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Abstract

In this paper, a Grossman-Helpman-Romer-type endogenous growth model is developed with two regions in which there are mobile workers and linkage between consumption goods and differentiated intermediate goods. The economy has the potential to reach the following spatial configuration: full agglomeration, partial agglomeration, and segmented agglomeration. In perfect agglomeration, the innovation sector and intermediate goods sector agglomerate in one region. In partial agglomeration, intermediate goods firms partially agglomerate in the region where the innovation sector agglomerates perfectly. In segmented agglomeration, the innovation sector agglomerates in the region where both intermediate goods sector and final good sector do not agglomerate perfectly.

In addition, we show the comparison of the welfare of skilled workers in each steady state. Not surprisingly, the welfare of the skilled in full agglomeration is always the highest. However, even though there are transportation costs of final good, the welfare in segmented agglomeration is not necessarily the lowest.

Key words: knowledge spillovers, transportation costs, inter-regional trade,

JEL classification: F43: O18: R11

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1 Introduction

In this paper, a Grossman-Helpman-Romer-type endogenous growth model is developed with two regions, in which there are two types of workers (i.e., skilled and unskilled) and a linkage between the production of consumption good and that of differentiated intermediate inputs. The activity of the innovation sector results in the expansion of the differentiated goods. In this paper, the term " growth " means the expansion of differentiated goods. The innovation sector uses only skilled workers. We assume that the productivity of each skilled worker residing in a certain region is affected by the knowledge capital, which is determined by an interaction among the skilled. Knowledge capital in one region increases as the number of skilled workers residing in this region increases. The productivity of the skilled increases as knowledge capital increases. Thus, the productivity of skilled workers increases as their number increases. Due to this assumption, skilled workers tend to agglomerate in one region. In this setting, we investigate spatial configurations and the welfare of skilled workers.

Agglomeration of innovation activity is notable in economic activity. Furthermore, agglomeration of the innovation sector does not always occur in the region where manufacturing sectors agglomerate. The statistics presented by the Japan Statistics Bureau show that the share of the number of workers who work at academic research institutions in three metropolitan areas (i.e., Tokyo and Kanagawa Prefectures, Aichi Prefecture, and Osaka and Hyogo Prefectures) is about 37%. In addition, the data presented at the Japan Patent Office shows that the share of patent registrations in the three metropolitan areas is more than 80%. Moreover, an examination of the research and development sector of the Japanese motor industry shows that the innovation sectors of the automobile industry are located in Kanto or Tokai area.

Turning to the United States, Audretsh and Feldman (1996) report that the distribution of innovation is not uniform and that the cause of the agglomeration of innovation is not merely the agglomeration of production. The industry where knowledge spillovers are more widespread has a stronger propensity for agglomeration in one region. However, the region in which agglomeration of the innovation activity occurs might not be necessary where manufacturing activity is occurring. This is shown by comparing Audretsh and Feldman (1996) with Hanson (1998). Both papers discuss the distribution of innovation activity in the United States. Audretsh and Feldman (1996) indicate that the agglomeration of the innovation sector in New England is notable. On the other hand, Hanson (1998) shows that the share of the manufacturing sector of New England is low. In short, this implies that the places where innovation occurs may be different from those where production takes place.

From a theoretical viewpoint, in terms of the migration of skilled workers and the production of patents, our model is close to that of Fujita and Thisse (2003). Moreover, in terms of the supply side, our model is close to that of Yamamoto (2003). In Fujita and Thisse (2003), the linkage between manufacturing firms and workers and knowledge spillovers lead to the agglomeration of firms and workers. On the other hand, we show that the linkage between final good firms and intermediate goods firms leads to agglomeration of firms in both sectors. In Yamamoto (2003), the transaction between intermediate goods sector and manufactured goods sector creates a linkage. In addition, there is a linkage between the innovation sector and manufactured goods sector due to the assumption that the innovation sector uses manufactured goods as inputs. However, there is no linkage between the innovation sector and manufactured goods sector in our model. Furthermore, there is a difference between Yamamoto (2003) and our model in terms of mobility. Although, in Yamamoto (2003), workers are identical and immobile, we assume that there are two types of workers and mobile skilled workers. By integrating Fujita and Thisse (2003) and Yamamoto (2003), a richer implication is obtained.

In our model, the economy has two trade patterns, one in which interregional trade of final good occurs, and one in which it does not. The economy has both trade patterns simultaneously or only the latter trade pattern. Which economy will emerge depends on the relationship between transportation costs of final good and those of intermediate goods. When transportation costs of intermediate goods are relatively high, the former economy occurs. When transportation costs of intermediate goods are relatively low, the latter economy emerges. It is possible for the economy to reach the following spatial configurations: full agglomeration, partial agglomeration, and segmented agglomeration. In full agglomeration, the innovation sector and intermediate goods sector agglomerate in one region. In partial agglomeration, intermediate goods firms partially agglomerate in the region where the innovation sector agglomerates perfectly. In segmented agglomeration, the innovation sector agglomerates in one region, and all intermediate goods firms agglomerate perfectly in the other. The innovation sector always agglomerates in one region. This is because the more skilled workers agglomerate in one region, the higher their wage rate due to knowledge spillovers among skilled workers.

Partial agglomeration and full agglomeration occur in both trade patterns. However, segmented agglomeration does not always emerge. This agglomeration is accomplished when transportation costs of final good are sufficiently low and those of intermediate goods are relatively high. The reason that this agglomeration might occur is as follows. Since there are transportation costs for intermediate goods, final good firms and intermediate goods firms tend to locate in one region. When transportation costs of final good are sufficiently low, under some situations, it might be profitable for skilled workers to agglomerate in the region where final good firms do not locate and to consume final good while paying transportation costs.

We investigate the welfare of skilled workers in each agglomeration. The welfare in full agglomeration is always the highest. However, the welfare in segmented agglomeration may not necessarily be the lowest in spite of requiring transportation costs for final good. When transportation costs of final good are not too low and if the initial distribution of skilled workers in the region where the final good sector does not operate is high, skilled workers will tend to agglomerate in this region, even if they can increase their welfare by means of agglomeration in the region where final good firms agglomerate.

This paper is organized as follows. In the next section, the basic model is presented, and the two patterns relating to final good trade are described. In section 3, the migration behavior of skilled workers is described. In section 4, a spatial configuration under fixed distribution of skilled workers is presented. In section 5, a spatial configuration when migration is allowed is shown. In section 6, the welfare of the skilled workers is investigated. Section 7 is the conclusion.

2 The model with migration

2.1 The model

The economy consists of regions A and B. There are four production sectors: agricultural, final good, intermediate goods, and innovation. Moreover, there are two types of labor, skilled and unskilled. The innovation sector uses only skilled labor to produce patents, which are needed for production of intermediate goods. While agricultural sector uses only unskilled worker, final good is produced by using unskilled labor and a set composed of intermediate goods. Intermediate goods are also produced by using unskilled workers and patent. We chose the price of agricultural goods as the numeraire. Shipping of final good and intermediate goods incurs transportation costs that are of the ice-berg cost form.

There exists L units of unskilled labor in the economy, which is immobile between two regions, and L is constant over time. Each region is an identical amount of unskilled labor. Each skilled worker can move between two regions by incurring positive cost. The total number of skilled workers in the economy is constant over time. Without loss of generality, the number of skilled workers is normalized to 1. Although the number of skilled workers is constant, growth occurs by knowledge capital.

Since our model is symmetric, we focus on the specification of region A. The inter-temporal utility function of the consumer who consumes agricultural goods and final good is given by the following form:

$$U = \int_0^\infty e^{-\rho t} \ln u(t) dt, \tag{1}$$

where $u(t)$ is the instantaneous utility at time t and ρ is the subject discount rate. The instantaneous utility function is as follows: $u(t) = M(t)^\alpha A(t)^{1-\alpha}$, where $M(t)$ and $A(t)$ represent the consumption of homogeneous final good and that of agricultural good, respectively, $\alpha (0 < \alpha < 1)$ is the share of expenditures devoted to final good. Solving consumer's problem derives the indirect utility function of a type j consumer residing in A, v_{jA}:

$$v_{jA} = \alpha^\alpha (1-\alpha)^{1-\alpha} p_{MA}^{-\alpha} \epsilon_{jA}, \tag{2}$$

where ϵ_{jA} represents the expenditure for a type j worker residing in A.

Moving from one region to the other at time t incurs cost $C(t)$, which is expressed in terms of an individual lifetime utility. We define the lifetime utility of consumer j at time 0 as follows:

$$U_j(0) \equiv V_j(0) - \sum_h e^{-\rho t} C(t_h), \tag{3}$$

where t_h represents the sequence of time in which a consumer moves and

$$V_j(0) \equiv \int_0^\infty e^{-\rho t} \ln[v_j(t)] dt. \tag{4}$$

This expresses lifetime utility obtained by consumption of goods.

We assume that there exists a global and perfectly competitive capital market in which bonds bear an interest rate equal to $r(t)$ at time t. This capital market makes interest rates in both regions equate: $r_A(t) = r_B(t) = r(t)$. We have to specify consumer j's intertemporal budget constraint. First, the present value of wage income of consumer j is given as follows:

$$W_j(0) = \int_0^\infty e^{-\bar{r}(t)} w_{jA(t)}(t) dt, \tag{5}$$

where $w_{jA(t)}(t)$ is the wage of consumer j residing in region A at time t and $\bar{r}$ is the average interest rate between 0 and t: $\bar{r} = (1/t) \int_0^t r(\tau) d\tau$. Using this presents an inter-temporal budget constraint from the flow of the budget constraint:

$$\int_0^\infty \epsilon_j(t)e^{-rt}dt = a_j(0) + W_j(0), \tag{6}$$

where $a_j(0)$ is the value of initial assets. Given any location path $s_j(t)$, $s = A, B$, if $\epsilon_j(t)$ is the expenditure path, such that the lifetime utility is maximized subject to an inter-temporal budget constraint, $\dot{\epsilon}_j(t)/\epsilon_j(t) = r(t) - \rho$, at time t. Since this relationship applies to any consumer, we show

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho, \qquad t \geq 0, \tag{7}$$

where $E(t)$ is an expenditure in the economy at time t.

Turn to the supply side. First, agricultural good is produced by constant returns to scale technology. For simplicity, suppose that the input coefficient in the production of this good is 1. Because there is no transportation cost, the nominal wages equate between two regions, and $w = 1$, where w represents the wage rate of unskilled labor. We assume that the demand for agricultural good is sufficiently large so that the demand for agricultural good is not satisfied when this good is produced exclusively in one region. From this assumption, this good is produced in both regions at equilibrium.

Then, final good is produced by constant returns to scale technology under perfect competition: $M_A(t) = I_A^\beta(t)L_{MA}^{1-\beta}(t)$, where $M_A(t)$ is the quantity of final good produced in region A, $L_{MA}(t)$ is the demand for unskilled labor in final good sector in region A, and $\beta(0 < \beta < 1)$ is the share of the expenditure for the set of intermediate goods to the total expenditure for input requirement. $I_A(t)$ is the composite of intermediate goods, which is given as follows:

$$I_A(t) = \Big[\int_0^{N(t)} z_i(t)^\theta di\Big]^{1/\theta}, \tag{8}$$

where $z_i(t)$ is an intermediate input indexed by i, $N(t)$ is the total number of varieties available in the economy in time t, and θ represents the intensity of the input requirement for a variety of differentiated intermediate goods. If we set $\sigma = 1/(1-\theta)$, σ is the elasticity of substitution between each variety of intermediate goods. [1]

Finally, differentiated intermediate goods are produced in a monopolistically competitive sector with an increasing return-to-scale technology in each variety. Suppose that there is no cost of discrimination. This makes each

[1]The sufficient condition such that agricultural good is produced in both regions in equilibrium is $\alpha < \sigma/(2\sigma - \beta)$.

firm produce only one variety. This implies that the number of the firms is equal to the number of varieties available. We assume that a unit of a patent is required when production is started and that a μ unit of unskilled labor is also needed marginally in the production. Hence, the optimal pricing is

$$p_{iA} = p_{iB} = \frac{\sigma\mu}{\sigma - 1}, \tag{9}$$

where p_{iA} (p_{iB}) represents the prices of intermediate goods produced in region A (B). Because of equality in the price between the two regions, we denote that $p_{iA} = p_{iB} = p$. The operating profit is as follows:

$$\pi = px - \mu x = \frac{\mu x}{\sigma - 1}, \tag{10}$$

where x is the size of the firms in intermediate goods sector.

Solving problems of final good firm yields the demand for intermediate goods locating in region A:

$$z_{iA} = \beta[p_{MA}M_A p^{-\sigma} G_A^{\sigma-1} + \tau_I^{1-\sigma} p_{MB} M_B^{-\sigma} G_B^{\sigma-1}], \tag{11}$$

where p_{MA} (p_{MB}) is the price of final good produced in region A (B), τ_I is transportation costs of intermediate goods, and $G_A(G_B)$ represents the price index of intermediate goods in region A (B), which is given as follows:

$$G_A = [n_A p^{1-\sigma} + \tau_I^{1-\sigma} n_B p^{1-\sigma}]^{1/(1-\sigma)}, \tag{12}$$

where $n_A(n_B)$ is the number of intermediate goods firms locating in region A (B) and $n_A + n_B = N$. (12) says that, given the number of intermediate goods firms, the region with more intermediate goods firms has the lower price index of intermediate goods. Due to the zero profit of final good firm, the price of final good produced in region A is given as follows:

$$p_{MA} = \beta^{-\beta}(1 - \beta)^{-(1-\beta)} G_A^{\beta}. \tag{13}$$

From our assumption on trade cost and the demand for agricultural goods, the wage rates of unskilled workers in two regions are 1. Thus, final good is produced in the region that has more intermediate goods firms since the more intermediate goods firms locate, the lower the price index of intermediate goods is.

An innovation is needed to produce a new variety. We assume that, once the innovation is performed, an entrepreneur can have monopoly power on the variety produced and the choice to relocate the production facilities freely across regions at no cost. Since a unit of patent is needed to start production intermediate goods, the total number of varieties is equal to that of patents.

2.2 The innovation sector

We will now turn to the innovation sector, which produces a patent by using only skilled labor. Following Fujita and Thisse (2003), the production function of a patent in region A takes the following form:

$$\dot{n}_A = \frac{K_A}{b}\lambda_A, , \tag{14}$$

where K_A represents the existing knowledge in region A, b is the requirement of skilled labor for the production of a unit of patent, and λ_A represents the number (share) of skilled workers residing in region A. Furthermore, the knowledge stock existing in each region is determined by the interaction among skilled workers. When we denote h_j as the knowledge which worker j has, the available knowledge in region A is [2]

$$K_A = \Big[\int_0^{\lambda_A} h_j^{\psi} dj + \eta \int_0^{\lambda_B} h_j^{\psi} dj\Big]^{\frac{1}{\psi}}, \tag{15}$$

where $\psi(0 < \psi < 1)$ represents a measurement of the complementarity among the skilled with regard to the creation of knowledge and $\eta(0 \leq \eta \leq 1)$ is the index of knowledge spillovers between two regions. For simplicity, we assume that the knowledge that worker j has is proportionate to the number of existing patents in the economy, i.e., $h_j = N$. [3] From this assumption, we can rewrite (15) as follows: [4]

$$K_A = N[\lambda_A + \eta\lambda_B]^{\frac{1}{\psi}}. \tag{16}$$

To simplify the notation, we define $k_A = [\lambda_A + \eta\lambda_B]^{1/\psi}$. Substituting (16) into (14), we find that $\dot{n}_A = (N/b)[\lambda_A + \eta(\lambda_B)]^{1/\psi}\lambda_A$. Supposing that the length of a patent is infinite, the firm producing differentiate intermediate goods has monopoly power forever. This leads to the next motion:

$$\dot{N} = \dot{n}_A + \dot{n}_B = \frac{N}{b}[\lambda k_A(\lambda) + (1-\lambda)k_B(\lambda)], \tag{17}$$

[2]Audretsh and Feldman (1996) show that industries in which knowledge plays a more important role tend to cluster spatially. Therefore, we think that this formation is not unnatural.

