Azarnert (2006) assumes that child mortality depends on parents' investments in education and Fioroni (2010) assumes it positively depends on parents' human capital. These two latter studies show that, if parents have a sufficiently high human capital, they have better knowledge about health and are willing to improve their children's health. Strulik and Weisdorf (2014) assume that child mortality depends on the geographical location and food expenditure per child. In contrast with these studies, the method of introducing child mortality in the present study simplifies the analysis as follows. The government levies a lump-sum tax on each household. The collected taxes reduce child mortality through public health investment by the government, which includes new medical facilities, sanitation improvement, disease control, and inoculation programs.

Further, we examine three alternative policies: a policy that improves child mortality, a ban on child labor, and an education policy. Strulik (2004) states that the ban on child labor is not effective in escaping from economic stagnation, compared to compulsory schooling and improvement in health policies. In his model, compulsory schooling and a health policy can make parents invest in education for their children, which leads to sustained economic growth. However, even if the ban on child labor is executed by the government, parents can choose not to invest in the education of their children because such investments in education are still costly. Conversely, Dessy (2000) and Hazan and Berdugo (2002) assert that the prohibition of child labor is effective and is the only way to escape from the poverty trap. However, these two studies do not consider the existence of child mortality. The setup of our model is closer to that of Strulik (2004) rather than those of Dessy (2000) and Hazan and Berdugo (2002); however, it is different from Strulik's (2004) in that we consider the heterogeneity of individuals by paying attention to the differential wages and abilities that constrain parents' investments in the education of their children. Since both skilled and unskilled parents can invest in education in the proposed model, the effect of the health policy differs from the results of Strulik (2004). The health policy in the proposed model spurs economic stagnation by lowering the per capita output. The model also shows that the ban on child labor policy is not sufficient for economic growth. However, the education policy, which supports both skilled and unskilled workers' investments in the education of their children by building schools, increases the proportion of skilled workers and the average income in the economy. The government executes this education policy, as well as its public health expenditure, by using its tax revenue.

The findings are as follows. First, an improvement in child mortality makes skilled workers have higher quality input regarding their children, thereby reducing child labor and the number of children. Conversely, an improvement in child mortality makes unskilled workers have more children, thus increasing the child labor supply. Second, skilled parents have a lower fertility rate and child labor supply than unskilled ones. We find that if the upper bound of the investments in education for unskilled parents decreases, the differences between fertility rates and child labor supplies become large between the two groups. Third, an improvement in child mortality can decrease the proportion of skilled individuals and the average income in the economy. Fourth, the ban on child labor also reduces the proportion of skilled workers and the average income in the economy. Finally, the child education policy is the most promising among the three, since it increases the ratio of skilled workers, as well as the average income in the economy. As a result, the findings are partly related to those of

Strulik (2004), while the ban on child labor policy goes against the findings of Dessy (2004) and Hazan and Berdugo (2002).

The remainder of this paper is organized as follows. Section 2 develops the basic model and considers how an improvement in child mortality affects individuals' fertility, child labor supply, and investments in education. Section 3 focuses on population dynamics and analyzes its steady state. Section 4 examines three alternative policies: a ban on child labor, an improvement in child mortality, and child education. We demonstrate that only the child education policy increases the proportion of skilled workers and the average income in the economy. Section 5 provides several concluding remarks.

2. The model

2.1. Environments

We consider a three-period overlapping generations model: childhood, adulthood, and old age. Time is discrete and infinite. As we consider a small open economy, the interest rate is equal to the world interest rate. Childhood consists of two sub-periods: early childhood and school age. Children may die between early childhood and school age[1]. During childhood, individuals do not make any decisions. During adulthood, individuals give birth to their children, raise and educate them, and determine the supply of child labor. During old age, individuals retire and consume only. An individual dies at the beginning of school age with probability $1 - \pi^c \in [0, 1)$ and lives through school age with probability $\pi^c \in (0, 1]$.

The economy consists of two sectors: a traditional sector and a modern one. Production takes place in either or in both sectors. Both sectors produce a single good and the same output, but employ different factors. The traditional sector employs unskilled workers and child labor from both unskilled and skilled households, while the modern sector employs skilled workers and physical capital.

[1] See, for example, Azarnert (2004) and Fironi (2010).

The government imposes a lump-sum tax on each individual at the end of adulthood. Collected taxes are used for public health expenditure to improve medical facilities, sanitation, disease control, and inoculation programs. Public health expenditure improves child health and, thus, reduces child mortality.

2.2. Production

Because the output level in the traditional sector is based only on unskilled workers and child labor, the production function of the traditional sector is

$$Y_t^u = w^u L_t^u. \tag{1}$$

Let Y_t^u, w^u, and L_t^u represent production in this sector, the wage rate of the unskilled labor, and the amount of unskilled labor, respectively. L_t^u includes child labor from both skilled and unskilled households. Conversely, production in the modern sector employs skilled labor and physical capital. The production function of the skilled sector is

$$Y_t^s = F(K_t, L_t^s). \tag{2}$$

Let Y_t^s, K_t, and L_t^s express production in its sector, physical capital, and the amount of skilled labor, respectively. We assume the production function of the skilled sector satisfies all neoclassical characteristics. The firm's profit maximization problem is

$$\max_{\{K_t, L_t^s\}} F(K_t, L_t^s) - \bar{r}K_t - w_t^s L_t^s, \tag{3}$$

where $\bar{r}$ and w_t^s are the world interest rate and the wage rate for the skilled labor. Maximizing the objective function (3) with respect to K_t and L_t^s yields the following conditions:

$$\bar{r} = f'(k_t), \tag{4}$$

$$w_t^s = f(k_t) - f'(k_t)k_t, \tag{5}$$

where $k_t \equiv \frac{K_t}{L_t^s}$. Here, k_t and w_t^s are constant. These values are denoted as $\bar{k}$ and w^s and we also assume $w^s > w^u$.[2]

2.3. Individuals

We consider the behavior of individuals. Each individual $i \in \{s, u\}$ derives his/her utility from the number of surviving children $(\pi_t^c n_t^i)$, rearing (n_t^i) children, children's leisure $(1-\ell_{c,t}^i)$, the investments in education for each child (e_t^i), his/her own consumption in adulthood (c_t^i), and his/her future consumption in old age (c_{t+1}^i). The utility function of the individual i belonging to generation t is

$$\begin{aligned} u_t^i = &\gamma[\log(\pi_t^c n_t^i) + \phi \log(1 - \ell_{c,t}^i) + \beta \log(e_t^i)] \\ &+ (1-\gamma)[\log(c_t^i) + \delta \log(c_{t+1}^i)], \quad \gamma, \phi, \beta, \delta \in [0, 1), \end{aligned} \tag{6}$$

where γ stands for the extent to which each individual values her children relative to his/her own lifetime consumption. ϕ represents the weight on the leisure of each child. β represents the weight of investments in education for the surviving children. Component $\gamma\beta \log(e_t^i)$ represents either the warm glow of giving (Andreoni 1989) or a preference of having higher-quality children (Becker 1960). Since ϕ and β are bellow one, the individual values her utility from having surviving children more than the surviving children's leisure and investments in education. This assumption indicates that having a family is more important than investing in education for children and children's leisure.[3] δ is the discount rate that gives priority to present consumption over future consumption.

