Discussion Papers In Economics And Business

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この研究は「大学院経済学研究科・経済学部記念事業」
基金より援助を受けた、記して感謝する。

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Technology, Preference Structure, and the Growth Effect of Money Supply*

Jun-ichi Itaya[†] and Kazuo Mino[‡]

December 9, 2005

Abstract

This paper studies the growth effect of money supply in the presence of increasing returns and endogenous labor supply. By using a simple model of endogenous growth with a cash-in-advance constraint, it is shown that the growth effect of money supply depends on the specifications of preference structures as well as on the production technology. Either if the production technology exhibits strong non-convexity or if the utility function has a high elasticity of intertemporal substitution, then there may exist dual balanced-growth equilibria and the impact of a change in money growth depends on which steady state is realized in the long run. It is also shown that there is no systematic relationship between the growth effect of money supply and local determinacy of the balanced growth path.

JEL Classification Numbers: E31, E52, O.42.

Keywords: monetary growth, indeterminacy, increasing returns, non-separable utility.

[*] We are grateful to an anonymous referee for the comments that have been very helpful for revising the paper. We also thank Chong Kee Yip and Shin-ichi Fukuda for their useful comments on earlier versions of this paper. This reseach is financially supported by Grant-in-Aid for Scientific Research (# 1653017).

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1 Introduction

The growth effect of money supply is one of the central research concerns in the money and growth literature. In particular, if the economy allows endogenous growth and if money is not supernuetral, then monetary policy may have profound effects on the long-term behavior of the economy. Most of the existing studies on money and endogenous growth have concluded that in economies with infinitely-lived agents, a higher money growth depresses the long-run growth rate of the economy. The literature has demonstrated that this conclusion holds in a wide variety of monetary endogenous growth models: see, for example, Howitt (1990), De Gregorio (1993), Jones and Manuelli (1995), Marquis and Leffitt (1991 and 1995), Pecorino (1995) and Mino (1997). The mechanism that generates the negative relationship between monetary and real growth in the infinitely-lived, representative agent economy has an intuitive appeal: a higher growth of money supply raises the inflation tax on the rate of returns to physical (and human) capital, so that investment will be reduced to yield a lower growth rate of the economy.[1]

The purpose of this paper is to re-examine the fundamental observation mentioned above. We add two extensions to the conventional models employed by the existing studies on money and endogenous growth. First, the model economy examined in this paper is subject to increasing returns to scale. Second, we use a more general utility function in which consumption and leisure can be non-separable. In the real business cycle studies, it has been known that both production technology and preference structure are crucial for policy impacts as well as for the dynamic behavior of the economy. We demonstrate that this conclusion can be confirmed in the monetary growth models as well. Namely, both the production technology and the preference structure may play essential roles in determining the growth effect of money supply. The analytical framework of our discussion is based on the model examined by Benhabib and Farmer (1994) that has been frequently used to discuss sunspot-driven business cycles. We introduce money into an endogenous growth version of the base model via a cash-in-advance constraint. Since the model allows

[1] In overlapping generations models with endogenous growth, however, inflation and growth may be positively correlated: see, for example, Mino and Shibata (2000). Itaya and Mino (2003) and Jha et al. (2002) examine endogenous monetary growth models based on transactions cost approach to money.

labor-leisure choice, money is not superneutral in the long-run equilibrium, even though the cash-in-advance constraint applies to consumption alone.[2]

Our main finding is that the growth effect of money supply hinges on the degree of increasing returns characterizing production technology as well as on the elasticity of intertemporal substitution in consumption. More specifically, if the elasticity of intertermporal substitution in consumption is lower than one and if production externalities are relatively small, we obtain the conventional results, that is, the balanced-growth path is uniquely given and a rise in money growth reduces the long-term growth rate. In contrast, if the elasticity of intertemporal substitutability is smaller than one but the degree of increasing returns is high enough, then there may exist dual balanced-growth paths. In this case an increase in money growth decreases (increases) the growth rate on the balanced-growth path with a higher (lower) growth rate. Those results are, however, reversed if the elasticity of intertemporal substitution in felicity exceeds one. Under such a preference characterization, if the degree of increasing returns is relatively small, then dual balanced-growth paths may exit. If this is the case, a rise in monetary growth increases (decreases) the long-term growth rate in the high-(low-) growth equilibrium. Moreover, if both the degree of increasing returns and the elasticity of intertemporal substitutability are high, the balanced-growth path can be unique and a higher money growth accelerates long-run growth.

In addition to the growth effect of money supply, we investigate dynamic properties of the model economy out of the balanced-growth equilibrium. We find that, as well as in the real business cycle models without money, the uniqueness of transitional path of the monetary economy heavily depends upon specifications of production technology and preference structure. For example, in the presence of unique balanced-growth equilibrium, the converging equilibrium path is uniquely determined when both the degree of increasing returns and the elasticity of intertemporal substitutability are relatively small. By

[2]It is to be noted that Fukuda (1996) also introduces money into the Benhabib and Famer model with a separable utility function. Since we use a more general preference structure, our discussion involves Fukuda's main conclusions as special cases. Since Fukuda (1996) mainly focuses on the case of exogenous growth in which long-term growth cannot be sustained, our study overlaps a part of his paper that treats the case of endogenous growth.

contrast, when either the degree of increasing returns or the elasticity of intertemporal substitution in consumption is high, a continuum of converging paths emerges around the balanced-growth equilibrium. Additionally, it is shown that if there are dual balanced-growth equilibria, one of them is determinate and the other is indeterminate. Which balanced-growth path satisfies determinacy again depends upon the specifications of technology and preferences. Based on those findings, we consider the relationship between the determinacy of equilibrium and the growth effect of money supply.

The rest of the paper is organized as follows. Section 2 sets up an analytical framework for the subsequent discussions. Sections 3 contains our main discussions. It considers the steady-state effects of a change in the money growth rate as well as the local dynamic behavior of the balanced-growth under alternative conditions on the parameter values characterizing the technology and preferences. Section 4 presents intuitive implications of our main findings. Section 5 concludes the paper.

2 The Analytical Framework

2.1 Production

In formulating the production side of the economy, we follow Benhabib and Farmer (1994). There are many identical firms whose number is normalized to one. The production technology of each firm is specified as

$$y = k^a l^{1-a} \bar{k}^{\alpha - a} \bar{l}^{\beta - 1 + a}, \quad 0 < a < 1, \quad \alpha > a, \quad \beta > 1 - a,$$

where y is output, k is capital and l is labor. Here, $\bar{k}^{\alpha-a}$ and $\bar{l}^{\beta-1+a}$ represent positive external effects associated with the capital and labor of the economy at large. We assume that the private technology satisfies constant returns, but the social technology involving the external effects exhibits increasing returns to scale.

The final goods and factor markets are competitive, so that the rate of return to capital, r, and the real wage rate, w, are equal to the marginal products of private capital and labor, respectively:

$$r = ak^{a-1} l^{1-a} \bar{k}^{\alpha-a} \bar{l}^{\beta-1+a},$$

$$w = (1-a)\, k^a l \bar{k}^{1-a} \bar{l}^{\beta-1+a}.$$

Since we have assumed that the total number of firms is one, it always holds that $\bar{k} = k$ and $\bar{l} = l$ in equilibrium. In this paper we focus on the situation in which endogenous growth is sustained by capital externalities. This means that $\alpha = 1$, and hence the aggregate production function at the social level is

$$y = kl^{\beta} \tag{1}$$

and the equilibrium levels of return to capital and real wage rate are respectively expressed as

$$r = al^{\beta}, \tag{2}$$

$$w = (1 - a)\, kl^{\beta - 1}. \tag{3}$$

2.2 Households

There is no population growth and the number of households is also normalized to be one. The representative household maximizes a discounted sum of utilities

$$U = \int_0^{\infty} e^{-\rho t} u\,(c, l)\, dt, \quad \rho > 0$$

subject to

$$\dot{a} = ra + wl - c + \tau - im,$$

$$a = k + m,$$

$$c \leq m.$$

In the above, c is consumption, a total asset, m real money balances, i the nominal interest rate and τ denotes lump-sum transfers from the government. We assume that the government does not issue debt so that the total wealth of the household consists of capital and money holdings. In this paper, we assume that the cash-in-advance constraint is applied to consumption spending alone.