[3]This is interpreted as the knowledge that an individual has is the entire stock accumulated in the past.

[4]When $\eta = 1$, $K_A = N$. This implies that knowledge is interpreted as a pure public good. On the contrary, when $\eta = 0$, $K_A = N\lambda_A^{\frac{1}{\psi}}$. This describes a situation in which knowledge is a local public good. Accordingly, η is interpreted as a measure of the localness of knowledge.

where $\lambda \equiv \lambda_A$. For simplicity of notation, we set $f(\lambda) = \lambda k_A(\lambda) + (1-\lambda)k_B(\lambda)$. Using $f(\lambda)$, we rewrite (17) as $\dot{N} = [f(\lambda)/b]N$. Thus, when the distribution of the skilled is λ, the growth rate of the number of varieties is: [5]

$$g(\lambda) = \frac{f(\lambda)}{b}. \tag{18}$$

Recalling (14), since K_A is exogenous for the innovation sector, the marginal product of skilled workers (or the marginal product of migration) is K_A/b. Therefore, the unit cost for the production of a patent is bw_A/Nk_A. Free entry and zero profit in the innovation sector make the value of a patent developed in region A $\Pi_A = bw_A/Nk_A$. Denoting the wage rate of skilled workers in region A on equilibrium as w_A^*, the wage is represented as follows:

$$w_A^* = \frac{\Pi_A N k_A}{b}. \tag{19}$$

In the intermediate goods sector, the value of a patent is equal to that of the firm due to free entry at every moment t.

2.3 Trade pattern

Now, we investigate the location of the final good sector. As shown in Yamamoto (2003), there are two patterns regarding the location or trade of the final good sector in the economy: final good is produced in both regions, or final good is produced in one region.

Since the economy is symmetric, we focus on the case $1/2 \leq \gamma \leq 1$, where γ is the share of intermediate goods firms locating in region A. This implies that, if the agglomeration of a final good firm (perfect or not) emerges, it will occur in region A. This is because, when $1/2 \leq \gamma \leq 1$, the price index of intermediate goods in region A is lower. From (13), it is clear that $p_{MA} \leq p_{MB}$ if $G_A \leq G_B$. Moreover, if final good is produced in both regions, the price of final good produced in region B must be lower than the price including transportation costs of this good produced in region A. In short, the trade pattern of final good is stated as follows.

1. (pattern 1) If $p_{MA}\tau_M \geq p_{MB}$, final good is produced in both regions and there is no trade of final good. [6]

[5] It is verified that $g(\lambda)$ is symmetric about 1/2 and such that $g(0) = g(1) = 1/b$. In addition, it is clear that $\partial g(\lambda)/\partial \eta > 0$ and $\partial^2 g(\lambda)/\partial \eta^2 > 0$. Therefore, the growth rate of the number of varieties is the increasing function of η, and, when $\eta = 1$, the growth rate reaches its maximum value. That is, the strength of inter-regional knowledge spillovers affects the growth rate.

[6] More precisely, we have to state that, if $p_{MA} > p_{MB}$, final good is produced in both

2. (pattern 2) If $p_{MA}\tau_M < p_{MB}$, the production of final good is operated only in region A and there is trade of final good.

From the above discussion, rearranging $p_{MA}\tau_M < p_{MB}$ clarifies that, if γ is larger than a critical value, $\hat{\gamma}$, then final goods sector operates only in region A, where $\hat{\gamma}$ is:

$$\hat{\gamma} \equiv \frac{1 - \tau_I^{1-\sigma}\tau_M^{\frac{1-\sigma}{\beta}}}{(1 + \tau_M^{\frac{1-\sigma}{\beta}})(1 - \tau_I^{1-\sigma})}, \tag{20}$$

This demonstrates that $\hat{\gamma} > 1/2$. Moreover, it is obvious that, if $\hat{\gamma} > 1$, then the following relationship holds:

$$\tau_M > \tau_I^{\beta}. \tag{21}$$

When (21) holds, perfect agglomeration of final good firm in region A cannot occur. This is due to the relatively high transportation costs of final good compared to those of intermediate goods.

Then, we investigate the prospect for the economy when the trade is in pattern 2. If intermediate goods firms work in both regions, the operating profit (the output) must be equal between two regions. In short, $1/2 < \gamma < 1$ if $x_A = x_B$, and $\gamma = 1$ otherwise. Substituting $n_A > 0$ and $n_B = 0$ into (11) indicates that $\pi_A > \pi_B$ ($x_A > x_B$). Therefore, when the trade is in pattern 2, $\gamma = 1$ holds.

Now, we examine what happens when the trade is in pattern 1. In pattern 1, intermediate goods firms locate in both regions, which implies that the output is equal between two regions. This shows the following relationship:

$$G_A^{\beta+\sigma-1}E_A = G_B^{\beta+\sigma-1}E_B, \tag{22}$$

where E_A (E_B) is the expenditure of region A (B). If $\gamma = 1/2$ and $\lambda = 1/2$, $G_A = G_B$ and $E_A = E_B$. That is, $n_A = n_B$. Therefore, it is obvious that, when $\gamma = 1/2$ and $\lambda = 1/2$, (22) holds. However, we would like to focus on the equilibrium that $1/2 < \gamma \leq 1$ and $1/2 < \lambda \leq 1$. Here, note that the expenditure in each region cannot be determined until λ is satisfied.

Rearranging (22) reveals the share of intermediate goods firms located in a region, γ, which is given as follows.

$$\gamma = \frac{\frac{E_A}{E_B} - \tau_I^{1-\sigma}}{(1 - \tau_I^{1-\sigma})(\frac{E_A}{E_B} + 1)}. \tag{23}$$

regions, and there is no trade of final good. However, suppose that if $p_{MA} = p_{MB}$, final good is also produced in both regions and there is no trade final good.

This shows that, if the expenditure ratio and transportation costs of intermediate goods rise, the share of intermediate goods firms locating in region A increases. This is due to transportation costs of intermediate goods and scale economy of producing intermediate goods.

From the above discussion, it is found that all equilibria are in pattern 1 or pattern 2. This implies that there is no equilibrium such that intermediate goods firms agglomerate in a region in which final good sector does not agglomerate. This is because final good can be produced with lower cost if final good firm locates in the region in which the number of intermediate goods firms is higher.

We will now turn to the labor market. Since unskilled labor is used in all sectors except the innovation sector, the clearing condition for the unskilled labor market is

$$L^A + L_{MA} + L_{MB} + L_{IA} + L_{IB} = L, \tag{24}$$

where L^A is the demand for unskilled labor in the agricultural sector, L_{MA} (L_{MB}) is the demand for unskilled labor in the final good sector in segion A (B), L_{IA} (L_{IB}) is the demand for unskilled labor in the intermediate goods sector in region A (B), and L is the amount of unskilled labor supply in the economy. Deriving the clearing condition for unskilled labor markets in each sector and substituting these outcomes into (24), we show the relationship between unskilled labor and expenditure in the economy: $E = \sigma L/(\sigma - \alpha\beta)$. This implies that, if L is constant, then E is also constant. Hence, E is constant due to the assumption that L is constant. This leads to $r(t) = \rho$ from (7). We find that the equilibrium interest rate is equal to the subjective discount rate.

Here, we need to specify the income of each worker. An intertemporal budget constraint derives $\epsilon_j = \rho[a_j(0)+W_j(0)]$. We assume that intermediate goods firms are equally possessed among only skilled workers. [7] This implies $a_L = 0$. From (5), the present value of the wage income of the unskilled is $\int_0^\infty e^{-\rho t}w(t)dt = 1/\rho$, since $w = 1$. The income of skilled workers is also represented as follows:

$$\epsilon_H = \rho[a_H(0) + W_H(0)]. \tag{25}$$

From the assumption, the initial endowment of the skilled is given the following form:

$$a_H(0) = n_A(0)\Pi_A + n_B(0)\Pi_B. \tag{26}$$

[7]Picard et al. (2004) have shown that the spatial configuration is affected by the structure of the ownership of firms working under monopolistic competition. In our model, the simplest case as to ownership of the firms is considered.

Note that $\epsilon_H(0)$ is determined by the present value of income and the equilibrium wage rate of skilled workers.

3 Migration behavior

Recall the assumption that migration between two regions incurs the loss of utility. Following Fujita and Thisse (2003), the migration cost takes the following form: [8]

$$C(t) = \frac{|\dot{\lambda}(t)|}{\Psi}. \tag{27}$$

The left-hand side represents the cost in terms of the utility loss for migrant time at t. As for the right-hand side, $\dot{\lambda}$ represents the flow of skilled workers moving from one region to the other and Ψ is a positive constant interpreted as an adjustment cost. When skilled workers move from region B to A, $\dot{\lambda}$ is positive. On the contrary, when skilled workers move from region A to B, $\dot{\lambda}$ is negative.

Consider the following case, which will be relevant for the stability of analysis of the steady state at $\tilde{\lambda} \in [0, 1]$. Let the initial distribution of the skilled be lower than $\tilde{\lambda}$. Suppose that $T > 0$ exists such that the flow of skilled workers from B to A starts at time 0 and stops at time T. Briefly, this is stated as follows:

$$\begin{array}{rcll} \dot{\lambda}(t) & > & 0 & t \in (0, T) \\ \lambda(t) & = & \tilde{\lambda} & t \leq T \end{array} \tag{28}$$

In this case, the equilibrium dynamics of skilled workers under the expectation (28) is derived as the following form:

$$\dot{\lambda}(t) = \frac{\Psi e^{\rho t}}{\rho} \ln \left[\frac{a_H(0) + W_H(0; t)}{a_H(0) + W_H(0; T)} \right] - \alpha \Psi e^{\rho t} \int_t^T e^{-\rho s} \ln \frac{p_{YA}(s)}{p_{YB}(s)} ds \tag{29}$$

for any $t \in (0, T)$. The process of deriving (29) is shown in Appendix 1.

In the next section, we investigate the equilibrium when λ is fixed.

4 The steady-state growth path when λ is fixed.

In this section, we investigate the steady-state growth path under fixed λ.

[8]The migration cost is interpreted as a congestion cost.

4.1 Location of the intermediate goods sector

The objective in this subsection is to find the location of intermediate sector, γ, given the distribution of the skilled, λ. To show this, some preparation is required.

Here, we find the profit (the output) of intermediate goods firms. Note that intermediate goods firms are free to chose the region where they work because patents are perfectly mobile. First, we derive the profits when intermediate goods firms locate in both regions. In this case, $x_A = x_B$ holds. Using (22), $n_A + n_B = N$ and $E_A + E_B = E$ yield the number of intermediate goods firms in each region:

$$n_A = \frac{E_A - \tau_I^{1-\sigma} E_B}{(1-\tau_I^{1-\sigma})E} N, \qquad n_B = \frac{E_B - \tau_I^{1-\sigma} E_A}{(1-\tau_I^{1-\sigma})E} N. \tag{30}$$

It is obvious that $n_A > 0$ and $n_B > 0$ if and only if $\tau_I^{1-\sigma} < (E_A/E_B) < 1/\tau_I^{1-\sigma}$. (11) reveals the output of intermediate goods firms in both regions. This is

$$x_A = x_B = \frac{\alpha\beta(\sigma-1)}{\sigma\mu N} E.$$

Then, it is shown that $n_A = N$ and $n_B = 0$ if and only if $1/\tau_I^{1-\sigma} \leq (E_A/E_B)$. In this case, the output of intermediate goods firms in each region is

$$x_A = \frac{\alpha\beta(\sigma-1)}{\sigma\mu N} E \geq \frac{\alpha\beta(\sigma-1)}{\sigma\mu N}(\tau_I^{1-\sigma} E_A + \tau_I^{\sigma-1} E_B) = x_B.$$

Finally, it is also found that $n_A = 0$ and $n_B = N$ if and only if $\tau_I^{1-\sigma} \geq (E_A/E_B)$. In this case, the output of intermediate goods firms in each region is

$$x_A = \frac{\alpha\beta(\sigma-1)}{\sigma\mu N}(\tau_I^{\sigma-1} E_A + \tau_I^{1-\sigma} E_B) \leq \frac{\alpha\beta(\sigma-1)}{\sigma\mu N} E = x_B.$$

Since intermediate goods firms choose the region in which their own profit is larger, the equilibrium profits of intermediate goods firms are

$$\pi = max\{\pi_A, \pi_B\} = \frac{\alpha\beta}{\sigma N} E. \tag{31}$$

From (17) and (18), it is clear that the total number of intermediate goods in the economy is $N(t) = N(0)e^{g(\lambda)t}$, where $N(0)$ is the initial number of varieties in the economy. Since the value of each intermediate goods firm is $\Pi(t) = \int_t^\infty e^{-\rho(\tau - t)}\pi(\tau)d\tau$ and E is constant, the aggregate value of intermediate goods firms in the economy is given as follows and denoted as $\tilde{a}(\lambda)$;

$$N(t)\Pi(t) = \frac{\alpha\beta E}{\sigma(\rho + g(\lambda))} \equiv \tilde{a}(\lambda). \tag{32}$$

Substituting (32) into (19) derives the wage rate of skilled workers in each region. Note that the aggregate expenditure in each region at any time is $E_s(\lambda) = (L/2) + \lambda_s \epsilon_s(\lambda)$. Using this yields the ratio of the aggregate expenditure in region A to that in region B as follows:

$$\frac{E_A(\lambda)}{E_B(\lambda)} = \frac{\frac{L}{2} + \lambda\tilde{a}(\lambda)\big[\rho + \frac{(\lambda+\eta(1-\lambda))^{1/\psi}}{b}\big]}{\frac{L}{2} + (1-\lambda)\tilde{a}(\lambda)\big[\rho + \frac{(\eta\lambda+1-\lambda)^{1/\psi}}{b}\big]}. \tag{33}$$

Substituting $\lambda = 1$, $1/2$, and 0 into this shows that $E_A(1)/E_B(1) > 1$, $E_A(1/2)/E_B(1/2) = 1$, $E_A(0)/E_B(0) < 1$. [9]

Here, we show the relationship between the expenditure ratio and λ. The process used to explore this relationship is shown in Appendix 2. As a result, the following relationship holds:

$$\frac{d[E_A(\lambda)/E_B(\lambda)]}{d\lambda} > 0, \qquad \lambda \in (0,1). \tag{34}$$

This suggests that γ is an increasing function of the expenditure ratio and the expenditure ratio increases monotonously as λ rises. This is described in the figures. Figure 1 describes the case in which $\tau_I^{-(1-\sigma)} < (\sigma+\alpha\beta)/(\sigma-\alpha\beta)$. In this case, intermediate goods firms locate in both regions for any λ. On the other hand, Figure 2 represents the case $\tau_I^{-(1-\sigma)} \geq (\sigma+\alpha\beta)/(\sigma-\alpha\beta)$. In this case, there are two configurations. One is that in which intermediate goods are produced in both regions. The other is that in which intermediate goods sector works in one region. The former will emerge if λ is sufficiently close to 1/2, (i.e., $\lambda'' < \lambda < \lambda'$). [10] The latter will occur if λ is sufficiently close to 1 or 0 (i.e., $\lambda'' < \lambda$ or $\lambda < \lambda'$). In other words, if the difference of the number of skilled workers is large, intermediate goods firms will agglomerate in the region in which more skilled workers reside, and if the difference is not large, intermediate goods firms will locate in both regions and will partially

[9] Writing more precisely, it is clear that

$$\frac{E_A(1)}{E_B(1)} = \frac{\sigma+\alpha\beta}{\sigma-\alpha\beta} > 1, \; \frac{E_A(1/2)}{E_B(1/2)} = 1, \; \frac{E_A(0)}{E_B(0)} = \frac{\sigma-\alpha\beta}{\sigma+\alpha\beta} < 1.$$

[10] λ' and λ'' are the values associated with $E_A(\lambda')/E_B(\lambda') = \tau_I^{-(1-\sigma)}$ and $E_A(\lambda'')/E_B(\lambda'') = \tau_I^{1-\sigma}$, respectively.

agglomerate in the region in which more skilled workers reside. Agglomeration of intermediate goods firms is derived from the home market effect through the demand for final good.