Each individual and child have one unit of time. During adulthood, the individual gives birth to children, educates surviving children, and determines the child labor supply.

[2]See, for example, Galor and Zeira (1993) and Hazan and Berdugo (2002).
[3]See, for example, Efrlich and Lui (1991), Zhang et al. (2001), and Hashimoto and Tabata (2016).

The individual then collects the income earned from child labor and uses it for his/her own consumption. The individual pays a lump-sum tax τ to the government. Here, all individual's incomes are called family income, being described as $(1 - zn_t^i - e_t^i \pi_t^c n_t^i)w^s + d\ell_{c,t}^i \pi_t^c n_t^i w^u - \tau$. The individual also divides his/her family income into present consumption and savings. Hence, the budget constraint of individual i at time t is given by:

$$c_t^i = (1 - zn_t^i - e_t^i \pi_t^c n_t^i)w^i + d\ell_{c,t}^i \pi_t^c n_t^i w^u - \tau - s_t^i, \tag{7}$$

$$c_{t+1}^i = (1 + \bar{r})s_t^i, \tag{8}$$

$$\bar{e} \geq e_t^u, \tag{9}$$

where z is the time to bear and raise children.[4] Following Becker and Lewis (1973), Rosenzweig and Wolpin (1980), Galor and Weil (2000), and Moav (2005), we assume that this time cost z does not vary with family size. $d \in (0, 1)$ indicates that children can only provide $d\ell_{c,t}^i$ units of parents' labor because children's physical ability is inferior to that of adults and they can only work in the traditional sector. We assume that unskilled parents are less productive in terms of education and always choose the upper bound and their marginal contribution of their time to children's education is substantially lower than that of skilled parents, that is, $e_t^u = \bar{e} > 0$.[5] When parents school their children at home, they must possess basic abilities such as writing, reading, and arithmetics. We assume that only skilled parents have these abilities. This assumption and its implications originate from empirical evidences as follows. Lynch (2000) and Ramey and Ramey (2010) show that, in the USA, mothers who had a higher education level spend much more time educating their children and give them an overall better education than mothers who did not have a high education level. Ballantine (2001) also supports the stylized fact that better educated parents spend more time educating their children from the sociological side.

[4]See, for example, Becker (1965).

[5]See, for example, Moav (2005) and Fan and Zhang (2013) for the theoretical background.

Maximizing (6) subject to (7) and (8) for skilled workers yields the following solutions:

$$s_t^s = \frac{\delta(1-\gamma)(w^s-\tau)}{(1-\gamma)(1+\delta)+\gamma} \equiv s^s, \tag{10}$$

$$n_t^s = \frac{\gamma(\phi+\beta-1)(w^s-\tau)}{[(1-\gamma)(1+\delta)+\gamma](d\pi_t^c w^u - zw^s)}, \tag{11}$$

$$\ell_{c,t}^s = 1 - \frac{\phi(d\pi_t^c w^u - zw^s)}{(\phi+\beta-1)d\pi_t^c w^u}, \tag{12}$$

$$e_t^s = \frac{\beta(d\pi_t^c w^u - zw^s)}{(\phi+\beta-1)\pi_t^c w^s}, \tag{13}$$

$$c_t^s = \frac{(1-\gamma)(w^s-\tau)}{(1-\gamma)(1+\delta)+\gamma} \equiv c^s. \tag{14}$$

Maximizing (6) subject to (7) to (9) for unskilled workers yields the following solutions:

$$s_t^u = \frac{\delta(1-\gamma)(w^u-\tau)}{(1-\gamma)(1+\delta)+\gamma} \equiv s^u, \tag{15}$$

$$n_t^u = \frac{\gamma(1-\phi)(w^u-\tau)}{[(1-\gamma)(1+\delta)+\gamma][z-(d-\bar{e})\pi_t^c]w^u}, \tag{16}$$

$$\ell_{c,t}^u = 1 - \frac{\phi[z-(d-\bar{e})\pi_t^c]}{(1-\phi)d\pi_t^c}, \tag{17}$$

$$c_t^u = \frac{(1-\gamma)(w^u-\tau)}{(1-\gamma)(1+\delta)+\gamma} \equiv c^u. \tag{18}$$

Because all variables are required to be non-negative, the following conditions must be satisfied:

$$d\pi_t^c w^u - zw^s > 0, \tag{19}$$

$$z-(d-\bar{e})\pi_t^c > 0, \tag{20}$$

$$w^s > \tau, \tag{21}$$

$$w^u > \tau, \tag{22}$$

$$\phi+\beta > 1. \tag{23}$$

We explore how an improvement in child mortality affects skilled and unskilled workers. Hereafter, we make the following assumption on educational investments of unskilled workers:

Assumption 1.

$$0 < \bar{e} < \min\left\{\frac{\beta(d\pi_t^c w^u - zw^s)}{(\phi+\beta-1)\pi_t^c w^s}, d\right\}. \tag{24}$$

Inequality $\bar{e} < \frac{\beta(d\pi_t^c w^u - zw^s)}{(\phi+\beta-1)\pi_t^c w^s}$ ensures that skilled parents spend more time educating their children than unskilled ones, while inequality $\bar{e} < d$ ensures that an improvement in child mortality always makes unskilled workers increase the number of their children. As a result, we obtain the following proposition.

