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Growth Cycles with Emerging Industries*

Ryo Horii[†] (Osaka University)

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Abstract

This paper develops a theory of endogenous growth cycles focusing on the interaction between consumers' desire to satisfy an indefinite range of wants and firms' incentive to utilize knowledge from past production experiences. We show that firms endogenously form a number of distinguishable industries as accumulated knowledge induces them to agglomerate in the technology space. Knowledge accumulation in existing industries reduces production costs, but, as the diminishing returns from learning sets in, some firms start to adopt previously unexplored technologies so that their new goods fit consumers' unsatisfied wants and attract large demand. Thus, sporadic emergence of new industries generates growth cycles, where both the timing and the new technology to be adopted are endogenously determined. New industries based on new technology reduce the rate of per capita GDP growth in the initial phase, but nonetheless are indispensable for sustained economic growth in the long run.

JEL Classification Numbers: O31, O33, O41.
Keywords: endogenous growth, technology choice, wants, knowledge, cycles.

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1 Introduction

The notion that long-term economic growth is primarily the result of the growth of technological knowledge is now a widely held view among growth economists. Not entirely surprisingly, this perception existed in the 1960s when Schmookler published his influential volume entitled "Invention and Economic Growth," in which new additions to knowledge were termed invention or subinvention according to the degree of novelty. The focus of the book was on the determinants of production of new knowledge, still a central concern of the current growth literature. He wrote as follows.

> The very definition of an invention as a novel combination of pre-existing knowledge to satisfy some want better suggests the possible causes for its occurrence. ... Since it is based on prior knowledge, the received stock of knowledge must also play a role. And since it is calculated to better serve human wants, these too must also affect invention. ... [O]ur chief inquiry concerned the comparative influence of wants and past knowledge on the inventive process. (Schmookler 1966, p. 197)

Given that there are broad classes of technological knowledge, his insight suggests two mechanisms by which past economic activities affect the pattern of knowledge accumulation today. The stock of knowledge upon which a new firm can rely is often obtained through the past experiences of existing firms. Thus, a new firm has an incentive to adopt the same technology as existing firms to utilize that class of knowledge, which again gives subsequent firms the same, and further fortified, incentive.[1] This force gives rise to a group of firms—which we call an industry in this paper—that utilize the common knowledge base of a particular technology to minimize their operating

[1] Jovanovic and Nyarko (1996) and Redding (2002) examined this issue and showed that technological lock-in may occur if agents accumulated too much knowledge with respect to a particular technology. In our model, technological lock-in does not persist because of a conflicting force that eventually dominates; namely, human wants.

costs.

Human wants affect the decisions of firms in a different way. Any technology is subject to an obvious limitation regarding the range of wants to which it can be adequately applied, and this fact influences a firm's technology choice. While this aspect of technology is not considered explicitly by Schmookler, its importance was afterward pointed out by Rosenberg (1974).

> Oddly enough then, science and technology play a subordinate role in influencing the *direction* of inventive activity within Schmookler's analysis, not because his analysis downgrades their historical significance, but rather because he regards science and technology in the modern age as being, in a significant sense, omnicompetent. ... Now this is precisely the aspect of Schmookler's argument which seems to be most inadequate. ... Many important categories of human wants have long gone either unsatisfied or very badly catered for in spite of a well-established demand. (pp. 94, 97)

Given the indefinite range of human wants, Rosenberg's view implies that there will always be a set of wants that are not well matched by existing technologies. Firms can attract large demand if they can adopt previously unexplored technologies so that their new goods fit consumers' unsatisfied wants. Thus, profit maximizing firms may find it optimal to adopt technologies significantly different from other firms, rather than to exploit the benefits of accumulated knowledge within existing industries.

There is an apparent conflict between knowledge and wants—or cost and demand—in the choice of technology by firms, and therefore in the direction in which knowledge grows.[2] This paper develops a theoretical framework that explicitly captures this trade-

[2]Empirically, it has been difficult to distinguish between these two forces because the strength of both forces depends on closeness between firms. Nonetheless, Bloom, Schankerman and Van Reenen (2005) recently developed two measures of a firm's position in technology space and product market space and found that both technology spillovers (i.e., past knowledge) and product market rivalry (i.e., human wants) affect a firm's value.

off and investigates the pattern of economic growth, based on variety-expansion models of endogenous growth (e.g., Romer 1990; Grossman and Helpman 1989). We show that firms endogenously form a number of distinguishable industries, as accumulated knowledge induces firms to agglomerate in technology space. Knowledge accumulation in existing industries reduces production costs, but, as the diminishing returns from learning set in, some firms start to adopt new and significantly different technologies to capture demands for unsatisfied wants. Both the timing and the new technology to be adopted are endogenously determined in equilibrium. As a result, the equilibrium dynamics are characterized by the sporadic emergence of new industries, rather than a smooth increase in the number of symmetric products.

This type of dynamics naturally causes the rate of economic growth to fluctuate. In particular, it captures the observed tendency that the emergence of a new industry that utilizes a new technology—e.g., electricity and information technology—reduces the rate of per capita GDP growth in the initial phase, a phenomenon known as the "productivity slowdown puzzle."[3] It is shown that this slowdown occurs because emerging industries diversify the GDP share of individual industries and diminish the benefits of the agglomeration economy that comes from knowledge accumulation within an industry.

The model also shows that new industries nonetheless disproportionately contribute to the economy's productivity growth after becoming large.[4] As knowledge concerning the new technology accumulates, more firms enter the new industry, further accelerat-

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[3]Hornstein and Krusell (1996) and Greenwood and Yorukoglu (1997) found a negative effect of the arrival of information technology (IT) on productivity, and attributed the productivity slowdown to the learning cost associated with new technologies. Jovanovic and Rousseau (2005) found that a productivity-growth slowdown occurred not only in the initial phase of the IT era, but also in the initial phase of electrification.

[4]In the case of the revivals in total factor productivity (TFP) growth during the period 1995–99, Gordon (2000) argued that productivity increased only within the computer industry and associated sectors, which together comprise only about 12 percent of the private business economy.

ing knowledge accumulation. Production costs and prices of products fall faster, and eventually cause a spurt in overall GDP growth. It is shown that, at every point in time, only a small number of young industries account for quite a large proportion of the overall growth rate in the model. Without the sporadic emergence of new industries induced by human wants, cost reductions within existing industries would eventually come to an end.

As a study of cyclical growth, the present paper is closely related to Helpman and Trajtenberg (1998). They considered an exogenous process in which new general purpose technologies (GPTs) arrive sporadically and discretely, and showed that GPTs trigger recurrent cycles in the growth rate. Our study improves the understanding of this type of dynamics in that both the timing of arrival (emergence of new industries) and the degree of discreteness (the difference between the technologies) are endogenized.

This paper is also related to the literature that integrates endogenous fluctuation, sometimes called "natural volatility" as opposed to exogenous shocks, with endogenous long-run growth. Freeman, Hong and Peled (1999), Wälde (2002, 2005), Maliar and Maliar (2004), and Phillips and Wrase (2005) developed models in which activity of agents causes major improvements in the TFP to occur sporadically, thereby generating fluctuations in the rate of economic growth. A common feature of these studies is that they *assume* that productivity improvements in each sector are discrete, and that the number of sectors are finite so that the law of large numbers does not work. In contrast, we model a continuous-time economy with a near continuum of technologies, whose productivity improves always continuously. Growth cycles occur in our model because spillover of knowledge generates a centripetal force that induces firms to choose only a finite number of technologies, and consumers' wants generate a centrifugal force that expands the set of technologies used in discrete steps.

The rest of this paper is organized as follows. Sections 2 and 3 respectively present the model of human wants and that of past knowledge. Section 4 derives the equilibrium distribution of firms at each instant, given the distribution of knowledge accumulated by that time. Section 5 investigates how the interaction between wants and past knowledge drives the evolution of firms, and hence determines the pattern of knowledge accumulation. Section 6 examines the fluctuations in growth rate. Section 7 concludes.

2 Modeling Human Wants

Preferences

In the model, the economy consists of a continuum of identical consumers with measure L_t, which grows at an exogenous rate $\lambda > 0$. They have an unbounded range of wants, represented by real number $r \in (-\infty, \infty)$. Each want can be satisfied by consuming differentiated goods and, as in Dixit and Stiglitz (1977), consumers benefit from consuming a large variety of differentiated goods to satisfy each want. In our model, however, some goods are more suitable for satisfying a particular want than others, because goods are produced using different technologies and therefore a great variability exists in the wants that the goods best satisfy.

Let goods be indexed by $i \in [0, I_t]$, where I_t is the measure of the total number of goods at time t, which is to be determined in equilibrium. Let $\widetilde{s}_t(i) \in (-\infty, \infty)$ denote the technology by which good i is produced. Technologies are indexed so that goods produced by technology $\widetilde{s}_t(i)$ are best suited for satisfying want $r = \widetilde{s}_t(i)$. The consumer can also arbitrarily divide good i to use part of it to satisfy any want $r \neq \widetilde{s}_t(i)$, but the good becomes gradually unfit as want r and technology $\widetilde{s}_t(i)$ diverge (See Figure 1).

