Discussion Papers In Economics And Business

Availability of Higher Education and Long-Term Economic Growth

Ryo Horii Akiomi Kitagawa Koichi Futagami

Discussion Paper 03-14-Rev.

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Availability of Higher Education and Long-Term Economic Growth

Ryo Horii Akiomi Kitagawa Koichi Futagami

Discussion Paper 03-14-Rev.

September 2005

この研究は「大学院経済学研究科・経済学部記念事業」
基金より援助を受けた、記して感謝する。

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Availability of Higher Education and Long-Term Economic Growth

Ryo Horii* Akiomi Kitagawa† Koichi Futagami‡

September 8, 2005

Abstract

This paper examines the economic growth effects of limited availability of higher education in a simple endogenous growth model with overlapping generations. With limited availability, the scarcity of human capital keeps its price high and distributes a larger share of the aggregate output to young households. Under certain conditions, it leads to greater aggregate savings in each period, thereby enabling the economy to grow faster than without any limitation. In such cases, an excessive expansion in the availability causes a temporary boom followed by a serious deficiency in investible funds, resulting in a substantial slowdown in economic growth.

JEL Classification Numbers: O41.

Keywords: Endogenous Growth; Human Capital; Slowdown; Intergenerational Income Distribution.

---

*correspondence: Graduate School of Economics, Osaka University,1-7 Machikaneyama, Toyonaka, Osaka 560-0043, JAPAN; E-mail: horii@econ.osaka-u.ac.jp; Tel: +81-6-6850-5254; Fax: +81-6-6850-5274.

†Graduate School of Economics and Management, Tohoku University.

‡Graduate School of Economics, Osaka University.

1 Introduction

Does education promote economic growth? Although this may appear to have an obvious answer, deeper examination has proved to be an uneasy task for economists, primarily because there is no direct measure for the output of education. Despite the difficulty, recent empirical micro studies find strong evidence supporting the view that individual private returns to education are quite high (see the survey by Card, 1999). Given that human capital is individually productive, existing models of economic growth predict that education should enhance growth (e.g., Barro and Sala-i-Martin, 2004). The growth accounting literature devotes enormous effort to confirming this prediction using aggregate data, but so far, most studies only find weak and elusive connections between education and economic growth.[1]

There are three candidates, at least, that may cause this discrepancy between the micro-macro evidence. First, if education works as a signaling device, its positive private returns would not correspond to individual productivity of human capital (Spence, 1973; Weiss, 1995). Second, the amount and quality of human capital as inputs to the production process have not yet been fully measured (see the survey by Caselli, 2005). The third possibility, on which little work has been done so far but is the focus of this paper, is that the individual benefits of education might be drowned by negative external

[1]Using the Solow (1956) growth model, Mankiw, Romer and Weil (1992) argue that a large part of cross-country differences in steady-state income is explained by a certain measure of human capital. Islam (1995) finds, however, that once differences in technologies (individual country effects) are accounted for by a dynamic panel data model, the role of human capital becomes insignificant. Romer (1989), De Gregorio (1992), Barro and Lee (1994), Benhabib and Spiegel (1994), and Pritchett (2001) also report that the direct effect of human capital on growth is either insignificant or even negative. Topel (1999) and Temple (2001) argue that the growth effect of education is found to be positive under more sophisticated specifications, but both admit that the literature connecting human capital investment to economic growth is still inconclusive.

effects operating at the aggregate level, even when human capital is truly productive and appropriately measured at the micro level.

Specifically, this paper investigates how the long-term rate of economic growth is affected by the pecuniary externality of an increased supply of human capital.[2] A rise in the aggregate supply in human capital would lower its price compared to other factors summarized as physical capital. Since the distribution of production factors is not uniform across cohorts of different age—that is, human capital belongs to working-age generations whereas physical capital is largely owned by older generations—the implied change in factor prices shifts the distribution of income from the young to the old. If the aggregate saving rate is adversely affected by the reduced income of the young generation, the shift in income distribution would be critical for long-term growth. Using a simple endogenous growth model with overlapping generations, this paper shows theoretically that, under a certain condition, the long-term rate of growth may be reduced by an increased supply of human capital.

To examine changes in the supply of human capital, we focus on the availability of higher education. In many countries, aggregate investment in human capital is constrained by the number of higher education institutions and the enrollment capacity of each institution. These are not entirely determined by market forces but are also determined by non-economic factors such as history, culture, and the social system of the country, as well as the government's education policies.[3] The limited availability of

[2]A majority of endogenous growth models assumes the presence of *technological* externalities to knowledge or human capital, as also does the model we present in the next section (see the survey by Klenow and Rodrìguez-Clare, 2005). A widespread perception, however, is that they are positive externalities and therefore do not explain the micro-macro discrepancy.

[3]For example, Duan (2003) reports that only 2.4 percent of Chinese university candidates could gain a place in 1981 due to the government's strict control on the number of enrollments for higher education institutions. In Japan, Kaneko (1997) documents that the Ministry of Education used to control the

higher education puts an upper bound on the rate at which the economy can accumulate human capital, hence potentially restricts the rate of economic growth. At the same time, however, the scarcity of human capital keeps its price high and thereby enables young households to earn a larger share of the aggregate output than without such a limitation.[4] Their increased savings contribute to maintaining a high rate of accumulation of both physical and human capital and therefore of growth.

The relative significance of the two opposing effects is shown to depend on the stage of development to which the economy in question belongs. In agrarian countries, or more precisely in economies where the nature of existing knowledge allows it to be transferred intergenerationally largely without higher education, the savings-enhancing effect is marginal and therefore expanding enrollment capacity promotes growth in the long run.[5] Conversely, in industrialized economies where the transfer of existing knowledge is substantially dependent on higher education, there is a range of levels of the availability within which the savings-enhancing effect dominates the growth-restricting effect. In this case, the economy has a balanced growth path on which human capital accumulation is

number of private universities through the rigorous interpretation of university establishment standards. Private institutions did not argue against such a discretionary policy because it allowed them to operate as a virtual cartel to maintain their prestige.

[4]This argument implicitly assumes that graduates, not higher education institutions, receive a large part of the rent that is generated by the limited availability of higher education. Kaneko (1997) notes that this was in fact the case in Japan since universities were often reluctant to raise their tuition for fear of losing students of higher academic ability.

[5]This prediction is consistent with the recent experience of rapid economic growth and drastic expansion of higher education in China. The proportion of university candidates that could be admitted was increased dramatically from 2.4 per cent in 1981 to 52 per cent by 2003, mainly as a result of the government's policy of reducing the gap between strong demand for higher education and limited access. The total enrollment in government-controlled higher education institutions almost doubled in only three years, from 6.53 million in 1998 to 12.14 million in 2001 (Duan, 2003).

constrained by limited availability but nonetheless grows faster than in the case without such a limitation. It implies, however, that a further expansion in enrollment capacity will lower the long-term rate of growth since it causes a regime change beyond which the limited availability is no longer binding.[6] After the regime change, young households no longer enjoy rent from the limited aggregate supply of human capital and therefore their savings cannot maintain that high rate of growth in the long run. We examine the dynamic properties of the slowdown process and show that it is characterized by a temporary boom followed by a serious deficiency in investible funds. This slowdown is substantial not only in magnitude but also in that, due to its hysteresis, the recovery from the slowdown involves a major restructuring of educational institutions.

In the literature on human capital accumulation and endogenous growth, the issue of the limited availability of education has been examined in the context of credit market imperfections. De Gregorio (1996) constructs an endogenous growth model in which the availability of education is limited by the borrowing constraints imposed on the young generation, and argues that relaxing these constraints has two opposing effects on growth. First, it makes possible a rapid accumulation of human capital through increased participation in education, which has a positive effect on long-term growth. Second, it enables young households to enjoy more consumption by borrowing more, which reduces the aggregate saving rate and therefore has a negative effect on growth.[7]

[6] For example, a recent report by the Japanese government predicts that a steady increase in the number of private universities will cause total enrollment capacity to exceed the number of all university applicants by 2007.

[7] Some authors have already pointed out the possibility that severe borrowing constraints rather accelerate economic growth by encouraging aggregate savings. Modigliani (1986), for example, argues that credit market imperfections prevent households from borrowing as much as would be required to carry out an unconstrained optimum consumption plan, which has the general effect of postponing consumption and increasing wealth as well as savings. Based on this argument, Jappelli and Pagano (1994)

If the second effect dominates the first, relaxing the borrowing constraint reduces the rate of economic growth. De Gregorio and Kim (2000), Christou (2001), and Azariadis and de la Croix (2003) have extended this argument to the case in which each generation contains heterogeneous households that are subject to differential borrowing constraints.

Distinct from these studies, this paper directly imposes a limitation on the level of human capital investment, so that a relaxation of the limitation does not have a negative effect on aggregate savings through changing the consumption behavior of young households. In our model, however, it does have a negative effect on aggregate savings through encouraging human capital accumulation, because human capital affects the distribution of income between generations. The cumulative effect eventually causes a catastrophic slowdown, pushing the rates of both physical and human capital accumulation, and hence economic growth, to a considerably lower level. This is in sharp contrast to the aforementioned models that only emphasize the growth-enhancing effects of human capital accumulation.

The rest of the paper is organized as follows. After presenting the model in Section 2, temporary equilibrium is characterized in Section 3. Section 4 examines equilibrium dynamics and steady states for possible situations. Section 5 evaluates the growth effects of a gradual increase in the availability of higher education. Section 6 gives some concluding remarks. Proofs of propositions are contained in an additional appendix, which is available from the corresponding author upon request, or can be downloaded from `http://www2.econ.osaka-u.ac.jp/~horii/`.

