A Dynamic International Trade Model with Endogenous Fertility

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Abstract

This paper examines a two-country dynamic general equilibrium model with endogenous fertility. We show that the introduction of child rearing behavior brings about new properties in long run dynamics. After analysis about the existence, uniqueness, and local stability of long run equilibrium, we investigate the international trade pattern and comparative statics in order to see the differences with the standard dynamic international trade model.

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1. Introduction

This paper presents a dynamic two-country model of international trade with endogenous fertility. Especially, we analyze the properties of dynamic system, that is the existence, the uniqueness, and the stability, and international trade pattern among between two large countries.

The two-sector international trade model has a long tradition. "Heckscher-Ohlin-Samuelson (H-O-S) model" is the static international trade model in the neoclassical framework. The framework is characterized by constant returns to scale and perfect competition. The Rybczynski therem and the Stolper-Samuelson theorem are the important properties of the framework, which are applied to a variety of economic theories. This two-sector framework was introduced into the closed growth theory: the neoclassical growth model by Uzawa (1961,1963) and the optimal growth model by Uzawa (1964), Srinivasan (1964). Moreover, Lucas (1988), Mino (1996), and Bond, Wang, and Yip (1996) extended two-sector growth model to the endogenous growth theory.

The dynamic H-O-S model originates in Oniki and Uzawa (1965). Oniki and Uzawa (1965) assumed two country, two goods, two factors, and the neoclassical growth framework, while Stiglitz (1970), Baxter (1992), and Chen (1992) assumed the optimal growth framework. These optimal growth models, however, lead to unrealistic results as follows;

- There is a crucial price with long-run diversified production, in other word, this economy is likely to specialize completely in one good in the steady state.
- The long-run equilibria with diversified production is not unique.
- There is no transition dynamics for the long-run equilibria with diversified production.

These properties contrast the static H-O-S model, in which the slight difference in the conditions like technologies, preferences, or economic policies among countries does not cause specialization. Baxter (1992, p.714) argued for these properties that "The predictions of this model regardeing patterns of specialization and trade are markedly different from those of H-O-S model but are very much in the spirit of the traditional Ricardian model. That is, the equilibrium pattern of specialization and trade depends on comparative advantage and there is a corresponding presumption of specialization." The determinant of long-run comparative advantage in the dynamic H-O-S model mentioned above is the countries' exogenous time preference rate and population growth rate. The incomplete specialization holds only if time preference rates and fertility rates, which are exogenous and constant, are strictly identical between the economy and the rest of world. This property in dynamic H-O-S model is not apparently satisfactory. One of the purpose in this paper is to construct a dynamic international trade model in which leads more realistic results.

To achieve this purpose, we introduce "the endogenous fertility" into the dynamic international trade model. The analysis presented in this paper reveals the importance of two features. If agents choose fertility rate endogenously, the steady state with incomplete specialization exhibits uniqueness. After analysis about the local stability of long run equilibrium, we can investigate the international trade pattern and comparative statics in order to see the differences with the standard dynamic international trade model. The results obtained by Golar and Lin (1997), Hu and Shimomura (2007), and Chen et al. (2008) is that a patient country will postpone its current consumption for capital accumulation. Then, a patient country acts just like a capital abundant country and exports the capital–intensive goods. On the other hand, in our study, a patient country will not necessarily be capital abundant country, and export labor–intensive goods under some conditions.

[Table 1 about here.]

The rest of the paper is organized as follows: Section 2 presents the model and derives the optimality conditions. In section 3, we characterize the steady state and equilibrium dynamics in autarkic economy. In section 4, we characterize the symmetric steady state and equilibrium dynamics in open economy. In section 5, we derive the trade pattern properties in the case that both countries are not symmetric. Section 6 concludes the paper.

2. Model

This section specifies a continuous-time Ramsey model of a two-country open economy with perfective competitive world. The economy consists of many infinitely lived identical agents (households and firms). There are two sectors in this economy that produce a pure consumption good (good 1) and a pure investment good (good 2). We choose good 2 as the numeraire. We assume each country's initial number of people to N_0 in home country and N_0^* in foreign country. Moreover, in each country, population grows at an endogenous rate $n(t)$ in home country and $n^*(t)$ in foreign country. Following the tradition of the dynamic international trade theory, we assume that while the two goods are tradable, the production factors are not mobile internationally.

2.1. Firms

The output of good 1 and good 2 are generated by using capital and labor. Technology is specified by two production functions which are homogenous of degree one in both factors, and is stationary over time. The production function of each producer can be written in terms of outputs per worker as follows:

$$f^i(k_i, l_i), \quad i = 1, 2, \tag{1}$$

where k_i is the per worker capital-input in sector i, and $l_i \in [0,1]$ is the per worker labor supply employed in sector i at time $t \in [0, \infty)$, $i = 1, 2$, respectively. We assume there are no factor intensity reversals among both production sector.

The per capita GDP function can be defined as:[1]

$$\begin{aligned} g(k, l, p) \equiv \max_{\{y_i\},\{k_i\},\{l_i\}} \{ & py_1 + y_2 : \; y_i \leq f^i(k_i, l_i), \; k_1 + k_2 \leq k, \; l_1 + l_2 \leq l, \\ & y_i \geq 0, \; k_i \geq 0, \; l_i \geq 0, \quad i = 1, 2 \}, \end{aligned} \tag{P}$$

where p is the price of consumption goods in terms of investment goods. In the case of incomplete specialization, $g(k, l, p) = r(p)k + w(p)l$, where $r(p)$ and $w(p)$ are factor prices and expressed with consumption goods price. Moreover, by using the envelope theorem, we have the following properties for the GDP function:

$$\begin{aligned} y_1(k, l, p) &= r'(p)k + w'(p)l, \\ y_2(k, l, p) &= [r(p) - pr'(p)]k + [w(p) - pw'(p)]l = \left[1 - \frac{pr'(p)}{r(p)}\right] r(p)k + \left[1 - \frac{pw'(p)}{w(p)}\right] w(p)l. \end{aligned}$$

[1]Needless to say, the aggregated GDP function is defined as $G(K, L, p) \equiv N \cdot g(k, l, p)$, where $K \equiv Nk$ and $L \equiv Nl$.

Finally, in this economy, we have Stolper-Samuelson theorem as follows:

Lemma 1 (Stolper–Samuelson)**.** *Suppose that incomplete specialization is satisfied.*

1. *If* $k_2/l_2 < k_1/l_1$*, then* $\dfrac{pr'(p)}{r(p)} > 1, \quad \dfrac{pw'(p)}{w(p)} < 0,,$

2. *If* $k_2/l_2 > k_1/l_1$*, then* $\dfrac{pr'(p)}{r(p)} < 0, \quad \dfrac{pw'(p)}{w(p)} > 1.$

2.2. Households

In this economy, the population size grows an endogenously determined rate of expansion. This is obtained by inserting the fertility rate into the utility function of the representative agent and allowing it to be endogenously chosen; see for the same economic structure, Parivos et al. (1993), for example.

