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Ryo Horii† and Tatsuro Iwaisako‡

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Abstract

This paper examines the growth effects of intellectual property right (IPR) protection in a quality-ladder model of endogenous growth. Stronger IPR protection, which reduces the imitation probability, increases the reward for innovation. However, stronger protection also gradually reduces the number of competitive sectors, in which innovation is easier than in monopolistic sectors. With free entry to R&D, the number of researchers in each remaining competitive sector increases, but the concentration of R&D activity raises the possibility of unnecessary duplication of innovation, thereby hindering growth. Consequently, imperfect rather than perfect protection maximizes growth. Welfare and scale effects are also examined.
Keywords: intellectual property rights, endogenous growth, quality ladder, imitation, leapfrogging, duplication.
JEL Classification: O31, O34, O41
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1 Introduction

In the last two decades, a number of countries have reformed their patent systems in order to strengthen the protection of intellectual property rights (hereafter, IPRs). Such reforms are often justified by the view that stronger IPR protection should enhance economic growth by increasing the returns to innovation and, hence, incentives to innovate. However, the relationship between IPR protection and growth is not as clear as widely believed.[1] Figure 1 illustrates a scatter plot of the average growth rate and the level of IPR protection, using Rapp and Rozek's (1990) cross-country data on the level of patent protection. As shown, although there is substantial variability across countries in both the level of IPR protection and the rate of economic growth, it is difficult to determine a clear relationship between these two variables.[2]

Given that stronger IPRs unambiguously provide greater incentives to innovate, the weak relationship observed between IPR protection and the rate of economic growth suggests that IPRs may have a negative effect on economic growth. A well-known drawback of stronger IPRs is that they provide innovators, on average, with longer monopoly periods and therefore tend to increase the number of monopolistic sectors within the economy as a whole. In light of this tendency, earlier studies have demonstrated that consumer welfare is not necessarily improved when IPRs are strengthened, because monopolists limit their output below the socially desirable level. In this paper, we present a mechanism through which stronger IPRs negatively

[1] Applications for patents in the United States have increased drastically since 1985; however, there is little evidence that this increase was caused by the strengthening of IPR protection. Changes in the management of R&D and the shift to more applied activities appear to have spurred patenting (Kortum and Lerner, 1998).

[2] Gould and Gruben (1996) ran regressions using Rapp and Rozek ' s (1990) index and the average growth rate between 1960 and 1988. They found a positive but weak relationship between IPR protection and economic growth (see Table 2 of their paper). In addition, they pointed out that, on average, countries with level "three" IPR protection grew more slowly than those with weaker (level "two") IPR protection.

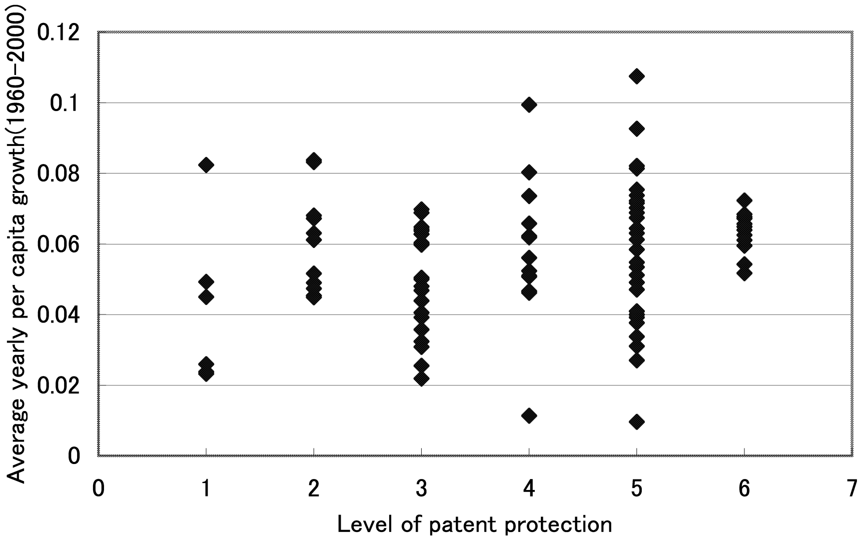

Figure 1: Average per capita growth rate 1960–2000 and the level of patent protection. The horizontal axis ranks the level of patent protection from one to six, where one corresponds to nations with no patent protection law and higher values indicate countries with patent laws that are more consistent with the minimum standards proposed by the United States Chamber of Commerce (1987). The average growth rate is from Heston *et al.* (2002).

affect the long-term rate of economic growth, by focusing on the following two R&D properties.

The first property is the difference in the environment for R&D provided by monopolistic and competitive sectors. In a monopolistic sector, where an incumbent holds the exclusive right to produce a state-of-the-art good, the incumbent has little incentive to improve the product further, because it can secure monopoly profits without such efforts (the Arrow effect, 1962). Thus, in monopolistic sectors, innovative efforts are made only by outside firms, which can succeed only when they create a good of higher quality than the incumbent's. Such *leapfrogging* innovations are more difficult to achieve than innovations in competitive sectors because the outside firms have no experience in producing state-of-the-art-quality goods. In addition, production of a new superior good often involves a similar process to the production of current state-of-the-art goods, and therefore, the incumbent's IPRs may result in the production of the new superior goods being banned. Thus, *ceteris paribus*, it is more difficult for outside firms to invent new high-quality goods in monopolistic sectors than in competitive sectors.[3]

Another important property of R&D projects is that they take time, and their outcomes (successes or failures) are revealed only after the projects are completed. This means that individual innovators must initiate R&D projects without knowing whether other innovators' projects will eventually succeed. Thus, there is a non-negligible probability that more than two innovators may independently succeed in developing a similar intermediate good of the same quality. Given the nonrivalry of the knowledge obtained by innovation, this *duplication* of innovations not only is futile from the viewpoint of economic growth but also reduces the profits and incentives of innovators.[4]

Incorporating these two properties of R&D explicitly into the quality-ladder model of endogenous growth, this paper shows that IPR protection inhibits growth when it is too strict. At the same time, a moderate degree of IPR protection is required to provide the incentives to innovate. As a result, it is shown that the long-term growth rate is maximized by the imperfect, rather than the perfect, protection of IPRs.

The remainder of the paper is organized as follows. After intuitively explaining in the next section the reason why stronger IPR may inhibit growth, Section 3 sets up a formal model. Section 4 explains how tightening patent protection affects the growth rate, assuming that leapfrogging is prohibitively difficult. Section 5 derives

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[3]Empirically, this statement is confirmed by Blundell *et al.* (1995) and Nickell(1996), who found that innovation is less active in more concentrated sectors. See discussion in Section 5.

[4] Earlier studies stressed the importance of duplication in R&D. For example, Kortum (1993) found evidence of significant diminishing returns in R&D attributable to duplication. Lambson and Phillips (2005) found that the probability of duplication is not low for most industries. The possibility of duplication also plays an important role in R&D-based growth models by Jones (1995) and Jones and Williams (2000), although they use a reduced form setting, whereas we explicitly consider the time required to innovate.

the balanced growth path (BGP) in a general setting and presents the main results of the paper. Section 6 compares our results with those obtained by earlier studies. Section 7 discusses the scale effects and the impact on welfare. Section 8 concludes the paper. The proofs of the lemmas and propositions are provided in the appendix.

2 Overview of the Mechanism

Before presenting a formal model, this section provides a brief intuition for the reasoning why the two properties discussed in the introduction jointly imply that stronger IPRs may inhibit growth.

Consider a quality-ladder model of endogenous growth and suppose that innovations are imitated over time. If IPR protection is strengthened, the probability of imitation declines. Thus, in the long run, stronger IPR protection increases the number of monopolistic sectors in which the state-of-the-art-quality goods have not yet been imitated. Because the environment for R&D is assumed to be better in competitive sectors than in monopolistic sectors, the number of R&D workers per sector (research intensity) is higher in competitive sectors. Therefore, as stronger IPR protection reduces the number of competitive sectors, the aggregate R&D activity declines if the research intensity in each type of sector is fixed (see the horizontal arrow in Figure 2).

While this provides a central mechanism in this analysis, this finding is not conclusive since research intensity actually changes. Specifically, since increased monopolization lowers aggregate labor demand, the equilibrium market wage falls.[5] This means a lower cost of R&D, which induces more research firms to operate in every sector employing new R&D workers, as illustrated by the vertical arrows in Figure 2. In a standard setting with an inelastic labor supply and constant returns to R&D activities, entry would continue until it totally offset the initial decline in aggre-

[5]This is an example of the general-equilibrium effects of IPR policy discussed in O'Donoghue and Zweimüller (2004, Section 5).

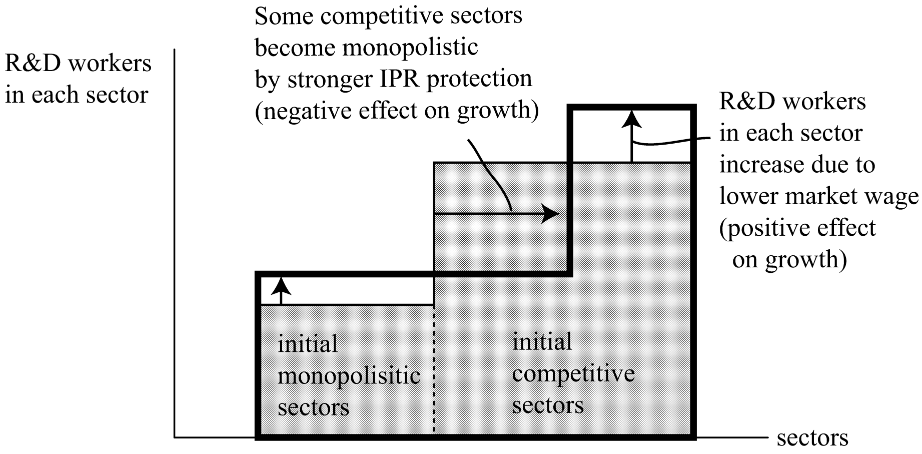

Figure 2: Reallocation of R&D workers when IPR protection is strengthened.

gate R&D activities caused by the increased monopolization. However, when the time required for innovation is considered explicitly, the possibility of duplication of innovations becomes more prominent as more researchers enter each field. Since this reduces the expected return on R&D, entry stops before the negative effect of increased monopolization on growth is canceled totally.

Accordingly, the net effect of monopoly on growth tends to be negative. Of course, stronger IPR protection not only intensifies monopolization but also spurs innovation directly by guaranteeing more rewards for successful innovators. The aim of this paper is to examine the relative significance of the growth-enhancing and -reducing effects of IPRs by explicitly incorporating the two key components into an otherwise standard quality-ladder model of endogenous growth.

3 Model

This section sets up a discrete-time version of a quality-ladder model as based on Grossman and Helpman (1991). After describing, in the first subsection, households and production sectors, in the second subsection, we explain how the environments for R&D differ between competitive and monopolistic sectors. The final subsection considers the evolution of the value of innovation and the number of monopolistic sectors in the economy, allowing for the possibility of imitation.

3.1 Households and Production Technologies

We consider a closed economy consisting of homogeneous and infinitely lived households of size L. Each household is endowed with a unit of labor in each period. The nominal market value of labor (i.e., the wage) is denoted by w_t, which is to be determined in equilibrium. The utility function of the representative household is given by:

$$U = \sum_{t=0}^{\infty} \beta^t \ln c_t, \tag{1}$$

where $\beta \in (0, 1)$ is a constant subjective discount factor and c_t is consumption of the final good in period t. This is maximized subject to the intertemporal budget constraint $a_{t+1} - a_t = r_t a_t + w_t - P_t c_t$, where a_t denotes the per capita nominal financial asset,[6] r_t is the interest rate, and P_t is the price of the final good, which is normalized such that the aggregate consumption expenditure becomes unity in each period: $P_t c_t L = 1$ for all t. The Euler equation under this normalization shows that the interest rate must be constant at $r_t = \beta^{-1} - 1 \equiv r$.

