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Abstract

We examine how an introduction of education subsidies affects growth rates, incorporating an uncertain lifetime. We demonstrate that the introduction of subsidies engenders higher growth rates in aging economies, except when the education-tax rate is sufficiently low.

Keywords: Education subsidies; Social security; Uncertain lifetime

JEL: H31; H52; H55

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1 Introduction

Industrial countries are aging rapidly. When we intend to maintain pay-as-you-go financed pension systems with moderate benefits, the tax burdens of the young-working generation become much heavier. Therefore, young-working households who save some after-tax income for a longer retired period cannot afford to invest much in their children. In this paper, we examine effects of education subsidies to complement parental educational investment, considering aging effects.

The effectiveness of education subsidies has been widely discussed by Zhang and Casagrande (1998), Kaganovich and Zilcha (1999), and Wigger (2004), among others. Kaganovich and Zilcha (1999) show that the introduction of education subsidies can engender higher growth rates only if (i) parents have overly low preference levels for their children to invest privately in the children's education, and (ii) parents happen to have a medium preference level for children, and the government has large education revenue (and relatively small pension revenue). In other cases, the usual negative effects of subsidies, which crowd out parental private investment, are so large that the introduction of education subsidies has a negative effect, or none at all, on the growth rate.

Our motivation is to determine the effectiveness of education subsidies in aging countries where parents cannot afford to invest much in their children's education.

Incorporating an uncertain lifetime to a model that is based on Kaganovich and Zilcha (1999), we demonstrate that, in the case of aging economies, the introduction of education subsidies engenders a higher growth rate, except when the education-tax rate is sufficiently low.

2 The model

We consider an overlapping-generations model of endogenous growth, incorporating an uncertain lifetime. The life of a representative individual is divided into three periods: a childhood and a young-working period (each with fixed duration), and a retirement period (of uncertain length). The individual is alive at the beginning of the third period with probability $p \in (0, 1]$. Let N_t denote the number of working-aged individuals, who have n children, in period t. Savings of individuals who have died at the onset of the third period are distributed among the retired individuals as an actuarially fair annuity.[1] The expected rate of return to savings is $(1+\rho_{t+1}) = \frac{(1+r_{t+1})}{p}$, where $(1+r_{t+1})$ is the return of direct holdings of capital. The government supplies education to children and social security benefits to retired individuals by taxing the labor income of young-working individuals. First, we examine an education-subsidy policy (ESP), by which the government provides

[1]This is a simplified version of Blanchard's (1985) model.

both education subsidies and public schools. Secondly, we show the equilibrium of a public-school policy (PSP) in which the government supplies public schools alone. Thereafter, we compare the two growth rates in the next section.

In childhood, individuals only accumulate human capital. Young individuals receive a wage income, which is taxed away. They divide their income among education expenditures for their children, ne_t, their current consumption, c_t^y, and investing in annuities, a_t, for their post-retirement consumption, c_{t+1}^o. Subsequently, living individuals obtain principal and interest from their annuities and consume them with their pension benefits, T_{t+1}, after retirement.

Let h_{t+1} be the human capital level of each individual who is born at time t and called generation $t+1$. Human capital is accumulated according to:

$$h_{t+1} = (e_t + \nu_t)^{\gamma}(e_t^g)^{1-\gamma}.$$

In that equation, $\gamma \in (0,1)$ denotes the efficiency of education input provided privately, such as textbooks and tutors. The education subsidy given by the government is ν_t, and e_t^g is the public-school quality provided by the government.[2] The subsidies are used directly for human capital accumulation as in the methods

[2]We call e_t^g "public-school quality", as do Glomm and Ravikumar (1992), to distinguish strictly between ν_t, which is subsidized to private parental investment, and e_t^g. We can also regard e_t^g as other public educational investment like libraries, museums, and so on.

of Kaganovich and Zilcha (1999). Individuals determine the amount of private investment for their children, taking the value of ν_t and e_t^g as given.

The budget constraints of a member of generation t when young and retired are given, respectively, as $(1-\tau_t-\omega)w_t h_t = c_t^y + ne_t + a_t$ and $(1+\rho_{t+1})a_t + T_{t+1} = c_{t+1}^o$, where τ_t is the social-security-tax rate, and ω is the education-tax rate. The lifetime utility function of generation t is represented as[3]

$$u_t = \ln c_t^y + p\ln c_{t+1}^o + \delta n \ln h_{t+1}.$$

By solving individuals' optimization problems, the optimal values are given as $e_t = \frac{\gamma\delta}{(1+p+\gamma\delta n)}I_t - \frac{(1+p)}{(1+p+\gamma\delta n)}\nu_t$, $c_t^y = \frac{1}{(1+p+\gamma\delta n)}I_t$, $c_{t+1}^o = \frac{p}{(1+p+\gamma\delta n)}(1+\rho_{t+1})I_t$, and $a_t = \frac{p}{(1+p+\gamma\delta n)}I_t - \frac{T_{t+1}}{(1+\rho_{t+1})}$, where $I_t \equiv (1-\tau_t-\omega)w_t h_t + \frac{T_{t+1}}{(1+\rho_{t+1})}$.