Proposition 1. *An improvement in child mortality makes skilled workers decrease fertility, the child labor supply, and increase investments in education. However, unskilled workers increase both their fertility and the child labor supply.*

Proof. We can obtain the partial derivatives of (11), (12), (13), (16), and (17) with respect to π_t^c as follows:

$$\frac{\partial n_t^s}{\partial \pi_t^c} = -\frac{\gamma(\phi+\beta-1)(w^s-\tau)dw^u}{[(1-\gamma)(1+\delta)+\gamma](d\pi_t^c w^u - zw^s)^2} < 0, \tag{25}$$

$$\frac{\partial \ell_{c,t}^s}{\partial \pi_t^c} = -\frac{\phi z w^s}{(\phi+\beta-1)dw^u(\pi_t^c)^2} < 0, \tag{26}$$

$$\frac{\partial e_t^s}{\partial \pi_t^c} = \frac{\beta z}{(\phi+\beta-1)(\pi_t^c)^2} > 0, \tag{27}$$

$$\frac{\partial n_t^u}{\partial \pi_t^c} = \frac{\gamma(w^u-\tau)(1-\phi)(d-\bar{e})}{[(1-\gamma)(1+\delta)+\gamma](1-\gamma+\gamma w^u)w^u[z-(d-\bar{e})\pi_t^c]^2} > 0, \tag{28}$$

$$\frac{\partial \ell_{c,t}^u}{\partial \pi_t^c} = \frac{\phi z}{(1-\phi)d(\pi_t^c)^2} > 0. \tag{29}$$

□

Since skilled workers freely choose to invest in education, an improvement in child mortality makes them have better educated children and this effect dominates the budget and time constraints of skilled individuals. Higher education increases the cost of having children and, thereby, reduces the number of children because investments in education are time-consuming. Hence, skilled workers are inclined to have a smaller family, which makes children's contributions to the family income smaller and, thus, reduces the child labor supply. Conversely, since unskilled workers have an upper bound for investments in the education of their children, an improvement in child mortality contributes to having more children, which leads to a larger family. As child survival and fertility rates increase, the income from child labor also increases, compensating for the higher disutility of making their children work. Therefore, unskilled agents increase the child labor supply.

In addition, we compare the fertility rates and child labor supply between skilled and unskilled workers. Then, we obtain the following proposition.

Proposition 2. *Skilled parents have lower fertility rates and child labor supply compared to unskilled ones:*

$$n_t^u > n_t^s, \tag{30}$$

$$\ell_{c,t}^u > \ell_{c,t}^s. \tag{31}$$

Proof. We examine the sign of differential fertility $n_t^u - n_t^s$ by using (11) and (16):

$$\begin{aligned}
n_t^u - n_t^s &= \frac{\gamma(1-\phi)(w^u-\tau)}{[(1-\gamma)(1+\delta)+\gamma][z-(d-\bar{e})\pi_t^c]w^u} - \frac{\gamma(\phi+\beta-1)(w^s-\tau)}{[(1-\gamma)(1+\delta)+\gamma](d\pi_t^c w^u - zw^s)}, \\
&= \frac{\gamma[(1-\phi)(w^u-\tau)(d\pi_t^c w^u - zw^s) - (\phi+\beta-1)(w^s-\tau)(z-d\pi_t^c+\bar{e}\pi_t^c)w^u]}{[1-\gamma)(1+\delta)+\gamma][z-(d-\bar{e})\pi_t^c](d\pi_t^c w^u - zw^s)w^u}, \\
&\quad \text{(Here, we employ Assumption 1: } \bar{e} < e_t^s.\text{)} \\
&> \frac{\gamma[(1-\phi)(w^u-\tau)(d\pi_t^c w^u - zw^s) - (\phi+\beta-1)(w^s-\tau)(z-d\pi_t^c+e_t^s\pi_t^c)w^u]}{[1-\gamma)(1+\delta)+\gamma][z-(d-\bar{e})\pi_t^c](d\pi_t^c w^u - zw^s)w^u}, \\
&\quad \text{(Here, we employ (13) and } w^s > w^u.\text{)} \\
&> \frac{\gamma[(1-\phi)(w^u-\tau)(d\pi_t^c w^u - zw^s) - (\phi+\beta-1)(w^s-\tau)(z-d\pi_t^c)w^u - \beta(w^s-\tau)(d\pi_t^c w^u - zw^s)]}{[1-\gamma)(1+\delta)+\gamma][z-(d-\bar{e})\pi_t^c](d\pi_t^c w^u - zw^s)w^u} \\
&= \frac{\gamma\{(1-\phi)[(w^u-\tau)(d\pi_t^c w^u - zw^s) + (w^s-\tau)(d\pi_t^c - z)w^u] + \beta z(w^s-\tau)(w^s-w^u)\}}{[1-\gamma)(1+\delta)+\gamma][z-(d-\bar{e})\pi_t^c](d\pi_t^c w^u - zw^s)w^u}, \\
&> 0.
\end{aligned}$$

In the last inequality, we employed (19), (21), (22), (23), and $w^s > w^u$. Similarly, we examine the sign of child labor supply $\ell_{c,t}^u - \ell_{c,t}^s$ employing (12) and (17):

$$\begin{aligned}
\ell_{c,t}^u - \ell_{c,t}^s &= 1 - \frac{\phi[z-(d-\bar{e})\pi_t^c]}{(1-\phi)d\pi_t^c} - \left[1 - \frac{\phi(d\pi_t^c w^u - zw^s)}{(\phi+\beta-1)d\pi_t^c w^u}\right], \\
&= \frac{\phi\{[-z+(d-\bar{e})\pi_t^c](\phi+\beta-1)w^u + (d\pi_t^c w^u - zw^s)(1-\phi)\}}{(1-\phi)(\phi+\beta-1)dw^u\pi_t^c}, \\
&\quad \text{(Here, we employ Assumption 1: } \bar{e} < e_t^s.\text{)} \\
&> \frac{\phi\{[-z+(d-e_t^s)\pi_t^c](\phi+\beta-1)w^u + (d\pi_t^c w^u - zw^s)(1-\phi)\}}{(1-\phi)(\phi+\beta-1)dw^u\pi_t^c}, \\
&\quad \text{(Here, we employ (13) and } w^s > w^u.\text{)} \\
&> \frac{\phi[(\phi+\beta-1)(d\pi_t^c - z)w^u - \beta w^u(d\pi_t^c w^u - zw^s) + (1-\phi)(d\pi_t^c w^u - zw^s)]}{(1-\phi)(\phi+\beta-1)dw^u\pi_t^c}, \\
&= \frac{\phi(\phi+\beta-1)(w^s-w^u)z}{(1-\phi)(\phi+\beta-1)dw^u\pi_t^c}, \\
&> 0.
\end{aligned}$$

In the last inequality, we employed (23) and $w^s > w^u$. □

If Assumption 1: $\bar{e} < e_t^s$ is satisfied, the fertility rate of unskilled workers is always greater than that of skilled workers. Further, the child labor supply of unskilled workers is always greater than that of skilled workers. Therefore, condition $\bar{e} < e_t^s$ is sufficient to determine the signs of $n_t^u - n_t^s$ and $\ell_{c,t}^u - \ell_{c,t}^s$. Since unskilled workers cannot provide their children with a sufficient level of investments in education and their wage rate is small in contrast to that of skilled workers, unskilled workers have a large family and depend on child labor income.