The representative agent makes consumption, fertility, and savings decisions in order to maximize the following Millian social welfare function[2]

$$\begin{aligned} U &\equiv \int_0^\infty \log De^{-\rho t}dt, \quad D \equiv c^\alpha n^{1-\alpha},\ \alpha \in (0,1) \\ &= \int_0^\infty \left[\alpha \log c + (1-\alpha)\log n\right] e^{-\rho t}dt, \end{aligned} \tag{2}$$

where c is consumption and ρ is the subjective discount rate. That is, this type of social welfare function is baced on average utilitarism, which implies maximisation of the average or utility per capita.

Her/His flow budget constraint and time allocation constraint is as follows:

$$\dot{k} = rk + wl - (\delta + n)k - pc, \tag{3}$$

$$l + zn = 1, \quad (n, l) \in (0,1) \times (0,1), \tag{4}$$

where δ denotes the rate of capital depreciation, and z is time spent for child–rearing, or child–rearing cost.

[2]For Millian social welfare function, see Razin and Sadka (1995).

The representative agent solves the following household's maximizing problem:

$$\begin{cases} \max\limits_{\{c\},\{n\}} & \int_0^\infty \left[\alpha \log c + (1-\alpha) \log n\right] e^{-\rho t} dt \\ \text{s.t.} & \dot{k} = (r-\delta) k + w - \left[pc + (zw+k) n\right] \\ & k(0) = k_0 \equiv K_0/N_0 : \text{ given} \end{cases} \tag{H}$$

The problem has both intratemporal and intertemporal components. First, intratemporal optimization behavior is summarized by the expenditure functions: $E \equiv \min\limits_{\{c\},\{n\}} \left\{pc + (zw+k) n : D \leq c^{\alpha} n^{(1-\alpha)}\right\}$. Then, we can obtain consumption expenditure $c = \alpha E/p$ and fertility cost $n = (1-\alpha) E/(zw+k)$. Second, the intertemporal optimization problem is as follows:

$$\begin{cases} \max\limits_{\{E\}} & \int_0^\infty \left[\log E - \alpha \log p - (1-\alpha) \log (zw+k) + \log \alpha^{\alpha} (1-\alpha)^{(1-\alpha)}\right] e^{-\rho t} dt \\ \text{s.t.} & \dot{k} = (r-\delta) k + w - E \\ & k(0) = k_0 \equiv K_0/N_0 : \text{ given.} \end{cases} \tag{H$'$}$$

The first–order conditions for intertemporal optimization problem are as follows:

$$\dot{E} = \left[\left\{r - \left[\delta + (1-\alpha) \frac{E}{zw+k}\right]\right\} - \rho\right] E, \tag{5a}$$

$$\dot{k} = (r-\delta) k + w - E \tag{5b}$$

$$\lim_{t \to \infty} k \lambda e^{-\rho t} = 0, \tag{5c}$$

where λ is the costate variables of k. The flow budget constraint (3) and the time allocation constraint (4) must also be satisfied at the optimum.

3. Autarkic Equilibrium

In this section, we consider about autarkic equilibrium. To close the model, we are left to describe the consumption goods market clearing condition. Given the autarkic price of consumption goods, we can express per capita consumption goods production as $g_p(k, l, p)$. Hence the excess demand function for consumption goods is

$$Z \equiv c - y_1(k, l, p) = c - g_p(k, l, p).$$

Moreover, in autarkic equilibrium, this economy has to produce both goods necessarily; That is, $g(k,l,p) = r(p)k + w(p)l$. Therefore, we can derive the excess demand functions in both markets as follows:

$$Z = \frac{\alpha E}{p} - \left[r'(p)k + w'(p)\left\{ 1 - z(1-\alpha)\frac{E}{zw(p)+k} \right\} \right]. \tag{6}$$

We obtain the effect for Z:

$$\begin{aligned}
&\frac{\partial Z}{\partial p} = -\Sigma + \Phi, &(7)\\
&\Sigma \equiv \left[\frac{\alpha E}{p^2} + \left(r''k + w''l \right) + \frac{z^2 w'^2 n^2}{(1-\alpha)E} \right] > 0,\\
&\Phi \equiv \left[-\phi_k \frac{\partial k}{\partial p} + \phi_E \frac{\partial E}{\partial p} \right], \quad \phi_k \equiv r' + \frac{zw'n^2}{(1-\alpha)E}, \quad \phi_E \equiv \frac{\alpha}{p} + \frac{zw'n}{E},
\end{aligned}$$

where $-\Sigma$ shows the sum of consumption and production substitution effect and Φ is that of income effects. From Eq.(3), we can obtain the Walras's law; pZ plus the excess demand for good 2 equal zero. Therefore, the market for good 1 is clearing, that is $Z = 0$, we can show the excess demand for good 2 is zero necessarily.

Combaining these market clearing conditions with the optimizing conditions for households and firms, we can obtain a closed-form dynamic system with respect to E, k, and p;

$$\dot{E} = \left[\left\{ r(p) - \left[\delta + (1-\alpha)\frac{E}{zw(p)+k} \right] \right\} - \rho \right] E, \tag{8}$$

$$\dot{k} = (r(p) - \delta)k + w(p) - E, \tag{9}$$

$$\frac{\alpha E}{p} - \left[r'(p)k + w'(p)\left\{ 1 - z(1-\alpha)\frac{E}{zw(p)+k} \right\} \right] = 0. \tag{10}$$

Suppose that $\bar{p}$ is steady state price. Then, the autarkic steady state values are defined as satisfying $\dot{E} = \dot{k} = 0$:

$$\rho = r(\bar{p}) - (\delta + \bar{n}), \tag{11a}$$

$$\bar{l} = 1 - z\bar{n}, \tag{11b}$$

$$\bar{c} = \alpha \frac{\bar{E}}{\bar{p}}, \tag{11c}$$

$$\bar{n} = (1 - \alpha) \frac{\bar{E}}{zw(\bar{p}) + \bar{k}}, \tag{11d}$$

$$\bar{E} = (r(\bar{p}) - \delta)\bar{k} + w(\bar{p}). \tag{11e}$$

From Eqs. (11a)–(11e), we can solve these system for $\bar{p}$:

$$\bar{n} = r(\bar{p}) - (\rho + \delta), \tag{12a}$$

$$\bar{l} = 1 - z[r(\bar{p}) - (\rho + \delta)], \tag{12b}$$

$$\bar{c} = \left(\frac{\alpha}{\bar{p}}\right) \frac{(z\rho - \bar{l})\bar{n}}{(1-\alpha)\rho - \alpha\bar{n}} w(\bar{p}) = \left(\frac{\alpha}{\bar{p}}\right) \frac{[z(r(\bar{p}) - \delta) - 1][r(\bar{p}) - (\rho + \delta)]}{(1-\alpha)\rho - \alpha[r(\bar{p}) - (\rho + \delta)]} w(\bar{p}), \tag{12c}$$

$$\bar{E} = \frac{(z\rho - \bar{l})\bar{n}}{(1-\alpha)\rho - \alpha\bar{n}} w(\bar{p}) = \frac{[z(r(\bar{p}) - \delta) - 1][r(\bar{p}) - (\rho + \delta)]}{(1-\alpha)\rho - \alpha[r(\bar{p}) - (\rho + \delta)]} w(\bar{p}), \tag{12d}$$

$$\bar{k} = \frac{\alpha - \bar{l}}{(1-\alpha)\rho - \alpha\bar{n}} w(\bar{p}) = \frac{\alpha - [1 - z\{r(\bar{p}) - (\rho + \delta)\}]}{(1-\alpha)\rho - \alpha[r(\bar{p}) - (\rho + \delta)]} w(\bar{p}). \tag{12e}$$

We make following assumption about steady state.

