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Discussion Paper 04-03-Rev.

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Dominant Strategy Implementation in Pure Exchange Economies*

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Abstract

In this paper, we consider dominant strategy implementation in classical pure exchange economies with free disposal. We show that quasi-strong-non-bossiness and strategy-proofness together are necessary and sufficient for dominant strategy implementation *via the direct revelation mechanism*. Moreover, we prove that strategy-proofness is sufficient for dominant strategy implementation, by using an augmented revelation mechanism similar to the one devised by Jackson et al. (1994). This implies that, in classical pure exchange economies, dominant strategy implementability by a certain indirect mechanism is equivalent to truthful implementability in dominant strategy equilibria.

Keywords: Quasi-strong-non-bossiness, Strategy-proofness, Augmented Revelation Mechanism, The Revelation Principle.

JEL Classification Numbers: D51, C72, D71, D78.

1 Introduction

The revelation principle asserts that if a social choice function is implementable in dominant strategy equilibria, then it is *truthfully* implementable in dominant strategy equilibria (i.e., *truthful* revelation by each agent is a dominant strategy equilibrium of the direct revelation mechanism). However, the direct revelation mechanism might have *untruthful* dominant strategy equilibria, so it might fail to *fully* implement the social choice function (e.g., see Dasgupta et al. (1979) for dominant strategy implementation, and Postlewaite and Schmeidler (1986), Repullo (1986), and Palfrey and Srivastava (1989) for Bayesian implementation). Since, as is well-known, this *multiple equilibrium problem* depends on the possibility of indifference in preferences, the problem can arise in a classical pure exchange economy.

Example 1 (A Multiple Equilibrium Problem in an Exchange Economy). Consider a two-agent, two-good exchange economy with initial endowment point $\omega = ((6,4),(4,6))$. Each agent $i \in \{1,2\}$ has two possible types θ_i, θ'_i. Agent 1 has preferences represented by

$$u_1((x_1^1, x_1^2); \theta_1) = x_1^1 \left(x_1^2\right)^2 \quad \text{and} \quad u_1((x_1^1, x_1^2); \theta'_1) = \left(x_1^1\right)^2 x_1^2;$$

and agent 2 has preferences represented by

$$u_2((x_2^1, x_2^2); \theta_2) = x_2^1 x_2^2 \qquad \text{and} \quad u_2((x_2^1, x_2^2); \theta'_2) = \left(\left(x_2^1\right)^{1/2} + \left(x_2^2\right)^{1/2}\right)^2.$$

Consider an individually rational social choice function f defined by

$$f = \begin{array}{cc|c} \theta_2 & \theta'_2 & \\ \hline z & \omega & \theta_1 \\ \omega & \omega & \theta'_1 \end{array}$$

where $z = ((4,6),(6,4))$. The social choice function can be fully implemented in dominant strategy equilibria by an indirect mechanism $\Gamma = \left(\left\{m_1, m'_1, m''_1\right\} \times \left\{m_2, m'_2\right\}, g\right)$ such that

$$g = \begin{array}{cc|c} m_2 & m'_2 & \\ \hline z & \omega & m_1 \\ \omega & \omega & m'_1 \\ \omega & y & m''_1 \end{array}$$

where $y = ((7.5, 1.5), (2.5, 8.5))$.[1] In the mechanism Γ, agent 1's dominant strategy is m_1 if her type is θ_1, and m'_1 if θ'_1; and agent 2's dominant strategy

[1] This type of indirect mechanism is called *an augmented revelation mechanism* (see Mookherjee and Reichelstein (1990)).

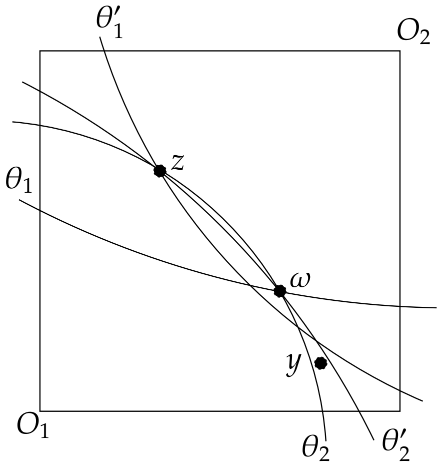

Figure 1: A Two-agent, Two-good Exchange Economy

is m_2 if her type is θ_2, and m_2' if θ_2'. Hence, Γ fully implements f in dominant strategy equilibria.

Since f is dominant strategy implementable, the revelation principle implies that telling the truth is a dominant strategy of the direct revelation mechanism $\Gamma' = \left(\left\{\theta_1, \theta_1'\right\} \times \left\{\theta_2, \theta_2'\right\}, f\right)$. However, the direct revelation mechanism Γ' *cannot* fully dominant strategy implement f, because an *untruthful* revelation as well as the truthful announcement is always a dominant strategy of Γ' for agent 2, and because, when she plays the untruthful dominant strategy, the equilibrium outcome does not coincide with the social goal.[2] ■

Another important topic on the revelation principle is to explore the relationship between implementation in dominant strategy equilibria and truthful implementation in dominant strategy equilibria. The revelation principle states that, in order for social choice functions to be implemented in dominant strategy equilibria, it is necessary for them to be truthfully implemented in dominant strategy equilibria. However, it is still open whether truthful implementability (i.e., strategy-proofness) is a sufficient condition for dominant strategy implementation in pure exchange economies.

[2] The multiple equilibrium problem in dominant strategy implementation might not be disturbing, because, to quote Dasgupta et al. (1979), "[i]n direct mechanisms where telling the truth is one of several dominant strategies, it may be reasonable to suppose that players will in fact tell the truth." (See also Chapter 23 in Mas-Colell et al. (1995).) However, since each agent is *indifferent* among dominant strategies irrespective of messages of the other agents, there is no guarantee that she will play truthful dominant strategies. Indeed, recent experimental results indicate that agents can choose a *spiteful strategy* (e.g., see Saijo (2005)).

