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Abstract

We examine how child-allowance policies with pay-as-you-go systems affect fertility and growth rates. A current method to subsidize child-rearing households, which determines benefits according to family's number of children, increases the fertility rate but decreases the growth rate. This study also demonstrates that when a government initiates a child-allowance policy using some part of the pension budget, the fertility rate declines in aging economies.

Keywords: Fertility, Social security, Uncertain lifetime.

JEL: D91, H55, J13.

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1 Introduction

Declining fertility rates threaten the feasibility of current social security systems, thereby posing a serious problem to many advanced countries. On the other hand, as the active life after retirement becomes longer through a higher life-expectancy, the importance of public pensions which stabilize a retired lifestyle is increasing. Pay-as-you-go financed (PAYG-) public pensions, which are the dominant scheme in countries with aging populations, are supported by working-age people. For that reason, reproduction of the immediately subsequent generation poses a social problem as long as people anticipate receiving PAYG-pension benefits after their own retirement. However, for raising children, who will be contributors to future PAYG-pension systems, people virtually depend on the expenses of individuals with their own children.[1]

This paper describes how child-allowance policies with PAYG systems affect fertility and growth rates by incorporating an uncertain lifetime. This model has an endogenous growth mechanism where the engine of growth is human capital accumulation by parental-teaching education. Children are treated as consumer durables, as in Becker and Barro (1988) and Barro and Becker (1989).[2] Children's human capital is also assumed to be a source

[1]Folbre (1994) describes this as " Individuals who devote relatively little time or energy to child-rearing are free-riding on parental labor".

[2]When we consider motives for having children, we abstract from the old-age security hypothesis, because most low-fertility countries have already constructed social security systems and the degree of such motives in developed countries is much lower relative to

of utility to the parent, as in Eckstein and Zilcha (1994). Groezen et al. (2003) show that a child-support policy stimulates fertility rates in a small open economy but disregard human capital accumulation. We show that a child-allowance policy increases the fertility rate, but decreases the growth rate by retarding human capital accumulation.[3]

In addition, the social security budget for elderly people is commonly much larger than that for young families with children in low-fertility countries, such as Japan, Italy, and Spain.[4] Therefore, we also analyze an effect when the government initiates a child-allowance policy by employing some portion of revenue which has been used for pensions. That is, the government diminishes the size of pension systems and redistributes it among the working generation, who are rearing future contributors. This policy change leads to a lower fertility rate in high life-expectancy economies with a larger opportunity cost of having children. This is true because a smaller pension size in aging economies compels individuals to increase the labor supply to provide for their post-retirement consumption. Therefore, even if individuals receive child allowances, they decrease the number of children they choose to have.

The remainder of this paper is organized as follows. Section 2 presents

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that in developing economies.

[3]Zhang (1997) examines how a child-allowance policy affects fertility and growth rates in a simple model without savings, pensions, and an uncertain lifetime.

[4]The Japanese social security budget in 2001 for " children and families" is 3.7% in contrast to 68.7% for " old-age people".

the model. Section 3 explains effects of a child-allowance policy with PAYG-systems. Section 4 shows effects of the child-allowance policy by diminishing the size of pension systems. Concluding remarks appear in Section 5.

2 The economy

We use an overlapping-generations model of endogenous growth, incorporating an uncertain lifetime. The life of a representative individual is divided into three periods: a childhood and a young-working period, both with fixed durations, and a retirement period of uncertain length. For simplicity, we assume that the individual is either alive or dead at the beginning of the third period. The probability that the individual is alive at the beginning of the retirement period is denoted as $p \in (0, 1]$, which is known and common to individuals in the same generation.

Assuming that insurance companies are risk-neutral and that private annuities markets are competitive, the insurance companies promise individuals a payment of $\frac{(1+r_{t+1})}{p} a_t$ in exchange for having an estate a_t accruing to the companies. In the absence of bequest motives, individuals are willing to invest their assets in such insurance.[5]

The government levies a tax $\tau \in [0, 1)$ on wages of young individuals and redistributes μ part of the revenue to those young child-rearing individuals as

[5] This is a simplified version of Blanchard's (1985) model.

child allowances, and $(1-\mu)$ part of it to retired individuals as PAYG-public pensions. The value of the allowance per child at time t is described as Q_t and the payment per retired person at time t is described as A_t.

