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Abstract

We consider a two-period overlapping generations model where agents face the uncertainty of intergenerational transfers from their children. To avoid this kind of risk, agents have an incentive to share the risk within the same generation. However, there exists an information asymmetry about the realization of the old period's income between the insurance company and old agents. By analyzing economies with and without risk sharing, we find that risk sharing decreases the rate of economic growth and accelerates social welfare when the rate of social time preference is sufficiently large.

JEL Classification Numbers: D81, D82, G22, O40

Keywords: gifts economy, risk sharing, information asymmetry, economic growth, overlapping generations

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1 Introduction

It has been widely recognized that intergenerational transfers account for a significant part of aggregate saving (e.g., Kotlikoff (1988)), and they have been modeled in a number of different ways since the seminal works by Becker (1965) and Barro (1974), which explain transfers as arising from altruistic behavior. For example, under fully altruistic behavior, agents' utility ultimately depends on the utility of all of their descendants (e.g., Becker and Barro (1988) and Barro and Becker (1989)). Under semialtruistic behavior, agents derive utility from the consumption of their offspring and/or the consumption of their parents, or even simply from the sizes of the transfers they make (e.g., Bernheim and Ray (1987)). From a no-altruism perspective, transfers may arise out of pure self-interest because of the perceived mutual benefits of exchange between progenitors and progeny (e.g., Cox (1987)). Alternatively, the existence of bequests may simply be an accident as in Abel (1985), and the existence of gifts may be the result of social norms or customs that obligate children to provide some form of material support to their parents during old age, as argued by Morand (1999).

Despite the substantial research on intergenerational transfers, the uncertainty of intergenerational transfers has not been paid much attention. This is because in the previous research, altruism is formalized by the assumption of fully altruistic or semialtruistic behavior. As Blackburn and Cipriani (2005) showed, however, the direction of intergenerational transfers shifts from children to parents to parents to children according to the state of economic development.[1] In this process, it is thought that uncertainty may exist as to whether parents have received gifts from their children. In fact, the percentage of older agents who live with their children or grandchildren decreases with economic development: it is 75% in developing countries, whereas it is only 27% in developed countries (see United Nations (2006)). Thus, this paper tries to complement the existing literature by studying the uncertainty of intergenerational transfers, especially of gift transfers.[2]

To analyze these issues, we employ a two-period overlapping generations model, and assume that young agents are divided into two types according to their attitude to their parents.

[1] Blackburn and Cipriani (2005) showed that at low levels of development, fertility is high and the flow of net intergenerational transfers is from children to parents. At high levels of development, fertility is low and the flow of net transfers runs from parents to children.

[2] Studies of gifts economies have been conducted by O'Connell and Zeldes (1993) and Wigger (2001). O'Connell and Zeldes (1993) found that intergenerational transfers of gifts from children to parents are dynamically inefficient. However, the result of Wigger (2001)'s investigation was that if growth is endogenous, the gift economy is dynamically efficient.

One group constitutes agents who are dutiful or “good” to their parents and the other constitutes selfish or “bad” agents. Good children are different from bad ones as they care about their parents and derive utility from giving them a part of their wage income as a gift, whereas bad children do not. We assume that the type of children is uncertain ex ante. Thus, income during the old period is stochastic. If each agent is risk averse, he or she has an incentive to share the risk relating to income; that is, he or she may not receive any intergenerational transfer from their children, within the same generation.[3] Thus, the agents who have the ability to monitor the contracts agents act as insurance companies. Each agent can insure himself or herself against fluctuations of income by making a contract with an insurance company. Our purpose is to analyze how the risk sharing methods that cover the uncertainty of intergenerational transfers affect the rate of economic growth and welfare.

We begin by considering the economy without risk sharing. In this regime, the intergenerational transfer is subject to uncertainty. Thus, each agent has precautionary motive for saving, as in Leland (1968), Sandmo (1970) and Kimball (1990).

Next, we consider the economy with risk sharing. If the insurance company can observe the income of old agents, that is, whether their children are dutiful to their parents, the insurance company offers the full insurance to old agents. However, there exists information asymmetry between the insurance company and old agents about the income realization of the old agents. Thus, we first construct the schedule of risk sharing by using the costly state verification theorem, as in Townsend (1979), Williamson (1986), Bernanke and Gertler (1989), and Bhattacharya (1997), and verify the lifecycle activity under the risk sharing schedule.

By analyzing the model, we find the following: the economy grows at a positive constant rate under both regimes. Second, as to the effect of risk sharing, risk sharing can smooth the fluctuation of income, which decreases the precautionary motive for saving, which in turn results in a lower level of economic growth. In addition to this, an increase in the number of parents with selfish children decreases the rate of economic growth. Finally, risk sharing accelerates the welfare level of the current generation, whereas that of the future generation deteriorates. Thus, its effect on the social welfare level depends on the social time preference rate: when the social time preference rate is sufficiently high (low), the risk sharing regime accelerates (reduces) the social welfare level.

