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Abstract

This paper presents an overlapping generations model with technology choice and credit market imperfections, in order to investigate a possible source of underdevelopment. The model shows that a better financial infrastructure that provides stronger enforcement of contracts facilitates the development of financial markets, which, in turn, enables firms to switch to more productive and capital-intensive technologies, thereby promoting economic development. In the presence of credit rationing, however, this technological switch widens inequality. Therefore, risk-averse agents would not be willing to improve the financial infrastructure to the level at which the technological switch occurs, resulting in a development trap. A remedy is to facilitate small firms’ adoption of the currently used technology rather than the new one.

JEL Classification Numbers: O14, O16.

Key words: Enforcement; Technological Switch; Income Distribution; Credit Rationing; Development Trap; Institutions.

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1 Introduction

Although earlier studies point to the accumulation of physical and human capital as the main determinant of economic development, recent empirical analysis increasingly suggests that there remains a large gap in per capita income between rich and poor countries even after controlling for differences in those factors.\(^1\) The search for missing determinants has been conducted from several directions. First, there is a convincing argument that the unexplained gap implies the existence of some barrier that prevents poor countries from adopting and efficiently using better technologies (e.g., Parente and Prescott, 2000, 2005). Second, as surveyed extensively by Levine (1997, 2005), a line of research supports the view that a country’s financial system plays an important role in economic development.\(^2\) Developed financial markets contribute to distributing productive factors more efficiently across production units so that more output can be produced from given aggregate inputs. Third, institutions and government policies also appear to be important factors since they determine the economic environment of individuals and firms and affect how they behave. Hall and Jones (1999) call those factors “social infrastructure,” which, they argue, causes a large portion of the observed international per capita income differences.

These alternative explanations are not necessarily in conflict with each other, but they are all integral aspects of the process of economic development. Historical evidence shows that the timing of technology adoption is critically dependent on the development of financial markets. Sylla (2002) documents that, in most developed economies, ‘financial revolutions’ preceded major technological changes — the Industrial Revolution in the case of England — and subsequent rapid economic development.\(^3\) Financial markets, in turn, are affected by the social infrastructure. La

\(^1\)For example, see Mankiw, Romer, and Weil (1992) and Islam (1995), respectively, for earlier and more recent views.

\(^2\)Christopoulos and Tsionas (2004) argue that the causality runs from finance to economic development and not vice versa.

\(^3\)Sylla (2002) reports “The Dutch financial revolution had occurred by the first decades of the seventeenth century, before the Dutch Golden Age... The British financial revolution in the late
Porta et al. (1997, 1998) and Levine, Loayza, and Beck (2000) show evidence that the development of financial markets is strongly influenced by legal and accounting systems—i.e., by creditor rights, contract enforcement, and accounting standards, all of which are parts of the social infrastructure.

This paper develops an overlapping generations model with technology choice and credit market imperfections, which are used to investigate a possible source of underdevelopment. Consistent with the observations above, the model shows that the enforceability of contracts (a part of social infrastructure) determines the development of financial markets, which, in turn, affects the choice and utilization of technologies and, hence, economic performance. More specifically, stronger enforcement of financial contracts facilitates the productive use of capital through two mechanisms. First, it makes credit accessible to an increased number of agents and, thus, raises the aggregate capital-labor ratio imputed to a given technology. Second, while the agents must rely on labor intensive technologies when obtaining credit is difficult, improved enforcement of financial contracts enables them to choose from wider set of technologies, including those with higher capital intensities and higher productivities. Both imply that stronger contract enforcement, or, more broadly, a better financial infrastructure, improves productivity and, therefore, raises the average income.

In this framework, the issue of economic development, or, equivalently, that of income differences, can be restated in terms of the differences in the financial infrastructure that determines the enforceability of contracts. A fundamental question is why some countries are stuck with poor enforcement even though it results in primitive financial markets and unproductive technologies. From the point of view of political economy, socially optimal reforms are not always adopted because of

seventeenth and early eighteenth centuries, before the English industrial revolution. The U.S. financial revolution occurred ..., before the U.S. economy accelerated its growth in the ‘statistical dark age’ of the early nineteenth century”. He also notes “In the early Meiji era of the 1870s and 1880s, Japan had a financial revolution ... Once their financial revolution was in place, the Japanese were off and running.” See also Dickson (1967) for similar arguments.
conflicts between interest groups. While such conflicts may have played important roles in a number of cases, it theoretically remains open whether it is in fact optimal to upgrade an economy’s financial infrastructure. While stronger enforcement of financial contracts improves the per capita income, as explained above, it also changes the distribution of income among agents in either desirable or undesirable directions. In this paper, we show that, in certain situations, people are reluctant to improve their financial infrastructure because they know that such an improvement would undermine their welfare by widening income inequality.

The intuition behind this mechanism is as follows. When enforcement of financial contracts is imperfect, credit rationing may be required to give borrowers the incentive to repay rather than to default. As long as the same technology is employed, stronger enforcement weakens the necessity of credit rationing, thereby reducing the inequality. However, if improved contract enforcement triggers a technological switch and the new technology requires a larger scale of operation, as is often the case, then the number of agents who obtain credit may decline. As a result, the surplus from the better technology is monopolized by a small number of agents, and, therefore, inequality rises.

In a circumstance in which the means of redistribution are limited, the foreseeable rise in inequality gives a strong disincentive for risk-averse people to achieve economic development through improving their institutions for enforcing financial contracts. In this paper, we demonstrate that stronger enforcement actually worsens the (Benthamian) social welfare, if the minimum size of investment required

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4Rajan and Zingales (2003) suggest that industrial incumbents may want to leave financial infrastructure underdeveloped in order to limit competition. Erosa and Hidalgo-Cabrillana (2005) argues that if entrepreneurs, as a class, have the power to impede financial development, they would do since it allows them to extract rents from the factor servicing they hire. See Drazen (2000) for general discussions about the conflicting interests in economic reforms.

5In England, for example, the technological switch from cottage technologies to manufacturing technologies occurred in the textile industry and many others between 1759 and 1801, during which the nominal Gini index was raised from 52.2 to 59.3 (Hobsbawm 1968; Lindert 1999). See also Lindert and Williamson (1985).
for adoption of the currently used technology is within a certain range.\textsuperscript{6} Given the considerable variability across countries in both social and regulatory circumstances that affect the minimum requirement, our finding provides one possible reason why some economies face difficulties in achieving industrialization due to opposition from their residents, while other economies successfully industrialize without major conflicts.\textsuperscript{7} Another implication of the model is that it may not be a good idea to force developing countries to adopt ‘better’ institutions when the non-adoption of such institutions is caused by welfare concerns rather than political reasons. If this is the case, we show that an appropriate remedy is to facilitate adoption by small firms of the currently used technology, not the new one, until credit rationing is resolved.

In the literature, a number of theoretical works explain the rise of income inequality at early stages of economic development using models with credit market imperfections. Banerjee and Newman (1998) and Greenwood and Jovanovic (1990) derive the Kuznets curve as a result of the technological shift.\textsuperscript{8} If agents can only gradually shift to the new sector where they receive a higher income, the distribution of income within the whole economy widens during the period when agents are divided between the new and old sectors. This inequality, however, is always welfare-improving as those who moved to the new sector are better off because they voluntarily chose to do so while those who remained in the old sector are never worse off because reduced labor supply in the old sector increases their wage. Aghion and Bolton (1997) also shows that the rise in inequality in the early phases of development is beneficial to the poor since accumulation of capital by the rich makes lending terms more favorable to poor borrowers. Thus, in these models, the fore-

\textsuperscript{6}Galor and Zeira (1993) and Matsuyama (2000) have already shown that the minimum investment requirement gives rise to inequality when the credit market is imperfect. Our new point is that the degree of inequality after the technological switch is critically dependent on indivisibility before the switch, mainly because the latter affects the timing of the switch.

\textsuperscript{7}For example, as reported by Mokyr (1990), resistance to new technologies delayed their adoption in 19th Century Continental Europe, where the old urban guilds restricted new entrants. In Britain, where such guilds were not common, new technologies were adopted with little resistance.

\textsuperscript{8}See also Barro (2000, p. 9) for a survey of related work.
seeable rise in inequality does not retard economic development since its benefit is
every individual. By contrast, inequality in our model can be welfare-reducing since
it is caused by a particular sort of crowding-out effect: once improvement in the
financial infrastructure allows adoption of the new technology, those who are not
given an entrepreneurial chance to start projects with that technology are no longer
even able to start projects with the old technology because demand for productive
factors by the entrepreneurs with the new technology crowds the old out of factor
markets. Thus, people resist the technological shift, thereby causing a development
trap.

The impact of financial markets on technological choice is also stressed by Saint-
Paul (1992), Castro, Clementi and MacDonald (2005), and Bencivenga, Smith and
market, risk-averse agents may choose less specialized and less productive technolo-
gies. Castro et al. (2005) demonstrate that stronger investor protection facilitates
economic development given that the technology for producing investment goods
involves higher idiosyncratic risk than the technology for producing consumption
goods. In contrast, Bencivenga et al. (1995) show that a technological switch led by
a better financial infrastructure may reduce the growth rate if the new technology
requires a longer length of periods for which investments must be committed. Those
studies are closely related to ours in motivation, but our primary focus is to examine
the effects of financial development on inequality and welfare when technologies are
different in capital intensities and the sizes of minimum investments.

The rest of this paper is organized as follows. In Section 2, a model of technology
choice and credit rationing is constructed, in which we demonstrate how inequality
arises in equilibrium even when agents are almost homogenous ex ante. Section 3
contains a clarification of how enforcement of financial contracts affects the choice of
technology in equilibrium and when credit rationing occurs. Section 4 is a long-term
analysis, in which the effects of improved enforcement on aggregate consumption,
inequality, and the welfare of agents are investigated. Using the results obtained,
Section 5 presents a discussion of the possibility of a development trap and effective
remedies. The paper is concluded in Section 6. The appendix contains the proofs for selected propositions, while proofs of the other propositions are available from the corresponding author upon request.

2 The Model

In this section, a model of technology choice and credit rationing is presented in order to explain why credit rationing occurs when the enforceability of financial contracts is limited and characterize the equilibrium distribution of income among agents. One problem in deriving equilibrium is that the demand functions of production factors may become discontinuous if individuals change the choice of technology all at once in response to a marginal change in factor prices. We overcome this problem by first introducing small \textit{ex ante} heterogeneity among agents and then considering the limit in which \textit{ex ante} heterogeneity almost vanishes.

2.1 Economic Environments

An overlapping generations economy is considered, in which time is discrete and extends from zero to infinity \((t = 0, 1, 2, \ldots)\). In each period, there are two generations of two-period-lived agents. All generations are identical in size, and each contains a continuum of agents with unit mass, indexed by \(i \in [0, 1]\). In the first period of his life, each agent born in period \(t\) supplies one unit of labor inelastically to the competitive labor market and receives \(w_t\) units of consumption goods as wage income at the end of that period. He (or she) is also endowed with a small exogenous income, \(\varepsilon_{it}\), which is chosen randomly from a uniform distribution between 0 and \(\tau > 0\).