---

constructed an endogenous growth model without human capital, in which the borrowing constraint encourages physical capital accumulation and economic growth.

2 Model

The model is a variant of Diamond's (1965) overlapping generations model. Time is discrete and extends from one to infinity. A continuum of households of measure one are born at the beginning of each period, and every household lives for two periods. Thus, in any period, there are just two age groups of households of an equal size, which we call the young and old households.

In each period, a single final good, the *numeraire*, is competitively produced from physical and human capital according to a Cobb-Douglas technology, $Y_t = AK_t^{\alpha} H_t^{1-\alpha}$, where $A > 0$, $\alpha \in (0, 1)$, Y_t, K_t and H_t, respectively, represent the total factor productivity, the share of physical capital, the aggregate output of the final good, the aggregate input of physical capital, and that of human capital. Factor markets are perfectly competitive, so that both kinds of capital earn their marginal values, i.e.,

$$r_t = A\alpha(K_t/H_t)^{\alpha-1}, \quad w_t = A(1-\alpha)(K_t/H_t)^{\alpha}, \tag{1}$$

where r_t and w_t, respectively, denote the market price of physical capital and human capital. The good produced in one period can be either consumed in that period or saved for the production in the next period. Once saved, the good cannot be consumed, but can be used as either physical capital or an input to human capital investment, which we call higher education. Moreover, the saved good perishes in one period, so that physical capital existing in the current period cannot be carried over to the next period.

The household maximizes a time-separable logarithmic utility function:

$$U_t = (1-\sigma)\log c_{1,t} + \sigma \log c_{2,t+1}, \tag{2}$$

where $c_{1,t}$ and $c_{2,t+1}$, respectively, represent the amounts of consumption in the young and old ages, and $\sigma \in (0, 1)$ is a parameter specifying the patience of households. This formulation suggests that, given that households earn labor income only when young, they would save σ percentage of that income for their old age.

The main difference between our model and that of Diamond is that newly born households are not only endowed with some amount of human capital through home training and free (costless) primary education, but also with the ability to augment their human capital by participating in higher education. Higher education is described as a process that produces human capital using saved goods, relying upon the previous generation's aggregate stock of knowledge. Specifically, when the previous generation possesses amount H_{t-1} of human capital in aggregate (or equivalently, on average), a young household can obtain

$$h_t = G(e_t, H_{t-1}) \tag{3}$$

units of human capital by investing $e_t \geq 0$ units of the saved goods.[8]

We assume that the education production function $G(\cdot, \cdot)$ is homogeneous of degree one with respect to e_t and H_{t-1}, but that its output cannot increase unboundedly with the value of e_t because investment opportunities are limited by the current availability of higher education. To specify this feature we assume that $G(\cdot, \cdot)$ has the following simple functional form,

$$G(e_t, H_{t-1}) \equiv \delta H_{t-1} + \min[e_t, \gamma H_{t-1}], \tag{4}$$

where δ and γ are parameters satisfying $0 < \delta < 1$ and $\gamma > 0$. The first term in the right-hand side of (4) represents the amount of human capital transferred from the parent's generation through home training and primary education. Parameter δ measures the percentage of the previous generation's knowledge that can be transferred through such channels. The value of δ tends to be smaller in industrialized economies than in agrarian economies, since agricultural knowledge is often passed down from parents to children whereas the transfer of industrial knowledge often requires higher education. The second term is the amount of human capital obtained through higher education. By

[8]Similar specifications are found, for example, in Azariadis and Drazen (1990) and De la Croix and Michel (2002, section 5.2).

participating in higher education, young households can produce human capital linearly with the input of saved goods,[9] but the limited availability of higher education puts an upper bound on the individual investment in human capital at γH_{t-1}. The availability is measured by parameter γ, which is determined by such factors as the number of higher education facilities that offer graduate and undergraduate courses and the enrollment capacity in each facility. Due to those limitations, there is an upper limit on the rate of aggregate human capital growth;[10] i.e., $G(e_t, H_{t-1})/H_{t-1}$ is bounded by $\delta + \gamma$.

The life of a household born at the beginning of a generic period t proceeds as follows. In period t, this household is endowed with δH_{t-1} units of human capital. In addition, she purchases some amount of the saved goods from the physical capital market (the market for the saved goods) to augment her own human capital through higher education. Then she supplies the sum of endowed and augmented human capital to the human capital market, and in return receives $w_t h_t$ units of the newly produced good at the end of that period. Since the household has to pay $r_t e_t$ units of the produced good for the purchase of the saved goods made at the beginning of that period, her net income becomes

$$w_t h_t - r_t e_t = w_t \delta H_{t-1} + (w_t - r_t) e_t \quad \text{for } e_t \leq \gamma H_{t-1}. \tag{5}$$

At the end of period t, she consumes the $1 - \sigma$ percentage of the net income and saves $s_t = \sigma(w_t h_t - r_t e_t)$ units of the produced good for her old age. In period $t+1$, the saved

---

[9] In this respect, we follow the simplest setting employed in the growth literature. A more realistic production function of human capital would take both goods and human capital (i.e., time) as inputs. In that case, one can interpret (4) as the *net* production function. The one-to-one production function is equivalent to the one that produces x units of human capital from one unit of saved good and $x - 1$ units of (the learner's and/or teachers') human capital.

[10] There are more 'well-behaved' production functions that satisfy this property as well as homogeneity of degree one, twice differentiability, and strict quasi-concavity; e.g., $G(e_t, H_{t-1}) = \delta H_{t-1} + \gamma e_t / [\gamma + (e_t/H_{t-1})]$. The savings-enhancing effect of limited availability can also be derived by assuming those functions, but analytical results are harder to obtain than in the current linear setting.

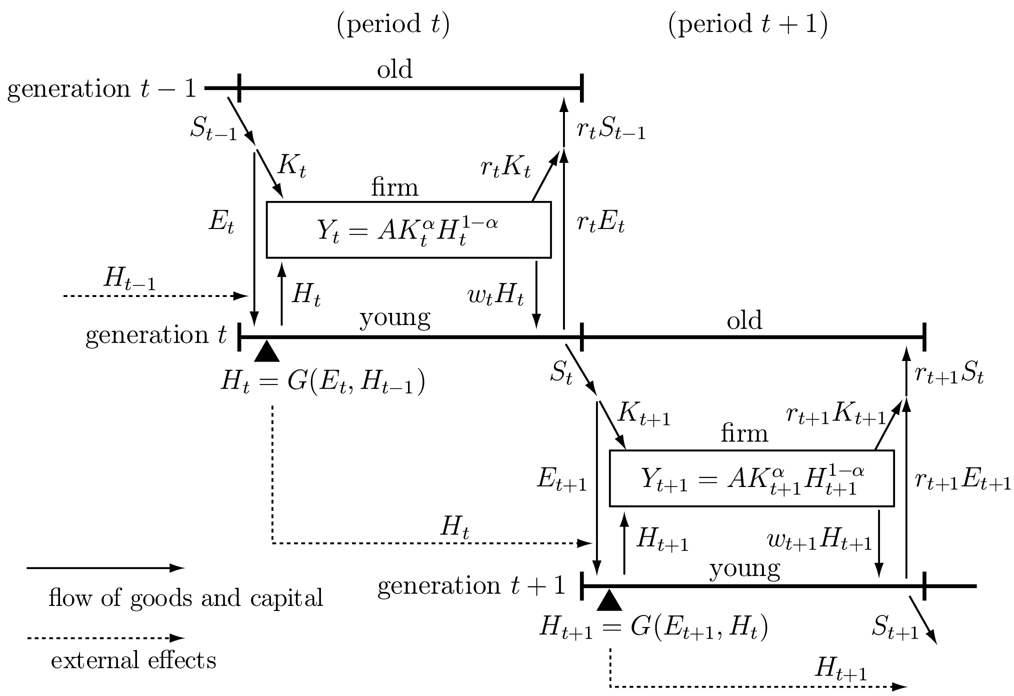

Figure 1: Time structure of the model

goods are supplied to the physical capital market, being used for either the production of the final good or that of human capital. At the end of that period, the household consumes $r_{t+1}s_t$ units of the produced good which is received in compensation for her supply of the saved goods, and dies. Figure 1 summarizes the timing of transactions, where H_t, E_t and S_t represent the aggregate levels of h_t, e_t and s_t, respectively.

Taking the values of w_t, r_t and H_{t-1} as given, every household born in period t chooses a level of e_t so as to maximize the net income at the end of period t, given by (5), because this choice leads to the maximization of her lifetime utility. Note that from the viewpoint of a newly born household, w_t represents the rate of return from higher education whereas r_t is the interest rate that applies to the intra-period education loan.

According to their relative magnitude, each household sets the level of e_t as

$$e_t = \begin{cases} 0 & \text{if } w_t/r_t < 1, \\ \forall e \in [0, \gamma H_{t-1}] & \text{if } w_t/r_t = 1, \\ \gamma H_{t-1} & \text{if } w_t/r_t > 1. \end{cases} \tag{6}$$

Once the level of e_t is determined, so are the levels of h_t, s_t, $c_{1,t}$ and $c_{2,t+1}$.

To close the model, we assume that, in period 1, there are old households of measure one, each of whom is endowed with S_0 units of the saved goods. The young households born in that period are assumed to have access to a human capital production function $h_1 = G(e_1, H_0)$, where H_0 is a positive constant. The old households supply the endowed good to the physical capital market, and in return receive $r_1 S_0$ units of the produced good. They consume those goods and die at the end of that period. Their lifetime utilities are given by $U_0 = \sigma \log c_{2,1} = \sigma \log r_1 S_0$.

