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Abstract

Using an overlapping generations model, this note shows that an improvement in the efficiency of human capital production decreases the net income of the young household while increasing that of the old. Without compensating redistribution, it deteriorates lifetime utilities of all generations except for the initial old households.

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1 Introduction

The importance of the education environment has a prominent place in the literature of endogenous growth, where the education environment determines the efficiency of human capital production and thereby critically affects the long-term path of aggregate output.[1] This note demonstrates that the education environment also has a nontrivial effect on the intergenerational distribution of income and that a better education environment does not necessarily improve the welfare of households in the long run. We present an overlapping generations model in which young households invest in the process of human capital production (i.e., education). A better education environment increases the supply of human capital. The increased supply of human capital, on one hand, raises the productivity of physical capital and the interest rate, thereby raising the income of old households. On the other hand, the net income of young households is reduced because the increased supply lowers the price of their labor endowments (i.e., wages) while the increased revenue from selling human capital is offset by education expenses.[2] Although the initial old households unilaterally benefit from increased income, the overall welfare effect on subsequent generations is shown to be negative under plausible parameter values.

2 Model

Production Technology. A version of Diamond's (1965) overlapping generations model is considered, in which time is divided into periods $t = 1, 2, \cdots, \infty$. In each period, a single final good, denoted by Y_t, is competitively produced from physical capital K_t and human capital H_t by a Cobb-Douglas technology. The production function is $Y_t = AK_t^{\alpha}H_t^{1-\alpha}$,

[1]In a model with a representative agent, Lucas (1988) and many others have shown that a better education environment accelerates economic growth. In our separate paper (Kitagawa, Horii and Futagami, 2003), we showed in an overlapping generations model that an expansion in the availability of education (i.e., the maximum amount of human capital that can be accumulated through education) has non-monotonic effects on the long-term rate of growth.

[2]In a static model of child labor, Basu and Van (1998) also showed that an exogenous expansion of labor-supply capacity does not necessarily benefit its suppliers.

where $A > 0$ and $\alpha \in (0, 1)$, respectively, represent total factor productivity and the share of physical capital. Factor markets are perfectly competitive, so that the market price of physical capital r_t and that of human capital w_t, in terms of the final good, are determined by their marginal productivities:

$$r_t = A\alpha(K_t/H_t)^{\alpha-1}, \quad w_t = A(1-\alpha)(K_t/H_t)^{\alpha}. \tag{1}$$

The final goods produced in a certain period can either be consumed in that period or be saved for production in the next period. Once saved, the good can either be used as an input to human capital production or be used directly as physical capital, which implies that the price of the saved good (i.e. the interest rate) is r_t. Human and physical capital depreciates within one period and therefore cannot be carried over to subsequent periods.

Households. At each period, there are two generations of households, which we call the young and the old. Each generation contains a unit mass of households and lives for two periods. The objective of the generation t households (those born at period t) is to maximize their lifetime utility

$$u_t = (1-\beta)\ln c_{1t} + \beta \ln c_{2\,t+1}, \tag{2}$$

where $\beta \in (0, 1)$ is a parameter specifying the patience of agents and c_{1t} and $c_{2\,t+1}$ represent their consumption in youth and old age. The young households are endowed with one unit of human capital. In addition, they can augment their human capital through education, which must be financed by borrowing from the old. Let $e_t \geq 0$ be the amount of saved goods borrowed from the old generation to invest in this process. Then the amount of their human capital is

$$H_t = 1 + \gamma e_t, \tag{3}$$

where parameter $\gamma \geq 0$ represents the quality of the education environment, which determines the efficiency of human capital production. They sell off their human capital at market price w_t and in return receive $w_t(1+\gamma e_t)$ units of the final good. After repaying $r_t e_t$ units of the final good, they consume part of their net income at the end of that period and save the remainder for consumption in their old age. The intertemporal budget constraint is

$$c_{1t} + c_{2\,t+1}/r_{t+1} = w_t(1+\gamma e_t) - r_t e_t. \tag{4}$$

For every $t \geq 0$, the generation t households choose e_t, c_{1t} and $c_{2\,t+1}$ so as to maximize (2) under constraint (4). Net income in the right hand side of (4) is maximized by choosing

$$e_t \begin{cases} = 0 & \text{if } w_t - \gamma r_t < 0; \\ \in [0, +\infty) & \text{if } w_t - \gamma r_t = 0; \\ = +\infty & \text{if } w_t - \gamma r_t > 0. \end{cases} \tag{5}$$

Since human capital investment must be finite in equilibrium, condition (5) implies that

$$w_t - r_t \leq 0 \text{ with equality whenever } e_t > 0. \tag{6}$$

From (2), (4) and (6), the consumption and savings of generation $t \geq 0$ households in equilibrium are written in terms of factor prices:

$$c_{1t} = (1-\beta)w_t,\ c_{2\,t+1} = \beta r_{t+1} w_t,\ S_t = \beta w_t. \tag{7}$$

At period 0, the initial old (generation -1) households are endowed with $S_0 > 0$ units of saved goods and consume $r_0 S_0$ units of final goods in exchange for their endowment.