3. Population dynamics

The total public health expenditure is determined by the total tax revenue, that is,

$$G_t = \tau L_t,$$

where L_t denotes the number of adult workers in the economy. We define public health services per family as follows:

$$g = \tau. \tag{32}$$

Because the lump-sum tax per family is constant over time, g also becomes constant. We assume that the child survival probability is a monotonically increasing function of public health services. Therefore, children's survival probability function is given by

$$\pi_t^c = \pi^c(g_t) = \pi^c(\tau) \equiv \pi^c, \tag{33}$$

where $\pi'(g_t) > 0, \pi''(g_t) < 0, \lim_{g_t \to 0} \pi(g_t) = 0, \lim_{g_t \to \infty} \pi(g_t) < 1$, and $\lim_{g_t \to \infty} \pi(g_t) < \infty$. Substituting (33) into (11), (12), (13), (16), and (17) yields

$$n_t^s = \frac{\gamma(\phi + \beta - 1)(w^s - \tau)}{[(1-\gamma)(1+\delta) + \gamma](d\pi^c w^u - z w^s)} \equiv n^s, \tag{34}$$

$$\ell_{c,t}^s = 1 - \frac{\phi(d\pi^c w^u - z w^s)}{(\phi + \beta - 1) d\pi^c w^u} \equiv \ell_c^s, \tag{35}$$

$$e_t^s = \frac{\beta(d\pi^c w^u - z w^s)}{(\phi + \beta - 1)\pi^c w^s} \equiv e^s, \tag{36}$$

$$n_t^u = \frac{\gamma(1-\phi)(w^u - \tau)}{[(1-\gamma)(1+\delta) + \gamma][z - (d - \bar{e})\pi^c] w^u} \equiv n^u, \tag{37}$$

$$\ell_{c,t}^u = 1 - \frac{\phi[z - (d - \bar{e})\pi^c]}{(1-\phi) d\pi^c} \equiv \ell_c^u. \tag{38}$$

We now derive population dynamics. Let λ_t and p represent the proportion of skilled workers at time t and the probability of a child whose parent is skilled or unskilled to become skilled, respectively. We further assume that p is a function of investments in education by parents: $p^s = p(e^s) \in (0,1)$ and $p^u = p(\bar{e}) \in (0,1)$.[6] These probability functions are strictly increasing with investments in education and are concave. Since unskilled parents are less productive in terms of investments in education based on Assumption 1, $p^s > p^u$ always holds. The total number of workers at time t is $\pi^c n^s \lambda_t L_t + \pi^c n^u (1-\lambda_t) L_t$. The total number of skilled workers at time $t+1$ is $p^s \pi^c n^s \lambda_t L_t + p^u \pi^c n^u (1-\lambda_t) L_t$. Therefore, the proportion of skilled workers at time $t+1$ is

$$\begin{aligned} \lambda_{t+1} &= \frac{p^s \pi^c n^s \lambda_t L_t + p^u \pi^c n^u (1-\lambda_t) L_t}{\pi^c n^s \lambda_t L_t + \pi^c n^u (1-\lambda_t) L_t} \\ &= \frac{p^s n^s \lambda_t + p^u n^u (1-\lambda_t)}{n^s \lambda_t + n^u (1-\lambda_t)}. \end{aligned} \tag{43}$$

We consider whether population dynamics (43) has at least one stable steady state. Then, we obtain the following proposition.

[6] de la Croix and Michel (2002), Anger and Heineck (2010), and Becker (2009) state that family background and parental academic achievements and human capital have substantial effects on an individual's educational attainment based on empirical evidence. See, for example, Cremer and Pestrieau (2006), Mookherjee and Napel (2007), and Fan and Stark (2008) for the theoretical background of this assumption.

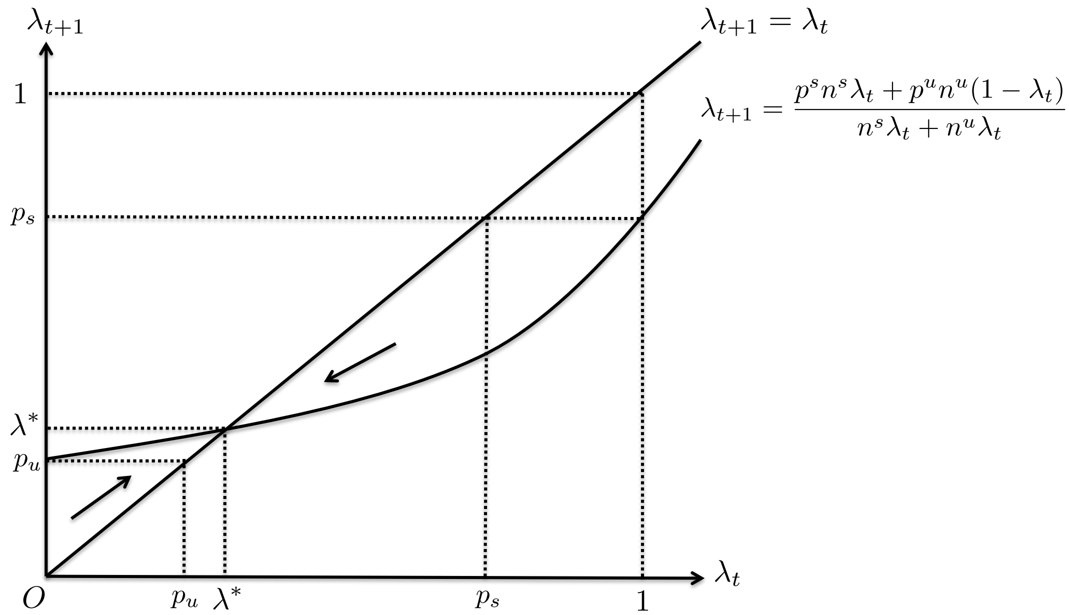

Figure 1: The transition process of λ_t when $n^s < n^u$ holds.