These problems do not arise when indifference in preferences is not allowed, because three notions of dominant strategy implementation—truthful implementation, implementation by the direct revelation mechanism, and implementation with indirect mechanisms—are equivalent in the sense of implementability. In this case, the revelation principle has a more powerful implication: in order to find out dominant strategy implementable social choice functions, it is necessary and sufficient to search for social choice functions that are truthfully implementable in dominant strategy equilibria.[3] However, when indifference is possible, the equivalence does not in general hold. Thus, the revelation principle loses the powerful implication, because not all truthfully implementable social choice functions may be able to be *fully* implemented in dominant strategy equilibria not only by the direct revelation mechanism but also by any indirect mechanism. In this paper, we explore the relationships among three notions of dominant strategy implementation to examine the implications of revelation principle in classical pure exchange economies with free disposal.

First, we search for conditions that are necessary and sufficient for a social choice function to be implemented[4] in dominant strategy equilibria by the direct revelation mechanism. In Theorem 1, we show that *quasi-strong-non-bossiness* is a necessary condition for dominant strategy implementation *via the direct revelation mechanism* in pure exchange economies. Quasi-strong-non-bossiness requires that if a change in one agent's type keeps her utility unchanged *irrespective of the other agents' types*, then the consumption bundle of each agent should not be changed. This is a version of *non-bossiness*,[5] the requirement that if a change in one agent's type retains her *consumption bundle* unchanged, then the bundle of each agent should be unchanged. Note that quasi-strong-non-bossiness is *not* a necessary condition for dominant strategy implementation if indirect mechanisms can be used. Example 1 above demonstrates the existence of a social choice function that fails to satisfy quasi-strong-non-bossiness but which is implementable in dominant strategy equilibria *via an indirect mechanism*.

In Theorem 2, we prove that quasi-strong-non-bossiness and strategy-proofness together are sufficient conditions for dominant strategy implementation by the direct revelation mechanism. Therefore, it follows from the revelation principle and Theorems 1 and 2 that a social choice function is dominant strategy implementable via the direct revelation mechanism in pure exchange economies if and only if it satisfies both quasi-strong-non-bossiness and strategy-proofness (Corollary 1).[6] A well-known example

[3]This is because every social choice function that is truthfully implementable in dominant strategy equilibria can be *fully* implemented in dominant strategy equilibria.

[4]The term "implement" should be interpreted as "fully implement" hereafter.

[5]Non-bossiness was first introduced by Satterthwaite and Sonnenschein (1981).

[6]Saijo et al. (2004) has independently obtained necessary and sufficient conditions for dominant strategy implementation via the direct revelation mechanism.

of social choice functions satisfying both quasi-strong-non-bossiness and strategy-proofness is the so-called *fixed-price rule* (see Barberà and Jackson (1995) for details of the fixed-price rule).

Next, we look for a sufficient condition for dominant strategy implementation when *indirect* mechanisms are allowed to be used. In Theorem 3, we construct an augmented revelation mechanism similar to the one that Jackson et al. (1994) devised for undominated Nash implementation, so that we prove that strategy-proofness is a sufficient condition for social choice functions to be dominant strategy implemented in pure exchange economies. This theorem answers the open question mentioned above.

Together with the revelation principle, Theorem 3 leads to Corollary 2: every strategy-proof social choice function is dominant strategy implementable in pure exchange economies. This reveals a stark contrast between the set of social choice functions that can be implemented in dominant strategy equilibria with a certain indirect mechanism, and that of social choice functions that can be implemented in dominant strategy equilibria with the direct revelation mechanism. An important social choice function that satisfies strategy-proofness but violates quasi-strong-non-bossiness is an *inversely dictatorial* social choice function,[7] i.e., one where there exists some agent who always receives nothing. It might appear that inversely dictatorial social choice functions cannot be implemented in dominant strategy equilibria. But, Corollary 2 indicates that they are dominant strategy implementable by a certain *indirect* mechanism.

The robustness of implementation has recently deserved attention in implementation theory (e.g., see Bergemann and Morris (2003) for details). From this point of view, dominant strategy implementation has an advantage, because it does not need strong assumptions on the information structure: it needs neither common knowledge about each other's types nor common knowledge about prior beliefs over type profiles.

The Related Literature

This paper closely relates to both Jackson et al. (1994) and Sjöström (1994), which showed that almost all social choice functions are implementable in undominated Nash equilibria by a bounded mechanism. One might guess that all of the strategy-proof social choice functions are implementable in dominant strategy equilibria by their mechanisms. Indeed, for almost every strategy-proof social choice function, it is easy to check that the truthful revelation of own type by each agent is the unique dominant strategy equilibrium of their mechanisms; thus their mechanisms can implement it in

[7]Inversely dictatorial social choice functions are considered important in the sense that Zhou (1991) conjectured that it is only inversely dictatorial social choice functions that satisfy both strategy-proofness and Pareto efficiency in pure exchange economies with three or more agents.

dominant strategy equilibria. However, not all strategy-proof social choice functions are dominant strategy implementable by their mechanisms. In fact, their mechanisms cannot dominant strategy implement inversely dictatorial social choice functions, because their mechanisms cannot implement any social choice function that gives the zero consumption bundle to some agent.[8] In contrast, our mechanism constructed in the proof of Theorem 3 can implement *every* social choice function whenever the social choice function satisfies strategy-proofness.

This paper is also related to implementation in undominated strategies with a bounded mechanism. In fact, our bounded mechanism can implement strategy-proof social choice functions via undominated strategies in pure exchange economies. Jackson (1992) considered implementation in undominated strategies with a bounded mechanism, and showed that strategy-proofness is a necessary condition for bounded implementation in undominated strategies. Thus, combining the result of Jackson (1992), we can conclude that strategy-proofness is both necessary and sufficient for bounded implementation via undominated strategies in pure exchange economies (Theorem 4). The result indicates that, in pure exchange economies, dominant strategy implementation is equivalent to bounded implementation in undominated strategies, in the sense that every dominant strategy implementable social choice function can be boundedly implemented in undominated strategies, and vice versa. This is the same result as in voting environments with only strict preferences (e.g., see Jackson (1992) and Jackson and Srivastava (1996) for details).

This paper is organized as follows. Section 2 describes the model and gives some definitions. We explore dominant strategy implementation by the direct revelation mechanism in Section 3. In Section 4, dominant strategy implementation by an indirect mechanism is considered. Section 5 contains some concluding remarks.