The instantaneous utility function, $u\,(c, l)$, is assumed to be monotonically increasing in consumption and decreasing in labor. In what follows we use a specific form of function such that

$$u\,(c, l) \equiv \frac{c^{1-\sigma}}{1-\sigma} \Lambda\,(l)\,, \quad \sigma > 0, \;\; \sigma \neq 1, \tag{4}$$

where $\Lambda\,(l)$ is strictly positive and it satisfies the following conditions:

$$\frac{\Lambda'\,(l)}{1-\sigma} < 0, \qquad \frac{\Lambda''\,(l)}{1-\sigma} < 0, \qquad \frac{\sigma}{1-\sigma} \Lambda\,(l)\, \Lambda''\,(l) + \Lambda'\,(l)^2 < 0. \tag{5}$$

These conditions ensure that the felicity function decreases with labor and it is strictly concave in consumption and labor.

To obtain the necessary conditions for an optimum, let us define the current-value Hamiltonian function:

$$H = \frac{c^{1-\sigma}}{1-\sigma}\Lambda(l) + \lambda[ra + wl - c + \tau - im] + \mu(m - c) + \gamma(a - k - m),$$

where λ is the costate variable of a, and μ and γ are Lagrange multipliers. If we focus on an interior solution, the necessary conditions for an optimum are given by the following:

$$\partial H/\partial c = c^{-\sigma}\Lambda(l) - (\lambda + \mu) = 0, \tag{6}$$

$$\partial H/\partial l = \frac{c^{1-\sigma}}{1-\sigma}\Lambda'(l) + \lambda w = 0, \tag{7}$$

$$\partial H/\partial m = -\lambda i + \mu - \gamma = 0, \tag{8}$$

$$\partial H/\partial k = -\gamma = 0, \tag{9}$$

$$\dot{\lambda} = \rho\lambda - \partial H/\partial a = \lambda(\rho - r) - \gamma, \tag{10}$$

together with the transversality condition: $\lim_{t\to\infty} a\lambda e^{-\rho t} = 0$ and the initial condition on a.

2.3 Money Growth and Market Equilibrium

Since this paper does not focus on comparison of the alternative money supply rules, we assume the simplest money supply regime. Namely, the growth rate of nominal stock of money is kept constant and the newly created money is distributed to the households as lump-sum transfers. Denoting the growth rate of money supply by θ, we see that the real money balances behave in accordance with

$$\dot{m} = m(\theta - \pi). \tag{11}$$

The government's flow budget constraint is thus given by $\tau = \theta m$.

The market equilibrium condition for the final good is

$$y = c + \dot{k}. \tag{12}$$

For notational simplicity, we assume that capital does not depreciate. Finally, the nominal interest rate is determined by the Fisher equation:

$$i = r + \pi, \tag{13}$$

where π denotes the rate of inflation.

2.4 The Dynamic System

Equations (8) and (9) yield $\mu/\lambda = i$. Thus using (6), (7), (8) and (9), we obtain

$$-\frac{c}{1-\sigma}\frac{\Lambda'(l)}{\Lambda(l)} = \frac{w}{1+i}. \tag{14}$$

This states that the marginal rate of substitution between labor and consumption is equal to the relative price between leisure and consumption. Notice that when the cash-in-advance constraint is binding, an additional unit of consumption needs an additional unit of real money balances so that the effective price of consumption is one plus the nominal interest rate, i, which represents an opportunity cost of money holding. Therefore, $w/(1+i)$ expresses the real wage rate in terms of the effective price of consumption. Using (2), (3), (13) and (14), we find that the rate of inflation is expressed as

$$\pi = -(1-a)(1-\sigma)\frac{\Lambda(l)\, l^{\beta-1}}{\Lambda'(l)}x - al^{\beta} - 1 \equiv \pi(x, l), \tag{15}$$

where $x \equiv k/c$.

As we focus on an interior equilibrium, it holds that $c = m$ for all $t \geq 0$. Hence, from (11) and (15) consumption changes according to

$$\frac{\dot{c}}{c} = \theta - \pi(x, l). \tag{16}$$

Equations (3) and (7) give the following:

$$-\frac{c^{1-\sigma}}{1-\sigma}\Lambda'(l) = \lambda(1-a)\, kl^{\beta-1}.$$

Logarithmic differentiation of both sides of the above equation with respect to time yields

$$(1-\sigma)\frac{\dot{c}}{c} + \eta(l)\frac{\dot{l}}{l} = \frac{\dot{\lambda}}{\lambda} + \frac{\dot{k}}{k} + (\beta - 1)\frac{\dot{l}}{l},$$

where $\eta(l) \equiv \Lambda''(l)\, l/\Lambda'(l) > 0$. Therefore, using (9), (10), (12) and (16), we obtain

$$\frac{\dot{l}}{l} = \frac{1}{\eta(l) + 1 - \beta}\left[(\sigma - 1)(\theta - \pi(x, l)) + (1 - a)\, l^{\beta} + \rho - \frac{1}{x}\right], \tag{17}$$

in which we assume that $\eta(l) + 1 - \beta \neq 0$. Additionally, by use of (1), (12), (16) and $\dot{x}/x = \dot{k}/k - \dot{c}/c$, we can derive

$$\frac{\dot{x}}{x} = l^{\beta} - \frac{1}{x} - \theta + \pi(x, l). \tag{18}$$

To sum up, (17) and (18) constitute a complete dynamic system with respect to l and $x\,(\equiv k/c)$.

3 Growth Effect of Money Supply

3.1 The Balanced-Growth Equilibrium

From (17) and (18) the steady-state levels of l and x satisfy the following conditions:

$$(\sigma - 1)(\theta - \pi(x, l)) + (1 - a)\, l^{\beta} + \rho - \frac{1}{x} = 0, \tag{19}$$

$$l^{\beta} - \frac{1}{x} - \theta + \pi(x, l) = 0. \tag{20}$$

The steady-state conditions given above show that on the lanced-growth path capital consumption and real money balances grow at a common rate, while the level of employment stays constant over time. Manipulating (15), (19) and (20), we obtain

$$\begin{aligned} & -(1 - a)(1 - \sigma)\frac{\Lambda(l)\, l^{\beta - 1}}{\Lambda'(l)} \\ = \quad & \left[\left(1 - \frac{a}{\sigma}\right) l^{\beta} + \frac{\rho}{\sigma}\right]\left[a\left(1 - \frac{1}{\sigma}\right) l^{\beta} + \frac{\rho}{\sigma} + \theta + 1\right]. \end{aligned} \tag{21}$$

If this equation has a positive solution, then there exists a steady-state value of l. In order to conduct a graphical analysis of (21), let us define the following functions:

$$F(l) \equiv -(1 - a)(1 - \sigma)\frac{\Lambda(l)\, l^{\beta - 1}}{\Lambda'(l)}, \tag{22}$$

$$G(l) \equiv \left[\left(1 - \frac{a}{\sigma}\right) l^{\beta} + \frac{\rho}{\sigma}\right]\left[a\left(1 - \frac{1}{\sigma}\right) l^{\beta} + \frac{\rho}{\sigma} + \theta + 1\right]. \tag{23}$$

A positive solution of $F(l) - G(l) = 0$ gives the steady-state level of l.

Since the capital share of income, a, may be about 0.3 in reality, if $\sigma < a$, we should assume that the intertemporal elasticity of consumption, $1/\sigma$, takes an implausibly high value. Thus, in order to focus on economically plausible cases, we assume that $\sigma > a$. Inspecting the graph of $G(l)$ function defined above, we can easily confirm that if $\sigma > 1$ (resp. $a < \sigma < 1$) and if $F(l)$ is monotonically decreasing (resp. increasing) for all feasible l, then $F(l) - G(l) = 0$ has a unique positive solution. Otherwise, there may exist two solutions at most. The exact shape of $F(l)$ cannot be determined without further specifications. Therefore, we use the following function as a typical example to obtain clear results:

$$\Lambda(l) = \frac{1}{\chi}(1-l)^{\chi(1-\sigma)}, \quad 0 < \chi < 1, \tag{24}$$

which is defined on $l \in [0,1]$. Since $1-l$ expresses leisure, (24) means that the instantaneous felicity of the household is given by a standard Cobb-Douglas function of consumption and leisure in such a way that

$$u(c,l) = \frac{\chi^{-1}\left(c(1-l)^{\chi}\right)^{1-\sigma}}{1-\sigma}. \tag{25}$$

Note that this function fulfills the concavity conditions in (5) for $\sigma > \chi/(1+\chi)$. Since we have assumed that $\sigma > a$, in what follows we assume that $\sigma > \hat{\sigma} \equiv \max\left(\chi/(1+\chi)/\chi, a\right)$.