Assumption 1 (positive steady state). *Suppose $(p_{\min}, p_{\max})$ is the range of $\bar{p}$ for incomplete specialization. Then, $\exists \bar{p} \in (p_{\min}, p_{\max})$, $(\bar{n}, \bar{l}, \bar{c}, \bar{E}, \bar{k}) \gg 0$ under given $(\rho, \alpha, r(\bar{p}), w(\bar{p}), z)$.*

Remark 1. *Yip and Zhang (1997) attempts to compute child-rearing cost with the reference to the U.S. data because there are no empirical estimates available for it, and they get $z = 56.9$. Then, for example $\rho = 0.025$, $z\rho = 1.4225 > 1 \geq l$. Therefore, we consider the property $z\rho - \bar{l} > 0$ seems to be plausible intuitively.*

Thereafter, we concentrate the analysis in the case of $z\rho > \alpha > \bar{l} > 0$ and $(1-\alpha)\rho - \alpha\bar{n} > 0$ to satisfy Assumption 1.

In the next step, we can obtain the effect of the change in $\bar{p}$ in $\bar{n}$, $\bar{l}$, $\bar{E}$, and $\bar{k}$ at the steady state as follows:

$$\frac{\partial \bar{n}}{\partial \bar{p}} = r'(\bar{p}), \quad \mathrm{sgn}\left(\frac{\partial \bar{n}}{\partial \bar{p}}\right) = \mathrm{sgn}\left(r'(\bar{p})\right), \tag{13a}$$

$$\frac{\partial \bar{l}}{\partial \bar{p}} = -zr'(\bar{p}), \quad \mathrm{sgn}\left(\frac{\partial \bar{l}}{\partial \bar{p}}\right) = \mathrm{sgn}\left(w'(\bar{p})\right), \tag{13b}$$

$$\frac{\partial \bar{E}}{\partial \bar{p}} = \frac{\left(z\rho - \bar{l}\right)\bar{n}}{(1-\alpha)\rho - \alpha\bar{n}} w'(\bar{p}) + \frac{\left[(1-\alpha)\rho - \alpha\bar{n}\right]\left[z(\rho + \bar{n}) - \bar{l}\right] + \alpha\left(z\rho - \bar{l}\right)\bar{n}}{\left[(1-\alpha)\rho - \alpha\bar{n}\right]^2} w(\bar{p})\, r'(\bar{p}), \tag{13c}$$

$$\frac{\partial \bar{k}}{\partial \bar{p}} = \frac{\alpha - \bar{l}}{(1-\alpha)\rho - \alpha\bar{n}} w'(\bar{p}) + \frac{z\left[(1-\alpha)\rho - \alpha\bar{n}\right] + \alpha\left(\alpha - \bar{l}\right)}{\left[(1-\alpha)\rho - \alpha\bar{n}\right]^2} w(\bar{p})\, r'(\bar{p}). \tag{13d}$$

The effect for $\bar{c}$ is

$$\frac{\partial \bar{c}}{\partial \bar{p}} = -\frac{\alpha \bar{E}}{\bar{p}^2} + \left(\frac{\alpha}{\bar{p}}\right)\frac{\partial \bar{E}}{\partial \bar{p}}, \tag{14}$$

where the first term in the right hand side is the substitution effect and necessarily negative. On the other hand, the second term is income effect and the sign of this effect depends on the factor intensity ranking in the production sectors. Moreover, we can rearrenge $\bar{Z}$ in the steady state:

$$\bar{Z} = \left(\frac{\alpha}{\bar{p}}\right)\frac{\left(z\rho - \bar{l}\right)\bar{n}}{(1-\alpha)\rho - \alpha\bar{n}} w(\bar{p}) - \left[r'(\bar{p})\frac{\alpha - \bar{l}}{(1-\alpha)\rho - \alpha\bar{n}} w(\bar{p}) + w'(\bar{p})\,\bar{l}\right] = 0. \tag{15}$$

We put following assumption for the property of excess demand function around the autarkic steady state.

Assumption 2 (Naito and Ohdoi (2008), Naito and Zhao (2009))**.** *Around the autarkic steady state,* $\partial Z/\partial p|_{p=\bar{p}} = -\bar{\Sigma} + \bar{\Phi} < 0$.

Then, we can obtain the following proposition about autarkic steady state immediately.

Proposition 1. *Suppose that Assumption 1 and 2 hold. Then, there exists the unique steady state equilibrium with incomplete specialization in autarkic economy.*

Proof. *Existence*: If $p \in (0, p_{\min}]$, the economy specializes in the investment good, which means $Z > 0$. Moreover, if $p \in [p_{\max}, \infty)$, the economy specializes in the consumption good, which means $Z < 0$. Finally, if $p \in (p_{\min}, p_{\max})$, then $\lim_{p \to +p_{\min}} Z > 0$ and $\lim_{p \to -p_{\max}} Z < 0$. Because function Z is continuous in p, there exists at least one of $\bar{p}$ such that $\bar{p} \in (p_{\min}, p_{\max})$. *Uniqueness*: Following Assumption 2, $\partial Z/\partial p|_{p=\bar{p}} < 0$ because $\bar{p} \in (p_{\min}, p_{\max})$. If there is not unique p which satisfy $Z = 0$, at least one of these necessarily satisfy $\partial Z/\partial p|_{p=\bar{p}} > 0$ because function Z is continuous. This is contradictory to Assumption 2. Therefore, $\bar{p}$ is unique. $\square$

[Figure 1 about here.]

3.1. Local Stability Analysis in Autarkic Economy

In this section, we will examine the stability around the steady state derived above. Linearizing the system (9) – (10) around the steady state yeilds

$$
\begin{aligned}
\dot{k} &= (\bar{n}+\rho)\left(k-\bar{k}\right) - \left(E-\bar{E}\right) + \left(r'(\bar{p})\,\bar{k} + w'(\bar{p})\right)(p-\bar{p}), \\
\dot{E} &= \frac{\bar{n}^2}{1-\alpha}\left(k-\bar{k}\right) - \bar{n}\left(E-\bar{E}\right) + \bar{E}\bar{\phi}_k\,(p-\bar{p}), \\
0 &= -\bar{\Sigma}\,(p-\bar{p}) - \bar{\phi}_k\left(k-\bar{k}\right) + \bar{\phi}_E\left(E-\bar{E}\right).
\end{aligned}
$$

Using third equation to eliminate $(p-\bar{p})$, we have the following two-dimensional dynamic system:

$$
\begin{bmatrix} \dot{k} \\ \dot{E} \end{bmatrix} = \begin{bmatrix} K_k & K_E \\ E_k & E_E \end{bmatrix} \begin{bmatrix} k-\bar{k} \\ E-\bar{E} \end{bmatrix}, \tag{16}
$$

where

$$
\begin{aligned}
K_k &\equiv \left[(\bar{n}+\rho) - \frac{\bar{\phi}_k\left(r'(\bar{p})\,\bar{k}+w'(\bar{p})\right)}{\bar{\Sigma}}\right], \quad K_E \equiv \left[-1 + \frac{\bar{\phi}_E\left(r'(\bar{p})\,\bar{k}+w'(\bar{p})\right)}{\bar{\Sigma}}\right], \\
E_k &\equiv \left[\frac{\bar{n}^2}{1-\alpha} - \frac{\bar{E}\bar{\phi}_k^2}{\bar{\Sigma}}\right], \quad E_E \equiv \left[-\bar{n} + \frac{\bar{E}\bar{\phi}_k\bar{\phi}_E}{\bar{\Sigma}}\right].
\end{aligned}
$$