The representative consumer decides the amount of good i to be consumed to satisfy want r at each point in time. Let $x_t(r, i)$ denote this amount, or more precisely consumption density defined for all wants $r \in (-\infty, \infty)$ and all goods $i \in [0, I_t]$. Then

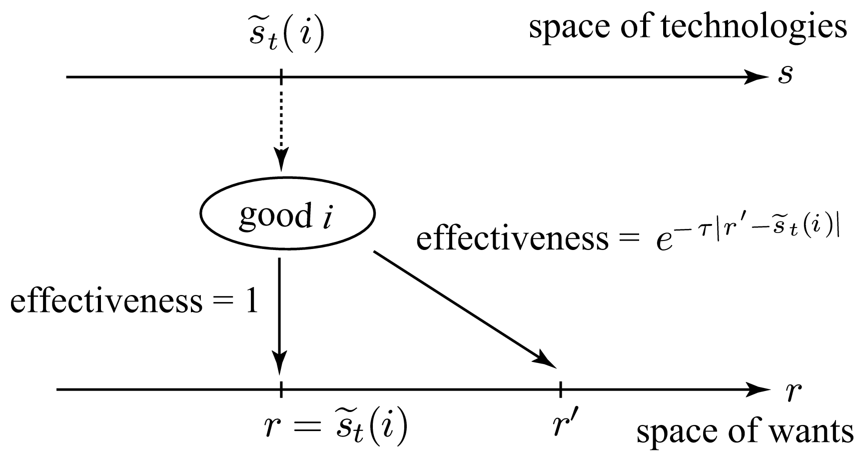

Figure 1: Technologies and wants.

the level of satisfaction of want r is:

$$v_t(r) = \left[\int_0^{I_t} \left(x_t(r,i)e^{-\tau|r-\widetilde{s}_t(i)|}\right)^{(\sigma-1)/\sigma} di\right]^{\sigma/(\sigma-1)}, \tag{1}$$

where $\tau > 0$ is a coefficient that measures how rapidly a good becomes unfit as its technology diverges from r, and $\sigma > 1$ represents the elasticity of substitution between goods. The consumer's overall instantaneous utility at time t is given as an aggregation over $v(r)$:

$$u_t = \left[\int_{-\infty}^{\infty} v_t(r)^{(\beta-1)/\beta} dr\right]^{\beta/(\beta-1)}, \tag{2}$$

where $\beta \in (1, \sigma)$ is the elasticity of substitution between wants. The assumption of $\beta < \sigma$ implies that substituting one want with another is more difficult than substituting one particular variety of good with another to satisfy the same want. Later we will show that this property makes the consumer value novel goods that are made with technologies significantly different from existing goods.

Each consumer inelastically supplies one unit of labor, and prices are normalized so that the nominal wage at each instant is 1. Every good is perishable and cannot be stored. In addition, as will be seen below, there is no opportunity to invest. Thus, the homogeneity of consumers implies that the credit market involves no trade and every consumer spends all income at each instant. Let $p_t(i)$ denote the price of good

i. The consumer then maximizes the instantaneous utility (2) with (1) under the instantaneous budget constraint:

$$\int_0^{I_t} \left(\int_{-\infty}^{\infty} x_t(r,i)\, dr \right) p_t(i)\, di = 1. \tag{3}$$

Consumer Demand

Here we derive the demand function of consumers for each good. Since the utility function is a two-stage CES function, we adopt a two-step method. The first step is to maximize the subutility of each want $v(r)$ defined by (1) with respect to $x_t(r,i)$, under the constraint $\int_0^{I_t} p_t(i) x_t(r,i)\, di = y_t(r)$. Here $y_t(r)$ is the density of expenditure for want r, which in this step we take as given. For each want r and good i, the solution to this problem and the maximized value are:

$$x_t(r,i) = y_t(r) q_t(r)^{\sigma-1} p_t(i)^{-\sigma} e^{-(\sigma-1)|r-\widetilde{s}_t(i)|}, \tag{4}$$

$$v_t(r) = y_t(r)/q_t(r), \tag{5}$$

$$\text{where} \quad q_t(r) \equiv \left[\int_0^{I_t} \left(p_t(i) e^{\tau|r-s_t(i)|} \right)^{-(\sigma-1)} di \right]^{-1/(\sigma-1)}. \tag{6}$$

Function $q_t(r)$ represents the amount of expenditure required to increase one unit of $v_t(r)$, and therefore we call it the 'price index' of want r. Substituting the indirect subutility function (5) for (2) gives the instantaneous utility in terms of the expenditure density $y_t(\cdot)$. The second step is to maximize this utility function subject to the instantaneous budget constraint $\int_{-\infty}^{\infty} y_t(r)\, dr = 1$. The optimal expenditure density and the maximized instantaneous utility, respectively, are:

$$y_t(r) = Q_t^{\beta-1} q_t(r)^{-(\beta-1)} \tag{7}$$

$$u_t = 1/Q_t, \tag{8}$$

$$\text{where} \quad Q_t \equiv \left[\int_{-\infty}^{\infty} q_t(r)^{-(\beta-1)} dr \right]^{-1/(\beta-1)}. \tag{9}$$

Q_t is the expenditure function required to attain a unit instantaneous utility, which can be interpreted as the 'average price index' over all wants. Substituting (7) into (4) yields the amount of good i used to satisfy want r as:

$$x(r,i) = Q_t^{\beta-1} q_t(r)^{\sigma-\beta} p_t(i)^{-\sigma} e^{-(\sigma-1)\tau|r-\widetilde{s}_t(i)|}. \tag{10}$$

Integrating the demand for good i in (10) across all wants and then multiplying it by the population L_t yields the demand for good i:

$$X_t(i) = L_t Q_t^{\beta-1} p_t(i)^{-\sigma} \int_{-\infty}^{\infty} q_t(r)^{\sigma-\beta} e^{-(\sigma-1)\tau|r-\widetilde{s}_t(i)|} dr. \tag{11}$$

As shown by (11), the price elasticity of demand is constant. Therefore, the inverse demand function for good i can be expressed as:

$$p_t(i) = d_t(\widetilde{s}_t(i)) X_t(i)^{-1/\sigma}, \tag{12}$$

where $d_t(s)$ is the *unit demand price*, which represents the price level at which any good made with technology s sells a unit quantity. From (11), it is:

$$d_t(s) \equiv \left[L_t Q_t^{\beta-1} \int_{-\infty}^{\infty} q_t(r)^{\sigma-\beta} e^{-(\sigma-1)\tau|r-s|} dr \right]^{1/\sigma}. \tag{13}$$

Centrifugal Force

Equation (13) implies that the price consumers are willing to pay for a variety of good depends on its technology s, and this dependence comes from variations in the price index $q_t(r)$ across wants.[5] More specifically, from $\sigma > \beta > 1$, (13) shows that the unit demand price for a certain technology s is higher when the price index for the wants that are close to s is higher, and, from (6), this is the case when there are fewer goods produced by technologies that are close to s. This property has a natural interpretation: because of diminishing marginal utility from each want, consumers are willing to pay more for a variety of good that fits wants that are not well met by the other goods. Figure 2 depicts an example of the shape of function $d_t(s)$, which we call the unit demand price curve.

[5] Observe that, if $q_t(r)$ is the same for all r, then $d_(s)$ is also the same for all s.

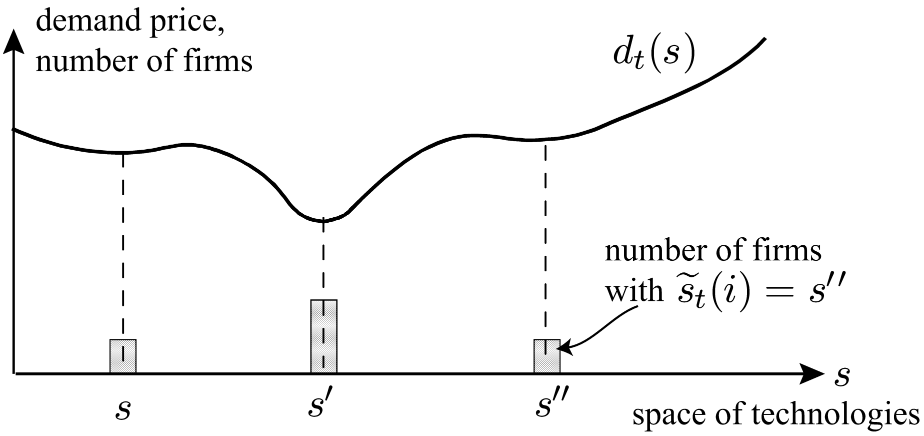

Figure 2: Unit demand price curve. This example shows the shape of function $d_t(s)$ when goods are produced with either of the three technologies (s, s' or s''). The height of the gray bars represents the number of varieties produced by each technology.

From the viewpoint of producers, which will be introduced shortly, they can sell the same amount of their products at a higher price when they choose a technology that is more distant in technology space from that of other firms since they will then face little competition from producers who are using the same or similar technologies. Thus, producers have an incentive to choose technologies that are as distant as possible from others; creating a centrifugal force working in the technology space. This might imply that the range of choices of technologies by profit-maximizing producers should explode in the technology space. In reality, however, we do not observe such explosion because, at a given date, the costs of producing goods extremely different from others are prohibitively high, if not impossible, due to scarcity of relevant knowledge.[6] In the following section, we describe the production side of the economy and how it is affected by the stock of knowledge that is accumulated in the past.

[6]For example, even though a tour to Mars is quite different from any other leisure services and it is not theoretically impossible, currently no business firm offers such a tour, probably due to the extremely high estimated costs that stem from scarcity of knowledge or experience.

3 Modeling Past Knowledge

Production Technologies

Each variety of goods is produced by a distinct firm. With a flow fixed cost that must be paid throughout the period of its operation (such as the cost of maintaining the production line), any firm can enter the goods market.[7] Firms can also exit the market costlessly.

Each firm chooses one technology from a set of usable technologies denoted by $\mathbf{s} \subset (-\infty, \infty)$. The number of potentially (or theoretically) usable technologies is very large and there are many analogous but slightly different technologies in the set, even though only a small subset of $\mathbf{s}$ would be in use at any point in time. Specifically, define $\mathbf{s}$ by a countable set $\{s_j\}_{j=-\infty}^{\infty}$, where $s_j = j\varepsilon$ with $\varepsilon > 0$ being a very small constant. Set $\mathbf{s}$ does not change over time, and therefore it is defined as broadly as possible.[8] Thus, our focus is on the evolution of knowledge about how to use technologies, while abstracting from the growth of knowledge about what technologies exist.[9]

Goods are produced from labor with a constant marginal cost. Suppose that a firm, call it firm i, produces an amount $X_t(i)$ of its goods with technology $\widetilde{s}_t(i) \in \mathbf{s}$. We normalize the units for the quantity of goods and for the measure of firms so that

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[7]Most existing R&D-based growth models consider a lumpy setup cost (e.g., the cost of innovation) while ignoring the flow fixed cost. Obviously, the reality incorporates both. The difference between the two specifications is not so large as it might appear, because the flow fixed cost can also be interpreted as the interest payments or dividends for the investors who financed the initial setup cost. Nonetheless, the assumption that entry is reversible significantly simplifies the analysis since it enables us to abstract from the forward-looking aspects of firms' decisions.

[8]At the limit to which $\varepsilon \to 0$, $\mathbf{s}$ approaches the set of all real numbers, $(-\infty, \infty)$, which coincides with the space of wants. The only reason that we maintain the assumption that $\mathbf{s}$ is countable is to prove the existence of the unique equilibrium in a tractable manner.