When the location is pattern 2, intermediate goods firms perfectly agglomerate in the region where final good firms agglomerate, as mentioned before.

5 The steady-state growth path when migration is allowed.

In this section, we investigate the steady state when migration of skilled workers is allowed. To do this, the lifetime utility of skilled workers is necessary in two regions associated with the growth path under any fixed λ. In the course of this process, the values of λ in equilibrium are acquired. The lifetime utility of skilled workers in region s is shown as follows:

$$V_s = \int_0^\infty e^{-\rho t} \ln v_s(t; \lambda, \gamma) dt, \tag{35}$$

where V_s is the lifetime utility of skilled workers in region s and $v_s(t; \lambda, \gamma)$ is the instantaneous utility at time t. From this, the difference of the lifetime utility of skilled workers between the two regions is

$$V_A(0; \lambda, \gamma) - V_B(0; \lambda, \gamma) = \int_0^\infty e^{-\rho t} \ln \left[\frac{v_A(t; \lambda, \gamma)}{v_B(t; \lambda, \gamma)} \right]. \tag{36}$$

The expenditure for skilled workers in each region is represented as follows:

$$\epsilon_s(\lambda) = \tilde{a}(\lambda) \left[\rho + \frac{k_s(\lambda)}{b} \right]. \tag{37}$$

First, an effort is made to determine the locations in which pattern 1 appears.

5.1 The steady-state growth path when trade is in pattern 1

In this pattern, final good firm does not engage in trade. Deriving the ratio between the indirect utility in region A to that in region B from (2) and (13) and setting $\Phi_1(\lambda, \gamma) = v_A(t; \lambda, \gamma)/v_B(t; \lambda, \gamma)$, the difference of the lifetime utility is rewritten as follows: [11]

$$V_A(0;\lambda,\gamma) - V_B(0;\lambda,\gamma) = \int_0^\infty e^{-\rho t} \ln \Phi_1(\lambda,\gamma) = \frac{1}{\rho} \ln \Phi_1(\lambda,\gamma). \quad (38)$$

This implies that V_A will be larger (smaller) than V_B if Φ_1 is larger (smaller) than 1. Recall that $G_A = G_B$ and $k_A = k_B$ when $\gamma = 1/2$ and $\lambda = 1/2$. Thus, it is obvious that $V_A(0;1/2,1/2) = V_B(0;1/2,1/2)$ holds. This implies that full dispersion ($\lambda = 1/2, \gamma = 1/2$) is always the steady-state equilibrium.

Recall that $\partial k_A(\lambda)/\partial\lambda > 0$, $\partial k_B(\lambda)/\partial\lambda < 0$, and $k_A(\lambda) = k_B(\lambda) = 1$ for $\eta = 1$. Hence, for any $\eta > 0$,

$$\frac{\partial \Phi_1(\lambda,\gamma)}{\partial\lambda} \geq 0, \qquad \frac{\partial \Phi_1(\lambda,\gamma)}{\partial\gamma} \geq 0, \qquad \lambda \in (0,1), \gamma \in (0,1).$$

Hence, it is determined that V_A will be larger (smaller) than V_B if γ and λ are larger (smaller) than 1/2. This states that the economy is on a steady-state equilibrium under $\lambda = 0, 1/2, 1$. We denote λ in steady-state equilibrium as $\tilde{\lambda}$. It is shown in Appendix 3 that $\tilde{\lambda} = 1/2$ is unstable and $\tilde{\lambda} = 0, 1$ is stable.

As shown in Fujita and Thisse (2001), the self-fulfilling nature of the migration process makes stability more difficult to define. It is possible that several perfect foresight solution under the same initial distribution of skilled worker, λ_0 in our model. As a result, for a given steady-state growth path under $\tilde{\lambda}(= 0, 1/2, 1)$, there might be a neighborhood Λ of $\tilde{\lambda}$ such that, for any $\lambda_0 \in \lambda$, the equilibrium path based on a particular expectation converges to this steady-state growth path while another equilibrium path diverges from the same steady-state growth path. To avoid such a difficulty, following Fujita and Thisse (2001), we impose some restrictions on the expectations that have to be satisfied when an equilibrium path converges into a steady-state growth path. Due to perfect foresight, this is amounts to imposing a restriction on the equilibrium path itself. Hence, we introduce the following restrictions.

Suppose that $\tilde{\lambda} \in [0,1]$ and $\lambda_0 \in [0,1]$ are not equal. If $\lambda(t)$ is an equilibrium path satisfying the initial condition, λ_0, this path satisfies the monotonic convergence hypothesis under $\tilde{\lambda}$ (we call this the mc-hypothesis) when $0 < T \leq \infty$ such that,

$$when\ \lambda_0 < \tilde{\lambda}, \quad \dot{\lambda}(t) > 0, \quad for\ t \in (0,T),$$

[11]From (2) and (13), we find that the ratio of the indirect utility in region A to that in region B is as follows:

$$\frac{v_A(t;\lambda,\gamma)}{v_B(t;\lambda,\gamma)} = \left[\frac{\gamma + \tau_I^{1-\sigma}(1-\gamma)}{\tau_I^{1-\sigma}\gamma + 1 - \gamma}\right]^{-\alpha\beta/(1-\sigma)} \frac{b\rho + k_A(\lambda)}{b\rho + k_B(\lambda)}.$$

$$\begin{aligned} \lambda(t) &= \tilde{\lambda}, \quad for\ t \leq T, \qquad (39) \\ when\ \lambda_0 > \tilde{\lambda}, \quad \dot{\lambda}(t) &< 0, \quad for\ t \in (0,T), \\ \lambda(t) &= \tilde{\lambda}, \quad for\ t \leq T. \qquad (40) \end{aligned}$$

The steady-state growth path under $\tilde{\lambda}$ will be stable if there is a neighborhood Λ of $\tilde{\lambda}$ such that, for each $\lambda_0 \in \Lambda$ with $\lambda_0 \neq \tilde{\lambda}$, there is an equilibrium path that satisfies the mc-hypothesis under $\tilde{\lambda}$. The steady-state growth path will not be stable otherwise. We show in Appendix 3 that $\lambda = 0, 1$ is stable under $\tilde{\lambda}$.

The location of intermediate goods firms in the case of $\lambda = 1$ remains to be studied. As mentioned in 4.1, when $\lambda = 1$, the configuration of intermediate goods firms is determined by the relationship between the expenditure ratio and transportation costs of intermediate goods.

First, we investigate the location of the intermediate goods sector when $\tau_I^{1-\sigma} < (\sigma - \alpha\beta)/(\sigma + \alpha\beta)$. In this case, the definitions of λ' and λ'' expose the following relationship:

$$\frac{E_A(\lambda')}{E_B(\lambda')} = \frac{1}{\tau_I^{1-\sigma}} > \frac{\sigma + \alpha\beta}{\sigma - \alpha\beta} = \frac{E_A(1)}{E_B(1)}, \ \frac{E_A(\lambda'')}{E_B(\lambda'')} = \tau_I^{1-\sigma} < \frac{\sigma - \alpha\beta}{\sigma + \alpha\beta} = \frac{E_A(0)}{E_B(0)}.$$

Due to $d[E_A(\lambda)/E_B(\lambda)]/d\lambda > 0$, it is clear that $\lambda' > 1$ and $\lambda'' < 0$. In addition, for $\lambda \in (0,1)$, $\tau_I^{1-\sigma} < E_A(\lambda)/E_B(\lambda) < \tau_I^{-(1-\sigma)}$ holds. This situation is described in Figure 3. Note that λ moves gradually and γ is able to jump due to the assumption that skilled workers can move while incurring costs and intermediate goods firms can move freely. The points on the curve in Figure 3 satisfy the location equilibrium. This is so because of the following reason. Below the curve, $G_A^{\beta+\sigma-1} M_A < G_B^{\beta+\sigma-1} M_B$ holds, which implies that $x_A < x_B$ holds. That is, the profits of intermediate goods firms in region B are higher than those in region A. Hence, the vertical line through $\gamma = 1$ does not satisfy the location equilibrium. Above the curve, the opposite is true. [12] In short, we find that, when $\tau_I^{1-\sigma} < (\sigma - \alpha\beta)/(\sigma + \alpha\beta)$, a configuration in which intermediate goods firms always locate in both regions is chosen as the steady state. We denote this steady state as *partial agglomeration under no trade in region s, (s=A, B)* (PAN in s), where s represents the region in which the innovation sector locates.

Next, we consider the case $\tau_I^{1-\sigma} \geq (\sigma - \alpha\beta)/(\sigma + \alpha\beta)$ in the same way. It is obvious that $\lambda' \leq 1$ and $\lambda'' \geq 0$. [13] This situation is described in Figure 4.

[12] Of course, due to the symmetry of the model, the points satisfy $\gamma = 0$ but not the location equilibrium.

[13] Written more precisely, it is clear that

$$\frac{E_A(\lambda')}{E_B(\lambda')} = \tau_I^{-(1-\sigma)} \leq \frac{\sigma + \alpha\beta}{\sigma - \alpha\beta} = \frac{E_A(1)}{E_B(1)}, \ \frac{E_A(\lambda'')}{E_B(\lambda'')} = \tau_I^{1-\sigma} \geq \frac{\sigma - \alpha\beta}{\sigma + \alpha\beta} = \frac{E_A(0)}{E_B(0)}.$$

The points on the curve satisfy the location equilibrium. The reason is given above. In this case, the steady state occurs when all intermediate goods firms and the innovation sector agglomerate in one region. We denote this steady state as *full agglomeration under no trade in region s, (s=A, B)* (FAN in s).

We summarize the above discussion as a proposition.

Proposition 1 *When there is no trade of final good ($\tau_I^\beta \leq \tau_M$), the stable equilibrium is as follows:*

- *if transportation costs of intermediate goods are sufficiently high ($\tau_I^{1-\sigma} < [\sigma - \alpha\beta]/[\sigma + \alpha\beta]$), PAN in s (s=A, B) occurs.*
- *if transportation costs of intermediate goods are sufficiently low ($\tau_I^{1-\sigma} \geq [\sigma - \alpha\beta]/[\sigma + \alpha\beta]$), FAN in s (s=A, B) emerges.*
- *As transportation costs of intermediate goods become lower, this change occurs more smoothly.*

5.2 The steady-state growth path when trade is in pattern 2

We investigate this in the same manner shown in the previous subsection. Setting $\Phi_2(\lambda) = v_A(t; \lambda, \gamma)/v_B(t; \lambda, \gamma)$, the difference of the utility of skilled workers in region A and region B is represented as follows: [14]

$$V_A(0; \lambda, \gamma) - V_B(0; \lambda, \gamma) = \int_0^\infty e^{-\rho t} \ln \Phi_2(\lambda) dt = \frac{1}{\rho} \ln \Phi_2(\lambda). \tag{41}$$

It is clear that V_A is larger (smaller) than V_B if $\Phi_2(\lambda)$ is larger (smaller) than 1. We denote the value associated with $\Phi_2(\lambda) = 1$ as $\hat{\lambda}$. As mentioned before, it is shown that $\partial k_A(\lambda)/\partial\lambda > 0$ and $\partial k_B(\lambda)/\partial\lambda < 0$ and that, when $\eta = 1$, $k_A(\lambda) = k_B(\lambda) = 1$ for all λ. Thus, it is clear that, for any $\eta > 0$,

$$\frac{d\Phi_2(\lambda)}{d\lambda} \geq 0, \qquad \lambda \in (0, 1). \tag{42}$$

[14]When the difference in the indirect utility of skilled workers in each region is derived by the same manner presented in previous subsection, this is represented as follows:

$$\frac{v_A(t; \lambda, \gamma)}{v_B(t; \lambda, \gamma)} = \frac{\epsilon_{jA}}{\tau_M^{-\alpha} \epsilon_{jB}} = \frac{b\rho + k_A(\lambda)}{b\rho + k_B(\lambda)} \frac{1}{\tau_M^{-\alpha}}.$$

Note that this is dependent on only λ.

This implies that $V_A(\lambda)$ is larger (smaller) than $V_B(\lambda)$ if λ is larger (smaller) than $\hat{\lambda}$.

Now, we need to investigate the property of $\hat{\lambda}$. From the definition of $\hat{\lambda}$, $b\rho + k_A(\hat{\lambda}) = [b\rho + k_B(\hat{\lambda})]\tau_M^{-\alpha}$ must hold. Rearranging this by using the definition of k_s, $s = A, B$ yields the following relationship:

$$(1 - \tau_M^{-\alpha})b\rho = \tau_M^{-\alpha}(\eta\hat{\lambda} + 1 - \hat{\lambda})^{\frac{1}{\psi}} - (\hat{\lambda} + \eta(1 - \hat{\lambda}))^{\frac{1}{\psi}}.$$

Differentiating this equation reveals the following relationship:

$$\frac{d\hat{\lambda}}{d\tau_M} < 0. \tag{43}$$

In addition, it is clear that, when $\tau_M \geq \hat{\tau}_M$, $\hat{\lambda} \leq 0$, where

$$\hat{\tau}_M = \left(\frac{b\rho + \eta^{\frac{1}{\psi}}}{b\rho + 1}\right)^{-\frac{1}{\alpha}} \tag{44}$$

If transportation costs of final good are higher than $\hat{\tau}_M$, $\hat{\lambda}$ will be less than 0, that is, $\hat{\lambda}$ will not exist. In this case, residing in region B is not profitable for the skilled, since transportation costs of final good are not sufficiently low. Hence, the skilled do not locate in region B. However, if transportation costs of final good decrease so that the value of τ_M exceeds $\hat{\tau}_M$, $\hat{\lambda}$ will be larger than zero. When this happens, there is a range of λ such that locating in region B is profitable for skilled workers due to sufficiently low transportation costs of final good. In this case, if the initial share of the skilled workers in region A, λ_0, is larger than $\hat{\lambda}$, the steady state will be a case in which all sectors agglomerate in region A, and, if that is lower than $\hat{\lambda}$, final good sector and all intermediate goods firms will agglomerate in one region, and the skilled workers will agglomerate in the other. We denote the former steady state as *full agglomeration under trade in region A* (FAT in A) and the latter one as *segmented agglomeration under trade in region B* (SAT in B).