Note that only k is a state variable here. The determinant of matrix above is:

$$
\begin{aligned}
\mathrm{Det} \equiv &\left[(\bar{n}+\rho) - \frac{\bar{\phi}_k\left(r'(\bar{p})\,\bar{k}+w'(\bar{p})\right)}{\bar{\Sigma}}\right]\cdot\left[-\bar{n} + \frac{\bar{E}\bar{\phi}_k\bar{\phi}_E}{\bar{\Sigma}}\right] \\
&- \left[-1 + \frac{\bar{\phi}_E\left(r'(\bar{p})\,\bar{k}+w'(\bar{p})\right)}{\bar{\Sigma}}\right]\cdot\left[\frac{\bar{n}^2}{1-\alpha} - \frac{\bar{E}\bar{\phi}_k^2}{\bar{\Sigma}}\right] \\
= &\frac{\left(z\rho-\bar{l}\right)\bar{n}^2 w(\bar{p})}{(1-\alpha)\,\bar{E}\bar{\Sigma}}\cdot\left.\frac{\partial Z}{\partial p}\right|_{p=\bar{p}},
\end{aligned} \tag{17}
$$

because of the definition in Eq. (7). Therefore, the dynamic property is saddle-point stable in the case of $\partial Z/\partial p|_{p=\bar{p}} < 0$, and is unstable or indeterminate in the case of $\partial Z/\partial p|_{p=\bar{p}} > 0$.

On the other hand, the trace of matrix above is:

$$\begin{aligned}\mathrm{Tr} &\equiv \left[(\bar{n}+\rho) - \frac{\bar{\phi}_k\left(r'\left(\bar{p}\right)\bar{k} + w'\left(\bar{p}\right)\right)}{\bar{\Sigma}}\right] + \left[-\bar{n} + \frac{\bar{E}\bar{\phi}_k\bar{\phi}_E}{\bar{\Sigma}}\right] \\ &= \frac{1}{\bar{\Sigma}}\left[\rho\bar{\Sigma} - \bar{\phi}_k\left(r'\left(\bar{p}\right)\bar{k} + w'\left(\bar{p}\right) - \bar{E}\bar{\phi}_E\right)\right]. \end{aligned} \tag{18}$$

The first term in braces refrects the negative substitution effect for the excess demand function, whereas the second term capture the income effect, whose signs are ambiguous. If Σ is sufficiently large, then we have $\mathrm{Tr} > 0$, and hence both characteristic roots have positive real parts.

The results above are summarized in the following proposition.

Proposition 2. *Suppose that Assumption 1 holds.*

1. *If $\partial Z/\partial p|_{p=\bar{p}} < 0$, that is, Assumption 2 holds, then the steady state of the autarkic economy is saddle-point stable.*

2. *If $\partial Z/\partial p|_{p=\bar{p}} > 0$, that is, Assumption 2 violates, and*

   a. *Σ is sufficiently large, then the steady state of the autarkic economy is unstable.*

   b. *Σ is sufficiently small relative for income effect, then the steady state of the autarkic economy is locally indeterminate.*

Therefore, if Assumption 1 and 2 hold, the autarkic steady state is unique and saddle-point stable.

4. Two-Country Economy

We examine the International trade pattern when the country open trade. A similar anarysis above applies to the foreign country, with corresponding variables of the foreign country indicated with the asterisk. Moreover, we can put the relative prices in both countries are same value under free trade, that is $p = p^*$. The equilibrium conditions for the both countries are as follows;

$$
\begin{aligned}
\dot{k} &= \left(r\left(p\right) - \delta\right)k + w\left(p\right) - E, \\
\dot{E} &= \left[\left\{r\left(p\right) - \left[\delta + \left(1-\alpha\right)\frac{E}{zw\left(p\right)+k}\right]\right\} - \rho\right]E, \\
\dot{k}^* &= \left(r^*\left(p\right) - \delta^*\right)k^* + w^*\left(p\right) - E^*, \\
\dot{E}^* &= \left[\left\{r^*\left(p\right) - \left[\delta^* + \left(1-\alpha^*\right)\frac{E^*}{z^*w^*\left(p\right)+k^*}\right]\right\} - \rho^*\right]E^*, \\
& NZ + N^*Z^* \\
& \quad = N\left[\frac{\alpha E}{p} - \left\{r'\left(p\right)k + w'\left(p\right)\left[1 - z\left(1-\alpha\right)\frac{E}{zw\left(p\right)+k}\right]\right\}\right] \\
& \qquad + N^*\left[\frac{\alpha^* E^*}{p} - \left\{r^{*\prime}\left(p\right)k^* + w^{*\prime}\left(p\right)\left[1 - z^*\left(1-\alpha^*\right)\frac{E^*}{z^*w^*\left(p\right)+k^*}\right]\right\}\right] = 0.
\end{aligned}
$$

On the next step, we can difine a variable $\mu \equiv N^*/N$ and modify the world market clearing condition for consumption goods as follows:

$$
\begin{aligned}
& \left[\frac{\alpha E}{p} - \left\{r'\left(p\right)k + w'\left(p\right)\left[1 - z\left(1-\alpha\right)\frac{E}{zw\left(p\right)+k}\right]\right\}\right] \\
& \quad + \mu\left[\frac{\alpha^* E^*}{p} - \left\{r^{*\prime}\left(p\right)k^* + w^{*\prime}\left(p\right)\left[1 - z^*\left(1-\alpha^*\right)\frac{E^*}{z^*w^*\left(p\right)+k^*}\right]\right\}\right] = 0.
\end{aligned} \tag{19}
$$

Moreover, by using the demographic change in each country, we derive the defferential equation about μ,

$$
\left.\begin{aligned}
\dot{N} &= nN \\
\dot{N}^* &= n^*N^*
\end{aligned}\right\} \quad \Rightarrow \quad \frac{\dot{\mu}}{\mu} = n^* - n, \quad \mu\left(0\right) = N^*\left(0\right)/N\left(0\right).
$$

Following the studies Chen (1992), Galor and Lin (1997), Hu and Shimomura (2007), and Chen, Nishimura, and Shimomura (2008), we make following "symmetric steady state" assumption to compare with static H-O-S model which the difference in (initial) factor endowments is the source of international trade:

Assumption 3. *Intertemporal preference parameters, depreciation rates, production technologies, and child-rearing costs are all same between two countries:* $\left(\rho,\ \alpha,\ \delta,\ r\left(p\right),\ w\left(p\right),\ z\right) = \left(\rho^*,\ \alpha^*,\ \delta^*,\ r^*\left(p\right),\ w^*\left(p\right),\ z^*\right)$. *Then, the steady state derived under this circumstance is said to symmetric steady state.*

