Discussion Papers In Economics And Business

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Discussion Paper 23-12

December 2023

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Optimal Subsidy for Investment in Extraction Technology in a Small Open Economy with a Non-renewable Natural Resource*

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Abstract

We develop a small open economy model with non-renewable natural resource, where firms can improve the extraction technology by investment. We analytically derive the equilibrium investment in extraction technology and then derive the equilibrium duration of non-renewable resource extraction. We show that a subsidy for investment in extraction technology promotes the investment and extends the duration. Moreover, we assume that the subsidy is financed by the income tax, and examine the effect of the investment subsidy. As a result, the higher the share of natural resource earnings in total output, the weaker the positive effect of subsidy on the duration is. The optimal subsidy which maximizes the profit from natural resource sales is lower as the ratio of resource sales to non-resource output is higher.

Keywords: non-renewable natural resource, small open economy, optimal subsidy
JEL classification: H21, O40, Q32

* I would like to extend my sincere thanks to Koichi Futagami, Tatsuro Iwaisako, and Kazuhiro Yamamoto for their valuable comments and recommendations. All remaining errors are of course my own.
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1. Introduction

There is no consensus in economic literature about the optimal rate of depletion of non-renewable natural resource, and the importance of intertemporal conservation for sustainable development. Any conclusion depends on the aspects of the economic models under consideration. For example, in one setup Stiglitz (1976) concludes that monopoly owned resource extraction will exhaust in slower than optimal rate, while in a different setup Lewis et. al (1979) show that monopolist depletes natural resource faster than optimal. Although it is not considered in every study in this topic, the presence of the government is important in the analysis of optimal behavior in dynamic setting. Dasgupta and Heal (1979) presented a comprehensive study about the role of different taxes in the economies with exhaustible resources, since taxes generally are the essential element of the presence of the government in the analysis of non-renewable resources. The role of subsidies has also been subject of extensive studies in economies with natural resource stock. Bergstrom et.al (1981) study the effects of subsidies on monopolist suppliers of natural resource extraction. They conclude that, if the present value of subsidy rate is an increasing function of time, the monopolist will receive more subsidy the sooner the resource is extracted. The monopolist can alter the present value of total subsidies over the course of extraction by altering the time path of extraction. Groth and Schou (2007) use a similar economic model introduced in Stiglitz (1974) and conclude that, contrary to the predictions of standard endogenous growth theory neither a tax on interest income nor a subsidy to capital accumulation affect the long run growth rate when non-renewable resource is an input in the sector where growth is ultimately generated. Nakamoto and Futagami (2016) study investment subsidy in a dynamic model of small open economy with a renewable resource. They conclude that increasing subsidy has different outcomes, depending on whether this increase is temporary or permanent. That is, if the increase is temporary, in the long run the level of natural resource always decreases and if the increase is permanent, the effects can be positive.

Most of the literature studying different economic problems related to non-renewable natural resource focus on private financing of extraction. In Aliyev (2023) we have considered the role of government policy in financing the extraction of natural resource where income tax serves as the source of extraction investment. However, that model does not cover the situation when the private investment can also serve as the source to finance the extraction. Therefore, the research question of this paper is determining whether subsidization of extraction is effective for welfare maximization if along with natural resource export, the economy produces another good inputting labor and physical capital only. Stiglitz (1974) analyzed the optimal growth where the natural resource stock exists along with the stock of physical capital. However, that model allows the domestic consumption. More recently, Nakamoto and Futagami (2016) study the subsidy policy in dynamic setting of small open economy endowed with stock of natural resource along with physical capital. We chose a different setup to analyze the situation when the extracted natural resource is neither consumed, nor inputted to produce another output and instead, completely exported. This approach would better describe the situation when the domestic demand for the natural resource either does not exist or negligible. Our model has similar approach with Nakamoto and Futagami (2016) in that, natural resource is exchanged into foreign assets, however it differs in the following. First, the natural resource stock is non-renewable in our model and households do not derive any utility from holding or conserving the natural resource for the future. Another difference is how the government runs the budget. In Nakamoto and Futagami (2016) the government runs a balanced budget, where tax is imposed on natural resource sector only and the difference of collected taxes and provided subsidies are transferred to foreign assets constraint. Instead, the government finances subsidies by taxing the whole output of the economy running an intertemporal budget eliminating transfers in our model. The role of the government in encouraging efficient extraction of natural resource is one of important implications of our model. One such model was proposed in Nystad (1985) where the author considers a petroleum taxation system encouraging natural resource extraction while securing the economic rent. His model is different from ours in that, the tax policy determines how much of natural resource will be recoverable, hence the stock size is not fixed. Although the stock size is fixed in our model, the total amount obtained as extraction differs depending on the path of investment. This is a desirable characteristics of extraction technology for our model. However, in contrast with multifactor extraction technologies in Nystad (1985), we prefer simple function which depends only on investment. Our key finding is how the changes in subsidy rate affects the duration of extraction and particularly, how efficient it is depending on the share of resource export in total output. Another important finding from policy optimization is the relation between the share of natural resource export and subsidy rate.