From the above discussion, it is clear that the economy in this pattern is on a steady state equilibrium under $\lambda = 0$, $\hat{\lambda}$, and 1 when $\tau_M \leq \hat{\tau}_M$ and on a steady state under $\lambda = 1$ when $\tau_M > \hat{\tau}_M$. We denote the value of λ on steady state equilibrium as $\tilde{\lambda}$. Using the same technique shown in Appendix 3, it is demonstrated that, if $\tau_M \leq \hat{\tau}_M$, $\tilde{\lambda} = \hat{\lambda}$ is unstable and $\tilde{\lambda} = 0$ and 1 are stable; if $\tau_M > \hat{\tau}_M$, $\lambda = 1$ is stable. That is, the stable steady state is FTA or SAT. The steady state that the economy reaches is determined by transportation costs of final good and the initial distribution of skilled workers. Due to the symmetry of the model, there is the value of λ, which resembled $\hat{\lambda}$. We denote this value as $\bar{\lambda}$.

We summarize the above discussion as the following proposition. [15]

Proposition 2 *When there is trade of final good, the stable steady state is FAT in A or SAT in B. Which steady state occurs depends on transportation costs of final good and the initial distribution of skilled workers.*

- *If $\tau_M \geq \hat{\tau}_M$, FAT in A occurs regardless of the initial distribution of the skilled.*
- *If $\tau_M < \hat{\tau}_M$ and $\hat{\lambda} < \lambda_0$, FAT in A will emerge.*
- *If $\tau_M < \hat{\tau}_M$ and $\hat{\lambda} > \lambda_0$, SAT in B will occur.*

It is interesting to investigate what happens in the entire economy. It must be recalled that the trade is only in pattern 1 when $\tau_I^\beta \leq \tau_M$ and both trade patterns occur simultaneously in the economy when $\tau_I^\beta > \tau_M$. The economy under pattern 1 of trade has already been shown in the previous subsection. Therefore, studying an economy in which both trade patterns occur simultaneously is a very interesting proposition.

First, the relationship among τ_I, τ_M, and $\hat{\gamma}$ is shown. From (20), it is obvious that $\partial\hat{\gamma}/\partial\tau_I < 0$, $\partial\hat{\gamma}/\partial\tau_M > 0$. In addition, (23) and $E_A(1)/E_B(1) = (\sigma - \alpha\beta)/(\sigma + \alpha\beta)$ yield that the values of $\gamma = \hat{\gamma}$ and $\lambda = 1$ hold if $\tau_M = \bar{\tau}_M$, where

$$\bar{\tau}_M \equiv \left(\frac{\sigma - \alpha\beta}{\sigma + \alpha\beta}\right)^{\beta/(1-\sigma)}. \tag{45}$$

As mentioned before, if the value of γ reaches $\hat{\gamma}$, then the trade pattern will change instantaneously, which implies the number of the possible steady states which the economy can reach is determined by transportation costs of final good. That is, the relationship among τ_M, $\hat{\tau}_M$, and $\bar{\tau}_M$ affects the number of steady states that the economy reaches.

The relationship between $\hat{\tau}_M$ and $\bar{\tau}_M$ is then presented. Using (44) and (45), it is shown that $\hat{\tau}_M > \bar{\tau}_M$ holds if the expenditure share for final good, α, the subject discount rate, ρ, and the expenditure share for intermediate goods in final good sector, β, are low and the elasticity of substitution between any varieties of intermediate goods, σ, is high. In this paper, we focus on a developed economy. Therefore, we assume $\hat{\tau}_M \leq \bar{\tau}_M$.

The above relationships present the economy under $\hat{\tau}_M \leq \bar{\tau}_M < \tau_M$ in Figure 5. Recall that λ moves gradually and γ can jump. In this case, the

[15] Consider the pattern in which the final good sector locates in region B. When $\tau_M > \hat{\tau}_M$, the steady state is FAT in B regardless of the initial distribution of skilled workers. When $\tau_M < \hat{\tau}_M$, if $\lambda_0 < \bar{\lambda}$, the steady state is FAT in B, and, if $\lambda_0 > \bar{\lambda}$, the steady state is SAT in A.

economy has four multiple equilibria (PAN in A, PAN in B, FAT in A, and FAT in B) regardless of the initial distribution of the skilled. The steady state chosen by the economy is determined by expectation. [16]

Proposition 3 *When $\tau_I^\beta > \tau_M$, if $\hat{\tau}_M \le \bar{\tau}_M < \tau_M$, the stable steady state is PAN in s, or FAT in s (s=A, B), regardless of the initial distribution of the skilled.* [17]

Finally, the value λ associated with $\gamma = \hat{\gamma}$ when $\tau_M < \bar{\tau}_M$ is found. Denote this value as λ^*. It is clear from (23) that [18]

$$\frac{E_A(\lambda^*)}{E_B(\lambda^*)} = \tau_M^{-\frac{1-\sigma}{\beta}}.$$

This implies that $E_A(\lambda^*)/E_B(\lambda^*) < E_A(1)/E_B(1)$, which derives $\lambda^* < 1$ due to the fact that $d[E_A(\lambda)/E_B(\lambda)]/d\lambda > 0$. [19] In addition, it is found that there is a value of λ that is analogous to λ^*, which is called λ^{**}, and it is obvious that $\lambda^{**} > 0$ since the model is symmetric. [20]

In Figure 7, we show the economy when $\hat{\tau}_M \le \tau_M \le \bar{\tau}_M$. In this case, the economy has two multiple equlibria (FAT in A and FAT in B) regardless of the initial distribution of the skilled. The steady state that the economy chooses is determined by expectation.

In Figure 8, we show the economy when $\tau_M < \hat{\tau}_M \le \bar{\tau}_M$. In this case, the economy has four steady states (FAT in A, FAT in B, SAT in A, and SAT in B). When the initial distribution of the skilled is lower than $\hat{\lambda}$, it is possible

[16] We describe the economy in Figure 6 under the condition that $\bar{\tau}_M < \tau_M < \hat{\tau}_M$ holds. In this case, the economy has six multiple equilibria (PAN in A, PAN in B, FAT in A, FAT in B, SAT in A, and SAT in B). The steady state that the economy might reach is determined by the initial distribution of skilled workers and the expectation. When $\lambda_0 \le \hat{\lambda}$, there are three steady states that the economy might reach (PAN in B, FAN in B, and SAT in A). When $\lambda_0 \ge \bar{\lambda}$, the number of steady states that the economy might reach might be three (PAN in A, FAN in A, and SAT in B). When $\hat{\lambda} < \lambda_0 < \bar{\lambda}$, the economy has the potential to reach each steady state. Given the initial distribution of skilled workers, the steady state is determined by expectation.

[17] When $\tau_I^\beta \le \tau_M$, if $\bar{\tau}_M < \tau_M < \hat{\tau}_M$ and $\lambda_0 \le \hat{\lambda}$, the steady state is FAT in B, PAN in B, or SAT in A; if $\bar{\tau}_M < \tau_M < \hat{\tau}_M$ and $\lambda_0 \ge \bar{\lambda}$, the steady state is FAT in A, PAN in A, or SAT in B; if $\bar{\tau}_M < \tau_M < \hat{\tau}_M$ and $\hat{\lambda} < \lambda_0 < \bar{\lambda}$, the economy has the potential to reach each steady state.

[18] Of course, from this equation, it is also shown that, when $\tau_M = \bar{\tau}_M$, $E_A(\lambda^*)/E_B(\lambda^*) = (\sigma+\alpha\beta)/(\sigma-\alpha\beta)$ holds. Hence, we find that $\lambda^* = 1$, in this case.

[19] This is because of $E_A(\lambda^*)/E_B(\lambda^*) < (\sigma+\alpha\beta)/(\sigma-\alpha\beta)$ and $E_A(1)/E_B(1) = (\sigma+\alpha\beta)/(\sigma-\alpha\beta)$.

[20] More precisely, λ^{**} is the value at which intermediate goods firms begin to agglomerate in region B and the trade pattern changes in the case that $\tau_M < \bar{\tau}_M$.

for the economy to reach one of two steady states (FAT in B and SAT in A). When the initial distribution of the skilled is higher than $\bar{\lambda}$, there are two steady states that the economy might reach (FAT in A and SAT in B). When $\hat{\lambda} < \lambda_0 < \bar{\lambda}$, the economy has the potential to reach four steady states. Given the initial distribution of the skilled, the steady state is determined by expectation.

We summarize the above discussion as follows.

Proposition 4 *When $\tau_I^{\beta} > \tau_M$,*

- *if $\hat{\tau}_M \leq \tau_M \leq \bar{\tau}_M$, the stable steady state is FAT in s (s=A, B), regardless of the initial distribution of the skilled;*
- *if $\tau_M < \hat{\tau}_M \leq \bar{\tau}_M$ and $\lambda_0 < \hat{\lambda}$, steady state is FAT in B or SAT in A;*
- *if $\tau_M < \hat{\tau}_M \leq \bar{\tau}_M$ and $\lambda_0 > \bar{\lambda}$, the steady state is FAT in A or SAT in B;*
- *if $\tau_M < \hat{\tau}_M \leq \bar{\tau}_M$ and $\hat{\lambda} < \lambda_0 < \bar{\lambda}$, the economy will have the potential to reach four steady states.*

5.3 Discussion

In this section, we discuss the effect of changes in transportation costs of final good, τ_M, and those of intermediate goods, τ_I, on the spatial configuration. When transportation costs of intermediate goods are relatively higher than those of final good, $\tau_I^{\beta} > \tau_M$. In this case, both trade patterns emerge in the economy. Then, the economy will reach partial agglomeration or full agglomeration (Figure 5). Which spatial configuration it will reach is determined by expectation.

Suppose that transportation costs of final good are reduced. The reductions of those costs results in partial agglomeration ceasing to exist. Thus, the economy will reach only full agglomeration (Figure 7). The relatively high transportation costs of intermediate goods cause the linkage between final good firm and intermediate goods firms to be strong, which leads to full agglomeration. Moreover, skilled workers think that it is preferable to reside in the region where final good sector locates since transportation costs of final good are not sufficiently low. We think that this situation is representative of the fact that urban population growth is linked with the development of transportation technology, shown in Bairoch (1988).

Now, suppose that transportation costs of intermediate goods decline. We think that this is possible because of the development of communication technology. When the reductions in transportation costs of intermediate goods are sufficient to hold $\tau_I^\beta \leq \tau_M$, the trade pattern changes, and trade of final good, consequently, there is no trade of final good. This is because transportation costs of final good are relatively high. Thus, final good sector tends to locate in both regions to save on transportation costs. The reduction of transportation costs of intermediate goods in 1950 is shown in Baldwin et al. (2001). In addition, a regional distribution of manufacturing employment from 1850-1990 is seen in Hanson (1993). It was then shown that, in 1950, the progress of dispersion in manufacturing was faster.

How the economy is affected by the change of transportation costs in this situation is stated. At first, the economy will accomplish partial agglomeration under no trade (Figure 3) due to the linkage between final good firm and intermediate goods firms. However, as transportation costs of intermediate goods continue to decline, the steady state is only full agglomeration under no trade (Figure 4). This is because it is profitable for intermediate goods firms to agglomerate in the region where skilled workers locate. This is derived from the indirect effect through final good, which are consumed more in the region where skilled workers agglomerate. Now, suppose that transportation costs of final good decline and $\tau_I^\beta > \tau_M$ thus holds. This might be derived by, for instance, the improvements in the technology associated with the delivery of goods. The steady state is full agglomeration under trade or segmented agglomeration under trade (Figure 8). Which steady state is accomplished is determined by expectation. If the expectation leads to segmented agglomeration in the economy, this spatial configuration will be fulfilled. This case is described in Audretsh and Feldman (1996), who showed that the agglomeration of production does not attract the agglomeration of innovation. Workers who have a special ability tend to agglomerate in one region to enjoy spillovers that are the most important for innovation activity. The region in which the innovation sector agglomerates is not necessarily the one in which the activities of production of goods agglomerate.

6 Welfare

In this subsection, we investigate the welfare under FAT in A, PAN in A, and SAT in B when both trade patterns emerge. [21] The welfare gap between each equilibrium is shown by using (2) and (13). [22] For simplicity, we omit

[21]Since the model is symmetric, we focus on the case in which final good sector agglomerates in region A when this sector agglomerates.

[22]Note that workers in the region in which final good firms locate can always acquire final good without transportation costs and the wage rate of skilled workers does not

the superscript, which represents skilled workers, H. First, the welfare under FAT in A with that under PAN in A is compared. The ratio of the welfare between two steady states is given by the following form;

$$\frac{v_A(t;1,1)}{v_A(t;1,\gamma)} = \left[\frac{G_A(1)}{G_A(\gamma)}\right]^{-\alpha\beta} = [\gamma + \tau_I^{1-\sigma}(1-\gamma)]^{\frac{\alpha\beta}{1-\sigma}} > 1.$$

Thus, it is clear that $V_A(0;1,1) - V_A(0;1,\gamma) > 0$. This implies that, for skilled workers residing in region A, welfare in a case in which intermediate goods firms agglomerate in region A is higher than that in a case in which intermediate goods firms locate in both regions. This is because the price index is lower in the region that has more intermediate goods firms.

Next, the relationship between the welfare of PAN in A and that of SAT in B is as follows. [23]

$$\frac{v_A(t;1,1)}{v_B(t;0,1)} = \left[\frac{p_{YA}}{p_{YB}}\right]^{-\alpha} = \tau_M^{\alpha} > 1.$$

Therefore, it is obvious that $V_A(0;1,1) - V_B(0;0,1) > 0$. This is due to the fact that workers residing in region A can acquire final good without transportation costs.

Finally, the welfare of PAN in A with that of SAT in B is shown. This is written as follows:

$$\frac{v_A(t;1,\gamma)}{v_B(t;0,1)} = [\gamma + \tau_I^{1-\sigma}(1-\gamma)]^{\frac{\alpha\beta}{\sigma-1}}\tau_M^{\alpha} \geq (<)1 \qquad as\ \gamma \geq (<)\gamma^*.$$

where

$$\gamma^* = \frac{\tau_M^{\frac{1-\sigma}{\beta}} - \tau_I^{1-\sigma}}{1-\tau_I^{1-\sigma}}.$$

Whether the ratio of the welfare in this case is larger than 1 or not depends on the value of γ at PAN in A. Using (23) and $E_A(1)/E_B(1) = (\sigma+\alpha\beta)/(\sigma-\alpha\beta)$ derives the value of γ under $\lambda = 1$:

$$\gamma = \frac{\sigma + \alpha\beta - \tau_I^{1-\sigma}(\sigma - \alpha\beta)}{2\sigma(1-\tau_I^{1-\sigma})}.$$

Computing using the two equations above reveals the following relationship:

$$V_A(0;1,\gamma) - V_B(0;0,1) \leq (>)0 \qquad as\ \tau_I^{1-\sigma} \geq (<)\frac{2\sigma}{\sigma+\alpha\beta}\tau_M^{\frac{1-\sigma}{\beta}} - 1.$$

change if the value of λ does not vary. In addition, the price of final good is affected by the location of intermediate goods firms.