2.1 Households

In childhood, individuals only accumulate human capital by receiving parental-teaching. Individuals are endowed with one divisible unit of time in their young periods, reproduce asexually, and allocate their time toward labor and raising children. To raise a child, they are compelled to spend their time in child-rearing and parental-teaching, which helps their children to accumulate human capital.[6] They receive wage income, which is taxed away, and allowances according to the number of children in the end of their young periods. They consume part of their income and invest the rest of their income in annuities. Subsequently, living individuals obtain the principal and interest from their annuities and consume them with their pension benefits after retirement.

Each individual who is born at time t, and called generation $t+1$, accumulates human capital through education by parents at time t. Given parents' human capital, h_t, when a parent allocates time e_t toward parental-teaching

[6] In this model, as in Eckstein and Zilcha (1994), the decision of education time is based on parental altruism. Chamley (1993) and Göcke (2004), among others, present a model in which adult agents divide their own time between schooling and working.

for one child, the child accumulates human capital h_{t+1} according to

$$h_{t+1} = \theta e_t^{\gamma} h_t,$$

where $\theta \geq 1$ and $\gamma \in (0, 1)$.[7]

The endowment time in the young period is normalized to unity. The time constraint of generation t is then given by

$$1 = l_t + n_{t+1}(q + e_t),$$

where l_t, n_{t+1}, and q respectively denote the labor time, the number of children, and the rearing time per child.[8]

The budget constraints of a member of generation t when young and retired are given, respectively, by $(1-\tau)w_t h_t l_t + Q_t n_{t+1} = c_t^y + a_t$ and $\frac{(1+r_{t+1})}{p} a_t + A_{t+1} = c_{t+1}^o$, where c_t^y is the consumption when young, a_t is the annuity, A_{t+1} is the pension benefit, and c_{t+1}^o represents post-retirement consumption.

All individuals have identical preferences. The utility function of genera-

[7] Glomm and Ravikumar (1992) employ a similar structure.

[8] Another type of endogenous labor-supply model where agents choose their labor length considering social security benefits is analyzed in Hu (1979), Momota (2003), and Cremer et al. (2004).

tion $t(\geq 0)$ is represented by[9]

$$u_t = (1-\sigma)\varphi \ln c_t^y + p(1-\sigma)(1-\varphi)\ln c_{t+1}^o + \sigma \ln n_{t+1}h_{t+1}. \tag{1}$$

The parameter $\sigma \in (0,1)$ can be interpreted as the degree of preference over " total human capital" of their children. The parameter $\varphi \in (0,1)$ describes the degree of time preference for consumption while young. By solving individuals' optimization problems, the optimal values are given by:

$$n_{t+1} = \frac{\kappa\sigma(1-\gamma)}{\{q(1-\tau)w_t h_t - Q_t\}} I_t,$$

$$l_t = 1 - \kappa\sigma \frac{\{q(1-\tau)w_t h_t - \gamma Q_t\}}{\{q(1-\tau)w_t h_t - Q_t\}} \frac{1}{(1-\tau)w_t h_t} I_t,$$

$$e_t = \frac{\gamma q}{1-\gamma} - \frac{\gamma Q_t}{(1-\gamma)(1-\tau)w_t h_t},$$

$$c_t^y = \kappa(1-\sigma)\varphi I_t,$$

$$c_{t+1}^o = \kappa p(1-\sigma)(1-\varphi)\frac{(1+r_{t+1})}{p} I_t,$$

$$a_t = \kappa p(1-\sigma)(1-\varphi)I_t - \frac{pA_{t+1}}{(1+r_{t+1})},$$

where $\kappa \equiv \frac{1}{\{1-(1-p)(1-\sigma)(1-\varphi)\}}$ and $I_t \equiv (1-\tau)w_t h_t + \frac{pA_{t+1}}{(1+r_{t+1})}$.

[9] With an uncertain lifetime, $p \in (0,1]$, this utility form is employed by Pecchenino and Pollard (1997), Pecchenino and Utendorf (1999), and Yakita (2001), among others.