3 Temporary Equilibrium

We begin the analysis by showing that the equilibrium of our model exhibits backward-looking dynamics, or equivalently, that temporary equilibria of the markets for human and physical capital in one period are uniquely determined by those in the previous period.

Consider the human capital market equilibrium at a generic period $t \geq 1$. From (4) and (6), we can derive the aggregate supply of human capital as a function of the ratio of factor prices, w_t/r_t,

$$H_t^S = \begin{cases} \delta H_{t-1} & \text{if } w_t/r_t < 1, \\ \forall h \in [\delta H_{t-1}, (\delta + \gamma) H_{t-1}] & \text{if } w_t/r_t = 1, \\ (\delta + \gamma) H_{t-1} & \text{if } w_t/r_t > 1. \end{cases} \tag{7}$$

Equation (1) implies that, given the aggregate level of physical capital used in the production of the final good, K_t, the aggregate demand for human capital can also be

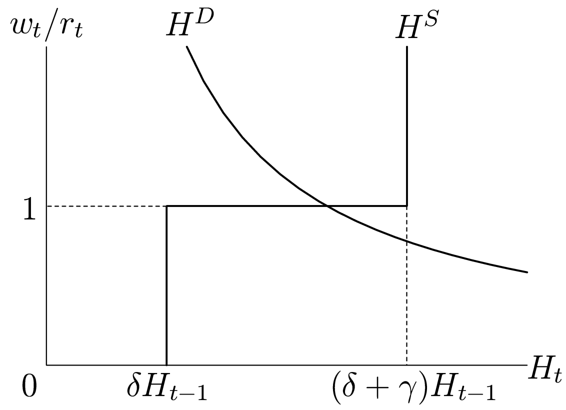

Figure 2: Demand and supply of human capital

expressed as a function of w_t/r_t,

$$H_t^D = \frac{1-\alpha}{\alpha}\frac{r_t}{w_t}K_t. \tag{8}$$

Figure 2 depicts the graphs of (7) and (8), the supply and demand curves of human capital. As shown in figure 2, the two graphs have a unique intersection for any value of $K_t > 0$. When K_t takes a value between $\alpha\delta H_{t-1}/(1-\alpha)$ and $\alpha(\delta+\gamma)H_{t-1}/(1-\alpha)$, the demand curve intersects with the flat segment of the supply curve, and the prices of human and physical capital are equalized; i.e., $w_t/r_t = 1$. This is not a surprising result, since both kinds of capital are produced from the saved goods on a one-to-one basis.[11] In this case, the factor intensity is determined as $H_t/K_t = (1-\alpha)/\alpha$, at which the marginal products of human and physical capital equate with each other. When K_t takes a value outside that interval, the demand curve intersects with one of the vertical sections of the supply curve, and the prices of human and physical capital differ in equilibrium. In this case, the value of H_t has reached either the upper or lower limit, so that the factor intensity cannot be adjusted to the level at which the marginal products are equalized.

[11] The same result is also obtained by Barro and Sala-i-Martin (2004, section 4.2).

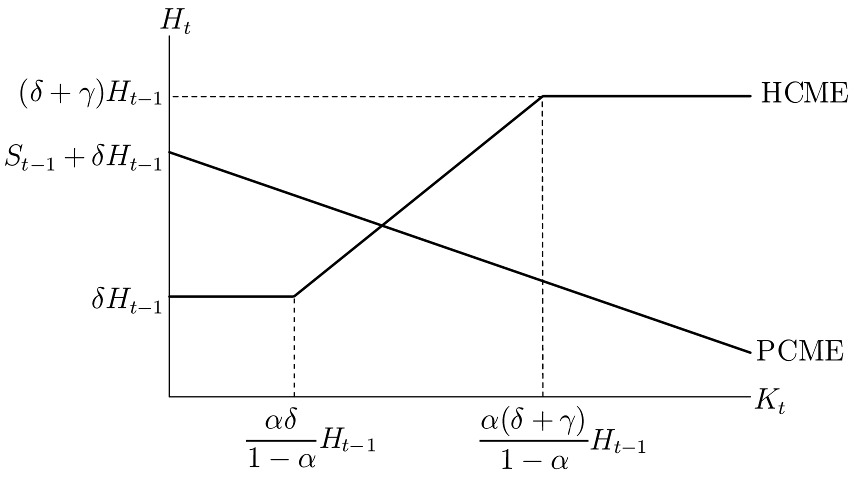

Figure 3: The HCME and PCME curves

To summarize, the human capital market determines the value of H_t as a function of K_t:

$$H_t = \begin{cases} \delta H_{t-1} & \text{if } K_t < \frac{\alpha\delta}{1-\alpha}H_{t-1}, \\ \frac{1-\alpha}{\alpha}K_t & \text{if } K_t \in [\frac{\alpha\delta}{1-\alpha}H_{t-1}, \frac{\alpha(\delta+\gamma)}{1-\alpha}H_{t-1}], \\ (\delta+\gamma)H_{t-1} & \text{if } K_t > \frac{\alpha(\delta+\gamma)}{1-\alpha}H_{t-1}. \end{cases} \tag{9}$$

Next, consider the physical capital market equilibrium at period t. While the aggregate supply of physical capital (the saved goods) equals the aggregate savings in the previous period, S_{t-1}, the aggregate demand for physical capital consists of E_t and K_t, the former of which represents the demand arising from the investment into human capital and the latter the demand arising from the production of the final good. Because the level of E_t is given by $E_t = H_t - \delta H_{t-1}$, the equilibrium condition for the physical capital market can be expressed as

$$H_t - \delta H_{t-1} + K_t = S_{t-1}. \tag{10}$$

Given the values of H_{t-1} and S_{t-1}, the equilibrium values of H_t and K_t are uniquely determined by (9) and (10). To see this, we only need to draw the graphs of these conditions in the (K_t, H_t) space. As depicted in figure 3, the graph of (9), which we call the

HCME (human capital market equilibrium) curve, is a piece-wise linear curve consisting of an upward-sloping segment in the middle and two flat segments corresponding to the upper and lower limits on H_t, whereas the graph of (10), which we call the PCME (physical capital market equilibrium) curve, is a downward-sloping line with a gradient of -1. Note that locations of these curves are fully determined by the values of H_{t-1} and S_{t-1}, and that the two curves intersect each other only once. This means that the equilibrium values of H_t and K_t are uniquely determined, being expressed as

$$H_t = H(H_{t-1}, S_{t-1}), \tag{11}$$

$$K_t = K(H_{t-1}, S_{t-1}). \tag{12}$$

We can also show that the aggregate level of savings in period t, S_t, is a function of H_{t-1} and S_{t-1}. As seen in the previous section, young households save the σ percentage of their net income, and thus the individual level of savings in period t, s_t, is given by

$$s_t = \sigma(w_t h_t - r_t e_t) = \sigma\{w_t \delta H_{t-1} + (w_t - r_t)e_t\} \text{ for } e_t \leq \gamma H_{t-1}. \tag{13}$$

Aggregating (13) over young households in period t and using the fact that $E_t = H_t - \delta H_{t-1}$, we obtain $S_t = \sigma\{w_t H_t - r_t(H_t - \delta H_{t-1})\}$. As implied by (1), (11) and (12), the equilibrium values of r_t, w_t and H_t are uniquely determined, given the values of H_{t-1} and S_{t-1}. Thus the equilibrium value of S_t can be expressed as

$$S_t = S(H_{t-1}, S_{t-1}). \tag{14}$$

Note that (11) and (14) constitute a mapping from (H_{t-1}, S_{t-1}) to (H_t, S_t), which governs the evolution of this economy. Given the values of H_0 and S_0, it uniquely determines the equilibrium sequence, $\{(H_t, S_t)\}_{t=1}^{\infty}$, which, in turn, determines the sequence of other aggregate variables, K_t, r_t and w_t through (1) and (12). Thus, to investigate the dynamical system of this economy, we need to derive an explicit form of that mapping. This task necessitates dividing all possible situations into three cases by the value of S_{t-1}/H_{t-1}.

Case 1. When $S_{t-1}/H_{t-1} < \alpha\delta/(1-\alpha)$, the PCME curve intersects with the bottom flat segment of the HCME curve, and so the equilibrium values of H_t and K_t are, respectively, determined as $K_t = S_{t-1}$ and

$$H_t = \delta H_{t-1}. \tag{15}$$

Substitution of these values into (1) yields

$$w_t = A(1-\alpha)\left(\frac{S_{t-1}}{\delta H_{t-1}}\right)^{\alpha} < A\alpha\left(\frac{S_{t-1}}{\delta H_{t-1}}\right)^{\alpha-1} = r_t.$$

Recall that w_t can be viewed as the rate of return from higher education. When this rate of return is smaller than the interest rate r_t, young households in period t are not willing to participate in higher education and merely supply the endowed human capital to the market, receiving in return $w_t\delta H_{t-1}$ units of the produced good. They save the σ percentage of that income, and thus the aggregate level of savings in period t is given by

$$S_t = \sigma w_t \delta H_{t-1} = \sigma A(1-\alpha)(\delta H_{t-1})^{1-\alpha} S_{t-1}^{\alpha}. \tag{16}$$

Case 2. When $\alpha\delta/(1-\alpha) \leq S_{t-1}/H_{t-1} \leq (\alpha\delta+\gamma)/(1-\alpha)$, the PCME curve intersects with the upward-sloping segment of the HCME curve. In this case, factor intensity and prices are determined as $K_t/H_t = \alpha/(1-\alpha)$ and $w_t = r_t = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}$. From this result and (10), we obtain

$$H_t = (1-\alpha)(S_{t-1} + \delta H_{t-1}). \tag{17}$$

Since $w_t = r_t$, the cost and return of higher education are equalized with each other, implying that the level of human capital investment through education does not affect the net income of young households in period t. In this case, the net income is determined solely by the earnings of endowed human capital, $w_t\delta H_{t-1}$. They save the σ percentage of that income, and so the aggregate level of savings in period t is given by

$$S_t = \sigma w_t \delta H_{t-1} = \sigma A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\delta H_{t-1}. \tag{18}$$