The final good is produced competitively using a continuum of different types of intermediate goods, indexed by $i \in [0, 1]$. As a result of past product innovations, each type of intermediate good potentially has several quality grades, indexed by integers $j \geq 0$, which are perfectly substitutable as inputs to the production of the final good and have different marginal productivities. For each i and j, let $\tilde{x}_{it}(j)$ denote the units of the type i intermediate good of quality j, which are used in the final-good production. Then, the output Y_t is determined by the following production function:

$$Y_t = \exp\left\{ \int_0^1 \ln \left[\sum_{j=0}^{q_{it}} \lambda^j \tilde{x}_{it}(j) \right] di \right\}, \tag{2}$$

[6]The initial value of a_0 is determined by the initial value of the IPRs: $a_0 = \mu_0 V_0 / L$, where the definitions of μ_0 and V_0 are given later. In fact, we do not need to keep track of the consumer's budget constraint owing to Walras' law.

where $\lambda > 1$ represents the size of the quality improvement obtained from one innovation and $q_{it} \geq 0$ is the highest (state-of-the-art) quality of the type i intermediate good (i.e., it represents the cumulative innovations that have occurred in sector i by date t). As the intermediate goods of the same type are perfect substitutes, final good producers use the single quality that has the lowest quality-adjusted price, $\tilde{p}_{it}(j)/\lambda^j$, for every type of intermediate good.

Each type of intermediate good is produced in a separate intermediate good sector. In every sector, production of one unit of the intermediate good of any quality requires one unit of labor: this means that the marginal cost of production equals the nominal wage, w_t. As the costs are the same for all qualities, only the state-of-the-art technology is used in equilibrium. Let $x_{it} \equiv \tilde{x}_{it}(q_{it})$ and $p_{it} \equiv \tilde{p}_{it}(q_{it})$ denote the amount and price of the state-of-the-art good in sector i. Then the profit maximization of the final goods sector, under the normalization of P_t as described, implies that the demand function for each type of intermediate good is $x_{it} = 1/p_{it}$.

The equilibrium prices and quantities in the intermediate good sectors depend on whether they are monopolistic or competitive. We call an intermediate good competitive when the technology to produce the highest quality intermediate good is publicly available. In a competitive sector, firms compete with each other, pushing the price down to the marginal cost, $p_{it} = w_t$. Given the demand function $x_{it} = 1/p_{it}$, the output of a competitive sector is $x_{it} = 1/w_t$. An intermediate good sector is monopolistic when there is a firm that holds an IPR giving it the exclusive right to produce the highest quality intermediate goods. For simplicity, we assume that only the highest quality technology is protected by IPR, and the technology to produce the next-best-quality good is always publicly available. In this case, the monopoly firm maximizes profits by employing the limit-pricing strategy, $p_{it} = \lambda w_t$, which cannot be undercut by the next-best-quality good in terms of the quality-adjusted price. Since the demand function is $x_{it} = 1/p_{it}$, the monopoly firm sells amount $x_{it} = 1/(\lambda w_t)$, obtaining the monopoly profit $\pi = (\lambda - 1)/\lambda$.

3.2 R&D and the Labor Market Equilibrium

In this model, economic growth is driven by innovations, which raise the quality of intermediate goods. Innovation may occur in competitive and monopolistic sectors as a result of R&D activities by workers. However, the environments for such activities differ, depending upon whether R&D is undertaken in a competitive or a monopolistic sector.

Let us start by describing the environment for R&D in the competitive sectors. When one worker conducts R&D activity for one period in a particular competitive sector i, he or she has a small probability, $\widehat{a} > 0$, of successfully creating an innovated good. Any R&D activity is assumed to take one period to complete. Hence, the innovated intermediate good can be produced only from the next period onwards. We assume away coordinating behaviors among researchers. Then, given that the total number of researchers in the competitive sector is n_{it}, the probability that some of them successfully innovate is:

$$G(n_{it}) = 1 - (1 - \widehat{a})^{n_{it}} = 1 - \exp(-a n_{it}), \tag{3}$$

where $a \equiv -\ln(1 - \widehat{a})$. As $a \simeq \widehat{a}$, given that $\widehat{a}$ is sufficiently small, we use parameter a interchangeably with $\widehat{a}$ in the following. Let V_{t+1} denote the value of monopolizing this innovation in period $t+1$. Note that this value must be discounted by $\beta = 1/(1+r)$, because the fruits from innovation may be reaped only in the next period, whereas the costs of the innovation must be incurred immediately. Then, the expected payoff per worker from R&D is written as $\beta V_{t+1} g(n_{it})$, where:

$$g(n_{it}) \equiv \frac{G(n_{it})}{n_{it}} = \frac{1 - \exp(-a n_{it})}{n_{it}} \leq a, \tag{4}$$

with $g'(\cdot) < 0$, $g(0) = a$, and $\lim_{n \to \infty} g(n) = 0$.

Equation (4) implies that the payoff per worker decreases as the total number of researchers competing with each other in the sector increases. This is due to the possibility of *duplication*; even when two workers simultaneously innovate in the same sector, their total payoff is at most βV_{t+1}, rather than $2\beta V_{t+1}$, because of the nonrival nature of knowledge. We may assume that successful innovators share the profits from the innovation, βV_{t+1}, equally because they fear entering into Bertrand competition from the next period onwards and ending with zero profits if they fail to agree upon a division of profits. Alternatively, we can consider the case where one fortunate innovator obtains the sole right to use that innovation, while others are prohibited. In either case, the expected return from engaging in R&D activities in sector i is $\beta V_{t+1} g(n_{it})$.

Innovation may also occur in monopolistic sectors. Note, however, that the incumbent monopolist has no incentive to innovate in its own sector, since its profit, $\pi = (\lambda - 1)/\lambda$, does not change, even when innovation is successful.[7] Therefore, any R&D that occurs is carried out by outsiders who try to leapfrog the current monopolist without the experience of producing goods with state-of-the-art technology. The lack of experience limits these firms' ability to attain the information required to improve upon the current state-of-the-art technology. In addition, outside researchers must restrict their methods of innovation to avoid infringing the patents of the incumbent firm. For these reasons, the probability that an outside researcher in a monopolistic sector succeeds in innovation, denoted by $\widehat{b} \in (0, \widehat{a})$, is well below the probability of success of innovation in a competitive sector.

Let m_{it} denote the total number of researchers in monopolistic sector i. Then, the probability that at least one of the researchers succeeds is:

$$H(m_{it}) = 1 - (1 - \widehat{b})^{m_{it}} = 1 - \exp(-b m_{it}), \tag{5}$$

where $b \equiv -\ln(1 - \widehat{b}) \simeq b$. The expected payoff that each researcher receives is $\beta V_{t+1} h(m_{it})$. Here, $h(m_{it}) \equiv H(m_{it})/m_{it}$, with $h'(\cdot) < 0$, $h(0) = b$, and $\lim_{m \to \infty} h(m) =$

[7] This property, commonly called the Arrow effect, depends on the simplifying assumption that only state-of-the-art technology is protected. If the incumbent firm can retain the exclusive right to use the current technology, in addition to the innovated technology, it may be willing to continue research activity in order to widen the technological advantage between itself and other firms (see, for example, Aghion *et al.* 2001). When the incumbent firm's technology is j-steps ahead of other firms, the profit flow is $(\lambda^j - 1)/\lambda^j$, and an additional innovation can increase it by $(\lambda - 1)/\lambda^{j+1}$. However, since the profit increment gradually shrinks as j gets large, the incumbent eventually loses the incentive to innovate. Thus, even in this generalized setting, the Arrow effect holds.

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The number of researchers in each sector, competitive or monopolistic, is determined by the free-entry condition. We assume a complete insurance market such that the risk of engaging in R&D activities can be fully diversified. Free entry to research activities in each type of sector then implies that the following conditions must be satisfied in equilibrium: $\beta V_{t+1} g(n_{it}) \leq w_t$ with equality when $n_{it} > 0$ for all competitive intermediate good sectors, and $\beta V_{t+1} h(m_{it}) \leq w_t$ with equality when $m_{it} > 0$ for all monopolistic sectors. These requirements determine n_{it} and m_{it} as follows:[8]

$$n_{it} = n_t = N(w_t/\beta V_{t+1}) \equiv \begin{cases} g^{-1}(w_t/\beta V_{t+1}) & \text{if } w_t/\beta V_{t+1} < a, \\ 0 & \text{if } w_t/\beta V_{t+1} \geq a, \end{cases} \tag{6}$$

$$m_{it} = m_t = M(w_t/\beta V_{t+1}) \equiv \begin{cases} h^{-1}(w_t/\beta V_{t+1}) & \text{if } w_t/\beta V_{t+1} < b, \\ 0 & \text{if } w_t/\beta V_{t+1} \geq b. \end{cases} \tag{7}$$

Equations (6) and (7) show that the number of researchers in each sector is determined by $w_t/\beta V_{t+1}$, which represents the cost of R&D relative to the discounted value of innovation that a researcher can obtain if he or she succeeds. Researchers participate in R&D activities in competitive sectors when this relative cost is lower than the probability of success, a, and they carry out R&D also in monopolistic sectors if the relative cost is lower than the probability of success there, b. Observe that given the discounted value of innovation, βV_{t+1}, the number of researchers in every competitive or monopolistic sector, is a decreasing function of the market wage. That is, if the cost of R&D rises, the number of R&D competitors must decrease so that the reduced possibility of duplication compensates for the increase in the cost of R&D.

The equilibrium market wage in each period is determined so that the aggregate labor demand for both production and R&D is equalized to the aggregate labor

[8] As expression $w_t/\beta V_{t+1}$ appears frequently: to minimize notation, we write it this way rather than $w_t/(\beta V_{t+1})$.

supply. Recall that the demand for production workers in each intermediate good sector is determined by the amount of sales, which is $1/w_t$ in the competitive sectors and $1/(\lambda w_t)$ in the monopolistic sectors. Let μ_t denote the number of monopolistic sectors in the economy. Labor market clearing requires that:[9]

$$(1-\pi\mu_t)/w_t + (1-\mu_t)N(w_t/\beta V_{t+1}) + \mu_t M(w_t/\beta V_{t+1}) = L. \qquad (8)$$

The LHS of (8) gives the aggregate labor demand, with the first term representing the number of production workers and the second and third terms representing the total number of researchers in the competitive and the monopolistic sectors, respectively. It is easily confirmed that the aggregate labor demand is downward sloping with respect to w_t and that it increases unboundedly when $w_t \to 0$, and contracts toward zero as $w_t \to \infty$. Those properties guarantee that, given μ_t and V_{t+1}, there exists a unique level of w_t at which the aggregate labor demand coincides with the aggregate labor supply, L. Once the equilibrium value of w_t is obtained, n_t and m_t are found from (6) and (7).

3.3 Evolution of the Economy

The state of the economy is characterized by the number of monopolistic sectors, μ_t, and the value of innovation, V_t. This subsection describes the evolution of the economy over time.

There are two reasons why a firm's monopoly in a particular intermediate good sector ends. The first is leapfrogging. If a successful innovation occurs in a monopolistic sector, the incumbent monopolist is replaced by the successful innovator in the following period. As shown by (5), leapfrogging occurs with a probability of $H(m_t)$ during each period. The second cause is imitation. Even though the IPRs of a monopoly firm are, to a certain extent, protected, other firms may find different methods of producing similar goods without infringing upon the monopolist's patent.

[9]Recall that $\pi = (\lambda-1)/\lambda$. The total number of production workers is $(1-\mu_t)/w_t + \mu_t/(\lambda w_t) = (1-\mu_t+\mu_t/\lambda)/w_t = (1-\pi\mu_t)/w_t$.

We assume that such imitation occurs with a constant probability of $\delta \in [0, 1]$ in each period and that it allows any firm to produce the state-of-the-art-quality intermediate good from the next period onwards. Parameter δ measures the weakness of IPR protection; in an economy with strong IPR protection, δ is close to zero and imitation rarely occurs, whereas, with weak IPR protection, δ is large and imitation is frequent.