[9]By the definition in Mokyr (2002), the former is called prescriptive knowledge while the latter is propositional knowledge.

the labor requirement for this activity is written by:[10]

$$\ell_t(i) = \left(\frac{\sigma-1}{\sigma}X_t(i) + \frac{1}{\sigma}\right)c_t(\widetilde{s}_t(i)). \tag{14}$$

In equation (14), the marginal and fixed flow costs of production in terms of efficiency units of labor are normalized, respectively, to $\sigma/(\sigma-1)$ and $1/\sigma$. The last term, $c_t(\widetilde{s}_t(i))$, represents the number of workers required to generate one efficiency unit of labor. Since the nominal wage is normalized to 1, $c_t(\widetilde{s}_t(i))$ also represents the expenditure required for one efficiency unit of labor, and therefore we call it unit cost. This cost depends on technology and is negatively related to the amount of knowledge based upon that technology, as we now explain.

Learning by Doing

As people use a certain technology more, they learn how to use it better, and costs associated with the use of that technology fall. Here we formalize this 'learning-by-doing' process.[11]

For each $s \in \mathbf{s}$, let $k_t(s)$ denote the cumulative past experience of technology s. The value of $k_t(s) \geq 0$ can be interpreted as the amount of technology-specific knowledge on s. This paper considers aggregate or social knowledge in that it is

[10] Since our instantaneous utility function exhibits homogeneity of degree one in consumption density, we can freely choose measurement units for output quantity. We can also normalize the number (measure) of firms: if we double the whole distribution of firms and simultaneously halve the output of each firm and the fixed requirement, the instantaneous utility will simply be multiplied by $2^{1/(\sigma-1)}$ without substantially affecting the behavior of agents. This normalization is similar to the one employed in Fujita, Krugman and Venables (1999, Chapter 4).

[11] Of course, investment in knowledge often precedes production, for example in the case of R&D, and many workers invest actively in their human capital. Although these processes are also important, there is empirical evidence that supports our specification as a close approximation. Jovanovic (1995) reports that even the most advanced countries spend far more on the adoption of existing technologies than on the invention of new ones, with his rough estimate that in the U.S. adoption costs outweigh invention costs by a factor of 20 or 30 to 1.

totally nonappropriable and every firm in the economy has access to it. For every $s \in \mathbf{s}$, technology-specific knowledge grows according to:

$$\dot{k}_t(s) = \int_{\widetilde{s}_t(i)=s} X_t(i)\, di - \delta k_t(s), \tag{15}$$

where the first term represents the aggregate amount of production using technology s and $\delta \geq 0$ is the rate of depreciation (or forgetting) of experience.

Past knowledge enables firms to produce goods with a fewer number of workers. In this process, we focus on two important properties. The first property is spillovers of knowledge across different technologies.[12] Many of the technical and managerial advances brought about by experience in the production of certain goods have applications elsewhere. That is, productivity increases using a particular technology are not only a consequence of productive activity using that technology, but also the result of spillovers from learning-by-doing using other technologies. We assume that the amount of experience that a firm using technology s can rely on is proportional to $\sum_{s' \in \mathbf{s}} e^{-\nu|s-s'|} k_t(s')$, where term $e^{-\nu|s-s'|}$ represents the extent to which the experience with some technology can be applied to the production process with another technology.[13] Parameter $\nu > (\sigma - 1)\tau/\sigma$ represents how rapidly this applicability deteriorates with the distance in the technology space.

Second, the learning-by-doing process eventually runs into diminishing returns.[14]

[12] Young (1991, 1993) pointed out that there are substantial spillover effects in the development of knowledge across industries. See also Bloom, Schankerman and Van Reenen (2005)

[13] Young (1993) focused on $\sum_{s' \leq s} k_t(s')$, implicitly assuming that the applicability of knowledge does not depend on the distance between technologies. A more general specification is considered in Stokey (1988).

[14] For example, Gordon (2000, p.63) wrote: "Numerous industries have run into barriers to steady growth in productivity, most notably the airline industry when jet aircraft reached natural barriers of size and speed, and the electric utility industry when turbogenerator/boiler sets reached natural barriers of temperature and pressure. The apparent dearth of productivity growth in the construction and home maintenance industry reflects that electric portable power tools could only be invented once and have been subject to only marginal improvements in recent decades."

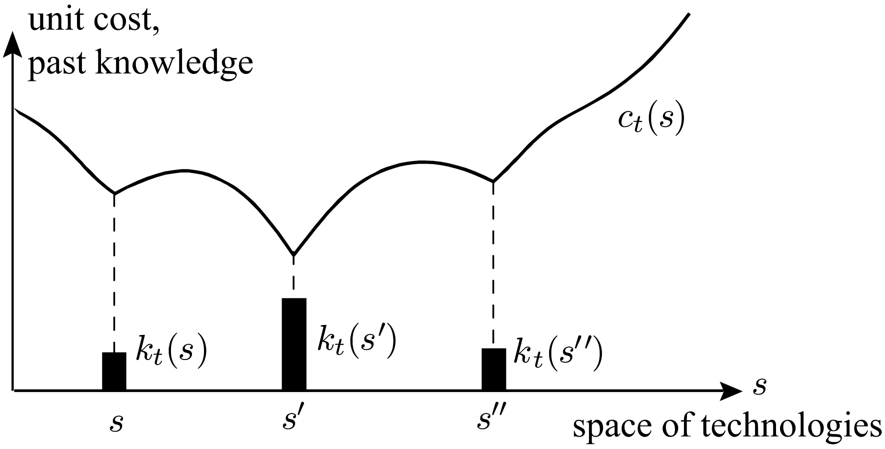

Figure 3: Unit cost curve. This example shows the shape of function $c_t(s)$ when past knowledge is accumulated on three technologies (s, s' or s'')

Specifically, intertemporal spillovers of experience generate a learning curve, in which the unit cost starts high, decreases rapidly on initial units, and then begins to level out. A simple specification we employ is:

$$c_t(s) = 1 + \left(\sum_{s' \in \mathbf{s}} e^{-\nu|s-s'|} k_t(s') \right)^{-1}. \tag{16}$$

Equation (16) means that, while the unit cost of using technology s is a decreasing function of usable knowledge $\sum_{s' \in \mathbf{s}} e^{-\nu|s-s'|} k_t(s')$, the marginal contribution of knowledge becomes smaller when the production cost approaches the lower bound, which is normalized to unity.

Figure 3 shows an example of the shape of function $c_t(s)$, which we call the unit cost curve. Accumulated past production experiences of a particular technology reduce the unit cost of production by using the same technology. In addition, thanks to spillovers across technologies, such experiences also contribute to lowering the cost of using other technologies, although to a lesser extent. Since this spillover effect diminishes with the distance in the technology space, past experiences enable firms to produce goods by similar (or 'near') technologies with lower labor costs. This mechanism generates a centripetal force that provides firms with an incentive to agglomerate with each other over time.

Behavior of Firms

Now we are ready to examine firms' profit maximization behavior. At each date t, a firm's problem is twofold: it must choose technology $\widetilde{s}_t(i)$ and then price $p_t(i)$.

Let us start with the second stage: the choice of price given the choice of technology $\widetilde{s}_t(i)$. Note that, since there is a continuum of firms, no single firm's behavior will change price indices $q_t(r)$ or Q_t and thus firms take unit demand price (13) as given. In addition, since knowledge is nonappropriable, they do not consider the impact of their activities upon the accumulation of knowledge; i.e., they also take unit cost (16) as given. Therefore, at each date, firm i chooses price $p_t(i)$ so as to maximize its current profit $\pi(i) = p_t(i)X_t(i) - \ell_t(i)$, subject to inverse demand function (12) and production technology (14). The profit maximizing price and quantity, and the maximized profit are:

$$p_t(i) = c_t(\widetilde{s}_t(i)), \tag{17}$$

$$X_t(i) = \left(d_t(\widetilde{s}_t(i))/c_t(\widetilde{s}_t(i))\right)^{\sigma}, \tag{18}$$

$$\pi_t(i) = \sigma^{-1} c_t(\widetilde{s}_t(i))^{1-\sigma} \left(d_t(\widetilde{s}_t(i))^{\sigma} - c_t(\widetilde{s}_t(i))^{\sigma}\right). \tag{19}$$

Given those results, the problem in the first stage is straightforward: firms choose the technology that maximizes the right-hand side of (19). Note that the amount of profit, given by (19), is decreasing in the unit cost $c_t(\widetilde{s}_t(i))$. Thus, firms have an incentive to behave in a similar manner to other firms, since this would enable them to fully utilize the past stock of knowledge. However, since the amount of profit is decreasing in the unit demand price $d_t(\widetilde{s}_t(i))$, they also have an incentive to behave as differently as possible from others in order to capture consumer's unsatisfied wants. In the next section, we examine how these two forces uniquely determine the equilibrium choice of technologies at each instant.

4 Instantaneous Equilibrium

In this section, we characterize the market equilibrium at each instant, taking as given the accumulated stock of past knowledge. Since the amount produced and the price charged by each firm is already obtained in (17), the main task of this section is to derive the equilibrium pattern of the choice of technology by firms. We then check the equilibrium of the labor market.

Recall that, in this economy, firms do not incur any cost when they enter or exit the market. By the assumption of free entry, there must be no opportunity in equilibrium to attain positive profit. Using (19), this condition is written as: $d_t(s) \leq c_t(s)$ for all $s \in \mathbf{s}$. In addition, the profit of any firm must not be negative because they can exit costlessly. That is, $d_t(\widetilde{s}_t(i)) = c_t(\widetilde{s}_t(i))$ for all $i \in [0, I_t]$.

Recall that the unit cost function $c_t(s)$ is determined by past knowledge, which is taken as given at each instant. On the other hand, the unit demand price function $d_t(s)$ is determined by the current pattern of the choices of technologies, or the current distribution of firms on the technology space. Therefore, the search for the market equilibrium is tantamount to the search for the distribution of firms such that the implied shape of function $d_t(s)$ satisfies the free entry/exit conditions, given the shape of function $c_t(s)$.

Equilibrium Conditions

In order to be explicit about the dependence of $d_t(s)$ upon the distribution of firms, let n_{jt} denote the number of firms adopting technology s_j. Then, infinite dimensional vector $\mathbf{n}_t \equiv \{n_{jt}\}_{j=-\infty}^{\infty}$ represents the distribution of firms in the technology space.[15] Let $c_{jt} \equiv c_t(s_j)$ denote the unit cost of producing a good with technology s_j so that vector $\mathbf{c}_t \equiv \{c_{jt}\}_{j=-\infty}^{\infty}$ represents the schedule of unit costs. Then, using (17) in (6)

[15]Throughout this paper, we use variables and functions in bold face to represent (infinite dimensional) vectors.