[23] We have already shown that the wage rate of the skilled when $\lambda = 1$ is equal to that when $\lambda = 0$.

This is interpreted as follows. $\tau_I^{1-\sigma} > [2\sigma/(\sigma+\alpha\beta)]\tau_M^{\frac{1-\sigma}{beta}} - 1$ means transportation costs of intermediate goods are relatively low. While skilled workers residing in region A can acquire final good without paying transportation costs, those residing in region B can consume final good while incurring transportation costs. In addition, the cost of final good producer when the economy is under PAN in A is higher than when the economy is under SAT in B. From the discussion above, the inequality is determined by the relationship between transportation costs of final good and those of intermediate goods.

Note that if $\tau_I^{1-\sigma} < [2\sigma/(\sigma+\alpha\beta]\tau_M^{(1-\sigma)/\beta} - 1$, the welfare under SAT in B is lowest. However, the skilled workers tend to agglomerate in region B even if they can increase their welfare by means of agglomeration in region A.

The discussion above is summarized as follows:

Proposition 5 *The welfare of the skilled on FAT in A is the highest. The welfare on SAT in B is higher than that on PAN in A, if the following relationship holds:*

$$\tau_I^{1-\sigma} > \frac{2\sigma}{\sigma+\alpha\beta}\tau_M^{\frac{1-\sigma}{\beta}} - 1.$$

7 Conclusion

In this paper, a Grossman-Helpman-Romer-type endogenous growth model is developed with two regions. We show that there are multiple equilibria in our economy. In addition, we investigate the welfare of skilled workers at the steady state.

In our economy, there always exist multiple equilibria. Note that the economy has the potential to achieve segmented agglomeration. This describes the situation in which agglomeration of an innovation sector and agglomeration of production in manufacturing sector are spatially separate.

The relationship between transportation costs of final good and those of intermediate goods determines the number of steady states that there are in the economy. The steady state that the economy achieves is determined by expectation and the initial distribution of the skilled. This implies that, in the case in which expectation plays an important role, the government of a region that would like to have agglomeration of firms and workers needs to implement some coordination.

We investigate the level of welfare of skilled workers. Not surprisingly, the welfare in full agglomeration is the highest. However, the relative magnitude between the welfare in partial agglomeration and that in segmented agglomeration depends on the relationship between the transportation costs of final good and those of intermediate goods. It is possible for the economy to reach segmented agglomeration even if their welfare at this agglomeration is lowest. This shows that coordination failure occurs. From the point of welfare analysis, some policies are needed to acquire the highest level of welfare.

Appendix

Appendix 1

First, using (2), (4), and (25), we present the lifetime wage of the skilled who migrates from B to A at time t as follows:

$$W(0;t) = \int_0^t e^{-\rho\tau} w_B(\tau) d\tau + \int_t^\infty e^{-\rho\tau} w_A(\tau) d\tau. \tag{A.1}$$

Then, using (3) and (27), the lifetime utility is given by

$$U(0;t) = V(0;t) - e^{\rho t} \frac{\dot{\lambda}(t)}{\Psi}, \tag{A.2}$$

where $V(0;t)$ is the lifetime utility gross of migration costs. $V(0;t)$ is shown as follows:

$$V(0;t) = \frac{1}{\rho} \ln \rho + \frac{1}{\rho} \ln[a_H(0) + W_H(0)] - \alpha \left[\int_0^t e^{-\rho\tau} \ln[p_{YB}(\tau)] d\tau + \int_t^\infty e^{-\rho\tau} \ln[p_{YA}(\tau)] d\tau \right] + \frac{\alpha}{\rho} \ln \alpha + \frac{1-\alpha}{\rho} \ln(1-\alpha). \tag{A.3}$$

From Fukao and Benabou (1993), it is found that

$$\lim_{t \to T} C(t) = 0. \tag{A.4}$$

Therefore, by using (A.2), (A.3), and (A.4), we have

$$U(0;T) = V(0;T)$$

$$U(0;T) = \frac{1}{\rho} \ln \rho + \frac{1}{\rho} \ln[a_H(0) + W_H(0)] - \alpha \left[\int_0^T e^{-\rho\tau} \ln[p_{YB}(\tau)] d\tau + \int_T^\infty e^{-\rho\tau} \ln[p_{YA}(\tau)] d\tau \right] + (1-\alpha) \int_0^\infty e^{-\rho\tau} \ln p^A(\tau) d\tau + \frac{\alpha}{\rho} \ln \alpha + \frac{1-\alpha}{\rho} \ln(1-\alpha). \tag{A.5}$$

Since, in equilibrium, all migrants are indifferent to their migration time, it is shown that $U(0;t) = V(0;t)$ for all $t \in (0,T)$. Thus, rearranging (A.2) and substituting (A.5) and $U(0;t) = V(0;t)$ into (A.2), we find that

$$\begin{aligned} \dot{\lambda}(t) &= \frac{\Psi e^{\rho t}}{\rho}[V(0;t) - V(0;T)] \\ &= \frac{\Psi e^{\rho t}}{\rho} \ln\left[\frac{a_H(0) + W_H(0;t)}{a_H(0) + W_H(0;T)}\right] - \alpha\Psi e^{\rho t}\int_t^T e^{-\rho\tau} \ln\frac{p_{YA}(\tau)}{p_{YB}(\tau)} d\tau, \end{aligned} \tag{A.6}$$

for any $t \in (0,T)$.

Appendix 2

We investigate the relationship between the expenditure ratio, E_A/E_B, and the distribution of skilled workers, λ. Recall that the aggregate expenditure in the economy is constant. Thus, it is sufficient to investigate the relationship between $E_A(\lambda)$ and λ. To investigate this relationship, it is necessary to specify $\tilde{a}(\lambda)$.

First, we derive the aggregate expenditure in region A;

$$E_A = \frac{L}{2} + \tilde{a}(\lambda)\left[\rho + \frac{k_A}{b}\right] \tag{A.7}$$

Now, using (32) for finding $\tilde{a}(\lambda)$, we can identify it as follows:

$$\tilde{a}(\lambda) = \frac{\alpha\beta L}{(\sigma - \alpha\beta)(\rho + g(\lambda))}. \tag{A.8}$$

Next, we show the derivative of $g(\lambda)$ and $\tilde{a}(\lambda)$ in terms of λ:

$$g'(\lambda) = \frac{1}{b}[k_A(\lambda) + \lambda k'_A(\lambda) - k_B(\lambda) + (1-\lambda)k'_B(\lambda)],$$

and

$$\frac{d\tilde{a}(\lambda)}{d\lambda} = -\frac{\alpha\beta L g'(\lambda)}{(\sigma - \alpha\beta)(\rho + g(\lambda))^2}.$$

From (A.7) and these equations, it is verified that $dE_A(\lambda)/d\lambda > 0$, $\lambda \in (0,1)$. Therefore, it is shown that [24]

$$\frac{d(E_A/E_B)}{d\lambda} > 0, \qquad \lambda \in (0,1).$$

[24]Since $E_A + E_B = E$ is constant, it is sufficient to verify $dE_A/d\lambda > 0$.

Appendix 3

As shown before, the economy follows a steady-state growth path under different values of λ. When the economy is in pattern 1, the values of λ at the steady state are 0, 1/2, and 1. On the contrary, when the economy is in pattern 2, the values of λ at the steady state are 0, $\hat{\lambda}$, and 1. Here, the analysis of stability is shown.

Now, we show that, when trade is in pattern 1, the steady-state growth path under $\tilde{\lambda} = 1/2$ is unstable, while the steady-state growth path under $\tilde{\lambda} = 1$ is stable. Consider the case $\tilde{\lambda} = 1/2$. Since two regions are in symmetry, we focus on a value of λ_0 that is lower than 1/2. In this case, the equilibrium migration dynamics of the skilled worker is represented by (55).

Now, using the asset value of intermediate goods firms and the growth rate, we find that

$$\frac{N(t)}{N(\tau)} = e^{-\int_t^\tau g[\lambda(v)]dv}.$$

Using this equation, at each $t \geq 0$, the following relationship reveals

$$a(t) = N(t)\Pi(t) = \frac{\alpha\beta E}{\sigma}\int_t^\infty e^{-\int_t^\tau [\rho+g[\lambda(v)]]dv}d\tau. \tag{A.9}$$

In addition, from dynamics,

$$a(T) = \frac{\alpha\beta E}{\sigma}\frac{1}{\rho + g(\tilde{\lambda})}. \tag{A.10}$$

Hence, it is found that $a(t)$ is not dependent on $N(0) = N_0$. This implies that N_0 does not affect the equilibrium values of variables except $N(t)$.

Then, the wage of skilled workers in region s at time t is as follows:

$$w_s(t) = a(t)\frac{k_s(t)}{b}. \tag{A.11}$$

By definition, we write as follows, for $t \geq T$,

$$\begin{aligned} W(0;t) &= \int_0^t e^{-\rho\tau}w_B(\tau)d\tau + \int_t^T e^{-\rho\tau}w_A(\tau)d\tau \\ &= W(0;T) - \int_t^T e^{-\rho\tau}w_B(\tau)d\tau + \int_t^T e^{-\rho\tau}w_A(\tau)d\tau \\ &= W(0;T) + \int_t^T e^{-\rho\tau}a(\tau)\frac{k_A(\tau) - k_B(\tau)}{b}d\tau, \end{aligned} \tag{A.12}$$

where

$$W(0;T) = \int_0^T e^{-\rho\tau}a(\tau)\frac{k_A[\lambda(\tau)]}{b}d\tau + \frac{a(T)k_A(\hat{\lambda})e^{-\rho T}}{b}.$$

If the workers who locate in region s do not migrate, then, it is shown that

$$W_s(0) = \int_0^\infty e^{-\rho\tau} a(\tau) \frac{k_s[\lambda(\tau)]}{b} d\tau. \tag{A.13}$$

Now, we write the expenditure of region A as follows:

$$E_A(t) = \frac{L}{2} + \lambda(0)\rho[a(0) + W_A(0)] + \int_0^t \dot{\lambda}(\tau)\rho[a(0) + W(0;\tau)]d\tau.$$

The last term in this equation represents the expenditure of workers that move from region B to region A between 0 and t. Since $E(t) = E_A(t)+E_B(t)$, $E_B(t) = E - E_A(t)$. Therefore, the following equations hold:

$$E_B(T) = \frac{L}{2} + (1 - \tilde{\lambda})\rho[a(0) + W_B(0)].$$

and

$$\dot{E}_B(t) = -\dot{E}_A(t) = -\dot{\lambda}(t)\rho[a(0) + W(0;t)].$$

In addition, we write the expenditure of region B for each $t \geq T$,

$$\begin{aligned} E_B(t) &= E_B(T) - \int_t^T \dot{E}_B(\tau)d\tau \\ &= \frac{L}{2} + (1 - \tilde{\lambda})\rho[a(0) + W_B(0)] + \int_t^T \dot{\lambda}(\tau)\rho[a(0) + W(0;\tau)]d\tau. \end{aligned}$$

From the discussion above, we show that

$$\begin{aligned} E_A(t) - E_B(t) = \lambda(0)\rho[a(0) + W_A(0)] + \int_0^t \dot{\lambda}(\tau)\rho[a(0) + W(0;\tau)]d\tau \\ - (1 - \tilde{\lambda})\rho[a(0) + W_B(0)] - \int_t^T \dot{\lambda}(\tau)\rho[a(0) + W(0;\tau)]d\tau. \end{aligned} \tag{A.14}$$

Proposition A.1 *The steady-state growth path under $\tilde{\lambda}$ is stable.*

Proof. Under dynamics and $\tilde{\lambda} = 1/2$,

$$\begin{aligned} \lambda(t) &< \frac{1}{2}, \qquad for\ t < T, \\ \lambda(t) &= \frac{1}{2}, \qquad for\ t \geq T. \end{aligned} \tag{A.15}$$

Since $1/2 \leq \lambda \leq 1$, it is clear that

$$k_A[\lambda(t)] \equiv [\lambda(t) + \eta(1 - \lambda(t))]^{1/\psi} \leq [1 - \lambda(t) + \eta\lambda(t)]^{1/\psi} \equiv k_B[\lambda(t)]. \tag{A.16}$$

Since k_A is an increasing function of λ, $\eta \leq 1$, and $a(t) > 0$ for $t \geq 0$, it is found from (A.12) that

$$W(0;t) \leq W(0;T).$$

This implies that

$$\frac{a_H + W(0;t)}{a_H + W(0;T)} = \frac{a(0) + W(0;t)}{a(0) + W(0;T)} \leq 1. \tag{A.17}$$

From (A.12) and (A.16), it is obvious that $W_A(0) \leq W_B(0)$. Hence, (A.14) is rewritten as follows:

$$\begin{aligned} E_A(t) - E_B(t) &< \lambda(0)\rho[a(0) + W_A(0)] - (1 - \frac{1}{2})\rho[a(0) + W_B(0)] \\ &+ \int_0^T \dot{\lambda}(\tau)\rho[a(0) + W(0;\tau)]d\tau \\ &< (\lambda_0 - \frac{1}{2})\rho[a(0) + W_B(0)] + \int_0^T \dot{\lambda}(\tau)\rho[a(0) + W(0;\tau)]d\tau. \end{aligned} \tag{A.18}$$

In addition, from the definition, it is shown that, for $t < T$,

$$\begin{aligned} W_B(0) - W(0;\tau) &= \int_\tau^\infty e^{-\rho s}[w_B(v) - w_A(v)]dv \\ &= \int_\tau^\infty e^{-\rho v} a(v) \frac{k_B(\lambda(v)) - k_A(\lambda(v))}{b} dv \geq 0. \end{aligned} \tag{A.19}$$

Given $\lambda(T) = 1/2$, we rewrite (A.18) as follows:

$$\begin{aligned} E_A(t) - E_B(t) &< (\lambda_0 - \frac{1}{2})\rho[a(0) + W_B(0)] + \left(\int_0^T \dot{\lambda}(\tau)d\tau\right)\rho[a(0) + W_B(0)] \\ &= (\lambda_0 - \frac{1}{2})\rho[a(0) + W_B(0)] + (\frac{1}{2} - \lambda_0)\rho[a(0) + W_B(0)] = 0. \end{aligned}$$

Therefore, we state that

$$E_A(t) < E_B(t) \qquad for\ t \leq T \tag{A.20}$$

From the discussion in 4.1, (13) and (23), we obtain that

$$\frac{p_{YA}(t)}{p_{YB}(t)} = min\left\{ \left(\frac{E_A(t)}{E_B(t)}\right)^{\frac{\beta}{1-\sigma}}, \tau_I^\beta \right\}.$$

Thus, we rewrite (A.6) as follows:

$$\dot{\lambda}(t) = \frac{\Psi e^{\rho t}}{\rho} \ln\left[\frac{a(0)+W(0;t)}{a(0)+W(0;\tau)}\right] + \frac{\alpha\beta\Psi e^{\rho t}}{\sigma-1}\int_t^T e^{-\rho v}\ln\left[min\left\{\left(\frac{E_A}{E_B}\right), \tau_I^{1-\sigma}\right\}\right]dv. \tag{A.21}$$

Therefore, it is found that RHS in (A.21) is negative from (A.20). Hence, this implies that, under $\tilde{\lambda} = 1/2$, there is no equilibrium to satisfy the mc hypothesis, that is, the steady-state growth path under $\tilde{\lambda} = 1/2$ is unstable. Q.E.D.