If imitation occurs, it converts a monopolistic sector into a competitive sector. A subtle point is that both leapfrogging and imitation may simultaneously occur within one period. In this case, a new monopoly firm emerges in the following period and the imitation of the current state-of-the-art product has no impact on the new monopolist. Without systematic correlation between these two events, the coincidence occurs with a probability of $\delta H(m_t)$. Thus, the number of monopolistic sectors that change to competitive ones in the following period is $\delta(1 - H(m_t))\mu_t$. On the other hand, out of $1 - \mu_t$ competitive intermediate good sectors, $G(n_t)(1 - \mu_t)$ sectors develop into monopolistic ones through successful innovations. The net change in μ_t for one period is determined by the difference between these two flows:

$$\mu_{t+1} - \mu_t = (1 - \mu_t)G(n_t) - \delta\mu_t(1 - H(m_t)). \tag{9}$$

Let us now examine how the value of innovation evolves over time. Recall that V_t represents the value of holding a valid (not imitated) IPR at period t. If a monopoly firm holds this right, it earns a profit of $\pi = (\lambda - 1)/\lambda$. In addition, this IPR is still valid at period $t + 1$ if neither imitation nor leapfrogging has occurred during period t. Its present value is βV_{t+1}, and this value is realized with a probability of $(1 - \delta)(1 - H(m_t))$. In sum, V_t is determined by a backward dynamics, as follows:

$$V_t = \pi + \beta(1 - \delta)(1 - H(m_t))V_{t+1}, \tag{10}$$

together with a transversality condition, $\lim_{T\to\infty} \beta^T V_T = 0$, which is required for ruling out bubble prices.

The equilibrium dynamics is characterized by equations (9) and (10). Once n_t and m_t are eliminated by (6), (7), and (8), we see that they constitute an autonomous system of difference equations in terms of μ_t and V_t.[10] The initial number of monopolistic sectors, μ_0, is historically given, whereas the initial value of an IPR, V_0, is determined such that the transversality condition is not violated. Along the equilibrium path, the growth rate of output is obtained by taking the difference in the log of Y_t. From (2), it is:

$$\gamma_{Y_{t+1}} \equiv \ln Y_{t+1} - \ln Y_t = (\ln \lambda) \int_0^1 (q_{i\,t+1} - q_{it}) di + \int_0^1 (\ln x_{i\,t+1} - \ln x_{it}) di. \tag{11}$$

Equation (11) shows that the growth rate can be decomposed into changes in qualities and changes in quantities. On the right-hand side (RHS) of (11), the integral in the first term corresponds to the total number of sectors in which innovation occurs at period t. It is the sum of the number of leapfroggings, $\mu_t H(m_t)$, and the number of innovations in the competitive sectors, $(1 - \mu_t)G(n_t)$. The second term is the aggregate change of (the log of) output in the intermediate good sectors. Recall that the intermediate good output is $1/(\lambda w_t)$ in a monopolistic sector, and $1/w_t$ in a competitive sector. By substituting these quantities into (11), the growth rate can be expressed as:

$$\gamma_{Y_{t+1}} = (\ln \lambda)[(1 - \mu_t)G(n_t) + \mu_t H(m_t)] - (\ln \lambda)(\mu_{t+1} - \mu_t) - (\ln w_{t+1} - \ln w_t). \tag{12}$$

The growth effects of IPR policies can be studied by examining how changes in δ affect the endogenous variables in expression (12). The following two sections are devoted to this task; after analyzing a special but still informative case in the next section, Section 5 examines the effects of IPR protection in a general setting.

4 Growth without Leapfrogging

This section examines the growth effects of IPR policies by focusing on an extreme case in which leapfrogging is prohibitively difficult. Note that, from (8) and (10),

[10]Let us define function $W(\mu_t, V_{t+1})$ as the equilibrium level of w_t given μ_t and V_{t+1}, as determined by (8). Then, (9) and (10) can be written as $\mu_{t+1} = \big[1-\delta+\delta H\big(M(W(\mu_t, V_{t+1})/\beta V_{t+1})\big)\big]\mu_t + G\big(N(W(\mu_t, V_{t+1})/\beta V_{t+1})\big)(1 - \mu_t)$, and $V_t = \pi + \beta(1 - \delta)\big[1 - H\big(M(W(\mu_t, V_{t+1})/\beta V_{t+1})\big)\big]V_{t+1}$.

there is a lower bound for the cost of R&D relative to the discounted value of innovation:[11]

$$w_t/\beta V_{t+1} \geq r/((\lambda-1)L) \equiv z^{\min}. \tag{13}$$

As a benchmark, this section considers a case in which the probability of successful leapfrogging, b, is even lower than this lower bound.

Assumption 1. $b < z^{\min}$.

Under Assumption 1, (7) and (13) imply that $m_t = 0$ for all t; that is, no researchers operate in the monopolistic sectors, and therefore leapfrogging never occurs. Without the risk of being leapfrogged, the value of an innovation is influenced only by the probability of being imitated, which is directly controlled by IPR policies. Substituting $m_t = 0$ for all t into (10) and then applying the transversality condition, we find that the value of innovation is constant over time:

$$V_t = \frac{\pi}{1-\beta(1-\delta)} \equiv \overline{V}(\delta) \quad \text{for all } t. \tag{14}$$

The number of researchers in each competitive sector, n_t, is determined by the free-entry condition (6) and the labor-market-clearing condition (8). Given that $m_t = 0$ and $V_t = \overline{V}(\delta)$ for all t, these conditions can be restated as follows:

$$\frac{1-\pi\mu_t}{L-(1-\mu_t)n_t} \geq \beta\overline{V}(\delta)g(n_t), \quad \text{with equality if } n_t > 0. \tag{15}$$

Condition (15) has an intuitive interpretation. The LHS of (15) represents the production workers' wage. Note that the number of production workers, $L-(1-\mu_t)n_t$ in the denominator, decreases as n_t rises. Then, the nominal wage that each production worker receives increases because the total aggregate consumption expenditure is normalized to unity. Hence, the production workers' wage is upward sloping, as shown by Figure 3. The RHS of (15) is the expected payoff of R&D in the competitive

[11] Note that, from (8), the equilibrium wage cannot be lower than $(1-\pi)/L$, as otherwise the labor demand for production, $(1-\pi\mu_t)/w_t$, exceeds the labor supply, L. Note also that $V_{t+1} \leq \pi/(1-\beta) = (\lambda-1)/(\lambda(1-\beta))$ from (10) and the transversality condition. Therefore, using $r = \beta^{-1}-1$ and $\pi \equiv (\lambda-1)/\lambda$, we have $w_t/\beta V_{t+1} \geq r/((\lambda-1)L) \equiv z^{\min}$.

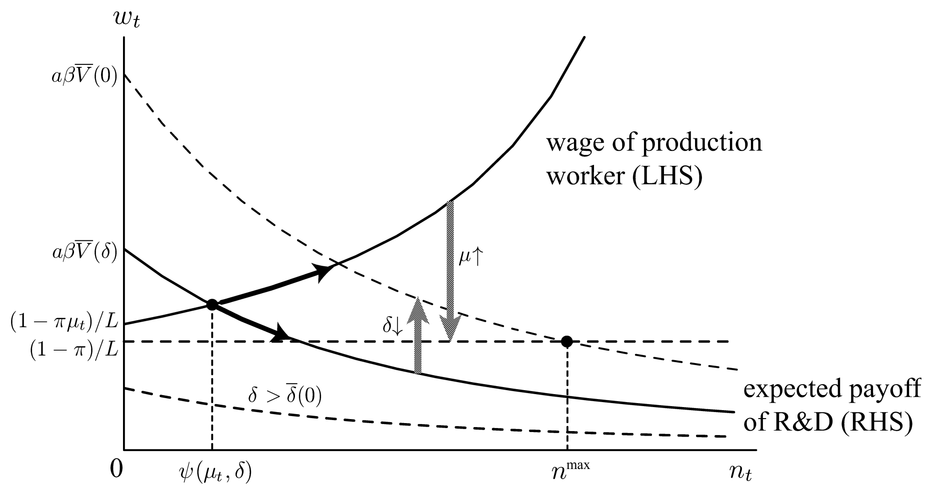

Figure 3: Determination of n_t and w_t when R&D occurs only in competitive sectors. The dashed curve at the bottom of the figure shows that there is no interior solution when IPR protection is too weak ($\delta \geq \overline{\delta}(0)$). The expected payoff of R&D shifts up as δ decreases toward 0, and the wage for production shifts down as μ increases toward 1. The result of both changes is an increase of n_t toward $n^{\max} \equiv g^{-1}(z^{\min})$.

sectors. It is downward sloping with respect to n_t because of the increasing possibility of duplication. If IPR protection is very weak, or if the possibility of imitation δ is very high, the expected payoff of R&D is lower than the wage for any positive value of n_t. Therefore, no worker engages in R&D (i.e., $n_t = 0$). A straightforward calculation shows that this is the case when $\delta \geq \overline{\delta}(\mu_t)$, where:[12]

$$\overline{\delta}(\mu_t) \equiv \frac{a\pi L}{1 - \pi\mu_t} - r \tag{16}$$

gives the level of IPR protection required to activate R&D activities. Observe from (16) that if $a \leq r/(\pi L)$, no R&D occurs regardless of the degree of IPR protection, which means that growth is not possible. Conversely, if $a \geq (1+r)/(\pi L)$, innovation is so easy that R&D take place without any IPR protection. Therefore, the realistic

[12]Note that $\delta \geq \overline{\delta}(\mu_t)$ is equivalent to $(1-\pi\mu_t)/L \geq a\beta\overline{V}(\delta) = \beta\overline{V}(\delta)g(0)$. As function $g(n_t)$ is strictly decreasing with respect to n_t, the above inequality implies that $(1-\pi\mu_t)/(L-(1-\mu_t)n_t) \geq (1-\pi\mu_t)/L > \beta\overline{V}(\delta)g(n_t)$ for all $n_t > 0$.

case is:

$$r/(\pi L) < a < (1+r)/(\pi L), \tag{17}$$

so that $\overline{\delta}(0) \equiv a\pi L - r \in (0,1)$. We assume (17) throughout the paper.

Under (17), R&D take place provided that IPR protection is reasonably tight.[13] When $\delta < \overline{\delta}(\mu_t)$, a unique interior intersection exists between the two curves depicted in Figure 3, because the expected return from R&D is higher than the production workers' wage when $n_t = 0$, and it converges to zero as $n_t \to \infty$. At the intersecting point, denoted by $n_t = \psi(\mu_t, \delta)$, workers are indifferent between these two types of activity: the free-entry condition for R&D is satisfied. The figure also shows that the point of intersection, i.e., the equilibrium number of researchers, depends on the number of monopolistic sectors in the economy, μ_t. Note that as the number of monopolistic sectors increases, the aggregate labor demand declines because no researcher is working in monopolistic sectors and because the monopoly firm hires fewer production workers than those employed in a competitive sector. As a result, as μ_t increases, the curve representing the market wage in Figure 3 shifts down (see the LHS of equation 15), and the point of intersection moves toward the right. This means that the equilibrium number of researchers, $n_t = \psi(\mu_t, \delta)$, is increasing with respect to μ_t.

With this result in hand, let us investigate the dynamics of the economy. The left panel of Figure 4 depicts the phase diagram of the economy in (μ_t, n_t) space. One of the upward sloping curves represents the equilibrium relationship between μ_t and n_t, given by $n_t = \psi(\mu_t, \delta)$. Another upward sloping curve in the panel represents the number of researchers where the number of monopolistic sectors is stationary, as explained below. From (9) and $m_t = 0$, μ_t is constant if and only if the aggregate flow of innovation, $(1-\mu_t)G(n_t)$, coincides with the aggregate flow of imitation, $\delta\mu_t$. Note that, since $G(n_t) < 1$, this condition can be satisfied only if $\mu_t < 1/(1+\delta)$. Using the definition of $G(n_t) = 1 - e^{-an_t}$, the stationary condition can be solved for

[13] Note that, because $\overline{\delta}'(\mu_t) > 0$, (17) implies $\overline{\delta}(\mu_t) \geq \overline{\delta}(0) > 0$ for all $\mu_t \in [0,1]$.