Given the above specification, we find that $F(l)$ is written as

$$F(l) = \frac{1}{\chi}(1-a)(1-l)\, l^{\beta-1}.$$

Figures 1 (a) and (b) depict the graphs of $F(l)$ and $G(l)$ under $\sigma > 1$. Figure 1 (a) shows the case of $\beta < 1$. In this situation, $F(l)$ is monotonically decreasing and satisfies that $\lim_{l\to+0} F(L) = +\infty$ and $F(1) = 0$. In addition, $G(l)$ is a monotonically increasing and strictly concave function for $l \geq 0$. Thus $F(l) - G(l) = 0$ has a unique, positive solution. Figure 1 (b) assumes that $\sigma > 1$ and $\beta > 1$. Under these conditions, $F(0) = F(1) = 0$ and sign $[F'(l)] =$ sign $\left[1 - \frac{1}{\beta} - l\right]$. In addition, $G(l)$ is increasing, strictly convex and $G(0) = (\rho/\sigma)(\rho/\sigma + \theta + 1) > 0$. Thus if $F(l) - G(l) = 0$ has a solution, it involves two roots for $l \geq 0$.

Figures 2 (a), (b), (c) and (d) display the graphs of $G(l)$ and $F(l)$ for $\hat{\sigma} < \sigma < 1$. In these cases, the graph of $G(l)$ is inverse-U shaped and $G(\bar{l}) = 0$, where $\bar{l} = \left[\frac{\rho+\sigma(\theta+1)}{a(1-\sigma)}\right]^{1/\beta}$. Thus when $\beta < 1$, there may exist dual steady-state levels of l if $\bar{l} < 1$, while the steady state is uniquely determined if $\bar{l} > 1$: see Figures 2 (a) and (b). On the other hand, in the case of $\beta > 1$, $F(l) - G(l) = 0$ has a unique positive solution if $\bar{l} < 1$, and there are two solutions at most if $\bar{l} > 1$: see Figures 2 (c) and (d).

3.2 Effects of Monetary Expansion

From (1), (12), (19) and (20) the balanced-growth rate and the steady-state rate of inflation are respectively given by

$$g = \frac{1}{\sigma}\left(al^{\beta} - \rho\right), \tag{26}$$

$$\pi = \theta - g = \theta + \frac{1}{\sigma}(\rho - al^{\beta}), \tag{27}$$

where g denotes the balanced growth rate of y, c, k and m. Hence, the balanced-growth rate increases with l and the rate of inflation decreases with l. This implies that the balanced-growth path with a higher employment level attains a higher growth rate and a lower rate of inflation, while the other steady state with a lower employment exhibits a lower growth rate and a higher rate of inflation.

Considering the steady-state characterization displayed above, we can easily examine the effects of an increase in the rate of money growth, θ, on the steady-state rates of income growth and inflation. Notice that an increase in θ causes an upward shift of the graph of $G(l)$. Since the graph of $F(l)$ is unaffected by a change in θ, it is easy to establish the following comparative statics results:

Proposition 1 *If functions $F(l)$ and $G(l)$ given by* (22) *and* (23) *satisfies $G'(l) > F'(l)$ at the steady state, a rise in money growth rate lowers the balanced-growth rate. If $G'(l) < F'(l)$ at the steady state, an increase in money growth raises the balanced-growth rate.*

In the context of graphical analysis of $G(l)$ and $F(l)$ functions, we find that if $G'(l) > F'(l)$, the negative relation between θ and l holds in the balanced-growth equilibrium. In contrast, if $G'(l) < F'(l)$ is satisfied, the steady-state level of l (so the balanced-growth rate, g) is positively related to the money growth rate, θ. It is worth emphasizing that these graphical relations do not depend on our specification of $\Lambda(l)$ function. Since the shapes of $G(l)$ and $F(l)$ functions are characterized by the parameters concerning preferences and technology, the above propositions clearly demonstrates that the growth effect of money supply relies critically on the preference structure as well as on the production technology.

If $\Lambda(l)$ function is given by (24), then in the presence of a unique balanced-growth path, a rise in money growth rate, θ, lowers the long-term growth rate of income and raises the rate of inflation if $\beta < 1$: see Figures 1 (a) and 2 (b). If $\beta > 1$, then, as shown by Figure 2 (b), a higher θ increases the growth rate of income. If $\sigma > 1$ and if dual balanced-growth paths exist, then a rise in θ depresses the growth rate of income and increases the rate of inflation at the high-growth equilibrium, while it increases the growth rate of income at the low-growth equilibrium: see Figure 1 (b). If $\hat{\sigma} < \sigma < 1$, the opposite results hold (i.e. Figure 2 (b)). Table 1 summarizes these findings. In this table, *High* and *Low* respectively denote the balanced growth path (BGP) with a higher and lower growth rate. In addition, $\hat{\sigma} = \max\ (\chi/(1+\chi), a)$ and $\bar{l} = \left[\frac{\rho+\sigma(\theta+1)}{a(1-\sigma)}\right]^{1/\beta}$.[3]

	$\sigma > 1$	$\hat{\sigma} < \sigma < 1$ and $\bar{l} < 1$	$\hat{\sigma} < \sigma < 1$ and $\bar{l} > 1$
$\beta < 1$	unique BGP, $dg/d\theta < 0$	dual BGPs *High* : $dg/d\theta > 0$ *Low* : $dg/d\theta < 0$	unique BGP $dg/d\theta < 0$
$\beta > 1$	dual BGPs *High* : $dg/d\theta < 0$ *Low* : $dg/d\theta > 0$	unique BGP $dg/d\theta > 0$	dual BGPs *High* : $dg/d\theta < 0$ *Low* : $dg/d\theta > 0$

Table 1

3.3 Dynamics

Next, let us examine the dynamic property of our model economy around the balanced-growth equilibrium. When we linealize the dynamic system consisting of (17) and (18) at the stationary point, the coefficient matrix of the resulting linearized system is given by

$$J = \begin{bmatrix} \frac{l}{\eta(l)+1-\beta}\left[(1-\sigma)\pi_l(l,x) + (1-a)\beta l^{\beta-1}\right] & \frac{l}{\eta(l)+1-\beta}\left[(1-\sigma)\pi_x(l,x) + \frac{1}{x^2}\right] \\ x\left[\beta l^{\beta-1} + \pi_l(l,x)\right] & x\pi_x(l,x) + \frac{1}{x} \end{bmatrix}, \tag{28}$$

[3] The definition of $\bar{l}$ states that $\bar{l}$ is relatively large when θ, ρ and σ have large values and it is relatively small when β and a are high.

where l and x indicate their steady-state values. Since we treat an endogenous growth model with an Ak technology, neither of the initial values of l nor x ($\equiv k/c$) are predetermined. This means that if the linearized system is completely unstable around the steady state (i.e. the stationary point is a source), the economy can always stay on the balanced growth path so that local determinacy of equilibrium is established. However, if the steady state of the dynamic system is either a saddle-point or a sink, then there exists a continuum of converging paths and hence local indeterminacy emerges. Keeping these facts in mind, we can show the following analytical results:

Proposition 2 *The balanced-growth equilibrium has a local saddlepoint property, either if* $\eta(l) + 1 - \beta > 0$ *and* $F'(l) > G'(l)$ *or if* $\eta(l) + 1 - \beta < 0$ *and* $F'(l) < G'(l)$ *at the steady state.*

The proof of this proposition is shown in Appendix 1 of the paper. It is worth emphasizing that the stability property is related to the sign of $\eta(l) + 1 - \beta$, while the characterization of the steady state and the growth effect of money supply depend critically on the sign of $1 - \beta$. Therefore, if we consider both the stability and steady state characterization, we should classify the cases into $\beta < 1$, $1 < \beta < 1 + \eta(l)$ and $\beta > 1 + \eta(l)$. Applying these results to the case of Cobb-Douglas utility given by (25), we find that local determinacy of equilibrium at each steady state may be summarized in Table 2. In the table D and I respectively indicate local determinacy and indeterminacy of BGPs. Appendix 2 presents the detail to drive the results shown in the table.