Using this assumption, we can modify Eq. (19) as follows[3]:

$$\begin{aligned}&\left(\frac{1}{E}\right)\left[\frac{\alpha E}{p}-\left\{r'(p)\,k+w'(p)\left[1-z(1-\alpha)\frac{E}{zw(p)+k}\right]\right\}\right]\\&+\left(\frac{1}{E^*}\right)\left[\frac{\alpha E^*}{p}-\left\{r'(p)\,k^*+w'(p)\left[1-z(1-\alpha)\frac{E^*}{zw(p)+k^*}\right]\right\}\right]=0\end{aligned}\tag{20}$$

The steady state is characterized by the set of conditions

$$\begin{aligned}\bar{c}&=\alpha\frac{\bar{E}}{\bar{p}}, & \bar{c}^*&=\alpha\frac{\bar{E}^*}{\bar{p}},\\ \bar{E}&=(r(\bar{p})-\delta)\,\bar{k}+w(\bar{p}), & \bar{E}^*&=(r(\bar{p})-\delta)\,\bar{k}^*+w(\bar{p}),\\ \bar{l}+z\bar{n}&=1, & \bar{l}^*+z\bar{n}^*&=1,\\ \rho&=r(\bar{p})-(\delta+\bar{n}), & \rho&=r(\bar{p})-(\delta+\bar{n}^*),\\ \bar{n}&=(1-\alpha)\frac{\bar{E}}{zw(\bar{p})+\bar{k}}, & \bar{n}^*&=(1-\alpha)\frac{\bar{E}^*}{zw(\bar{p})+\bar{k}^*},\end{aligned}$$

$$\begin{aligned}&\left(\frac{1}{\bar{E}}\right)\left[\frac{\alpha\bar{E}}{\bar{p}}-\left\{r'(\bar{p})\,\bar{k}+w'(\bar{p})\left[1-z(1-\alpha)\frac{\bar{E}}{zw(\bar{p})+\bar{k}}\right]\right\}\right]\\&+\left(\frac{1}{\bar{E}^*}\right)\left[\frac{\alpha\bar{E}^*}{\bar{p}}-\left\{r'(\bar{p})\,\bar{k}^*+w'(\bar{p})\left[1-z(1-\alpha)\frac{\bar{E}^*}{zw(\bar{p})+\bar{k}^*}\right]\right\}\right]=0.\end{aligned}$$

We can derive the following proposition about long-run equilibrium in this economy.

Proposition 3. *Suppose that Assumption 1 – 3 hold, and there is a difference only in the initial per-capita capital endowments and populations, k_0, k_0^*, N_0, and N_0^*, in both countries. Then, international trade does not take place and populations in both countries are equal in the steady state.*

Proof. From the above calculations, we can obtain following steady state value in the world economy: $\bar{n}=\bar{n}^*=r(\bar{p})-(\rho+\delta)$, $\bar{l}=\bar{l}^*=1-z(r(\bar{p})-(\rho+\delta))$, $\bar{k}=\bar{k}^*=\left(\alpha-\bar{l}\right)w(\bar{p})/[(1-\alpha)\rho-\alpha\bar{n}]$, $\bar{c}=\bar{c}^*=\alpha\left(z\rho-\bar{l}\right)\bar{n}w(\bar{p})/\bar{p}\,[(1-\alpha)\rho-\alpha\bar{n}]$, $\bar{E}=\bar{E}^*=\left(z\rho-\bar{l}\right)\bar{n}w(\bar{p})/[(1-\alpha)\rho-\alpha\bar{n}]$, and $\overline{Z}=\overline{Z}^*=0$. Therefore, international trade does not take place and populations in both countries are equal in the steady state. □

Atokeson and Kehoe (2000), which is multi-country dynamic general equilibrium model with exogenous time preferences rates and populations, says as follows: "A country that be-

[3]We can derive (20) as follows: If both countries are symmetric, we can derive $\frac{\dot{E}}{E}-\frac{\dot{E}^*}{E^*}=\frac{\dot{N}^*}{N^*}-\frac{\dot{N}}{N}=n^*-n$. This equation implies $\mu\equiv\frac{N^*}{N}=\frac{E}{E^*}$. Therefore, we can modify Eq. (19) to $\left(\frac{1}{E}\right)Z+\left(\frac{1}{E^*}\right)Z^*=0$.

gins the development process later than most of the rest of the world –a late-bloomer– ends up with a permanently lower level of income than the early-blooming countries that developed earlier. This is true even though the late-bloomer has the same preferences, technology, and initial capital stock that the early-bloomers had when they started the process of development". On the other hand, Hu and Shimomura (2007), which is two-country dynamic general equilibrium model with status-seeking agents, says that under some conditions, there uniquely exists an incompletely specialized symmetric steady state, and therefore catching-up and overtaking phenomena seen in economic development can be explained. In our study, which is two-country dynamic general equilibrium model with endogenous fertility, there also uniquely exists an incompletely specialized catching-up and overtaking long-run equilibrium.

We can linearize the dynamic system as follows;

$$
\begin{aligned}
&\dot{k} = (\bar{n}+\rho)\left(k-\bar{k}\right) - \left(E-\bar{E}\right) + \left(r'\left(\bar{p}\right)\bar{k} + w'\left(\bar{p}\right)\right)\left(p-\bar{p}\right)\\
&\dot{k}^{*} = (\bar{n}+\rho)\left(k^{*}-\bar{k}\right) - \left(E^{*}-\bar{E}\right) + \left(r'\left(\bar{p}\right)\bar{k} + w'\left(\bar{p}\right)\right)\left(p^{*}-\bar{p}\right)\\
&\dot{E} = \frac{\bar{n}^{2}}{1-\alpha}\left(k-\bar{k}\right) - \bar{n}\left(E-\bar{E}\right) + \bar{E}\bar{\phi}_{k}\left(p-\bar{p}\right)\\
&\dot{E}^{*} = \frac{\bar{n}^{2}}{1-\alpha}\left(k^{*}-\bar{k}\right) - \bar{n}\left(E^{*}-\bar{E}\right) + \bar{E}\bar{\phi}_{k}\left(p-\bar{p}\right)\\
&0 = \left(\frac{1}{\bar{E}}\right)\left[-\bar{\Sigma}\left(p-\bar{p}\right) - \bar{\phi}_{k}\left(k-\bar{k}\right) + \bar{\phi}_{E}\left(E-\bar{E}\right)\right] + \left(\frac{1}{\bar{E}}\right)\left[-\bar{\Sigma}\left(p-\bar{p}\right) - \bar{\phi}_{k}\left(k^{*}-\bar{k}\right) + \bar{\phi}_{E}\left(E^{*}-\bar{E}\right)\right]
\end{aligned}
$$

We define the new variables $\tilde{k}$ and $\tilde{E}$ as $\tilde{k} = (k+k^{*})/2$ and $\tilde{E} = (E+E^{*})/2$. Using these variables, we can simplify the dynamic system.

$$
\begin{bmatrix} \dot{\tilde{k}} \\ \dot{\tilde{E}} \end{bmatrix} = \begin{bmatrix} K_k & K_E \\ E_k & E_E \end{bmatrix} \begin{bmatrix} \tilde{k}-\bar{\tilde{k}} \\ \tilde{E}-\bar{\tilde{E}} \end{bmatrix}, \tag{21}
$$

where K_i and E_i $(i = k, E)$ are mentioned in Eq. (16). Then, We obtain the following proposition as same as Proposition 2.