The agent maximizes his utility, which depends only on the amount of consumption in the second period, \(c_{i,t+1}\). In order to finance this consumption, he makes use of his wealth, \(w_t + \varepsilon_{it}\), in either of two alternative ways. First, he may save his entire

\footnote{\(\varepsilon_{it}\) gives a small perturbation to the realized incomes of otherwise identical agents. Later, we show that this term is necessary for establishing the existence of equilibrium and that it can be arbitrarily small for that end. The limiting case where \(\tau \to 0\) is considered at the end of this section.}
wealth and consume \( c_{t+1} = r(w_t + \epsilon_t) \) in the second period. Interest rate \( r \geq 1 \) is constant either because the economy considered is a small open economy or because there is a storage technology that yields the gross rate of return \( r \). In the latter case, we assume that the demand for capital never exceeds the amount of aggregate savings.\(^{10}\)

His second option is to become an entrepreneur and start a project. Each agent can run at most one project, and a project cannot be shared by multiple entrepreneurs due to information and enforcement problems among them. When starting a project, an agent can choose from several types of technology, the set of which is finite and denoted by \( \mathcal{J} \). Every technology produces a homogeneous consumption good from capital and labor with constant returns to scale. Specifically, if agent \( i \) adopts technology \( j \), where \( j \in \mathcal{J} \), his project produces the consumption good according to

\[
y_{it+1} = \begin{cases} 
  k_{it+1} f_j(\ell_{it+1}/k_{it+1}), & \text{if } k_{it+1} \geq I_j, \\
  0, & \text{if } k_{it+1} < I_j,
\end{cases}
\]

(1)

where \( f_j(\cdot) \) is the per unit capital production function of technology \( j \), \( \ell_{it+1} \), and \( k_{it+1} \) are the amounts of labor and capital inputs, respectively. Equation (1) shows that exploiting the potential of each technology requires at least a certain amount of investment. The minimal required amount of capital, denoted by \( I_j \geq 0 \), differs across technologies depending on technical aspects (e.g., the scope of scale economy for that technology) and various barriers to the adoption of those technologies, which may be specific to each economy. Capital depreciates completely within one period and \( f_j(\cdot) \) satisfies the standard Inada conditions for all \( j \in \mathcal{J} \).

Capital to be used at period \( t+1 \) is obtained by converting the consumption good

\(^{10}\)The open-economy assumption enables us to focus on the role of financial markets in determining the demand for capital and its composition rather than the supply of capital, which is given by the amount of savings in the closed-economy setting. Consistent with our assumption, recent empirical studies suggest that financial markets promote economic development not by enhancing overall capital accumulation but by efficiently allocating capital across sectors (e.g., Wurgler 2000). The assumption of storage technology is more suitable for low income countries, where inventories are the principal substitutes for investment (see discussions by Bencivenga and Smith 1993, Section 5).
at the end of period $t$ on a one-to-one basis. Thus, if the wealth of agent $i$ at the end of his first period is larger than $I_j$, then he can manage the investment for the project, adopting technology $j$ using his own funds.\footnote{Whenever an agent starts a project, he invests all of his wealth in the project since it is always optimal to do so.} If the agent’s wealth, $w_t + \epsilon_{it}$, falls short of the minimum required amount of capital, $I_j$, he must finance the gap by borrowing from the competitive financial intermediaries, which we call banks. Banks can borrow from the international credit market at the constant world interest rate $r$, while agents cannot do so because of the issue of limited enforcement, as explained below.

In order to obtain the loan that is needed to finance the investment, $k_{i,t+1} - w_t - \epsilon_{it}$, the agent applies to banks by announcing the plan of his project, consisting of a triple $(j, k_{i,t+1}, w_t + \epsilon_{it})$. The choice of technology, $j$, the size of investment, $k_{i,t+1}$, and the amount of his own fund, $w_t + \epsilon_{it}$, are verifiable and, thus, contractable. If the agent is approached by several banks, he chooses a loan contract from the bank that offers the lowest gross interest rate, denoted by $r_{it}$. If the agent is denied the loan at any interest rate—i.e., if he is credit rationed—he gives up becoming an entrepreneur and lends his entire wealth to the credit market.

At period $t + 1$, an entrepreneur (an agent who has successfully obtained credit or has managed his investment fully using his own funds) hires a positive number of workers, denoted by $\ell_{i,t+1}$, at the market wage rate $w_{t+1}$. The revenue from his project at the end of this period is

$$\rho_{i,t+1} = y_{i,t+1} - w_{t+1}\ell_{i,t+1},$$

where $y_{i,t+1}$ is given by (1). The entrepreneur is obliged to repay the loan from this revenue, but he has an option to default at a certain cost. We assume that the cost of default is proportional to the revenue from the project, $\lambda \rho_{i,t+1}$, where $\lambda \in (0, 1)$ is a parameter representing the quality of enforcing financial contracts. This is equivalent to assuming that only $100\lambda$ percent of the cash flow from any project can be collateralized. If he defaults, his end of period consumption becomes $c_{i,t+1} = (1 - \lambda)\rho_{i,t+1}$; otherwise, he repays the loan and consumes $c_{i,t+1} = \rho_{i,t+1} -$
\( r_{it+1}(k_{it+1} - w_t - \epsilon_{it}) \) units of the good.

2.2 Behaviors of Households and Banks

This subsection examines the rational behaviors of generation-\( t \) households (who become entrepreneurs at period \( t+1 \) if they obtain credit) and banks, taking as given the market wage rate \( w_{t+1} \). The decision processes are sequential and, therefore, can be solved backward from the final decision: the decision at period \( t+1 \) of an entrepreneur who has already chosen technology \( j \) and the amount of capital \( k_{it+1} \geq I_j \). The remaining variable to be determined is the number of workers to hire. Whether to default or not, the entrepreneur’s objective at this stage is to maximize \( \rho_{it+1} \) with respect to \( `_{it+1} \).

Differentiating (2) by \( `_{it+1} \) yields

\[
(3) \quad f'_j (\ell_{it+1}/k_{it+1}) = w_{t+1},
\]

which shows that there is a unique solution to this maximization problem,

\[
\ell_{it+1} = \tilde{\ell}_j(w_{t+1})k_{it+1}, \quad \rho_{it+1} = \tilde{\rho}_j(w_{t+1})k_{it+1},
\]

where \( \tilde{\ell}_j(w) \equiv f^{-1}_j(w) \), \( \tilde{\rho}_j(w) \equiv f_j(\tilde{\ell}_j(w)) - w\tilde{\ell}_j(w) \).

Out of the maximized revenue \( \tilde{\rho}_j(w_{t+1})k_{it+1} \), he repays the loan when it is in his interest to do so. That is, the loan will be paid back if and only if

\[
(\tilde{\rho}_j(w_{t+1}) - r_{it+1})k_{it+1} + r_{it+1}(w_t + \epsilon_{it}) \geq (1 - \lambda)\tilde{\rho}_j(w_{t+1})k_{it+1}. \tag{4}
\]

Now let us focus on the decisions made by banks. Banks offer loans to potential entrepreneurs if and only if entrepreneurs are expected to repay and banks can earn interest at least as large as the market interest rate \( r \). As long as repayment is expected, competition among banks brings the interest rate down to \( r \). Banks are assured of the repayment if a prospective entrepreneur’s planned project, summarized by \( (j, k_{it+1}, w_t + \epsilon_{it}) \), satisfies condition (4) at interest rate \( r_{it+1} = r \) as well as the minimum requirement for the size of investment. These conditions are summarized in terms of the size of investment for the proposed project:

\[
k_{it+1} \in \begin{cases} 
I_j, \frac{w_t + \epsilon_{it}}{1 - \lambda \tilde{\rho}_j(w_{t+1})/r} & \text{if } \lambda \tilde{\rho}_j(w_{t+1}) < r, \\
[I_j, \infty) & \text{if } \lambda \tilde{\rho}_j(w_{t+1}) \geq r,
\end{cases} \tag{5}
\]
If the plan fails to satisfy (5), the project cannot obtain credit at any interest rate.\footnote{Note that a higher interest rate makes condition (4) more strict and gives borrowers more incentive to default. Thus, banks cannot make (zero) profit by offering a loan for projects that do not satisfy (5) with an interest rate higher than $r$.}

We then go back to the decision of a prospective entrepreneur at the end of his first period. He chooses technology $j$ and the size of investment $k_{i,t+1}$ to maximize his second-period consumption,

$$c_{i,t+1} = \begin{cases} (\tilde{\rho}_j(w_{t+1}) - r) k_{i,t+1} + r(w_t + \epsilon_{it}) & \text{if the plan satisfies (5)}, \\ r(w_t + \epsilon_{it}) & \text{otherwise}. \end{cases} \tag{6}$$

Let us first examine whether it is feasible to adopt technology $j$. Note that the equilibrium wage, $w_{t+1}$, must satisfy $\lambda \tilde{\rho}_j(w_{t+1}) < r$, or, equivalently,

$$w_{t+1} > \max_{j \in \mathcal{J}} \tilde{\rho}_j^{-1}(r/\lambda) \equiv w(\lambda), \tag{7}$$

because violation of (7) will result in infinite amounts of investment and, therefore, in excess labor demand.\footnote{Suppose that $\lambda \tilde{\rho}_j(w_{t+1}) \geq r$ for some $j \in \mathcal{J}$. Then, from the second line of (5), entrepreneurs can obtain an infinite payoff by investing an infinite amount of capital and hiring an unbounded number of workers, which, necessarily, results in excess demand in the labor market. Thus, in equilibrium, $w_{t+1}$ must be in a range where $\lambda \tilde{\rho}_j(w_{t+1}) \geq r$ does not hold for any $j$, which is guaranteed under (7).} Under (7), the amount of investment must be within the finite interval, as designated by the first line of (5), which implies that technology $j$ can be adopted only when this interval is not an empty set. This condition is equivalent to

$$w_t + \epsilon_{it} \geq \left(1 - \frac{\lambda \tilde{\rho}_j(w_{t+1})}{r}\right) I_j \equiv \eta_j(w_{t+1}; \lambda, I_j), \tag{8}$$

where function $\eta_j(\cdot)$ represents the minimum amount of own funds required to borrow from banks to start a project with technology $j$. Since function $\eta_j(w_{t+1}; \lambda, I_j)$ is positive and upward-sloping under (7), condition (8) can be inverted to yield

$$w_{t+1} \leq \tilde{\rho}_j^{-1} [(r/\lambda) (1 - (w_t + \epsilon_{it})/I_j)] \equiv \omega_{jb}(w_t + \epsilon_{it}; \lambda, I_j). \tag{9}$$

$$\Delta$$
We call (9), or, equivalently, (8), the *borrowing constraint* for technology \( j \). Any technology is adoptable for agent \( i \) if and only if (9) is satisfied.