	$\sigma > 1$	$\hat{\sigma} < \sigma < 1$ and $\bar{l} < 1$	$\hat{\sigma} < \sigma < 1$ and $\bar{l} > 1$
$\beta < 1$	unique BGP, D	dual BGPs *High* : I *Low* : ?	unique BGP ?
$1 < \beta < 1 + \eta(l)$	dual BGPs *High* : ? *Low* : I	unique BGP I	dual BGPs *High* : ? *Low* : I
$\beta > 1 + \eta(l)$	dual BGPs *High* : I *Low* : D	unique BGP D	dual BGPs *High* : I *Low* : D

Table 2

Tables 1 and 2 demonstrate that there are various patterns of the relationship between the growth effect of money supply and determinacy of equilibrium. That is, we may have the counter-intuitive, positive relation between money growth and real growth, even if determinacy is satisfied around the balanced-growth path. At the same time, the standard negative relation between money growth and real growth can be established in the long-run equilibrium with local indeterminacy. For example, when $\sigma > 1$ and $\beta < 1$, the unique balanced growth path exhibits local determinacy. In contrast, although there is a unique balanced-growth equilibrium, it is locally indeterminate if $1 < \beta < 1 + \eta(l)$ and $\sigma < 1$. However, if β exceeds $1 + \eta(l)$, the long-run equilibrium becomes locally determinate. In these cases, the growth effect of money supply is negative if $\sigma > 1$ and $\beta < 1$, while it is positive if $\beta > 1$ and $\sigma < 1$.

3.4 Separable Utility

In order to highlight the role of the non-separable utility in our discussion, it is useful to examine a simple model with an additively separable utility function. Suppose that the instantaneous utility is given by

$$u(c, l) = \log c - \Phi(l), \tag{29}$$

where $\Phi'(l) > 0$ and $\Phi''(l) > 0$. It is easy to confirm that the dynamic equation of l in this case may be obtained by setting $\sigma = 1$ in (17), while the behavior of x is the same as (18). Thus the complete dynamical system in the case of separable utility consists of the following differential equations:

$$\begin{aligned} \frac{\dot{l}}{l} &= \frac{1}{\eta(l) + 1 - \beta}\left[(1-a)\,l^{\beta} + \rho - \frac{1}{x}\right], \\ \frac{\dot{x}}{x} &= l^{\beta} - \frac{1}{x} - \theta + \pi(x, l), \end{aligned}$$

where $\eta(l) = \Phi''(l)\,l/\Phi'(l)\,(> 0)$ and the rate of inflation is given by

$$\pi(l, x) = \frac{(1-a)\,l^{\beta-1}}{\Phi'(l)}x - al^{\beta} - 1.$$

As a result, the steady-state conditions under which $\dot{l} = \dot{x} = 0$ can be reduced to the following equation whose solution gives the steady state value of l:

$$\frac{(1-a)\, l^{\beta-1}}{\Phi'(l)} = (1+\rho+\theta)\left[(1-a)\, l^{\beta} + \rho\right]. \tag{30}$$

We see that the left-hand side of (30) is monotonically decreasing (resp. increasing), if $\eta(l) + 1 - \beta > 0$ (resp. $\eta(l) + 1 - \beta < 0$) for all $l > 0$. Figures 3 (a) and (b), in which $\hat{F}(l) \equiv (1-a)\, l^{\beta-1}/\Phi'(l)$ and $\hat{G}(l) \equiv (1+\rho+\theta)\left[(1-a)\, l^{\beta} + \rho\right]$, show the existence of the steady-state level of l. As Figure 3 (a) depicts, if $\eta(l)+1-\beta > 0$, the balanced-growth equilibrium is uniquely given. If $\eta(l)+1-\beta < 0$, then there are at most two steady states: see Figure 3 (b). A typical example of $\Phi(l)$ function is

$$\Phi(l) = \frac{l^{\eta+1}}{\eta+1},$$

where η is a positive constant. This specification of disutility of labor has been frequently employed in the real business cycle literature. Since η us constant for all $l > 0$ in this case, the global argument given above always holds.

It is also to be noted that in the case of unique balanced-growth path, it is locally determinate. In the case of dual steady states, the BGP with a lower growth rate (a lower level of l) is locally indeterminate and that with a higher growth rate (a higher level of l) is locally determinate. The positive relation between the balanced-growth rate and the monetary expansion rate, θ, holds only at the low-growth BGP in the case of dual steady states. These results mean that, as compared to the case of non-separable utility, in the model with a separable utility function the degree of labor externalities, i.e. the magnitude of β, plays a predominant role in determining the growth effect of money supply as well as the determinacy of equilibrium.

To sum up, we have shown:

Proposition 3 *If the utility function is given by (29) and if $\beta < 1 + \eta(l)$ for all $l > 0$, then the balanced-growth equilibrium is unique and globally determinate. If $\beta > 1 + \eta(l)$ for all l, there may exist dual steady states at most. In the presence of dual steady states, the balanced-growth equilibrium with a lower growth rate is locally determinate and the one with a higher growth rate is locally indeterminate, provided that dual balanced-growth equilibria exist.*

4 Discussion

4.1 The Roles of Technology and Preference

In this section we present intuitive implications of the roles of production technology and preference structure in determining the growth effect of money supply. To do so, we assume that the economy initially stays on the balanced-growth path and that there is a permanent rise in the growth rate of nominal money supply, θ.

Notice that the steady-state rate of nominal interest rate satisfies $i = r + \pi = (\sigma - 1)\, g + \rho + \theta$, because $r = \sigma g + \rho$ and $\pi = \theta - g$ in the balanced-growth equilibrium. First, consider the case of separable utility given by (29). As was shown, in this case the growth effect of money depends mainly on the magnitude of β. Noting that under the separable utility assumption $i = \rho + \theta$ on the balanced-growth path, we can rewrite (30) as

$$\Phi'(l)\left[(1-a)\, l^{\beta} + \rho\right] = \frac{(1-a)\, l^{\beta-1}}{1+\rho+\theta}. \tag{31}$$

The left-hand side of (31) is the steady-state expression for the marginal rate of substitution between consumption and labor, while the right-hand side shows the steady-state level of real wage rate in terms of effective price.[4] We may interpret the left- and right-hand sides of (31) as the steady-state representations of the labor supply and labor demand curves, respectively. Figure 4 illustrates the labor supply and demand curves in (l, w) space. A permanent rise in θ increases the steady-state rate of nominal interest rate, which causes a downward shift of the labor demand curve. If $\beta < 1$, the labor demand curve has a negative slope. Thus an increase in θ reduces the steady-state value of employment, l, which lowers the balanced growth rate because of the relation given by (26): see panel (a) in Figure 4. In words, a higher rate of inflation caused by an increase in the money growth rate raises the opportunity cost of money holding, and hence the effective cost of consumption also rises due to a tighter cash-in-advance constraint. Consequently, the households substitute consumption with leisure, which reduces labor supply so that the balanced-growth rate declines.

If $\beta > 1$, then the labor demand curve has a positive slope. As shown by Proposition

[4] More precisely, the left hand side of (31) is the ratio of the marginal rate of substitution and capital stock, while the right hand side expresses the ratio of the (effective) real wage and capital stock.

1, in this case there are two steady states and it is straightforward to confirm that the long-run labor demand curve is steeper (resp. flatter) than the labor supply curve at the steady state with a lower (resp. higher) level of l.[5] Therefore, an downward shift of the labor demand curve caused by a rise in θ increases (resp. decreases) the steady-state level of l at the low-(resp. high-) growth steady state. Figure 3 (b) shows the case of lower-growth steady state where a rise in the nominal interest rate caused by a higher θ enhances employment and thus income growth. By contrast, Figure 3 (c) depicts the case of high-growth steady state where a higher θ lowers employment and the balanced-growth rate.