Then, consider the case $\tilde{\lambda} = 1$. Suppose that the equilibrium path that satisfies the dynamics under $\tilde{\lambda} = 1$ given $\lambda_0 \in [1/2, 1)$. Under the mc hypothesis and $\tilde{\lambda} = 1$, for $\lambda_0 \geq 1/2$, we write

$$\begin{aligned} \frac{1}{2} < \lambda(t) < 1, \qquad & for\ t \in (0, T), \\ \lambda(T) = 1. & \end{aligned} \tag{A.22}$$

Note that, since $1/2 < \lambda \leq 1$, $k_A[\lambda(t)] \geq k_B[\lambda(t)]$. This implies that

$$W_A(0) \equiv W(0;0) \geq W(0;t) \geq W(0;T), \qquad t \geq T.$$

From this relationship, we show that

$$\frac{a_H + W(0;t)}{a_H + W(0;T)} = \frac{a(0) + W(0;t)}{a(0) + W(0;T)} \geq 1, \qquad t \geq T. \tag{A.23}$$

From (A.12) and $\tilde{\lambda} = 1$, it is shown that, for $t \in (0, T)$,

$$\begin{aligned} E_A(t) - E_B(t) &= \lambda_0\rho[a(0) + W_A(0)] + \int_0^t \dot{\lambda}(\tau\rho[a(0) + W(0;\tau)]d\tau \\ &- \int_t^T \dot{\lambda}(\tau)\rho[a(0) + W(0;\tau)]d\tau \\ &> \lambda_0\rho[a(0) + W_A(0)] - \int_0^T \dot{\lambda}(\tau)\rho[a(0) + W_A(0)]d\tau \\ &\geq \lambda_0\rho[a(0) + W_A(0)] - \left(\int_0^T \dot{\lambda}(\tau)d\tau\right)\rho[a(0) + W_A(0)]. \end{aligned}$$

Since $\int_0^T \dot{\lambda}(\tau)d\tau = 1 - \lambda_0$, this equation is rewritten as follows:

$$E_A(t) - E_B(t) = (2\lambda_0 - 1)\rho[a(0) + W_A(0)] \geq 0.$$

This implies that, when $\lambda_0 \geq 1/2$,

$$E_A(t) \geq E_B(t) \tag{A.24}$$

In the same approach, since $E_A(t)/E_B(t) \geq 1$, the difference in the price of final good produced in each region is as follows:

$$\frac{p_{YA}}{p_{YB}} = min\left\{\left(\frac{E_A(t)}{E_B(t)}\right)^{\frac{\beta}{1-\sigma}}, \left(\frac{1}{\tau_I}\right)^{\beta}\right\} \leq 1 \quad for\ t \in (0, T] \tag{A.25}$$

Here, we define as follows: for $t \in [0, T]$

$$\begin{aligned} \Delta V(t) &\equiv e^{\rho t}[V(0;t) - V(0;T)] \\ &= \frac{e^{\rho t}}{\rho} \ln\left[\frac{a(0) + W(0;t)}{a(0) + W(0;T)}\right] + \frac{\alpha\beta\Psi e^{\rho t}}{\sigma - 1}\int_t^T e^{-\rho\tau} \ln\left[min\left\{\frac{E_A(t)}{E_B(t)}, \left(\frac{1}{\tau_I}\right)^{\beta}\right\}\right] d\tau. \end{aligned} \tag{A.26}$$

Using this definition, we find that, given each $\lambda_0 \in (1/2, 1)$,

$$\begin{gathered} \Delta V(t) > 0 \qquad for\ t \in [0, T) \\ \Delta V(T) = 0 \end{gathered} \tag{A.27}$$

Hence, from (A.6) and (A.30), it is shown that

$$\dot{\lambda}(t) = \Psi\Delta V(t) > 0 \tag{A.28}$$

By assumption $\lambda_0 \in (1/2, 1]$, and even though $\dot{\lambda}(t)$ is positive when t is close to 0:

$$\dot{\lambda}(0) \equiv \lim_{t\to 0} \dot{\lambda}(t) > 0. \tag{A.29}$$

Therefore, when $\lambda_0 = 1/2$, $\dot{\lambda} > 0$. This implies that expectation plays an important role in the economy.

Next, we show that, starting from $\lambda_0 \in (1/2, 1]$, the economy reaches the point $\lambda = 1$ for a finite time.

From (A.9) and (A.10), we have the following representation:

$$\frac{\alpha\beta E}{\sigma}\frac{1}{\rho + 1} \leq a(t) \leq \frac{\alpha\beta E}{\sigma}\frac{1}{\rho + g(\frac{1}{2})} \tag{A.30}$$

and

$$a(T) = \frac{\mu E}{\sigma}\frac{1}{\rho + 1}. \tag{A.31}$$

In addition, by using (A.13), we rewrite the following form:

$$
\begin{aligned}
W_A(0) &= \int_0^\infty e^{-\rho\tau} a(\tau) \frac{k_A[\lambda(\tau)]}{b} d\tau \geq \frac{\alpha\beta E}{\sigma} \frac{1}{\rho+1} \int_0^\infty e^{-\rho\tau} \frac{k_A[\lambda(\tau)]}{b} d\tau \\
&\geq \frac{\alpha\beta E}{\sigma} \frac{1}{\rho+1} \frac{\frac{1}{2}(1+\eta)}{\rho b}. \qquad \text{(A.32)}
\end{aligned}
$$

As shown before, $E_A(t) \geq E_B(t)$, which implies that $E_B(t) \leq E/2$ for $t \leq T$. Using (7), the ratio between the expenditure of region A and that of region B is

$$
\frac{E_A(t)}{E_B(t)} > 1 + \frac{2(\lambda_0 - 1)\rho[a(0) + W_A(0)]}{E_B(t)} \geq 1 + 2(2\lambda_0 - 1) \frac{\alpha\beta E}{\sigma} \frac{\rho + \frac{\frac{1}{2}(1+\eta)}{b}}{\rho + 1}. \qquad \text{(A.33)}
$$

By the definition of ΔV, (A.30) is represented by the following equation:

$$
\begin{aligned}
\Delta V &= \frac{e^{\rho t}}{\rho} \ln \left[\frac{a(0) + W(0;t)}{a(0) + W(0;T)} \right] \\
&+ \frac{\alpha\beta\Psi e^{\rho t}}{\sigma - 1} \int_t^T e^{-\rho\tau} \ln \left[min \left\{ 1 + 2(2\lambda_0 - 1) \frac{\alpha\beta E}{\sigma} \frac{\rho + \frac{\frac{1}{2}(1+\eta)}{b}}{\rho + 1}, \frac{1}{\tau_I^{1-\sigma}} \right\} \right] d\tau \\
&> \frac{\alpha\beta\Psi e^{\rho t}}{\sigma - 1} \int_t^T e^{-\rho\tau} \ln \left[min \left\{ 1 + 2(2\lambda_0 - 1) \frac{\alpha\beta E}{\sigma} \frac{\rho + \frac{\frac{1}{2}(1+\eta)}{b}}{\rho + 1}, \frac{1}{\tau_I^{1-\sigma}} \right\} \right] d\tau \\
&= \frac{1 - e^{-\rho(T-t)}}{\rho} J(\lambda(0)), \qquad \text{(A.34)}
\end{aligned}
$$

where

$$
J(\lambda_0) = \frac{\alpha\beta\Psi}{\sigma - 1} \ln \left[min \left\{ 1 + 2(2\lambda_0 - 1) \frac{\alpha\beta E}{\sigma} \frac{\rho + \frac{\frac{1}{2}(1+\eta)}{b}}{\rho + 1}, \frac{1}{\tau_I^{1-\sigma}} \right\} \right]. \qquad \text{(A.35)}
$$

It is clear that $J(\lambda_0) > 0$ when $\lambda(0) > 1/2$. In addition, it is shown that $dJ(\lambda(0))/d\lambda_0 \geq 0$. Rewriting (A.30) by using (A.38), it is shown that

$$
\dot{\lambda}(t) = \Psi \Delta V(t) > \frac{\Psi J(\lambda_0)}{\rho} [1 - e^{-\rho(T-t)}]. \qquad \text{(A.36)}
$$

Integrating both sides in (A.36) from t to T expose that

$$
\frac{\rho^2}{\Phi} \frac{1 - \lambda_0}{J(\lambda_0)} > \rho T - (1 - e^{-\rho T}). \qquad \text{(A.37)}
$$

Here, we denote the solution of the following equation as $T_{sup}(\lambda_0)$:

$$\frac{\rho^2}{\Phi}\frac{1-\lambda_0}{J(\lambda_0)} = \rho T - (1 - e^{-\rho T}). \tag{A.38}$$

It is illustrated that, for each $\lambda_0 \in (1/2, 1)$, there is a single solution, $T_{sup}(\lambda_0)$, which is positive, continuous, and decreasing on $(1/2, 1)$, while

$$\lim_{\lambda_0 \to 1} T_{sup}(\lambda_0) = 0. \tag{A.39}$$

Therefore, it is said that the value of T associated with the equilibrium path that starts from λ_0 is less than $T_{sup}(\lambda_0)$. Thus, we conclude as follows.

Lemma A.1 *Suppose $\tilde{\lambda} = 1$, and assume that the mc hypothesis holds. There is a function $T_{sup}(\lambda_0)$ defined on $(1/2, 1)$. This is positive, continuous, and decreasing. In addition, this function is such that the equilibrium path that starts from $\lambda_0 \in (1/2, 1)$ at time 0 reaches $\tilde{\lambda} = 1$ before $T_{sup}(\lambda_0)$, where $\lim_{\lambda_0 \to 1} T_{sup}(\lambda_0) = 0$.*

Due to the fact that $\lambda(1/2)$, the function $T_{sup}(\lambda_0)$ has the following property.

$$\lim_{\lambda_0 \to 1/2} T_{sup}(\lambda_0) = \infty \tag{A.40}$$

However, the time that λ reaches $\tilde{\lambda} = 1$ is finite. Since, when $\lambda_0 = 1/2$, $\Delta V(t) > 0$ and $\Delta V(t)$ is continuous on $(0, T)$, it is shown that $\dot{\lambda}(t) > 0$, which implies that $\lambda(t) > 1/2$ along the equilibrium path starting from $\lambda_0 = 1/2$.

Now, we show the existence of a neighborhood, Λ, in such a way that there is an equilibrium path that leads to $\tilde{\lambda} = 1$ for any $\lambda_0 \in \Lambda$. We define this as follows:

$$\epsilon(t) \equiv \rho[a(0) + W(0; t)]. \tag{A.41}$$

If $(\lambda(t), \Delta V(t), a(t), \epsilon(t), E_A(t))_{t=0}^{T}$ is the equilibrium path that reaches $\tilde{\lambda} = 1$ at time T and starts from the initial distribution, λ_0, at time 0, then the dynamics is as follows: for $t \in (0, T)$,

$$\dot{\lambda} = \Phi \Delta V,$$

$$\dot{\Delta V} = \rho \Delta V - \frac{a(t)}{\epsilon(t)}[k_A(\lambda) - k_B(\lambda)] - \frac{\alpha\beta\rho\Psi}{\sigma - 1}\ln\left[min\left\{\left(\frac{E_A}{E_B}\right), \left(\frac{1}{\tau_I^{1-\sigma}}\right)\right\}\right].$$

$$\dot{a} = [\rho + g(\lambda)]a - \frac{\alpha\beta E}{\sigma},$$

$$\dot{\epsilon} = -\rho e^{-\rho t} a \frac{k_A(t) - k_B(t)}{b},$$

$$\dot{E}_A = \dot{\lambda}\rho[a(0) + W(0;t)] = \Phi\Delta V \epsilon.$$

The terminal conditions are as follows:

$$\lambda(0) = \lambda_0, \quad \lambda(T) = 1, \tag{A.42}$$

$$V(T) = 0,$$

$$a(T) = \frac{\alpha\beta E}{\sigma}\frac{1}{\rho+1},$$

$$E_A(T) = E - \frac{1}{2},$$

$$\epsilon(T) = \rho\left[a(0) + \int_0^T e^{-\rho\tau} a(\tau)\frac{k_B(\tau)}{b} d\tau + \frac{\alpha\beta E}{\sigma\rho}\frac{e^{-\rho T}}{\rho+1}\right]. \tag{A.43}$$

The set of terminal conditions is unusual in two points. First, λ is specified at both end points. Second, $\epsilon(T)$ is a complex condition involving integrals. Hence, it is not straightforward to show the existence of an equilibrium path that starts from each $\lambda_0 \in \Lambda$, whose Λ is neighborhood of $\tilde{\lambda} = 1$. Therefore, we take a different way to reach the desired result. This is that given terminal conditions are specified at time T, and we move backward from $t = T$ to $t = 0$ by introducing new time variables:

$$\tau \equiv T - t$$

In addition, instead of specifying λ_0, we specify T and obtain the associated λ_0. Using the new variable, we rewrite the dynamics as follows: for $\tau \in (0, T)$

$$\dot{\lambda} = -\Phi\Delta V, \tag{A.44}$$

$$\dot{\Delta V} = -\rho\Delta V + \frac{a}{\epsilon}[k_A(\lambda) - k_B(\lambda)] + \frac{\alpha\beta\rho\Psi}{\sigma - 1}\ln\left[min\left\{\left(\frac{E_A}{E_B}\right), \left(\frac{1}{\tau_I^{1-\sigma}}\right)\right\}\right],$$

$$\dot{a} = -(\rho + g(\lambda))a + \frac{\alpha\beta E}{\sigma},$$

$$\dot{\epsilon} = \rho e^{-\rho(T-t)} a \frac{k_A(\lambda) - k_B(\lambda)}{b},$$

$$\dot{E}_A = -\Phi\Delta V \epsilon,$$

where

$$\lambda(0) = 1,$$

$$\Delta V(0) = 0,$$
$$a(0) = \frac{\alpha\beta E}{\sigma}\frac{1}{\rho+1},$$
$$E_A(0) = E - \frac{L}{2},$$
$$\epsilon(0) = \rho\left[a(T) + \int_0^T e^{-\rho\tau}a(\tau)k_B[\lambda(\tau)]d\tau + \frac{\alpha\beta E}{\sigma\rho}\frac{e^{-\rho T}}{\rho+1}\right]. \tag{A.45}$$

Since $\epsilon(T)$ is complex, we denote it as $\epsilon(0) = \epsilon_0$, where ϵ is a parameter that can be chosen appropriately. It can be shown that, for each sufficiently small T, there is a closed interval, $I_\epsilon(T)$, in the positive part of a real number, such that, for each $\epsilon_0 \in I_\epsilon(T)$, the above system and ϵ_0 have a unique solution, denoted as

$$\lambda[(s;T,\epsilon_0), \Delta(s;T,\epsilon_0), a(s;T,\epsilon_0), E_A(s;T,\epsilon_0)]_{s=0}^T.$$