Proposition 3. *Population dynamics* (43) *has a stable steady state that satisfies* $p^u < \lambda^* < p^s$. *Moreover, this steady state is given by*

$$\lambda^* = \frac{(1+p^u)n^u - p^s n^s - (\Phi)^{\frac{1}{2}}}{2(n^u - n^s)}, \tag{44}$$

where $\Phi = [(1+p^u)n^u - p^s n^s]^2 - 4p^u(n^u - n^s)n^u > 0$.

Proof. We can obtain the first and second order derivatives of (43) as follows:

$$\frac{\partial \lambda_{t+1}}{\partial \lambda_t} = \frac{(p^s - p^u)n^u n^s}{\{n^s \lambda_t + n^u(1-\lambda_t)\}^2} > 0, \tag{45}$$

$$\frac{\partial^2 \lambda_{t+1}}{\partial \lambda_t^2} = \frac{2n^u n^s (p^s - p^u)(n^u - n^s)}{\{n^s \lambda_t + n^u(1-\lambda_t)\}^3} > 0. \tag{46}$$

When $\lambda_t = 0$, then λ_{t+1}=p^u. Conversely, when $\lambda_t = 1$, then λ_{t+1}=p^s. Therefore, we obtain population dynamics as per in Figure 1. Hence, the steady state that satisfies $p^u < \lambda^* < p^s$ is globally stable. □

We consider the characteristics of the steady state and how an increase in fertility rates, the probability for children to become skilled, and an improvement in child mortality affects the steady state.

Proposition 4. *In the steady state, we have the following inequalities:*

$$\frac{\partial \lambda^*}{\partial z} < (\geq) 0, \quad \frac{\partial \lambda^*}{\partial \bar{e}} > 0, \quad \frac{\partial \lambda^*}{\partial \tau} < (\geq) 0. \tag{47}$$

Proof. We can obtain the partial derivatives of equation (43) in the steady state with respect to z, $\bar{e}$, and π^c, respectively, as follows:

$$\frac{\partial \lambda^*}{\partial z} = \frac{-(1-\lambda^*)(\lambda^* - p^u)\frac{\partial n^u}{\partial z} + \lambda^*(p^s - \lambda^*)\frac{\partial n^u}{\partial z} n^s \lambda^* \frac{\partial p^s}{\partial z}}{(\Phi)^{\frac{1}{2}}} < (\geq) 0, \tag{48}$$

$$\frac{\partial \lambda^*}{\partial \bar{e}} = \frac{-(1-\lambda^*)(\lambda^* - p^u)\frac{\partial n^u}{\partial \bar{e}} + (1-\lambda^*) n^u \frac{\partial p^u}{\partial \bar{e}}}{(\Phi)^{\frac{1}{2}}} > 0, \tag{49}$$

$$\frac{\partial \lambda^*}{\partial \pi^c} = \frac{-(1-\lambda^*)(\lambda^* - p^u)\frac{\partial n^u}{\partial \pi^c} + (p^s - \lambda)\lambda^* \frac{\partial n^s}{\partial \pi^c} + n^s \lambda^* \frac{\partial p^s}{\partial \pi^c}}{(\Phi)^{\frac{1}{2}}} < (\geq) 0, \tag{50}$$

where $\frac{\partial n^s}{\partial z} > 0$, $\frac{\partial n^u}{\partial z} < 0$, $\frac{\partial p^s}{\partial z} < 0$, $\frac{\partial n^u}{\partial \bar{e}} < 0$, $\frac{\partial p^u}{\partial \bar{e}} > 0$, $\frac{\partial \pi^c}{\partial \tau} > 0$, $\frac{\partial n^u}{\partial \pi^c} > 0$, $\frac{\partial n^s}{\partial \pi^c} < 0$, $\frac{\partial p^s}{\partial \pi^c} > 0$. Therefore, we obtain the following inequalities:

$$-(1-\lambda^*)(\lambda^* - p^u)\frac{\partial n^u}{\partial z} + (p^s - \lambda^*)\lambda^* \frac{\partial n^s}{\partial z} < -n^s \lambda^* \frac{\partial p^s}{\partial z}, \tag{51}$$

$$(1-\lambda^*)(\lambda^* - p^u)\frac{\partial n^u}{\partial \pi^c} - (p^s - \lambda^*)\lambda^* \frac{\partial n^s}{\partial \pi^c} > n^s \lambda^* \frac{\partial p^s}{\partial \pi^c}. \tag{52}$$

An increase in the time cost to bear and rear children and the health policy leading to an improvement in child mortality negatively affect the proportion of skilled agents in the steady state. □

The amount of time, (z), represents parents' opportunity cost of the time spent away from work to raise their children. If z increases, skilled parents choose to decrease investments in education (e^s) and increase the child labor supply (ℓ_c^s) to compensate for these opportunity costs. As such, skilled parents increase their number of children (n^s). Conversely, an increase in z makes it difficult for unskilled parents to bear and rear their children. As a result, they choose to decrease their number of children (n^u). If the effect on the probability (p^s) dominates the effect of skilled and unskilled fertility rates, as in inequality (51), an increase in the time cost can decrease the proportion of skilled workers in the steady state. An increase in the upper bound of investments in education ($\bar{e}$) indicates that unskilled parents provide better education to their children. Hence, they require more time for education and they have to decrease n^u. Moreover, the increase in $\bar{e}$ positively affects the probability (p^u). Therefore, the increase in $\bar{e}$ positively affects the proportion of skilled agents in the steady state. As per Proposition 1, an improvement in child mortality increases n^u and e^s and decreases n^s. An increase in (e^s) positively affects the probability (p^s). If the effect on the fertility rates (n^u, n^s) dominates the influence on the probability (p^s), as in inequality (52), the government's health policy can decrease the proportion of skilled agents in the steady state.

4. Policy implications

We examine three alternative policies: a ban on child labor, improvement in child mortality, and child education. Then, we examine whether these policies can increase the ratio of skilled workers.

4.1. Improvement in child mortality policy and a ban on child labor policy

We investigate how the ban on child labor affects skilled and unskilled individuals, assuming that the government imposes the abolition of child labor, that is, $\ell_c^i = 0$.[7] We employ the following assumption to ensure that both skilled and unskilled individuals choose investments in the education of their children as an interior solution because we focus on the case where

[7] See, for example, Strulik (2004).

the ban decreases the investments in the education of skilled individuals to the level where $e^s < \bar{e}$ holds and $\bar{e}$ is no longer the upper bound for unskilled individuals.