We should note that the growth effect of money supply in the case of separable utility essentially has the same effect of labor income taxation in the absence of cash-in-advance constraint. If there is no money and if real factor income is subject to distortionary taxation, the budget constraint for the household's optimization problem is replaced with

$$\dot{k} = (1 - \tau_k)\, rk + (1 - \tau_l)\, wl - c,$$

where τ_k and τ_l denote the tax rates on capital and labor income, respectively. Given this constraint, the optimization condition (14) becomes

$$c\Phi'(l) = (1 - \tau_l)\, w.$$

Thus the labor market equilibrium condition corresponding to (31) is modified as

$$[(1 - a)\, l^{\beta} + \rho]\Phi'(l) = (1 - \tau_l)(1 - a)\, l^{\beta - 1}.$$

Replacing $1 - \tau_l$ with $(1 + \theta + \rho)^{-1}$ in the above equation, we see that a rise in the rate of labor income tax, τ_l, yields the same growth effect as that of an increase in money growth rate, θ. Hence, when the preference structure satisfies additive separability between

[5] The slope of the graph of LHS in (31) is $(1-a)\,\beta l^{\beta-1}\Phi'(l) + \left[(1-a)\, l^{\beta} + \rho\right]\Phi''(l)$, and that of RHS is $(1-a)(\beta-1)\, l^{\beta}/(1+\rho+\theta)$. Remember that the graphs given in Figure 3 satisfies

$$\begin{aligned} \hat{F}'(l) &\equiv (1-a)(1-\beta)\, l^{\beta-2}/\Phi'(l) - (1-a)\, l^{\beta-1}\Phi''(l)/\phi'^2(l) \\ \hat{G}'(l) &= (1\_r+\theta)(1-a)\,\beta l^{\beta-1}. \end{aligned}$$

Thus, using $\hat{G}(l) = \hat{F}(l)$, we find that if $\hat{F}'(l) > \hat{G}'(l)$ at the intersection, the slope of demand curve is steeper than that of the labor supply curve, and vise versa.

consumption and labor, the inflation tax plays the role exactly analogous to that played by the labor income taxation.[6]

When the utility function is not separable between consumption and labor, the nominal interest rate in the steady state is $i = (\sigma - 1) g + \rho + \theta$. By use of $g = (1/\sigma)(r - \rho)$, we find that the relation between the long-term nominal interest rate and the steady-state level of employment is

$$i = \left(1 - \frac{1}{\sigma}\right) al^{\beta} + \rho + \theta. \tag{32}$$

This equation demonstrates that if $\sigma \geq 1$, the nominal interest rate increases with the employment level (so the balanced-growth rate) in the steady state, while it decreases with l if $\sigma < 1$. In view of (32), the equality between the marginal rate of substitution and the real wage rate in the steady state can be written as

$$-\frac{1}{1-\sigma}\Lambda'(l)\left[(1-a)l^{\beta} + \rho\right] = \frac{(1-a)\,l^{\beta-1}}{1+i}, \tag{33}$$

where i is given by (32). Since the case of $\sigma \geq 1$ yields essentially the same outcomes as those obtained in the model with separable utility, we examine the case of $\hat{\sigma} < \sigma < 1$. In order to employ graphical interpretation displayed in Figure 4, we take i as a given parameter. Inspecting the graph of (33) reveals that if $\beta > 1$, the labor demand curve has a positive slope. Therefore, if the labor supply curve has a positive slope and if the demand curve is steeper than the supply curve, the situation is depicted by Figure 4 (b). Again, a rise in the money growth rate, θ, shifts the labor demand curve downward, which increases the steady-state level of employment and long-term growth rate.

We should note that if $\hat{\sigma} < \sigma < 1$, the labor supply curve may have a negative slope. As shown by Figures 4 (d) and (e), if the labor demand curve is less steeper than the labor supply curve, then a downward shift of the labor demand due to a rise in θ increases the steady state levels of employment and growth rate. This gives an intuitive implication for the positive growth effect of money supply obtained in the case of $\hat{\sigma} < \sigma < 1$.[7] Since

[6] Here, we implicitly assume that income tax is not levied on leisure.

[7] The effect of a change in θ on the steady-state rate of nominal interest is

$$\frac{di}{d\theta} = 1 + \left(1 - \frac{1}{\sigma}\right)\beta l^{\beta-1}\frac{dl}{d\theta}.$$

Thus if $dl/d\theta < 0$, the nominal interest rate in the steady state increases when $\sigma < 1$. If $dl/d\theta > 0$, we could obtain $di/d\theta < 0$. Figures 4 (d) and (f) ignore this possibility.

under the assumption of non-separable utility the steady-state level of nominal interest rate is endogenized and thus the effect of inflation tax is not so straightforward as that in the case of separable utility under which the long-term nominal interest rate, $i = \rho + \theta$, is independent of the level of l. These observations imply that the preference structure plays an essential role in determining the growth effect of money supply when the utility function is not additively separable between consumption and labor.

4.2 Equilibrium Determinacy and the Growth Effect of Money

We have shown that if there are dual balanced-growth paths, at least one of them is locally indeterminate in the sense that there are a continuum of converging paths around the balanced-growth equilibrium.[8] Moreover, even when the balanced-growth path is uniquely given, there may exist a continuum of converging paths around the balanced-growth equilibrium. The presence of real indeterminacy means that sunspot-driven expectations affect the dynamic behavior of real variables. In this case, even if the money growth rate is kept constant, a change in the anticipated rate of inflation produced by a sunspot-driven change in expectations may have real effects. Such a disturbance, of course, will not affect the balanced-growth equilibrium, but it may bring the economy out of the balanced-growth path. Since there exists a continuum of converging paths at least around the balanced-growth equilibrium, the economy may converge to the original position. During the transition towards the balanced-growth path, the growth rate of the economy may change. Therefore, in the presence of equilibrium indeterminacy, any extrinsic uncertainty about the money growth rate may have the growth effect.

As emphasized before, an important conclusion of our discussion is that there is no systematic link between the growth effect of money supply and the local determinacy of equilibrium. According to the corresponding principle in the sense of Samuelson (1983,

[8] Many authors have demonstrated that complex preference structures may yield indeterminacy of equilibrium in the monetary dynamic models based on the money-in-the-utility function formulation: see, for example, De Fiore (2000), Carlstrom and Fuerst (2001), Matheny (1998) and Matsuyama (1991). Since we assume the presence of a cash-in-advance constraint on consumption alone, the relation between consumption and real money balances is extremely simple in our setting. Hence, indeterminacy caused by the preference structure in our model mainly stems from the labor supply behavior rather than money demand behavior of the households.

Chapter 9), one may conjecture that the counter-intuitive policy effect, i.e. a rise in money growth rate promotes economic growth despite the presence of cash in advance constraint, is associated with indeterminacy of equilibrium. We have, however, seen that a higher growth of money supply may have a positive effect on the long-term growth rate of income even when the balanced-growth equilibrium satisfies local determinacy. Conversely, the negative relation between monetary expansion and real growth would prevail even in the presence of equilibrium indeterminacy.

It should be also noted that if the balanced-growth equilibrium exhibits local indeterminacy, we may not observe a one-to-one relation between inflation and growth in the long run. If expectations-driven fluctuations may emerge, the economy easily diverges from the balanced-growth path without any monetary disturbance. Therefore, when the equilibrium indeterminacy holds, the long-run relationship between money supply and real growth derived by the comparative statics exercise does not help so much to identify the actual relationship between inflation and growth. Although empirical researches on the relation between inflation and growth have not yet reached a consensus, both in the cross-country regressions and in the time-series analyses the following finding has been widely accepted: among the group of low inflation countries, it is hard to find statistically meaningful relationships between the growth rate of income and the rate of inflation.[9] One of the interpretations of these findings is that there is no long-term relation between money and growth at least in low-inflation countries, that is, money is superneutal in the long-run. Alternatively, we may also reason that such an absence of clear relation between inflation growth can be attributed to equilibrium indeterminacy emerged in the low-inflation economies. Since our model with nonseparable utility can yield various combinations of the growth effect of money supply and equilibrium determinacy, it may provide us with a useful analytical framework to interpret various empirical results within a single model.

[9] See, for example, Barro (1996) and Bruno and Easterly (1996). Temple (2000) surveyed the empirical as well as theoretical studies on inflation and growth. He summarized controversies and unresolved problems in this field.