Proposition 4. *If Assumption 1 – 3 hold in each countries, the symmetric steady state of the free trade economy has the properties of uniqueness and saddle-point stable.*

5. Long-Run International Trade Pattern and Comparative Statics

In this section, we investigate the properties about steady state in the world economy. We can rewrite the world market equilibrium condition

$$\begin{aligned}
&\left(\frac{1}{\bar{E}}\right)\bar{Z}+\left(\frac{1}{\bar{E}^*}\right)\bar{Z}^* \\
&\quad=\left(\frac{1}{\bar{E}}\right)\left[\frac{\alpha\bar{E}}{\bar{p}}-\left\{r'(\bar{p})\,\bar{k}+w'(\bar{p})\left[1-z(1-\alpha)\frac{\bar{E}}{zw(\bar{p})+\bar{k}}\right]\right\}\right] \\
&\qquad+\left(\frac{1}{\bar{E}^*}\right)\left[\frac{\alpha\bar{E}^*}{\bar{p}}-\left\{r'(\bar{p})\,\bar{k}^*+w'(\bar{p})\left[1-z(1-\alpha)\frac{\bar{E}^*}{zw(\bar{p})+\bar{k}^*}\right]\right\}\right]=0,
\end{aligned}$$

where $\bar{Z}$ and $\bar{Z}^*$ is the both country's per capita excess demand function on the consumption goods. Therefore, if one country exports (imports) consumption goods, that is $\bar{Z}/\bar{E}<(>)\,0$, the other country does necessarily import (export) that goods, $\bar{Z}^*/\bar{E}^*>(<)\,0$. From the above, straightforward calculation yields

$$\left.\frac{\partial\left(\bar{Z}/\bar{E}\right)}{\partial\bar{p}}\right|_{p=\bar{p}}=\frac{\left(\partial\bar{Z}/\partial\bar{p}\right)\bar{E}-\bar{Z}\left(\partial\bar{E}/\partial\bar{p}\right)}{\bar{E}^2}=\frac{1}{\bar{E}}\left(-\bar{\Sigma}+\bar{\Phi}\right)-\frac{\bar{Z}}{\bar{E}^2}\left(\frac{\partial\bar{E}}{\partial\bar{p}}\right),\quad \bar{\Sigma}>0,\ \bar{\Phi}\gtreqless 0. \tag{22}$$

Using these relations, we obtain following proposition:

Proposition 5. *Suppose that Assumption 1 – 3 hold. Then, the excess demand function of consumption goods in home (resp. foreign) country $\bar{Z}/\bar{E}$ (resp. $\bar{Z}^*/\bar{E}^*$) is decreasing in $\bar{p}$, that is $\partial\left(\bar{Z}/\bar{E}\right)/\partial\bar{p}\big|_{p=\bar{p}}<0$ (resp. $\partial\left(\bar{Z}^*/\bar{E}^*\right)/\partial\bar{p}\big|_{p=\bar{p}}<0$), around the symmetric steady state, that is, $\bar{Z}=0$.*

This property is different from the study with constant time preference rate and exogenous fertility without endogenous fertility[4]. In these studies, the excess demand of consumption goods does not have positive or negative slope under incomplete specialization. Therefore, the long-run equilibria with diversified production are not unique. On the other hand, in our study, there is a unique long-run equilibrium with diversified production in the two-country economy.

[4]See Stiglitz (1970) for the properties of the excess demand function in the study with with constant time preference rate and exogenous fertility.

5.1. The Effect of Increasing in the Subjective Dicount Rate

In this section, we investigate the effect of increasing in the subjective discount rate in home country. If the parameters in both country is not symmetric, then the autarkic prices and the world price do not also satisfy equality. We difine the autrkic price in home country as p^a, the autarkic price in foreign country as p^{a*}, and the world price as p^W. We can obtain the effect for each country's endogenous valiables of change in ρ within only home country under the symetric steady state as follows:

$$\begin{aligned}\frac{\partial(\cdot)}{\partial\rho} &= \left.\frac{\partial(\cdot)}{\partial\rho}\right|_{p=\bar{p}} + \frac{\partial(\cdot)}{\partial\bar{p}}\cdot\frac{\partial\bar{p}}{\partial\rho},\\ \frac{\partial(\cdot)^*}{\partial\rho} &= \frac{\partial(\cdot)^*}{\partial\bar{p}}\cdot\frac{\partial\bar{p}}{\partial\rho}.\end{aligned}$$

The first equation shows the effects in home country. The first-term in the right-hand side of it is the direct effect and second-term is the indirect effect through the change of consumption good's world price. The second equation means the effect in foreign country and there is not the direct effect. In the case of symmetric steady state, note that $\partial(\cdot)/\partial\bar{p} = \partial(\cdot)^*/\partial\bar{p}$. Therefore, the difference of the effects between both countries is given by

$$\begin{aligned}\frac{\partial(\cdot)}{\partial\rho} - \frac{\partial(\cdot)^*}{\partial\rho} &= \left.\frac{\partial(\cdot)}{\partial\rho}\right|_{p=\bar{p}} + \frac{\partial\bar{p}}{\partial\rho}\left(\frac{\partial(\cdot)}{\partial\bar{p}} - \frac{\partial(\cdot)^*}{\partial\bar{p}}\right)\\ &= \left.\frac{\partial(\cdot)}{\partial\rho}\right|_{p=\bar{p}}.\end{aligned}$$

From the property above, we can derive the effects in $\bar{n}-\bar{n}^*$, $\bar{l}-\bar{l}^*$, and $\bar{k}-\bar{k}^*$ as follows:

$$\frac{\partial\bar{n}}{\partial\rho} - \frac{\partial\bar{n}^*}{\partial\rho} = \left.\frac{\partial\bar{n}}{\partial\rho}\right|_{p=\bar{p}} = -1 < 0, \tag{25a}$$

$$\frac{\partial\bar{l}}{\partial\rho} - \frac{\partial\bar{l}^*}{\partial\rho} = \left.\frac{\partial\bar{l}}{\partial\rho}\right|_{p=\bar{p}} = z > 0, \tag{25b}$$

$$\frac{\partial\bar{k}}{\partial\rho} - \frac{\partial\bar{k}^*}{\partial\rho} = \left.\frac{\partial\bar{k}}{\partial\rho}\right|_{p=\bar{p}} = -\frac{(1-\alpha)\left(z\rho-\bar{l}\right)}{\left[\alpha\bar{n}-(1-\alpha\rho)\right]^2}w(\bar{p}) < 0. \tag{25c}$$

The effect derived above shows that home country, which is a less patient country, is a labor abundant country with smaller populations. Moreover, we can obtain the effect for the excess demand in home country around the symmetric steady state:

$$\left.\frac{\partial\left(\bar{Z}/\bar{E}\right)}{\partial\rho}\right|_{p=\bar{p}}=-\frac{1}{\bar{E}}\left[\left(\frac{\bar{n}+\rho}{\bar{p}}\right)\left(\frac{\bar{p}r'\left(\bar{p}\right)}{r\left(\bar{p}\right)-\delta}-\alpha\right)\frac{\partial\bar{k}}{\partial\rho}+zw'\left(\bar{p}\right)\right],\quad \operatorname{sgn}\left(\frac{\partial\left(\bar{Z}/\bar{E}\right)}{\partial\rho}\right)=\operatorname{sgn}\left(r'\left(\bar{p}\right)\right), \tag{26}$$

because $\partial\bar{c}/\partial\rho|_{p=\bar{p}}=\left[\alpha\left(\bar{n}+\rho\right)/\bar{p}\right]\cdot\partial\bar{k}/\partial\rho\big|_{p=\bar{p}}<0$ and $\partial\bar{y}_1/\partial\rho|_{p=\bar{p}}=r'\left(\bar{p}\right)\cdot\partial\bar{k}/\partial\rho\big|_{p=\bar{p}}+zw'\left(\bar{p}\right)$, that is, $\operatorname{sgn}\left(\partial\bar{y}_1/\partial\rho|_{p=\bar{p}}\right)=\operatorname{sgn}\left(w'\left(\bar{p}\right)\right)$.