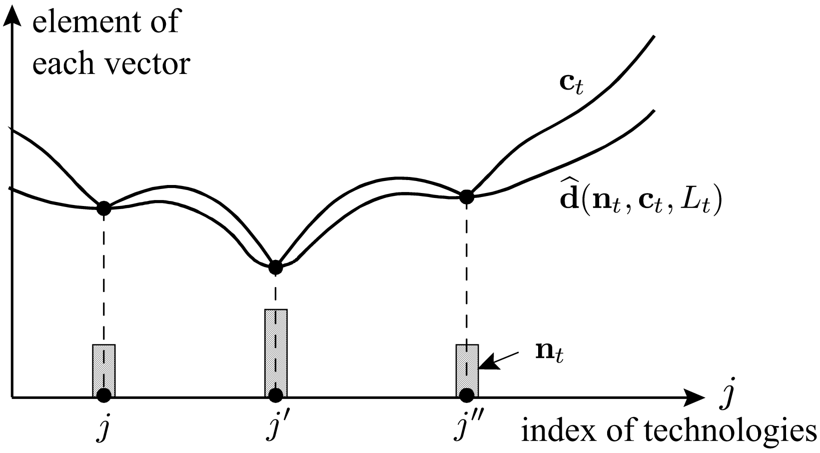

Figure 4: Free entry/exit conditions

and (9), the price indices are written in terms of these vectors:

$$q_t(r) = \widehat{q}(r; \mathbf{n}_t, \mathbf{c}_t) \equiv \left[\sum_{j=-\infty}^{\infty} n_{jt} \left(c_{jt} e^{\tau|r-s_j|} \right)^{-(\sigma-1)} \right]^{-1/(\sigma-1)}, \tag{20}$$

$$Q_t = \widehat{Q}(\mathbf{n}_t, \mathbf{c}_t) \equiv \left[\int_{-\infty}^{\infty} \widehat{q}(r; \mathbf{n}_t, \mathbf{c}_t)^{-(\beta-1)} dr \right]^{-1/(\beta-1)}. \tag{21}$$

From (20) and (21), the unit demand price for each s_j in (13) can be expressed in terms of $\mathbf{n}_t$, $\mathbf{c}_t$, and population L_t:

$$d_t(s_j) = \widehat{d}_j(\mathbf{n}_t, \mathbf{c}_t, L_t) \equiv \left[L_t \widehat{Q}(\mathbf{n}_t, \mathbf{c}_t)^{\beta-1} \int_{-\infty}^{\infty} \widehat{q}(r; \mathbf{n}_t, \mathbf{c}_t)^{\sigma-\beta} e^{-(\sigma-1)\tau|r-s_j|} dr \right]^{1/\sigma}. \tag{22}$$

Let $\widehat{\mathbf{d}}(\mathbf{n}_t, \mathbf{c}_t, L_t) \equiv \{\widehat{d}_j(\mathbf{n}_t, \mathbf{c}_t, L_t)\}_{j=-\infty}^{\infty}$ be the vector of unit demand prices. Then, the free entry/exit conditions can be stated in the form of a complementary slackness condition:[16]

$$(\mathbf{c}_t - \widehat{\mathbf{d}}(\mathbf{n}_t, \mathbf{c}_t, L_t)) \cdot \mathbf{n}_t = 0, \ \mathbf{c}_t - \widehat{\mathbf{d}}(\mathbf{n}_t, \mathbf{c}_t, L_t) \geq \mathbf{0}, \ \mathbf{n}_t \geq \mathbf{0}. \tag{23}$$

Since $\mathbf{c}_t$ and L_t are predetermined variables, (23) is a condition that determines $\mathbf{n}_t$ in equilibrium at each instant. Figure 4 depicts an example of the distribution of firms that satisfies condition (23). In this example, every firm concentrates at either of three points in the technology space, and the unit demand price curve implied by this

[16]A dot between vectors indicates their inner product; e.g., $\mathbf{n}_t \cdot \mathbf{c}_t \equiv \sum_{j=-\infty}^{\infty} n_{jt} c_{jt}$.

distribution of firms touches the unit cost curve exactly at these three points. Each of these points in the technology space is considered as an *industry* where a continuum of firms monopolistically compete with each other selling goods that are differentiated yet aimed at the same consumer want. Except for these three technologies, the unit demand price is lower than the unit cost and therefore no firm operates.

Using vectors $\mathbf{n}_t$ and $\mathbf{c}_t$ introduced above, the equilibrium condition for the labor market can be briefly stated. The aggregate demand of labor is $\int_0^{I_t} \ell_t(i)di$ while the aggregate supply is L_t. From (14), (18) and (23), each firm produces a unit quantity, using amount $c_t(\widetilde{s}_t(i))$ of labor. Thus, the labor market clears if:

$$\mathbf{n}_t \cdot \mathbf{c}_t = L_t. \tag{24}$$

The equilibrium of the economy at each instant is characterized by the distribution of firm $\mathbf{n}_t$ that satisfies the equilibrium condition for the goods market (23) and one for the labor market (24). Natural questions are whether such a distribution actually exists and, if so, whether it is uniquely determined. In addition, it is of interest to know the range of technologies that are actually chosen by firms, and the desirability of such a choice of technologies in terms of the welfare of consumers. The remainder of this section examines these issues.

Boundedness

This subsection shows that the distribution of firms in equilibrium has a bounded support, in the sense that no firm chooses technologies extremely distant from others. Note that function $d_t(s)$ in (13) is well defined for all real number s. Differentiating it with respect to s and comparing the result with $d_t(s)$ gives an upper bound for the curvature of $d_t(s)$:

$$\left| \frac{1}{d_t(s)} \frac{\partial d_t(s)}{\partial s} \right| < \frac{\sigma - 1}{\sigma} \tau. \tag{25}$$

This property can easily be interpreted. Recall that consumers are willing to pay more for 'better suited' goods because such goods provide them with more utility than other poorly matched goods, where the difference depends on the distance in the technology space. The unit demand price curve thus cannot have slopes steeper than the value that corresponds to this difference.

Boundedness of equilibrium would be guaranteed whenever the cost of adopting very distant technology rises with the distance more rapidly than the price that consumers are willing to pay. For the latter claim to be valid, since the production cost is inversely related to the amount of usable knowledge, it must be the case that firms do not have good knowledge about very distant technologies. Stated formally, we assume that

Assumption 1 *At date t, there exist a finite value $\widehat{s}_t$ such that $k_t(s) = 0$ whenever $|s| > \widehat{s}_t$.*

Assumption 1 states that the technological frontier is finite in that no firm has adopted technologies outside $[-\widehat{s}_t, \widehat{s}_t]$ by date t. Under assumption 1, the difference in costs provides a centripetal force strong enough to make all firms choose from a bounded set of technologies.

Proposition 1 *Suppose that assumption 1 holds. Then, condition (23) implies that there exists a finite $\overline{s}_t > \widehat{s}_t$ such that:*

$$n_{jt} = 0 \text{ whenever } |s_j| > \overline{s}_t. \tag{26}$$

Proof: in Appendix.

Observe that, since $\overline{s}_t > \widehat{s}_t$, Proposition 1 permits the possibility that firms choose a technology that has not been adopted by any firms; i.e., the technology frontier may advance over time. At the same time, however, it says that the new frontier is bounded since $\overline{s}_t$ is finite. The latter property is necessary for proving the existence and uniqueness of equilibrium.

Desirability

Observe from (8), (20) and (21) that the instantaneous utility of consumers is the reciprocal of the average price index Q_t and that the index is determined by $\mathbf{n}_t$ and $\mathbf{c}_t$. Thus, given $\mathbf{c}_t$, the consumer benefits from a pattern of the distribution of firms that minimizes the average price index. Here we show that the distribution of firms in instantaneous equilibrium is desirable in the sense that it actually minimizes the average price index under a certain resource constraint.

Let us consider the problem of minimizing $Q_t = \widehat{Q}(\mathbf{n}_t, \mathbf{c}_t)$ with respect to $\mathbf{n}_t$ under resource constraint (24). In addition, we impose condition (26) to this problem in order to reduce the number of technologies that must be considered in the problem. This minimization problem can be solved by the standard Kuhn–Tucker method, yielding the following proposition.

Proposition 2 *Suppose Assumption 1 holds and consider the problem of minimizing $\widehat{Q}(\mathbf{n}_t, \mathbf{c}_t)$ under (24) and (26) with respect to $\mathbf{n}_t \geq \mathbf{0}$. Then, the necessary and sufficient condition for the solution is (23) and (24).*
Proof: in Appendix.

Since condition (26) is implied by condition (23) as shown by Proposition 1, (26) is no longer required for defining the minimizing, or the derisible, allocation. Recall that the distribution of firms, $\mathbf{n}_t$, constitutes an equilibrium if and only if it satisfies conditions (23) and (24). Therefore, Proposition 2 states that the equilibrium distribution of firms exactly coincides with the desirable distribution that minimizes the average price index (and hence maximizes instantaneous utility) under conditions (24) and (26).

Existence and Uniqueness

Now we are ready to derive the main result of this section, which relies on the following proposition.

Proposition 3 *The solution to the minimizing problem in Proposition 2 uniquely exists.*
Proof: in Appendix.

Given the coincidence between the solution and the equilibrium, Proposition 3 implies that the instantaneous equilibrium is also unique and existent. Note that the minimizing problem depends (only) on $\mathbf{c}_t$ and L_t. It means that the solution to the problem, which coincides with the instantaneous equilibrium, is a function of $\mathbf{c}_t$ and L_t. Therefore, the equilibrium distribution at each instant can be written as $\mathbf{n}_t = \mathbf{n}^*(\mathbf{c}_t, L_t)$.

5 Dynamics

The analysis in the previous section showed that, given the size of population, the equilibrium distribution of firms at each instant is uniquely determined by the pattern of unit costs. Section 3 described how the distribution of firms in turn affects the costs of production in the future through accumulation of knowledge. This section examines the dynamic interaction between the distribution of firms and the pattern of accumulated knowledge.