An agent also examines whether it is desirable to become an entrepreneur (i.e., to choose some \( j \) and set \( k_{i,t+1} > 0 \)) rather than simply save his entire wealth \( (k_{i,t+1} = 0) \). From (6), it is optimal to invest a positive amount of capital in technology \( j \) only when \( \tilde{p}_j(w_{t+1}) \geq r \). This condition is equivalent to

\[
w_{t+1} \leq \frac{e^{\rho_j}(w_{t+1})}{r} \equiv \omega_{jp},
\]

which we call the *profitability constraint*. The constant \( \omega_{jp} \) represents the level of wage at which a project with technology \( j \) breaks even.

Combining (9) and (10), we see that technology \( j \) satisfies both the borrowing and profitability conditions if and only if

\[
w_{t+1} \leq \min \{ \omega_{jp}, \omega_{jb}(w_{t+1}; \lambda, I_j) \} \equiv \phi_j(w_{t+1}; \lambda, I_j).
\]

Function \( \phi_j(w_{t+1}; \lambda, I_j) \) gives the upper bound for \( w_{t+1} \), at which the adoption of technology \( j \) is both feasible and desirable for an agent with a given amount of own funds. Accordingly, we find that there is at least one technology that is both adoptable and profitable for agent \( i \) if and only if

\[
w_{t+1} \leq \max_{j \in J} \phi_j(w_{t+1}; \lambda, I_j) \equiv \theta(w_{t+1}; \lambda, I),
\]

where \( I \equiv \{ I_j \}_{j \in J} \) represents the list of minimum requirements. Note that the value of function \( \theta(\cdot) \) on the right-hand side differs among agents due to a random term \( \epsilon_{it} \). According to the relative magnitude between \( w_{t+1} \) and the individual value of \( \theta(\cdot) \), the optimal behavior of each agent can be categorized into three cases.

First, if \( w_{t+1} < \theta(w_{t+1}; \lambda, I) \) for agent \( i \), then there must be some technology that he can adopt whose rate of return \( \tilde{p}_j(w_{t+1}) \) exceeds \( r \). He becomes an entrepreneur,

\[\text{14}\]In this case, (11) and (12) imply that there must be some \( j \) such that \( w_{t+1} < \theta(w_{t+1}; \lambda, I) = \min \{ \omega_{jp}, \omega_{jb}(w_{t+1}; \lambda, I_j) \} \). It follows that \( w_{t+1} < \omega_{jb}(w_{t+1}; \lambda, I_j) \) and \( w_{t+1} < \omega_{jp} \); that is, technology \( j \) is adoptable, and its rate of return is strictly larger than \( r \).
adopting that technology,\(^\text{15}\) and invests
\[
k_{it+1} = \frac{w_t + \epsilon_{it}}{1 - \frac{\lambda \rho_j(w_{t+1})}{r}} = \frac{w_t + \epsilon_{it}}{\eta_j(w_{t+1}; \lambda, I_j)} I_j
\] (13)
units of capital because (6) implies it is optimal to invest as much as permitted by (5). Note that \((w_t + \epsilon_{it})/\eta_j(w_{t+1}; \lambda, I_j)\) in equation (13) represents the ratio of actual own funds to the amount required to obtain the credit and, therefore, must be above 1. From (3) and (6), consumption of the entrepreneur and the individual labor demand from this project are
\[
\ell_{it+1} = k_{it+1} \tilde{\ell}_j(w_{t+1}) = \frac{w_t + \epsilon_{it}}{\eta_j(w_{t+1}; \lambda, I_j)} I_j \tilde{\ell}_j(w_{t+1}),
\] (14)
\[
c_{it+1} = r (w_t + \epsilon_{it}) + (\tilde{p}_j(w_{t+1}) - r) \frac{w_t + \epsilon_{it}}{\eta_j(w_{t+1}; \lambda, I_j)} I_j.
\] (15)
The second term in (15) is the extra income, or rent, obtained by becoming an entrepreneur. Second, if \(w_{t+1} > \theta(w_t + \epsilon_{it}; \lambda, I)\), the rate of return from any adoptable technology falls short of \(r\). Then, it is best for the agent to save his entire wealth \((k_{it+1} = \ell_{it+1} = 0)\) and receive \(c_{it+1} = r (w_t + \epsilon_{it})\). Finally, if \(w_{t+1} = \theta(w_t + \epsilon_{it}; \lambda, I)\), he may act similarly to the first case or may be indifferent as to which option to choose.\(^\text{16}\) If he is indifferent, investment \(k_{it+1}\) can be anywhere between the minimum amount \(I_j\) and (13); the labor demand is \(\ell_{it+1} = k_{it+1} \tilde{\ell}_j(w_{t+1})\), and any choice results in \(c_{it+1} = r (w_t + \epsilon_{it})\).

### 2.3 Inequality in Equilibrium

The analysis above implicitly gives individual labor demand as a function of the market wage rate, \(w_{t+1}\). This subsection establishes the existence of an equilibrium

\(^{15}\)In principle, there may be two or more technologies whose rate of return is larger than \(r\). In that case, the entrepreneur chooses the most profitable technology. In the limiting case of \(\epsilon \to 0\) which we will consider later, however, there is generically only one technology that satisfies this condition and, therefore, this possibility can be ignored.

\(^{16}\)When \(w_{t+1} = \theta(w_t + \epsilon_{it}; \lambda, I)\), there must be some \(j\) such that \(\min\{\omega_{jp}, \omega_{jb}(w_t + \epsilon_{it}; \lambda, I_j)\} = w_{t+1}\). This means that either the borrowing or profitability condition is binding. It is possible that \(\omega_{jp} > w_{t+1}\) and the agent strictly prefers to start a project, in which case the borrowing constraint should be binding. Otherwise, the profitability condition is binding, and he is indifferent as to whether to start a project or not.
wage rate at which the aggregate supply of and demand for labor are equalized. We then examine the extent of inequality arising in equilibrium.

Aggregate labor demand is obtained by summing the decisions of individual agents,

\[ L_{t+1}^D(w_{t+1}; w_t) \equiv \int_0^1 \ell_{i t+1} \, di, \quad (16) \]

which explicitly shows its dependence on \( w_{t+1} \) and \( w_t \). Recall that \( \theta(w_t + \epsilon_{it}; \lambda, I) \) is the threshold level of \( w_{t+1} \) for starting a project. Individual labor demand may differ across agents because they have heterogeneous amounts of own funds, \( w_t + \epsilon_{it} \), and, therefore, the threshold \( \theta(w_t + \epsilon_{it}; \lambda, I) \) may vary across agents. A rise in \( w_{t+1} \) decreases the number of agents whose \( \theta(w_t + \epsilon_{it}; \lambda, I) \) is above \( w_{t+1} \), which causes aggregate labor demand (16) to decrease in \( w_{t+1} \). Intuitively, an increase in \( w_{t+1} \) strengthens both the borrowing constraint (9) and the profitability constraint (10) and thus reduces the number of agents who are both able and willing to start projects. A subtle point is that \( L_{t+1}^D(w_{t+1}; w_t) \) may be set-valued at certain levels of \( w_{t+1} \); that is, the demand curve of labor may have flat segments at which a mass of agents are indifferent as to whether to save or starts a project.\(^{17}\) If the set of labor demand at a certain wage level, \( L_{t+1}^D(w_{t+1}; w_t) \), includes the level of labor supply, 1, then \( w_{t+1} \) is an equilibrium wage level. The following proposition establishes the existence of an equilibrium wage level.

**Proposition 1** Suppose that \( I_j > \bar{\ell}_j(\bar{w}_{j p})^{-1} \) for all \( j \in J \) and that there are, at most, a finite number of intersections between functions \( \bar{\rho}_j(w) \) and \( \bar{\rho}_{j'}(w) \) for any \( j \neq j' \). Then, for any \( w_t > 0 \), there is an equilibrium level of \( w_{t+1} \in [\underline{\theta}, \bar{\theta}] \) with which \( 1 \in L_{t+1}^D(w_{t+1}; w_t) \) holds, where \( \underline{\theta} \equiv \theta(w_t; \lambda, I) \) and \( \bar{\theta} \equiv \theta(w_t + \bar{\epsilon}; \lambda, I) \).

**Proof:** in Appendix

Condition \( I_j > \bar{\ell}_j(\bar{w}_{j p})^{-1} \) means that the minimum size of each project is such that it requires hiring more than one worker, which we reasonably assume to be satisfied. Note that, on the one hand, \( w_{t+1} < \underline{\theta} \) cannot be an equilibrium since, in that case, all agents start projects and therefore aggregate labor demand would

\(^{17}\)Another possibility is that they are indifferent as to which technology to adopt.
Figure 1: An Example of the $\theta$ Curve. It depicts a case of two technologies, $\mathcal{J} = \{A, M\}$. From (11) and (12), it is easily confirmed that function $\theta(\cdot)$ is weakly upward-sloping and bounded below by $\max_{j \in \mathcal{J}} \phi_j(0, I_j) = \max_{j \in \mathcal{J}} \rho^{-1}(\lambda/r) = \bar{\omega}(\lambda)$ and above by $\max_{j \in \mathcal{J}} \bar{\varphi}_j$. The $\theta$ curve consists of several flat segments and upward-sloping segments: each upward-sloping segment corresponds to the borrowing constraint for a certain technology, (9), while each flat segment corresponds to the profitability constraints, (10).

exceed the labor supply. On the other hand, if $w_{t+1} > \bar{w}_t$, then no agent would start a project and, therefore, the aggregate labor demand would become zero. Moreover, we prove in the Appendix that aggregate labor demand is continuous with respect to $w_{t+1}$, relying upon heterogeneity in the amounts of own funds among prospective entrepreneurs.\[18\] It follows that there must be a level of $w_{t+1}$ between $\overline{\theta}_t$ and $\overline{\bar{\theta}}_t$ at which aggregate labor demand coincides with the supply.

In the remainder of this section, we present an intuitive explanation of how and when a significant income inequality arises among old agents in equilibrium.\[19\] Figure 1 depicts a typical shape of function $\theta(\cdot)$ against the amount of own funds, $w_t + $.