The rest of the paper is organized as the following. Section 2 describes the basic model setup. The optimal path of variables in dynamic model is derived in section 3 where we find out that the extraction and consumption become constant over time, although the resource stock exhausts in finite time. Section 4 shows the policy optimization behavior of the government in basic model determining the welfare maximizing rate of subsidy. Concluding remarks are given at the end.

2. Basic model

We consider a small open economy setting where the population is constant. The preference of households is expressed as the following:

$$u[0] = \int_0^\infty u(c_t)e^{-\rho t}dt\,, \tag{1}$$

where ρ denotes the subjective discount rate. In such generalization the consumption, c_t is seen in broader terms covering the expenses of every aspect of life necessary for maintaining the desired characteristics of the labor force including education, healthcare, settlement and so on. We assume the production function is given by $F(K,L)$, where K and L denote capital stock and labor respectively. The production function has constant returns to scale and thus the per capita output is given by $\frac{F(K,L)}{L} = F(\frac{K}{L},1) \equiv f(k)$, where $k \equiv \frac{K}{L}$. The world interest rate r is assumed to be constant. Then, the profit maximization of firms ensures that $r = f'(k)$, making the domestic capital stock constant in value. Hence, the wage rate becomes constant too, $w = f(k) - kf'(k)$. Households have option of consuming or investing into natural resource extraction the goods produced by inputting the labor and capital. The extracted natural resource X_t is exchanged to foreign assets b_t in the world market. Then, the accumulation of foreign assets in the economy can be expressed as the following:

$$\dot{b}_t = rb_t + (1-\tau^y)[f(k) + pX_t] - c_t - (1-\tau^a)a_t, \tag{2}$$

where a_t denotes investment into natural resource extraction and p denotes the relative price of exported good, measured by the price of non-resource output, $f(k)$. p is exogenous since this is a small open economy. τ^y and τ^a represent the income tax and investment subsidy rates respectively.

The natural resource is given as the finite stock S_t, which decreases as extracted. We assume that the economy is insignificant supplier of natural resource and cannot affect the equilibrium prices of the world market. Depending on the efficiency of technology the natural resource decreases more than obtained extraction, as given by the following:

$$\dot{S}_t = -\Gamma(a_t)X_t. \tag{3}$$

The extraction technology is $\Gamma(a_t) > 1$ in above natural resource constraint describing unrecoverable loss separate from extraction. The technology improves its efficiency as investment a_t increase. It is twice differentiable and has the following characteristics:

$$\Gamma'(a_t) < 0, \qquad \Gamma''(a_t) > 0.$$

The extraction technology we use is in line with the approach to petroleum extraction technology given in Nystad (1985) where the author points that, despite a number of factors in practice, the more inputted, the higher is the amount of extractable resource.

Tax and subsidy rates are declared beforehand and remain unchanged during the period of natural resource extraction. The government keeps intertemporal budget to support extraction, where the discounted sum of subsidies equals the discounted sum of income tax as given in the following constraint:

$$\int_0^T e^{-rt}\ \tau^y[f(k) + pX_t]dt \ = \int_0^T e^{-rt}\,\tau^a\ a_t\ dt\,. \tag{4}$$

The representative household maximizes its lifetime utility subject to the constraint of the non-renewable natural resource and the budget constraint by choosing a_t, c_t and X_t.