5 Conclusion

In the context of a simple model of endogenous growth with a cash-in-advance constraint, we have explored the growth effect of money supply in the long-run equilibrium. We have shown that the magnitude of the elasticity of intertemporal substitution in consumption as well as the degree of increasing returns are crucial for determining the growth effect of money supply. We have also confirmed that local indeterminacy of equilibrium hinges on these two key parameters. At the same time, we have found that there is no systematic link between the growth effect of money supply and the equilibrium determinacy around the balanced-growth path.

The present paper has assumed that the growth rate of nominal money supply is kept constant. The recent studies on indeterminacy in monetary dynamic models emphasize the role of money supply rule. In particular, the interest rate control rules may generate indeterminacy even in models with convex production technologies and separable utility functions.[10] The existing investigations on this issue, however, have been mostly concerned with models without capital accumulation. Clarifying the relation between technology, preference and money supply rules in the presence of capital formation would deserve further investigation.[11]

Appendices

Appendix 1 Proof of Proposition 2

From (15) and (22) the rate of inflation is written as

$$\pi(l,x) = -(1-a)(1-\sigma)\frac{\Lambda(l)\,l^{\beta-1}}{\Lambda'(l)}x - al^{\beta} - 1 = F(l)\,x - al^{\beta} - 1,$$

implying that $\pi_l(l,x) = F'(l)\,x - a\beta l^{\beta-1}$ and $\pi_x(l,x) = F(l)$. Hence, the determinant of the coefficient matrix J can be written as

$$\det J = \frac{xl}{\eta(l)+1-\beta}\left\{(\sigma-a)\,\beta l^{\beta-1}\left(F+\frac{1}{x^2}\right) - \frac{\sigma}{x^2}\left[(1-a)\,\beta l^{\beta-1} + F'(l)\,x\right]\right\}. \quad \text{((A1))}$$

[10]See, for example, Benhabib et al. (2000).

[11]Some authors have considered the relationship between the interest rate rules and determinacy of equilibrium in models with capital formation: see Carlstrom and Fuerst (2005), Meng and Yip (2004) and Itaya Mino (2004).

Note that

$$
\begin{aligned}
&(\sigma - a)\,\beta l^{\beta-1}\left(F + \frac{1}{x^2}\right) - \frac{\sigma}{x^2}\left[(1-a)\,\beta l^{\beta-1} + F'(l)\,x\right] \\
= \;& \frac{\sigma}{x}\left\{\left[\left(1-\frac{1}{\sigma}\right)F(l)\,x + \frac{1}{x}\left(1-\frac{1}{\sigma}\right)\right]\beta l^{\beta-1} - F'(l)\right\}. \qquad \text{((A2))}
\end{aligned}
$$

Since $G(l)$ is given by $G(l) \equiv \left[\left(1-\frac{a}{\sigma}\right)l^{\beta} + \frac{\rho}{\sigma}\right]\left[a\left(1-\frac{1}{\sigma}\right)l^{\beta} + \frac{\rho}{\sigma} + \theta + 1\right]$, we obtain:

$$
\begin{aligned}
G'(l) \;=\;& \beta l^{\beta-1}\left[\left(1-\frac{a}{\sigma}\right)\left(a\left(1-\frac{1}{\sigma}\right)l^{\beta} + \frac{\rho}{\sigma} + \theta + 1\right)\right. \\
&\left. + a\left(1-\frac{1}{\sigma}\right)\left(\left(1-\frac{a}{\sigma}\right)l^{\beta} + \frac{\rho}{\sigma}\right)\right]. \qquad \text{((A3))}
\end{aligned}
$$

By using the fact that $F(l)\,x = \pi + al^{\beta} + 1$, $\pi = \theta - g = \theta - (1/\sigma)\left(al^{\beta} - \rho\right)$ and $1/x = l^{\beta} - \theta + \pi = (1 - a/\sigma)\,l^{\beta} + \rho/\sigma$, in equation (A2) we can show

$$
\begin{aligned}
&\left(1-\frac{1}{\sigma}\right)F(l)\,x + \frac{1}{x}\left(1-\frac{1}{\sigma}\right) \\
= \;& \left(1-\frac{a}{\sigma}\right)\left[a\left(1-\frac{1}{\sigma}\right)l^{\beta} + \frac{\rho}{\sigma} + \theta + 1\right] + a\left(1-\frac{1}{\sigma}\right)\left(\left(1-\frac{a}{\sigma}\right)l^{\beta} + \frac{\rho}{\sigma}\right). \qquad \text{((A4))}
\end{aligned}
$$

From (A1), (A2), (A3) and (A4), we finally obtain:

$$
\det J = \frac{\sigma l}{\eta(l) + 1 - \beta}\left[G'(l) - F'(l)\right].
$$

Therefore, it holds that

$$
\text{sign } \det J = \text{sign } \left[\eta(l) + 1 - \beta\right]\left[G'(l) - F'(l)\right],
$$

which confirms the proposition.

Appendix 2 Local stability when the utility function is given by (25).

First, notice that when the utility function is (25), we obtain

$$
\eta(l) = \frac{\Lambda''(l)\,l}{\Lambda'(l)} = \left[1 - \chi(1-\sigma)\right]\frac{l}{1-l}.
$$

Hence, if $\beta > 1$, then $\beta > 1 + \eta(l)$ for $0 \le l \le \frac{\beta-1}{\beta-\chi(1-\sigma)}$. In the following, when considering the case of $\beta > 1 + \eta(l)$, we restrict our attention to the region of $0 \le l \le \frac{\beta-1}{\beta-\chi(1-\sigma)}$. The trace of the coefficient matrix J is written as

$$
\text{Tr } J = \frac{l}{\eta(l) + 1 - \beta}\left[(1-\sigma)\left(xF'(l) - a\beta l^{\beta-1}\right) + (1-a)\,\beta l^{\beta-1}\right] + xF(l) + \frac{1}{x}
$$

Thus in the case of Figure 1 (a) ($\sigma > 1$ and $\beta < 1$) in which $F'(l) < 0$, we see that on the balanced-growth path det $J > 0$ and Tr $J > 0$. This means that the steady state is a source and local determinacy is established. If $\beta > 1 + \eta$ and $\sigma > 1$ (Figure 1 (b)), the high-growth steady state where $G' > F'$, we obtain: $\det J < 0$ and thus the steady state is a saddlepoint, implying that the BGP is locally indeterminate. In contrast, in the low-growth steady state it is easy to confirm that Tr $J > 0$ and $\det J > 0$, so that local determinacy holds. In the case of Figure 2 (a), we cannot determine the sign of Tr J without imposing further restrictions. Since $\det J > 0$ in this case, the steady state can be either a source or a sink. Similarly, in Figure 2 (b) the stability property of the low-growth steady state cannot be determined unless we use numerical examples, while the high growth steady state is a saddlepoint (i.e. locally indeterminate) because $\det J < 0$. Note that if $1 < \beta < 1 + \eta(l)$ holds at the steady state, the sign of Tr J cannot be determined at the high-growth steady state both for the cases $\sigma > 1$ and $\sigma < 1$. Since $\det J > 0$ in these steady states, they may be either sinks or sources. By contrast, the low-growth steady state satisfies $\det J < 0$ and thus it is locally indeterminate.