Case 3. When $S_{t-1}/H_{t-1} > (\alpha\delta+\gamma)/(1-\alpha)$, the PCME curve intersects with the top flat segment of the HCME curve, and so the equilibrium values of H_t and K_t are, respectively, determined as $K_t = S_{t-1} - \gamma H_{t-1}$ and

$$H_t = (\delta+\gamma)H_{t-1}. \tag{19}$$

Substitution of these values into (1) yields

$$w_t = A(1-\alpha)\left(\frac{S_{t-1}-\gamma H_{t-1}}{(\delta+\gamma)H_{t-1}}\right)^{\alpha} > A\alpha\left(\frac{S_{t-1}-\gamma H_{t-1}}{(\delta+\gamma)H_{t-1}}\right)^{\alpha-1} = r_t.$$

This inequality shows that the rate of return from higher education exceeds the interest rate. Thus, young households in period t are willing to participate in higher education as much as availability permits, implying that their net income equals $w_t(\delta+\gamma)H_{t-1} - r_t\gamma H_{t-1}$. Since they save the σ percentage of that income, the aggregate level of savings in period t is given by

$$\begin{aligned} S_t &= \sigma(w_t(\delta+\gamma)H_{t-1} - r_t\gamma H_{t-1}) \\ &= \sigma A\left((1-\alpha)S_{t-1} - \gamma H_{t-1}\right)\left(\frac{(\delta+\gamma)H_{t-1}}{S_{t-1}-\gamma H_{t-1}}\right)^{1-\alpha}. \end{aligned} \tag{20}$$

4 Dynamics and Steady States

Now we are in a position to analyze the dynamic behavior of this economy. As shown in the previous section, the equilibrium dynamics is completely described by the movement of two variables H_t and S_t, which evolves according to (15)-(20) given the initial values of H_0 and S_0. In this section, we further simplify the dynamic system by redefining variable and parameters as follows:

$$X_t \equiv \frac{S_t}{\sigma A H_t}, \quad \Delta \equiv \frac{\delta}{\sigma A}, \quad \Gamma \equiv \frac{\gamma}{\sigma A}. \tag{21}$$

By definition, the values of X_t, Δ and Γ are all positive. The new variable X_t summarizes how households born in period t contribute to the increased production of goods in period $t+1$. Recall that their activities can increase the production in period $t+1$ both through enhancing the supply of the saved goods in that period (S_t) and through the external effect of aggregate human capital on the next generation's achievement in higher education (H_t). A large value of X_t suggests that a greater part of their contributions has been made through the first channel, rather than the second channel. The new parameters, Δ and Γ, respectively, are normalized measures of δ and γ.

Using (21), we can rewrite (15)-(20) as a dynamic system with one variable and two parameters,

$$X_t = \Psi(X_{t-1}; \Delta, \Gamma) \quad \text{for all } t \geq 1. \tag{22}$$

In (22), the shape of function Ψ is given by a combination of three curves:

$$\Psi(X_{t-1}; \Delta, \Gamma) \quad \equiv \begin{cases} \Psi_1(X_{t-1}; \Delta) & \text{if } X_{t-1} \in \left(0, \frac{\alpha\Delta}{1-\alpha}\right), \\ \Psi_2(X_{t-1}; \Delta) & \text{if } X_{t-1} \in \left[\frac{\alpha\Delta}{1-\alpha}, \frac{\alpha\Delta+\Gamma}{1-\alpha}\right], \\ \Psi_3(X_{t-1}; \Delta, \Gamma) & \text{if } X_{t-1} \in \left(\frac{\alpha\Delta+\Gamma}{1-\alpha}, \infty\right), \end{cases}$$

where those three curves are defined by

$$\Psi_1(X_{t-1}; \Delta) \equiv \frac{(1-\alpha)X_{t-1}^{\alpha}}{\Delta^{\alpha}},$$

$$\Psi_2(X_{t-1}; \Delta) \equiv \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \frac{\Delta}{X_{t-1}+\Delta}, \tag{23}$$

$$\Psi_3(X_{t-1}; \Delta, \Gamma) \equiv \frac{(1-\alpha)X_{t-1} - \Gamma}{(\Delta+\Gamma)^{\alpha}(X_{t-1}-\Gamma)^{1-\alpha}}. \tag{24}$$

Equilibrium sequences of X_t, H_t and S_t are derived as follows. Given the initial value $X_0 \equiv S_0/(\sigma A H_0)$, equation (22) uniquely determines the equilibrium sequence $\{X_t\}_{t=1}^{\infty}$ in a recursive manner. Then, given the obtained sequence of X_t, the equilibrium sequences of H_t and S_t are determined by the relations,

$$H_t/H_{t-1} = \begin{cases} \Delta\sigma A & \text{if } X_{t-1} \in \left(0, \frac{\alpha\Delta}{1-\alpha}\right), \\ (1-\alpha)(\Delta + X_{t-1})\sigma A & \text{if } X_{t-1} \in \left[\frac{\alpha\Delta}{1-\alpha}, \frac{\alpha\Delta+\Gamma}{1-\alpha}\right], \\ (\Delta+\Gamma)\sigma A & \text{if } X_{t-1} \in \left(\frac{\alpha\Delta+\Gamma}{1-\alpha}, \infty\right). \end{cases} \tag{25}$$

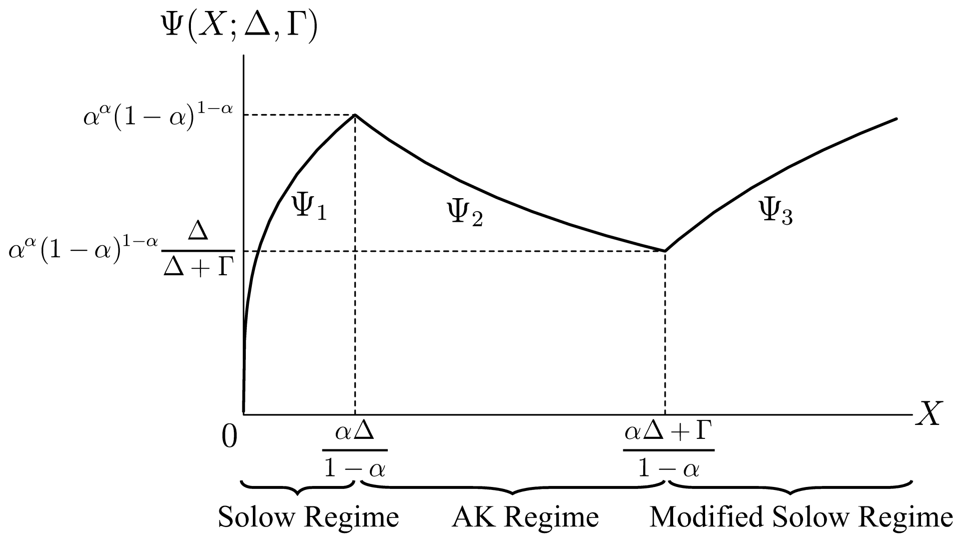

Figure 4: The Ψ curve

and $S_t = \sigma A H_t X_t$ for all $t \geq 1$, both of which are derived from (15)-(21). Since the steps of deriving the sequences of H_t and S_t are straightforward, we concentrate on the behavior of X_t in the rest of this section.

Figure 4 depicts the graph of function Ψ, which we call the Ψ curve. It is comprised of three segments called Ψ_1, Ψ_2 and Ψ_3 curves. When $X_{t-1} \in [\frac{\alpha\Delta}{1-\alpha}, \frac{\alpha\Delta+\Gamma}{1-\alpha}]$, the arbitrage between two sorts of capital equalizes their marginal productivities with each other, keeping the factor intensity constant.[12] In this case, the aggregate production is linear in the amount of overall capital stock $(K_t + H_t)$, which is equal to the sum of saved goods and transferred human capital: $(K_t + H_t) = S_{t-1} + \delta H_{t-1}$ (see equation 10). This representation suggests that, as long as X_t stays within this interval, the workings of our model are basically similar to the AK-type endogenous growth models.[13] In this *AK Regime* the Ψ curve is downward sloping, which can be explained as follows. Note

[12]Recall *Case 2* in the previous section.

[13]To see this point, define $\mathcal{K}_t \equiv K_t + H_t$ and $\mathcal{A} \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}A$. Then, the amount of production can be expressed as $Y_t = \mathcal{A}\mathcal{K}_t$.

that a large value of $X_{t-1}(\equiv S_{t-1}/\delta A H_{t-1})$ means that the supply of saved goods, S_{t-1}, constitutes a large portion of overall capital in period t. As shown by equations (17) and (18), however, a large S_{t-1} raises H_t but not S_t, reducing the percentage of saved goods in the overall capital supplied for the next period. Therefore, a large X_{t-1} is followed by a small X_t.