3. The analysis

Rearranging the budget constraint we can separate the household's maximization problem into 2 steps. The first step is maximization of utility from consumption with respect to foreign assets constraint. However, foreign assets constraint is identified as the source of investment into extraction to obtain revenue from natural resource sales in this model. Hence, there is a step 2 below the step 1, where households maximize profit from resource sales with respect to natural resource constraint, which in turn is a constituent part of foreign assets constraint at step 1 as maximized natural resource earnings. Formally, Step 1 can be stated as the following:

$$\max_{c_t} \int_0^\infty u(c_t)e^{-\rho t}dt \tag{1}$$

subject to:

$$\dot{b}_t = rb_t + (1-\tau^y)[f(k) + pX_t] - c_t - (1-\tau^a)a_t$$

$$\Rightarrow \int_0^\infty c_t e^{-rt}\,dt = b_0 + \int_0^\infty (1-\tau^y)f(k)\,e^{-rt}dt + Z,$$

$$Z \equiv \int_0^T [(1-\tau^y)pX_t - (1-\tau^a)a_t]e^{-rt}dt\,, \tag{2'}$$

where Z defined in (2') denotes the sum of natural resource earnings. Step 2 of this problem is given as the profit maximization from natural resource as the following:

$$\max_{a_t} \int_0^T [(1-\tau^y)pX_t - (1-\tau^a)a_t]e^{-rt}dt$$

subject to:

$$\dot{S}_t = -\Gamma(a_t)X_t\,, \qquad S_T = 0. \tag{3}$$

Households solve Step 2 of this problem as the following:

Step 2.

We let H_t denote the Hamiltonian, where μ_t is the costate variable for the natural resource stock which is given by

$$H_t = [(1-\tau^y)pX_t - (1-\tau^a)a_t]e^{-rt} - \Gamma(a_t)X_t\mu_t.$$

The necessary conditions of this Hamiltonian are as follows.

$$\frac{\partial H_t}{\partial X_t} = (1-\tau^y)pe^{-rt} - \Gamma(a_t)\mu_t,$$

which shows that Hamiltonian is linear in X_t. Then:

$$X_t = \begin{cases} \bar{X} \\ [0,\bar{X}] \quad if \quad (1-\tau^y)pe^{-rt} \overset{>}{\underset{<}{=}} \Gamma(a_t)\mu_t. \\ 0 \end{cases} \tag{5}$$

Rationality requires the natural resource be extracted only on condition that revenue from extraction sales outweigh the cost of extraction. The proof of this condition is provided after the statement of all necessary conditions. Thus, the optimal value is the highest possible, denoted by $\bar{X}$. Since the natural resource stock is finite, the value of $\bar{X}$ is finite and it is restricted by exogenous factors which we do not attempt to explain in this analysis.

$$\frac{\partial H_t}{\partial a_t} = 0\text{:}\;\; (1-\tau^a)e^{-rt} - \Gamma'(a_t)X_t\mu_t = 0$$

$$\Rightarrow \Gamma'(a_t)\mu_t = -\frac{(1-\tau^a)e^{-rt}}{X_t}. \tag{6}$$

$$\frac{\partial H_t}{\partial S_t} = -\dot{\mu}_t\text{:}\;\; \dot{\mu}_t = 0, \tag{7}$$

which means it is constant and we drop the time script from μ hereafter. Since μ represent the shadow value of the natural resource, its constant value means that over the course of extraction the scarcity of the natural resource stock does not grow for this economy. The time path of natural resource stock does not cause any concerns and it can be completely exhausted. This situation is explained as isoperimetric problem[1].

$$H_T = 0: \quad [(1-\tau^y)pX_T - (1-\tau^a)a_T]e^{-rT} = \Gamma(a_T)X_T\mu_T. \tag{8}$$