The aggregate production function at time t is given as $Y_t = AK_t^\alpha(h_t N_t)^{1-\alpha}$, where Y_t, A, K_t, and $\alpha\in(0,1)$ respectively denote the aggregate output, the productivity parameter, the physical capital that fully depreciates in the production process, and the share of physical capital. Because the factor markets are presumed to be perfectly competitive, the firms take factor prices as given: $w_t = A(1-\alpha)\left(\frac{K_t}{h_t N_t}\right)^\alpha$, $(1+r_t) = A\alpha\left(\frac{K_t}{h_t N_t}\right)^{\alpha-1}$.

The government allocates the $\mu \in [0,1)$ portion of the education-tax revenue to

[3]With an uncertain lifetime, $p\in(0,1]$, this utility form is employed by Pecchenino and Pollard (2002) and Yakita (2001), among others.

education subsidies and the $(1-\mu)$ portion of it to public schools.[4] Here, μ is treated as a predetermined parameter and is constant over time. Budget constraints of the government at time t are

〔Education subsidies〕; $\nu_t = \frac{\mu\omega}{n} w_t h_t,$

〔Public schools〕; $e_t^g = \frac{(1-\mu)\omega}{n} w_t h_t.$

Social security payments are specified as a replacement rate, $\phi \in (0,1)$, on current workers' wage income, as

$$T_t = \phi w_t h_t. \tag{1}$$

Total pension benefits must be balanced by pension revenue:

$$T_t p N_{t-1} = \tau_t w_t h_t N_t. \tag{2}$$

From (1) and (2), the social-security-tax rate is determined as

$$\tau_t = \frac{p\phi}{n} \equiv \tau.$$

[4] When $\mu = 0$, it means PSP is operated.

Note that this contribution rate is increasing in the degree of aging, p.

By employing the capital market-clearing condition, $K_{t+1} = a_t N_t$, the human capital level of generation $t+1$ is represented as

$$h_{t+1} = (\frac{s_\nu^*\{1+\frac{(1-\alpha)}{\alpha}\tau\}\gamma\delta}{p})^\gamma(\frac{(1-\mu)\omega}{n})^{1-\gamma} w_t h_t \equiv h_\nu^* w_t h_t,$$

where

$$s_\nu^* \equiv \frac{p\{1-\tau-(1-\mu)\omega\}}{\{(1+p+\gamma\delta n)+(1+\gamma\delta n)\frac{(1-\alpha)}{\alpha}\tau\}}.$$

Consequently, the per-capita growth rate at time t is constant over time:

$$(1+g_{\nu,t}) \equiv \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}} = \frac{A(1-\alpha)}{n} s_\nu^{*\alpha}(h_\nu^* n)^{1-\alpha} \equiv (1+g_\nu). \qquad : ESP$$

When there are no education subsidies, $\mu = 0$, the per-capita growth rate at time t is given as

$$(1+g_t) \equiv \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}} = \frac{A(1-\alpha)}{n} s^{*\alpha}(h^* n)^{1-\alpha} \equiv (1+g), \qquad : PSP$$

where

$$s^* \equiv \frac{p(1-\tau-\omega)}{\{(1+p+\gamma\delta n)+(1+\gamma\delta n)\frac{(1-\alpha)}{\alpha}\tau\}},$$

and

$$h^* \equiv (\frac{s^*\{1+\frac{(1-\alpha)}{\alpha}\tau\}\gamma\delta}{p})^{\gamma}(\frac{\omega}{n})^{1-\gamma}.$$

3 Education subsidies vs. Public schools

We shall set the ratio of the growth rate of ESP, $(1+g_\nu)$, to the growth rate of PSP, $(1+g)$, as

$$G(p) \equiv \frac{(1+g_\nu)}{(1+g)} = (1-\mu)(\frac{1-\tau-(1-\mu)\omega}{(1-\tau-\omega)(1-\mu)})^{\alpha+(1-\alpha)\gamma}.$$

Inferring that the social-security-tax rate, τ, is influenced by the degree of aging, p, we obtain the following proposition.

Proposition.

(1) When the education-tax rate is higher than ω_H, the ESP implies a higher growth rate than the PSP.

(2) When the education-tax rate is medium, as $\omega_L \leq \omega \leq \omega_H$, the ESP implies:

(2a) a higher growth rate if the degree of aging is higher than $\hat{p}$

(2b) a lower growth rate if the degree of aging is lower than $\hat{p}$

than the PSP.

(3) When the education-tax rate is lower than ω_L, the ESP implies a lower growth

rate than the PSP.