When $X_{t-1} < \frac{\alpha\Delta}{1-\alpha}$ or $X_{t-1} > \frac{\alpha\Delta+\Gamma}{1-\alpha}$, the growth rate of H_t becomes constant.[14] In this case, we can interpret H_t as the efficiency unit of labor per worker growing at a constant rate, and $Y_t = (AH_t^{1-\alpha})K_t^{\alpha}$ as a production function of the neoclassical growth model with exogenous labor-augmenting technological progress. When $X_{t-1} < \frac{\alpha\Delta}{1-\alpha}$, in particular, our model becomes essentially the same as that of Solow (1956). In this *Solow Regime*, a fixed percentage of the aggregate output in period $t-1$ is saved and used entirely as physical capital in period t, i.e., $K_t = S_{t-1} = \sigma(1-\alpha)Y_{t-1}$. This structure, which is also observed in the Solow model, defines the shape of the Ψ_1 curve in such a way that it is increasing, concave, and starting from the origin. In the *Modified Solow Regime*, where $X_{t-1} > \frac{\alpha\Delta+\Gamma}{1-\alpha}$, a certain amount of the saved goods are used for higher education and the remainder becomes physical capital. This makes Ψ_3 curve located below the extension of the Ψ_1 curve, but the increasing and concave properties of the curve are preserved.[15]

The global dynamics of X_t is critically dependent on intersecting patterns of the Ψ curve and the 45-degree line, which are summarized in Figure 5. Intersection points of the Ψ curve and the 45-degree line represent steady states of this economy, in which

[14]Recall *Cases 1 and 3* in the previous section.

[15]Our approach of dividing the patterns of the dynamics into the neoclassical and endogenous growth regimes is similar to that of Matsuyama (1999), who classified the dynamics into the Solow and Romer regimes. A novel property of our model is that the economy behaves as described in the neoclassical growth models not only when X_t is small but also when it is large, giving rise to the possible existence of multiple steady states.

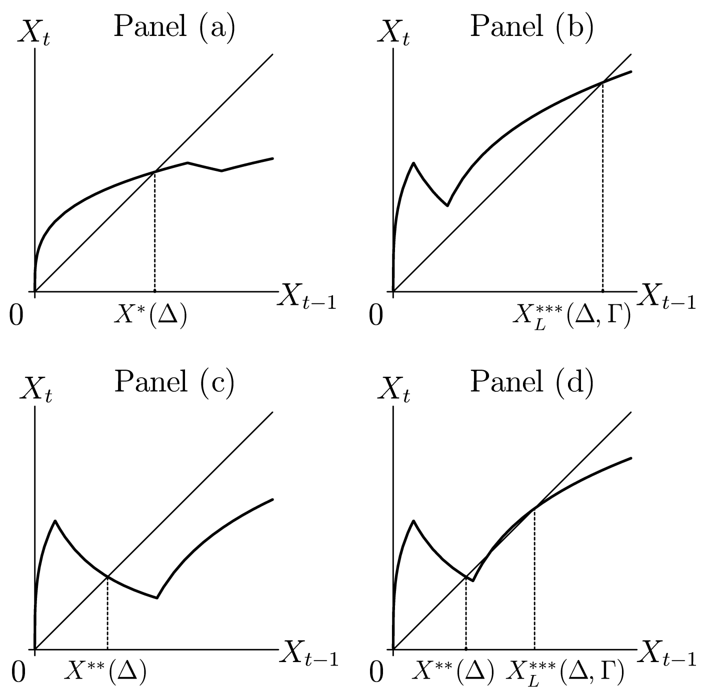

Figure 5: Four patterns of dynamics

the aggregate amounts of output, consumption, physical capital, and human capital are all growing at a common rate. Our model has at least one steady state, since $\Psi(\cdot)$ is a continuous function satisfying $\lim_{X\to+0}\Psi'(X;\Delta,\Gamma)=\infty$ and $\lim_{X\to\infty}\Psi'(X;\Delta,\Gamma)=0$ for all values of Δ and Γ. Parameter values determine the number of steady states. As depicted in figure 5, the Ψ curve and the 45-degree line can intersect with each other as many as three times, dependent on the values of Δ and Γ. Thus, to examine the long-run behavior of the economy, we need to clarify how the parameter values restrict patterns of dynamics.

A straightforward calculation shows that the economy converges to a steady state in the Solow regime as shown by panel (a) whenever[16]

$$\Delta \geq \alpha^{\alpha-1}(1-\alpha)^{2-\alpha} \equiv \overline{\Delta}. \tag{26}$$

[16]Condition (26) is derived from $\alpha^{\alpha}(1-\alpha)^{1-\alpha} < \alpha\Delta/(1-\alpha)$. See Figure 4.

We ignore this possibility by assuming that $\Delta < \overline{\Delta}$, since the convergence to the Solow regime implies that no human capital is accumulated in the long run and therefore the availability of higher education is irrelevant for those economies.

4.1 Economies with Insufficient Availability

Let us consider the dynamics for economies where the availability of higher education is low. If Γ is below a threshold level defined by[17]

$$\overline{\Gamma}(\Delta) \equiv \frac{\Delta}{2}\left(-(1+\alpha)+(1-\alpha)\sqrt{1+\frac{4}{\Delta}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}}\right), \tag{27}$$

the economy converges to a steady state in the modified Solow regime from any initial state, as depicted by panel (b) of figure 5. The unique steady state $X_L^{***}(\Delta,\Gamma) \in [\frac{\alpha\Delta+\Gamma}{1-\alpha}, +\infty)$ is given by the larger root of equation $\Psi_3(X;\Delta,\Gamma) = X$, where $\Psi_3(X;\Delta,\Gamma)$ is defined by (24).

In the steady state, the aggregate amounts of output, consumption, human, and physical capital are growing at a constant rate of $\delta+\gamma = (\Delta+\Gamma)\sigma A$. In these economies, the availability of higher education is too limited relative to the economy's ability to supply the saved goods. The limited availability keeps the rate of return from higher education greater than the interest rate in the long run, which gives an incentive for young households to participate in higher education as much as the availability permits. Thus the long-term rate of economic growth as well as the rate of human capital accumulation is determined by the availability of higher education.

4.2 Possibility of Multiple Steady States

When $\Gamma \geq \overline{\Gamma}(\Delta)$, the dynamical system (22) has a steady state in the AK regime (See panels (c) and (d) of Figure 5). Solving equation $\Psi_2(X;\Delta) = X$, where $\Psi_2(X;\Delta)$

[17] $\overline{\Gamma}(\Delta)$ is the unique positive root of equation $(\alpha\Delta+\Gamma)/(1-\alpha) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\Delta/(\Delta+\Gamma)$. See Figure 4.

is defined by (23), we see that the steady state is located at

$$X^{**}(\Delta) \equiv \frac{\alpha\Delta + \overline{\Gamma}(\Delta)}{1-\alpha}. \tag{28}$$

In this steady state, the aggregate amounts of output, consumption, human, and physical capital are growing at the rate of $\delta + \sigma A\overline{\Gamma}(\Delta) = (\Delta + \overline{\Gamma}(\Delta))\sigma A$, which is between δ and $\delta+\gamma$. In this steady state, marginal products of human and physical capital are equalized with each other, and the amount of education is reaching neither zero nor the maximum limit implied by the availability. Thus, a small change in γ has no influence on the long-term growth rate.

As panel (d) suggests, however, $X^{**}(\Delta)$ is not necessarily the unique steady state of the economy. The 45-degree line may intersect not only with the Ψ_2 curve but also with Ψ_3, giving rise to further two steady states in the modified Solow regime. For the existence of multiple steady states, both Δ and Γ must satisfy certain conditions. First, note that the smaller parameter Δ is, the steeper the Ψ_3 curve becomes (see the definition of function Ψ_3 in equation 24). Since the Ψ_3 curve is concave, for it to intersect with the 45-degree curve twice, Δ must be small enough so that the slope of the Ψ_3 curve at the left end (i.e. at $X_{t-1} = \frac{\alpha\Delta+\Gamma}{1-\alpha}$) must be greater than one. In particular, Δ must be smaller than[18]

$$\widehat{\Delta} \equiv \left(3/2 - \alpha + \sqrt{\alpha^{-1} - 3/4}\right)^{-1} \overline{\Delta} \tag{29}$$

for the case in which the left end of the Ψ_3 curve is exactly on the 45-degree curve, as illustrated by point A in Figure 6 (that is, when $\Gamma = \overline{\Gamma}(\Delta)$). As will be clear from Figure 6 and the discussion below, for every $\Gamma \geq \overline{\Gamma}(\Delta)$, condition $\Delta < \widehat{\Delta}$ is necessary for the existence of multiple steady states.[19]

[18]This condition is obtained from $1 < \Psi_3'((\alpha\Delta + \overline{\Gamma}(\Delta))/(1-\alpha); \Delta, \overline{\Gamma}(\Delta))$. Note that $\widehat{\Delta}$ is smaller than $\overline{\Delta}$, since $3/2 - \alpha + \sqrt{\alpha^{-1} - 3/4} > 3/2 - 1 + \sqrt{1 - 3/4} = 1$. A detailed derivation of $\widehat{\Delta}$ is available

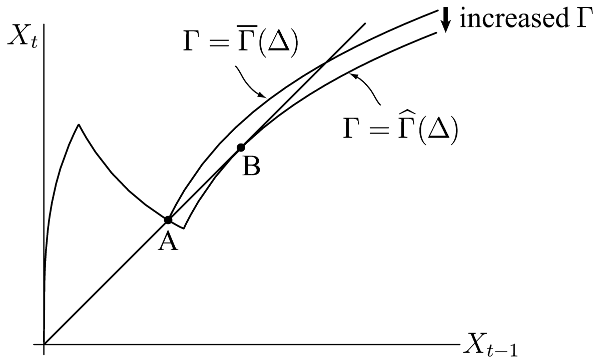

Figure 6: Existence of multiple steady states. It depicts the Ψ curve for two values of Γ, keeping the value of Δ fixed at $\Delta < \widehat{\Delta}$. If $\Delta > \widehat{\Delta}$, the slope of Ψ_3 curve at point A becomes less steeper than the 45-degree line, and the stady state is always unique.