Write $k_{jt} \equiv k_t(s_j)$ and let $\mathbf{k}_t \equiv \{k_{jt}\}_{j=-\infty}^{\infty}$ represent the distribution of knowledge in the technology space. Then (16) implies that the pattern of unit costs is a function of $\mathbf{k}_t$ and can be written as[17] $\mathbf{c}_t = \widehat{\mathbf{c}}(\mathbf{k}_t)$. Using these notations and equations (15), (18) and (23), the dynamics of the economy can be described as an autonomous system in terms of $\mathbf{k}_t$ and L_t:

$$\dot{\mathbf{k}}_t = \mathbf{n}^*(\widehat{\mathbf{c}}(\mathbf{k}_t), L_t) - \delta \mathbf{k}_t, \quad \dot{L}_t = \lambda L_t. \tag{27}$$

[17] The jth element of function $\widehat{\mathbf{c}}(\mathbf{k}_t)$ is defined by $\widehat{c}_j(\mathbf{k}_t) \equiv 1 + ((\mathbf{T}\mathbf{k}_t)_j)^{-1}$, where $\mathbf{T}$ is a matrix representing knowledge spillover whose (j, m) element is given by $T_{jm} = e^{-\nu|s_j - s_m|}$. Note that we treat vector $\mathbf{k}_t$ (and any other vector) as a column vector and $(\mathbf{T}\mathbf{k}_t)_j$ denotes the jth element of vector $\mathbf{T}\mathbf{k}_t$.

Given an initial stock of knowledge $\mathbf{k}_0$ and the initial size of population L_0, equations in (27) determine the evolution of knowledge, and therefore the equilibrium path of the distribution of firms, for all $t \geq 0$.

This section analyzes the dynamic evolution of the economy in three steps: first, we derive the equilibrium distribution of firms at time 0 given an initial stock of knowledge on one technology. Second, we examine when and how this initial pattern of firm distribution changes. Third, the evolution of the economy thereafter is investigated with the aid of numerical simulations.

Equilibrium at Time 0

Suppose that the initial population L_0 is endowed with a small amount of 'innate' knowledge about using one technology.[18] Let this technology be $s_0 = 0$ without loss of generality. Then, each element of the initial stock of knowledge $\mathbf{k}_0$ is given by:

$$k_{00} > 0, \text{ and } k_{j0} = 0 \text{ for all } j \neq 0, \tag{28}$$

Substituting (28) for (16) yields the initial unit costs $\mathbf{c}_0$, whose elements are:

$$c_0(s_j) = c_{0j} = 1 + {k_{00}}^{-1} e^{\nu |s_j|}. \tag{29}$$

Observe that the unit cost is lowest at technology s_0 and therefore, as far as production cost is concerned, every firm has an incentive to choose it.

To find the equilibrium distribution of firms at $t = 0$, suppose that every firm chooses technology s_0. Then, the labor market clearing condition (24) requires that $\mathbf{n}_0$ should be:

$$n_{00} = L_0/(1 + {k_{00}}^{-1}), \text{ and } n_{j0} = 0 \text{ for all } j \neq 0. \tag{30}$$

[18] It is obvious from (16) that if $\mathbf{k}_0 = \mathbf{0}$, no production activity takes place and therefore experience never accumulates in our model. Thus, the economy must start with a positive k_{j0} for at least one technology. How k_{j0} emerges in the first place is an interesting problem, but it is beyond the scope of this paper.

Substituting (29) and (30) into (20), (21) and (22) gives an explicit expression for the unit demand price $\widehat{\mathbf{d}}(\mathbf{n}_0, \mathbf{c}_0, L_0)$:

$$d_0(s_j) = \widehat{d}_j(\mathbf{n}_0, \mathbf{c}_0, L_0) = \left(1 + {k_{00}}^{-1}\right) \psi(s_j), \text{ where}$$
$$\psi(s) = \left[\frac{\int_{-\infty}^{\infty} \exp\{(1-\sigma)\tau|r-s| + (\sigma-\beta)\tau|r|\}\, dr}{\int_{-\infty}^{\infty} \exp\{(1-\beta)\tau|r|\}\, dr} \right]^{1/\sigma}. \tag{31}$$

Function $\psi(s)$ is well defined for all real numbers s, and is a smooth and symmetric function with the following properties.

Lemma 1 *(i)* $\psi(0) = 1$. *(ii)* $1 < \psi(s) < \exp\{((\sigma - \beta)/\sigma)\tau|s|\} < e^{\nu|s|}$ *for all* $s \neq 0$.
Proof: in Appendix.

Property *(i)* implies that, for any value of k_0, the unit demand price at $s_0 = 0$ is equalized to the unit cost, confirming that each firm choosing technology s_0 exactly breaks even. The remaining equilibrium condition is that the unit demand price must never exceed the unit cost for any other technologies. From property *(ii)* of Lemma 1, (29) and (31), this condition is expressed as:

$$k_{00} \leq \frac{e^{\nu|s_j|} - \psi(s_j)}{\psi(s_j) - 1} \equiv \kappa(s_j) \quad \text{for all } j \neq 0. \tag{32}$$

The following proposition guarantees that condition (32) is satisfied if the initially endowed experience is sufficiently small.

Proposition 4 *(i)* $\lim_{|s|\to 0} \kappa(s) = \lim_{|s|\to\infty} \kappa(s) = \infty$.
(ii) There exists a finite $\underline{\kappa} > 0$ *and* $s^* \in \mathbf{s} \setminus s_0$ *such that* $\underline{\kappa} = \kappa(s^*) = \min_{j\neq 0} \kappa(s_j)$.
Proof: in Appendix.

Property *(ii)* of Proposition 4 means that, if and only if $k_{00} \leq \underline{\kappa}$, the one-industry structure represented by (30) satisfies the free entry/exit conditions (23) and therefore is the unique instantaneous equilibrium. We assume that $k_{00} \leq \underline{\kappa}$.

Emergence of New Industries

Given the initial distribution of knowledge (28) with $k_{00} \leq \underline{\kappa}$, the economy starts from a one-industry structure, where all firms agglomerate at one point ($s_0 = 0$) in the technology space. This structure is self-sustaining for a certain time period because experience accumulates only in that industry, which induces firms in subsequent periods to agglomerate again at the same technology.

As long as production experience accumulates only on technology s_0 and the accumulated knowledge k_{0t} does not exceed $\underline{\kappa}$, the analysis in the previous subsection applies and, similar to (30), the equilibrium distribution of firms is given by $n_{0t} = L_t/(1+{k_{0t}}^{-1})$ and $n_{jt} = 0$ for all $j \neq 0$. Substituting this into (27) gives the evolution of knowledge within the initial industry:

$$\dot{k}_{0t} = L_t/(1 + k_{0t}^{-1}) - \delta k_{0t}. \tag{33}$$

Since population L_t grows at a positive constant rate λ, (33) implies that the amount of knowledge k_{0t} exceeds the critical value $\underline{\kappa} = \kappa(s^*)$ in a finite time period. If all firms continue to choose the initial technology $s_0 = 0$ when $k_{0t} > \kappa(s^*)$, it follows that $d_t(s^*) = (1 + k_{0t}^{-1})\psi(s^*) > 1 + k_{0t}^{-1} e^{\nu|s^*|} = c_t(s^*)$ from (29), (31) and (32). This means that firms can make positive profits by choosing technology s^* rather than staying in the initial industry.[19] Therefore, the one-industry structure no longer satisfies the free entry condition, and firms begin to adopt the new profitable technology s^* (and $-s^*$). This lowers the unit demand price at s^*, and the number of firms adopting the new technology is determined so that the unit demand price at s^* is equalized to the unit cost there. Note that property (i) of Proposition 4 implies that the newly chosen technologies (which have the lowest $\kappa(s_j)$) are significantly different, but not extremely distant, from the currently used technology.

In this way, accumulation of experience in the initial industry eventually gives rise to new industries that are based on significantly but not extremely different technologies. This structural change can be explained in terms of two forces that affect the technology choice. In a one-industry economy, the relative unit demand price between a pair of goods with different technologies is $\psi(s_j)/\psi(s_{j'})$ from (31). Since it is independent of

[19]Since function $\psi(\cdot)$ is symmetric around zero, choosing $-s^*$ is also profitable.

k_{0t}, the centrifugal force induced by the desire of consumers to satisfy a wider range of wants is unaffected by the accumulation of experience. As experience accumulates, however, the centripetal force induced by the differences in the unit cost gradually gets weaker for the following reasons. Recall that equation (16) shows that experience reduces the unit cost but at a diminishing rate. As the unit cost at the initial industry approaches the lower bound, which exists at 1, any additional experience within the industry has little impact on the unit cost there. Spillovers of knowledge, however, significantly reduce the costs of somewhat distant technologies where there still remains a relatively large gap between the current unit cost and the lower bound. As a result, the relative cost between the initial industry and other technologies gradually shrinks, which weakens the centripetal force operating in the technology space.

The preceding analysis has shown that the centripetal force globally dominates the centrifugal force while the amount of accumulated experience in the initial industry is below a certain critical value $\underline{\kappa}$. This global dominance comes to an end when k_{00} exceeds $\underline{\kappa}$. Nonetheless, the centripetal force is locally dominant in the neighborhood of the initial industry because the unit cost curve (29) has a downward kink at $s = 0$ while the unit demand price curve (31) is smooth.[20] On the other extreme, Proposition 1 means that the centripetal force always dominates the centrifugal force at infinitely distant technologies. Therefore, the new industry emerges at an intermediate distance from the initial industry when k_{0t} reaches the threshold.

Evolution of Industrial Structure

After the one-industry structure becomes unstable, we must deal directly with system (27) to track the evolution of the distribution of firms. Although the previous section confirmed that $\mathbf{n}^*(\widehat{\mathbf{c}}(\mathbf{k}_t), L_t)$ is well defined, it does not have an explicit representation. Thus, it seems sensible at this point to turn to a numerical simulation.

Figure 5 depicts a simulated evolution of the distribution of firms. Black loci in

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[20] Recall that (25) shows that the curvature of the unit demand price curve is bounded.