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\[18\] Since $L^D_{t+1}(\cdot)$ is a correspondence, the notion of continuity is different from that for a function. Precisely, we show in the Appendix that $L^D_{t+1}(\cdot)$ is convex-valued, non-empty, and upper hemi-continuous, which implies that the graph of labor demand in $(L^D_{t+1}, w_{t+1})$ space is jointed.

\[19\] We concentrate on the income of the old agents by two reasons. First, the income of young agents is uniformly distributed between $[w_1, w_t + \tau]$, the inequality among whom is negligible when $\tau$ is sufficiently small. Second, in our model, the welfare of agents is determined solely by the amount of consumption ($= \text{income}$) in their old age.
Proposition 1 implies that what is important in determining the equilibrium is the shape of the \( \theta \) curve only on a short interval \([w_t, w_t + \bar{\epsilon}]\). One possibility is that the curve is entirely flat in that interval. In this case, the labor demand curve is completely elastic (flat) at \( w_{t+1} = \bar{\theta}_t = \bar{\theta}_t \), and the equilibrium wage is uniquely determined at this level. In equilibrium, the rate of return on the most profitable technology among all feasible technologies is equal to the exogenous interest rate.\(^{20}\) All agents are indifferent as to whether to become entrepreneurs or save their wealth. Either way, they obtain \( c_{t+1} = r(w_t + \epsilon_t) \). Given that the magnitude of random income \( \epsilon_t \) is marginal, the inequality of consumption in the second period is also marginal.

We have a different distributional consequence, however, when the \( \theta \) curve is upward-sloping in interval \([w_t, w_t + \bar{\epsilon}]\). The labor demand curve is downward-sloping for \( w_{t+1} \in [\theta_t, \bar{\theta}_t] \) because the level of \( w_{t+1} \) affects the number of entrepreneurs who can obtain credit. In this case, the profitability constraint is not (generically) binding, and, therefore, the rate of return from starting a project is strictly higher than \( r \). This means that every agent strictly prefers to start a project, but the overall labor demand would exceed the aggregate supply if all agents start projects. Thus, the equilibrium wage \( w_{t+1} \) must be between \( \theta_t \) and \( \bar{\theta}_t \), so that some agents (whose \( \theta(w_t + \epsilon_t; \lambda, I) \) is below \( w_{t+1} \)) do not satisfy the borrowing constraint. In other words, some agents must be rationed from the credit market. Consumption of those credit-rationed agents is lower than entrepreneurs.

The income inequality that emerged in this way would not be removed even when the \textit{ex ante} heterogeneity, represented by \( \bar{\epsilon} \), is infinitesimally small. The following proposition explicitly makes this point.

**Proposition 2** Suppose that the assumptions in Proposition 1 hold and the \textit{ex ante} heterogeneity is vanishingly small \((\bar{\epsilon} \to 0)\). Then, equilibrium at the limit is charac-

\(^{20}\)To understand this, note that the flat segments of the \( \theta \) curve correspond to \( \omega_{jp} \) for some \( j \). In equilibrium, \( w_{t+1} = \bar{\theta}_t = \bar{\theta}_t = \omega_{jp} \) means \( \tilde{\rho}_j(w_{t+1}) = r \) from the definition of \( \omega_{jp} \). For any technology that is more profitable than \( j \), that is, for all \( j' \) such that \( \omega_{jp} > \omega_{jp} \), definitions (11) and (12) imply that \( \omega_{j'p}(w_t + \epsilon_t; \lambda, I_{j'}) < \theta(w_t + \epsilon_t; \lambda, I) = \omega_{jp} = w_{t+1} \) for almost all agents and, therefore, such a technology cannot be adopted because of the borrowing constraint.
terized as follows:

(i) the equilibrium wage is determined by \( w_{t+1} = \theta(w_t; \lambda, I) \).

(ii) the technology used in equilibrium is in argmax_{j \in J} \phi_j(w_t; \lambda, I_j).

(iii) If \( j^* = \text{argmax}_{j \in J} \phi_j(w_t; \lambda, I_j) \) and \( \omega_{j^*b}(w_t; \lambda, I_{j^*}) < \sigma_{j^*p} \), then the economy specializes in technology \( j^* \), and credit rationing occurs. The number of entrepreneurs and their consumption are given by

\[
n_{t+1} = \left( I_{j^*} \bar{\ell}_{j^*}(\omega_{j^*b}(w_t; \lambda, I_{j^*})) \right)^{-1} < 1, \quad (17)
\]

\[
c_{t+1} = rw_t + r((1 - \lambda)I_{j^*} - w_t)/\lambda > rw_t. \quad (18)
\]

Proof: in Appendix.

(17) and (18) clearly show that, even at the limit where almost no \textit{ex ante} heterogeneity exists, a significant number of agents (i.e., \( 1 - n_{t+1} \) of them) are credit rationed, and their consumption \( rw_t \) is significantly lower than those lucky enough to obtain credit. In the remainder of the paper, we focus on this limiting case in order to examine the degree of inequality arising from credit rationing rather than that arising from \textit{ex ante} heterogeneity among agents.\footnote{Note that, however, we could not start the analysis of the model without the random term \( \epsilon_{it} \). Since, in that case, Proposition 1 does hold and there may be no equilibrium. Fortunately, the proof of Proposition 1 in Appendix is valid regardless of how small \( \sigma > 0 \) is. Therefore, at the limit, we can rely on Proposition 1 without actually considering the random term.}

3 Technology Choice and Credit Regime

The objective of this paper is to clarify how the economy’s financial infrastructure, represented by the enforceability of financial contracts \( \lambda \), affects the equilibrium distribution of income and the welfare of agents. From (17) and (18) in Proposition 2, we can see that, other things being equal, a better financial infrastructure (larger \( \lambda \)) results in more equal income distribution (i.e., in larger \( n_{t+1} \) and smaller \( c_{t+1} \)) by easing credit rationing. However, Proposition 2 also implies that there are other mechanisms through which \( \lambda \) affects income distribution in direct or indirect ways.
First, as property (i) says, $\lambda$ affects the dynamics of equilibrium wage $w_{t+1}$. Second, property (ii) shows that $\lambda$ is a determinant of the equilibrium choice of technology by affecting the highest level of wage that can be offered by entrepreneurs using each type of technology. Third, property (iii) implies that $\lambda$ determines credit regime; that is, credit rationing occurs only when the borrowing constraint, which obviously depends on $\lambda$ as represented by $\omega_{j^*b}(w_t; \lambda, I_{j^*})$, is tighter than the profitability constraint, $\omega_{j^*p}$.

From among these issues, this section examines those of technology choice and credit regime, leaving the analysis of dynamic issues for the next section.

### 3.1 Credit Regime

Let us start by considering whether or not credit rationing occurs in equilibrium, given the choice of technology $j^*$. As shown by property (iii) of Proposition 2, credit rationing occurs if borrowing constraint (9) is stronger than profitability constraint (10). A comparison of these conditions, where $\epsilon_{it} \to 0$, gives

$$\omega_{j^*b}(w_t; \lambda, I_{j^*}) \leq \omega_{j^*p} \iff w_t \leq (1 - \lambda)I_{j^*}. \tag{19}$$

(19) shows that there is a threshold level of the amount of own funds below which entrepreneurs are constrained by the borrowing limit. This condition can also be written as $\lambda \leq \frac{1 - w_t}{I_{j^*}}$. That is, given the amount of own funds $w_t$, credit rationing occurs if and only if the enforceability of financial contract is below $1 - w_t/I_{j^*}$. This explains one mechanism through which stronger enforcement (a larger $\lambda$) leads to more equality. Note that, however, the threshold level $1 - w_t/I_{j^*}$ depends on the equilibrium choice of technology $j^*$. Therefore, we need to clarify how $\lambda$ affects the technology choice, which is to be examined below.

### 3.2 Technology Choice

The technology used in equilibrium changes, or a technological switch occurs, when the technology with the largest $\phi_j(w_t; \lambda, I_j)$ is overtaken by another technology (recall property (ii) of Proposition 2). The number of usable technologies is potentially large, and it is not unusual for any economy to experience a technological switch several
times during the process of economic development. Nonetheless, to see how income distribution is affected at each instance of a technological switch, it generically suffices to focus on the two technologies involved, i.e., that with the largest $\phi_j(w_t; \lambda, I_j)$ and that with the second largest one. To examine the role played by the difference in capital intensity, let us suppose that these two are Cobb-Douglas technologies,

$$f_j(\ell/k) = A_j(\ell/k)^{1-\alpha_j} \quad \text{where } \alpha_A < \alpha_M.$$  \hspace{1cm} (20)

The one with a lower share of capital is called technology $A$ (e.g., agriculture), while the other technology is called $M$ (e.g., manufacturing). Substituting (20) into (3) and then into (10) and (9) gives the profitability and borrowing constraints for each technology:

$$w_{t+1} \leq (1 - \alpha_j) (\alpha_j/r)^{\hat{\alpha}_j} A_j^{\hat{\alpha}_j+1} \equiv \varpi_{jp},$$ \hspace{1cm} (21)

$$w_{t+1} \leq \varpi_{jp} (\lambda I_j/(I_j - w_t))^{\hat{\alpha}_j} \equiv \omega_{jb}(w_t; \lambda, I_j),$$ \hspace{1cm} (22)

where $\hat{\alpha}_j \equiv \alpha_j/(1 - \alpha_j) > 0$. The value of $\phi_j(w_t; \lambda, I_j)$ is given by the smaller of $\varpi_{jp}$ and $\omega_{jb}(w_t; \lambda, I_j)$ in (21) and (22).

While our concern is when a marginal increase in $\lambda$ causes a technological switch, it is insightful to see how the pattern of technological choice is affected by large changes in $\lambda$. In particular, when the economy’s financial infrastructure is quite primitive ($\lambda \to 0$), then the borrowing constraint becomes very tight ($\omega_{jb}(w_t; \lambda, I_j) \to 0$ for every $j$), which means that $\phi_j(w_t; \lambda, I_j)$ is determined by $\omega_{jb}(w_t; \lambda, I_j)$. In addition, (22) shows that the higher the capital intensity is, the more rapidly $\omega_{jb}(w_t; \lambda, I_j)$ converges to 0. This means that, with a sufficiently low $\lambda$, the economy specializes in labor-intensive technology. Intuitively, if the enforcement of financial contracts is weak, only a small number of agents obtain funds due to tight credit rationing. In that situation, entrepreneurs who have successfully obtained funds can hire a large number of workers at a low wage level, for which case the labor-intensive technology is more profitable. Conversely, when the enforcement of financial contracts is nearly perfect ($\lambda \to 1$), the borrowing constraint becomes weaker than the profitability constraint ($\omega_{jb}(w_t; \lambda, I_j) > \varpi_{jp}$ for every $j$), which means $\phi_j(w_t; \lambda, I_j)$ is determined by $\varpi_{jp}$. Hence, the economy specializes in the technology with the highest $\varpi_{jp}$, which
Figure 2: Technology choice and credit regime. The economy specializes in technology A in regions Ab and Ap and in technology M in Mb and Mp. The borrowing constraint is binding in regions Ab and Mb, while the profitability constraint is binding in Ap and Mp. This result is valid within a range of \((\lambda, w_t)\), in which technologies A and M have higher \(\phi_j(w_t; \lambda, I_j)\)'s than others. The parameters are \(\alpha_A = 0.20\), \(\alpha_M = 0.45\), \(r = 2.0\), \(\omega_{Ap} = 1.20\), and \(\omega_{Mp} = 2.25\) (these values are used for all following figures). The figure shows the case of \(I_A < I_M\) (specifically, \(I_A = 1.8, I_M = 3.5\)). Region Ap disappears if \(I_A > I_M\).

is largely determined by the TFP of that technology \(A_j\). Intuitively, when credit is easy to obtain, many entrepreneurs start businesses, and wages go up. In that circumstance, entrepreneurs choose a more productive technology using which they can break even with a higher wage level. Thus, when development in financial infrastructure triggers a technological change, the new technology tends to have higher capital intensity and higher productivity.\(^{22}\)

Reflecting this observation, we focus on the case in which TFP \(A_j\) is such that technology M can break even at a higher wage level. Specifically, \(\omega_{Mp} \geq \omega_{Ap}/(1 - \alpha_M)\) is assumed, which implies \(\omega_{Mp} > \omega_{Ap}\) since \(\alpha_M > 0.0\).\(^{23}\) In this setting, \(\phi_M(\cdot)\)

\(^{22}\)If the productivity of the new technology is lower, it is not adopted. If its capital intensity is lower, it should be adopted before the development of financial infrastructure. Within a large collection of potentially usable technologies, there are of course some technologies that are highly capital intensive but not very productive. Our argument is that, even though such technologies exist, they are unlikely to be chosen in equilibrium at any stage of the development of financial markets.