2.2 Production

Competitive firms produce a single final good by employing both physical capital and effective labor input. The aggregate production function at time t is given by $Y_t = F(K_t, h_t l_t N_t) = K_t^{\alpha}(h_t l_t N_t)^{1-\alpha}$, where Y_t, K_t, N_t, and $\alpha \in (0,1)$ respectively denote the aggregate output, the physical capital which fully depreciates in the production process, the working-age population, and the share of physical capital. The production function, in intensive form, can be expressed as

$$\tilde{y}_t = \tilde{k}_t^{\alpha},$$

where $\tilde{y}_t = \frac{Y_t}{h_t l_t N_t}$ is the output per effective-labor and $\tilde{k}_t = \frac{K_t}{h_t l_t N_t}$ is the physical capital per effective-labor.

The factor markets are presumed to be perfectly competitive. Therefore, the firms take factor prices as given. Firms hire labor inputs and physical capital up to the point where their marginal products equal their factor prices:

$$w_t = (1-\alpha)\tilde{k}_t^{\alpha},$$

$$(1+r_t) = \alpha \tilde{k}_t^{\alpha-1}.$$

2.3 Equilibrium

The government faces the budget constraints of the policies:

child allowances; $\mu\tau w_t h_t l_t N_t = Q_t n_{t+1} N_t$,

public pensions; $(1-\mu)\tau w_t h_t l_t N_t = A_t p N_{t-1}$.

Using capital market-clearing conditions, $K_{t+1} = a_t N_t$, we can get the values at equilibrium as follows.[10]

$$n_{t+1} = \frac{[\mu\tau\Psi + \kappa\sigma\{1+(1-\mu)\tau\frac{(1-\alpha)}{\alpha}\}\{(1-\gamma)(1-\tau)-\mu\gamma\tau\}]}{q\{1-(1-\mu)\tau\}\Psi} \equiv n_g,$$

$$l_t = \frac{(1-\tau)}{\{1-(1-\mu)\tau\}}[1 - \frac{\kappa\sigma\{1+(1-\mu)\tau\frac{(1-\alpha)}{\alpha}\}}{\Psi}] \equiv l_g,$$

$$e_t = \frac{\gamma q\kappa\sigma\{1-(1-\mu)\tau\}\{1+(1-\mu)\tau\frac{(1-\alpha)}{\alpha}\}}{[\mu\tau\Psi + \kappa\sigma\{1+(1-\mu)\tau\frac{(1-\alpha)}{\alpha}\}\{(1-\gamma)(1-\tau)-\mu\gamma\tau\}]} \equiv e_g,$$

where $\Psi \equiv [1+\kappa\{1-(1-\sigma)(1-\varphi)\}(1-\mu)\tau\frac{(1-\alpha)}{\alpha}]$.

The growth rate of per-capita output at time t is

$$(1+g_t) \equiv \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}} = (\frac{\tilde{k}_{t+1}}{\tilde{k}_t})^\alpha \frac{h_{t+1}}{h_t}.$$

The capital per effective labor becomes $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$ in the steady state.[11]

[10] Note that an increase in life expectancy lowers the fertility rate.

[11] The equilibrium sequence of capital per effective labor is

$$\tilde{k}_{t+1} = \frac{\kappa p(1-\sigma)(1-\varphi)(1-\alpha)}{E_g l_g n_g}\tilde{k}_t^\alpha,$$

where $E_g \equiv \theta e_g^\gamma$. This dynamics is always stable due to assumption of $\alpha \in (0,1)$. In the steady state, the physical capital per effective labor becomes

$$\tilde{k}^* = (\frac{\kappa p(1-\sigma)(1-\varphi)(1-\alpha)}{E_g l_g n_g})^{\frac{1}{1-\alpha}}.$$

Consequently, the per-capita growth rate in the balanced-growth path depends only on the parental-teaching time. It is given by

$$(1 + g^*) = \theta e_g^{\gamma}.$$

3 Policy effects

The following proposition summarizes effects of an introduction of the child-allowance policy and the PAYG-pension system.