2 The Model

Consider a *pure exchange economy with free disposal*. Let $N := \{1, 2, \ldots, n\}$ be the set of *agents*, where $2 \leq n < +\infty$. Let $L := \{1, 2, \ldots, l\}$ be the set of *goods*, where $2 \leq l < +\infty$. The set of *feasible allocations* is

$$A := \left\{ (a_1, a_2, \ldots, a_n) \in \mathbb{R}^l_+ \times \mathbb{R}^l_+ \times \cdots \times \mathbb{R}^l_+ \;\middle|\; \sum_{i \in N} a_i \leq \sum_{i \in N} \omega_i \right\},$$

where $a_i \in \mathbb{R}^l_+$ denotes agent i's *consumption bundle* and $\omega_i \in \mathbb{R}^l_{++}$ denotes agent i's *initial endowment*.

[8] This is why it has not yet been proved that strategy-proofness is a sufficient condition for dominant strategy implementation in pure exchange economies.

For each agent $i \in N$, let Θ_i be the set of all *types*, each of which prescribes her preference relation over $\mathbb{R}^l_+$. Each agent $i \in N$ is equipped with a *utility function* $u_i \colon \mathbb{R}^l_+ \times \Theta_i \to \mathbb{R}$. We assume that each agent is selfish, i.e., she cares only about her own consumption bundle. For each agent $i \in N$, let $\Theta^E_i \subset \Theta_i$ be agent i's set of *admissible* types: $\theta_i \in \Theta^E_i$ if and only if $u_i(\cdot;\theta_i)$ is (i) *continuous*, (ii) *strictly increasing*, and (iii) *strictly quasi-concave*. A *type profile* is a list $\theta = (\theta_1, \theta_2, \ldots, \theta_n) \in \Theta^E$, where $\Theta^E := \Theta^E_1 \times \Theta^E_2 \times \cdots \times \Theta^E_n$.

A *social choice function* is a single-valued function $f \colon \Theta^E \to A$ that assigns a feasible allocation $a \in A$ to each type profile $\theta \in \Theta^E$.

Let M_i denote a *message* or *strategy space* of agent $i \in N$. We call $m_i \in M_i$ a *message* or *strategy* of agent $i \in N$. A *mechanism* is a pair $\Gamma = (M, g)$, where $M := M_1 \times M_2 \times \cdots \times M_n$ and $g \colon M \to A$ is an *outcome function*. Given a mechanism (M, g), let g_i denote agent i's *outcome* of the mechanism. Given a social choice function f, a mechanism (M, g) is the *direct revelation mechanism* if $M_i = \Theta^E_i$ for every $i \in N$ and $g = f$. A *message* or *strategy profile* is denoted by $m = (m_1, m_2, \ldots, m_n) \in M$.

A strategy $m^*_i \in M_i$ is a *dominant strategy* of mechanism (M, g) at $\theta_i \in \Theta^E_i$ if $u_i(g(m^*_i, m_{-i}); \theta_i) \geq u_i(g(m'_i, m_{-i}); \theta_i)$ for all $m'_i \in M_i$ and all $m_{-i} \in M_{-i}$. For each agent $i \in N$, let $DS^\Gamma_i(\theta_i) \subseteq M_i$ be the set of her dominant strategies of mechanism Γ at $\theta_i \in \Theta^E_i$.

A strategy profile $m^* = (m^*_1, m^*_2, \ldots, m^*_n) \in M$ is a *dominant strategy equilibrium* of mechanism (M, g) at $\theta \in \Theta^E$ if, for any agent $i \in N$, $u_i(g(m^*_i, m_{-i}); \theta_i) \geq u_i(g(m'_i, m_{-i}); \theta_i)$ for any $m'_i \in M_i$ and any $m_{-i} \in M_{-i}$. Let $DSE^\Gamma(\theta) := DS^\Gamma_1(\theta_1) \times DS^\Gamma_2(\theta_2) \times \cdots \times DS^\Gamma_n(\theta_n) \subseteq M$ be the set of all dominant strategy equilibria of mechanism Γ at $\theta \in \Theta^E$. With an abuse of notation, we write $DSE^\Gamma(\theta) = (DS^\Gamma_1(\theta_1), DS^\Gamma_2(\theta_2), \ldots, DS^\Gamma_n(\theta_n))$ for any $\theta \in \Theta^E$. Let $g(DSE^\Gamma(\theta)) := \left\{ a \in A \,\middle|\, a = g(m) \text{ for some } m \in DSE^\Gamma(\theta) \right\}$ be the set of *dominant strategy equilibrium outcomes* of mechanism $\Gamma = (M, g)$ at $\theta \in \Theta^E$.

A mechanism $\Gamma = (M, g)$ *dominant strategy implements* a social choice function f if $g(DSE^\Gamma(\theta)) = f(\theta)$ for any $\theta \in \Theta^E$.[9] A social choice function f is *implementable in dominant strategy equilibria* if there exists a mechanism $\Gamma = (M, g)$ such that $g(DSE^\Gamma(\theta)) = f(\theta)$ for all $\theta \in \Theta^E$.

Remark 1. The single-valuedness of a social choice function, say f, implies that if a mechanism $\Gamma = (M, g)$ implements f in dominant strategy equilibria, then, for each $\theta \in \Theta^E$, $g(DSE^\Gamma(\theta))$ is a singleton, i.e., $g(m) = g(m')$ for any $m, m' \in DSE^\Gamma(\theta)$.

Remark 2. If a social choice function f is implementable in dominant strategy equilibria by the direct revelation mechanism $\Gamma = (M_1 \times M_2 \times \cdots \times M_n, g)$ where $M_i = \Theta^E_i$ for every $i \in N$ and $g = f$, then $\bigcup_{\theta_i \in \Theta^E_i} DS^\Gamma_i(\theta_i) = \Theta^E_i = M_i$

[9]To simplify notation, we write $g(DSE^\Gamma(\theta)) = \{f(\theta)\}$ as $g(DSE^\Gamma(\theta)) = f(\theta)$.

for any $i \in N$, because truth telling is a dominant strategy of the direct revelation mechanism for all $\theta_i \in \Theta_i^E$ and all $i \in N$.

Now we introduce two properties of social choice functions. *Strategy-proofness* is an incentive compatibility requirement that no agent should be able to change her type in a way that results in a direct gain to her, irrespective of the other agents' types.

Definition 1 (Strategy-proofness). A social choice function f satisfies *strategy-proofness* if, for all $\theta \in \Theta^E$ and all $i \in N$, there is no $\theta_i' \in \Theta_i^E$ such that

$$u_i(f(\theta_i', \theta_{-i}); \theta_i) > u_i(f(\theta); \theta_i).$$

Quasi-strong-non-bossiness, which is a version of *non-bossiness*,[10] requires that if a change in one agent's type does not affect her utility regardless of the other agents' types, then it does not change the consumption bundle everyone receives.