\(^{23}\)As shown by (21), the value of \(\omega_{jp}\) is determined by TFP \(A_j\), and this is the only way \(A_j\) affects
and $\phi_A(\cdot)$ can be compared by substituting (21) and (22) into (11). Since $\bar{\omega}_M > \bar{\omega}_A$, the value of $\phi_M(\cdot)$ becomes larger than $\phi_A(\cdot)$ when either $\omega_{Mb}(\cdot) \geq \bar{\omega}_A$ or $\omega_{Mb}(\cdot) \geq \omega_{Ab}(\cdot)$ holds. From (21) and (22), solving $\omega_{Mb}(w; \lambda, I_M) \geq \bar{\omega}_A$ gives

$$\lambda \geq \left( \frac{\bar{\omega}_A}{\bar{\omega}_M} \right)^{1/\bar{\alpha}_M} \left( 1 - \frac{w}{I_M} \right) \equiv \Lambda_1(w; I),$$

whereas $\omega_{Mb}(w; \lambda, I_M) \geq \omega_{Ab}(w; \lambda, I_A)$ holds if and only if $w < I_A$ and

$$\lambda \geq \left[ \frac{\bar{\omega}_A}{\bar{\omega}_M} \left( 1 - \frac{w}{I_M} \right) \left( 1 - \frac{w}{I_A} \right)^{-\bar{\alpha}_A} \right]^{1/(\bar{\alpha}_M - \bar{\alpha}_A)} \equiv \Lambda_2(w; I).$$

For convenience, define $\Lambda_2(w; I) = \infty$ when $w \geq I_A$. From (23) and (24), it turns out that

$$\phi_M(w_t; \lambda, I_M) \leq \phi_A(w_t; \lambda, I_A) \iff \lambda \leq \min\{\Lambda_1(w_t; I), \Lambda_2(w_t; I)\} \equiv \Lambda(w_t; I).$$

(25)

The representative shape of function $\Lambda(w_t; I)$ is calculated numerically in Figure 2. (25) shows the way in which the choice of technology in equilibrium depends on the enforceability of financial contracts. Since technology $M$ is both more productive and more capital intensive compared to $A$, there is a threshold level $\Lambda(w_t; I)$ in the enforceability of a financial contract at which a technological switch from $A$ to $M$ occurs.

Figure 2 also shows how the $(\lambda, w_t)$ space is separated by condition (19) in addition to (25). Observe that there is no simple relationship between the degree of contract enforcement and the existence of credit rationing. Specifically, economies in region $Mb$ experience credit rationing even though they have a better financial infrastructure than those in region $Ap$, where no such rationing occurs. While it may appear strange, this is not particularly at odds with reality. Credit rationing is not necessarily most prevalent at the initial stage of economic development when the financial infrastructure is weak. Our model shows that such non-monotonic behavior
arises because the degree of enforcement \( \lambda \) not only affects the difficulty of obtaining credit for a given technology but is also a determinant of the economy’s technology specialization.

4 Long-term Analysis

The previous section has shown that stronger enforcement of financial contracts enables the economy to adopt more productive technologies but, at the same time, may cause more inequality. As can be observed from Figure 2, the level of \( \lambda \) at which a technological switch occurs as well as the degree of inequality caused by the switch depends on the amount of own funds, \( w_t \), which is taken as given in the previous section. However, since \( w_t \) is determined by the previous period’s equilibrium in the labor market and since the latter is critically dependent on \( \lambda \), as we see in Propositions 1 and 2, an examination of the long-term consequence of a stronger enforcement on income distribution must take into account its effects on the dynamics of wages. The first half of this section investigates the dynamics of wages under different levels of enforcement. Then, in the second half, we examine how the level of enforcement affects income distribution and the welfare of agents in the long run.\(^{24}\)

4.1 Dynamics of Wages

To be consistent with historical experiences that financial markets affected economic performance, it is reasonable to consider situations in which the size of minimum investment is such that entrepreneurs must borrow part of their investments to start projects: specifically, \( I_j > \sigma_{jp} \), where \( \sigma_{jp} \) is the upper bound of the market wage when the economy specializes in technology \( j \). For notational simplicity, we retain the two-technology Cobb-Douglas setting introduced in the previous section.

\(^{24}\)We use the term ‘long-term’ or ‘long-run’ to mean the stationary state of the economy for a constant value of \( \lambda \). The consequences when the financial infrastructure is gradually improved (i.e., when \( \lambda \) increases gradually) are explained in the next section, where the implications of having more than three technologies are also discussed.
From property (i) of Proposition 2, \( w_t \) evolves over generations according to \( w_{t+1} = \theta(w_t; \lambda, I) \). Using (19) and (25), the pattern of movements of \( w_t \), given \( \lambda \) and other parameters, is summarized as follows. When the \((\lambda, w_t)\) pair is in region \(jp\) of Figure 2, where \(j \in \{A, M\}\), the equilibrium wage is determined by \( w_{t+1} = \bar{\omega}_{jp} \). In region \(jb\), it is given by \( w_{t+1} = \omega_{jb}(w_t; \lambda, I_j) \), which, from (22), implies that

\[
   w_{t+1} \geq w_t \iff \lambda \geq (1 - w_t/I_j)(w_t/\bar{\omega}_{jp})^{1/\alpha_j} \equiv \lambda_{hb}^*(w_t; I_j),
\]

where \( \lambda_{hb}^*(w_t; I_j) \) gives the level of enforcement at which \( w_t \) becomes stationary.

Figure 3 depicts representative patterns of the evolution of \( w_t \), numerically calculated under two different sets of parameters. A number of properties can be observed. First, there is at least one steady state for each level of \( \lambda \). Some steady states are unstable, but the lowest steady state (i.e., the steady state with the lowest \( w_t \)) is always stable. Second, in the long run, the economy specializes in the labor-intensive technology if enforcement of financial contracts is poor, whereas it specializes in the capital-intensive technology if enforcement is stronger than a certain threshold. Third, the steady-state income of young agents and, therefore, the amounts of their own funds grow with the enforcement of contracts. This property holds even at the threshold level of \( \lambda \), and the technology switch does not affect wages in a drastic
way. Fourth, the precise pattern of the evolution depends on the minimum size of investment. The locus of the steady state transits region $Ap$, as shown by panel (i), only if $I_A$ is smaller than a certain threshold $\zeta(I_M)$, which is defined by

$$\zeta(I_M) \equiv \omega_{Ap} \left(1 - \frac{\omega_{Ap}}{\omega_{Mp}}\right)^{1/\bar{\alpha}_M} \left(1 - \frac{\omega_{Ap}}{I_M}\right)^{-1}.$$ (27)

Intuitively, credit rationing in an economy specialized in technology $A$ cannot be resolved until that technology is abandoned if the size of the project is large. The following proposition establishes that those four properties are fairly robust.

**Proposition 3** Let $j^*(\lambda)$ and $w^*(\lambda)$ denote the choice of technology and wage rate at the lowest steady state. Suppose that increases in $\lambda$ do not cause $j^*(\lambda) \in \{A, M\}$ to switch more than twice.²⁵ Then, the following properties hold:

a. For any $\lambda \in (0, 1)$, there exists $w^*(\lambda) \in (0, \omega_{Mp}]$. In addition, if $w_T \leq w^*(\lambda)$ for some $T$, then $w_t$ converges to $w^*(\lambda)$;

b. There exists $\lambda^{uw} \in (0, 1)$ such that $j^*(\lambda) = A$ if $\lambda < \lambda^{uw}$ and $j^*(\lambda) = M$ if $\lambda > \lambda^{uw}$;

c. $w^*(\lambda)$ is weakly increasing in $\lambda$; in addition, it is continuous at $\lambda = \lambda^{uw}$;

d. $w^*(\lambda^{uw}) < \omega_{Ap}$ whenever $I_A > \zeta(I_M)$.

*Proof: Available upon request.*

### 4.2 Long-term Effects of Stronger Enforcement

Relying upon the above result, we now examine the impacts of stronger enforcement on income distribution among agents. Those considered are old agents since the utility of agents depends only on the amount of consumption when old. In region $jp$ (i.e., in region $Ap$ or $Mp$), no credit rationing occurs, and all old agents receive

²⁵By this, we ignore the possibility that the economy switches back to the old, labor-intensive technology as a result of improvements in the financial infrastructure because there is little evidence supporting such a possibility and also because it is also theoretically very unlikely to occur. Specifically, for this condition to be violated, functions $\lambda_{Ab}^*(w; I_A)$ and $\lambda_{Mh}^*(w; I_M)$ have multiple points of intersection in the range of $w \in (0, \omega_{Ap})$, which we numerically found to occur only for a very narrow range of parameters.
Figure 4: Aggregate consumption and the Gini coefficient at the lowest steady state. Parameters: $I_A = 1.5$, $I_M = 10$ (panel i); $I_A = 8.5$, $I_M = 10$ (panel ii).