Since the optimal extraction is constant over time, rewriting the natural resource constraint given by (3) we obtain the value of total extraction loss expressed by optimal extraction and initial resource stock:

$$\int_0^T \Gamma(a_t)dt = \frac{S_0}{\bar{X}}. \tag{9}$$

(9) shows that decreasing the integrated value of losses in natural resource extraction is essential to extend the duration of extraction until depletion. We define the function of extraction technology as

$$\Gamma(a_t) \equiv G a_t^{-\gamma}; \; G > 0, \gamma \in (0,1). \tag{10}$$

Plugging defined function of extraction technology (10) into the necessary condition for a_t given by (6) at time T as well as considering (5) and (7) gives us

$$\gamma G a_T^{-\gamma-1}\mu = \gamma \frac{\Gamma(a_T)}{a_T}\mu = \frac{1-\tau^a}{\bar{X}}e^{-rT}, \tag{11}$$

which then substituted into $H_T = 0$ given by (8) and rearranged to obtain

$$\Gamma(a_T)\mu = \frac{(1-\tau^y)p\bar{X}e^{-rT}}{1+\gamma}. \tag{12}$$

We can rearrange (6) and (11) respectively as

$$\gamma G a_t^{-\gamma-1} = \frac{1-\tau^a}{\mu\bar{X}}e^{-rt},$$

$$\gamma G a_T^{-\gamma-1} = \frac{1-\tau^a}{\mu\bar{X}}e^{-rT},$$

and divide both sides of former by those of latter to obtain:

$$\frac{a_t}{a_T} = e^{\frac{r}{1+\gamma}(t-T)}, \qquad \therefore \frac{\Gamma(a_t)}{\Gamma(a_T)} = e^{-\frac{\gamma}{1+\gamma}r(t-T)}. \tag{13}$$

We can write the shadow value of extraction losses $\Gamma(a_t)\mu$ as the following:

$$\Gamma(a_t)\mu = \frac{\Gamma(a_t)}{\Gamma(a_T)}\Gamma(a_T)\mu\,. \tag{14}$$

Substituting (13) and (12) into the first and second term in the right-hand side of (14) respectively gives us the following value of extraction technology after rearranging:

$$\Gamma(a_t)\mu = \frac{1}{1+\gamma}e^{-\frac{1}{1+\gamma}r(t-T)} \times (1-\tau^y)pe^{-rt}.$$

The value of the first term in the right-hand side is less than one. Hence, from (5), we prove that $(1-\tau^y)pe^{-rt} > \Gamma(a_t)\mu$, that is $X_t = \bar{X}$ for $\forall t \leq T$. QED

[1] See Alpha C. Chiang Elements of Dynamic Optimization. 1992. Chapter 10.1 Constraints involving control variables, p280

By substituting the shadow value of natural resource from (12) into the necessary condition determining the optimal investment given by (6) we obtain its optimal path expressed by extraction technology, tax and subsidy rates, and discounted natural resource earnings as the following:

$$a_t^* = \frac{\gamma}{1+\gamma}\frac{\Gamma(a_t^*)}{\Gamma(a_T^*)}\frac{1-\tau^y}{1-\tau^a}p\bar{X}e^{-r(T-t)}, \tag{15}$$

and when $t = T$:

$$a_T^* = \frac{\gamma}{1+\gamma}\frac{1-\tau^y}{1-\tau^a}p\bar{X}. \tag{15'}$$

Rewriting (15) with (10) substitutes extraction technology with the parameter of extraction technology in the expression as the following:

$$a_t^* = \frac{\gamma}{1+\gamma}\frac{1-\tau^y}{1-\tau^a}p\bar{X}e^{-\frac{r}{1+\gamma}(T-t)}. \tag{16}$$

Once we have got the optimal path of investment a_t^*, we can implicitly obtain the duration of extraction from the equation showing total loss of natural resource resulting from applied extraction technology given by (9) as the following:

$$\frac{S_0}{\bar{X}} = \Omega\,\frac{1+\gamma}{\gamma r}\left(1-e^{-\frac{\gamma}{1+\gamma}rT}\right),\quad \Omega \equiv G\left[\frac{\gamma}{1+\gamma}\frac{1-\tau^y}{1-\tau^a}p\bar{X}\right]^{-\gamma}. \tag{17}$$

By this point, we have obtained the optimal values for natural resource extraction, investment rate over time and the shadow value of the resource as $\bar{X}, a_t^*, \mu^*$ and implicitly identified the duration of extraction. By rewriting the objective functional of Step 2 with the obtained optimal values gives:

$$\Rightarrow Z_{max} = (1-\tau^y)p\bar{X}\frac{1}{r}\left(1-e^{-\frac{r}{1+\gamma}T}\right). \tag{18}$$

Obtaining (18) concludes Step 2 of the households' maximization problem and enables us to proceed to Step1.

Step1.