Definition 2 (Quasi-strong-non-bossiness). A social choice function f satisfies *quasi-strong-non-bossiness* if, for all $i \in N$ and all $\theta_i, \theta_i' \in \Theta_i^E$, if $u_i(f(\theta); \theta_i) = u_i(f(\theta_i', \theta_{-i}); \theta_i)$ for all $\theta_{-i} \in \Theta_{-i}^E$, then

$$f(\theta) = f(\theta_i', \theta_{-i}) \text{ for all } \theta_{-i} \in \Theta_{-i}^E.$$

Remark 3. Quasi-strong-non-bossiness is weaker than *strong-non-bossiness*,[11][12] which is a stronger version of non-bossiness.

We can easily rewrite the definition of quasi-strong-non-bossiness as follows: a social choice function f satisfies *quasi-strong-non-bossiness* if, for all $\theta \in \Theta^E$, all $i \in N$, and all $\theta_i' \in \Theta_i^E$, if $u_i(f(\theta); \theta_i) = u_i(f(\theta_i', \theta_{-i}); \theta_i)$ for all $\theta_{-i} \in \Theta_{-i}^E$, then $f(\theta) = f(\theta_i', \theta_{-i})$. It is easy to check that this is equivalent to *weak non-bossiness* defined in Saijo et al. (2004).

3 Implementation by the Direct Mechanism

In this section, we identify a necessary and sufficient condition for a social choice function to be dominant strategy implemented via the direct revelation mechanism in pure exchange economies.

We begin by providing a necessary condition for dominant strategy implementation via the direct revelation mechanism.

[10] A social choice function f satisfies *non-bossiness* if, for all $i \in N$, all $\theta_i, \theta_i' \in \Theta_i^E$, and all $\theta_{-i} \in \Theta_{-i}^E$, if $f_i(\theta) = f_i(\theta_i', \theta_{-i})$, then $f(\theta) = f(\theta_i', \theta_{-i})$.

[11] A social choice function f satisfies *strong-non-bossiness* if, for all $i \in N$, all $\theta_i, \theta_i' \in \Theta_i^E$, and all $\theta_{-i} \in \Theta_{-i}^E$, if $u_i(f(\theta); \theta_i) = u_i(f(\theta_i', \theta_{-i}); \theta_i)$, then $f(\theta) = f(\theta_i', \theta_{-i})$.

[12] Ritz (1983) first introduced the notion of strong-non-bossiness, called *non-corruptibility*.

Theorem 1. *If a social choice function f is dominant strategy implementable by the direct revelation mechanism in pure exchange economies, then it satisfies quasi-strong-non-bossiness.*

Proof. Pick any $i \in N$ and any $\theta_i, \theta_i' \in \Theta_i^E$ such that

$$u_i(f(\theta); \theta_i) = u_i(f(\theta_i', \theta_{-i}); \theta_i) \text{ for all } \theta_{-i} \in \Theta_{-i}^E. \tag{1}$$

Since f is implementable in dominant strategy equilibria via the direct revelation mechanism $\Gamma = (\Theta^E, f)$, it holds that

$$f(DSE^\Gamma(\theta)) = f(\theta) \text{ for all } \theta \in \Theta^E, \tag{2}$$

where $f(DSE^\Gamma(\theta)) = \left\{a \in A \,\middle|\, a = f(\theta) \text{ for some } \theta \in DSE^\Gamma(\theta)\right\}$. Substituting (2) for f in (1), we obtain

$$u_i(f(DSE^\Gamma(\theta)); \theta_i) = u_i(f(DSE^\Gamma(\theta_i', \theta_{-i})); \theta_i) \text{ for all } \theta_{-i} \in \Theta_{-i}^E. \tag{3}$$

Suppose $\hat{\theta}_i \in DS_i^\Gamma(\theta_i')$. Then, it follows from (3) and Remark 1 that

$$\begin{aligned} u_i(f(DS_i^\Gamma(\theta_i), DS_{-i}^\Gamma(\theta_{-i})); \theta_i) &= u_i(f(DS_i^\Gamma(\theta_i'), DS_{-i}^\Gamma(\theta_{-i})); \theta_i) \\ &= u_i(f(\hat{\theta}_i, DS_{-i}^\Gamma(\theta_{-i})); \theta_i) \end{aligned}$$

for all $\theta_{-i} \in \Theta_{-i}^E$. This implies that

$$u_i(f(\hat{\theta}_i, \theta_{-i}); \theta_i) = u_i(f(DS_i^\Gamma(\theta_i), \theta_{-i}); \theta_i)$$

for any $\theta_{-i} \in \Theta_{-i}^E = \prod_{j \neq i}\left(\bigcup_{\theta_j \in \Theta_j^E} DS_j^\Gamma(\theta_j)\right)$ by Remark 2, so we obtain $\hat{\theta}_i \in DS_i^\Gamma(\theta_i)$. Therefore, $DS_i^\Gamma(\theta_i') \subseteq DS_i^\Gamma(\theta_i)$.

Since $DS_i^\Gamma(\theta_i') \subseteq DS_i^\Gamma(\theta_i)$, we have

$$f(DS_i^\Gamma(\theta_i'), DS_{-i}^\Gamma(\theta_{-i})) \subseteq f(DS_i^\Gamma(\theta_i), DS_{-i}^\Gamma(\theta_{-i})) \text{ for all } \theta_{-i} \in \Theta_{-i}^E. \tag{4}$$

Substituting (2) for f in (4), we have

$$f(\theta_i', \theta_{-i}) \subseteq f(\theta)$$

for all $\theta_{-i} \in \Theta_{-i}^E$. Therefore, by the single-valuedness of f, we obtain

$$f(\theta_i', \theta_{-i}) = f(\theta)$$

for all $\theta_{-i} \in \Theta_{-i}^E$. □

Next, we look for sufficient conditions for dominant strategy implementation by the direct revelation mechanism. If a social choice function satisfies both quasi-strong-non-bossiness and strategy-proofness, then there is always a *unique dominant strategy equilibrium outcome* in the direct revelation mechanism (although there may exist some dominant strategy equilibria in the mechanism). So, since the direct revelation mechanism rules out undesired equilibrium outcomes, the mechanism can implement the social choice function in dominant strategy equilibria.[13]