The remainder of this paper is organized as follows. Section 2 sets up the basic model.

[3] Krueger and Kubler (2002) stated that the role of the social security system plays a role in facilitating the allocation of aggregate risk among generations.

Section 3 derives the economic equilibrium without risk sharing. Section 4 sketches the schedule of risk sharing, and derives the economic equilibrium with risk sharing. Section 5 examines the relation between risk sharing and economic growth. Section 6 analyzes welfare. Section 7 contains some concluding remarks.

2 The Model

Consider a discrete time economy populated by an infinite sequence of two-period-lived overlapping generations and one-period-lived nonoverlapping generations. Time is indexed by $t = 1, 2, \cdots$. At each date, two sets of new generations, each consisting of a continuum of agents with unit measure, are born.

Two-period-lived agents

Agents of generation $t \geq 1$ live for periods t and $t+1$. They are endowed with one unit of labor when young and no labor when old. Within each generation, agents are divided into two types according to their attitude toward their parents. A fraction α of young agents are of type G, who are dutiful or "good" toward their parents. Type B agents, who constitute a fraction $1-\alpha$ of each generation, are undutiful or "bad" toward their parents. Type G agents differ from type B agents in that they care about their parents and derive satisfaction from giving them a gift, whereas type B agents do not. We assume that each agent of generation $t \geq 1$ has a utility function of the form:

$$u(c_t^t, q_t, c_{t+1}^t; \psi_t) = \psi_t \ln c_t^t + (1 - \psi_t) \ln q_t + \ln c_{t+1}^t, \tag{1}$$

where c_t^t is first-period (date t) consumption, q_t is a (date t) gift to the agent's parents, c_{t+1}^t is second-period (date $t+1$) consumption, and ψ_t is an index indicating an agent's type that takes a value of either 0 or 1 according to whether the agent is type G or type B. ψ_t, which is realized at the beginning of date t immediately after agents of generation t are born, is distributed independently and identically (across agents and time) with the probability distribution:

$$\psi_t = \begin{cases} 1 & \text{with} \quad \text{Prob.} \quad \alpha \\ 0 & \text{with} \quad \text{Prob.} \quad 1-\alpha, \end{cases} \tag{2}$$

where $0 < \alpha < 1$. In what follows, agents as consumers will also be referred to as "households".

At each date there is a single consumption good. Any old agent has access to a production technology given by:

$$y_t = A k_t^{\theta} (K_t l_t)^{1-\theta}, \tag{3}$$

where A indicates the parameter representing the technology level and K_t is the aggregate capital stock in the economy such that there is an externality in production of the type considered by Romer (1986). At each date $t \geq 1$, old agents acting as producers, whom we will call "firms", have the accumulated capital stock k_t, which is combined with l_t units of young labor in order to produce output y_t. We assume that capital depreciates completely in the process of production.

Finally, at date $t = 1$, there is an initial old generation (generation 0). These old agents are each endowed with $k_t > 0$ units of capital.

One-period-lived agents

At each date $t \geq 1$, in addition to two-period-lived agents, agents are born who are risk neutral and live only one period. We will call these one-period-lived agents "insurers", for reasons to be made clear. When risk sharing is allowed among (old) households within each generation, insurers set up insurance companies. We assume that the type of each (young) household and what may be called "the state of a (young/old) household"—how each young household spends his or her wage income and how much each old household spends on consumption—are private information; these are costlessly observable only to himself or herself and his or her parents. Following Townsend (1979), Williamson (1986), Bernanke and Gertler (1989), Bhattacharya (1997) and others, however, we assume that insurers have access to a costly state verification (monitoring) technology: an insurer can expend effort to learn the state of any household—in particular, how much an old household spends for his or her old age consumption. It requires $e \geq 0$ units of effort to observe the state of any one household. Note that the state of an old household is synomynous with the information about which type of child the old agent has.

Each insurer, who is endowed with an unbounded amount of effort, has a utility function of the form:

$$v(x_t, e_t) = x_t - e_t, \tag{4}$$

where x_t is consumption at date t and e_t is effort expended at date t. At each date $t \geq 1$, when risk sharing is permitted among (old) households within each generation, insurance companies are set up that are owned and operated by insurers.

2.1 Behavior of agents

Both producers (firms) and households behave competitively in goods and factor markets, taking the wage rate and the rental rate as given.