The problem can be restated as the following:

$$\max_{c_t}\int_0^\infty u(c_t)e^{-\rho t}dt$$

subject to:

$$\dot{b}_t = rb_t + (1-\tau^y)[f(k)+p\bar{X}] - c_t - (1-\tau^a)a_t^* \qquad \text{for } t \in [0,T],$$

$$\dot{b}_t = rb_t + f(k) - c_t \qquad \text{for } t \in (T,\infty).$$

Assigning the costate variable λ_t for the stock of foreign assets, the Hamiltonian function is given as

$$H_t = u(c_t)e^{-\rho t} + rb_t\lambda_t + (1-\tau^y)[f(k)+p\bar{X}]\lambda_t - c_t\lambda_t - (1-\tau^a)a_t^*\lambda_t,$$

and the following necessary conditions obtained:

$$\frac{\partial H_t}{\partial c_t} = 0:\ u(c_t)e^{-\rho t} = \lambda_t, \tag{19}$$

$$-\frac{\partial H_t}{\partial b_t} = \dot{\lambda}_t:\ \frac{\dot{\lambda}_t}{\lambda_t} = -r. \tag{20}$$

Taking the natural logarithm of (19) yields

$$-\rho + \frac{u''(c_t)}{u'(c_t)}\dot{c}_t = \frac{\dot{\lambda}_t}{\lambda_t},$$

and substituting (20) into it gives:

$$\frac{u''(c_t)}{u'(c_t)}\dot{c}_t = r - \rho. \tag{21}$$

We assume that $r = \rho$ in small open economy. Hence, the consumption becomes constant over time and for simplicity of notation we denote it $\bar{c}$ and drop the time index. After determining the path of consumption, we determine its value from foreign assets constraint with optimal values as the following:

$$\dot{b}_t - rb_t = f(k) - \bar{c} + (1-\tau^y)p\bar{X} - (1-\tau^a)a_t^*.$$

Multiply both sides by e^{-rt} and get

$$\frac{\partial b_t e^{-rt}}{\partial t} = (f(k) - \bar{c})e^{-rt} + (1-\tau^y)p\bar{X}e^{-rt} - (1-\tau^a)a_t^* e^{-rt}.$$

Integrating this gives us:

$$\int_0^\infty \frac{\partial b_t e^{-rt}}{\partial t} dt = (f(k) - \bar{c})\int_0^\infty e^{-rt}\, dt + Z_{max}.$$

Substituting (18) into the last term and solving gives us:

$$\bar{c} = rb_0 + f(k) + (1-\tau^y)p\bar{X}\left(1 - e^{-\frac{r}{1+\gamma}T}\right). \tag{22}$$

(22) determines the level of consumption as the sum of interest on initial foreign assets, non-resource output and discounted after tax resource earnings over the duration of extraction in small open economy producing non-resource output inputting labor and capital, as well as selling natural resource extraction at constant relative price. Households use the above level of constant consumption to maximize lifetime utility.

4. Policy optimality

The government determines the subsidy rate. Subsidization increases the investment into extraction in this economy. We can show it by differentiating (16) with respect to subsidy as the following:

$$\frac{\partial a_t^*}{\partial \tau^a} = \frac{\gamma}{1+\gamma}\frac{1-\tau^y}{(1-\tau^a)^2}p\bar{X}e^{-\frac{r}{1+\gamma}(T-t)} > 0 \tag{23}$$

Proposition 1. Rise in subsidy rate extends the duration of extraction.

Proof: Consider the sum of extraction losses as the function of T as the following:

$$A(T) \equiv \int_0^T \Gamma(a_t)dt = \Omega\,\frac{1+\gamma}{\gamma r}\left(1 - e^{-\frac{\gamma}{1+\gamma}rT}\right).$$

$A(T)$ is an increasing function of T, with positive first order

$$\frac{\partial A(T)}{\partial T} = \Omega e^{-\frac{\gamma}{1+\gamma} rT} > 0,$$

whereas a_t^*, which in turn is the argument of decreasing integrand, also is increasing function of subsidy. Hence, increasing τ^a decreases the value of $A(T)$. Using (17) we can graphically show that increasing subsidy rate extends the duration of extraction. In Figure 1, the black circle corresponds to (17) where the left-hand side is given as horizontal line and right hand side is the $A(T)$ curve. Increasing subsidy rate shifts the curve downwards, thus shifting the intersection point rightward. Hence, increasing subsidy rate increases the duration of extraction of natural resource. QED