Theorem 2. *If a social choice function f satisfies both quasi-strong-non-bossiness and strategy-proofness, then it is dominant strategy implementable by the direct revelation mechanism in pure exchange economies.*

Proof. Consider the direct revelation mechanism $\Gamma = (M, g)$ where $M_i = \Theta_i^E$ for every $i \in N$ and $g = f$. Since f satisfies strategy-proofness, $u_i(f(\theta); \theta_i) \geq u_i(f(\theta_i', \theta_{-i}); \theta_i)$ for all $\theta \in \Theta^E$, all $i \in N$, and all $\theta_i' \in \Theta_i^E$, implying that $\theta_i \in DS_i^\Gamma(\theta_i)$ for all $i \in N$ and all $\theta_i \in \Theta_i^E$; i.e., $\theta \in DSE^\Gamma(\theta)$ for any $\theta \in \Theta^E$. So, we have $f(DSE^\Gamma(\theta)) = \left\{a \in A \,\middle|\, a = f(\theta) \text{ for some } \theta \in DSE^\Gamma(\theta)\right\} \supseteq \{f(\theta)\}$ for every $\theta \in \Theta^E$.

Suppose $\bar{\theta}_i \in DS_i^\Gamma(\theta_i)$. Then, since $\theta_i, \bar{\theta}_i \in DS_i^\Gamma(\theta_i)$,

$$u_i(f(\theta_i, \theta_{-i}); \theta_i) \geq u_i(f(\bar{\theta}_i, \theta_{-i}); \theta_i) \text{ for all } \theta_{-i} \in \Theta_{-i}^E$$

and

$$u_i(f(\theta_i, \theta_{-i}); \theta_i) \leq u_i(f(\bar{\theta}_i, \theta_{-i}); \theta_i) \text{ for all } \theta_{-i} \in \Theta_{-i}^E.$$

These imply that

$$u_i(f(\theta); \theta_i) = u_i(f(\bar{\theta}_i, \theta_{-i}); \theta_i)$$

for all $\theta_{-i} \in \Theta_{-i}^E$. So, quasi-strong-non-bossiness implies that

$$f(\theta) = f(\bar{\theta}_i, \theta_{-i}) \text{ for any } \theta_{-i} \in \Theta_{-i}^E.$$

Thus, $f(\bar{\theta}_i, \theta_{-i}) = f(\theta)$ for $\bar{\theta}_i \in DS_i^\Gamma(\theta_i)$ and any $\theta_{-i} \in \Theta_{-i}^E$, which implies that $f(DS_i^\Gamma(\theta_i), \theta_{-i}) = f(\theta)$ for any $\theta_{-i} \in \Theta_{-i}^E$. Since similar arguments hold for any $\theta_i \in \Theta_i^E$, we obtain $f(DS_i^\Gamma(\theta_i), \theta_{-i}) = f(\theta)$ for any $\theta_i \in \Theta_i^E$ and any $\theta_{-i} \in \Theta_{-i}^E$, i.e., for any $\theta \in \Theta^E$.

Iteration of these arguments for further agents in N establishes that $f(DSE^\Gamma(\theta)) = f(\theta)$ for all $\theta \in \Theta^E$. □

[13] Laffont and Maskin (1982) proposed the notion of *strictly truthful implementation*, which requires that the direct revelation mechanism should have no untruthful dominant strategy equilibria. It is easy to see that not all strategy-proof and quasi-strong-non-bossy social choice functions are strictly truthfully implementable in dominant strategy equilibria because the direct revelation mechanisms may have untruthful dominant strategy equilibria.

By the revelation principle, if a social choice function is implementable in dominant strategy equilibria, then truth telling by each agent is a dominant strategy equilibrium of the direct revelation mechanism, i.e., the social choice function satisfies strategy-proofness. Therefore, combined with Theorems 1 and 2, this leads to the following corollary.

Corollary 1. *A social choice function f is dominant strategy implementable via the direct revelation mechanism in pure exchange economies if and only if it satisfies both quasi-strong-non-bossiness and strategy-proofness.*

Note that Corollary 1 holds even when the set of feasible allocations consists of all *balanced* allocations. It is easy to check that the social choice function defined in Example 1 satisfies strategy-proofness but violates quasi-strong-non-bossiness. Therefore, as seen in the example, the social choice function is not implementable in dominant strategy equilibria via the direct revelation mechanism. In contrast, fixed-price social choice functions (Barberà and Jackson (1995)), as well as dictatorial ones, satisfy both strategy-proofness and quasi-strong-non-bossiness, so they are dominant strategy implementable by the direct revelation mechanisms.

4 Implementation by an Indirect Mechanism

In Section 3, we show that strategy-proofness and quasi-strong-non-bossiness are both necessary and sufficient for dominant strategy implementation *via the direct revelation mechanism*. In this section, we search for a condition that is sufficient for social choice functions to be implemented in dominant strategy equilibria. Before proceeding, let us introduce two properties on environments and the definition of *weakly dominated strategies*.

Property 1 (A Common Worst Allocation). There exists $w = (w_1, w_2, \dots, w_n) \in A$ such that $u_i(a;\theta_i) \geq u_i(w;\theta_i)$ for all $i \in N$, all $\theta_i \in \Theta_i^E$, and all $a \in f(\Theta^E)$, where $f(\Theta^E) := \left\{a \in A \,\middle|\, a = f(\theta) \text{ for some } \theta \in \Theta^E\right\}$.

Property 1 says that there exists an allocation that is always considered to be the worst allocation for any agent. For instance, by setting $w = (0, 0, \dots, 0)$, Property 1 is satisfied in our environments.

Given $w \in A$ that satisfies Property 1, let

$$C_w^+ := \left\{(c^1, c^2, \dots, c^l) \in \mathbb{R}_+^l \,\middle|\, c^h \geq \max_{i \in N} w_i^h \text{ and } c^h \leq \sum_{i \in N} \omega_i^h \text{ for all } h \in L\right\}.$$

The next property is *strict value distinction* which was introduced by Palfrey and Srivastava (1991).

Property 2 (Strict Value Distinction). For all $i \in N$ and all $\theta_i, \theta_i' \in \Theta_i^E$, if $\theta_i \neq \theta_i'$, then there exist $c, \tilde{c} \in C_w^+$ such that $u_i(c; \theta_i) > u_i(\tilde{c}; \theta_i)$ and $u_i(c; \theta_i') < u_i(\tilde{c}; \theta_i')$.