Firms

There is no loss of generality in assuming that, at each date $t \geq 1$, there is a single competitive firm, which produces the total output of this economy using the production technology (3) by hiring young agents at wage rate w_t and renting capital at the rental rate R_t from old agents. As perfect competition prevails in both goods and factor markets and because $k_t = K_t$ and $l_t = 1$ are in equilibrium, the wage rate and the rental rate are given by the marginal productivity relations:

$$w_t = (1-\theta)AK_t^{1-\theta}k_t^{\theta}l_t^{-\theta} = (1-\theta)Ak_t, \tag{5}$$

$$R_t = r_t + 1 = \theta AK_t^{1-\theta}k_t^{\theta-1}l_t^{1-\theta} = \theta A, \tag{6}$$

where r_t is the rate of interest from date $t-1$ to date t.

Households

At each date $t \geq 1$, each young agent maximizes the expected value of (1) subject to the budget constraint:

$$\max_{c_t^t, q_t, c_{t+1}^t} U_t \equiv E_t u(c_t^t, q_t, c_{t+1}^t; \psi_t) = \psi_t \ln c_t^t + (1-\psi_t)\ln q_t + E_t \ln c_{t+1}^t, \tag{7}$$

s.t.

$$\psi_t c_t^t + (1-\psi_t)q_t + \frac{c_{t+1}^t}{1+r_{t+1}} = w_t + (1-\psi_{t+1})\frac{q_{t+1}}{1+r_{t+1}}. \tag{8}$$

When young, each agent earns the wage income by working for a firm, spends it either for his or her own consumption (if he or she is of type B), or gives some as a gift to his or her parents (if the agent is of type G), and saves the rest for his or her old age. When old, an agent uses the proceeds of his or her savings and a gift (if any) from his or her child to support his or her old age.

A few observations are in order. First, each household's second-period income is subject to uncertainty because whether the agent receives a gift from his or her child depends on the realization of the random variable ψ_{t+1}; it differs depending on which type of child is born in the family at date $t+1$. The expectation in (7) is taken with respect to ψ_{t+1}, the probability distribution of which is given by (2). Second, in maximizing the expected utility in (7), each household takes as given a gift from his or her child, as well as the wage rate and the real interest rate. Third, the first observation above also indicates that there is an incentive for risk sharing among agents within each generation. The budget constraint (8), however, assumes no intragenerational risk sharing. As a benchmark, we first describe a regime where risk sharing is prohibited. We will then consider one where risk sharing is permitted.

Insurers

There is no role for insurers to play in the economy without risk sharing. Every insurer is inactive, consuming nothing and expending zero units of effort. The autarkic level of utility for each insurer is zero.

3 Equilibrium without Risk Sharing

Let $s_t \equiv w_t - [\psi_t c_t^t + (1-\psi_t)q_t]$. Then, s_t is the saving of an agent of generation $t \geq 1$. The first-order condition for the maximization problem (7)-(8) is rearranged to give:

$$2(1+r_{t+1})s_t^2 + [(2-\alpha)q_{t+1} - (1+r_{t+1})w_t]s_t - (1-\alpha)q_{t+1}w_t = 0. \tag{9}$$

Note that the solution s_t to (9) implies that a young agent's saving does not depend on his or her type. In addition, as $\psi_t c_t^t + (1-\psi_t)q_t = w_t - s_t$, we have:

$$c_t^t = q_t = w_t - s_t, \tag{10}$$

for each $t \geq 1$.

Definition 1 *An equilibrium will satisfy the following three conditions at each date:*

- *Households and firms optimize, taking the wage rate and the rate of interest; that is, (5), (6), and (9) hold.*
- *The markets for goods, capital, and labor clear; that is, $s_t = k_t$ and $l_t = 1$ hold.*
- *The gift q_{t+1} from generation $t+1$, which is taken as given by each agent of generation $t \geq 1$ in his or her maximization problem (7)-(8), is realized.*

To express equilibrium in this economy in a compact manner, we note, first, that the third condition together with the second condition and (5) imply:

$$q_{t+1} = w_{t+1} - s_{t+1} = (1-\theta)Ak_{t+1} - k_{t+2}. \tag{11}$$

We then substitute (5), (6), and (11) as well as $s_t = k_{t+1}$ into (9) to obtain:

$$[2\theta A + (1-\alpha)(1-\theta)A]k_{t+1}^2 - [(1-\alpha)(1-\theta)^2A^2 + (1-\theta)\theta A^2]k_t k_{t+1}$$
$$-(2-\alpha)k_{t+1}k_{t+2} + (1-\alpha)(1-\theta)Ak_t k_{t+2} = 0. \quad (12)$$

An equilibrium path for k_t is one in which (12) is satisfied and the resource constraint:

$$k_{t+1} \leq (1-\theta)Ak_t \quad (13)$$

is met for each $t \geq 1$. Let $g_{t+1} \equiv k_{t+1}/k_t$ be the (gross) growth rate of the capital stock from date t to date $t+1$. For our purposes, it will be more useful to work with a transformed version of (12) and (13):

$$g_{t+2} = \frac{2-\alpha(1-\theta)A}{2-\alpha} + \frac{-\alpha\theta(1-\theta)A^2}{(2-\alpha)[(2-\alpha)g_{t+1} - (1-\alpha)(1-\theta)A]}, \quad (14)$$
$$g_{t+1} \leq (1-\theta)A. \quad (15)$$