Assumption 2.

$$\bar{e} > \frac{\beta z}{(1-\beta)\pi^c}. \tag{53}$$

Similarly to Section 2, maximizing (6) subject to (7) and (8) for skilled workers yields the following solutions:

$$s^s_{g,t} = \frac{\delta(1-\gamma)(w^s-\tau)}{(1+\delta)(1-\gamma)+\gamma} \equiv s^s_g, \tag{54}$$

$$n^s_{g,t} = \frac{\gamma(1-\beta)(w^s-\tau)}{[(1+\delta)(1-\gamma)+\gamma]zw^s} \equiv n^s_g, \tag{55}$$

$$e^s_{g,t} = \frac{\beta z}{(1-\beta)\pi^c} \equiv e_g, \tag{56}$$

$$c^s_{g,t} = \frac{(1-\gamma)(w^s-\tau)}{(1+\delta)(1-\gamma)+\gamma} \equiv c^s_g, \tag{57}$$

where the subscript g denotes that the government enforces the ban on child labor. Moreover, the optimal solutions for unskilled workers are given by

$$s^u_{g,t} = \frac{\delta(1-\gamma)(w^u-\tau)}{(1+\delta)(1-\gamma)+\gamma} \equiv s^u_g, \tag{58}$$

$$n^u_{g,t} = \frac{\gamma(1-\beta)(w^u-\tau)}{[(1+\delta)(1-\gamma)+\gamma]zw^u} \equiv n^u_g, \tag{59}$$

$$e^u_{g,t} = \frac{\beta z}{(1-\beta)\pi^c} \equiv e_g, \tag{60}$$

$$c^u_{g,t} = \frac{(1-\gamma)(w^u-\tau)}{(1+\delta)(1-\gamma)+\gamma} \equiv c^u_g. \tag{61}$$

Therefore, once the ban on child labor is executed, both skilled and unskilled workers choose the same amount of investments in the education of their children as an interior solution. Then, we obtain the following proposition, which is in contrast with Proposition 1.

Proposition 5. *When the government imposes a ban on child labor, an improvement in child mortality no longer affects the fertility rates of skilled workers and unskilled workers.*

Moreover, it makes skilled and unskilled workers decrease their investments in the education of their children.

Proof. We can obtain the partial derivatives of (55), (59), and (60) with respect to π^c as follows:

$$\frac{\partial n_g^s}{\partial \pi^c} = \frac{\partial n_g^u}{\partial \pi^c} = 0, \tag{62}$$

$$\frac{\partial e_g^s}{\partial \pi^c} = \frac{\partial e_g^u}{\partial \pi^c} = -\frac{\beta z}{(1-\beta)(\pi^c)^2} < 0. \tag{63}$$

□

Equation (62) follows from budget constraint (7), that is, an improvement in child mortality increases the cost of education for children, which in turn makes parents decrease the number of children. The marginal benefit of an improvement in child mortality and the marginal disutility of decreasing the number of children neutralize each other due to the characteristics of the log utility function (6), which implies unitary elastic demand for children with respect to the cost of having children. The negative sign of (63) also follows from budget constraint (7), that is, an improvement in child mortality increases the cost of education for children, which in turn makes parents decrease investments in the education of their children. This result comes from the characteristics of the utility function (6) that parents always prioritize an improvement in child mortality over investments in education because β is bellow one. Skilled and unskilled parents give birth to their children, expecting them to contribute to family income before the government enforces the ban on child labor. However, once the ban is in place, they can no longer obtain the rewards from child labor. Therefore, an improvement in child mortality gives parents a heavier burden of increased surviving children and less education for each child. [8]

We next examine how the ban on child labor affects fertility rates and clarify how the

[8]See, for example, Kalemli-Ozcan (2002, 2003, 2008). The findings here correspond to those of Kalemli-Ozcan (2008).

difference in the fertility rates between skilled and unskilled workers change. The findings, which are in contrast to Proposition 2, can be summarized as follows.

Proposition 6. *The ban on child labor overturns the fertility difference between skilled and unskilled workers. Moreover, it makes skilled workers increase the number of children and unskilled workers decrease it.*

$$n_g^u < n_g^s, \tag{64}$$

$$n^s < n_g^s, \tag{65}$$

$$n_g^u < n^u. \tag{66}$$

Proof. By using equations (55) and (59), we obtain

$$\begin{aligned} n_g^s - n_g^u &= \frac{\gamma(1-\beta)(w^s-\tau)}{[(1+\delta)(1-\gamma)+\gamma]zw^s} - \frac{\gamma(1-\beta)(w^u-\tau)}{[(1+\delta)(1-\gamma)+\gamma]zw^u}, \\ &= \frac{\gamma(1-\beta)\tau(w^s-w^u)}{[(1+\delta)(1-\gamma)+\gamma]zw^s w^u}, \\ &> 0. \end{aligned}$$

Assumptions 1 and 2 yield the following condition[9]:

$$(1-\beta)d\pi^c w^u > \phi z w^s. \tag{67}$$

[9]Assumptions 1 and 2 ensure that $e^s > e_g^s$ holds. Moreover, we can derive the value of $e^s - e^g$ from (36) and (56) as follows: $e^s - e^g = \frac{\beta}{\pi^c(\phi+\beta-1)w^s(1-\beta)}[(1-\beta)d\pi^c w^u - \phi z w^s]$.