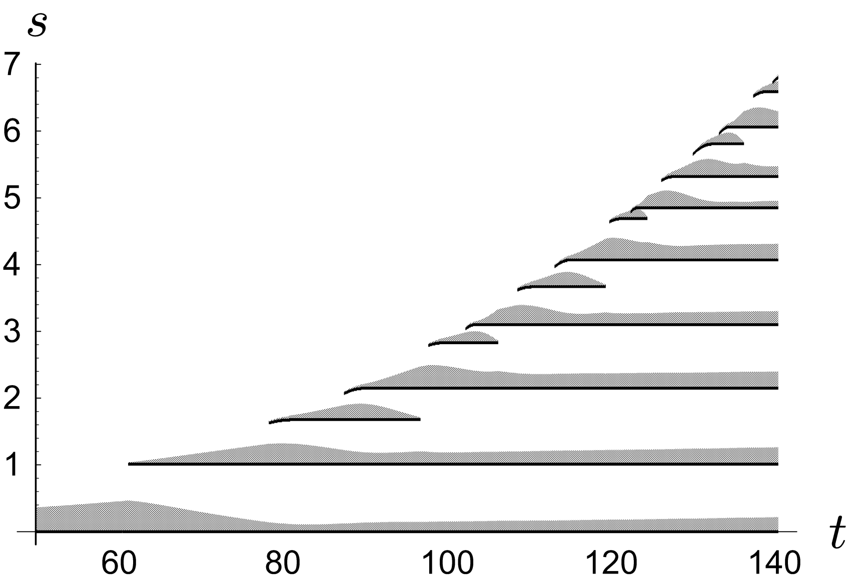

Figure 5: Evolution of industries. Since both $\mathbf{c}_t$ and $\mathbf{n}_t$ are symmetric around $s_0 = 0$, results are shown only for $s \geq 0$. Black loci represent the technologies with $n_{jt} > 0$, whereas the height of the gray area above each locus shows the magnitude of $n_{jt}c_{jt}$. Parameters are set to $\beta = 2$, $\sigma = 4$, $\tau = 1$, $\delta = 0.24$, $\nu = 1$, $\lambda = 0.024$, and $\varepsilon = 0.01$. Initial conditions are $k_{00} = 0.1$ and $N_0 = 0.1$.

the figure represent the evolution of the support of firm distribution, i.e., the set of technologies that are adopted by a positive measure of firms. Observe that the economy consists of a finite number of distinguishable industries, rather than one big industry that contains a wide range of technologies. Being induced by the centripetal force of past knowledge, all firms in each industry agglomerate in one point in the technology space, which creates a strong hysteresis in the choice of technology. However, as the diminishing returns in learning set in, the centripetal force gets weaker and is eventually dominated by the centrifugal force generated by consumers' wants when new industries emerge. Such a process is repeated cyclically, and as a result structural changes occur sporadically at distinct points in time. Thus, interacting forces of wants and past knowledge create a discrete system of industries while variations in their relative strength cause discrete structural changes, even though the underlying model does not assume such discreteness.[21]

In Figure 5, the height of the gray area above each locus represents the size of each industry measured by consumer expenditure, or equivalently the number of workers in that industry. Observe that after emergence the size of an industry initially rises and then falls,[22] for the following reasons. At the birth of an industry, the cost of adoption is very high, and the demand for the highly priced goods is low even though they well meet wants that are unsatisfied by existing industries. As experience accumulates, the price falls and the demand increases. Since there remains a wide margin for cost reduction in new industries the prices of goods fall more rapidly than those in the older industries through the accumulation of experiences, which eventually makes the new industry larger than typical existing industries. However, the demand for the new industry begins to decrease when a yet newer industry emerges, which better serves some wants that were previously served by the original industry. In the long run, some industries approach their steady state sizes, while other industries disappear.

Disappearance of some industries might require a further explanation. To see this, note that when a frontier industry emerges, it attracts a large unsatisfied demand from the wants beyond the frontier. Thus, it may be profitable to adopt a technology not so distant from existing industries. When the next frontier industries emerge, however, the demand that the industry previously on the frontier can attract significantly falls since it now has a technology too close to other industries on both sides. In some

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[21]Strictly speaking, the model for convenience assumes that firms choose technologies from an infinite but discrete set $\mathbf{s} \equiv \{s_j\}_{j=-\infty}^{\infty}$, where $s_j = j\varepsilon$, with $\varepsilon > 0$ being a very small constant. However, we confirmed that, given that ε is sufficiently small, the size of ε does not affect the simulation results, which implies that the discrete structural changes in the model do not depend on the discreteness of $\mathbf{s}$.

[22]This pattern is consistent with the finding by Wang (2004, Figures 3,4), who examined the evolution of the relative industry GDP (i.e., each industry's share of the GDP) for seven industries in the U.S. and found that the relative industry GDP initially rose and then fell, with only one exception: the nylon industry.

cases, the demand eventually becomes so small that no firm in this industry can break even. Are, then, such ephemeral industries futile from the viewpoint of economic growth? In fact, they play an important historical role: the knowledge accumulated by such industries spills beyond them and serves as a stepping stone to new, possibly everlasting, frontier industries.

6 Growth Cycles

In the previous section, simulation results illustrate that industries sporadically emerge and disappear as the economy grows. We now investigate how such continual changes in industrial structure affect the pattern of macroeconomic growth rate and the utility of the representative consumer.

Fluctuations in the GDP Growth Rate

In equilibrium, the total output quantity of goods with technology s_j is n_{jt} whereas their price is c_{jt}. Therefore, the per capita real GDP growth rate (i.e., the real GDP growth minus the population growth) in a conventional definition is given by:[23]

$$g_t = \frac{\dot{\mathbf{n}}_t \cdot \mathbf{c}_t}{\mathbf{n}_t \cdot \mathbf{c}_t} - \lambda. \tag{34}$$

Differentiating both sides in (24) with respect to time gives $\dot{\mathbf{n}}_t \cdot \mathbf{c}_t + \mathbf{n}_t \cdot \dot{\mathbf{c}}_t = \dot{L}_t$. Applying it to (34) enables us to express the growth rate in terms of the rate of cost reduction:

$$g_t = -\frac{\mathbf{n}_t \cdot \dot{\mathbf{c}}_t}{\mathbf{n}_t \cdot \mathbf{c}_t} = \sum_{j=-\infty}^{\infty} \left(\frac{n_{jt} c_{jt}}{L_t} \right) \left(-\frac{\dot{c}_{jt}}{c_{jt}} \right). \tag{35}$$

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[23]Equation (34) implicitly assumes that vector $\mathbf{n}_t$ changes continuously. This can be confirmed in the following way. As shown by Proposition 2, $\mathbf{n}_t$ coincides with the solution to the problem of minimizing $\widehat{Q}(\mathbf{n}_t, \mathbf{c}_t)$ under (24) and (26) with respect to $\mathbf{n}_t$. The constraint of this problem is compact and changes continuously as $\mathbf{c}_t$ moves according to (36). Then we can apply the Theorem of the Maximum (Stokey and Lucas 1988, Chapter 3) to be assured of the continuity of $\mathbf{n}_t$.

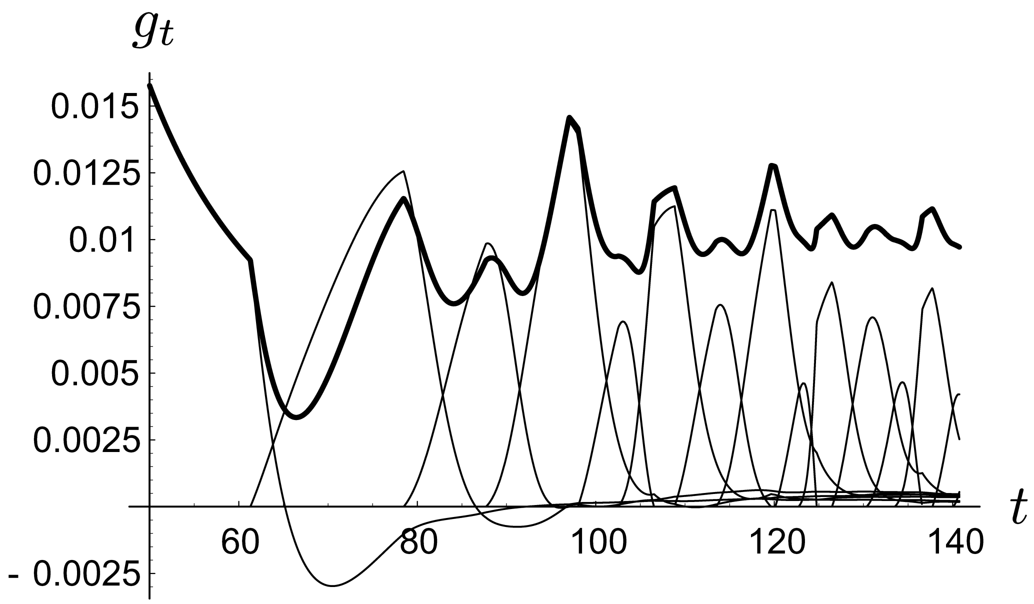

Figure 6: The per capita real GDP growth and its decomposition. The overall per capita real GDP growth rate is represented by the thick curve, whereas the thin curves give its decomposition into contributions from each industry according to formula (35). Parameter values and initial conditions are the same as in Figure 5.

Note that $n_{jt}c_{jt}/L_t$ represents the market share of industry j. Thus, the last expression in (35) gives a clear decomposition of the growth rate in each industry: g_t is the weighted sum of the rates of cost reduction in each industry, where the weights are their market shares.

Equation (35) enables us to calculate numerically the evolution in the GDP growth rate, the result of which is shown in Figure 6. We can observe three conspicuous features of the figure. First, at each point in time usually only one or two, at most three, young industries explain quite a large portion of the overall growth rate, as Gordon (2000) pointed out in the case of computer and related industries. In other words, mature industries contribute little to economic growth, implying that structural changes, especially the emergence of new industries, are crucial for sustained economic growth. Second, the figure shows that the rate of economic growth falls immediately after a new industry emerges. That is, structural changes cause a temporary slowdown in growth although they are indispensable in the long run. This phenomenon corresponds to the well-known "productivity slowdown puzzles" (see Hornstein and Krusell 1996; Greenwood and Yorukoglu 1997; Jovanovic and Rousseau 2005). Third, the variation in the rate of economic growth is larger in the early stage of development and gradually stabilizes as the number of industries increases. Such a tendency was mentioned by Lucas (1988), and subsequently confirmed by data (e.g., Blanchard and Simon 2001; Koren and Tenreyro 2005).