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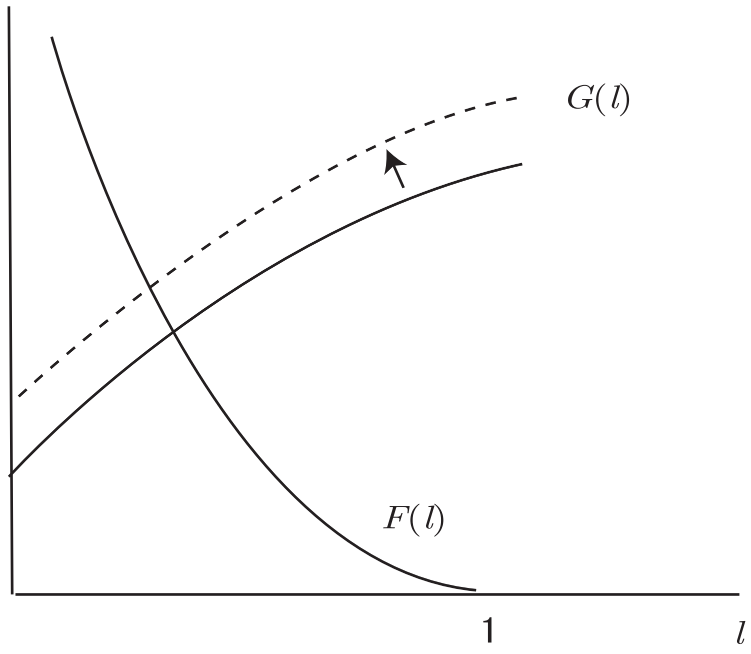
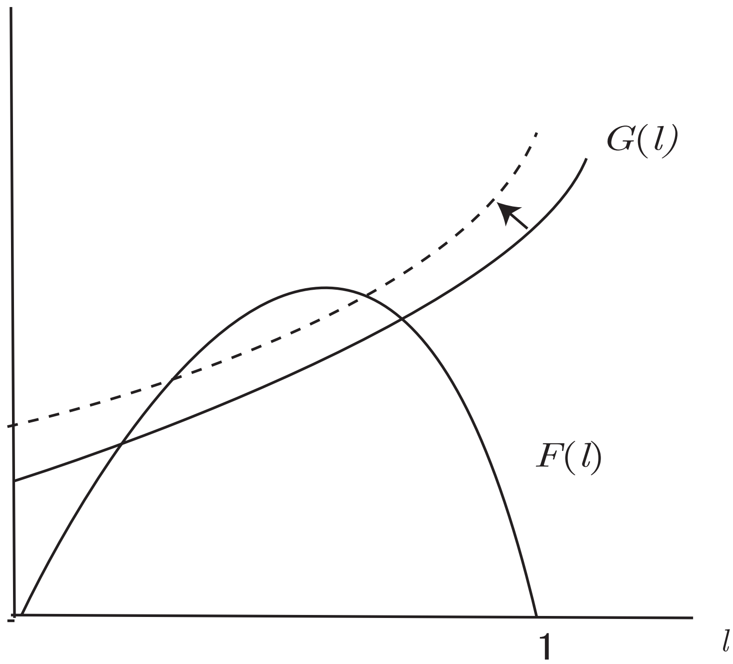
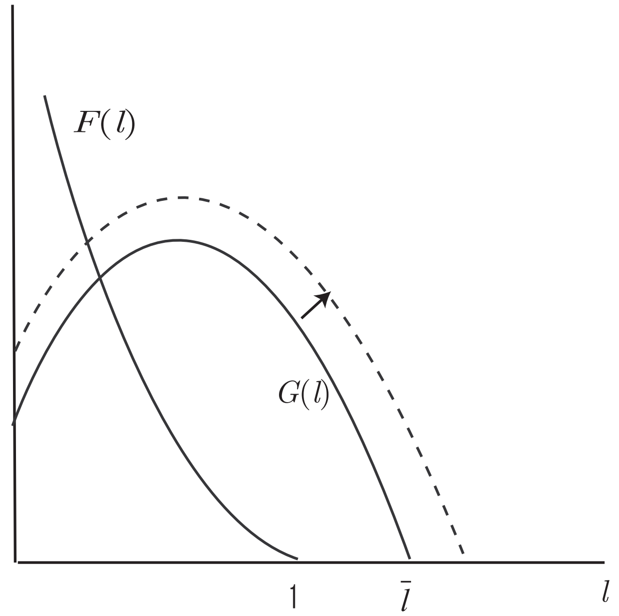


(a) $\sigma > 1, \ \beta < 1$

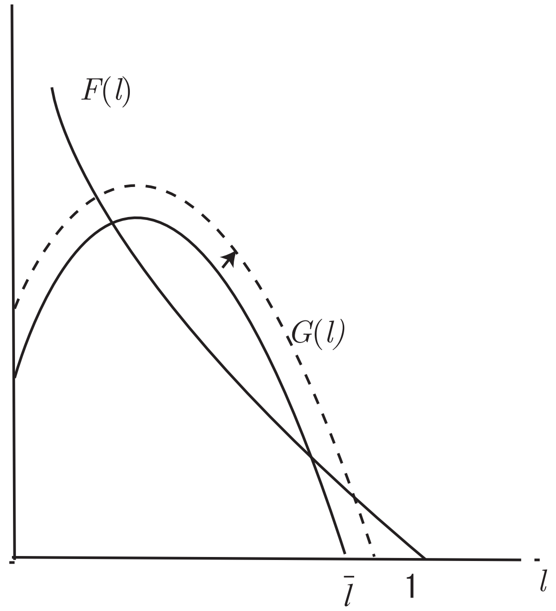


(b) $\sigma > 1, \ \beta > 1$

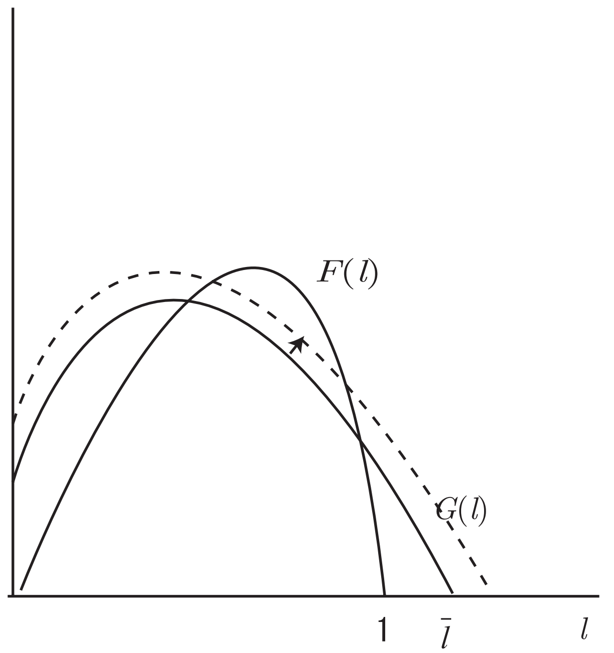
Figure 1



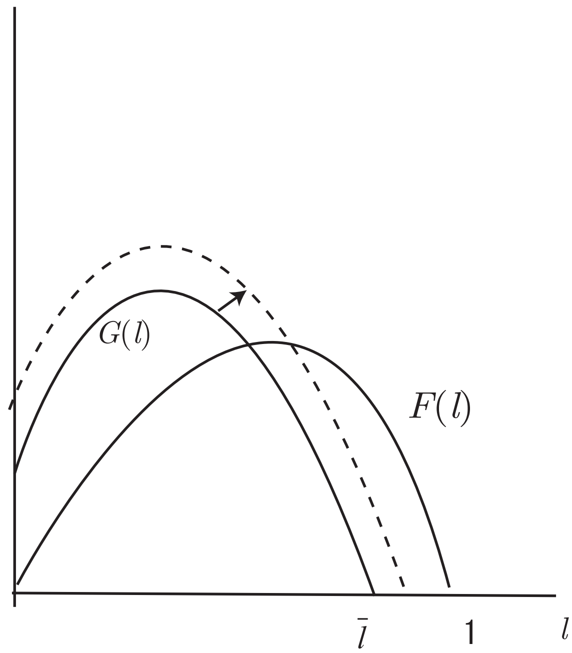
(a) $\sigma < 1$, $\beta < 1$, $\bar{l} > 1$