Second, observe from (24) that a larger Γ pushes the whole Ψ_3 curve downward.[20] Therefore, for the existence of multiple steady states, Γ should not be too large so that the position of the Ψ_3 curve is not too low. From the concavity of the Ψ_3 curve, there must be a value of $\Gamma \in (\overline{\Gamma}(\Delta), \infty)$ such that the Ψ_3 curve touches the 45-degree line at one point, as illustrated by point B in Figure 6. Letting $\widehat{\Gamma}(\Delta)$ denote this value, it is apparent that multiple steady states exist only when $\Gamma \in [\overline{\Gamma}(\Delta), \widehat{\Gamma}(\Delta)]$. The following proposition shows that this upper threshold is decreasing in Δ.

Proposition 1. *For all* $\Delta < \widehat{\Delta}$*,* $\widehat{\Gamma}(\Delta)$ *is differentiable and* $\widehat{\Gamma}'(\Delta) \in (-1, 0)$*. In addition,* $\lim_{\Delta \to \widehat{\Delta}} \widehat{\Gamma}(\Delta) = \overline{\Gamma}(\widehat{\Delta})$.

Proof: available upon request.

Intuitively, if Δ is larger, the Ψ_3 curve is flatter, and therefore there will be a narrower

upon request.

[19]If the gradient at the left end is less than unity when $\Gamma = \overline{\Gamma}(\Delta)$, then the concavity of Ψ_3 curve implies that there is no section of the Ψ_3 curve which is above the 45-degree line. Since the Ψ_3 curve shifts down when Γ increases, there would be no multiple steady states for any $\Gamma \geq \overline{\Gamma}(\Delta)$.

[20]Equation (24) can be written as $\Psi_3(X; \Delta, \Gamma) = (X - \Gamma)^{\alpha}(\Delta + \Gamma)^{-\alpha}(1 - \alpha X/(X - \Gamma))$. It is straightforward to confirm that this expression is decreasing in Γ for all $X \geq \frac{\alpha\Delta + \Gamma}{1-\alpha}$.

range of Γ for which the Ψ_3 curve intersects with the 45-degree line twice. The following two subsections in turn examine the dynamics of the economy without and with multiple steady states.

4.3 Economies with Excessive Availability

Now consider the dynamics of the economy where the availability of higher education is high. The analysis in the previous subsection implies that the pattern of the dynamics looks like panel (c) of figure 5 when $\Gamma > \widehat{\Gamma}(\Delta)$ in the case of $\Delta < \widehat{\Delta}$, and when $\Gamma \geq \overline{\Gamma}(\Delta)$ in the case $\Delta \in [\widehat{\Delta}, \overline{\Delta})$. The following proposition demonstrates that the economy converges to the unique steady state $X^{**}(\Delta)$ from any initial state.

Proposition 2. *Suppose that the values of Δ and Γ satisfy either of the conditions that $\Delta < \widehat{\Delta}$ and $\Gamma > \widehat{\Gamma}(\Delta)$, or that $\Delta \in [\widehat{\Delta}, \overline{\Delta})$ and $\Gamma \geq \overline{\Gamma}(\Delta)$. Then, the sequence of X_t converges to $X^{**}(\Delta)$ from any value of X_0.*
Proof: available upon request.

Starting from any initial state, the sequence of X_t enters a neighborhood of $X^{**}(\Delta)$ in finite time. Then, it begins to oscillate around $X^{**}(\Delta)$, but eventually converges to that steady state. This oscillation implies that, in converging to the steady state, the economy alternately experiences accelerated and decelerated growth. These economies have excessive availability of higher education not only in that the education facilities are not fully utilized, but also in that a higher rate of economic growth could have been sustained if the availability were not so high. The latter point will be confirmed immediately.

4.4 Economies with Intermediate Availability and Low Δ

When Γ is within the interval $[\overline{\Gamma}(\Delta), \widehat{\Gamma}(\Delta)]$ in an economy with $\Delta < \widehat{\Delta}$, the Ψ curve and the 45-degree line have three intersection points, as depicted in panel (d) of figure

5.[21] In addition to $X^{**}(\Delta)$, there are two steady states in the modified Solow regime. These are $X_S^{***}(\Delta, \Gamma)$ and $X_L^{***}(\Delta, \Gamma)$, respectively defined as the smaller and the larger roots of equation $X = \Psi_3(X; \Delta, \Gamma)$.

Panel (d) suggests that, in this case, an economy's destination depends critically on its initial state. If $X_0 > X_S^{***}(\Delta, \Gamma)$, then the sequence of X_t monotonically converges to $X_L^{***}(\Delta, \Gamma)$. This is similar to the dynamics observed in the case of insufficient availability. If X_0 is given in a neighborhood of $X^{**}(\Delta)$, the sequence of X_t follows dynamics essentially the same as the case of excessive availability, converging to that steady state. Although the dynamics become less obvious when the initial state is far from these two stable steady states, the following proposition shows that, from almost all initial states, the sequence of X_t eventually converges either to $X^{**}(\Delta)$ or to $X_L^{***}(\Delta, \Gamma)$, thus allowing us to regard these steady states as plausible long-run states.

Proposition 3. *Suppose that the values of Δ and Γ satisfy $\Delta < \widehat{\Delta}$ and $\Gamma \in [\overline{\Gamma}(\Delta), \widehat{\Gamma}(\Delta)]$. Then,*

a. X_t converges to one of $X^{**}(\Delta)$, $X_L^{***}(\Delta, \Gamma)$, and $X_S^{***}(\Delta, \Gamma)$.

b. From almost all initial states, X_t converges either to $X^{**}(\Delta)$ or $X_L^{***}(\Delta, \Gamma)$.

c. If $X_0 > X_S^{***}(\Delta, \Gamma)$, X_t monotonically converges to $X_L^{***}(\Delta, \Gamma)$.

d. $X^{**}(\Delta)$ is a locally stable steady state except for the case of $X^{**}(\Delta) = X_S^{***}(\Delta, \Gamma)$.

Proof: available upon request.

A comparison of the two stable steady states reveals that the economy is growing faster in steady state $X_L^{***}(\Delta, \Gamma)$ than in $X^{**}(\Delta)$.[22] In the former steady state, the

[21] Strictly speaking, panel (d) depicts the Ψ curve when $\Gamma \in (\overline{\Gamma}(\Delta), \widehat{\Gamma}(\Delta))$. When $\Gamma = \overline{\Gamma}(\Delta)$, $X_S^{***}(\Delta, \Gamma)$ coincides with $X^{**}(\Delta)$. When $\Gamma = \widehat{\Gamma}(\Delta)$, $X_S^{***}(\Delta, \Gamma)$ coincides with $X_L^{***}(\Delta, \Gamma)$.

[22] This may seem tricky because $X_L^{***}(\Delta, \Gamma)$ corresponds to the unique steady state in an economy where higher education is insufficiently available, whereas $X^{**}(\Delta)$ corresponds to one in an economy with excessive availability. The point is that the economy is growing fully utilizing existing facilities for

supply of the saved goods in one period is so large relative to the availability of higher education in the economy that the marginal products of human and physical capital cannot be equalized even if the availability is fully utilized. This keeps the rate of return from higher education above the interest rate, distributing a considerable percentage of the current output to the young generation, and thereby realizing a large supply of the saved goods in the next period. Due to this virtuous circle, this steady state maintains large amounts of savings and hence fast growth, the rate of which is as high as $(\Delta+\Gamma)\sigma A$.

In the latter steady state $X^{**}(\Delta)$, by contrast, the supply of the saved goods in one period is not so large relative to the availability of higher education. In this case, the marginal products of human and physical capital are equalized through higher education available in the economy. This equalizes the rate of return from higher education with the interest rate, distributing only a small percentage of the current output to the young generation, and thereby realizing a small supply of the saved goods in the next period. Due to this vicious circle, this steady state maintains only small amounts of savings and hence slower growth, the rate of which is as low as $(\Delta + \overline{\Gamma}(\Delta))\sigma A$.

Despite its restricting effects on the accumulation of human capital, the limited availability of higher education enables the economy to exhibit a fast growth through ensuring a sufficient supply of investible funds in each period. If there is no limit in the availability of higher education, the marginal products of human and physical capital would always be equalized to each other, and, as a result, $X^{**}(\Delta)$ would be the unique steady state in our model.

4.5 Summary

So far, we have seen how an economy's long-term growth rate is related to values of Γ and Δ. The observed relations are summarized by figure 7 and Table 1. Figure 7

higher education in steady state $X_L^{***}(\Delta, \Gamma)$, while the availability is not fully exploited in $X^{**}(\Delta)$.