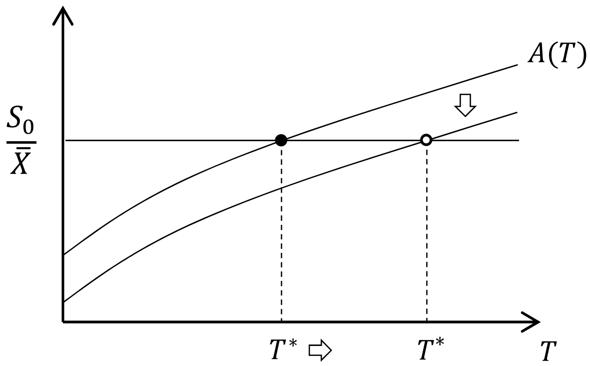

Figure 1. Change of extraction duration over change of subsidy rate.

Since the extraction rate becomes constant in this model, the higher the value of T, the larger is the contribution of natural resource stock into the foreign assets which is achieved through decreasing the technological loss in extraction. Hence, the government can control how efficiently the stock of natural resource depleted.

For the following analysis, assume that $r = 0$. Then, solving Step 2 of households' problem considering this assumption makes a_t constant over time and consequently solving the resource constraint (3) gives us:

$$T^* = \frac{S_0}{\Gamma(a)\bar{X}} = \frac{S_0}{G\bar{X}} a^\gamma. \tag{24}$$

Then we can state the objective functional of Step 2 as the following:

$$Z = \frac{S_0}{\bar{X}} \frac{a^\gamma}{G} [(1-\tau^y)p\bar{X} - (1-\tau^a)a].$$

Differentiating with respect to τ^a and rearranging for a gives us:

$$a^* = \frac{\gamma}{1+\gamma} \frac{1-\tau^y}{1-\tau^a} p\bar{X}. \tag{25}$$

Assuming $r = 0$ we can write the intertemporal budget constraint (4) as:

$$\tau^y = \frac{\tau^a a^*}{[f(k) + p\bar{X}]},$$

then substitute (25), rearrange and get:

$$\tau^y = \underbrace{\underbrace{\frac{\gamma}{1+\gamma}\frac{p\bar{X}}{[f(k)+p\bar{X}]}}_{\equiv\Phi}\frac{\tau^a}{1-\tau^a}}_{\equiv\Psi}(1-\tau^y). \tag{26}$$

Using the simplified notations Φ and Ψ and rearranging (26) gives us:

$$\tau^y = \frac{\Psi}{1+\Psi} = \Phi\frac{\tau^a}{1-(1-\Phi)\tau^a}, \tag{27}$$

where $\Phi < 1$.

Proposition 2. Even if the investment subsidy is financed by the income tax, the investment subsidy can extend the duration of non-renewable resource extraction. However, the higher the share of natural resource earnings in total output, the weaker the positive effect of subsidy on the duration is.

Proof: We can show it by deriving the optimal policy of the government. Substituting (27) into (25) gives us:

$$a^* = \frac{\gamma}{1+\gamma}p\bar{X}\frac{1}{1-(1-\Phi)\tau^a}. \tag{28}$$

Taking the natural logarithm of the above equation and (22), then differentiating both with respect to τ^a and rearranging gives us:

$$\frac{1}{a^*}\frac{\partial a^*}{\partial \tau^a} = \frac{1-\Phi}{1-(1-\Phi)\tau^a},$$

$$\frac{1}{T^*}\frac{\partial T^*}{\partial \tau^a} = \gamma\frac{1}{a^*}\frac{\partial a^*}{\partial \tau^a}.$$

$$\Rightarrow \frac{1}{T^*}\frac{\partial T^*}{\partial \tau^a} = \frac{1-\Phi}{1-(1-\Phi)\tau^a}\gamma \tag{29}$$

If the share of resource earnings in total output is high, then Φ is high and $\frac{1}{a^*}\frac{\partial a^*}{\partial \tau^a}$ is low, so is $\frac{1}{T^*}\frac{\partial T^*}{\partial \tau^a}$ in order (26) to hold. The left-hand side of (29) represents the percentage change of T^*, thus we conclude regarding the relationship between extraction duration and share of natural resource earnings of total output. QED