Using these effects, now we examine the international trade pattern in the case of good 1 (good 2) is labor (capital) intensive, that is, $r'(p)<0$ and $w'(p)>0$[5]. There are two way effects of rising subjective discount rate for international trade pattern.

- Supply–Side Effect

  The increasing of per capita labor l increases the output of good 1 and decreases that of good 2 because of the Rybczynski effect. The decreases the output of good 2, deccumulates the capital stock in this economy. Therefore, this economy acts just like a labor abundant country, and this economy has possibility to export good 1, that is $\partial\bar{Z}/\partial\rho<0$.

- Demand–Side Effect

  When the subjective discount rate is increasing, each agent increases her/his consumption level significantly at the present because the discount rate applied to the future utility will be higher. Because of the decreased per capita saving, individuals will decumulate their capital and decrease consumption expenditure in the long-run. Therefore, they have the possibility to export good 1, that is $\partial\bar{Z}/\partial\rho<0$.

We can derive the following proposition about the effect of rising ρ in international trade pattern.

Proposition 6. *Suppose that Assumption 1 – 3 hold and that the consumption (investment) good is more labor intensive. Then, less patient country export (import) the labor intensive good with smaller population.*

Figure 2 illustrates the international trade pattern in the case ρ in home country is larger than that in foreign country. In the case of symmetric steady state, the locus $(\bar{Z}/\bar{E})$ and $(\bar{Z}^*/\bar{E}^*)$ are same and the economy follows point E^S which p^S is the symmetric steady state

[5] We can also illustrate the case $r'(p)>0$ and $w'(p)<0$ in the same way.

price. The increase in ρ in only home country shifts $(\bar{Z}/\bar{E})$ locus leftward and home country follows point E'. Then, p increase over time. There is a trade equilibrium if $p = p^W$, because excess supply is zero in the world economy.

[Figure 2 about here.]

These results are in contrast to those in Stiglitz (1970). This study, which is two-country dynamic general equilibrium model with exogenous time preferences rates and populations without endogenous fertility, shows the possibility of perfect specialization and factor price inequalization caused exclusively by international differences in preference parameters such as the time preference rate. On the other hand, we have the properties of incomplete specialization and factor price equalization in our study.

Moreover, our results agree with that of the studies which satisfies incomplete specialization and the factor price equalization theorem. Golar and Lin (1997), which is two-country dynamic trade model with overlapping generations agents, Hu and Shimomura (2007), which is two-country dynamic trade model with status-seeking agents, and Chen, Nishimura, and Shimomura (2008), which is two-country dynamic trade model with endogenous time preference, show the property that the impatient country has lower capital stock and consumption level, and imports the capital-intensive goods. Then a patient country acts just like a capital abundant country and exports the capital-intensive good.

5.2. The Effect of Increasing in the Child-Rearing Cost

Finally, we investigate the effect of increasing in the child-rearing cost in home country in this section. We can derive the effects in $\bar{n} - \bar{n}^*$, $\bar{l} - \bar{l}^*$, and $\bar{k} - \bar{k}^*$ of the increase in z in only home country for each values in symmetric steady state are given by :

$$\frac{\partial \bar{n}}{\partial z} - \frac{\partial \bar{n}^*}{\partial z} = \left.\frac{\partial \bar{n}}{\partial z}\right|_{p=\bar{p}} = 0, \tag{27a}$$

$$\frac{\partial \bar{l}}{\partial z} - \frac{\partial \bar{l}^*}{\partial z} = \left.\frac{\partial \bar{l}}{\partial z}\right|_{p=\bar{p}} = -\bar{n} < 0, \tag{27b}$$

$$\frac{\partial \bar{k}}{\partial z} - \frac{\partial \bar{k}^*}{\partial z} = \left.\frac{\partial \bar{k}}{\partial z}\right|_{p=\bar{p}} = -\frac{\bar{n}}{\alpha \bar{n} - (1-\alpha)\rho} w(\bar{p}) > 0. \tag{27c}$$

The effect derived above shows that home country, which is a larger child-rearing cost country, is a capital abundant country and there are same populations between both countries. Moreover, we can obtain the effect for the excess demand in home country around the symmetric steady state:

$$\left.\frac{\partial\left(\bar{Z}/\bar{E}\right)}{\partial z}\right|_{p=\bar{p}} = -\frac{1}{\bar{E}}\left[\left(\frac{\bar{n}+\rho}{\bar{p}}\right)\left(\frac{\bar{p}r'\left(\bar{p}\right)}{r\left(\bar{p}\right)-\delta}-\alpha\right)\frac{\partial\bar{k}}{\partial z}-w'\left(\bar{p}\right)\bar{n}\right], \quad \operatorname{sgn}\left(\frac{\partial\left(\bar{Z}/\bar{E}\right)}{\partial z}\right) = \operatorname{sgn}\left(w'\left(\bar{p}\right)\right), \tag{28}$$

because $\left.\partial\bar{c}/\partial z\right|_{p=\bar{p}} = -\left[\alpha\bar{n}\left(\bar{n}+\rho\right)/\bar{p}\left\{\alpha\bar{n}-\left(1-\alpha\right)\rho\right\}\right]w\left(\bar{p}\right) > 0$ and $\left.\partial\bar{y}_1/\partial z\right|_{p=\bar{p}} = r'\left(\bar{p}\right)\cdot\partial\bar{k}/\partial z - w'\left(\bar{p}\right)\bar{n}$, that is, $\operatorname{sgn}\left(\partial\bar{y}_1/\partial z\right) = \operatorname{sgn}\left(r'\left(\bar{p}\right)\right)$.

Using these effects, we examine the international trade pattern in the case of good 1 (good 2) is labor (capital) intensive similar with the analysis in previous section.

- Supply–Side Effect

  The increasing of child-rearing time decreases the labor input. The decreasing of per capita labor l increases the output of good 2 and decreases that of good 1 because of the Rybczynski effect. The increases the output of good 1, accumulates the capital stock in this economy. Therefore, this economy acts just like a capital abundant country, and this economy has possibility to import good 1, that is $\partial\bar{Z}/\partial\rho > 0$.

- Demand–Side Effect

  When per capita labor input l is decreasing, each agent decreases her/his consumption level and increases saving significantly at the present. Because of the increased per capita saving, individuals will accumulate their capital and increase consumption expenditure in the long-run. Therefore, they have the possibility to import good 1, that is $\partial\bar{Z}/\partial\rho > 0$.