Equilibrium. Substituting (1) and (3) into condition (6) gives a relation between two kinds of capital,

$$H_t = \max\left\{\gamma\frac{1-\alpha}{\alpha}K_t, 1\right\}. \tag{8}$$

Aggregate demand for the saved good consists of demand for physical capital K_t and demand for input to human capital investment. From (3), the latter is $\gamma^{-1}(H_t - 1)$. Thus, the market-clearing condition for saved goods is

$$K_t + \gamma^{-1}(H_t - 1) = S_{t-1}, \tag{9}$$

where S_{t-1} is the savings of generation $t-1$ households. Given S_{t-1}, (8) and (9) determine the equilibrium (K_t, H_t) pair,

$$\big(K_t, H_t\big) = \begin{cases} \big(S_{t-1}, 1\big) & \text{if } S_{t-1} \leq \alpha/(\gamma(1-\alpha)); \\ \big(\alpha(S_{t-1} + 1/\gamma), (1-\alpha)(\gamma S_{t-1} + 1)\big) & \text{if } S_{t-1} > \alpha/(\gamma(1-\alpha)), \end{cases} \tag{10}$$

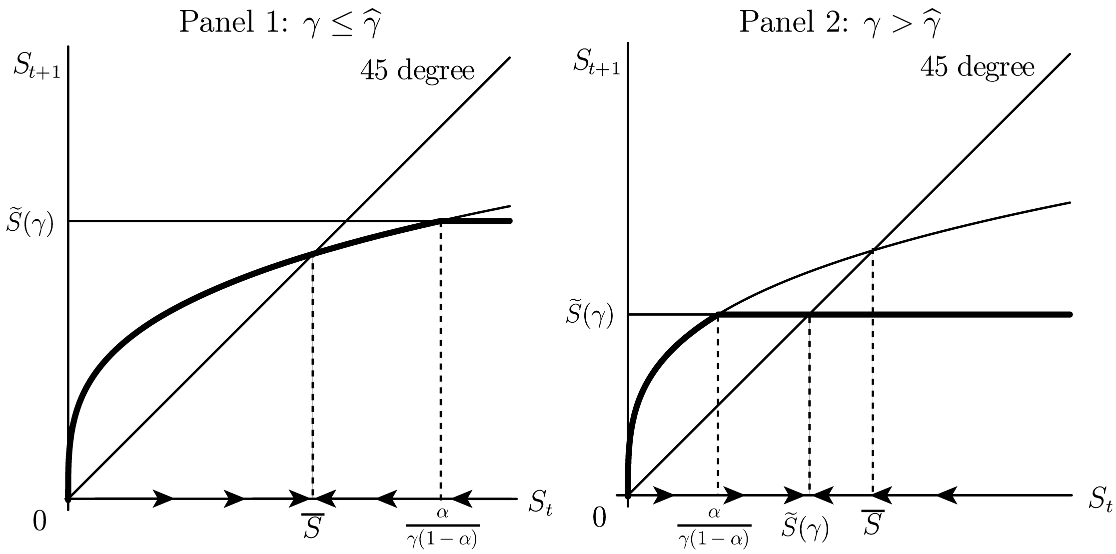

Figure 1: Saving Dynamics

which shows that young households invest in human capital if and only if $S_{t-1} > \alpha/(\gamma(1-\alpha))$. Substituting (1) and (10) into (7) yields the saving dynamics:

$$S_t = \begin{cases} \beta A(1-\alpha)S_{t-1}^{\alpha} & \text{if } S_{t-1} < \alpha/(\gamma(1-\alpha)); \\ \beta A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\gamma^{-\alpha} \equiv \widetilde{S}(\gamma) & \text{if } S_{t-1} \ge \alpha/(\gamma(1-\alpha)). \end{cases} \tag{11}$$

Given initial $S_0 > 0$, equation (11) generates the equilibrium sequence of aggregate saving. As shown by Figure 1, the sequence monotonically converges to

$$S^*(\gamma) = \begin{cases} (\beta A(1-\alpha))^{1/(1-\alpha)} \equiv \overline{S} & \text{if } \gamma \le \widehat{\gamma}; \\ \widetilde{S}(\gamma) & \text{if } \gamma > \widehat{\gamma}, \end{cases} \tag{12}$$

where $\widehat{\gamma} \equiv \alpha \left(\beta A(1-\alpha)^{2-\alpha}\right)^{-1/(1-\alpha)}$. Observe from Panel 2 of Figure 1 that the economy settles to the steady state in one period whenever $\gamma > \widehat{\gamma}$ and $e_t > 0$. The pair of factor prices in the steady state is obtained by substituting (12) for (1):

$$\left(w^*(\gamma), r^*(\gamma)\right) = \begin{cases} \left(\overline{S}/\beta, \alpha/((1-\alpha)\beta)\right) & \text{if } \gamma \le \widehat{\gamma}; \\ \left(\widetilde{S}(\gamma)/\beta, \gamma\widetilde{S}(\gamma)/\beta\right) & \text{if } \gamma > \widehat{\gamma}. \end{cases} \tag{13}$$