Property 2 requires that if types change, then indifference curves cross somewhere in C_w^+. Note that Properties 1 and 2 hold in our environments, i.e., in pure exchange economies with free disposal and with continuous, strictly increasing, and strictly convex preferences. In fact, these properties are satisfied in many environments (see Jackson et al. (1994) for other examples of environments where Properties 1 and 2 are satisfied).

A strategy $m_i \in M_i$ is a *weakly dominated strategy* of mechanism (M, g) at $\theta_i \in \Theta_i^E$ if there exists a strategy $m_i' \in M_i$ such that $u_i(g(m_i', m_{-i}); \theta_i) \geq u_i(g(m_i, m_{-i}); \theta_i)$ for all $m_{-i} \in M_{-i}$ and $u_i(g(m_i', m_{-i}); \theta_i) > u_i(g(m_i, m_{-i}); \theta_i)$ for some $m_{-i} \in M_{-i}$. In this case, m_i' is said to *weakly dominate* m_i at $\theta_i \in \Theta_i^E$.

We are now ready to state the main result regarding a sufficient condition for dominant strategy implementation in pure exchange economies.

Theorem 3. *If a social choice function f satisfies strategy-proofness, then it is dominant strategy implementable in pure exchange economies.*

Proof. For a strategy-proof social choice function f, pick $w = (w_1, w_2, \ldots, w_n) \in A$ in a way that satisfies Property 1. Let

$$C_w^{++} := \left\{ (c^1, c^2, \ldots, c^l) \in \mathbb{R}_{++}^l \;\middle|\; c^h > \max_{i \in N} w_i^h \text{ and } c^h \leq \sum_{i \in N} \omega_i^h \text{ for all } h \in L \right\}.$$

Now we construct an augmented revelation mechanism $\Gamma = (M, g)$, where $M = M_1 \times M_2 \times \cdots \times M_n$, by using ideas similar to those of the mechanism constructed by Jackson et al. (1994) for undominated Nash implementation. Agent i's message space is

$$M_i := \Theta_i^E \cup (C_w^+ \times C_w^+).$$

That is, each agent announces either own type or a pair of consumption bundles. A typical message for agent $i \in N$ is denoted by $m_i = \theta_i^i$ or $(c^i, \tilde{c}^i)$. The outcome function g is defined as follows.

Rule 1: If all agents announce their types, i.e., $m_i = \theta_i^i \in \Theta_i^E$ for all $i \in N$, then $g(m) := f(\theta_1^1, \theta_2^2, \ldots, \theta_n^n)$. Note that, by Property 1, $u_i(g(m); \theta_i^i) \geq u_i(w; \theta_i^i)$ for all $i \in N$ and all $\theta_i^i \in \Theta_i^E$.

Rule 2: Suppose that only one agent, say i, announces her type and all other agents $j \neq i$ announce a pair of consumption bundles, i.e., $m_i = \theta_i^i \in \Theta_i^E$ and $m_j = (c^j, \tilde{c}^j) \in (C_w^+ \times C_w^+)$ for any $j \neq i$. Then,

$$g_i(m) := \arg\max_{c \in \bar{C}} u_i(c; \theta_i^i), \text{ where } \bar{C} := \left\{ c^j, \tilde{c}^j \in C_w^+ \;\middle|\; j \neq i \right\},$$

and, for any $j \neq i$,

$$g_j(m) := z_j, \text{ where } z_j^h := \max\left\{\frac{w_j^h}{\sum_{j\neq i} w_j^h}\left(\sum_{i\in N} w_i^h - g_i^h(m)\right), 0\right\} \text{ for any } h \in L.$$

Note that ties are broken in some arbitrary manner unless $\arg\max_{c\in\bar{C}} u_i(c;\theta_i^i)$ is a singleton, and that $w_j^h \big/ \sum_{j\neq i} w_j^h := 0$ if $\sum_{j\neq i} w_j^h = 0$. Note also that $g_i(m) \geq w_i$ because $c \geq w_i$ for all $c \in \bar{C}$, and that $g_j(m) \leq w_j$ for all $j \neq i$ by construction.

Rule 3: In all other cases, $g(m) := w$.

We want to show that $DS_i^\Gamma(\theta_i) = \{\theta_i\}$ for all $i \in N$ and all $\theta_i \in \Theta_i^E$.

Step 1: For each agent, announcing a pair of consumption bundles is weakly dominated by the truthful revelation of own type.

Suppose that the true type profile is $\theta^* = (\theta_1^*, \theta_2^*, \ldots, \theta_n^*)$. Let $m_i \in M_i$ be such that $m_i = (c^i, \tilde{c}^i) \in (C_w^+ \times C_w^+)$, and $m_i' \in M_i$ be such that $m_i' = \theta_i^*$.

Case 1-1: $m_j = \theta_j^j \in \Theta_j^E$ for all $j \neq i$.

We have $g(m_i, m_{-i}) = w$ and $g(m_i', m_{-i}) = f(\theta_i^*, \theta_{-i}^{-i})$ by Rule 3 and by Rule 1, respectively. Since $u_i(f(\theta_i^*, \theta_{-i}^{-i}); \theta_i^*) \geq u_i(w; \theta_i^*)$ by Property 1, we obtain $u_i(g(m_i', m_{-i}); \theta_i^*) \geq u_i(g(m_i, m_{-i}); \theta_i^*)$ for any $m_{-i} \in \Theta_{-i}^E$.

Case 1-2: $m_j = (c^j, \tilde{c}^j) \in (C_w^+ \times C_w^+)$ for all $j \neq i$.