In the model, the last property is easy to interpret: as the number of industries increases, the contribution of the rise and fall of each industry to the economy-wide fluctuations in growth becomes smaller, and eventually the law of large numbers will apply. To understand the first two properties, we must explicitly look at the pattern of knowledge accumulation that drives economic growth. Recall that the rate of cost reductions in each industry is influenced by the pattern of knowledge accumulation, which in turn depends on the distribution of firms. Differentiating (16) with respect to time and then applying (27) to it yields:

$$\dot{\mathbf{c}}_t = (\mathbf{C}_t - \mathbf{I})^2 \mathbf{T} \mathbf{n}_t - \delta(\mathbf{c}_t - \mathbf{1}), \tag{36}$$

where $\mathbf{T}$ is a symmetric matrix with its (j, m) element being $T_{jm} = e^{-\nu|s_j - s_m|}$, $\mathbf{C}_t \equiv \mathrm{diag}(\mathbf{c}_t)$ is a diagonal matrix whose jth diagonal element is c_{jt}, $\mathbf{I}$ is an identity matrix, and $\mathbf{1}$ is a vector with all elements equal to 1. Substituting (24) and (36) into (35) gives the per capita real GDP growth in an intuitive form:

$$g_t = L_t^{-1} \mathbf{n}_t \cdot (\mathbf{C}_t - \mathbf{I})^2 \mathbf{T} \mathbf{n}_t - \delta(1 - L_t^{-1} \mathbf{1} \cdot \mathbf{n}_t). \tag{37}$$

The second term in (36) shows the extent to which depreciation of knowledge decelerates growth. As long as the rate of depreciation δ is not large, the significance of this term is limited.

The main body of g_t is determined by the first term, which represents the effect of knowledge accumulation through production experience on the macroeconomic growth rate. This term can be viewed as a quadratic form in $\mathbf{n}_t$ with $(\mathbf{C}_t - \mathbf{I})^2 \mathbf{T}$ being the matrix of coefficients. Since $(\mathbf{C}_t - \mathbf{I})^2$ is a diagonal matrix, any interaction effect between industries is described by matrix $\mathbf{T}$. Note that, reflecting the fact that experience in a certain industry is more useful within that industry than to other industries, the diagonal elements in $\mathbf{T}$ are larger than its off-diagonal elements. This suggests that there is an agglomeration economy: with a large market share an industry can accomplish a fast cost reduction, which in turn accounts for a large portion of the macroeconomic growth rate. When a new industry emerges, the individual market shares of industries become more diverse. The emergence of the new industry thus diminishes the benefit of agglomeration and lowers the rate of economic growth, at least in the short run during which matrix $(\mathbf{C}_t - \mathbf{I})^2$ can be regarded as approximately constant.[24]

In the long run, however, variations in matrix $(\mathbf{C}_t - \mathbf{I})^2$ play a crucial role in equation (37). This matrix implies that mature industries where $c_{jt} - 1$ is near zero contribute little to economic growth. Sustained growth thus requires continual emergence of new industries, and the new industries must adopt unexplored technologies in which there still remains room for further improvement. The learning-by-doing process within an industry affects the growth rate only transitorily, while spillover of knowledge across different technologies contributes to economic growth in the long run by paving the way for the emergence of ever newer industries.[25]

Welfare Effects

We have shown that, in terms of GDP, emergence of new industries temporarily decelerates growth, although they are necessary for long-term growth. GDP is a convenient measure, but it does not account for the benefit resulting from increases in the vari-

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[24]Note that vector $\mathbf{n}_t$ can change instantly, while $\mathbf{c}_t$ cannot.

[25]In this respect, our model contrasts clearly with and is complementary to the 'hybrid' endogenous growth models presented by Young (1998), Peretto (1998), and Dinopoulos and Thompson (1998), which combine the variety expansion model and the quality ladder model. In their models, the primary determinant of long-term growth is quality improvement within each industry, while the expansion of variety is treated as a transitory adjustment process that does not affect the long-term growth rate.

ety of goods. We now examine the evolution of the consumer's instantaneous utility and show that it also follows the same cyclical pattern as the GDP growth during the process of economic growth.

Note that instantaneous utility u_t is the reciprocal of the average price index Q_t as shown by (8). Thus, using (21), the rate of change in u_t can be decomposed into:

$$\frac{\dot{u}_t}{u_t} = -\frac{\dot{Q}_t}{Q_t} = \frac{\widehat{Q}_{\mathbf{c}}(\mathbf{n}_t, \mathbf{c}_t)}{\widehat{Q}(\mathbf{n}_t, \mathbf{c}_t)} \cdot (-\dot{\mathbf{c}}_t) - \frac{\widehat{Q}_{\mathbf{n}}(\mathbf{n}_t, \mathbf{c}_t)}{\widehat{Q}(\mathbf{n}_t, \mathbf{c}_t)} \cdot \dot{\mathbf{n}}_t, \tag{38}$$

where $\widehat{Q}_{\mathbf{c}}(\cdot)$ and $\widehat{Q}_{\mathbf{n}}(\cdot)$ denote the partial derivatives of function $\widehat{Q}(\cdot)$. The first term in (38) represents the welfare improvement that comes from the falling prices of goods. After some calculation and applying the equilibrium condition (23), we can show that this term coincides with the per capita GDP growth g_t. The second term in (38) represents the benefit from the increased variety of goods. Calculation shows that it equals $(g_t + \lambda)/(\sigma - 1)$; that is, $(\sigma - 1)^{-1}$ times the rate of GDP growth—not per capita GDP growth.[26] Intuitively, the increased aggregate purchasing power enables more firms to enter the market, which has a positive pecuniary externality on every consumer who values more variety.

Collecting these effects together, we now have a simple formula:

$$\frac{\dot{u}_t}{u_t} = \frac{\sigma}{\sigma - 1} g_t + \frac{\lambda}{\sigma - 1}. \tag{39}$$

Equation (39) implies that the utility growth of households evolves in parallel with per capita real GDP growth. Thus, the argument we have expounded with respect to the GDP growth applies also to the utility of households; the successive emergence of new industries is indispensable to maintain a sustained utility growth (above $\lambda/(\sigma - 1)$), but the rate of utility growth is temporarily reduced immediately after each industry emerges.

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[26]Since the size of firm is constant in equilibrium, the increased purchasing power proportionally increases variety. Substituting (20) and (21) into (8), we see that the electricity of instantaneous utility with respect to variety (i.e., $\mathbf{n}_t$) is $(\sigma - 1)^{-1}$. Then the result follows.

7 Conclusion

Consumers desire to satisfy an indefinite range of wants. This creates a large demand for goods made with novel technologies that meet the unsatisfied needs of consumers. However, firms face a high cost in adopting such previously unexplored technologies because relatively little is known about them. We investigate how this tradeoff between demand and cost, or between wants and knowledge, affects the distribution of firms in the technology space. We find that firms endogenously form a number of distinguishable industries and their dynamic evolution is characterized by the sporadic emergence of new industries. The emergence of a new industry temporarily reduces the rate of per capita GDP growth, because it diversifies the share of individual industries and diminishes the benefits of agglomeration economy that comes from knowledge accumulation within an industry. New industries are, however, indispensable for sustained growth since without them cost reduction within existing industries eventually comes to an end. We also confirm that the instantaneous utility of the consumer, which incorporates the benefit from increased variety, also follows the same pattern as the per capita GDP growth.

In the literature of endogenous growth, the majority of studies stressed the supply side (e.g., accumulation of knowledge) as the determinant of growth. However, our result highlights the importance of consumer demand in directing the economy to the right way to sustained economic growth.[27] Economic growth can be maintained only when knowledge is accumulated over an ever-expanding range of technologies. We show that firms willingly pay high costs of adopting previously unexplored technologies, even though they cannot appropriate knowledge generated by their behavior. Rather, their incentive is induced by the demand of consumers who desire to satisfy an ever-

[27]The importance of consumer demand in directing growth is also examined by Foellmi and Zweimüller (2004). They show that inequality may promote growth since rich consumers are able to satisfy a wider range of wants, although growth cycles are not their focus.

expanding range of wants.[28] The interaction between knowledge accumulation on the supply side and human wants on the demand side is thus an indispensable part of the sustained growth.

Appendix

Proof of Proposition 1

Consider the case of $s > \widehat{s}_t$. From Assumption 1 and (16),

$$\sum_{s' \in \mathbf{s}} e^{-\nu|s-s'|} k_t(s') = e^{-\nu(s-\widehat{s}_t)} \sum_{s' \in \mathbf{s}} e^{-\nu(\widehat{s}_t - s')} k_t(s') = e^{-\nu(s-\widehat{s}_t)} (c_t(\widehat{s}_t) - 1)^{-1}.$$

Substituting it for (16) gives:

$$c_t(s) = 1 + (c_t(\widehat{s}_t) - 1) e^{\nu(s-\widehat{s}_t)} \quad \text{for } s > \widehat{s}_t. \tag{A.1}$$

Note that $d_t(\widehat{s}_t) \leq c_t(\widehat{s}_t)$ from condition (23). Then, (25) implies that:

$$d_t(s) \leq c_t(\widehat{s}_t) e^{((\sigma-1)/\sigma)\tau(s-\widehat{s}_t)} \quad \text{for } s > \widehat{s}_t. \tag{A.2}$$

Since $c_t(\widehat{s}_t) > 1$ is predetermined and since $\nu > ((\sigma - 1)/\sigma)\tau$, there must be a finite value $\overline{s}_t > \widehat{s}_t$ such that the right-hand side of (A.1) is strictly larger than that of (A.2) for all $s > \overline{s}_t$. This means $c_{jt} = c_t(s_j) > d_t(s_j) = d_{jt}$ for all $s_j > \overline{s}_t$, from which condition (23) implies $n_{jt} = 0$ for all $s_j > \overline{s}$. A similar argument applies for $s < -\widehat{s}_t$.