3 Implications of a Better Education Environment

Welfare effects. Equation (13) implies that in the long run households receive education only if $\gamma > \widehat{\gamma}$. When γ is in this range, a better education environment (i.e., higher efficiency of human capital production) increases the interest rate but reduces the wage rate in the steady state. Then, from (7), consumption of young households decreases, whereas that of old households tends to increase.[3] Substituting (7) and (13) into the utility function (2) gives the lifetime utility of consumers in the steady state:

$$\begin{aligned} u^*(\gamma) &= \text{constant} + \ln w^*(\gamma) + \beta \ln r^*(\gamma) \\ &= \text{constant} + (\beta(1-\alpha) - \alpha) \ln \gamma. \end{aligned} \tag{14}$$

Lifetime utility is decreasing in γ if (and only if)

$$\beta < \alpha/(1-\alpha). \tag{15}$$

Recall that α is the share of physical capital while β is the young agents' propensity to save. Using a conventional value of 0.3 for α, condition (15) becomes $\beta < 0.428$, which is met under plausible values for β. In addition, (15) coincides with the condition for the economy to be dynamically efficient for all γ in the steady state because applying (15) for (13) gives $r^*(\gamma) \geq \alpha/((1-\alpha)\beta) > 1$.[4] Therefore, a better education environment reduces the lifetime utility of agents in the long run given that parameters are within an empirically plausible range or in a range that guarantees the economy's dynamic efficiency.

Who benefits? One may wonder why relaxing one of the resource constraints in the economy results in an adverse consequence. To be precise, the economy with a high γ is *not* Pareto inferior to the economy with a low γ, because consumption of the initial old households is higher in the economy with a high γ.[5] This implies that existing old

[3]When $\gamma > \widehat{\gamma}$, consumption of old households is $(\beta A \alpha^\alpha (1-\alpha)^{1-\alpha})^2 \gamma^{1-2\alpha}$, which is increasing in γ given that $\alpha < 1/2$. We assume, reasonably, that $\alpha < 1/2$ because α is the share of physical (non human) capital.

[4]Since population is constant over time, dynamic efficiency requires the gross interest rate to be higher than 1.

[5]Recall that initial old households consume $r_0 S_0$ units of goods, where r_0 is (weakly) increasing in γ

households in each period have an incentive to implement various policies that aim to improve the educational environment for young households whenever possible. Contrary to the usual perception, these seemingly altruistic policies actually benefit the old generation themselves while future generations are thrust into an 'educational rat race'.

Combination of policies. When combined with appropriate redistributional policies, however, a better education environment can improve the welfare of all generations. Substituting (10) into the production function and then differentiating it with respect to γ yields

$$\frac{\partial Y_t}{\partial \gamma} = \begin{cases} 0 & \text{if } S_{t-1} \leq \frac{\alpha}{\gamma(1-\alpha)}. \\ A\left(\frac{\alpha}{\gamma(1-\alpha)}\right)^{\alpha}\left(S_{t-1} - \frac{\alpha}{\gamma(1-\alpha)}\right) > 0 & \text{if } S_{t-1} > \frac{\alpha}{\gamma(1-\alpha)}. \end{cases} \tag{16}$$

Given S_{t-1}, (16) shows that aggregate output is increasing in γ whenever young households invest in human capital. Thus, old households benefit from a larger γ even when the authority implements a lump-sum redistribution policy that transfers income from the old to the young so that the income of young households (and therefore their savings) are unaffected by the increase in γ. When continued forever, this combination of a larger γ and the intergenerational transfer benefits all generations because they can enjoy more consumption when old while consumption in their youth is unchanged. In an economy with a highly developed education system, this argument legitimizes income transfers from old to young in the forms of grants and scholarships funded by taxes on the elder generation.[6]

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and S_0 is historically given.

[6]In the U.S., the percentage of students receiving grants and the average amounts received by students with grant aid have increased between 1990 and 2000, which seems to be mitigating the problem. (Choy, 2004). By contrast, the Japanese government has recently abolished a scholarship loan forgiveness program so that all scholarships received by Japanese residents must be repaid after graduation. Although the supply of scholarship loans has been increased in compensation, our model predicts that this change will aggravate intergenerational income inequality in the long run.

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