Then, we obtain the signs of $n_g^s - n^s$ and $n^u - n_g^u$ from (34), (37), (55), and (59).

$$\begin{aligned}
n^s - n_g^s &= \frac{\gamma(\phi+\beta-1)(w^s-\tau)}{[(1-\gamma)(1+\delta)+\gamma](d\pi^c w^u - zw^s)} - \frac{\gamma(1-\beta)(w^s-\tau)}{[(1+\delta)(1-\gamma)+\gamma]zw^s}, \\
&= \frac{\gamma(w^s-\tau)}{[(1-\gamma)(1+\delta)+\gamma](d\pi^c w^u - zw^s)zw^s}[\phi z w^s - (1-\beta)d\pi^c w^u], \\
&\text{(Here, we employ (67).)} \\
&< 0.
\end{aligned}$$

$$\begin{aligned}
n^u - n_g^u &= \frac{\gamma(1-\phi)(w^u-\tau)}{[(1-\gamma)(1+\delta)+\gamma][z-(d-\bar{e})\pi^c]w^u} - \frac{\gamma(1-\beta)(w^u-\tau)}{[(1+\delta)(1-\gamma)+\gamma]zw^u}, \\
&= \frac{\gamma(w^u-\tau)}{[(1+\delta)(1-\gamma)+\gamma](z-d\pi^c+\bar{e}\pi^c)zw^u}[(1-\beta)d\pi^c - \phi z + z - (1-\beta)(z+\bar{e}\pi^c)], \\
&\text{(Here, we employ (24).)} \\
&> \frac{\gamma(w^u-\tau)}{[(1+\delta)(1-\gamma)+\gamma](z-d\pi^c+\bar{e}\pi^c)zw^u}[(1-\beta)d\pi^c - \phi z + z - (1-\beta)(z+\frac{\beta(d\pi^c w^u - zw^s)}{(\phi+\beta-1)w^s})], \\
&\text{(Here, we employ } w^s > w^u \text{ and (23).)} \\
&> \frac{\gamma(w^u-\tau)}{[(1+\delta)(1-\gamma)+\gamma](z-d\pi^c+\bar{e}\pi^c)zw^u}[(1-\beta)d\pi^c - \phi z + z - (1-\beta)z + \beta(d\pi^c - z)], \\
&= \frac{\gamma(w^u-\tau)}{[(1+\delta)(1-\gamma)+\gamma](z-d\pi^c+\bar{e}\pi^c)zw^u}[(1-\beta)d\pi^c - \phi z + \beta z + \beta(d\pi^c - z)], \\
&\text{(Here, we employ } w^s > w^u\text{, (19), and (67).)} \\
&> 0.
\end{aligned}$$

□

The abolition of child labor significantly decreases unskilled parents' incentives to bear and rear their children more than skilled parents because the former have lower incomes than the latter. Therefore, the ban on child labor overturns the relative magnitude of fertility rates between skilled and unskilled parents. However, skilled parents increase their fertility rate since they decrease investments in education and they reallocate their time to having a larger

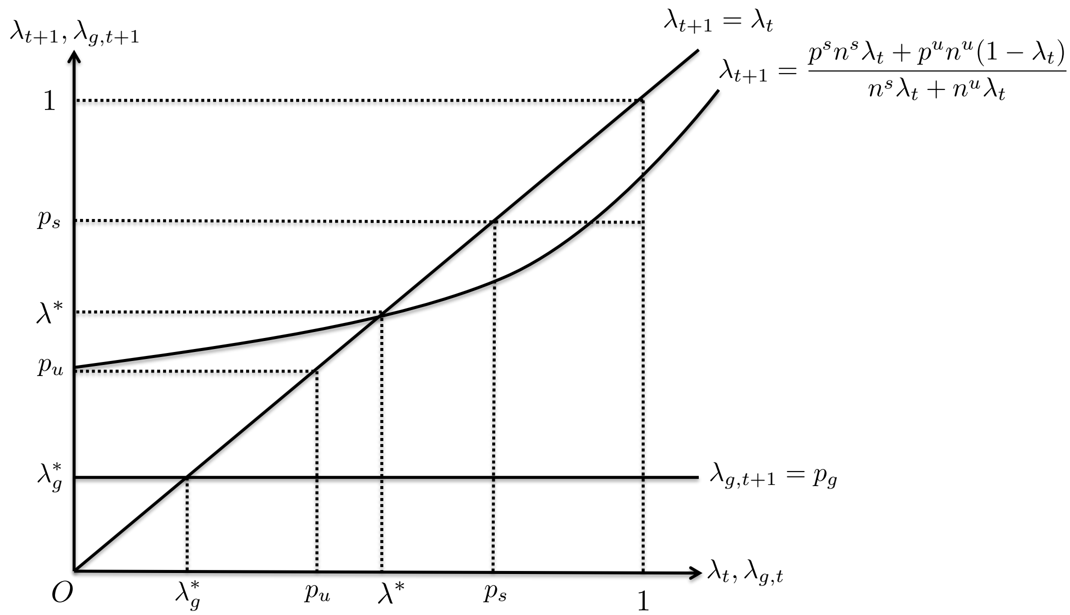

Figure 2: The transition process of λ_t where $n^s < n^u$ holds and the transition process of $\lambda_{g,t}$ when the ban on child labor is executed and $n_g^s > n_g^u$ holds.

family.

When the government imposes a ban on child labor, population dynamics becomes $\lambda_{g,t+1} = p(e_g) \equiv p_g$ by employing (55), (56), (59), and (60). We can obtain the figure of population dynamics as per Figure 2, which indicates that the ban on child labor policy decreases the proportion of skilled workers in the economy. Moreover, $\frac{\partial \lambda_g^*}{\partial \pi^c} < 0$ holds in the steady state because an improvement in child mortality decreases both skilled and unskilled parents' investments in education based on (63). Therefore, once the ban on child labor is implemented, an improvement in child mortality can no longer increase the proportion of skilled workers in the economy; however, it strictly decreases not only this proportion but also the average income in the economy. This result can be demonstrated as follows. We define y_t^p as the average income per person in the economy: $y_t^p = \lambda_t(w^s - w^u) + w^u$. Therefore, a decrease in the proportion of skilled workers reduces the average income.

4.2. Child education policy

Here, we examine how the child education policy affects the ratio of skilled workers and the average income in the economy. The child education policy can be interpreted as the provision of schools, which supports investments in the education of children from both skilled and unskilled. Although unskilled parents do not possess basic abilities, such as writing, reading, and arithmetic, they can help their children master these abilities at school. The government executes this education policy by dividing the total tax revenue (τL_t) into education policy and public health expenditure.