We can derive the following proposition about the effect of rising z in international trade pattern.

Proposition 7. *Suppose that Assumption 1 – 3 hold and that the consumption (investment) good is more labor intensive. Then, the country with higher child-rearing cost import (export) the labor intensive good. Moreover, there are not difference in population between both countries.*

Figure 3 illustrates the international trade pattern in the case z in home country is larger than that in foreign country. The important properties in our study is that in the economy with higher child-rearing cost, there are capital accumulation, the increases in expenditure, and the export of capital intensive goods. We notice that these properties are realistic in many developed countries.

[Figure 3 about here.]

6. Concluding Remarks

This paper examines a two-country dynamic general equilibrium model with endogenous fertility. We show that the introduction of child rearing behavior brings about new properties in long run dynamics. After deriving conditions about the existence, uniqueness, and local stability of long run equilibrium, we investigate the international trade pattern and comparative statics in order to see the differences with the standard dynamic international trade model.

Four remarks follow: First, we have not considered the Benthamite social welfare function. In Palivos and Yip (1993), They assumes the following utility function

$$\int_0^{\infty} \left[u\left(c\right)+v\left(n\right)\right] N^{1-\varepsilon} e^{-\rho t} dt = \int_0^{\infty} \left[u\left(c\right)+v\left(n\right)\right] N_0 e^{-\left[\rho-(1-\varepsilon)n\right]t} dt$$

where ε is an altruism parameter. If ε is equal to zero (one), then utility function above becomes the Benthamite (Millian) social welfare function. The Benthamite Social Welfare Function means following two implications:[6]

1. This type of social welfare function is based on the addition of utilities across the individuals of the society.

2. With all individuals carrying the same weight, this is equivalent to the total utility of the society.

Second, we have not considered agent's mortality rate. In the aging economy, we might obtain the different international trade pattern. Third, we have omitted the element of human capital accumulation. If agents accumulate human capital, there is possibility of endogenous growth in this economy. Finally, it would be interesting to discuss about the effect of the trade policy and gains from trade in the current framework. These are interesting issues in the future analysis.

[6]See Razin and Sadka (1995).

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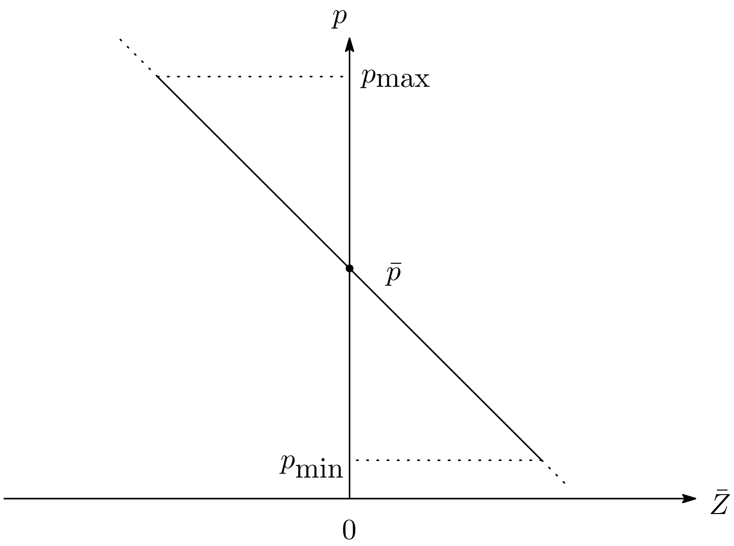

Figure 1: autarkic equilibrium

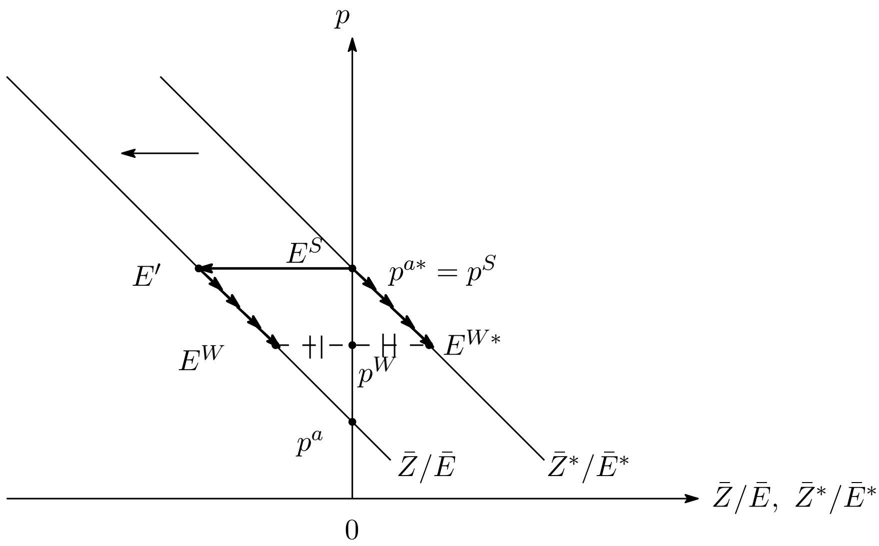

Figure 2: the effect of increasing in the subjective discount rate

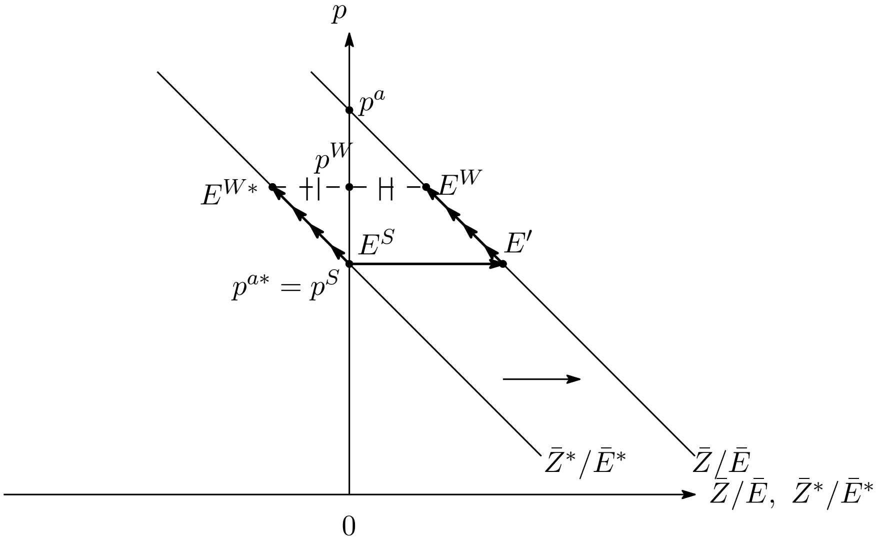

Figure 3: the effect of increasing in the child-rearing cost

	Static H-O-S model	Stiglitz (1970)	Chen (1992)	Karasawa and Yanase (2010) (this paper)
capital	fixed	endogenous	endogenous	endogenous
labor	fixed	fixed	endogenous	endogenous
population	fixed	fixed	fixed	endogenous

Table 1: factor endowments and population in the international trade models