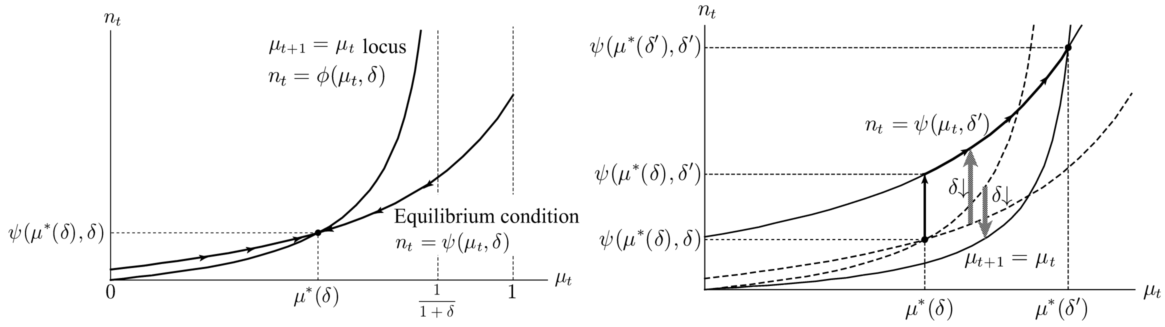

Figure 4: Phase diagram. The left panel shows the dynamics of μ_t and n_t for a given $\delta < \overline{\delta}(0)$. The right panel shows the transitional dynamics after IPR protection is strengthened.

n:

$$n_t = \frac{1}{a} \log \frac{1 - \mu_t}{1 - (1+\delta)\mu_t} \equiv \phi(\mu_t, \delta) \quad \text{for } \mu_t < \frac{1}{1+\delta}. \tag{18}$$

Equation (18) shows that the $\mu_{t+1} = \mu_t$ locus, given by $n_t = \phi(\mu_t, \delta)$, is increasing in μ_t as depicted in Figure 4. When the equilibrium number of researchers $n_t = \psi(\mu_t, \delta)$ is above this locus, the aggregate flow of innovation dominates the flow of imitation and therefore μ_t increases over time. The opposite holds if $n_t = \psi(\mu_t, \delta)$ is below the locus. Observe that, if $\delta < \overline{\delta}(0)$, the two curves have at least one intersecting point in $\mu_t \in (0, 1/(1+\delta))$ from $\psi(0, \delta) > 0 = \phi(0, \delta)$ and $\psi(1/(1+\delta), \delta) < \infty = \lim_{\mu \to 1/(1+\delta)} \phi(\mu, \delta)$. In the following, we assume that the intersecting point is unique, and we denote it by $\mu^*(\delta)$.[14] Then, as illustrated by the left panel of Figure 4, μ_t converges to the steady-state value $\mu^*(\delta)$ from any initial μ_0.

We are now ready to examine the effect of IPR policies on economic growth. Suppose that the economy is initially at the steady state $\mu_t = \mu^*(\delta)$ and then IPR protection is strengthened so that the probability of imitation δ falls to $\delta' < \delta$. Strengthened protection increases the value of innovation $\overline{V}(\delta)$, given by (14), and hence the expected payoff of R&D, given by the RHS of (15). Then the downward sloping curve in Figure 3 shifts up, increasing the equilibrium number of researchers in a competitive sector, n_t. Since this raises the probability that innovation occurs

[14]We experimented with various combinations of parameters and found that the intersecting point is, in fact, unique for every combination of parameters chosen.

in each competitive sector, $G(n_t)$, the rate of economic growth is enhanced, at least temporarily.

However, to obtain the long-term effect of stronger IPR protection, we must consider the transition to the new steady state. Note that the above discussion implies that function $n_t = \psi(\mu_t, \delta)$ shifts up when δ is decreased. In addition, the $\mu_{t+1} = \mu_t$ locus, given by (18), shifts down, since the stationarity of μ_t can be maintained with fewer researchers when the probability of imitation is low. As a result, the number of researchers in a competitive sector is now above the $\mu_{t+1} = \mu_t$ locus, as depicted by the right panel of Figure 4. The number of monopolistic sectors, μ_t, increases gradually toward the new steady state level $\mu^*(\delta') > \mu^*(\delta)$. This implies shrinkage in the number of competitive sectors where R&D activities are taken place, which gradually inhibits growth.

At the same time, the number of researchers in each competitive sector gradually increases during the transition, more or less offsetting the negative effect of increased monopolization on growth. As explained, an increase in μ_t reduces the competitive sectors where researchers are operating, lowering the aggregate labor demand and therefore the equilibrium wage level. Given a lower wage level, the number of researchers in each competitive sector, n_t, increases although there are fewer competitive sectors, $1 - \mu_t$. In effect, the reduction in the number of competitive sectors reallocates researchers into fewer sectors or research fields.

We can confirm those effects in mathematical terms by substituting $\mu_t = \mu_{t+1} = \mu^*(\delta)$, $w_{t+1} = w_t$, $n_t = \psi(\mu^*(\delta), \delta)$, and $m_t = 0$ into (12):

$$\gamma_Y^*(\delta)/(\ln \lambda) = (1 - \mu^*(\delta))G(\psi(\mu^*(\delta), \delta)). \tag{19}$$

Consistent with the above, equation (19) clarifies the three ways in which growth is affected by stronger IPR protection: first, $\psi(\cdot, \delta)$ is directly increased by smaller δ; second, $(1 - \mu^*(\delta))$ is reduced by larger $\mu^*(\delta)$; and, third, $\psi(\mu^*(\delta), \cdot)$ is increased by larger $\mu^*(\delta)$. As a result, the overall effect of stronger IPRs on growth can be either positive or negative depending on the relative magnitude of those three effects. In fact, the following proposition states that the relationship between IPR protection and long-term growth is nonmonotonic:

Proposition 1. *Under assumption 1, protection of IPRs should be neither too strict ($\delta = 0$) nor too loose ($\delta \geq \overline{\delta}(0)$), because in either case, the resulting long-term rate of growth would be zero. There exists a noncorner level of IPR protection, $\delta^{\max} \in (0, \overline{\delta}(0))$, which maximizes the total flow of innovation in competitive sectors and therefore the long-term rate of growth.*

The reason that IPR protection should not be too loose is obvious: if $\delta \geq \overline{\delta}(0)$, the expected reward from innovation is so low that no one will find it profitable to participate in R&D activities.[15] The reason that IPR protection should not be too strict requires some clarification. When IPR protection is extremely strengthened so that it shuts out the possibility of imitation (i.e., $\delta = 0$), the $\mu_{t+1} = \mu_t$ locus falls to the horizontal axis ($\phi(\mu_t, 0) = 0 < \psi(\mu_t, 0)$ for all $\mu_t < 1$), and therefore the number of monopolistic sectors gradually approaches 1. Although this means that the number of sectors where researchers operate converges to zero, it does not imply *per se* that growth is impossible if the number of researchers in each competitive sector can grow without bound such that the aggregate number of researchers is unaffected. However, recall that as more researchers operate in the same intermediate good sector, the risk of duplication rises. In particular, because of the increased risk of duplication, we can show that the positive effects cannot overturn the negative effect of increased monopolization on growth when IPR protection becomes very strict. To see this, observe that the number of researchers per sector is bounded by a finite number, $n^{\max} \equiv g^{-1}(z^{\min})$, from (6) and (13), because if more than $n^{\max}$ researchers operate in a sector, the probability that duplication will occur is so high that the expected return is below the lowest possible wage. This means that, if $\delta = 0$, the number of competitive sectors converges to zero, whereas the flow of innovation per sector is bounded, jointly implying that the aggregate flow of innovation is eliminated. The

[15] Suppose that $\delta \geq \overline{\delta}(0)$. Then, $\psi(0, \delta) = 0 = \phi(0, \delta)$ from the definition of $\overline{\delta}(0)$ and (18). This means that the unique steady state is $\mu^*(\delta) = 0$, and therefore $\gamma_Y^*(\delta)/(\ln \lambda) = G(\psi(0, \delta)) = G(0) = 0$ from (19).

latter half of the proposition is confirmed by noting that the rate of economic growth given by (19) is positive if and only if $\delta \in (0, \overline{\delta}(0))$ and that it is continuous in δ.

In all, Proposition 1 states that the growth-maximizing IPRs policy should be an intermediate one under the assumption that leapfrogging is impossible. However, how does the conclusion depend on this assumption? The following section investigates the general case in which R&D can be conducted in both the competitive and the monopolistic sectors and shows that the above result continues to hold in reasonable settings.

5 Growth with Imitation and Leapfrogging

In this section, we dispense with Assumption 1 and examine the effect of IPR protection on the long-term rate of growth in an economy where innovation can occur in both the competitive and the monopolistic sectors. Since this general case is considerably more complex, in that we need to keep track of the different intensities of R&D between the monopolistic sectors and the competitive sectors, we restrict the analysis to the long-run properties of the economy. On the BGP, the analysis can be kept more tractable by focusing on a single variable, $z_t \equiv w_t/\beta V_{t+1} > 0$, which represents the cost of R&D relative to the discounted value of innovation. The point is that both intensities are functions only of z_t; i.e., $n_t = N(z_t)$ and $m_t = M(z_t)$ from (6) and (7). This means that the flows of innovation in competitive or monopolistic are given by $G(N(z_t))$ and $H(M(z_t))$, both of which are determined only by z_t.

Given these flows, the number of monopolistic sectors in the long run is determined accordingly. Let z denote the long-run level of z_t. Then, by substituting $G(N(z))$ and $H(M(z))$ for (9) and equating μ_{t+1} to μ_t, the value of μ_t in the long run is obtained as a function of z and δ:[16]

$$\mu^*(z, \delta) = \frac{G(N(z))}{\delta(1 - H(M(z))) + G(N(z))}. \tag{20}$$

[16]Here, we recycle the notation $\mu^*(\cdot)$. Its definition differs from $\mu^*(\delta)$, as defined in the previous section.

Similarly, from (6), (7), (8), and (10), the values of V_t and w_t on the BGP are calculated as follows:

$$V^*(z,\delta) = \frac{\pi}{1-\beta(1-\delta)(1-H(M(z)))}, \tag{21}$$

$$w^*(z,\delta) = \frac{1-\pi\mu^*(z,\delta)}{L-(1-\mu^*(z,\delta))N(z)-\mu^*(z,\delta)M(z)} \quad \text{for } z > \underline{z}(\delta), \tag{22}$$

where $\underline{z}(\delta) \in (0,a)$ denotes the level of z at which the number of researchers, $(1-\mu^*(z,\delta))N(z)-\mu^*(z,\delta)M(z)$, coincides with L.[17]

Equations (20)-(22) express the BGP of the economy in terms of z and δ. As z is an endogenous variable, its dependence on δ needs to be clarified. From the definition of z, it must satisfy an identity:

$$z = w^*(z,\delta)/\beta V^*(z,\delta) \equiv Z(z,\delta). \tag{23}$$

That is, the value of z on the BGP is determined as a fixed point of function $Z(z,\delta)$. The following lemma establishes the existence of the fixed point and how it is affected by IPR policies.

Lemma 1. *For every $\delta \in [0,1]$, there exists a fixed point $z > \underline{z}(\delta)$ that satisfies (23). Suppose that the fixed point is unique and let it be denoted by $z^*(\delta)$. Then, $z^{*\prime}(\delta) > 0$ for all $\delta \in [0,1]$. In addition, there exists a continuous function $\delta^{LF}(b)$ such that:*

$$z^*(\delta) \begin{cases} \geq a & \text{if } \delta \geq \overline{\delta}(0); \\ \in [b,a) & \text{if } \delta \in [\delta^{LF}(b), \overline{\delta}(0)); \\ < b & \text{if } \delta < \delta^{LF}(b). \end{cases}$$

Function $\delta^{LF}(b)$ is zero for all $b \leq z^{\min}$, strictly increasing in b for all $b \in (z^{\min}, a)$, and $\lim_{b\to a} \delta^{LF}(b) = \overline{\delta}(0)$.

Proof: see the Appendix.

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[17] From (6) and (7), note that $\lim_{z\to 0} N(z) = \lim_{z\to 0} M(z) = \infty$ and $N(a) = M(a) = 0$. Therefore $\underline{z}(\delta) \in (0,a)$ is well defined. In equilibrium, z cannot be smaller than $\underline{z}(\delta)$, since otherwise the number of researchers exceeds the population.