The phase diagram for (14) is shown in Figure 1. The phase line is a rectangular hyperbola with its asymptotes given by $g_{t+1} = (1-\alpha)(1-\theta)A/(2-\alpha)$ and $g_{t+2} = [2-\alpha(1-\theta)A]/(2-\alpha)$. Setting $g_{t+2} = g_{t+1} = b$ in (14) and rearranging yields:

$$(2-\alpha)b^2 - (2\alpha\theta - 2\alpha - \theta + 3)Ab + (1-\theta)(1-\alpha+\alpha\theta)A^2 = 0. \quad (16)$$

We define the left-hand side of (16) as $h(b)$. Then, the roots of the quadratic equation $h(b) = 0$ give the fixed (stationary) points of (14), which are given by:

$$b_1 = A\left\{\frac{3-\theta-2\alpha(1-\theta)+\sqrt{(1+\theta)^2-4\alpha\theta(1-\theta)}}{2(2-\alpha)}\right\}, \quad (17)$$
$$b_1 = A\left\{\frac{3-\theta-2\alpha(1-\theta)-\sqrt{(1+\theta)^2-4\alpha\theta(1-\theta)}}{2(2-\alpha)}\right\}. \quad (18)$$

However, an examination of Figure 1 indicates that the dynamic system (14)–(15) possesses a unique stationary solution b_2, and that this stationary solution is the economy's saddle-path stable equilibrium. The following proposition formalizes this observation and its proof is given in Appendix A.

[Figure 1 here]

Proposition 1 *Suppose that:*

$$A > \frac{3-\theta-2\alpha(1-\theta)+\sqrt{(1+\theta)^2-4\alpha\theta(1-\theta)}}{2(1-\theta)[1-\alpha(1-\theta)]}, \quad (19)$$

Then, there exists a unique equilibrium such that $k_t = k_1(A\gamma)^{t-1}$*,* $A\gamma > 1$*, for each* $t \geq 1$*, where:* $\gamma = \frac{3-\theta-2\alpha(1-\theta)-\sqrt{(1+\theta)^2-4\alpha\theta(1-\theta)}}{2(2-\alpha)}$.

Proposition 1 makes it clear that if the productivity parameter A is sufficiently large, the economy without risk sharing grows at a positive (net) constant rate of $A\gamma - 1$. We can see from (3), (5), and (10) that output y_t, wages w_t, first-period consumption c_t^t, gifts q_t, and savings s_t also grow at the same constant (gross) rate $A\gamma$. The economy's growth rate depends on the parameters A, θ and α. In particular, the economy grows faster if it has a lower fraction of type G agents (see Section 5 below).

4 Risk Sharing

As mentioned above, all households are risk averse and have an incentive to insure themselves against the risk of having type B children, who do not care about their parents. At the beginning of each date, before children are born, insurance companies announce insurance contracts, letting old households choose among them. Contract announcements are made taking the announced contracts of other insurance companies as given.

4.1 Insurance contracts

At the beginning of date $t+1$, insurance companies offer old households contracts of the form $(p_{t+1}^G, p_{t+1}^B, \varphi_{t+1})$ such that an old agent will pay a premium p_{t+1}^G to an insurance company if a type G child is born, in return for which he or she will be paid p_{t+1}^B in the event that a type B child is born. An optimal contract between the insurance company and an old household must take into account the possibility that the latter may lie about the type of child who was born; he or she may misreport the type G child as a type B child. One can show that, in a two-state case like ours, under the optimal contract no verification (monitoring) occurs when it is announced that a type G child was born. Thus, the insurance company monitors only when the old household declares that a type B child was born. φ_{t+1} is the penalty imposed at date $t+1$ on an old household who misreports the type of his or her child.

Taking r_{t+1}, s_t and q_{t+1} as given, each insurance company chooses $(p_{t+1}^G, p_{t+1}^B, \varphi_{t+1})$ to maximize the expected utility of the old household subject to a set of constraints:

$$\max_{p_{t+1}^G, p_{t+1}^B, \phi_{t+1}} E_t[\ln c_{t+1}^t] = \alpha \ln\left[(1+r_{t+1})s_t + q_{t+1} - p_{t+1}^G\right] + (1-\alpha)\ln\left[(1+r_{t+1})s_t + p_{t+1}^B\right], \tag{20}$$

s.t.