Proposition 1. *Introduction of a child-allowance policy and a PAYG-pension system increases the number of children and decreases the labor time. When the government introduces the child-allowance policy, the education time per child and the per-capita growth rate decrease. When the government introduces the PAYG-pension system only, the introduction has no effects on the education time per child and the per-capita growth rate.*

Proof. Initially, the effect on the number of children is positive, as

$$\frac{\partial n_g}{\partial \tau}|_{\tau=0} = \frac{1}{q}[\mu(1-\kappa\sigma) + (1-\mu)\kappa^2 p\sigma(1-\sigma)(1-\varphi)(1-\gamma)\frac{(1-\alpha)}{\alpha}] > 0.$$

This sign is satisfied with any $\mu \in [0, 1]$. Consequently, this form of governmental intervention always increases the fertility rate.

Secondly, the effect on the labor time is negative, as

$$\frac{\partial l_g}{\partial \tau}|_{\tau=0} = -[\mu(1-\kappa\sigma) + (1-\mu)\kappa^2 p\sigma(1-\sigma)(1-\varphi)\frac{(1-\alpha)}{\alpha}] < 0.$$

This sign is also satisfied with any $\mu \in [0,1]$.

Lastly, the effect on the education time per child is given by

$$\frac{\partial e_g}{\partial \tau}|_{\tau=0} = -\mu\frac{(1-\kappa\sigma)\gamma q}{\kappa\sigma(1-\gamma)^2} \leq 0.$$

This sign is negative when $\mu \in (0,1]$, and zero when $\mu = 0$. That is, the intervention with child allowances ($\mu > 0$) decreases the education time per child. However, the introduction of a public pension without child allowances ($\mu = 0$) has no effects on the education time per child.

Therefore, the effect on the per-capita growth rate in the balanced-growth path depends on the per-child education time; it is given by

$$\frac{\partial(1+g_g^*)}{\partial \tau}|_{\tau=0} = \frac{\partial \theta e_g^\gamma}{\partial \tau}|_{\tau=0} \leq 0.$$

$\square$

These results are explained as follows. The prices, in labor terms, of raising and educating a child are, respectively, $q - \frac{Q_t}{(1-\tau)w_t h_t}$ and n_{t+1}. The number of children increases because of income effects and price effects by the intervention. The per-child education time decreases because negative price effects dominate positive income effects. When only the pension system exists, the negative price effects are proportional to positive income effects. Therefore, there is no change in education time.

4 Fertility and life expectancy

This section presents analysis of fertility effects when the government introduces a child-allowance policy using μ part of the revenue which has been spent on the PAYG-pension systems.[12] That is, the government partly diminishes the transfer from young to old and redistributes resources among young individuals to increase contributors in the future.[13]

The effect of such a policy change on the fertility rate can be recognized by the sign of the following formula.

$$\frac{\partial n_g}{\partial \mu}|_{\mu=0} = s^2p^2 - [\sigma(1-\gamma)(1-\tau)\frac{(1-\alpha)}{\alpha} - \{\sigma + 2(1-\sigma)\varphi\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}]sp$$
$$+(1-\sigma)\varphi(1-s)\{1+\tau\frac{(1-\alpha)}{\alpha}\}^2 \equiv f(p\,;\tau),$$

where $s \equiv (1-\sigma)(1-\varphi) > 0$.

This is a quadratic function of life expectancy, p. Herein, we shall see a

[12]We can easily see that the per-child education time always decreases, as in section 3. When the government introduces the child-allowance policy by levying a new labor tax for the sole purpose of supporting the policy, the fertility rate always increases.