The government expends a fraction $\theta \in (0,1)$ of the total tax revenue for the provision of schools and the remaining fraction $(1-\theta)$ for improving child mortality. We assume the following functions for the child survival probability: $\pi^c(\theta) \equiv \pi^c[(1-\theta)\tau]$: for the probability of a child whose parent is skilled to become skilled: $p^s(\theta) \equiv p[e^s(\theta), \theta\tau]$: and for the probability of a child whose parent is unskilled to become skilled: $p^u(\theta) \equiv p(\bar{e}, \theta\tau)$. Because $p^s(\theta)$ and $p^u(\theta)$ are strictly increasing with investments in education and concave, as in Section 3, these probability functions also satisfy the following properties: $\frac{\partial p^s(\theta)}{\partial e^s(\theta)} > 0$ and $\frac{\partial p^s(\theta)}{\partial(\theta\tau)} = \frac{\partial p^u(\theta)}{\partial(\theta\tau)} > 0$. We obtain the following results instead of (34), (36), and (37):

$$n^s(\theta) = \frac{\gamma(\phi+\beta-1)(w^s-\tau)}{[(1-\gamma)(1+\delta)+\gamma][d\pi^c(\theta)w^u - zw^s]}, \tag{68}$$

$$e^s(\theta) = \frac{\beta[d\pi^c(\theta)w^u - zw^s]}{(\phi+\beta-1)\pi^c(\theta)w^s}, \tag{69}$$

$$n^u(\theta) = \frac{\gamma(1-\phi)(w^u-\tau)}{[(1-\gamma)(1+\delta)+\gamma][z-(d-\bar{e})\pi^c(\theta)]w^u}. \tag{70}$$

The fertility rate of skilled parents increases with the ratio of child education expenditure (i.e., $\frac{dn^s(\theta)}{d\theta} > 0$). However, investments in education for skilled parents' children decrease with θ (i.e., $\frac{de^s(\theta)}{d\theta} < 0$). Conversely, the fertility rate of unskilled workers decreases with θ (i.e., $\frac{dn^u(\theta)}{d\theta} < 0$). We derive the population dynamics under this model in an analogous manner to (43) in Section 3:

$$\lambda_{t+1}(\theta) = \frac{p^s(\theta)n^s(\theta)\lambda_t(\theta) + p^u(\theta)n^u(\theta)[1-\lambda_t(\theta)]}{n^s(\theta)\lambda_t(\theta) + n^u(\theta)[1-\lambda_t(\theta)]}. \tag{71}$$

Then, we obtain the following proposition.

Proposition 7. *We assume that* $dp^s(\theta) = \frac{\partial p[e^s(\theta),\theta\tau]}{\partial e^s(\theta)} de^s(\theta) + \frac{\partial p[e^s(\theta),\theta\tau]}{\partial(\theta\tau)} d(\theta\tau) > 0$. *An increase in the ratio of child education expenditure has a positive effect on the proportion of skilled workers and the average income in the steady state.*

Proof. We consider population dynamics based on (71) in the steady state. Then, the following equation is satisfied in the steady state:

$$[n^u(\theta) - n^s(\theta)][\lambda(\theta)]^2 + \{-[1+p^u(\theta)]n^u(\theta) + p^s(\theta)n^s(\theta))\}\lambda(\theta) + p^u(\theta)n^u(\theta) = 0. \tag{72}$$

Differentiating (72) with respect to θ yields

$$\begin{aligned}
&[\frac{dn^u(\theta)}{d\theta} - \frac{dn^s(\theta)}{d\theta}][\lambda(\theta)]^2 + 2[n^u(\theta) - n^s(\theta)]\lambda(\theta)\frac{d\lambda(\theta)}{d\theta} \\
&+ \{-\frac{dp^u(\theta)}{d\theta}n^u(\theta) - [1+p^u(\theta)]\frac{dn^u(\theta)}{d\theta} + \frac{dp^s(\theta)}{d\theta}n^s(\theta) + p^s(\theta)\frac{dn^s(\theta)}{d\theta}\} \\
&+ \{-[1+p^u(\theta)]n^u(\theta) + p^s(\theta)n^s(\theta)\}\frac{d\lambda(\theta)}{d\theta} + \frac{dp^u(\theta)}{d\theta}n^u(\theta) + p^u(\theta)\frac{dn^u(\theta)}{d\theta} = 0.
\end{aligned}$$

Therefore, we obtain the following comparative static result:

$$\begin{aligned}
&\frac{d\lambda(\theta)}{d\theta} = \\
&\quad \frac{-[1-\lambda(\theta)][\lambda(\theta) - p^u(\theta)]\frac{dn^u(\theta)}{d\theta} + [1-\lambda(\theta)]n^u(\theta)\frac{dp^u(\theta)}{d\theta} + [p^s(\theta) - \lambda(\theta)]\lambda(\theta)\frac{dn^s(\theta)}{d\theta} + n^s(\theta)\lambda(\theta)\frac{dp^s(\theta)}{d\theta}}{[\Phi(\theta)]^{\frac{1}{2}}}, \\
&\quad > 0,
\end{aligned} \tag{73}$$

where $\lambda(\theta) \equiv \frac{[1+p^u(\theta)]n^u(\theta) - p^s(\theta)n^s(\theta) - [\Phi(\theta)]^{\frac{1}{2}}}{2[n^u(\theta) - n^s(\theta)]}$ and $\Phi(\theta) \equiv \{[1+p^u(\theta)]n^u(\theta) - p^s(\theta)n^s(\theta)\}^2 - 4p^u(\theta)[n^u(\theta) - n^s(\theta)]n^u(\theta)$. □

An increase in the ratio of child education expenditure has two opposite effects on the probability of skilled workers (i.e., $\frac{dp^s(\theta)}{d\theta} = \frac{\partial p[e^s(\theta),\theta\tau]}{\partial e^s(\theta)} \cdot \frac{de^s(\theta)}{d\theta} + \frac{\partial p[e^s(\theta),\theta\tau]}{\partial(\theta\tau)} \cdot \frac{d(\theta\tau)}{d\theta}$). One is the negative effect on investments in education through an increase in the child mortality rate (i.e., $\frac{de^s(\theta)}{d\theta} < 0$), while the other is the positive effect on investments in education through school education (i.e., $\frac{d(\theta\tau)}{d\theta} > 0$). Assumption $dp^s(\theta) > 0$ ensures that the positive effect of an increase in the ratio of child education expenditure dominates the effect of a decrease in investments in education through the child mortality rate increase. From (73), all four terms in the numerator take positive values. Since these four terms positively influence the proportion of skilled workers and average income in the economy, the child education policy is the most promising among the three policies.

5. Concluding remarks

We considered a three-period overlapping generations model to analyze how child mortality affects fertility, child labor, and investments in education in an economy where skilled and unskilled workers coexist. In the proposed model, an improvement in child mortality has different effects on the different worker types. A ban on child labor and an improvement in child mortality can decrease the proportion of skilled workers and the average income in the economy. However, a child education policy, which supports both skilled and unskilled workers' investments in the education of their children by building schools, can increase the proportion of skilled workers and the average income in the economy.

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