$$\ln\left[(1+r_{t+1})s_t + q_{t+1} - p^G_{t+1})\right] \geq \ln\left[(1+r_{t+1})s_t + q_{t+1} - \varphi_{t+1}\right], \tag{21}$$

$$q_{t+1} \geq p^G_{t+}, \tag{22}$$

$$q_{t+1} \geq \varphi_{t+1}, \tag{23}$$

$$\alpha p^G_{r+1} - (1-\alpha)(p^B_{t+1} + e) = 0 \tag{24}$$

$$\alpha \ln[q_{t+1} + (1+r_{t+1})s_t - p^G_{t+1}] + (1-\alpha)\ln[(1+r_{t+1})s_t + p^B_{t+1}]$$
$$\geq \alpha \ln\left[q_{t+1} + (1+r_{t+1})s_t\right] + (1-\alpha)\ln\left[(1+r_{t+1})s_t\right]. \tag{25}$$

Constraint (21) is the truth-telling or incentive constraint on the old household; it requires that the contract be such that the old household has no incentive to misrepresent the child as type B when he or she is in fact type G. Constraints (22) and (23) are the limited liability constraints, which state that neither the premium nor the penalty cannot exceed the amount of the gift that the old household receives from his or her child. (24) is the zero-profit condition on the insurance company that arises from competition among insurance companies; it takes into account the fact that the insurance company verifies that the truth has been told whenever it is reported that children of type B were born[4]. (25) is the participation constraint of an old household, who would not be willing to accept the contract unless he or she would be no worse off by taking out insurance than he or she would be otherwise. The solution to the maximization problem (20)–(25) yields the optimal contract.

Proposition 2 *At date $t+1$, an optimal insurance contract exists if and only if:*

$$e \leq \frac{1}{1-\alpha}\{(1+r_{t+1})s_t + \alpha q_{t+1} - [(1+r_{t+1})s_t + q_{t+1}]^{\alpha}[1+r_{t+1})s_t]^{1-\alpha}\}. \tag{26}$$

The optimal contract $(p^G_{t+1}, p^B_{t+1}, \varphi_{t+1})$ is such that:

$$p^G_{t+1} = (1-\alpha)q_{t+1} + (1-\alpha)e, \tag{27}$$

$$p^B_{t+1} = \alpha q_{t+1} - (1-\alpha)e, \tag{28}$$

$$\varphi \in [(1-\alpha)(q_{t+1}+e), q_{t+1}]. \tag{29}$$

[4]The left-hand side of (23) represents the insurance company's expected profits per contract. Insurance companies are willing to offer any number of contracts as long as they make nonnegative expected profits. Free entry into the insurance industry means that insurance companies earn zero expected profits in equilibrium. The law of large numbers, however, implies that an insurance company that makes a large number of contracts makes zero profits (rather than zero expected profits) with a probability of one in equilibrium. Insurers, who run insurance companies, consume $\alpha p^G_{t+1} - (1-\alpha)p^G_{t+1}$ units of the consumption good while supplying $(1-\alpha)e$ units of effort.

Proof. See Appendix B. ■

(26) indicates that, given r_{t+1}, s_t, and q_{t+1}, the verification cost must be small enough if old households are to insure themselves at date $t+1$. The insurance contract provides old households with the opportunity to insure themselves completely against the risk of having no gifts (no old-age care) from their children. The insurance company collects premiums summing to $(1-\alpha)(q_{t+1}+e)$ from old households with type G children and pays out compensation amounting to $\alpha q_{t+1} - (1-\alpha)e$ to those with type B children. In so doing, the insurance has the effect of redistributing the gifts from generation $t+1$ equally among old households of generation t. It is easy to see from (27) and (28) that, with a given level of gift, the insurance is more costly if the fraction of type G agents is smaller and the verification cost is greater.

4.2 Equilibrium with risk sharing

At each date $t \geq 1$, each young agent in the risk sharing regime maximizes the expected value of (1) in anticipation of the contract $(p^G_{t+1}, p^B_{t+1}, \varphi_{t+1})$ being placed at date $t+1$. That is, he or she solves the maximization problem:

$$\max_{c^t_t, q_t, c^t_{t+1}} U_t = \psi_t \ln c^t_t + (1-\psi) \ln q_t + E_t \ln c^t_{t+1}, \tag{30}$$

s.t.

$$\psi_t c^t_t + (1-\psi_t) q_t + \frac{c^t_{t+1}}{1+r_{t+1}} w_t = (1-\psi_{t+1}) \frac{q_{t+1}}{1+r_{t+1}} + \frac{1}{1+r_{t+1}} [\psi_{t+1} p^B_{t+1} - (1-\psi_{t+1}) p^G_{t+1}], \tag{31}$$

where p^G_{t+1} and p^B_{t+1} are given by (27) and (28), respectively.