$rw_t$. In region $jb$, substituting the equilibrium wage for (17) and eliminating $A_j$ and $\omega_{jb}(w_t; I_j)$ by (21) and (22) show that

$$N(j, w_t, \lambda) = \frac{\hat{\alpha}_j \omega_{jp}}{rI_j} \left( \frac{\lambda I_j}{I_j - w_t} \right)^{1+\hat{\alpha}_j}.$$  \hfill (28)

Agents start projects and earn $rw_t + r(1 - \lambda)I_j - w_t$ units of goods at the end of their second period, whereas $1 - N(j, w_t, \lambda)$ agents are rationed from the credit market and end up consuming $rw_t$ in a credit-constrained economy. The dependence of income distribution on $\lambda$ can be illustrated by focusing on aggregate consumption, denoted by $C$, and the Gini coefficient, $G$. Given technology $j$, current wage $w_t$, and enforcement $\lambda$, they are expressed as\(^{26}\)

$$C(j, w_t, \lambda) = rw_t + (r/\lambda)N(j, w_t, \lambda) \max [(1 - \lambda)I_j - w_t, 0],$$  \hfill (29)

$$G(j, w_t, \lambda) = (1 - N(j, w_t, \lambda))(1 - rw_t/C(j, w_t, \lambda)).$$  \hfill (30)

Their steady-state values for a given level of $\lambda$ are obtained by substituting $j = j^*(\lambda)$ and $w = w^*(\lambda)$; namely, $C^*(\lambda) \equiv C^*(j^*(\lambda), w^*(\lambda), \lambda)$ and $G^*(\lambda) \equiv G(j^*(\lambda), w^*(\lambda), \lambda)$.

In Figure 4, we calculated the representative shapes of $C^*(\lambda)$ and $G^*(\lambda)$ for the cases of $I_A > \zeta(I_M)$ and $I_A \leq \zeta(I_M)$, respectively. In either case, a stronger

\(^{26}\)In regions $Ap$ and $Mp$, where credit rationing is absent, condition (19) shows that $(1 - \lambda)I_j - w_t$ is negative. In that case, (29) shows that $C(j, w_t, \lambda) = rw_t$.  

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enforcement increases aggregate consumption while reducing inequality, except at the point at which a technology switch occurs. With strengthened enforcement, a larger number of agents become able to start projects both through the weakened borrowing constraint and their increased own funds (recall that \( w^*(\lambda) \) is increasing). Then, the improved allocation of resources increases the aggregate income of agents and, at the same time, reduces inequality among agents. Thus, policies that strengthen the enforcement of contracts unambiguously improve welfare as long as they do not cause a technological switch. The following proposition formally establishes those properties.

**Proposition 4**

a. \( C^*(\lambda) \) is weakly increasing in \( \lambda \) for all \( \lambda \);

b. \( G^*(\lambda) \) is weakly decreasing in \( \lambda \) for all \( \lambda \) except at \( \lambda = \lambda^{sw} \).

*Proof: Available upon request.*

The remaining task is to examine the welfare consequences of stronger enforcement when it causes a technological switch. Figure 4 shows that the degree of inequality increases discretely when \( \lambda \) reaches the threshold level \( \lambda^{sw} \). Since aggregate consumption also increases discretely at the same time, the welfare effect of a technological switch is, in general, ambiguous. In the following, we explicitly examine the distributions of income before and after a technological switch and show that the minimum investment size plays an important role in determining whether the technological switch is desirable or not in terms of welfare.

First, consider the case in which \( I_A \leq \zeta(I_M) \) (see panel (i) of Figure 3). When \( \lambda \) is slightly below the threshold \( \lambda^{sw} \), the economy is in region \( Ap \), with all old agents receiving \( r\omega_{Ap} \). Credit rationing occurs when \( \lambda \) is slightly above the critical level, since the economy is now in region \( Mb \). This creates income inequality, but, nonetheless, all agents receive at least \( r\omega_{Ap} \), which can be understood by noting that the steady-state level of \( w_t \) is weakly increasing in \( \lambda \) and that all old agents receive at least \( rw_t \) regardless of their entrepreneurial opportunities. Technically, the (steady-state) income distribution with \( \lambda > \lambda^{sw} \) first-order dominates the distribution with \( \lambda < \lambda^{sw} \), and, therefore, the overall welfare of agents is improving regardless of the choice of criteria unless the disutility caused by envy, which is ignored in our analysis,
Figure 5: Welfare consequence of the technological switch. The technological switch can worsen welfare if the \((I_A, I_M)\) pair is within the dark-gray area between the two thick curves. The \(\chi\) curve locates between the \(\zeta\) curve and the 45-degree line when \(I_M > T_M\), while the two curves overlap for \(I_M \leq T_M\). The curves are calculated numerically under the same parameters as in Figure 2 (\(\alpha_A = .20, \alpha_M = .45, r = 2.0, \omega_{Ap} = 1.20\), and \(\omega_{Mp} = 2.25\)).

is taken into account.

Next, let us consider the case in which \(I_A > \zeta(I_M)\), for which a comparison must be made between income distribution in region \(Ab\) and that in region \(Mb\). In this case, the above argument cannot be applied because some old agents earn an extra rent over \(rw_{Ap}\) in region \(Ab\). Specifically, if the number of agents who can get credit in region \(Mb\) is lower than that in region \(Ab\), then the distribution of income with \(\lambda > \lambda^{sw}\) would not first-order dominate one with \(\lambda < \lambda^{sw}\). The following proposition shows when this is the case.

**Proposition 5** When a slight increase in \(\lambda\) causes a technological switch, the number of old agents who earn more than \(rw_t\) in the lowest steady state decreases if and only if \(I_A \in (\zeta(I_M), \chi(I_M))\), where \(\chi(I_M)\) is a continuous function defined for \(I_M \geq T_M\), satisfying \(\zeta(I_M) < \chi(I_M) < I_M\) for \(I_M > T_M\), \(T_M = \omega_{Ap}(\hat{\alpha}_M/\hat{\alpha}_A - 1) / ((\omega_{Mp}/\omega_{Ap})^{1/\hat{\alpha}_M} - 1)\), and \(\chi(I_M) = \zeta(I_M)\) for \(I_M \leq T_M\).

**Proof:** in Appendix.

Figure 5 shows representative shapes of functions \(\zeta(I_M)\) and \(\chi(I_M)\), by which the \((I_A, I_M)\) space is divided into three areas. The technological switch, induced by
a stronger enforcement of contracts, is welfare-improving when the minimal size of investment for the labor-intensive technology is either sufficiently small \((I_A \leq \zeta(I_M))\) or sufficiently large \((I_A \geq \chi(I_M))\). In those cases, a technological switch enables more agents to obtain rents from entrepreneurial opportunities, while the remaining agents earn at least as much as the pre-switch levels (recall that \(rw(\lambda)\) is weakly increasing in \(\lambda\)). In addition, Proposition 4 implies that the aggregate amount of rents also increases. Therefore, the technological switch is desirable under standard welfare criteria.

When the size of investment is in an intermediate range, \(I_A \in (\zeta(I_M), \chi(I_M))\), by contrast, the number of rent-earners is discretely reduced by the technological switch even though it increases the amount of rent received by each entrepreneur. The reduced probability of obtaining credits (and therefore rents) lowers the expected utility of agents if they are highly risk-averse.\(^{27}\) This result can be explained as a certain type of *crowding-out* effect. Once enforcement is strengthened to a critical standard, banks can expect some agents (those with relatively large \(\varepsilon_\mu\)) to repay the debt when banks lend them enough funds to adopt the capital-intensive technology. Agents who are in the fortunate position to be able to borrow enough for the new technology employ workers by paying a wage rate marginally higher than any that could be offered by entrepreneurs using the old, labor-intensive technology within their respective borrowing constraints. In addition, they strictly prefer to do so because the new technology yields higher returns to their projects than the old technology. This implies, however, that agents without an entrepreneurial chance to

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\(^{27}\)We measure the expected utility before the random income is realized when all agents share an equal chance of obtaining credit. The technological switch caused by a marginal increase in \(\lambda\) affects the steady-state income distribution in three respects. First, the income of credit-rationed agents (i.e., savers) increases marginally. Second, the number of entrepreneurs falls discretely. Third, the amount of income received by each entrepreneur increases discretely. The first effect is a marginal one and is, therefore, dominated by the second and the third ones, unless risk aversion is infinite. The change of probability (the second effect) linearly affects the expected utility, while the third is subject to decreasing marginal utility. Therefore, when the degree of risk aversion is sufficiently high (but not infinite), the second effect dominates.
start projects with the new technology are no longer able even to start projects with
the old technology since they are, in effect, crowded out from the factor markets.
Thus, even when nobody wants a technological switch \textit{ex ante}, i.e., before their young
period incomes are revealed, they cannot stay with the old technology if $\lambda$ reaches
the threshold.

5 Discussion: Possibility of a Development Trap

Before successful industrialized countries achieved current levels of prosperity, history
witnessed a number of major technological changes. While, for notational simplicity,
the analysis in the previous section focused on a two-technology case, it can easily
be fitted into a multi-technology context. The result is qualitatively the same as
the two-technology case: whenever a marginal increase in $\lambda$ causes a technological
switch, welfare may either increase or decrease discretely depending on the size of
investment required for the pre-switch technology as well as other technological and
institutional properties.\footnote{Proposition 2 gives the choice of technology under given $\lambda$, from which we can identify each
time that a marginal increase in $\lambda$ causes a technological switch. Once we know the pre-switch and
post-switch technologies, Proposition 5 gives the welfare effect of the switch.} The size of the minimum investment required for a given
technology, in turn, depends not only on the technical features of that technology but
also on institutions, such as regulations and the forms of organizations, which vary
considerably across countries. Therefore, even when the set of available technologies
are the same across countries, countries may or may not face deteriorating welfare
at different stages in the development of their financial infrastructure.

Such a difference in the welfare effects gives people different degrees of incentive
(or disincentive) to improve their financial infrastructure. Let us illustrate how
development traps might emerge in some economies but not in others. Recall that
parameter $\lambda$ represents the enforceability of financial contracts, which relies on the
economy’s legal and accounting systems. Since such systems are quite complex in
every economy, their reforms take considerable time and have, in general, a gradual
nature.\textsuperscript{29} In the context of our model, these properties imply that a reform that aims to improve the enforceability of a financial contract (i.e., to increase $\lambda_{t+1}$ relative to $\lambda_t$) must be prepared at least one period in advance before it takes effect and that there is a small upper bound for the size of improvement that is feasible in one period (e.g., $\lambda_{t+1} - \lambda_t$ must be lower than some constant).\textsuperscript{30} This means that, unless the initial wealth $w_0$ is too high, we can approximate the evolution of the economy as being always at the lowest steady state (recall Proposition 3). Suppose that the current state of an economy is such that a small improvement in $\lambda$ is expected to cause a technological switch. Then, the analysis in the previous section implies that, even without the costs associated with a reform,\textsuperscript{31} society as a whole is reluctant to improve its financial infrastructure when there is a considerable (but not too huge) fixed cost in adopting the currently used technology, since, in that case, it expects the reform to result in a welfare-reducing technological switch.\textsuperscript{32} Thus, even though the continual upgrading of such systems is an integral part of achieving economic

\textsuperscript{29}Actually, Li, Squire, and Zou (1998) confirm that the degree of credit market imperfections can differ markedly across countries but change only slowly within countries.