(b) $\sigma < 1$, $\beta < 1$, $\bar{l} < 1$

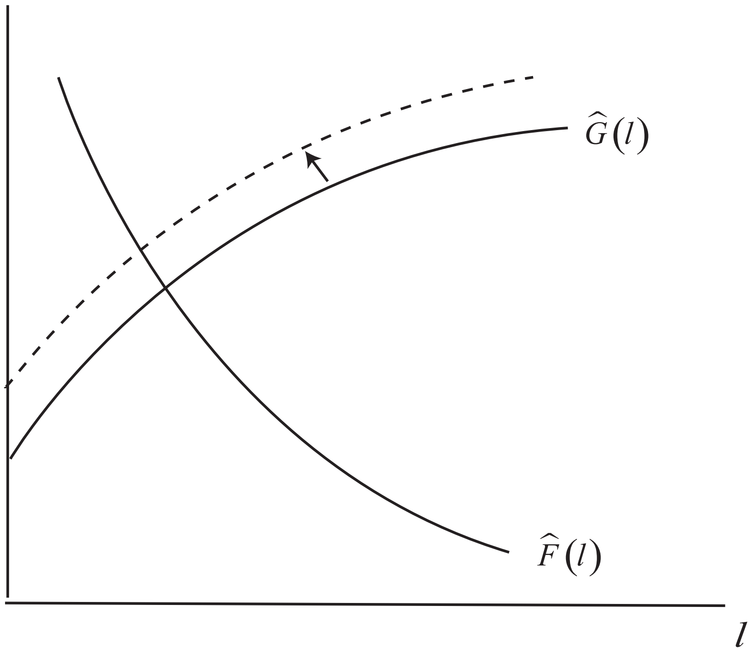


(c) $\sigma < 1$, $\beta > 1$, $\bar{l} > 1$

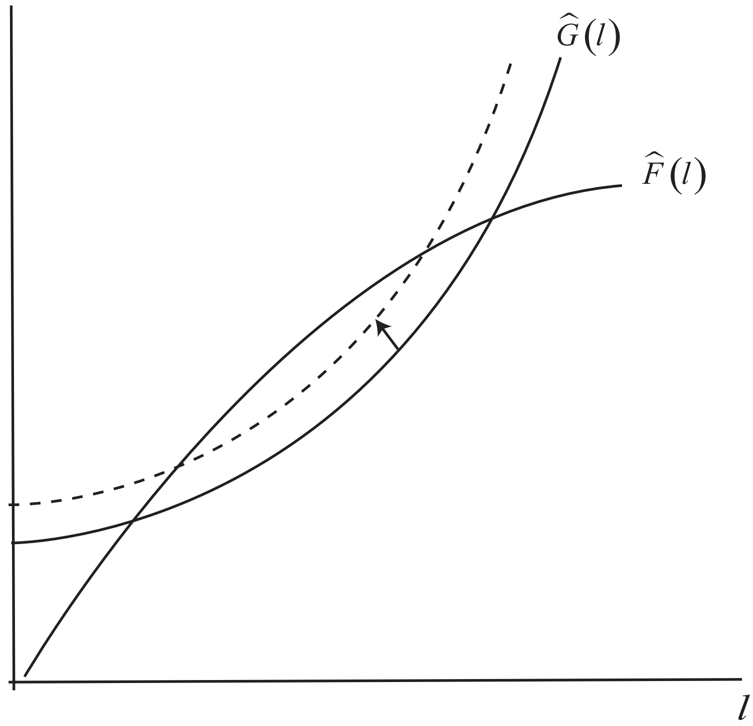


(d) $\sigma < 1$, $\beta > 1$, $\bar{l} < 1$

Figure 2

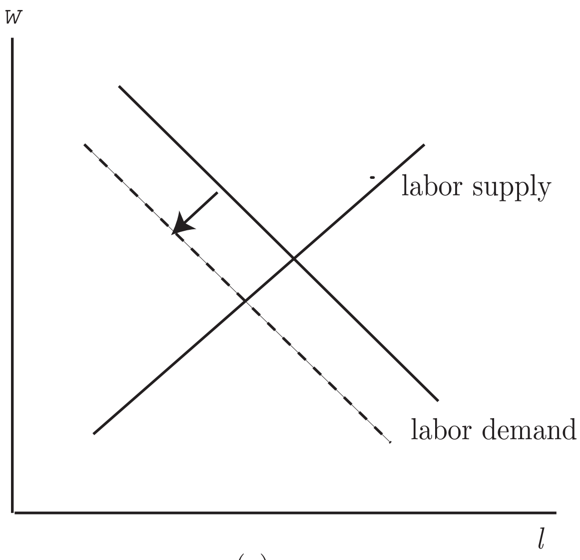


(a) $\beta < 1+\eta$

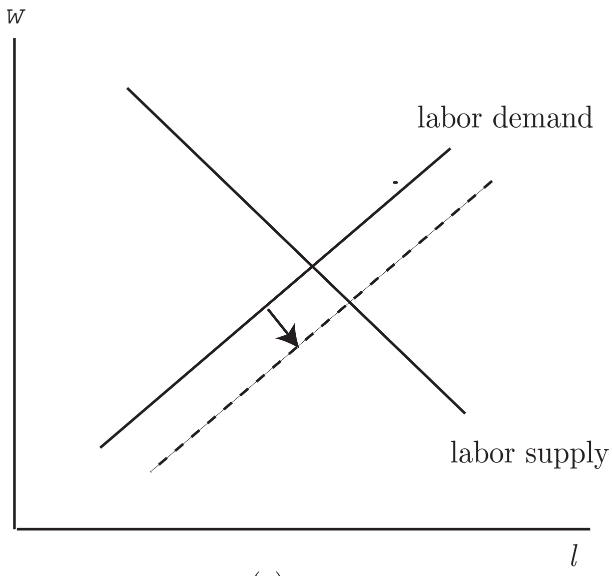


(b) $\beta > 1+\eta$

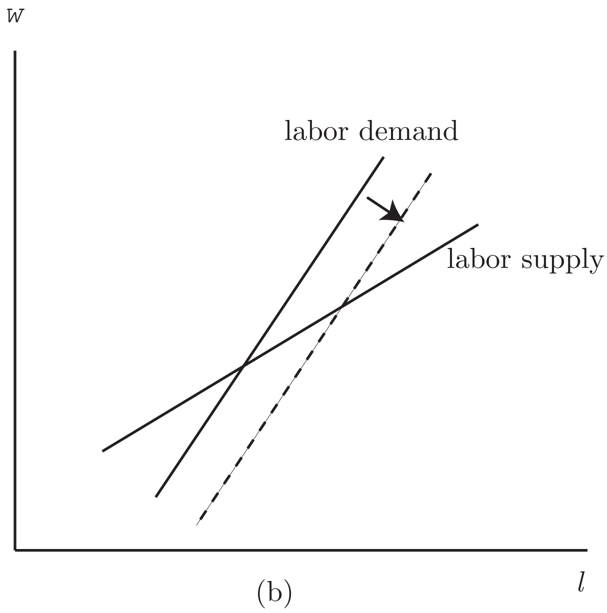
Figure 3



(a)



(e)



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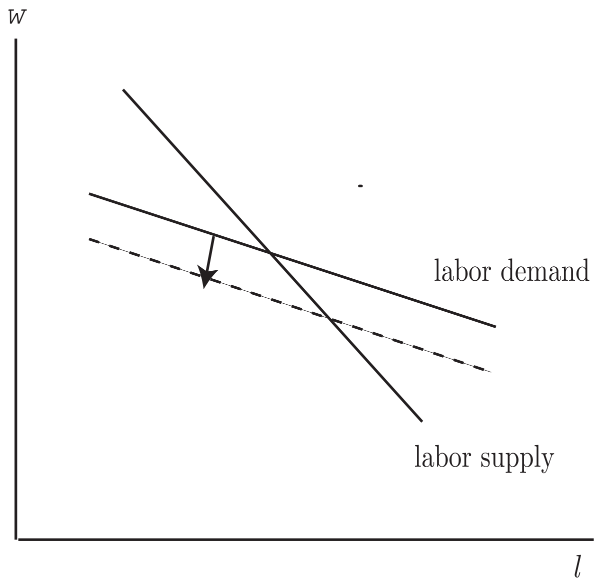
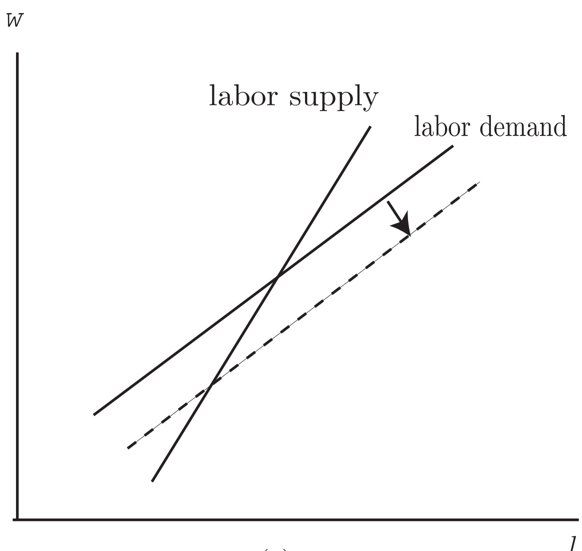


(f)



(c)

Figure 4