The first-order condition for (30)–(31) can be expressed in terms of saving as:

$$s_t = \frac{1}{2}\left[w_t - \frac{\alpha q_{t+1}}{1+r_{t+1}} + \frac{(1-\alpha)e}{1+r_{t+1}}\right]. \tag{32}$$

To find an equilibrium, we substitute the equilibrium conditions (5), (6), (11), and $s_t = k_{t+1}$ into (32) to obtain:

$$\alpha k_{t+2} - [2\theta + \alpha(1-\theta)] A k_{t+1} + (1-\theta)\theta A^2 k_t = -(1-\alpha)e. \tag{33}$$

An equilibrium path for k_t is one in which (33) is satisfied and the resource constraint (13) is met for each $t \geq 1$. The general solution to the second-order linear difference equation (33) is given by:

$$k_t = Z_1 b^t_1 + Z_2 b^t_2 + c, \tag{34}$$

where:

$$b_1 = A \left\{ \frac{\alpha(1-\theta) + 2\theta + \sqrt{\alpha^2(1-\theta)^2 + 4\theta^2}}{2\alpha} \right\}, \tag{35}$$

$$b_2 = A \left\{ \frac{\alpha(1-\theta) + 2\theta - \sqrt{\alpha^2(1-\theta)^2 + 4\theta^2}}{2\alpha} \right\}, \tag{36}$$

$$c = \frac{-(1-\alpha)e}{\alpha - [2\theta + \alpha(1-\theta)]A + (1-\theta)\theta A^2}, \tag{37}$$

with Z_1 and Z_2 as arbitrary constants. For later reference, we define the function $f(b)$ as:

$$f(b) \equiv \alpha b^2 - [2\theta + \alpha(1-\theta)]Ab + (1-\theta)\theta A^2. \tag{38}$$

Then, $f(b) = 0$ is the characteristic equation of (33), and (35) and (36) are its two (distinct) roots (characteristic roots).

Before stating the main result in this subsection, we prove the following lemma.

Lemma 1 *Suppose $A\xi > 1, f(1) > 0$, and $1-\theta > \xi$. We define the function $F : [0, \bar{\bar{\delta}}] \to R$ by:*

$$F(\delta) = \ln\left[\alpha(1-\xi) + (1-\alpha)\theta - (1+\alpha Q)\delta\right] - \alpha \ln(1-\xi-Q\delta) - (1-\alpha)\ln\theta, \tag{39}$$

where $\bar{\bar{\delta}} \equiv \frac{\alpha(1-\theta-\xi)}{1+\alpha Q}$ and $Q \equiv \frac{A\xi}{f(1)}$. Then, there exists a unique $\bar{\delta} \in (0, \bar{\bar{\delta}})$ with $F(\bar{\delta}) = 0$. Furthermore, $F(\bar{\delta}) > 0$ if and only if $\delta < \bar{\delta}$.

Proof. As

$$F(0) = \ln[\alpha(1-\xi) + (1-\alpha)\theta] - \alpha\ln(1-\xi) - (1-\alpha)\ln\theta > 0$$

and

$$F(\bar{\bar{\delta}}) = \alpha\Big\{ \ln\theta - \ln\Big[(1-\xi)\Big(1 - \frac{\alpha Q}{1+\alpha Q}\Big) + \theta\frac{\alpha Q}{1+\alpha Q}\Big]\Big\} < 0,$$

it suffices to show that $F'(\delta) < 0$ for each $\delta \in [0,\ \bar{\bar{\delta}}]$. However,

$$F'(\delta) = \frac{-(1+\alpha Q)(1-\xi) + \alpha Q[\alpha(1-\xi) + (1-\alpha)\theta] + (1-\alpha)(1+\alpha Q)Q\delta}{D(\delta)}, \tag{40}$$

where $D(\delta) \equiv [\alpha(1-\xi) + (1-\alpha)\theta - (1+\alpha Q)\delta](1-\xi-Q\delta)$. As $D(\delta) > 0$ for each $\delta \in [0, \bar{\bar{\delta}}]$ and because the numerator on the right-hand side of (40) is increasing in δ, it is sufficient to show that $F'(\bar{\bar{\delta}}) < 0$, which is certainly true because $F'(\bar{\bar{\delta}}) = -(1-\xi)/D(\bar{\bar{\delta}}) < 0$. ■

Proposition 3 *Suppose that:*

$$A > \frac{\alpha(1-\theta) + 2\theta + \sqrt{\alpha^2(1-\theta)^2 + 4\theta^2}}{2\theta(1-\theta)}, \tag{41}$$

and that:

$$e \leq \frac{\bar{\delta} A k_1}{1-\alpha}. \tag{42}$$

Then, there exists a unique equilibrium such that $k_t = (k_1 - c)(A\xi)^{t-1} + c, A\xi > 1, c < 0$, *for each* $t \geq 1$, *where* $\xi \equiv \frac{\alpha(1-\theta)+2\theta-\sqrt{\alpha^2(1-\theta)^2+4\theta^2}}{2\alpha}$.