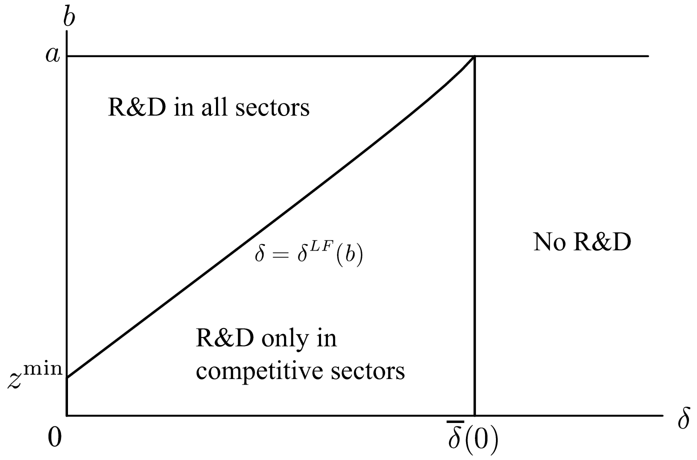

Figure 5: The degree of IPR protection and R&D activities. Calculated numerically by setting $\lambda = 1.5$, $\beta = 0.95$, $L = 10^8$, and $a = 1/L$. Under these parameter values, $\overline{\delta}(0) \approx 0.28$ and $z^{\min} \approx 0.11a$.

Throughout this section, we assume that mapping (23) has a unique fixed point (we confirmed this under various parameter values). Then, Lemma 1 proves that stronger IPR protection reduces the cost of innovation relative to the value of innovation (i.e., $z_t \equiv w_t/\beta V_{t+1}$ falls when δ is reduced). Recall that, from (6) and (7), researchers in competitive and monopolistic sectors are willing to incur the cost of R&D only when $z_t \equiv w_t/\beta V_{t+1}$ is lower than a and b, respectively. Thus, as discussed in the previous section, R&D occurs in competitive sectors only when IPR protection is stronger than $\overline{\delta}(0)$. Since b is smaller than a, the degree of IPR protection must be even tighter (i.e., $\delta < \delta^{LF}(b) < \overline{\delta}(0)$) to activate R&D activities in the monopolistic sectors, as illustrated in Figure 5,

To see the long-run effect of IPR policies on growth, we substitute $\mu_{t+1} = \mu_t = \mu^*(z^*(\delta), \delta)$, $n_t = N(z^*(\delta))$, $m_t = M(z^*(\delta))$, and $w_{t+1} = w_t$ into (12):

$$\gamma_Y^*(\delta)/\ln\lambda = (1 - \mu^*(z^*(\delta), \delta))G(N(z^*(\delta))) + \mu^*(z^*(\delta), \delta)H(M(z^*(\delta))), \qquad (24)$$

which is simply the sum of the innovation flow in the competitive sectors and the leapfrogging flow in the monopolistic sectors. Expression (24) depends on the degree of IPR protection functions $\mu^*(z^*(\delta), \delta))$, $N(z^*(\delta))$, and $M(z^*(\delta))$. Observe that, since $\mu_\delta^* < 0$, $\mu_z^* < 0$ from (20) and $z^{*\prime} > 0$ from Lemma 1, stronger IPR protection (or smaller δ) increases the number of monopolistic sectors, $\mu^*(z^*(\delta), \delta)$. Since the flow of innovation in a sector is higher when the sector is competitive rather than monopolistic (i.e., $G(N(z^*(\delta))) > H(M(z^*(\delta)))$ for any given $\delta < \overline{\delta}(0)$), this change in the composition of the economy reduces the aggregate research intensity as illustrated by the horizontal arrow in Figure 2, which is detrimental for growth.

Stronger protection also has positive effects on growth, since it reduces the ratio of the cost of innovation relative to the reward, $z^*(\delta) = w/\beta V$, which stimulates the flows of innovation in both types of sector, $N(z^*(\delta))$ and $M(z^*(\delta))$, from $z^{*\prime} > 0$, $N' < 0$ and $M' < 0$. Intuitively, there are two reasons why stronger IPR protection reduces this ratio. For one thing, since the demands for both production and R&D workers are smaller in a monopolistic sector than in a competitive sector, increased monopolization of the economy reduces the aggregate labor demand, and therefore the equilibrium wage level falls (observe that $w^*_\delta > 0$ from equation 22). Since this makes working as a researcher more attractive than working as a production worker, the number of R&D workers in every sector increases, as illustrated by the vertical arrows in Figure 2.

Therefore, the increased monopolization of the economy causes a reallocation of research workers across sectors, through which the distribution of research workers becomes more "skewed" (see, again, Figure 2). This reallocation would be growth neutral when both the aggregate number of researchers and the productivity of each researcher remain constant. However, if the possibility of duplication of innovations is explicitly considered, such a possibility will be higher when the distribution of researchers across sectors is more skewed. Therefore, the aggregate productivity of R&D after subtracting the losses from duplication decreases as the number of monopolistic sectors increases. Furthermore, the fall in productivity diverts workers from R&D activities.

As discussed, the net effect of increased monopolization, which is caused by stronger IPR protection, on growth tends to be negative. However, recall that there is yet another reason why stronger IPR protection stimulates R&D activities: it di-

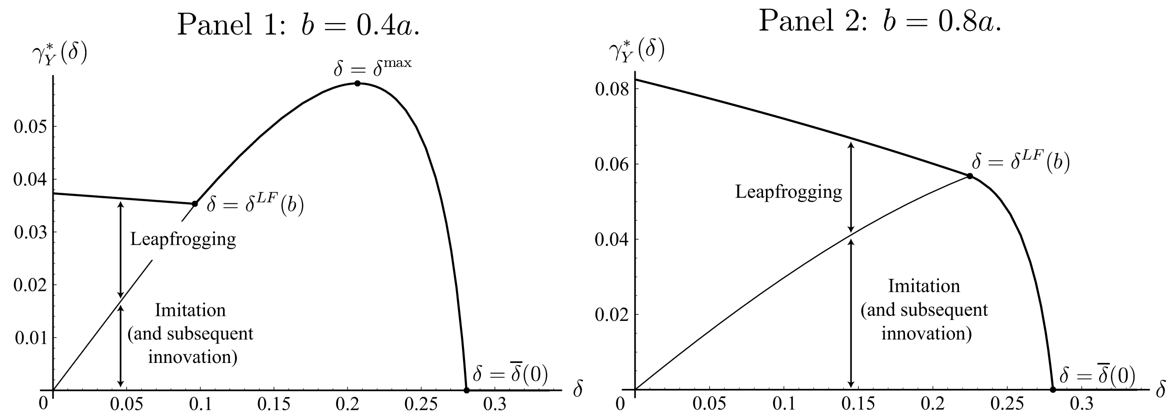

Figure 6: Growth rate on the BGP as a function of δ. When the parameters are given by $\lambda = 1.5$, $\beta = 0.95$, $L = 10^8$, and $a = 1/L$ (the same as in Figure 5), the long-run rate of growth is maximized by setting $\delta = \delta^{\max} = 0.207$ when $b \leq 0.578a$, whereas it is maximized at $\delta = 0$ when $b \geq 0.578a$.

rectly raises the reward for successful innovators. This positive effect can be seen by noting that the denominator of $z^*(\delta) = w/\beta V$ is increased by stronger IPR protection, since $V_\delta^* < 0$ from (21). Therefore, similarly to the analysis in the previous section, the total effect of stronger IPR on growth can be of either sign, depending on the relative magnitude of positive and negative effects. Note that the negative effect of monopoly on growth stems from the assumption that innovation is more difficult in monopolistic sectors than in competitive sectors. Therefore, the relative magnitude of opposing effects hinges critically upon the difficulty of leapfrogging compared with that of innovation in competitive sectors.

To evaluate the total effect, we numerically calculate the long-term rate of growth as a function of δ, for two different values of the probability of success in R&D in a monopolistic sector $b \in (z^{\min}, a)$.[18] Panel 1 of Figure 6 shows that, when b is considerably small, the flow of leapfrogging is small even when IPR protection is strongest ($\delta = 0$). In this case, the economy can grow faster with an intermediate level of IPR protection, $\delta^{\max} \in (0, \overline{\delta}(0))$, at which the total flow of innovation in competitive sector is maximized (see Proposition 1), even though no leapfrogging

[18]Recall that the case of $b < z^{\min}$ is already analyzed in Section 4.

occurs at this level of IPR protection. Panel 2 of Figure 6 illustrates the relationship between IPR protection and growth when b is high and there is only a small difference in the probability of success between the monopolistic and the competitive sectors. In this case, the growth rate is maximized at $\delta = 0$, which means that the economy need not rely on imitation to promote growth because the monopoly is not very harmful to R&D activities by outside researchers.

To interpret this result in more detail, recall from Lemma 1 and Figure 5 that when b is small, $\delta^{LF}(b)$ is small, and therefore $0 < \delta^{LF}(b) < \delta^{\max} < \overline{\delta}(0)$ holds (note that $\delta^{\max}$ does not depend on b). For $\delta > \delta^{LF}(b)$, leapfrogging does not occur, and as discussed in the previous section, there is a level of IPR protection at which the total flow of innovation in competitive sectors is maximized, which is $\delta^{\max}$. The positive effect of stronger IPR dominates when $\delta > \delta^{\max}$, whereas the opposite holds when $\delta \in (\delta^{LF}(b), \delta^{\max})$. When IPR protection is further strengthened beyond $\delta^{LF}(b)$, the simulation result shows that the positive effect again slightly dominates, but this does not affect the fact that $\delta = \delta^{\max}$ is growth maximizing, since $\delta^{LF}(b)$ is already near zero. However, when b is large, $\delta^{LF}(b)$ is higher than $\delta^{\max}$, and therefore the positive effect of stronger IPR protection globally dominates the negative effect. Then, why is stronger IPR protection more favorable for growth when $\delta < \delta^{LF}(b)$? There are two reasons. First, stronger IPR protection now encourages R&D, not only in competitive sectors but also in monopolistic sectors. Second, a worker who loses a job in a formerly competitive sector can now find another as a researcher, not only in a competitive sector but also in a monopolistic sector. This implies that the problem of the increased possibility of duplication is less serious than the case in which the job opportunities for R&D exist only in competitive sectors. From these observations, we obtain the following proposition.

Proposition 2. *If the probability of successful leapfrogging is low (with b sufficiently close to $z^{\min}$), the long-term rate of growth is maximized by allowing a certain positive probability of imitation, $\delta^{\max} \in (0, \overline{\delta}(0))$, so that the flow of imitation is maximized. If the probability of successful leapfrogging is high (with b sufficiently close to a), the*

long-term rate of growth is maximized by shutting out any imitation, $\delta = 0$*, so that the flow of leapfrogging is maximized.*
Proof: see the Appendix.

Proposition 2 states that the relationship between IPR protection and growth is quite different depending on the difficulty of leapfrogging in monopolistic sectors. Specifically, our simulation results suggest that there is typically a threshold level of b, below which growth is maximized by $\delta = \delta^{\max}$ and above which growth is maximized by $\delta = 0$. Therefore, to recommend an intermediate degree of IPR protection as a growth-maximizing policy, we must evaluate the extent to which monopoly is harmful for R&D activities. Although the validity of this assumption can vary considerably across economies in general, evidence from previous studies appears to support rather than reject it.

Using panel data from some 670 U.K. companies, Nickell (1996) examined how total factor productivity (TFP) growth is related to a survey-based measure of competition, specified as a dummy variable taking a value of one if the manager answered yes to the following question: "Have you more than five competitors in the market for your products?" After controlling for various factors, Nickell (1996) found a robust and significantly positive effect of competition on growth. In addition, he attempted to measure the size of this effect by ranking firms on the basis of rents, which are presumed to relate positively to the degree of monopolization. The result is that the growth of the TFP of a firm at the eightieth percentile in the ranking of rents is, *ceteris paribus*, 3.8–4.6 percentage points lower than that of a firm at the twentieth percentile, suggesting that monopolization significantly inhibits TFP growth.