These threshold values are:

$$\omega_H = \frac{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}}{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}}} \in (0,1),$$

$$\omega_L = \frac{(1-\frac{\phi}{n})\{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\}}{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}}} \in (0,1),$$

$$\hat{p} = \frac{n}{\phi}\frac{[(1-\omega)-\{1-(1-\mu)\omega\}(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}]}{\{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\}} \in (0,1).$$

Proof. See Appendix. □

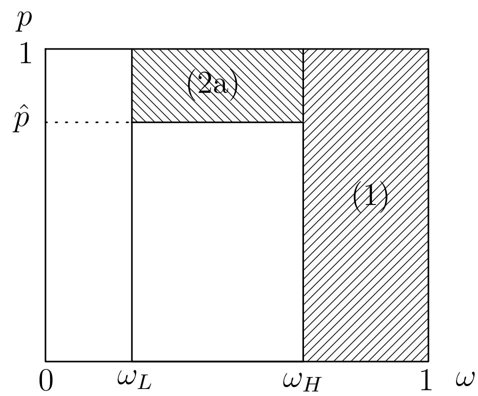

Figure 1: The growth rate of ESP is higher than that of PSP in shaded area.

If public-school investment by the government is overly large compared to private investment, transferring some part of education revenue from public-school investment to education subsidies that complement private investment leads to more efficient allocation and yields higher growth rates. In our model, parental investment becomes smaller in such economies in which parents have smaller after-tax income because of a higher education-tax rate (i.e. Proposition (1)), and those in which parents intend to save a larger part of income for a longer retirement period in spite of heavier social-security burdens (i.e. Proposition (2a)). In other words, except when the education-tax rate is sufficiently low ($0 < \omega < \omega_L$), parental private investment is much smaller than public-school investment in aging economies. In that event, the introduction of education subsidies leads to higher growth rates.

The implications of our results are similar to those of Kaganovich and Zilcha (1999): when private investment is not made sufficiently by parents, the complements to private investment using education subsidies engender higher growth rates under the same tax revenue.[5] However, although the threshold levels of parental preferences for their children hold important meaning for results revealed in Kaganovich and Zilcha (1999), the levels of preferences vary among individuals and are difficult to measure. Our contribution is to have demonstrated that two observable parameters – the degree of aging in the economy and the education-tax

[5]When parental investment is sufficiently large because of a lower education-tax rate and a lower aging degree (the lower left area in Fig. 1), the ESP engenders a lower growth rate than the PSP. This result differs from that of Kaganovich and Zilcha (1999), in which the introduction of education subsidies has no effect on the growth rate if parents invest much in their children because of larger preference for their children.

rates – become thresholds of determining the subsidies' effects by incorporating an uncertain lifetime into the model of Kaganovich and Zilcha (1999).

4 Concluding remarks

We have examined how the introduction of education subsidies affects growth rates, incorporating an uncertain lifetime. We have demonstrated that introducing subsidies engenders higher growth rates in aging economies, except when the education-tax rate is sufficiently low.

Appendix.

Proof. Using $\tau = \frac{p\phi}{n}$, we recognize that the ratio, $G(p)$, is increasing in aging degree, p, as

$$sign(\frac{\partial G(p)}{\partial p}) = sign(\frac{\phi}{n}\frac{\{\alpha + (1-\alpha)\gamma\}\mu\omega}{\{1-\tau-(1-\mu)\omega\}(1-\tau-\omega)}) > 0.$$

Initially, we shall see at $p = 0$. The range of the education-tax rate, which is satisfied with

$$G(p = 0) \geq 1,$$

is expressed as

$$\omega \geq \frac{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}}{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}}} \equiv \omega_H.$$

Therefore, when the education-tax rate is higher than ω_H, $G(p) > 1$ is always satisfied in $p \in (0, 1]$. The ESP always engenders a higher growth rate.

Secondly, in a similar fashion, we check at $p = 1$. The range of the education-tax rate, which is satisfied with

$$G(p = 1) \geq 1,$$

is given as

$$\omega \geq \frac{(1-\frac{\phi}{n})\{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\}}{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}}} \equiv \omega_L.$$

That is, when the education-tax rate is $\omega_L \leq \omega \leq \omega_H$, $G(p) > 1$ is satisfied at least at $p = 1$. In contrast, when the education-tax rate is $\omega < \omega_L$, $G(p)$ remains less than 1 over $p \in (0, 1]$. Consequently, when the education-tax rate is lower than ω_L, the PSP always leads to a higher growth rate.

Finally, in the case where education-tax rate is medium, as $\omega_L \leq \omega \leq \omega_H$, the value of $G(p)$, which is an increasing function in p, is less than 1 at $p = 0$ and more than 1 at $p = 1$. Here, a threshold value of p is satisfied with $G(p) = 1$. This threshold value is expressed as

$$\hat{p} = \frac{n}{\phi} \frac{\left[(1-\omega) - \{1-(1-\mu)\omega\}(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\right]}{\left\{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\right\}} \qquad (\omega_L \leq \omega \leq \omega_H).$$

Accordingly, when the economy's degree of aging is lower than $\hat{p}$, the value of $G(p)$ remains less than 1. When the degree of aging is higher than $\hat{p}$, $G(p)$ is greater than 1. Consequently, it is only in aging economies ($\hat{p} < p$) that the ESP leads to a higher growth rate when the education-tax rate is medium, as $\omega_L \leq \omega \leq \omega_H$. $\square$

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