Proof of Propositions 2 and 3

Since $\overline{s}_t$ is finite, we can choose a J_t such that $[-\overline{s}_t, \overline{s}_t] \cap \mathbf{s} = \{s_j\}_{j=-J_t}^{J_t}$. Note that, since $1 < \beta < \sigma$, minimization of $\widehat{Q}(\mathbf{n}_t, \mathbf{c}_t)$ is equivalent to maximization of $\widehat{Q}(\mathbf{n}_t, \mathbf{c}_t)^{-(\beta-1)}$. From (20) and (21), this equivalent problem can be written as:

$$\max_{n_{-J_t t}, \ldots, n_{J_t t}} \int_{-\infty}^{\infty} \left[\sum_{j=-J_t}^{J_t} n_{jt} \left(c_{jt} e^{\tau|r-s_j|} \right)^{-(\sigma-1)} \right]^{(\beta-1)/(\sigma-1)} dr, \tag{A.3}$$

[28] This property is consistent with a finding by Acemoglu and Linn (2004) that the size of potential demand is critical for the entry of new drugs and pharmaceutical innovation.

subject to (24). Observe that condition (26) is implicitly incorporated into problem (A.3). We set up a Lagrangian for this problem and differentiate it to obtain the first order condition. Let the Lagrange multiplier on (24) be ξ_t. Then, the resulting complementary slackness condition is:

$$\xi_t^{-1}\frac{\beta-1}{\sigma-1}\int_{-\infty}^{\infty}\widehat{q}(r;\mathbf{n}_t,\mathbf{c}_t)^{\sigma-\beta}e^{-(\sigma-1)\tau|r-s_j|}dr \leq c_{jt}^{\sigma}$$
$$\text{for all } |j| \leq J_t \text{ and with equality if } n_j > 0. \tag{A.4}$$

Substituting (A.4) into (24) and solving it for ξ_t gives:

$$\xi_t = \frac{1}{L_t}\frac{\beta-1}{\sigma-1}\int_{-\infty}^{\infty}\widehat{q}(r;\mathbf{n}_t,\mathbf{c}_t)^{-(\beta-1)}dr. \tag{A.5}$$

Substituting (A.5) back into (A.4) shows that the first order condition for problem (A.3) is exactly the same as (23).

The domain for $\{n_{-J_t t},\ldots,n_{J_t t}\}$ defined by (24) and (26) is compact and the object function in (A.3) is continuous and strictly concave in those variables. The existence and uniqueness of the solution follow from these properties (thus Proposition 3 holds). In addition, those properties guarantee that the first order condition (23), combined with constraints (24) and (26), is both necessary and sufficient. In fact, constraint (26) is not necessary to define the minimizing solution since it is implied by (23) and (24) from Proposition 1. This completes the proof of Proposition 2.

Proof of Lemma 1

For convenience, define function $\zeta(r,s) = (\sigma-\beta)\tau|r| - (\sigma-1)\tau|r-s|$, by which $\psi(\cdot)$ in (31) can be expressed as:

$$\psi(s) = \left[\frac{\int_{-\infty}^{\infty}\exp\zeta(r,s)\,dr}{\int_{-\infty}^{\infty}\exp\zeta(r,0)\,dr}\right]^{1/\sigma}. \tag{A.6}$$

Property (*i*): With expression (A.6), it is obvious that $\psi(0) = 1$ holds since the numerator and the denominator coincide when $s = 0$.

Property (*ii*): Consider the case of $s > 0$. It is straightforward to confirm $\zeta(r,s) \leq \zeta(r-s,0) + (\sigma-\beta)\tau s$ with strict inequality when $r < s$. Substituting this inequality into (A.6) shows $\psi(s) < \exp\{((\sigma - \beta)/\sigma)\tau s\}$. From $1 < \beta < \sigma$ and $\nu > ((\sigma - 1)/\sigma)\tau$, it follows that $\nu > ((\sigma - \beta)/\sigma)\tau$. Thus, we obtain $\psi(s) < e^{\nu s}$. Similarly, it can be confirmed that $\zeta(r, s) \geq \zeta(r - ((\sigma - 1)/(\beta - 1))s, 0)$ with strict inequality if $0 < r < ((\sigma - 1)/(\beta - 1))s$. Substituting this into (A.6), we have $\psi(s) > 1$. Collecting both results shows $1 < \psi(s) < \exp\{((\sigma - \beta)/\sigma)\tau s\} < e^{\nu s}$ for all $s > 0$. Since $\psi(s)$ is symmetric around zero, s can be replaced by $|s|$.

Proof of Proposition 4

Note that function $\kappa(s)$ defined in (32) is well defined for all real number s except at $s = 0$, and is symmetric, smooth, and positive for all $s \neq 0$. Since it is symmetric, it is sufficient to consider only the case of $s > 0$.

Property (i): When s tends toward zero from above, both the denominator and numerator of $\kappa(s)$ also tend to zero. Applying l'Hopital's Theorem, we have:

$$\lim_{s \to +0} \kappa(s) = \lim_{s \to +0} \frac{\nu - \psi'(s)}{\psi'(s)} = \frac{\nu - 0}{0} = \infty,$$

where we used $\psi'(0) = 0$ since $\psi(s)$ is everywhere differentiable and symmetric around zero. When s tends to infinity, we use Lemma 1 and divide both the denominator and numerator of $\kappa(s)$ by $\exp\{((\sigma - \beta)/\sigma)\tau s\}$ to obtain:

$$\lim_{s \to \infty} \kappa(s) \geq \lim_{s \to \infty} \frac{\exp\{(\nu - ((\sigma - \beta)/\sigma)\tau)s\} - 1}{1 - \exp\{-((\sigma - \beta)/\sigma)\tau s\}} = \frac{\infty - 1}{1 - 0} = \infty. \tag{A.7}$$

Property (ii): Note that, since $s_j = j\varepsilon$ and $\varepsilon > 0$, (A.7) also imply $\lim_{j \to \infty} \kappa(s_j) = \infty$. In addition, $\kappa(s_j) > 0$ for all $j > 0$. Thus, there exists a smallest element $\underline{\kappa} = \kappa(s^*) > 0$ in sequence $\{\kappa(s_j)\}_{j=1}^{\infty}$.

References

Acemoglu, D. and Linn, J. (2004), "Market Size in Innovation: Theory and Evidence from the Pharmaceutical Industry," *Quarterly Journal of Economics*, **119**(3), 1049–1090.

Blanchard, O.J. and J. Simon (2001), "The Long and Large Decline in US Output Volatility," *Brookings Papers on Economic Activity*, **1**, 135–174.

Bloom, N., Schankerman, M. and Van Reenen, J. (2005), "Identifying Technology Spillovers and Product Market Rivalry," *CEPR Discussion Paper* No. 4912.

Dinopoulos, E. and Thompson, P. (1998), "Schumpeterian Growth without Scale Effects," *Journal of Economic Growth*, **3**, 313–35.

Dixit, A.K. and Stiglitz, J.E. (1977), "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, **67**, 297–308.

Fujita, M., Krugman, P. and Venables, A.J. (1999) *The Spatial Economy: Cities, Regions and International Trade* (Cambridge: MIT Press).

Foellmi, R. and J. Zweimüller (2004), "Income Distribution and Demand-Induced Innovations," *IEW Working Paper* No. 212, University of Zurich.

Freeman, S., Hong, D.P. and Peled, D. (1999), "Endogenous Cycles and Growth with Indivisible Technological Developments," *Review of Economic Dynamics*, **2**(2), 402–432.

Gordon, R.J., (2000). "Does the 'New Economy' Measure Up to the Great Inventions of the Past?," *Journal of Economic Perspectives*, **14(4)**, 49–74.

Greenwood, J. and Yorukoglu, M. (1997), "1974," *Carnegie-Rochester Conference Series on Public Policy*, **46**, 49–95.

Grossman, G.M. and Helpman, E. (1989), "Product Development and International Trade," *Journal of Political Economy*, **97**, 1261–83.

Helpman, E. and Trajtenberg, M. (1998), "A Time to Sow and a Time to Reap: Growth Based on General Purpose Technologies," ch. 3 in E. Helpman (ed.) *General Purpose Technologies and Economic Growth* (Cambridge: MIT Press).

Hornstein, A. and Krusell, P. (1996), "Can Technology Improvements Cause Productivity Slowdowns?," in B.S. Bernanke and J. Rotemberg (eds.) *NBER Macroeconomic Annual 1996* (Cambridge: MIT Press).

Jovanovic, B. (1995), "Learning and Growth," NBER Working Paper No. 5383.

Jovanovic, B. and Y. Nyarko, (1996). "Learning by Doing and the Choice of Technology," *Econometrica*, **64(6)**, 1299–1310.

Jovanovic, B. and P.L. Rousseau, (2005). "General Purpose Technologies," *NBER Working Papers* 11093, National Bureau of Economic Research.

Koren, M. and S. Tenreyro (2005), "Technological Diversification," FRB of Boston Working Paper No. 05-1

Maliar, L. and S. Maliar, (2004), "Endogenous Growth and Endogenous Business Cycles," *Macroeconomic Dynamics*, **8**, 559–581.

Peretto, P. (1998), "Technological Change and Population Growth," *Journal of Economic Growth*, **3**, 283–311.

Phillips, K. and J. Wrase, (2005). "Is Schumpeterian Creative Destruction a Plausible Source of Endogenous Real Business Cycle Shocks," *forthcoming in Journal of Economic Dynamics and Control.*

Redding, S., (2002). "Path Dependence, Endogenous Innovation, and Growth," *International Economic Review*, **43(4)**, 1215–1248.

Romer, P.M. (1990), "Endogenous Technological Change," *Journal of Political Economy*, **98**, S71–102.

Rosenberg, N. (1974), "Science, Invention and Economic Growth," *Economic Journal*, **84**(127), 90–108.

Schmookler, J. (1966), *Invention and Economic Growth*, (Cambridge: Harvard University Press).

Stokey, N.L. (1988), "Learning by Doing and the Introduction of New Goods," *Journal of Political Economy*, **96**, 701–17.

Stokey, N.L. and Lucas, R.E. Jr., with Prescott, E.C. (1988) *Recursive Methods in Economic Dynamics* (Cambridge: Harvard University Press).

Wälde, K. (2002). "The Economic Determinants of Technology Shocks in a Real Business Cycle Model," *Journal of Economic Dynamics and Control*, **27(1)**, 1–28.

Wälde, K. (2005). "Endogenous Growth Cycles," *forthcoming in International Economic Review.*

Wang, Z. (2004) "Learning, Diffusion and the Industry Life Cycle," Payments System Research Working Paper PSR WP 04-01, Federal Reserve Bank of Kansas City.

Young, A. (1991), "Learning by Doing and the Dynamic Effects of International Trade," *Quarterly Journal of Economics*, **106**, 369–405.

Young, A. (1993), "Invention and Bounded Learning by Doing," *Journal of Political Economy*, **101**, 443–472.

Young, A. (1998), "Growth without Scale Effects," *Journal of Political Economy*, **106**, 41–63.