Our assumption is more directly supported by Blundell, Griffith and Van Reenen (1993, 1995), who estimated dynamic models of innovation using industry-level variables and the count of technologically significant and commercially important innovations commercialized by British firms between 1972 and 1982. Measuring the degree of concentration of an industry by the proportion of sales that is represented by the five largest domestic firms, they found a negative impact of concentration on the probability of innovation. On this basis, they concluded that firms in competitive industries tend to have a greater probability of innovation, and therefore firms with larger market shares that increase the level of industry concentration would depress the aggregate level of innovative activity. Geroski (1990) estimated a similar regression, which revealed that monopoly inhibits research activities, even after correcting for interindustry variations in technological opportunity and postinnovation returns.

Finally, note that the difficulty of leapfrogging stems not only from technical aspects but also from the incumbent's entry-deterring activities. For example, Bresnahan (1985, p.16) documented that when IBM and Litton entered the PPC (plain paper copier) market in 1972, Xerox sued to block entry with literally hundreds of patents. As a result, IBM spent millions to "invent around" Xerox's major patents—with 25 percent of the budget spent not on R&D, but on patent counsel. By 1974, free access to PPC technology was ensured through the FTC's antitrust action. After this, many firms entered the PPC market, and the transition period saw a great deal of innovative activity from both new entrants and Xerox.

6 Relationship with the Literature

Thus far, it has been shown that the long-term rate of economic growth can be maximized under intermediate IPR protection strength on the assumption that leapfrogging is considerably more difficult than innovation in competitive sectors. This study, of course, is not the first attempt to investigate the growth effects of IPR policies. In this section, we discuss the similarities and differences between our results and those obtained from earlier studies.

Using variety expansion models of endogenous growth, Kwan and Lai (2003) and Iwaisako and Futagami (2003) demonstrated that the long-term rate of growth is maximized when IPR policy is as stringent as possible. From the above analysis, it can be reckoned that their results follow from the fact that they did not consider the negative effects of monopoly on innovation, since the same result is obtained in our model in the extreme case of $b = a$. Intuitively, in an environment where outsiders can learn the state-of-the-art technology nearly as easily as insiders, stronger IPR does not inhibit innovation and therefore growth. In addition, the second half of Proposition 2 implies that their result is robust to small changes in that it continues to hold as long as the negative effects of monopoly on innovation are small (i.e., even when b is slightly below a).

While the majority of theoretical studies of IPR have concluded that stronger IPR protection enhances economic growth, there are notable exceptions. One important study was made by Michel and Nyssen (1998), who examined the impact of IPRs on growth in terms of patent length and found that the rate of economic growth is maximized when the patent length is finite rather than infinite. This result comes from their assumption that the knowledge spillover from past R&D, which makes creation of new goods easier, is limited during the term of patent protection. At the level of the aggregate economy, their assumption works similarly to ours in that IPR protection prevents outside firms from obtaining experience in the production of state-of-the-art-quality goods. However, since Michel and Nyssen used a variety-expansion setting in which R&D does not occur in existing sectors, they did not obtain different research intensities between monopolistic and competitive sectors, as reported by Blundell, Griffith and Van Reenen (1993, 1995) and Geroski (1990).

A stronger assumption is made in a class of models called *step-by-step* innovation, in which product innovations by outsiders can occur only after the state-of-the-art product has been imitated (i.e., leapfrogging is prohibited). Aghion *et al.* (1997, 2001) and Mukoyama (2003) used this setting and showed that the growth rate can increase with the ease of imitation and the rate of subsidy on imitation. In this paper, their setting corresponds to the analysis in Section 4, where leapfrogging is so difficult that trying to do so is not profitable, even when the degree of IPR protection is strongest (i.e., $b < z^{\min}$), and in Proposition 1 we obtained a result comparable to theirs. Moreover, the first half of Proposition 2 shows that Proposition 1 can be extended for the case in which b is slightly above $z^{\min}$. Therefore, the results from the step-by-step models, where facilitating imitation can be beneficial for growth when leapfrogging is impossible, is extended to the case in which leapfrogging is possible but considerably more difficult.

While in our model the probability of imitation is assumed to be directly controlled by the authority, studies of step-by-step innovation have an advantage in that they can explain the flow of imitation endogenously.[19] However, this was achieved at the cost of abstracting from reallocation of R&D workers across sectors through labor market. Specifically, Aghion *et al.* (1997, 2001) assumed an infinitely elastic supply of labor such that the wage level is constant, and therefore the number of competitive sectors does not affect research intensity in each sector.[20] Mukoyama (2003) ingeniously set up an innovation race game with two firms, in which the research intensity in a symmetric Nash equilibrium is determined independently of the labor market. In contrast, assuming free entry for innovative activities and inelastic aggregate labor supply, as is commonly assumed in standard endogenous growth models, we explicitly consider the process in which a reduction in the number of competitive sectors lowers the market wage and therefore increases the research intensity in both types of sector. Such reallocation appears to be realistic, but it can completely eliminate the negative effect of increased monopolization on growth, especially when innovation is subject to constant returns as is usually assumed. This is the reason why another realistic feature—the time required to innovate—must be introduced. As explained in Section 3, even when individual R&D is subject to constant returns, the time to innovate creates decreasing returns at the industry level.[21] In this setting, we have shown that widespread competition promotes growth

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[19]It is possible to incorporate endogenous imitation into our model by assuming that IPR protection, as in Dinopoulos and Segerstrom (2005), affects the ease of intentional imitation. However, it seriously complicates the analysis and provides vaguer implications.

[20] Aghion *et al.* (1997) briefly considered an extension with a fixed labor supply (see Section 5 in their paper). However, they analyzed only extreme cases in which $r + \delta = 0$ and $r + \delta \to \infty$.

[21] Note that decreasing returns at the level of individual firms, as assumed in Aghion *et al.* (1997, 2001), does not inhibit innovation at the industry level if free entry is allowed. At the same time, reallocation of research workers across sectors will be neutral for growth if decreasing returns to

by diversifying researchers across broader fields, thereby reducing the possibility of unnecessary duplication at each industry.

Finally, while the studies discussed restrict attention to a closed economy, another negative effect of IPR protection on growth can stem from the international division of labor. For example, in the framework of a North–South economy, Helpman (1993) showed that strengthening IPR protection (i.e., reducing the probability of imitation) in the South reduces the rate of innovation in the North in the long run, since IPR protection leads to more workers in the North in production activities rather than in R&D. This depends on the assumption of no movement of labor between the two countries, or equivalently, between the monopolistic (North) and competitive (South) sectors, while this paper considers a closed economy model without any restriction on the movement of workers across sectors. Obviously, the relevance of either model depends on the actual economy in which one is trying to apply a theory.

7 Discussion

This section briefly discusses two important topics in the literature of endogenous growth: scale effects and welfare issues.

7.1 Scale Effects

It is well known that the prototypes of R&D-based growth models exhibit strong scale effects in that the long-term rate of growth increases very sensitively with population size. For example, using our notations, the equilibrium growth rate derived in Grossman and Helpman (1991) can be written as $(\ln \lambda)\,(a\pi L - r/\lambda)$. This means that if the size of population, L, is doubled, the long-term rate of economic growth

innovation operate with respect to aggregate innovative activities as measured by the total number of R&D workers in the economy. Thus, on the assumption of free entry, the increased monopolization of the economy inhibits growth only when there are some forms of decreasing returns in innovation working at the industry level.

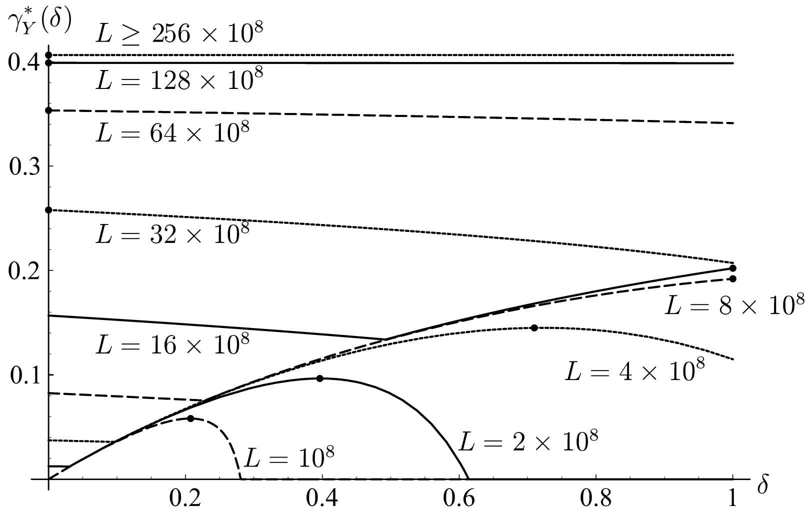

Figure 7: Relationship between IPR protection and growth with different sizes of population. Calculated numerically by changing $L = 2^i \times 10^8$, where $i = 0, 1, 2 \ldots$. Other parameters are fixed at $\lambda = 1.5$, $\beta = 0.95$, $L = 10^8$, $a = 1/10^8$, and $b = a/10$. The growth-maximizing level of δ for each size of L is designated by a black point.

is more than doubled.

Also in our model, greater population enables more labor to be allocated to R&D and therefore, given the fixed number of sectors, increases the possibility that innovation occurs in each sector.[22] However, since we explicitly consider the possibility of duplication, the rate of economic growth does not increase linearly with the aggregate number of R&D workers. Figure 7 illustrates how the relationship between the long-term rate of economic growth and the degree of IPR protection changes as the size of population is doubled, doubled again, and so on. Observe that although the economy with twice the population grows faster, it does not grow twice as fast. In particular, as the size of population increases, the possibility of duplication becomes more serious, and therefore the rate of economic growth cannot exceed a certain

[22]It is possible to eliminate the scale effects completely by incorporating the mechanism through which the number of sectors increases with the size of population (e.g., Young 1998). Alternatively, the scale effect can be removed by assuming that R&D difficulty increases with cumulative R&D efforts (e.g., Segerstrom 1998).

upper bound.

Figure 7 also suggests that the growth-maximizing IPR policy changes nonmonotonically with the size of population. When the size of population, L, is small, the market to which an innovated good can be sold is also small, and therefore strict IPR protection is required to give enough incentives for innovators to conduct R&D (observe that $\overline{\delta}(0) = a\pi L - r$ is increasing in L). Therefore, when the size of economy is small, the growth-maximizing level of δ is low, although it must be positive since R&D occurs only in competitive sectors given the small market size. As L increases, innovators become willing to conduct R&D at weaker levels of IPR protection, and therefore the rate of growth can be increased with weaker protection, since it will increase the number of competitive sectors where innovation occurs. In the example depicted in Figure 7, the growth-maximizing $\delta(= \delta^{\max})$ increases with L for $L \leq 16 \times 10^8$. However, when the size of population becomes sufficiently large ($L \geq 32 \times 10^8$), the market size is so large that innovators are also willing to conduct R&D in monopolistic sectors, regardless of the level of δ (i.e., $\delta^{LF}(b) \geq 1$). In that case, as discussed in Section 5, the positive effects of stronger protection on growth tend to dominate the negative effects, and therefore growth is maximized by the strongest possible level of IPR protection, $\delta = 0$.

This suggests that even when the technological circumstances are the same, the desirable level of IPR protection can differ significantly depending on the market size.

7.2 Welfare

Although we have so far discussed the desirability of IPR policies only in terms of economic growth, previous studies showed that the strength of IPR protection that is desirable in terms of welfare can differ from those that maximize growth. Specifically, Kwan and Lai (2003) and Iwaisako and Futagami (2003) showed that, while the long-term rate of growth is maximized by the strongest possible IPR protection, social welfare is higher when IPR protection is weaker.[23] Our model provides a very different implication in that growth is not necessarily maximized by the strongest possible IPR protection. What then is the relationship between the growth- and welfare-maximizing IPR policies? We shed some light on this issue by considering how the degree of IPR protection, represented by δ, affects the utility of the representative consumer on the BGP.[24]

On the BGP, the future path of output Y_t is given by $\ln Y_\tau = \ln Y_t + \gamma_Y^*(\delta)(\tau - t)$ for $\tau \geq t$, where $\gamma_Y^*(\delta)$ is given by (24). Substituting this expression and $c_t = Y_t/L$ into (1) gives:

$$U_t = \frac{1}{1-\beta}\ln Y_t + \frac{\beta}{(1-\beta)^2}\gamma_Y^*(\delta) - \frac{1}{1-\beta}\ln L. \tag{25}$$

Since the last term is constant, (25) shows that welfare is determined as the sum of a level effect (the first term) and a growth effect (the second term). Since the growth effect has been analyzed extensively in Section 5, here we examine the level effect of IPR protection. From (2), (22), and the fact that the outputs from competitive and monopolistic sectors are respectively $1/w_t$ and $1/(\lambda w_t)$, the level of $\ln Y_t$ on the BGP is given by:

$$\ln Y_t = -(\ln\lambda)\mu - \ln(1-\pi\mu) + \ln(L - (1-\mu)n - \mu m) + (\ln\lambda)\int_0^1 q_{it}\, di, \tag{26}$$

where we omit the subscripts on μ, n and m, since they are constant on the BGP. The last term of (26) represents the current technological level of the economy, or the aggregate TFP. Since this term is historically determined, we examine the effects of δ on the other terms.