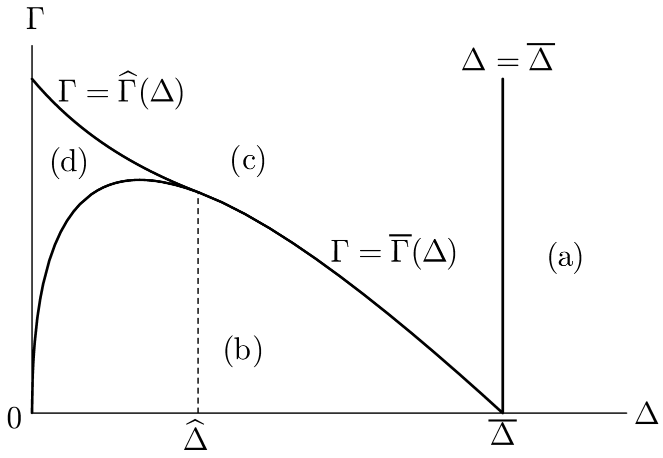

Figure 7: Division of the parameter space. The figure is numerically drawn by fixing the value of α at 0.3. Given the value of α, the locations of the three graphs are uniquely determined, since the functional forms of $\overline{\Gamma}(\cdot)$ and $\widehat{\Gamma}(\cdot)$, and hence the values of $\overline{\Delta}$ and $\widehat{\Delta}$, depend solely on α. In addition, Proposition 1 and the definitions of $\overline{\Delta}$ and $\overline{\Gamma}(\Delta)$ jointly guarantee that the shape and relative location of three graphs are maintained for other values of α.

region	stable steady states	growth rate	regime
(a)	$X^{*}(\Delta)$	$\Delta\sigma A$	Solow
(b)	$X_L^{***}(\Delta,\Gamma)$	$(\Delta+\Gamma)\sigma A$	modified Solow
(c)	$X^{**}(\Delta)$	$(\Delta+\overline{\Gamma}(\Delta))\sigma A$	AK
(d)	$X_L^{***}(\Delta,\Gamma)$	$(\Delta+\Gamma)\sigma A$	modified Solow
	$X^{**}(\Delta)$	$(\Delta+\overline{\Gamma}(\Delta))\sigma A$	AK

Table 1: Steady states and long-term growth rates in each region

depicts how the whole (Δ, Γ) space is divided into four regions by graphs of $\Gamma = \overline{\Gamma}(\Delta)$, $\Gamma = \widehat{\Gamma}(\Delta)$ and $\Delta = \overline{\Delta}$. Each region corresponds to a panel of the same label in figure 5, from which we can observe the pattern of the dynamics of the economy. Table 1 lists stable steady states and their long-term growth rates for each region.

5 Impacts of Expanded Higher Education Opportunities

We now evaluate how economic growth is affected by an increased enrollment capacity in each facility and the establishment of new higher education institutions.[23] In the model, those changes are described by gradual increases in parameter γ (and therefore $\Gamma \equiv \gamma/(\sigma A)$). Results are dependent on another parameter Δ.

Economies with high Δ[24]

When $\Delta \in [\widehat{\Delta}, \overline{\Delta})$, the pair of parameters (Δ, Γ) is moving upward from region (b) to region (c) in figure 7 as the availability expands. The economy always has a globally stable steady state and, according to Table 1, the long-term growth rate is $(\Delta + \Gamma)\sigma A$ when $\Gamma \leq \overline{\Gamma}(\Delta)$ and $(\Delta + \overline{\Gamma}(\Delta))\sigma A$ when $\Gamma > \overline{\Gamma}(\Delta)$. Therefore, the expansion of higher education opportunities raises the long-term growth rate up to $(\Delta + \overline{\Gamma}(\Delta))\sigma A$, and thereafter has no effect.

Recall that $\Delta \equiv \delta/\sigma A$ is a product of the low dependence on higher education in transferring existing knowledge over generations, a low saving rate, and also a low TFP. These are typical characteristics of developing agrarian countries. The conventional wisdom that the expansion of education opportunity is beneficial for growth, at least when the availability is binding, applies to these countries.

[23]Throughout this section, those changes in the availability of higher education are treated as exogenous. We assume, however, that the availability changes only gradually over generations.

[24]We mean 'high Δ' within the assumption that $\Delta \leq \overline{\Delta}$.

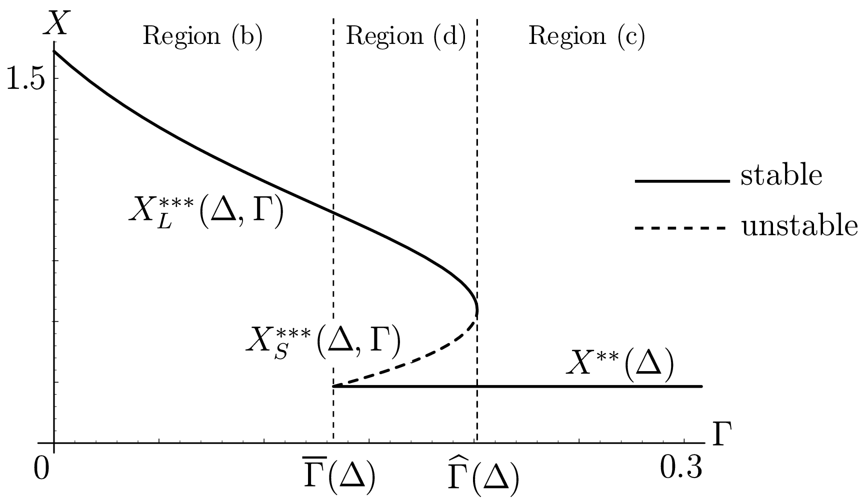

Figure 8: Evolution of steady states. Parameters: $\Delta = 0.1$, $\alpha = 0.3$

Economies with low Δ

We obtain different results when Δ is low, which is likely to be the case for more developed and industrialized economies. When $\Delta < \widehat{\Delta}$, the pair of parameters (Δ, Γ) is moving upward from region (b) through region (d) to region (c) in figure 7 as Γ takes a larger value. According to Table 1, the long-term growth rate is $(\Delta + \Gamma)\sigma A$ in region (b), whereas it is $(\Delta + \overline{\Gamma}(\Delta))\sigma A$ in region (c). Our remaining task is to determine the long-term growth rate when the pair belongs to region (d).

To this end, we must determine to which steady state the economy is converging because there are two stable steady states in region (d) and the long-term growth rate is different among them. Figure 8 is a bifurcation diagram that plots all steady states values of X_t against Γ. It suggests that until Γ reaches $\widehat{\Gamma}(\Delta)$, X_t stays in the neighborhood of $X_L^{***}(\Delta, \Gamma)$ as long as the increase in Γ is not too fast, because this steady state is (locally) stable. However, at the moment when Γ exceeds $\widehat{\Gamma}(\Delta)$, the steady state $X_L^{***}(\Delta, \Gamma)$ disappears and X_t is forced to converge to $X^{**}(\Delta)$, which is the unique and globally stable steady state in region (c). The implied variation in growth rate is striking. As long as Γ is below the threshold value $\widehat{\Gamma}(\Delta)$, a gradual expansion of higher education

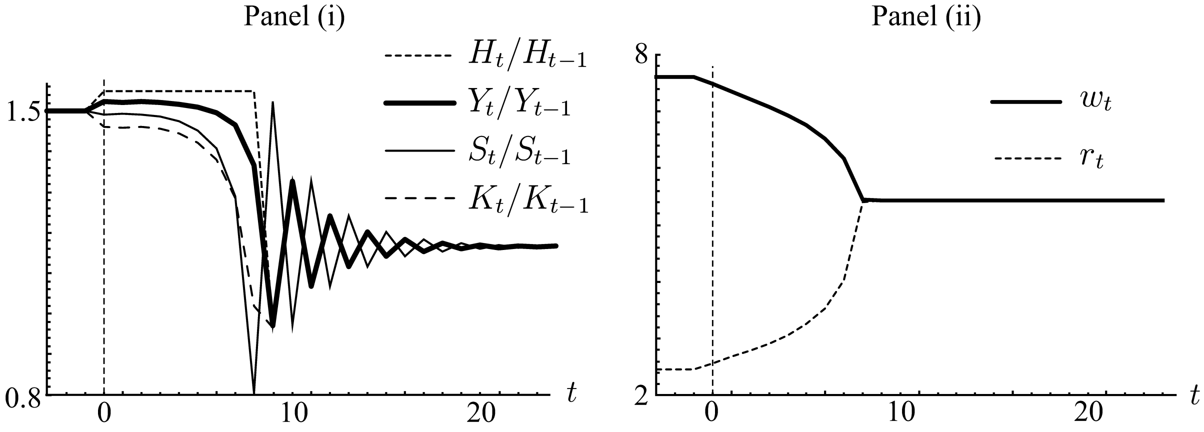

Figure 9: Responses of aggregate variables to a permanent expansion of higher education opportunities At date 0, γ is increased from 1 to 1.05 while other parameters are fixed at $A = 10$, $\alpha = 0.3$, $\sigma = 0.5$ and $\delta = 0.5$.

opportunities enhances the long-term growth rate, which is given by $(\Delta + \Gamma)\sigma A$ (see Table 1). However, once the availability exceeds the threshold value, the long-term growth rate drops sharply from $(\Delta + \widehat{\Gamma}(\Delta))\sigma A$ to $(\Delta + \overline{\Gamma}(\Delta))\sigma A$. The magnitude of slowdown is largest in highly industrialized economies with the smallest Δ.[25] It implies that the most serious slowdown occurs in economies that rely most heavily on higher education to maintain their growth. In addition, these economies are the ones in which the expansion of higher education opportunities is most effective, unless it exceeds the threshold.[26]

The movements of aggregate variables in transition help us understand the cause of this slowdown. Panel (i) of figure 9 depicts the evolution of the one-period growth rates of H_t, K_t, Y_t and S_t, while panel (ii) shows the movement of w_t and r_t against the

[25]The magnitude of slowdown is measured by $(\widehat{\Gamma}(\Delta) - \overline{\Gamma}(\Delta))\sigma A$, which is proportional to the vertical distance between $\widehat{\Gamma}(\Delta)$ curve and $\overline{\Gamma}(\Delta)$ curve in figure 7.