Proof. See Appendix C. ■

Suppose that:

$$e \leq \frac{\bar{\delta} A k_{t+1}}{1-\alpha} \tag{43}$$

holds at date $t+1$. Then, old households at that date have an incentive to participate in risk sharing. When (42) is satisfied, (43) will also be satisfied at each $t \geq 1$ (see the Appendix). If the monitoring cost is high enough to violate (42), the participation condition for old households will not be met, at least at date $t = 1$. The equilibrium path in such an economy will not be the one in the proposition. However, as the right-hand side of (43)—the critical value of e above which it is too costly for old households to insure themselves against the risk at date $t+1$—is increasing in the capital stock, (43) will be met (as long as e is finite) even though it is violated initially. From that time on, the economy will be on the path described in the proposition.

Suppose that (42) holds. Then, the economy grows at a (gross) rate of $g_{t+1} \equiv k_{t+1}/k_t = [(k_t - c)A\xi + c]/k_t = A\xi - c(A\xi - 1)/k_t$ at each date $t \geq 1$. Note that the economy's growth rate is no longer constant over time; because $c < 0$, it is higher than $A\xi$, but it converges monotonically to the latter over time. It is easy to see from (3) and (5) that both y_t and w_t grow at the same rate as k_t, whereas c_t^t, q_t, and s_t do not.

Before proceeding, it will be useful to have the following special case of Proposition 2 as another benchmark from which to evaluate the effects of private information on economic growth and welfare.

Proposition 4 *Suppose that* A *satisfies (41) and that* $e = 0$. *Then, there exists a unique equilibrium such that* $k_t = k_1(A\xi)^{t-1}$ *for each* $t \geq 1$.

Proposition 4 immediately follows from Proposition 3 by setting $e = 0$. This assumption is equivalent with assuming that insurers have full information about the state of any (old) household. From Proposition 3, the economy grows at a positive constant rate of $A\xi - 1$.

Another benchmark regime, the no-risk-sharing regime, could also be viewed as a special case, a regime that will prevail if the monitoring technology is prohibitively (or infinitely) costly (i.e., $e \to \infty$).

5 Risk Sharing and Economic Growth

The aim in this section is twofold. One is to consider what role risk sharing plays in accelerating or decelerating economic growth. As monitoring costs determine the structure of risk sharing, this question amounts to asking how differing monitoring costs will cause the economy to grow faster or slower. The other aim is to examine what effects young agents' altruism toward their parents have on economic growth. In particular, we ask ourselves whether the economy's growth rate will be higher or lower if a smaller fraction of young agents care about their parents in old age.

5.1 Risk sharing regimes and economic growth

Let superscripts n, a, and f denote, respectively, "the no-risk-sharing regime or the economy with $e \to \infty$", "the risk sharing regime with asymmetric information or the economy with $e < \infty$", and "the risk sharing regime with full information or the economy with $e = 0$". Our first result is somewhat surprising: monitoring costs have positive instead of negative effects on economic growth.

Lemma 2 *Let A meet the conditions in both Propositions 1 and 3 and let γ and ξ be defined as in Propositions 1 and 3. Then, the following statements hold:*

$$\begin{array}{ll} (a) & 1 < A\xi < A\gamma < A(1-\theta); \\ (b) & \dfrac{d(A\gamma)}{d\alpha} < 0, \quad \dfrac{d(A\xi)}{d\alpha} < 0. \end{array}$$

Proof. (a) The first inequality follows from Propositions 1 and 2. For proofs of the second and third inequalities, see Appendix D.

(b) As $A\gamma$ is the smaller root of $h(A\gamma; \alpha) = 0$, we use the implicit function theorem and differentiate $h(A\gamma; \alpha) = 0$ to obtain:

$$\frac{d(A\gamma)}{d\alpha} = \frac{A[A\gamma - A(1-\theta)]^2}{(2-\alpha)2A\gamma - (2\alpha\theta - 2\alpha - \theta + 3)}. \tag{44}$$

Substituting (18) into the denominator on the right-hand side of (44), the denominator becomes negative. Thus, $\frac{dA\gamma}{d\alpha} < 0$.

As $A\xi$ is the smaller root of $f(A\xi;\alpha)=0$, we also have that:

$$\frac{d(A\xi)}{d\alpha}=\frac{A\xi[A\xi-(1-\theta)A]}{[2\theta+\alpha(1-\theta)]A-2\alpha A\xi}. \tag{45}$$

From the resource constraints of (15), the numerator on the right-hand side of (45) is negative. Substituting (36) into the denominator on the right-hand side of (45) means the denominator becomes positive, and, thus, $\frac{dA\xi}{d\alpha}<0$. ■

We define the average (gross) growth rate $\bar{g}_{t+1}$ from date $t=1$ to date $t+1$ by:

$$\bar{g}_{t+1}\equiv(k_{t+1}/k_t)^{1/t}=(g_2,g_3\cdots g_{t+1})^{1/t},$$

where, recall, $g_2=k_2/k_1, g_3=k_3/k_2,\cdots,g_{t+1}=k_{t+1}/k_t$. That is, $\bar{g}_{t+1}$ measures the geometric average of the growth rates of capital stock from the initial date through date $t+1$. As $k_{t+1}=k_1(\bar{g}_{t+1})^t, \bar{g}_{t+1}$ also measures the level of capital stock outstanding at the beginning of date $t+1$. The following proposition compares the three regimes in terms of the average growth rates.