[13]Because individuals in each generation are assumed to be homogenous, this policy change exactly eases the tax burden of the young generation.

case in which the policy change engenders a lower fertility rate.[14] The sign of $f(p\,;\tau)$ is either positive or negative depending on p, if both the labor-capital distribution ratio and the labor-tax rate are

$$\frac{(1-\alpha)}{\alpha} > \alpha_1 \qquad\qquad and \qquad\qquad 0<\tau<\tau_1,$$

where $\alpha_1 \equiv \frac{1}{\sigma(1-\varphi)(1-\gamma)}$ and $\tau_1 \equiv \frac{-[\sigma(1-\gamma)s+(1-\sigma)\{\sigma(1-\varphi)+2\varphi\}]+\sqrt{\Upsilon}}{2(1-\sigma)\varphi(1-s)\frac{(1-\alpha)}{\alpha}}$.[15] The critical value of the old-age degree in the economy, p_1, which is an intersection of graph $f(p\,;\tau)$ with the x-axis, is the smaller solution of $f(p\,;\tau)=0$:[16]

$$p_1 \equiv \frac{[\sigma(1-\gamma)(1-\tau)\frac{(1-\alpha)}{\alpha} - \{\sigma+2(1-\sigma)\varphi\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}] - \sqrt{\Phi}}{2s}.$$

Therefore, we obtain the following proposition.

Proposition 2.

When the labor-capital distribution ratio is $\frac{(1-\alpha)}{\alpha} > \alpha_1$ *and the labor-tax rate is* $0<\tau<\tau_1$*, an introduction of the child-allowance policy using some part of pension revenue*

(3a) increases the fertility rate if $p \in (0, p_1)$*,*

(3b) decreases the fertility rate if $p \in (p_1, 1]$*,*

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[14] See Appendix for another case.

[15] $\Upsilon \equiv \sigma^2(1-\varphi)^2 + 4\sigma(1-\gamma)\varphi(1-\varphi)(1-s)\frac{(1-\alpha)}{\alpha}$
$+2(1-\sigma)\sigma(1-\gamma)s\{\sigma(1-\varphi)+2\varphi\} + \sigma^2(1-\gamma)^2 s^2 > 0$

[16] $\Phi \equiv \{\sigma^2 + 4(1-\sigma)\varphi(1-s)^2(1+s)\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}^2 + \sigma^2(1-\gamma)^2(1-\tau)^2(\frac{1-\alpha}{\alpha})^2$
$-2\sigma\{\sigma+2(1-\sigma)\varphi\}(1-\gamma)(1-\tau)\{1+\tau\frac{(1-\alpha)}{\alpha}\}\frac{(1-\alpha)}{\alpha} > 0$

(3c) has no effects on the fertility rate if $p = p_1$.

In the event of a larger labor share, which implies higher labor income, or a lower labor-tax rate, the opportunity cost of having children becomes large. In aging economies, where working-age individuals face a high probability that they will be alive in their retirement periods, pension and annuity amount per capita is relatively small by the larger old-population. Therefore, if the pension benefit is decreased by the policy change, individuals have an incentive to increase the labor supply for their retired-age consumption. For that reason, even if individuals receive child allowances for having children, they decrease the number of children those they have.

5 Concluding remarks

We have presented the effects of child-allowance policies. When child allowances are given to parents depending on the number of children, parents have an incentive to increase the " quantity" of children without maintaining the " quality" of each child. Some empirical studies discuss whether child-allowance policies actually increase the fertility rate or not. It might be important that individuals are guaranteed sufficient pension benefits to stimulate the fertility rate in aging economies.

Appendix.

The following proposition summarizes positive effects of an introduction of the child-allowance policy on the fertility rate.

Proposition.

An introduction of the child-allowance policy using some portion of pension revenue increases the fertility rate, when

(2a) the labor-capital distribution ratio is $\frac{(1-\alpha)}{\alpha} \leq \alpha_1$*,*

(2b) the labor-capital distribution ratio is $\frac{(1-\alpha)}{\alpha} > \alpha_1$ *and the labor-tax rate is* $\tau_1 < \tau < 1$*.*

The introduction has no effects on the fertility rate when the labor-capital distribution ratio is $\frac{(1-\alpha)}{\alpha} > \alpha_1$ *and the labor-tax rate is* $\tau = \tau_1$*.*

Proof. The following $f(p\,;\tau)$, which is a quadratic function of p, expresses the effect on the fertility rate.[17]

$$f(p\,;\tau) = \frac{\partial n_{t+1}}{\partial \mu}|_{\mu=0} = s^2[p-\Gamma]^2 + \Lambda.$$