\textsuperscript{30}Without this restriction, any reform would aim to improve $\lambda$ to one, since it is obviously the best: it means that 100 percent of the flow of future profits from any project can be collateralized, under which the most profitable technology can be adopted without borrowing limits. However, even equipped with state-of-the-art tools, modern financial intermediaries cannot monitor lenders perfectly and, therefore, do not usually lend too much. This implies the existence of a technological limit in the range of $\lambda$. In addition, the observed dependence of the current performance of the financial market on an economy’s colonial and legal origins (see La Porta et al., 1998) implies that the size of improvement within a given period is not very large.

\textsuperscript{31}Ando and Yanagawa (2004) consider a model in which a country has to hire workers to raise $\lambda_t$. The inclusion of such a cost to our model would further increase the possibility of a development trap.

\textsuperscript{32}Recall that there is no heterogeneity among young agents until the random income is realized at the end of their first period. Note also that the policy does not affect the utility of old agents because it does not take effect during their lives. By this setting, we consider an ideal situation with no conflict in choosing whether or not to implement the reform. In this respect, our study is complementary to the literature of political economy, in which the importance of conflicts between different parties is intensively studied (see Drazen, 2000).
development, the necessary reforms may not be implemented when the adoption of the currently used technology requires a relatively large amount of fixed investment, possibly due to some barriers for entry. Welfare losses can be prevented, but only at the cost of causing a development trap.

Some historical events are consistent with this observation. Mokyr (1990) argues that, in the 19th Century, the old guilds of Continental Europe, which restricted the entry of new firms into the market, became a fetter on technological progress. He documents workers’ riots against spinning machines imported from Britain and concludes that this resistance was one reason that the Industrial Revolution started in Britain rather than on the Continent. Unions can also be an obstacle to the adoption of new technologies, particularly, when they monopolize the currently used technology. Wolcot (1994) reports that the textile industry in India is an example.

One way implied by our theory to escape from this development trap is to facilitate the adoption of the currently used technology by small businesses so that the minimum size of investment is reduced. Such a policy is beneficial to people even without considering a technological switch, since it weakens credit rationing and thereby distributes the rent obtained by entrepreneurs to workers (in fact, it can be confirmed that the income of credit-rationed agents, \( rw^*(\lambda) \), increases as \( I_A \) falls). Moreover, if the minimum size can be reduced to a level below a threshold (specifically, when \( I_A < \zeta(I_M) \)), credit rationing disappears, and technological switch becomes Pareto improving.\(^{33}\) That is, even in a case in which the long-term object is to promote the adoption of the new technology, the immediate policy should be to remove barriers to the adoption of the currently used technology rather than the new technology. In the above example of Continental Europe, the introduction of new technologies was retarded until the entry costs to guilds fell substantially, when resistance eventually subsided. A more contemporary issue is the cost of entry associated with regulation. Djankov et al. (2002) reports that the official cost of following the

\(^{33}\)Note that the threshold level of \( \lambda \), given by \( \Lambda(w_t; I) \) in (25), gets larger when \( I_A \) is smaller. Thus, when \( I_A \) is small, the credit rationing immediately after switching to technology \( M \) is not as strong as when \( I_A \) is large. This is another reason that the switch becomes welfare improving.
procedures required to start up a simple firm averages 46 percent of annual per capita GDP in the world, with this number being systematically high in low-income countries. Our result implies that simplifications of procedures in low-income countries would be essential to get out of the development trap.

Another effective policy implied by our analysis is to protect the firms with the currently used technology, or even to deter the adoption of new technology, so that the technological switch does not occur until $\lambda_t$ becomes large enough to resolve credit rationing. Specifically, equation (19) shows that credit rationing is resolved if the adoption of technology $M$ is somehow deterred until $\lambda_t$ reaches $1 - \overline{\omega}_{Ab}/I_A$. Once this is accomplished, no party earns rents or loses from the adoption of the new technology. Although these kinds of policies are often criticized as protecting those with vested interests in the status quo and often also regarded as an obstacle in the way of economic development, our analysis has shown that, under certain conditions, they are effective for avoiding welfare losses due to technological switch and, thereby, the development trap. In the process of economic development after World War II, Okazaki (1996) reports that the Japanese government subsidized small-scale firms with primitive technologies while continually strengthening its financial infrastructure. Our model confirms that this combination of policies was effective in fostering economic development while keeping down social conflicts.

6 Concluding Remarks

We have presented a model in which improvements in the enforcement of credit contracts facilitate economic development. With the provision of improved enforcement, the economy switches from a labor-intensive technology to a more productive and capital-intensive technology, following which both the average income and the degree of inequality increase discretely. Our model shows that, in an economy where the credit market is imperfect, the scale of fixed investments has important meanings. If the scale of the fixed investments in the currently used technology is within a

34See, for example, Canton, de Groot, and Nahuis (2002).
certain range and economic agents are sufficiently risk-averse, industrialization worsens the welfare of agents due to the concentration of wealth on a small number of entrepreneurs. This creates the possibility of a development trap, in which society as a whole is reluctant to provide improved legal and accounting systems for fear of causing a welfare-reducing technological switch.

The theory implies that a way to escape from the development trap is to facilitate the adoption of the currently used (not new) technology by small businesses. It even legitimizes the protection of existing firms against new technologies for some time while promoting the development of financial markets. Such policies will gradually redistribute the rent received by existing entrepreneurs to the broader population and thus mitigate the welfare loss and opposition associated with the technological switch. There is a caveat, however. If the indivisibility of current technology is too high, or entry is too restricted, then this kind of protection is not only unnecessary but also causes a development trap.\textsuperscript{35} Therefore, policymakers should cautiously examine whether the demand from some parties to protect existing sectors is simply based on their vested interests or the protection is truly required to avoid economy-wide welfare losses.

\textsuperscript{35}If $I_A$ is originally higher than $\chi(I_M)$, policies that reduce $I_A$ cause an unnecessary trap if $I_A$ is pushed to below $\chi(I_M)$. 

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Appendix

Proof of Proposition 1

Here, we establish the equilibrium market wage at $t+1$, denoted simply by $w$, taking as given the predetermined market wage of the previous period $w_t$. As explained in the text, $\min L_{t+1}^D(w; w_t) > 1$ for all $w \in (w(\lambda)\bar{\theta}_t)$ and $L_{t+1}^D(w; w_t) = \{0\}$ for all $w > \bar{\theta}_t$. Thus, the intermediate value theorem implies the existence of $w \in [\bar{\theta}_t, \bar{\theta}_t]$ such that $L_{t+1}^D(w; w_t) \ni 1$ if the graph of $L_{t+1}^D(w; w_t)$ is joined for all $[\Theta, \overline{\Theta}]$, where $\Theta$ and $\overline{\Theta}$ are arbitrary constants with $\Theta \in (w(\lambda), \theta_\iota)$ and $\overline{\Theta} > \overline{\theta}$ (note that we can always choose such constants since $\overline{\theta}$ is finite and $\theta_\iota > w(\lambda)$ from $w_t > 0$). For this, it is sufficient to show that $L_{t+1}^D(w; w_t)$ is convex-valued, non-empty, and upper hemi continuous (hereafter u.h.c.) for all $w \in [\overline{\Theta}, \overline{\Theta}]$.

From the definition that $L_{t+1}^D(w; w_t)$ is an aggregation of individual labor demand, its non-emptiness and convexity are obvious; there is a continuum of agents, and the set of most preferred actions of each agent is non-empty and does not depend on the action of others given $w$. It is also easy to see that set $L_{t+1}^D(w; w_t)$ is compact. For any $w \in [\Theta, \overline{\Theta}]$, condition (7) implies that the size of investment of each project is finite. Then, there exists a finite upper bound in labor demand $\overline{\lambda}$ in $L_{t+1}^D(w; w_t)$, from (3). Since $L_{t+1}^D(w; w_t)$ is closed by construction, it is compact.

According to the definition of Stokey and Lucas (1989, p. 56), a compact-valued correspondence $L_{t+1}^D : [\Theta, \overline{\Theta}] \to [0, \overline{\lambda}]$ is u.h.c. at $w^*$, if, for every sequence $\{w_n\}$ such that $w_n \in [\Theta, \overline{\Theta}]$ and $w_n \to w^*$, and for every sequence $\{L_n\}$ such that $L_n \in L_{t+1}^D(w_n; w_t)$, there exists a convergent subsequence of $\{L_n\}$ whose limit point $L^*$ is in $L_{t+1}^D(w^*; w_t)$. To show that $L_{t+1}^D(w; w_t)$ is u.h.c., fix $w^* \in [\Theta, \overline{\Theta}]$ and pick any arbitrary sequences $\{w_n\}$ and $\{L_n\}$ such that $w_n \in [\Theta, \overline{\Theta}]$, $w_n \to w^*$, and $L_n \in L_{t+1}^D(w_n; w_t)$.

If $\bigcup_{n=1}^{\infty} w_n$ is finite, there must be some $N > 0$ such that $w_n = w^*$ for all $n \geq N$. Then, since $L_n \in L_{t+1}^D(w^*; w_t)$ for all $n \geq N$ and $L_{t+1}^D(w^*; w_t)$ is compact, there is a convergent subsequence of $\{L_n\}$ whose limit point $L^*$ is in $L_{t+1}^D(w^*; w_t)$. Therefore, the conditions for u.h.c. are satisfied.

The remaining case is that $\bigcup_{n=1}^{\infty} w_n$ is infinite. Note that, since $\tilde{\rho}_j(w)$'s intersect
with each other only a finite number of times, we can choose a subsequence of \( \{w_n\} \) so that each element in set \( \{\tilde{\rho}_j(w_n)\}_{j \in J} \cup r \) has a distinct value, i.e., so that there is always a strict ordering of profitability among technologies as well as between investment and saving. Since the set of technologies \( J \) is finite, the number of patterns in the ordering of profitability that possibly appear in that subsequence is also finite. Therefore, there is at least one pattern of the strict ordering of profitability that appears infinite times in sequence \( \{w_n\} \), from which we can construct a subsequence \( w_{nk} \to w^* \) such that

\[
\tilde{\rho}_1(w_{nk}) < \cdots < \tilde{\rho}_{j-1}(w_{nk}) < r < \tilde{\rho}_j(w_{nk}) < \cdots < \tilde{\rho}_J(w_{nk}) \text{ for all } k,
\]

where each of the available technologies is numbered in ascending order of profitability. In this particular ordering, \( J \) is the number of technologies, and \( \hat{j} \) is the index of the least profitable technology above saving.