Let us denote $\epsilon(0;T,\epsilon_0)$ as the value associated with RHS in $\epsilon(0)$. It is shown that

$$\epsilon(0;T,\epsilon_0) \equiv \rho\left[a(T;T,\epsilon_0) + \int_0^T e^{-\rho\tau}a(\tau;T,\epsilon_0)k_B[\lambda(\tau;T,\epsilon_0)]d\tau + \frac{\alpha\beta E}{\sigma\rho}\frac{e^{-\rho T}}{\rho+1}\right].$$

It is shown that the equation, $\epsilon(0;T,\epsilon_0) = \epsilon_0$, has a unique solution, denoted as $\epsilon_0(T)$. This derives the associated value of λ at $\tau = T$, denoted as $\lambda_0(T) \equiv \lambda(T;T,\epsilon_0)$. Finally, by showing that $\lambda_0(T)$ is a continuous function on the interval $(0,\hat{T}]$ and has the following property,

$$\lim_{T\to 0}\lambda_0(T) = 0,$$

we obtain the desired neighborhood of $\tilde{\lambda} = 1$, $[\lambda_0(\hat{T}), 1)$. This is sufficient to establish the stability of the steady-state growth path under $\tilde{\lambda} = 1$. We may conclude the following:

Proposition A.2 *The steady-state growth path under $\tilde{\lambda} = 1$ is stable.*

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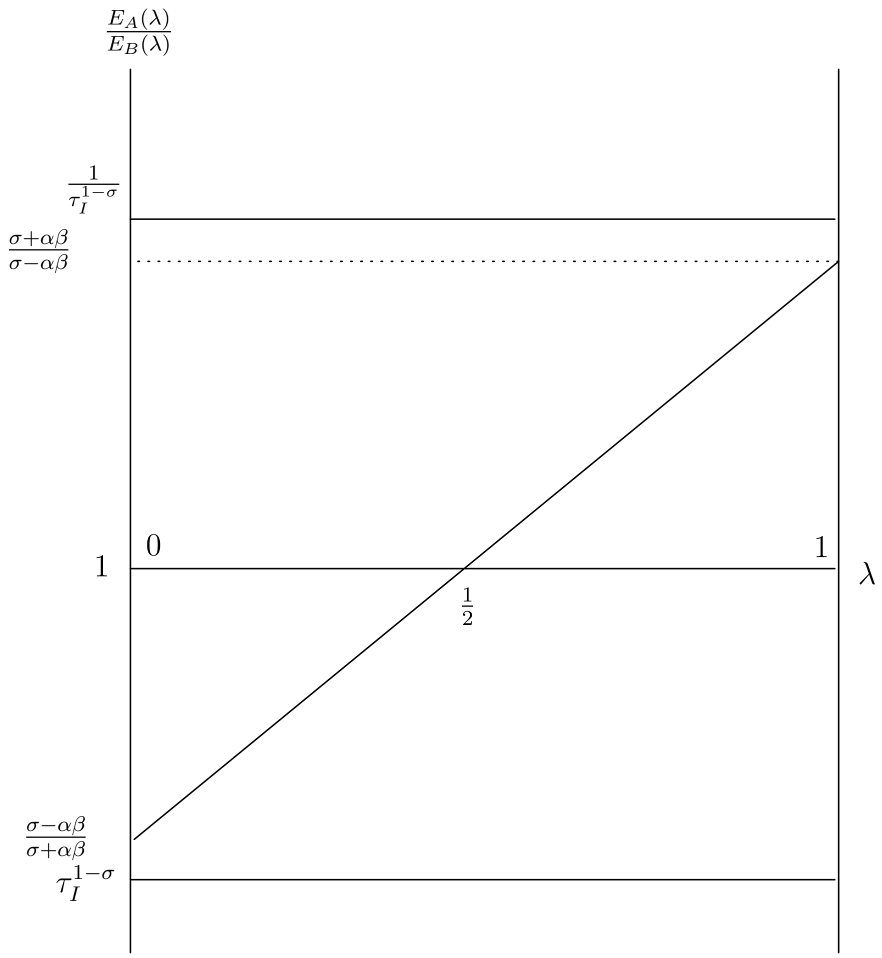

Figure 1: The case of $\tau_I < \frac{\sigma-\alpha\beta}{\sigma+\alpha\beta}$ when λ is fixed

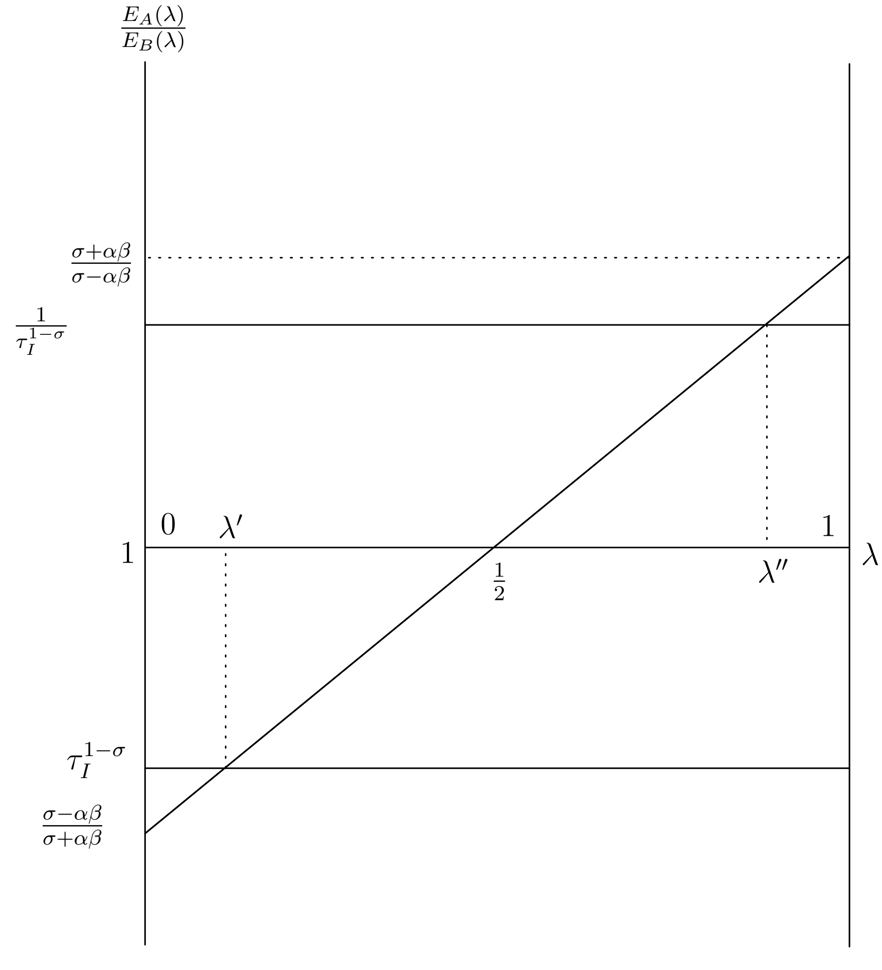

Figure 2: The case of $\tau_I \geq \frac{\sigma-\alpha\beta}{\sigma+\alpha\beta}$ when λ is fixed

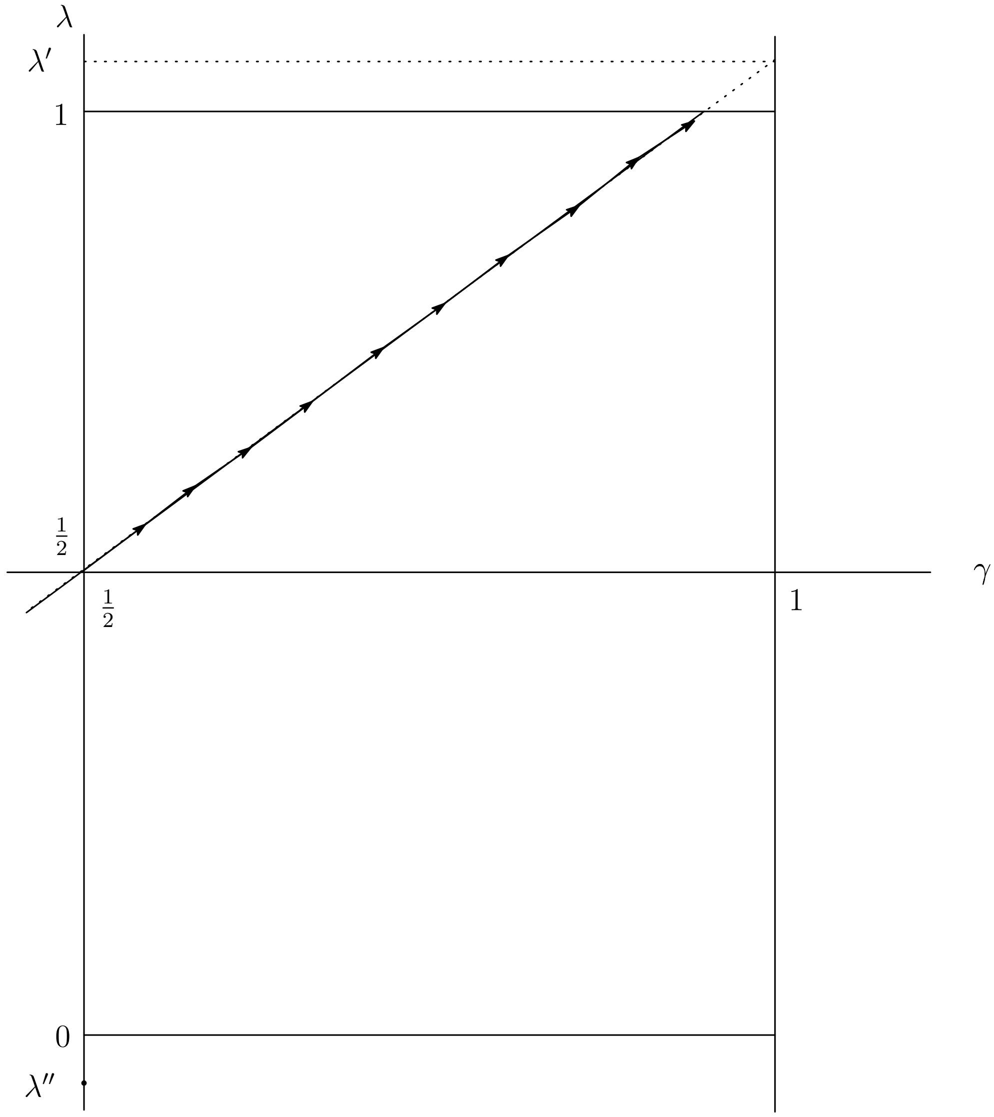

Figure 3: The case of $\tau_I^{1-\sigma} < \frac{\sigma-\alpha\beta}{\sigma+\alpha\beta}$ when the trade of final good does not occur

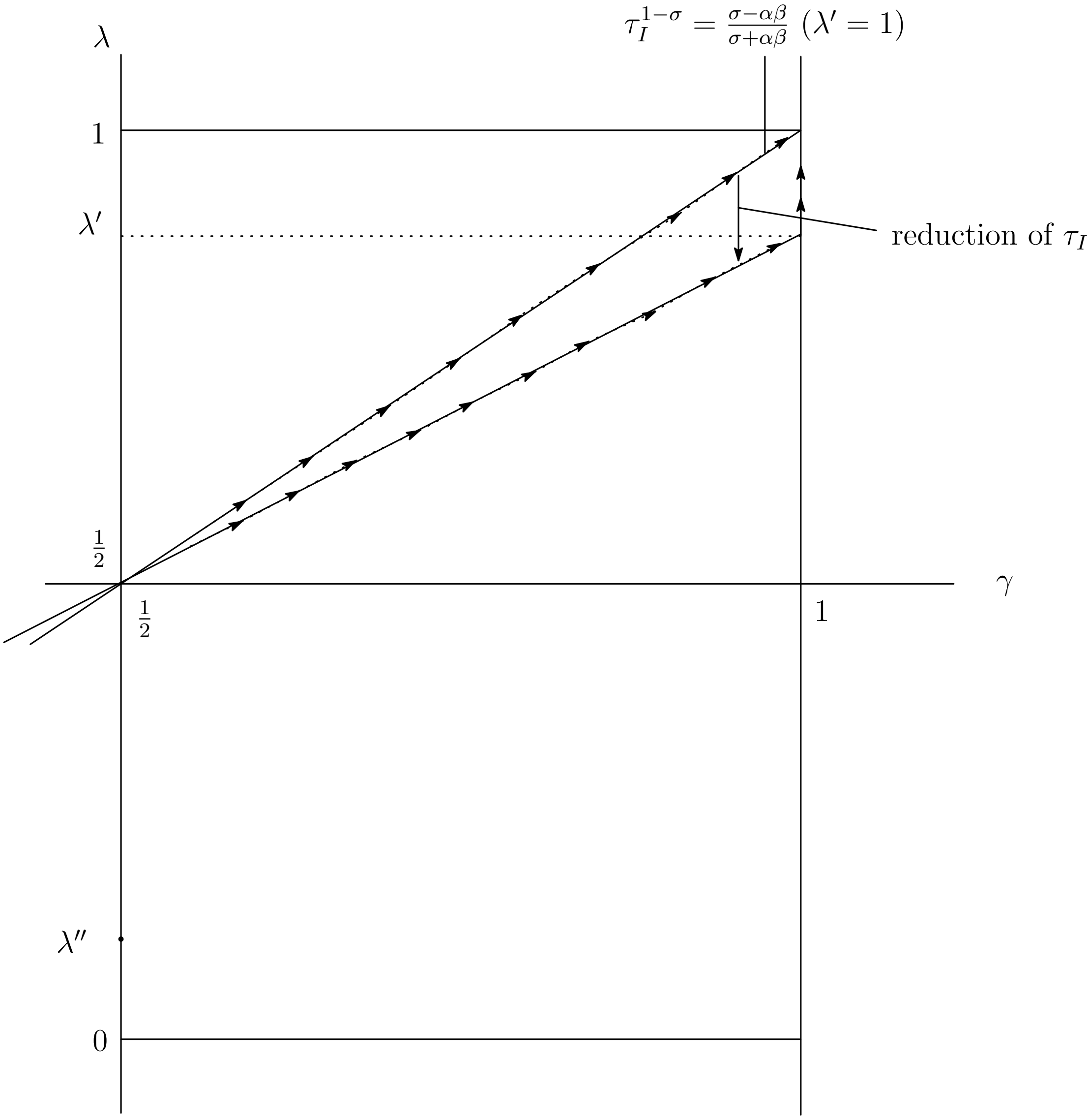

Figure 4: The case of $\tau_I^{1-\sigma} \geq \frac{\sigma-\alpha\beta}{\sigma+\alpha\beta}$ when the trade of final good does not occur