We have $g(m_i, m_{-i}) = w$ by Rule 3, while $g_i(m_i', m_{-i}) = \arg\max_{c\in\bar{C}} u_i(c; \theta_i^*)$ by Rule 2, where $\bar{C} = \left\{c^j, \tilde{c}^j \in C_w^+ \,\middle|\, j \neq i\right\}$. Since $c \geq w_i$ for all $c \in \bar{C}$ by construction, we obtain $g_i(m_i', m_{-i}) \geq g_i(m_i, m_{-i})$, which implies $u_i(g(m_i', m_{-i}); \theta_i^*) \geq u_i(g(m_i, m_{-i}); \theta_i^*)$ for any $m_{-i} \in (C_w^+ \times C_w^+)^{n-1}$. In particular, since $c > w_i$ for all $c \in \bar{C}$ by construction if $\bar{m}_j = (c^j, \tilde{c}^j) \in (C_w^{++} \times C_w^{++}) \subset (C_w^+ \times C_w^+)$ for all $j \neq i$, we have $g_i(m_i', \bar{m}_{-i}) > g_i(m_i, \bar{m}_{-i})$, implying $u_i(g(m_i', \bar{m}_{-i}); \theta_i^*) > u_i(g(m_i, \bar{m}_{-i}); \theta_i^*)$ for $\bar{m}_{-i} \in (C_w^{++} \times C_w^{++})^{n-1}$.

Case 1-3: $m_j = \theta_j^j \in \Theta_j^E$ for some $j \neq i$ and $m_k = (c^k, \tilde{c}^k) \in (C_w^+ \times C_w^+)$ for all $k \in N \setminus \{i, j\}$.

It follows from Rule 2 that $g_i(m_i, m_{-i}) = z_i$, where $z_i^h = \max\left\{\frac{w_i^h}{\sum_{i\neq j} w_i^h}\left(\sum_{i\in N} w_i^h - g_j^h(m)\right), 0\right\}$ for any $h \in L$. On the other hand, $g(m_i', m_{-i}) = w$ by Rule 3. Since $w_i \geq z_i$, we get $u_i(g(m_i', m_{-i}); \theta_i^*) \geq u_i(g(m_i, m_{-i}); \theta_i^*)$ for any $m_{-i} \in \Theta_j^E \times (C_w^+ \times C_w^+)^{n-2}$.

Case 1-4: All other cases.

Rule 3 implies $g(m_i, m_{-i}) = w$ and $g(m_i', m_{-i}) = w$, implying $u_i(g(m_i', m_{-i}); \theta_i^*) \geq u_i(g(m_i, m_{-i}); \theta_i^*)$.

The above cases together establish that m_i is weakly dominated by m'_i. Thus, announcing a pair of consumption bundles is weakly dominated by telling the truth.

Step 2: For every agent, misrepresenting own type is weakly dominated by the truthful announcement of own type.

Suppose that the true type profile is $\theta^* = (\theta^*_1, \theta^*_2, \ldots, \theta^*_n)$. Let $m_i \in M_i$ be such that $m_i = \theta^i_i \in \Theta^E_i \setminus \{\theta^*_i\}$, and $m'_i \in M_i$ be such that $m'_i = \theta^*_i$.

Case 2-1: $m_j = \theta^j_j \in \Theta^E_j$ for all $j \neq i$.

Rule 1 implies $g(m_i, m_{-i}) = f(\theta^i_i, \theta^{-i}_{-i})$ and $g(m'_i, m_{-i}) = f(\theta^*_i, \theta^{-i}_{-i})$. Since strategy-proofness implies $u_i(f(\theta^*_i, \theta^{-i}_{-i}); \theta^*_i) \geq u_i(f(\theta^i_i, \theta^{-i}_{-i}); \theta^*_i)$, we obtain $u_i(g(m'_i, m_{-i}); \theta^*_i) \geq u_i(g(m_i, m_{-i}); \theta^*_i)$ for any $m_{-i} \in \Theta^E_{-i}$.

Case 2-2: $m_j = (c^j, \tilde{c}^j) \in (C^+_w \times C^+_w)$ for all $j \neq i$.

It follows from Rule 2 that $g_i(m_i, m_{-i}) = \arg\max_{c \in \bar{C}} u_i(c; \theta^i_i)$ and $g_i(m'_i, m_{-i}) = \arg\max_{c \in \bar{C}} u_i(c; \theta^*_i)$, where $\bar{C} = \left\{c^j, \tilde{c}^j \in C^+_w \,\middle|\, j \neq i\right\}$. Since agent i's true type is θ^*_i, we obtain $u_i(g(m'_i, m_{-i}); \theta^*_i) \geq u_i(g(m_i, m_{-i}); \theta^*_i)$ for any $m_{-i} \in (C^+_w \times C^+_w)^{n-1}$. Suppose in turn that agent i's true type is θ^i_i. Then, since her true type is θ^i_i, $u_i(g(m'_i, m_{-i}); \theta^i_i) \leq u_i(g(m_i, m_{-i}); \theta^i_i)$ for any $m_{-i} \in (C^+_w \times C^+_w)^{n-1}$. Especially, by Property 2, we have

$$u_i(g(m'_i, \bar{m}_{-i}); \theta^*_i) > u_i(g(m_i, \bar{m}_{-i}); \theta^*_i)$$

and

$$u_i(g(m'_i, \bar{m}_{-i}); \theta^i_i) < u_i(g(m_i, \bar{m}_{-i}); \theta^i_i)$$

for some $\bar{m}_{-i} \in (C^+_w \times C^+_w)^{n-1}$; otherwise, for any $\bar{m}_{-i} \in (C^+_w \times C^+_w)^{n-1}$, (i) if $u_i(g(m'_i, \bar{m}_{-i}); \theta^*_i) > u_i(g(m_i, \bar{m}_{-i}); \theta^*_i)$, then $u_i(g(m'_i, \bar{m}_{-i}); \theta^i_i) = u_i(g(m_i, \bar{m}_{-i}); \theta^i_i)$, (ii) if $u_i(g(m'_i, \bar{m}_{-i}); \theta^i_i) < u_i(g(m_i, \bar{m}_{-i}); \theta^i_i)$, then $u_i(g(m'_i, \bar{m}_{-i}); \theta^*_i) = u_i(g(m_i, \bar{m}_{-i}); \theta^*_i)$, or (iii) $u_i(g(m'_i, \bar{m}_{-i}); \theta^*_i) = u_i(g(m_i, \bar{m}_{-i}); \theta^*_i)$ and $u_i(g(m'_i, \bar{m}_{-i}); \theta^i_i) = u_i(g(m_i, \bar{m}_{-i}); \theta^i_i)$, all of which contradict Property 2.[14]

Case 2-3: All other cases.

By Rule 3, we have $g(m_i, m_{-i}) = w$ and $g(m'_i, m_{-i}) = w$, which implies $u_i(g(m'_i, m_{-i}); \theta^*_i) \geq u_i(g(m_i, m_{-i}); \theta^*_i)$.