Let us derive \( \mathcal{L}_{t+1}^D(w_{nk}; w_t) \) using (31). Note that, from (5) and (6), it is optimal for each agent to invest in the most profitable technology (that with the largest index) under his specific borrowing constraint \( w_t + \epsilon_{it} \geq \eta_j(w_{nk}; \lambda, I_j) \) as long as there is such a technology above \( \hat{j} \). More specifically, he adopts technology \( j \) if and only if the amount of his own funds is within the range of \( w_t + \epsilon_{it} \in W_j(w_{nk}) \), where \( W_j(w_{nk}) \) is defined recursively for \( j = J, J - 1, \ldots, \hat{j} \) by

\[
W_j(w_{nk}) = \begin{cases} 
[\eta_j(w_{nk}; \lambda, I_j), \infty) & \text{for } j = J; \\
[\eta_j(w_{nk}; \lambda, I_j), \infty) \setminus W_{j+1}(w_{nk}) & \text{for } j = J - 1, \ldots, \hat{j}.
\end{cases}
\]

From (9) and (14), an entrepreneur with own funds \( w_t + \epsilon_{it} \) demands \( \tilde{\ell}(w_{nk})(w_t + \epsilon_{it})/(1 - \lambda \tilde{\rho}_j(w_{nk})/r) \) units of labor whenever \( w_t + \epsilon_{it} \in W_j(w_{nk}) \) for some \( j \geq \hat{j} \). Since \( \epsilon_{it} \) is distributed uniformly between 0 and \( \bar{\epsilon} \), the aggregate labor demand is given by

\[
\sum_{j=\hat{j}}^J \int_{w_t + \epsilon \in E_j(w_{nk}; w_t)} \frac{(w_t + \epsilon)\tilde{\ell}(w_{nk})}{1 - \lambda \tilde{\rho}_j(w_{nk})/r} \frac{d\epsilon}{\epsilon} = \mathcal{L}_{t+1}(w_{nk}; w_t),
\]

where set \( E_j(w_{nk}; w_t) = \{ \epsilon \in [0, \bar{\epsilon}] \mid w_t + \epsilon \in W_j(w_{nk}) \} \) represents the range of random incomes with which technology \( j \) is chosen. Note that \( \mathcal{L}_{t+1}(w_{nk}; w_t) \) is a function and, therefore, set \( \mathcal{L}_{t+1}^D(w_{nk}; w_t) \) has only one element. The only choice of sequence
\( \{ L_{nk} \} \) is such that \( L_{nk} = \tilde{L}_{t+1}(w_{nk}; w_t) \) for all \( k \). When viewed as a correspondence, \( E_j(w; w_t) \) is well defined and continuous for all \( w \in [\Theta, \overline{\Theta}] \). From (32), function \( \tilde{L}_{t+1}(w; w_t) \) is also well defined and continuous for all \( w \in [\Theta, \overline{\Theta}] \). Thus, given that \( w_{nk} \in [\Theta, \overline{\Theta}] \) converges to \( w^* \), \( L_{nk} = \tilde{L}_{t+1}(w_{nk}; w_t) \) converges to \( L^* = \tilde{L}_{t+1}(w^*; w_t) \).

The final task is to show that \( L^* \in L_{t+1}^D(w^*; w_t) \). Consider the relative profitability of each technology when the market wage is given by \( w^* \). Since \( \tilde{\rho}_j(w) \) is continuous, taking limit \( w_{nk} \to w^* \) in (31) implies

\[
\tilde{\rho}_1(w^*) \leq \cdots \leq \tilde{\rho}_{j-1}(w^*) \leq r \leq \tilde{\rho}_j(w^*) \leq \cdots \leq \tilde{\rho}_J(w^*). \tag{33}
\]

In the limit, we only have a weak ordering of profitability. Nonetheless, agents with own funds \( \epsilon_{it} \in W_j(w^*) \) for some \( j \geq \hat{j} \) find it at least weakly optimal to choose technology \( j \), while other agents find it at least weakly optimal to save. Thus, \( L_{t+1}^D(w^*; w_t) \supseteq \tilde{L}_{t+1}(w^*; w_t) = L^* \). It establishes that \( L_{t+1}^D(w; w_t) \) is u.h.c. at \( w = w^* \). Since \( w^* \in [\Theta, \overline{\Theta}] \) is arbitrary, the correspondence \( L_{t+1}^D(w; w_t) \) is u.h.c. for all \( w \in [\Theta, \overline{\Theta}] \). This completes the proof. \( \blacksquare \)

**Proof of Proposition 2**

Proportions (i) and (ii) are directly obtained by taking limit \( \tau \to 0 \) in Proposition 1. The following proves property (iii). Note that, from the continuity of function \( \omega_{jb}(\cdot) \) for all \( j \), we can choose sufficiently small \( \tau > 0 \) such that \( \theta(w_t + \epsilon_{it}; \lambda, I_{j^*}) = \omega_{j^b}(w_t + \epsilon_{it}; \lambda, I_{j^*}) \) for all \( \epsilon_{it} \in [0, \tau] \). Then, Proposition 1 says that \( w_{t+1} \in [\omega_{j^b}(w_t; \lambda, I_{j^*}), \omega_{j^b}(w_t + \tau; \lambda, I_{j^*})] \) or, equivalently, \( \eta_{j^*}(w_{t+1}; \lambda, I_{j^*}) \in [w_t, w_t + \tau] \). Now, consider the limiting case in which the degree of heterogeneity \( \tau \) is infinitesimally small (\( \tau \to 0 \)). In the limit, the previous relationships indicate \( w_{t+1} \to \omega_{j^*b}(w_t; \lambda, I_{j^*}) \) and \( \eta_{j^*}(w_{t+1}; \lambda, I_{j^*}) \to w_t \). Applying these for (14) yields \( l_{t+1} \to I_{j^*} \tilde{L}_{j^*}(\omega_{j^*b}(w_t; \lambda, I_{j^*})) \), which gives the limiting value of labor demand by each entrepreneur. Since the total labor demand should be 1 in equilibrium, it suggests that the number of entrepreneurs in the limit is \( \left( I_{j^*} \tilde{L}_{j^*}(\omega_{j^*b}(w_t; \lambda, I_{j^*})) \right)^{-1} \), which is smaller than the number of agents, 1, under \( I_j > \tilde{L}_j(\overline{\sigma}_{j^b})^{-1} \), as assumed in Proposition 1. Similarly, taking the limit in (15) and eliminating \( \omega_{j^*b}(\cdot) \) by (9) show that
the limiting value of consumption of entrepreneurs is \( rw_t + r((1 - \lambda)I_J - w_t)/\lambda \), where the second term is positive since, in any borrowing-constrained equilibrium, the amount of own funds \( w_t \) falls short of the portion of investment that cannot be collateralized, \((1 - \lambda)I_J.\)

\[ \frac{\text{Proof of Proposition 5}}{\text{It is sufficient to consider only the case of } I_A > \zeta(I_M) \text{ since, as explained in the text, we know that the number of rent earners before the technological switch is already zero whenever } I_A \leq \zeta(I_M). \text{ Let } w^{sw} \equiv w^s(\lambda^{sw}) \text{ denote the steady-state wage at the threshold. Using (28) and the continuity of } w^s(\lambda) \text{ at the threshold, the number of entrepreneurs at levels of enforcement slightly below and above the threshold is given by } N(A, w^{sw}, \lambda^{sw}) \text{ and } N(M, w^{sw}, \lambda^{sw}), \text{ respectively. Let } Q \text{ denote the ratio } N(M, w^{sw}, \lambda^{sw})/N(A, w^{sw}, \lambda^{sw}). \text{ Our concern is whether } Q \geq 1 \text{ or } Q < 1. \text{ Note that, from (26), the continuity of the steady state at the threshold means that } \lambda^{*A}(w^{sw}; I_A) = \lambda^{sw} = \lambda^{*M}(w^{sw}; I_M). \text{ Using this relationship and (28), the expression for } Q \text{ can be simplified as}
\]

\[
Q = \frac{N^{ss}(M, w^{sw})}{N^{ss}(A, w^{sw})} = \frac{\hat{\alpha}_M I_A - w^{sw}}{\hat{\alpha}_A I_M - w^{sw}}. \tag{34}
\]

From assumptions \( I_j > \varpi_{jp} \) and \( \varpi_{M_p} > \varpi_{Ap} \), it follows that both \( I_A - w^{sw} \) and \( I_M - w^{sw} \) are positive, guaranteeing \( Q > 0 \). Moreover, it is implied that \( Q > 1 \) whenever \( I_A \geq I_M \) (recall that \( \hat{\alpha}_A < \hat{\alpha}_M \)).

Let us examine how \( Q \) responds to changes in \( I_A \) when \( I_A < I_M \). Differentiating (34) with respect to \( I_A \) gives

\[
\frac{dQ}{dI_A} = \frac{\hat{\alpha}_M}{\hat{\alpha}_A(I_M - w^{sw})^2} \left[(I_M - w^{sw}) - (I_M - I_A) \frac{dw^{sw}}{dI_A}\right], \tag{35}
\]

the sign of which depends on that of \( dw^{sw}/dI_A \). Note that function \( \lambda^{*A}(w; I_A) \) and function \( \Lambda(w; I_A, I_M) \) intersect at the point \((\lambda^{sw}, w^{sw})\), as shown by panel b of Figure 3. Equations (26) and (25) show that, with an increase in \( I_A \), \( \lambda^{*A}(w; I_A) \) shifts to the right, whereas \( \Lambda(w; I_A, I_M) \) shifts to the left, pushing the intersecting point downward. This means that \( dw^{sw}/dI_A < 0 \), and, therefore, \( dQ/dI_A > 0 \) from (35).
We confirmed that $Q > 1$ when $I_A = I_M$ and that it gradually decreases as $I_A$ falls for all $I_A > \zeta(I_M)$. If $\lim_{I_A \to \zeta(I_M)} Q < 1$, the intermediate value theorem shows that there exists a value of $I_A$ below which $Q < 1$ holds. We now calculate the limiting value. From the definition $\zeta(I_M)$ and the continuity of $w^{sw}$ with respect to $I_A$, observe that $w^{sw} \to \overline{A}p$ when $I_A \to \zeta(I_M)$ (i.e., the point of technological switch approaches region $Ap$, where $w = \overline{A}p$). Substituting it into $Q = N^{ss}(M, w^{sw})/N^{ss}(A, w^{sw})$ and using the definition of $\zeta(I_M)$ in (27) show

$$\lim_{I_A \to \zeta(I_M)} Q = \frac{\alpha_M}{\alpha_A} \left( \frac{\overline{A}p}{\overline{A}p_M} \right)^{1/\alpha_M} \frac{\zeta(I_M)}{I_M} \leq 1 \Leftrightarrow I_M \geq \overline{T}.$$ 

If $I_M > \overline{T}_M$, there exists $\chi(I_M) \in (\zeta(I_M), I_M)$ such that $Q < 1$ for $I_A \in (\zeta(I_M), \chi(I_M))$. When $I_M \leq \overline{T}_M$, we can $\chi(I_M)$ should be equal to $\zeta(I_M)$ so that $(\zeta(I_M), \chi(I_M))$ is an empty set. Finally, since $Q$ is continuous with respect to $I_M$ from (34), the implicit function theorem guarantees that the value of $I_A$ at which $Q = 1$ changes continuously with respect to $I_M > \overline{T}_M$ and approaches $\zeta(I_M)$ as $I_M \to \overline{T}_M$. This implies the continuity of function $\chi(I_M)$.

\[ \blacksquare \]
References