Recall that, as shown in Section 5, if IPR protection is relaxed (if δ is increased), the research intensities in both types of sector decline; i.e., $dn/d\delta, dm/d\delta \leq 0$. Since

[23] Grossman and Lai (2005) also showed that stronger IPR protection is not necessarily welfare enhancing in a nonendogenous growth model.

[24] Because the transitional dynamics of the present model are too complex for analysis, we focus our analysis on the BGP following Iwaisako and Futagami (2003). The transitional dynamics of Kwan and Lai (2003) is more tractable and can be analyzed by linearizing the system around the steady state.

(26) is decreasing in n and m, these changes have positive effects on the current level of output, Y_t. Intuitively, since weaker IPR protection discourages R&D, a reallocation of labor from R&D to production occurs. Recall also that if IPR protection is relaxed, the number of monopolistic sectors declines; i.e., $d\mu/d\delta \leq 0$. Since goods are sold at higher markups in monopolistic sectors than in competitive sectors, this change affects the extent of output distortions, which can be measured by:

$$\left.\frac{\partial \ln Y_t}{\partial \mu}\right|_{BGP} = -\ln \lambda + \frac{\pi}{1-\pi\mu} + \frac{n-m}{L-(1-\mu)n-\mu m}. \tag{27}$$

To understand the difference between the growth- and welfare-maximizing IPR policy, let us examine the effect on welfare of a marginal increase in δ from the growth-maximizing level. As discussed in the previous sections, the growth-maximizing level can be either $\delta = 0$ or $\delta = \delta^{\max}$, depending on the difficulty of leapfrogging. First, consider the case in which the long-term rate of growth is maximized at $\delta = 0$, as in Panel 2 of Figure 6. If IPR protection is marginally relaxed from $\delta = 0$, the growth effect is negative since $\gamma_Y^*(\delta)$ is already maximized at the corner level of $\delta = 0$. In addition, if relaxing IPR protection reduces μ from 1, industries that become competitive start to sell goods at lower markups than other monopolistic sectors, creating output distortions.[25] In fact (27) is positive at $\mu = 1$ (note that the second term becomes $\lambda - 1 > \ln \lambda$), which implies that output level declines if μ is reduced. Since both the growth effect and output distortions undermine welfare, relaxing IPR protection can only be justified when the induced reallocation of labor from R&D to production substantially improves welfare. In other words, in a circumstance where the second half of Proposition 2 applies, the strongest possible IPR protection is welfare maximizing unless economic growth is too fast in terms of the trade-off between current consumption and future growth.[26]

Now consider the case in which $\delta = \delta^{\max}$ is the growth-maximizing policy. Then,

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[25]Note that if all sectors are monopolized, markups are the same across all sectors, and therefore there are no output distortions. This is another general-equilibrium effect pointed out by O'Donoghue and Zweimüller (2004) and Judd (1985).

[26]This trade-off is extensively analyzed by Kwan and Lai (2003). Note that, since both Kwan

a slight increase in δ from $\delta^{\max}$ yields no growth effect in the first order, since $\gamma_Y^*(\delta)$ is maximized at the interior level of $\delta = \delta^{\max}$. In addition, reducing μ does not necessarily worsen output distortions, especially when μ is already substantially small (observe that the second term of (27) becomes smaller than $\ln\lambda$ when $\mu = 0$). Moreover, reallocation of labor from R&D to production increases current output. As a result, it can be shown that the marginal relaxation of IPR protection at $\delta^{\max}$ improves welfare on the BGP provided that:[27]

$$\mu^* < f(\delta^{\max}) \equiv \frac{r + \delta^{\max}}{\pi r} + \frac{1}{\ln\lambda}\left(\frac{r}{r + \delta^{\max}} - 2\right). \tag{28}$$

To summarize, similarly to the growth-maximizing IPR policy, the desirable degree of IPR protection depends critically on the difficulty of leapfrogging relative to that of innovation in competitive sectors. If leapfrogging is so difficult that growth is maximized by an intermediate level of IPR protection ($\delta = \delta^{\max}$), the welfare-maximizing IPR protection tends to be even weaker. Conversely, if leapfrogging is not so difficult and therefore $\delta = 0$ is the growth-maximizing policy, then it is likely that the strongest possible protection also maximizes social welfare.

8 Conclusion

In a quality-ladder model of endogenous growth, we examined the extent to which growth is facilitated by stronger IPR protection by incorporating into our model two notable features of R&D: (i) for both technical and legal reasons, R&D is easier in competitive sectors where any firm can produce state-of-the-art goods than it is in monopolistic sectors where outsiders cannot produce state-of-the-art goods, and (ii) R&D projects take time to complete, which creates the risk of duplication of

and Lai (2003) and Iwaisako and Futagami (2003) assume that intermediate goods are produced from final goods rather than from labor, reductions in μ always improve welfare. Therefore, in their models, weaker IPR protection is more likely to be beneficial in terms of welfare than in our model.

[27]It can be confirmed that the RHS of (28) is positive from $\pi = (\lambda - 1)/\lambda < \ln\lambda$. Derivation of (28) is available from the corresponding author upon request.

innovation. In this setting, it is shown that stronger IPR protection that lowers the possibility of imitation has three effects on growth. First, it raises the expected profit from R&D, a positive effect. Second, it gradually reduces the number of competitive sectors where R&D is more active, a negative effect. Third, it lowers the market wage and therefore increases the number of R&D researchers employed in each competitive (or monopolistic) sector, a positive effect partially offsetting the second negative effect. The overall effect of stronger IPR on growth is determined by the relative magnitudes of these three effects.

The main finding is that the degree of IPR protection by which the long-term rate of economic growth is maximized differs significantly depending on the extent to which the existence of an incumbent monopolist raises the difficulty of R&D. If leapfrogging in monopolistic sectors is substantially more difficult than innovation in competitive sectors, it is shown that growth is maximized with an intermediate level of IPR protection. Conversely, if the difference in difficulty is small, the strongest possible protection maximizes growth. Both results are comparable to those from earlier studies employing different assumptions. In addition, we show that the dependence of a desirable level of IPR protection on the difficulty of leapfrogging is even more critical when the desirability is measured in terms of welfare. These results illustrate the importance of quantifying the adverse effect of monopoly, not only on output distortions but also on innovative activities. So far, empirical studies have found significant negative effects of monopoly on industry-level innovations, suggesting a case for the imperfect protection of IPR.

Appendix

Proof of Lemma 1

From (21)-(23), function $Z(z,\delta)$ is continuous, and it satisfies the conditions that $\lim_{z\to\underline{z}(\delta)} Z(z,\delta)-z=\infty-\underline{z}(\delta)>0$ and $\lim_{z\to\infty} Z(z,\delta)-z=1/(\beta\overline{V}(\delta)L)-\infty<0$, where $\overline{V}(\delta)$ is defined by (14). Thus, the intermediate value theorem guarantees that for every $\delta \in [0,1]$, there is at least one level of $z^*(\delta) \in (\underline{z}(\delta), \infty)$ such that $Z(z^*(\delta), \delta) - z^*(\delta) = 0$ holds, which is a fixed point of (23). In addition, the assumption that the fixed point is unique means that $Z(z,\delta) - z$ cuts the horizontal z axis from above and does so only once in $z \in (\underline{z}(\delta), \infty)$. Note that $Z_\delta(z,\delta) > 0$ from (20)-(23). Thus, the curve of $Z(z,\delta) - z$ shifts upward as δ increases. This implies that the point of intersection moves rightward as δ increases: i.e., $z^{*\prime}(\delta) > 0$.

Let us examine the condition under which $z^*(\delta)$ is smaller than a. Note that, because $Z(z,\delta) - z$ cuts the horizontal z axis from above and does so only once, the point of intersection (i.e., $z^*(\delta)$) is smaller than a if and only if $Z(a,\delta) - a < 0$. As $Z(a,\delta) = 1/(\beta\overline{V}(\delta)L)$ from (21)-(23), this condition is equivalent to $\delta < 1 - \beta^{-1} + a\pi L \equiv \overline{\delta}(0)$. Thus, $z^*(\delta) < a$ if and only if $\delta < \overline{\delta}(0)$.

Next, let us examine the condition under which $z^*(\delta)$ is smaller than b. Specifically, we want to find a threshold level $\delta^{LF}(b)$ such that $z^*(\delta) < b$ if and only if $\delta < \delta^{LF}(b)$, for a given level of b. Similarly to the above argument, $z^*(\delta) < b$ holds if and only if $Z(b,\delta) - b < 0$. From (22)-(23):

$$Z(b,\delta) = \frac{1}{\beta\overline{V}(\delta)} \frac{\delta + (1-\pi)G(N(b))}{(L - N(b))\delta + G(N(b))L}. \tag{29}$$

When $\delta = 0$, (29) implies that $Z(b,0) = (1-\beta)(1-\pi)/(\beta\pi L) \equiv z^{\min}$. Thus, when $b \leq z^{\min}$, $Z(b,\delta) - b \geq Z(b,0) - b \geq 0$ holds for all $\delta \in [0,1]$ because $Z(b,\delta)$ is increasing in δ. This means that $z^*(\delta) \geq b$ for all $\delta \in [0,1]$ and therefore that $\delta^{LF}(b) = 0$ for $b \leq z^{\min}$.

Now, consider the case of $b > z^{\min}$. Note that in this case, $Z(b,0) - b = z^{\min} - b < 0$. In addition, note that we have shown $Z(a, \overline{\delta}(0)) - a = 0$, which implies $Z(b, \overline{\delta}(0)) - b > 0$, because $b < a$ (recall that $Z(z, \overline{\delta}(0)) - z > 0$ cuts the horizontal z axis from above and does so only once). Thus, from the continuity of $Z(z,\delta) - z$ with respect to δ, the intermediate value theorem guarantees the existence of $\delta^{LF}(b) \in (0, \overline{\delta}(0))$ such that:

$$Z(b, \delta^{LF}(b)) - b = 0. \tag{30}$$

As $Z(b,\delta)$ is strictly increasing in δ, $\delta^{LF}(b)$ is uniquely determined, and $Z(b,\delta) - b < 0$ if and only if $\delta < \delta^{LF}(b)$. Thus, $z^*(\delta) < b$ if and only if $\delta < \delta^{LF}(b)$. The following proves that $\delta^{LF}(b)$ is increasing in b for all $b > z^{\min}$. Let us choose arbitrary values of b_1 and b_2 so that $z^{\min} < b_1 < b_2$. From the definition of $\delta^{LF}(b)$ in (30):

$$Z(b_1, \delta^{LF}(b_1); b_1) - b_1 = 0 = Z(b_2, \delta^{LF}(b_2); b_2) - b_2, \tag{31}$$

where we use expression $Z(z, \delta; b)$ to show explicitly the dependence of function $Z(z, \delta)$ on b. As $Z(z, \delta^{LF}(b_1); b_1) - z$ intersects the horizontal z axis from above and does so only once, the first equation of (31) implies that $Z(z, \delta^{LF}(b_1); b_1) - z < 0$ for all $z > b_1$. Thus, from $b_1 < b_2$ and (31):

$$Z(b_2, \delta^{LF}(b_1); b_1) - b_2 < 0 = Z(b_2, \delta^{LF}(b_2); b_2) - b_2. \tag{32}$$

Note from (7) that $M(z) = 0$ whenever $b \leq z$, which means that the value of function $Z(z, \delta; b)$ does not depend on b whenever $b \leq z$. Thus, when $z = b_2$ (which means that $b_1 < b_2 = z$):

$$Z(b_2, \delta^{LF}(b_1); b_1) = Z(b_2, \delta^{LF}(b_1); b_2). \tag{33}$$

From (32) and (33), we obtain $Z(b_2, \delta^{LF}(b_1); b_2) < Z(b_2, \delta^{LF}(b_2); b_2)$. As $Z_\delta(z, \delta; b) > 0$, this implies that $\delta^{LF}(b_1) < \delta^{LF}(b_2)$; i.e., $\delta^{LF}(b)$ is increasing in b.