[26]By expanding higher education opportunities, the long-term growth rate can be increased from $\Delta\sigma A$ up to $(\Delta + \widehat{\Gamma}(\Delta))\sigma A$. Since $\widehat{\Gamma}(\Delta)$ is a decreasing function of Δ (See proposition 1), the maximum contribution of higher education to long-term growth, $\widehat{\Gamma}(\Delta)\sigma A$, is larger in economies with smaller Δ.

number of periods elapsed after a sudden and permanent 5% increase in the value of γ, which suffices to push the pair (Δ, Γ) from region (d) to region (c). At the period when the availability is expanded, households fully utilize the increased availability of higher education, increasing the growth rate of human capital while decreasing that of physical capital. Observe that this reallocation of resources increases the output of the economy, creating a temporary boom. This is a natural response: since the constraint on human capital formation is relaxed, the resources are allocated more efficiently, at least in a static sense, and therefore a larger amount of the consumption good is produced from a given amount of the saved goods.

To see how this boom turns into a slowdown, we need to notice that the growth rates of savings and physical capital keep decreasing during that boom. This process is caused by the accelerated growth of human capital. Specifically, it initially lowers the rate of return from higher education, w_t,and raises the interest rate (see Panel ii). These movements shift income from the young households who participate in higher education to the old who receive interest earnings, thereby causing a slight slowdown in the growth of aggregate savings. Combined with the accelerated growth of human capital, the decelerated growth of savings lowers the growth rate of physical capital, which leads to further decline in the return from higher education and another slight slowdown in the growth of savings, and so forth. As this process runs on, the gap between the rate of return from higher education and the interest rate is narrowed gradually, but at an accelerated pace. Eventually the gap vanishes, and the young households lose incentives to utilize the availability of higher education up to its limit. Thus, given that the supply of saved goods has been significantly reduced by that time, the growth rate of human capital drops sharply. From that period on, the factor intensity as well as the aggregate savings rate is constant over time, and the amount of output, human, and physical capital exhibits a common oscillation, ending up at their new, lower steady state levels.

We close this section by noting the seriousness of the slowdown. While a marginal expansion of higher education opportunities causes a significant slowdown, the high rate of growth cannot be recovered by simply reducing the availability of higher education to the previous level. Figure 8 explains why the slowdown is irreversible. Once X_t is driven into the neighborhood of $X^{**}(\Delta)$ by an excessive expansion, it will be trapped near this stable steady state as long as this steady state exists. To restore a high rate of economic growth, it is not sufficient to reduce Γ to below $\widehat{\Gamma}(\Delta)$, the level at which the slowdown is triggered.[27] Figure 8 shows that Γ must temporarily fall below $\overline{\Gamma}(\Delta)$ so that the economy can return to $X_L^{***}(\Delta, \Gamma)$. Then Γ must be increased to the former level, perhaps to near $\widehat{\Gamma}(\Delta)$, in order to raise the long-term growth rate to the pre-slowdown level. In reality, such a drastic change in the availability of higher education would necessitate a major restructuring of educational institutions. This is especially true when Δ is small, since in that case the distance between $\widehat{\Gamma}(\Delta)$ and $\overline{\Gamma}(\Delta)$ is considerable.

6 Concluding Remarks

Empirical studies have shown that higher education raises individual earnings, but that its contribution to economic growth in the long run has been less clear. This paper demonstrates that even when education raises everyone's productivity, its aggregate consequence on growth in the long run is not always positive. An excessive availability of higher education may trigger a serious economic slowdown, by shifting the intergenerational income distribution toward the old and reducing aggregate savings. This possibility is more relevant in countries where the transfer of existing knowledge is highly dependent on higher education.

Throughout this paper, we treated the availability of higher education as exogenous.

[27] Recall that in steady state $X^{**}(\Delta)$, the availability of higher education is not fully utilized. Thus, a marginal change in the availability does not really affect the working of the economy.

If the government could restrict the expansion of higher education, is there any possibility that it would, willingly or not, cause the slowdown? There are two possibilities. First, the government might not have enough knowledge to detect the onset of the slowdown process during the period of temporary boom. When the economy experiences a decline in the rate of economic growth, the supply of investible funds is already too small for the economy to return to the fast-growing steady state. Second, even when the government correctly anticipates the forthcoming slowdown, it might willingly leave the slowdown process to take its own course because it usually places greater weight on the utility of current generations than on that of future generations. During the temporary boom, old households enjoy a rapid growth of consumption owing to the higher interest rate, while the growth rate of the young households' consumption is gradually declining due to the decreasing return from higher education. If the government heavily discounts the utility loss of future generations, it may be justified to pursue this temporary gain.[28]

We have shown that expansion of higher education opportunities may result in adverse consequences even without considering the costs of implementation. If these costs are explicitly introduced, such expansion can lead to a more serious slowdown in economic growth. However, when the associated costs are substantial, the authorities would be more reluctant, or simply unable, to establish sufficient opportunities to trigger the slowdown process. Similarly, a limited ability of households to learn may effectively prevent the speed of human capital accumulation from becoming too fast. There is no denying that those limitations have been preventing overinvestment in human capital, thereby contributing to sustained growth.

[28]We found that, in the settings given in Figure 9, social welfare is improved by the excessive expansion of higher education opportunities whenever the intergenerational discount factor is smaller than 0.72.

Acknowledgements

We are grateful to Fumio Hayashi, Yoshiyasu Ono, Makoto Saito, Yoshihiko Seoka, Akihisa Shibata, Yoshimasa Shirai, Katsuya Takii, Makoto Yano and seminar participants at Hitotsubashi University, ISER, Keio University, Kansai Macroeconomics Workshop, Kyoto Conference on Education and Training, the University of Tokyo, and the Workshop on Spatial and Public Economics for their helpful comments and suggestions. All remaining errors are naturally ours.

References

Azariadis, C. and D. De la Croix (2003) "Growth or Equality?" mimeo, Université catholique de Louvain.

Azariadis, C. and A. Drazen (1990) "Threshold Externalities in Economic Development," *Quarterly Journal of Economics*, **105**, 501-526.

Barro, R. (1991) "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics*, **106**, 407-444.

Barro, R. and J. H. Lee (1994) "Sources of Economic Growth," *Carnegie Rochester Conference Series on Public Policy*, **40**, 1-46.

Barro, R. and X. Sala-i-Martin (2004) *Economic Growth,* 2nd Ed. MIT Press, Cambridge, Mass.

Benhabib, J. and M. Spiegel (1994) "The Role of Human Capital in Economic Development: Evidence from Aggregate Cross-Country Data," *Journal of Monetary Economics*, **34**, 143-174.

Card, D. (1999) "The Causal Effect of Education on Earnings," *Handbook of Labor Economics*, Volume 3, Amsterdam, Elsevier Science.

Caselli, F. (2005) "Accounting for Cross-Country Income Differences," in P. Aghion and S. Durlauf (Eds.), *Handbook of Economic Growth*, Amsterdam, Elsevier Science.

Christou, C. (2001) "Differential Borrowing Constraints and Investment in Human Capital," *Journal of Macroeconomics*, **23**(1), 277-295.

Diamond, P. A. (1965) "National Debt in a Neoclassical Growth Model," *American Economic Review*, **55**, 1126-1150.

De Gregorio, J. (1992) "Economic Growth in Latin America," *Journal of Development Economics*, **39**(1), 59-84.

De Gregorio, J. (1996) "Borrowing Constraints, Human Capital Accumulation, and Growth," *Journal of Monetary Economics*, **37**, 49-71.

De Gregorio, J. and S. J. Kim (2000) "Credit Market with Differences in Abilities: Education, Distribution and Growth," *International Economic Review*, **41**(3), 579-607.

De la Croix, D. and P. Michel (2002) *A Theory of Economic Growth.* Cambridge University Press, Cambridge, U.K.

Duan, X. (2003) "Chinese Higher Education Enters a New Era," *Academe*, **89**(6), 22-27.

Islam, N. (1995), "Growth Empirics: A Panel Data Approach," *Quarterly Journal of Economics*, **110**(4), 1127-70.

Jappelli, T. and M. Pagano (1994) "Saving, Growth, and Liquidity Constraint," *Quarterly Journal of Economics*, **109**(1), 83-109.

Kaneko, M. (1997) "Efficiency and Equity in Japanese Higher Education," *Higher Education*, **34**, 165-181.

Klenow P.J. and A. Rodrìguez-Clare (2005) "Externalities and Growth," in P. Aghion and S. Durlauf (Eds.), *Handbook of Economic Growth*, Amsterdam, Elsevier Science.

Mankiw, N. G., D. Romer and D. N. Weil (1992) "A Contribution to the Empirics of Economic Growth," *Quarterly Journal of Economics*, **107**, 407-437.

Matsuyama, K. (1999) "Growing through Cycles," *Econometrica*, **67**, 335-347.

Modigliani, F. (1986) "Life Cycle, Individual Thrift, and the Wealth of Nations," *American Economic Review*, **76**(3), 297-313.

Pritchett, L. (2001) "Where Has All the Education Gone?" *World Bank Economic Review*, **15**, 367-391.

Romer, P. M. (1986) "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, **94**, 1002-37.

Romer, P. M. (1989) "Human Capital and Growth: Theory and Evidence," *NBER Working Papers*, No. 3173.

Solow, R. M. (1956) "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, **70**, 65-94.

Spence, A. M. (1973) "Job Market Signalling," *Quarterly Journal of Economics*, **87**, 355-74.

Temple, J. (2001) "Generalizations that Aren't? Evidence on Education and Growth," *European Economic Review*, **45**, 905-918.

Topel, R. (1999) "Labor Markets and Economic Growth," in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, Volume 3, Amsterdam, Elsevier Science.

Weiss, A. (1995) "Human Capital vs. Signalling Explanation of Wages," *Journal of Economic Perspectives*, **9**(4), 133-154.