The above cases together establish that m_i is weakly dominated by m'_i. Thus, misreporting own type is weakly dominated by truthful reporting.

Steps 1 and 2 together imply that the truthful revelation of own type weakly dominates both announcements of a pair of consumption bundles

[14]This is because, in any case, there are no $c, \tilde{c} \in C^+_w$ such that $u_i(c; \theta^*_i) > u_i(\tilde{c}; \theta^*_i) \geq u_i(w; \theta^*_i)$ and $u_i(\tilde{c}; \theta^i_i) > u_i(c; \theta^i_i) \geq u_i(w; \theta^i_i)$.

and false reports of own type. Thus, we establish that $DS_i^\Gamma(\theta_i) = \{\theta_i\}$ for all $i \in N$ and all $\theta_i \in \Theta_i^E$; i.e., $DSE^\Gamma(\theta) = \{\theta\}$ for all $\theta \in \Theta^E$. Therefore, we can conclude from Rule 1 that $g(DSE^\Gamma(\theta)) = f(\theta)$ for all $\theta \in \Theta^E$. □

Note that our mechanism works not only for $n > 3$ but also for $n = 2$, as the mechanism of Jackson et al. (1994) works. Note also that Theorem 3 is critically dependent on Property 1. This is because it is impossible to punish agents who do not announce their truthful types if Property 1 does not hold.

The following corollary follows immediately from the revelation principle and Theorem 3.

Corollary 2. *A social choice function f is dominant strategy implementable in pure exchange economies if and only if it satisfies strategy-proofness.*

Corollary 2 tells us that *every* strategy-proof social choice function is implementable in dominant strategy equilibria if *indirect* mechanisms are allowed to be used. This is in stark contrast to Corollary 1, because it states that only quasi-strong-non-bossy and strategy-proof social choice functions are dominant strategy implementable *by the direct revelation mechanism*.

A well-known social choice function that satisfies strategy-proofness but fails to satisfy quasi-strong-non-bossiness is an *inversely dictatorial* one, which was introduced by Zhou (1991). So, it follows from Corollaries 1 and 2 that inversely dictatorial social choice functions are not implementable in dominant strategy equilibria by the direct revelation mechanism, but are implementable in dominant strategy equilibria by the *indirect* mechanism constructed in the proof of Theorem 3.

As mentioned before, our mechanism appears to be similar to the one devised by Jackson et al. (1994). Indeed, their mechanism as well as the mechanism constructed by Sjöström (1994) can dominant strategy implement almost all social choice functions whenever the social choice functions satisfy strategy-proofness. However, an important difference is that our mechanism can implement inversely dictatorial social choice functions in dominant strategy equilibria, whereas their mechanisms cannot dominant strategy implement the social choice functions.[15] Thus, our mechanism has the advantage of being able to implement strategy-proof social choice functions that assign someone the zero consumption bundle (although the advantage is somewhat limited because our mechanism is not necessarily useful for undominated Nash implementation).

We conclude this section with a brief discussion of implementation in another equilibrium concept. Since our *bounded* mechanism constructed in the proof of Theorem 3 can implement strategy-proof social choice functions

[15] The difference stems from the fact that Jackson et al. (1994) and Sjöström (1994) needed a slightly stronger property than Property 1.

even in undominated strategies, Theorem 3 implies the following theorem when coupled with Corollary 1 in Jackson (1992) which says that strategy-proofness is necessary for implementation in undominated strategies by a bounded mechanism.

Theorem 4. *A social choice function f is implementable via undominated strategies with a bounded mechanism in pure exchange economies if and only if it satisfies strategy-proofness.*

Theorem 4 indicates that, in pure exchange economies, implementation in dominant strategy equilibria is equivalent to bounded implementation in undominated strategies, in the sense that the set of social choice functions implementable in dominant strategy equilibria coincides with that of social choice functions boundedly implementable in undominated strategies.

5 Conclusion

In this paper, we have considered the relationships among three notions of dominant strategy implementation in classical pure exchange economies with free disposal.[16] First, in Theorem 1, we have shown that quasi-strong-non-bossiness is necessary for social choice functions to be dominant strategy implemented by the direct revelation mechanism. Next, in Theorem 2, we have proved that quasi-strong-non-bossiness and strategy-proofness together are sufficient for dominant strategy implementation by the direct revelation mechanism. Finally, in Theorem 3, we have proved that strategy-proofness is sufficient for dominant strategy implementation, by using an augmented revelation mechanism.

Since there exists a social choice function that is strategy-proof but which is not quasi-strong-non-bossy (e.g., an inversely dictatorial social choice function), Theorems 1–3 together with the revelation principle imply the following relationships: from the point of view of implementability in classical pure exchange economies with free disposal, (i) dominant strategy implementation via the direct revelation mechanism implies, but is not implied by, dominant strategy implementation by a certain indirect mechanism, and (ii) dominant strategy implementation via a certain indirect mechanism is identical to truthful implementation in dominant strategy equilibria. As noted in the introduction, when indifference is possible, the revelation principle loses the powerful implication. However, our theorems indicate that, in classical pure exchange economies with free disposal, the

[16] It is worth noting that Theorems 1–3 hold not only in classical pure exchange economies with free disposal but also in other environments where Properties 1 and 2 introduced in Section 4 are satisfied; furthermore, Theorems 1 and 2 hold even without Properties 1 and 2.

revelation principle has the same powerful implication as in the case where indifference is impossible.

It is worth discussing relationships among dominant strategy implementation, Nash implementation, and bounded implementation in undominated Nash equilibria. It might seem at first glance that all of the dominant strategy implementable social choice functions are both Nash implementable and boundedly implementable in undominated Nash equilibria. However, there is no relationship among them, because inversely dictatorial social choice functions are dominant strategy implementable but are not boundedly implementable in undominated Nash equilibria as well as not being Nash implementable.

Our mechanism constructed in the proof of Theorem 3 is similar to the mechanism of Jackson et al. (1994); thus our mechanism possesses worrisome features as their mechanism possesses. For instance, as mentioned before, our mechanism makes frequent use of Property 1 to punish agents who do not truthfully announce their types. Property 1 is consistent with pure exchange economies with free disposal, but would hardly seem consistent with ones *without* free disposal. It is an interesting topic for further research to construct a more "natural" mechanism for dominant strategy implementation.

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