Finally, we show the continuity of $\delta^{LF}(b)$ and its boundary property. As $Z(b, \delta; b) - b$ is continuous in δ and b, $\delta^{LF}(b)$, as defined by (30), is continuous for all $b \in (z^{\min}, a)$. In addition, it is continuous at $b = z^{\min}$ because $\delta^{LF}(b) \to 0$ as $b \to z^{\min}$ from $Z(z^{\min}, 0) - z^{\min} = 0$ (recall that $\delta^{LF}(b) = 0$ for all $b \leq z^{\min}$). When $b \to a$, $\delta^{LF}(b) \to \overline{\delta}(0)$ because $Z(a, \overline{\delta}(0)) - a = 0$. This completes the proof.

Proof of Proposition 2

From (4)-(7), function $M(z)$ can be written as:

$$M(z) = (a/b)N((a/b)z). \tag{34}$$

Note that, since $N(\cdot)$ is a continuous function, equation (34) shows that function $M(z)$ changes continuously with respect to b. In addition, it implies that both functions $\mu^*(z, \delta)$ and $z^*(\delta)$ are continuous with respect to b, from (20)-(23) and the assumption that the fixed point of (23) is unique. Therefore, the long-term rate of growth $\gamma_Y^*(\delta)$, defined by (24), is also continuous with respect to b.

Utilizing the continuity of $\gamma_Y^*(\delta)$ with respect to b, we first prove that $\gamma_Y^*(\delta)$ is maximized at $\delta = \delta^{\max} \in (0, \overline{\delta}(0))$ whenever b is sufficiently close to $z^{\min}$. From Lemma 1 and (24):

$$\gamma_Y^*(\delta)/(\ln \lambda) = (1 - \mu^*(z^*(\delta), \delta))G(N(z^*(\delta))) \quad \text{whenever } \delta \geq \delta^{LF}(b), \tag{35}$$

since $M(z^*(\delta)) = 0$ in this case. Note that equation (35) is exactly the same as (19) since expressions $\mu^*(z^*(\delta), \delta))$ and $N(z^*(\delta))$ introduced in Section 5 correspond respectively to $\mu^*(\delta)$ and $\psi(\mu^*(\delta), \delta)$ in Section 4. Note also that $\delta^{LF}(z^{\min}) = 0$ from Lemma 1. Therefore, when $b = z^{\min}$, Proposition 1 shows that $\gamma_Y^*(\delta)$ is maximized at $\delta^{\max} \in (0, \overline{\delta}(0))$, and the maximized rate of growth $g^{\text{noleap}} \equiv \gamma_Y^*(\delta^{\max})$ is strictly positive. Now suppose that b is slightly larger than $z^{\min}$. Then $\delta^{LF}(b)$ is slightly larger than 0, since $\delta^{LF}(b)$ is continuous and strictly increasing from Lemma 1. Given that b is sufficiently close to $z^{\min}$, we have $\delta^{LF}(b) \leq \delta^{\max}$, and therefore, from (35), the growth rate of $g^{\text{noleap}} > 0$ is attainable by setting $\delta = \delta^{\max}$. This is the growth-maximizing IPR policy within the range of $\delta \geq \delta^{LF}(b)$, relying only on the flow of leapfrogging. On the other hand, given b, define $\eta(b)$ as the maximum value of $\gamma_Y^*(\delta)$ within the range of $\delta \in [0, \delta^{LF}(b)]$. Then $\eta(z^{\min}) = \gamma_Y^*(0) = 0$ from $\delta^{LF}(z^{\min}) = 0$. In addition, since $\gamma_Y^*(\delta)$ and $\delta^{LF}(b)$ are continuous with respect to δ and b, $\eta(b)$ is continuous with respect to b. From these we obtain $\eta(b) < g^{\text{noleap}}$ whenever b is sufficiently close to $z^{\min}$. This establishes the first half of Proposition 2.

Let us turn to the case in which b is large and close to a. If $b = a$, $M(z) = N(z)$ from (34), and therefore equation (24) reduces to $\gamma_Y^*(\delta)/(\ln \lambda) = G(N(z^*(\delta)))$. Recall from Lemma 1 that $z^*(\delta) < a$ and $z^{*\prime}(\delta) > 0$ for all $\delta \in (0, \overline{\delta}(0))$. In addition, $N'(z) < 0$ for all $z < a$ and $G'(\cdot) > 0$. Therefore, $\gamma_Y^*(\delta)$ is strictly decreasing in δ for all $\delta \in (0, \overline{\delta}(0))$, implying that $\delta = 0$ is the unique growth-maximizing IPR policy when $b = a$. Now suppose that b is slightly smaller than a. Since $\gamma_Y^*(\delta)$ changes continuously with respect to b, the growth-maximizing choice of δ must be in the neighborhood of $\delta = 0$ given that b is sufficiently close to a. Therefore, to prove that $\delta = 0$ is actually the growth-maximizing policy, it is sufficient to show that $\gamma_Y^*(\delta)$ is downward sloping at $\delta = 0$. Define $\widehat{G}(\delta) \equiv G(N(z^*(\delta)))$ and similarly $\widehat{H}(\delta) \equiv H(M(z^*(\delta)))$. Using (20), the long-term rate of growth (24) can be written as:

$$\gamma_Y^*(\delta)/(\ln \lambda) = \widehat{G}(\delta)\left(1 - \frac{\widehat{G}(\delta) - \widehat{H}(\delta)}{\delta(1 - \widehat{H}(\delta)) + \widehat{G}(\delta)}\right).$$

Differentiating the above expression with respect to δ and substituting $\delta = 0$ into the result yields:

$$\frac{d}{d\delta}\left(\frac{\gamma_Y^*(\delta)}{\ln \lambda}\right)\bigg|_{\delta=0} = \widehat{H}'(0) + \frac{\widehat{G}(0) - \widehat{H}(0)}{\widehat{G}(0)}(1 - \widehat{H}(0)), \tag{36}$$

where $\widehat{H}'(0) = H'(M(z^*(0))) \cdot M'(z^*(0)) \cdot z^{*\prime}(0)$. Note that, since we are considering the case of $b > z^{\min}$, Lemma 1 implies $\delta^{LF}(b) > 0$ and thus $z^*(0) < b$. Therefore $M'(z^*(0)) < 0$ holds, which means that $\widehat{H}'(0)$, the first term in (36), is strictly negative (this holds even at $b = a$). The second term is positive because $0 < \widehat{H}(0) < \widehat{G}(0) < 1$, but it approaches zero as $b \to a$, since in that case $\widehat{H}(0) \to \widehat{G}(0)$ from (34). Therefore, when b is sufficiently close to a, the slope of $\gamma_Y^*(\delta)$ at $\delta = 0$ is negative, which completes the proof of the latter half of Proposition 2.

References

[1] Aghion, P., Harris, C., Howitt, P., and Vickers, J. (2001). Competition, Imitation and Growth with Step-by-Step Innovation, *Review of Economic Studies* 68, 467–492.

[2] Aghion, P., Harris, C., and Vickers, J. (1997). Competition and Growth with Step-by-Step Innovation: An Example, *European Economic Review* 41, 771–782.

[3] Arrow, K. (1962). Economic Welfare and the Allocation of Resources for Invention. In: Nelson, R.R. (Ed.), The Rate and Direction of Inventive Activity: Economic and Social Factors. Princeton: Princeton University Press.

[4] Blundell, R., Griffith, R. and Van Reenen, J. (1993). Knowledge Stocks, Persistent Innovations and Market Dominance: Evidence from a Panel of British Manufacturing Firms, *IFS Working Paper* W93/19.

[5] Blundell, R., Griffith, R. and Van Reenen, J. (1995). Dynamic Count Data Models of Technological Innovation, *Economic Journal* 105, 333–344.

[6] Bresnahan, T.F.(1985). Post-Entry Competition in the Plain Paper Copier Market, *American Economic Review* 75, 15–19.

[7] Dinopoulos, E, and Segerstrom, P. (2005). A Theory of North-South Trade and Globalization. *mimeo*, Stockholm School of Economics.

[8] Geroski, P.A. (1990). Innovation, Technological Opportunity and Market Structure. *Oxford Economic Papers* 42, 586–602.

[9] Grossman, G., and Helpman, E. (1991). Quality Ladder in the Theory of Growth, *Review of Economic Studies* 58, 43–61.

[10] Grossman, G., and Lai, E. (2005). International Protection of Intellectual Property, *American Economic Review* 94(5), 1635–1653.

[11] Gould, D. M., and Gruben, W. C. (1996). The Role of Intellectual Property Rights in Economic Growth, *Journal of Development Economics* 48, 323–350.

[12] Helpman, E. (1993). Innovation, Imitation, and Intellectual Property Rights, *Econometrica* 61, 6, 1247–80.

[13] Heston, A., Summers, R., and Aten, B. (2002). Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania, October 2002.

[14] Iwaisako, T., and Futagami, K. (2003). Patent Policy in an Endogenous Growth Model, *Journal of Economics* (*Zeitschrift für Nationalökonomie*) 78, 239–258.

[15] Jones, C. I. (1995). R&D-based Models of Economic Growth, *Journal of Political Economy* 103, 759–784.

[16] Jones, C. I. and Williams, J. (2000). Too Much of a Good Thing? The Economics of Investment in R&D, *Journal of Economic Growth* 5(1), 65–85.

[17] Judd, K.L. (1985). On the Performance of Patents, *Econometrica* 53, 567–585.

[18] Kortum, S. (1993). Equilibrium R&D and the Patent-R&D Ratio: U.S. Evidence, *American Economic Association Papers and Proceedings* 83, 450–457.

[19] Kortum, S., and Lerner, J. (1998). Stronger Protection or Technological Revolution: What is Behind the Recent Surge in Patenting?, *Carnegie–Rochester Conference Series on Public Policy* 48, 247–304.

[20] Kwan, Y., and Lai, E. (2003). Intellectual Property Rights Protection and Endogenous Economic Growth, *Journal of Economic Dynamics and Control* 27, 853–873.

[21] Lambson, V., and Phillips, K. (2005). Market Structure and Schumpeterian Growth, *Journal of Economic Behavior and Organization*, forthcoming.

[22] Michel, P. and Nyssen, J. (1998). On Knowledge Diffusion, Patents' Lifetime, and Innovation-based Endogenous Growth, *Annales d'Economie et de Statistique* 49/50, 77–103.

[23] Mukoyama, T. (2003). Innovation, Imitation, and Growth with Cumulative Technology, *Journal of Monetary Economics* 50, 361–380.

[24] Nickell, S.J. (1996). Competition and Corporate Performance, *Journal of Political Economy* 104, 724–746.

[25] O'Donoghue, T. and Zweimüller, J. (2004). Patents in a Model of Endogenous Growth, *Journal of Economic Growth* 9(1), 81–123.

[26] Rapp, R.T., and Rozek, R.P. (1990). Benefits and Costs of Intellectual Property Protection in Developing Countries, *Journal of World Trade*, 24(5), 74–102.

[27] Segerstrom, P.S. (1998). Endogenous Growth without Scale Effects, *American Economic Review* 88, 1290–1310.

[28] United States Chamber of Commerce. (1987). Guidelines for Standards for the Protection and Enforcement of Intellectual Property Rights. Washington DC: United States Chamber of Commerce.

[29] Young, A. (1998). Growth without Scale Effects, *Journal of Political Economy* 106, 41–63.