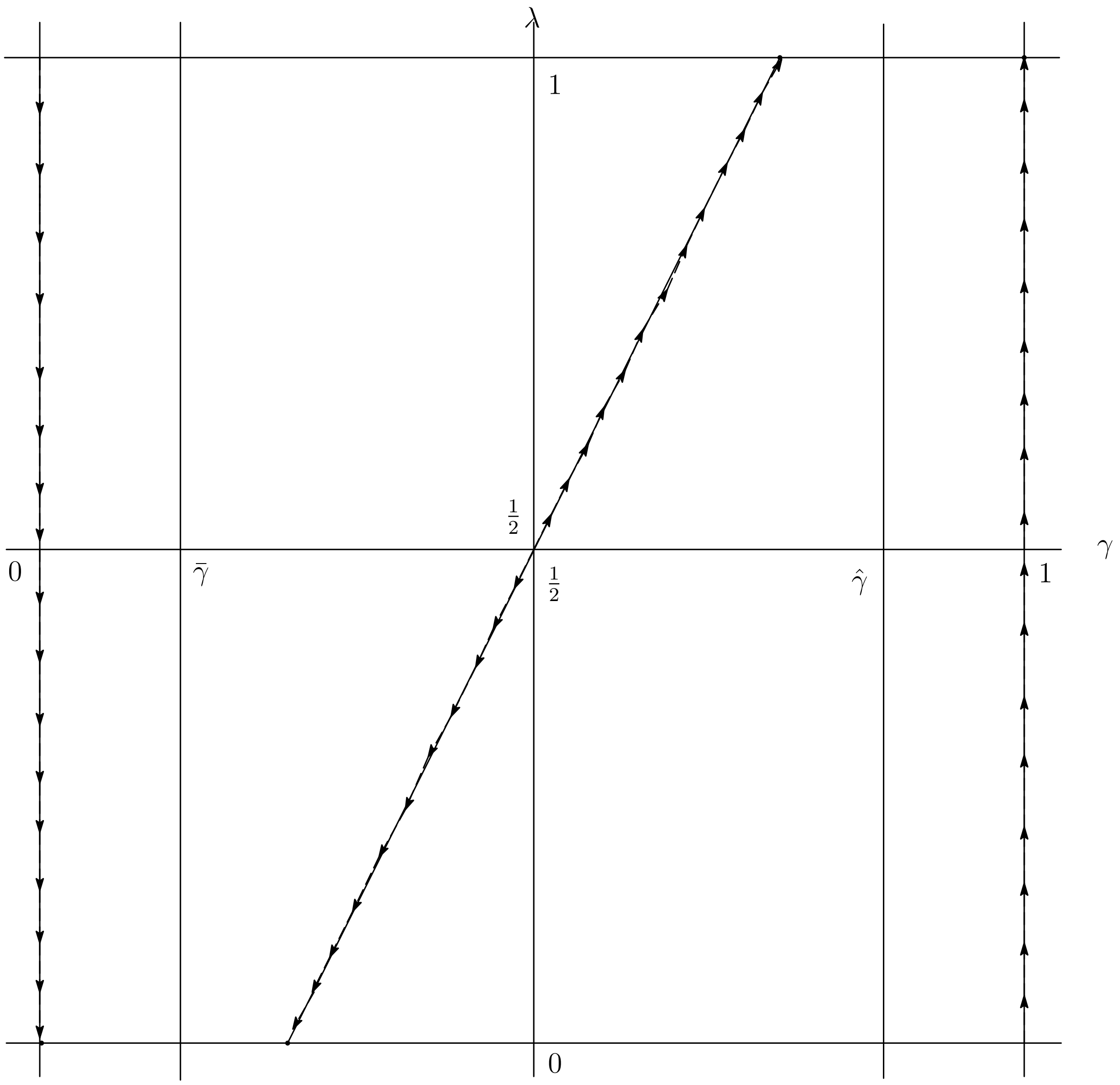

Figure 5: The case of $\bar{\tau}_M < \hat{\tau}_M < \tau_M$ or $\hat{\tau}_M < \bar{\tau}_M < \tau_M$ when both trade pattern occur.

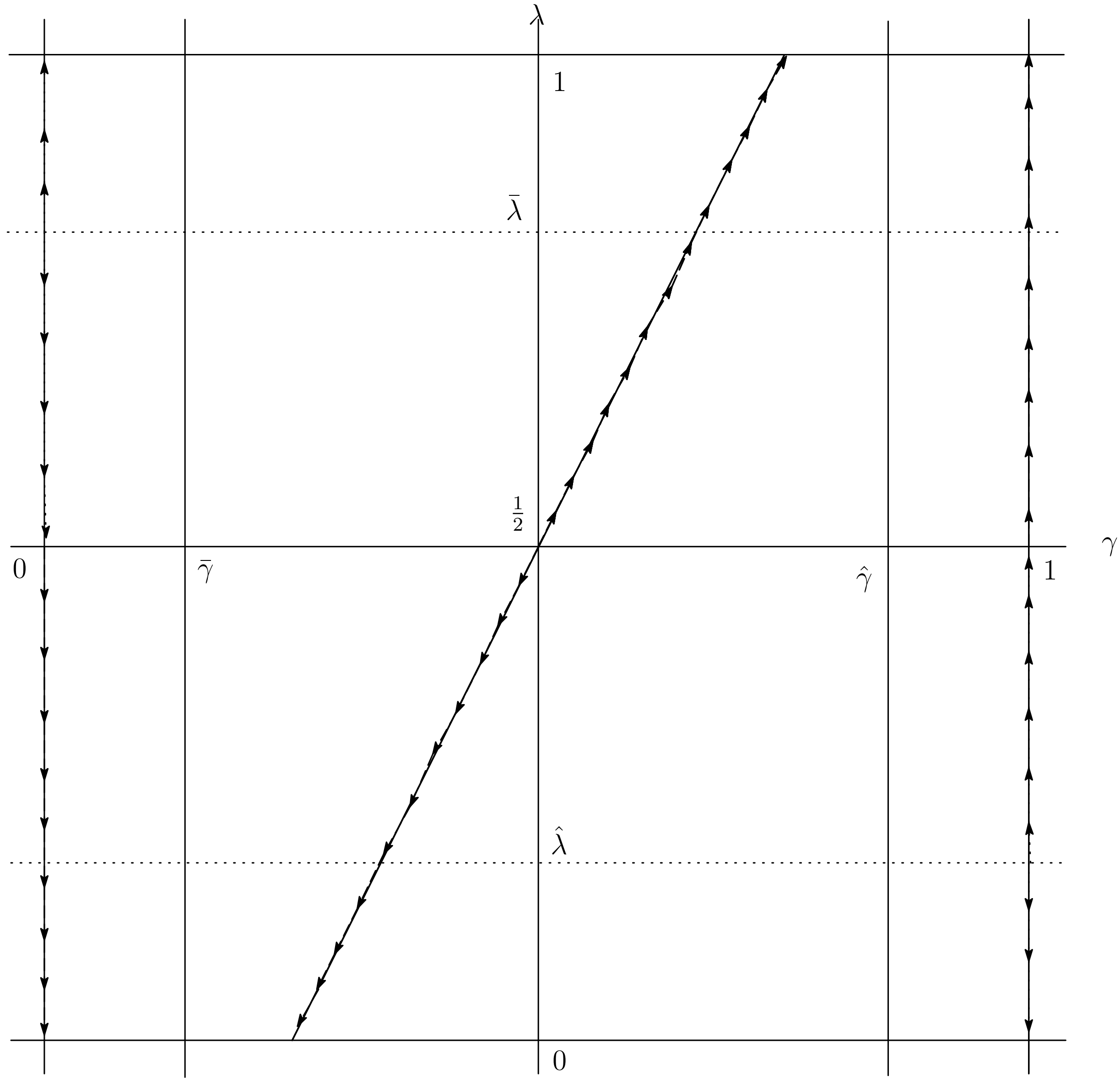

Figure 6: The case of $\tau_M < \bar{\tau}_M < \hat{\tau}_M$ when both trade pattern occur.

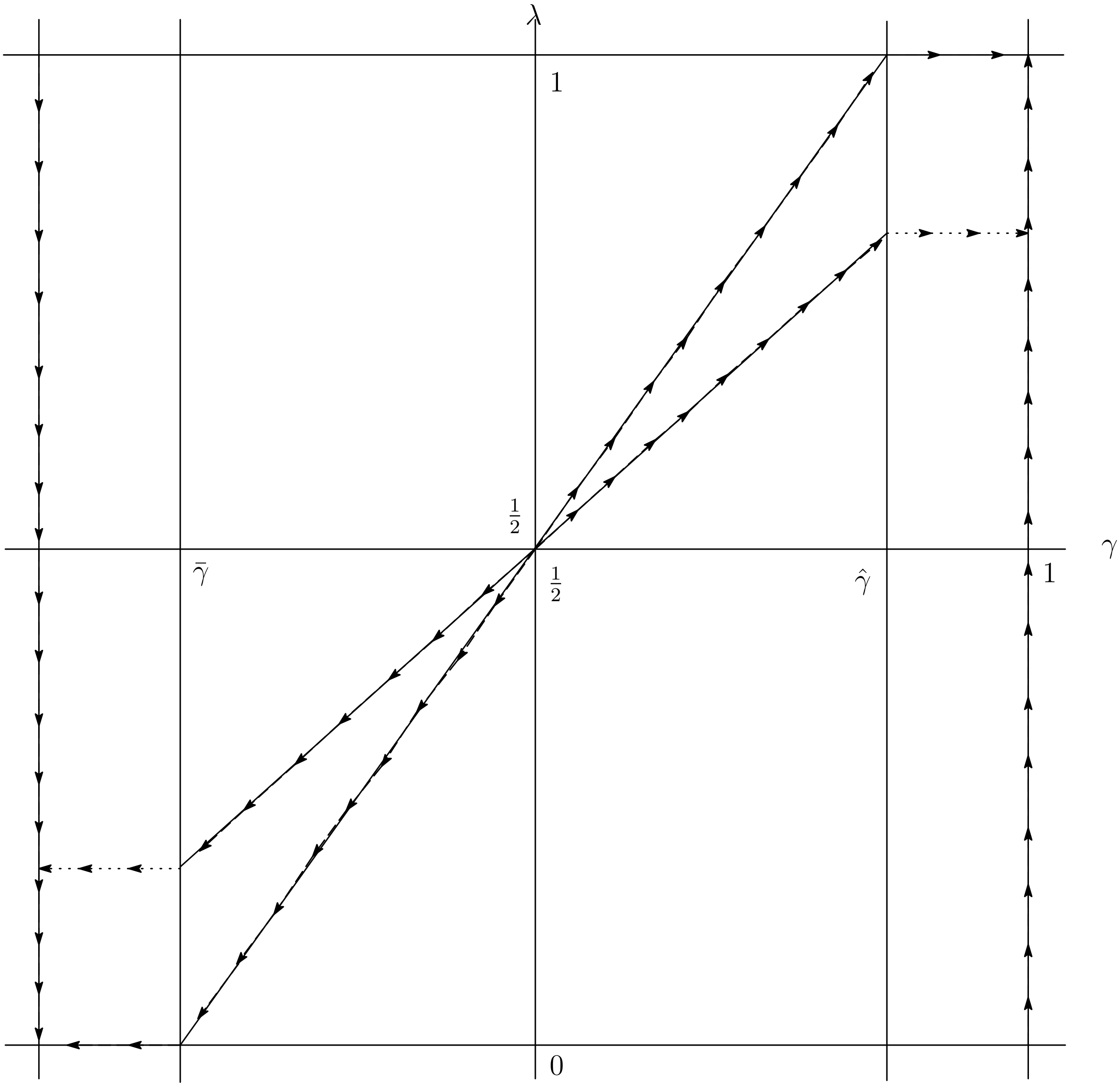

Figure 7: The case of $\hat{\tau}_M < \tau_M < \bar{\tau}_M$ when both trade patterns occur.

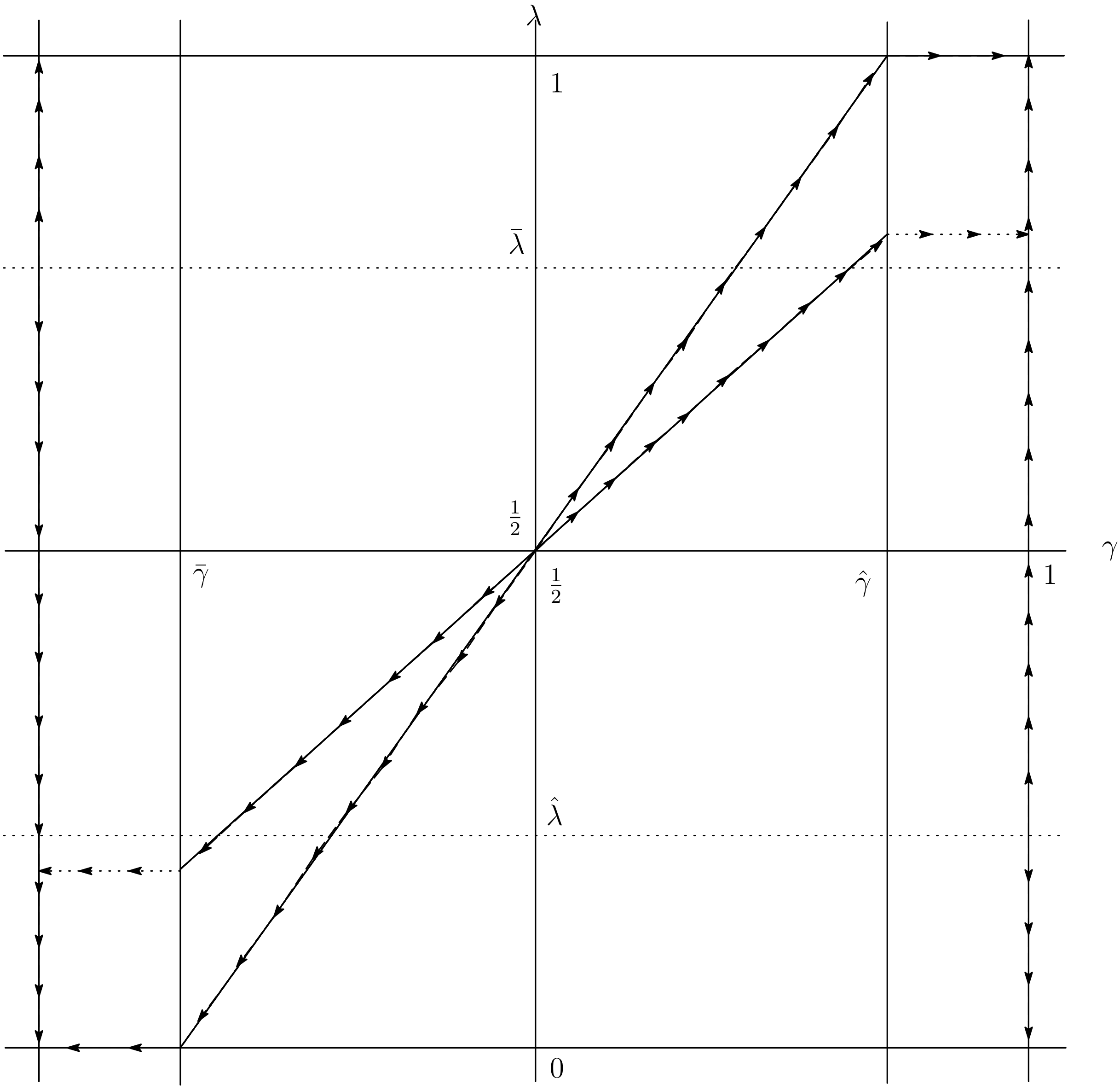

Figure 8: The case of $\tau_M < \hat{\tau}_M < \bar{\tau}_M$ or $\tau_M < \bar{\tau}_M < \hat{\tau}_M$ when both trade patterns occur.