The graph of $f(p\,;\tau)$ is convex downward and the value of $f(0\,;\tau)$ is positive.[18] The condition by which " the value of $f(1\,;\tau)$ is negative" allows the

[17] $\Gamma \equiv \frac{[\sigma(1-\gamma)(1-\tau)\frac{(1-\alpha)}{\alpha} - \{\sigma+2(1-\sigma)\varphi\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}]}{2s}$,
$\Lambda \equiv -\frac{[\sigma(1-\gamma)(1-\tau)\frac{(1-\alpha)}{\alpha} - \{\sigma+2(1-\sigma)\varphi\}\{1+\tau\frac{(1-\alpha)}{\alpha}\}]^2}{4} + (1-\sigma)\varphi(1-s)\{1+\tau\frac{(1-\alpha)}{\alpha}\}^2.$

[18] $f(0\,;\tau) = (1-\sigma)\varphi(1-s)\{1+\tau\frac{(1-\alpha)}{\alpha}\}^2 > 0.$

graph of $f(p\,;\tau)$ to intersect the x-axis once in $p\in(0,1]$.[19] .

The value of $f(1\,;\tau)$ is expressed by a function of τ:

$$
\begin{aligned}
f(1\,,\tau) = &(1-s)(1-\sigma)\varphi\{\frac{(1-\alpha)}{\alpha}\}^2\tau^2 \\
&+[(1-\sigma)\{\sigma(1-\varphi)+2\varphi\}+s\sigma(1-\gamma)]\frac{(1-\alpha)}{\alpha}\tau \\
&+[(1-\sigma)-s\sigma(1-\gamma)\frac{(1-\alpha)}{\alpha}].
\end{aligned}
$$

The sign of $f(1\,,1)$ is positive.[20] Because the first and second terms of $f(1\,,\tau)$ are positive, the sign of this function depends on the third term. When the labor-capital distribution ratio is

$$\frac{(1-\alpha)}{\alpha}\leq\frac{1}{\sigma(1-\varphi)(1-\gamma)}\equiv\alpha_1,$$

the third term is non-negative. Therefore, the sign of $f(1\,,0)$ is positive; thereby, that of $f(1\,;\tau)$ is positive for any $\tau(>0)$. Consequently, the value of $f(p\,;\tau)$ is always positive in $p\in(0,1]$.

When the third term is negative, as $\frac{(1-\alpha)}{\alpha}>\alpha_1$, the graph of $f(1\,,\tau)$ has an intercept in $\tau\in(0,1)$. The intersection is a larger solution of $f(1\,,\tau)=0$:[21]

$$\tau_1\equiv\frac{-[\sigma(1-\gamma)s+(1-\sigma)\{\sigma(1-\varphi)+2\varphi\}]+\sqrt{\Upsilon}}{2(1-\sigma)\varphi(1-s)\frac{(1-\alpha)}{\alpha}}.$$

[19] To evade intricacy, we eliminate the case in which the graph intersects the x-axis twice.

[20] $f(1\,,1)=(1-\sigma)[1+\{\sigma(1-\varphi)+2\varphi\}\frac{(1-\alpha)}{\alpha}+(1-s)\varphi(\frac{1-\alpha}{\alpha})^2]>0.$

[21] $\Upsilon\equiv\sigma^2(1-\varphi)^2+4\sigma(1-\gamma)\varphi(1-\varphi)(1-s)\frac{(1-\alpha)}{\alpha}$
$+2(1-\sigma)\sigma(1-\gamma)s\{\sigma(1-\varphi)+2\varphi\}+\sigma^2(1-\gamma)^2s^2>0$

The value of $f(1\,;\tau)$ is negative when $\tau\in(0,\tau_1)$ and positive when $\tau\in(\tau_1,1)$. Therefore, when $\frac{(1-\alpha)}{\alpha} > \alpha_1$ and $\tau_1 < \tau < 1$, the value of $f(p\,;\tau)$ is always positive in $p\in(0,1]$. When $\frac{(1-\alpha)}{\alpha} > \alpha_1$ and $0 < \tau < \tau_1$, the value of $f(p\,;\tau)$ is positive or negative depending on p. $\square$

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