(27) defines tax rate as the function of subsidy rate. Differentiating it with respect to subsidy rate gives us:

$$\frac{\partial \tau^y}{\partial \tau^a} = \frac{\Phi}{[1-(1-\Phi)\tau^a]^2}. \tag{30}$$

Considering (27) the objective functional of Step 2 can be stated as the following function of subsidy rate:

$$Z = \frac{S_0}{G\bar{X}}a(\tau^a)^\gamma\big[\big(1-\tau^y(\tau^a)\big)p\bar{X} - (1-\tau^a)a(\tau^a)\big].$$

Since the subsidy rate τ^a enters into the households' problem as an exogenous variable and the above objective function states endogenous variables as the function of it, by taking partial derivative we can find the effect of change in subsidy rate at optimized value applying the envelope theorem:

$$\frac{\partial Z}{\partial \tau^a} = \frac{S_0}{G\bar{X}}\underbrace{\big[\gamma a(\tau^a)^{\gamma-1}[(1-\tau^y)p\bar{X} - (1-\tau^a)a(\tau^a)] + a(\tau^a)^\gamma[-(1-\tau^a)]\big]}_{=0}\frac{\partial a(\tau^a)}{\partial \tau^a}$$

$$+\frac{S_0}{G\bar{X}}a(\tau^a)^\gamma\left[-\frac{\partial \tau^y}{\partial \tau^a}p\bar{X} + a(\tau^a)\right] = \frac{S_0}{G\bar{X}}a(\tau^a)^\gamma\left[-\frac{\partial \tau^y}{\partial \tau^a}p\bar{X} + a(\tau^a)\right].$$

Substituting (28) and (30) into above gives us:

$$\frac{\partial Z}{\partial \tau^a} = \frac{S_0}{G\bar{X}} a(\tau^a)^{\gamma} \frac{p\bar{X}}{[1-(1-\Phi)\tau^a]^2} \left[-\Phi + \frac{\gamma}{1+\gamma} [1-(1-\Phi)\tau^a] \right]. \tag{31}$$

From (31) we can show that

$$\frac{\partial Z}{\partial \tau^a} = 0:\ \tau^a = \frac{(1+\gamma)f(k)}{(1+\gamma)f(k) + p\bar{X}} < 1. \tag{32}$$

Thus, (32) shows the optimal rate of subsidy maximizing the resource earnings. Higher the resource revenue, lower is the optimal subsidy. We can summarize the results as the following proposition.

Proposition 3. If the subsidy for investment in extraction technology is financed by the income tax, the welfare-maximizing subsidy rate is given by (32). A rise in the ratio of the resource revenue to the production revenue reduces the subsidy rate.

5. Conclusion

This paper analyzes the welfare in a dynamic model of small open economy producing non-resource output and extracting natural resource from a non-renewable stock. The extracted natural resource itself is not a consumption good and seen as the means to finance the consumption. Consequently, the intertemporal resource conservation is neglected. Since the extraction technology is not affected by the remaining resource stock, the scarcity of natural resource does not change and the resource exhausts. This makes the level of extraction constant over time and since this is a small open economy without population growth, the consumption becomes constant too. The level of consumption is determined as the sum of interest on initial foreign assets, non-resource output and discounted after tax resource earnings over the duration of extraction.

The analyses of the government policy showed that the rise of subsidy increases the investment and extends the duration of natural resource extraction. In other words, subsidy policy affects the efficiency of natural resource extraction. The policy optimization behavior of the government also reveals that, even if the investment subsidy is financed by the income tax the investment subsidy can extend the duration of non-renewable resource extraction, although the positive effect is weaker as the resource share of total output is higher. Moreover, a rise in the ratio of the resource revenue to the production revenue reduces the optimal subsidy rate.

This model can be applied to optimize the dynamic processes in natural resource exporting economies, especially in those extracting with domestic expenses and where the technological loss of natural resources during the extraction is not recoverable, like crude oil or natural gas. The model can be further extended to incorporate the population growth, actual price of resource and aggregated non-resource output, and can be refined considering the actual world interest rate to describe the given economy more accurate.

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