Proposition 5 *$(a)\bar{g}^n_{t+1}>\bar{g}^f_{t+1}$ for each $t\geq 1$;*
$(b)\bar{g}^a_{t+1}>\bar{g}^f_{t+1}$ and $\frac{d\bar{g}^a_{t+1}}{de}>0$ for each $t\geq 1$;
$(c)\bar{g}^n_{t+1}>\bar{g}^a_{t+1}$ and $\bar{g}^n_{t+1}-\bar{g}^a_{t+1}$ is increasing with t for each $t\geq\bar{t}$, where $1\leq\bar{t}<\infty$.

Proof. (a) As Propositions 1 and 2 imply that $\bar{g}^n_{t+1}=A\gamma$ and $\bar{g}^f_{t+1}=A\xi$, the result follows immediately from Lemma 2.
(b) It suffices to show that $k^a_{t+1}>k^f_{t+1}$ and $\frac{dk^a_{t+1}}{de}>0$. As Propositions 3 and 4 imply that:

$$k^a_{t+1}=(k_1-c)(A\xi)^t+c=k_1(A\xi)^t-c[(A\xi)^t-1]>k_1(A\xi)^t=k^f_{t+1},$$

where, recall, $c=-(1-\alpha)e/f(1)<0$ (Proposition 3), the result follows.
(c) As $k^n_{t+1}=k_1(A\gamma)^t$ and $k^a_{t+1}-c=(k_1-c)(A\xi)^t$ from Propositions 1 and 3, we have:

$$\begin{aligned}k^n_{t+1}-k^a_{t+1} &= (k^a_{t+1}-c)\Big[\frac{k_1}{k_1-c}\Big(\frac{\gamma}{\xi}\Big)^t-1\Big]-c,\\ &= (k_1-c)(A\xi)^t\Big[\frac{k_1}{k_1-c}\Big(\frac{\gamma}{\xi}\Big)^t-1\Big]-c.\end{aligned}$$

As $c<0, A\xi>1$, and $\gamma>\xi$ from Proposition 3 and Lemma 2, it follows that if $\frac{k_1}{k_1-c}\Big(\frac{\gamma}{\xi}\Big)^t\geq 1$ or $t\geq\bar{t}\equiv\frac{\ln(k_1-c)-\ln k_1}{\ln\gamma-\ln\xi}$, then $k^n_{t+1}>k^a_{t+1}$, and $k^n_{t+1}-k^a_{t+1}$ is increasing with t for each $t\geq\bar{t}$, from which the result immediately follows. ■

Although, as noted above, the no-risk-sharing regime can be thought of as a special case of our economy, it differs from the risk sharing regimes in that each household has an uncertain (or stochastic) lifetime income. The extra saving caused by future income being subject to uncertainty is known as precautionary saving. Precautionary saving is associated with convexity of the marginal utility function or a positive third derivative of the utility function (see, among others, Leland (1968), Sandmo (1970), and Kimball (1990)). Households with a log utility function have a positive motive for precautionary saving. A rise in uncertainty about future income encourages private saving and capital accumulation. Given the technology with aggregate spillovers coming from firms' production activities, this effect manifests itself as increased economic growth. That is the mechanism behind Proposition 5(a).

However, the logic leading to part (b) of Proposition 5 differs somewhat from that behind part (a) because no risk is involved in the risk sharing regimes. As (32) makes clear, with a given level of gift, a higher verification cost means a lower second-period income, motivating households to respond to reduced second-period incomes by saving more. Therefore, the mechanism underlying part (b) is not so much a precautionary- saving motive as a consumption-smoothing motive; put differently, it is associated with a negative second derivative rather with than a positive third derivative of the utility function.

Given the result in part (c) of Proposition 5, we are now able to rank the three regimes in terms of (average) growth rates: $\bar{g}^n_{t+1} > \bar{g}^a_{t+1} > \bar{g}^f_{t+1}$ for each $t \geq \bar{t}$. Whatever the growth rates are in the early stages, the no-risk-sharing regime will eventually dominate the risk sharing regimes. We will explore some welfare implications of these differing growth rates in the following section. Before turning to the next section, it is worthwhile for our purposes to do one more comparative statics exercise. As it will turn out, a lower (higher) share of type G agents leads to higher (lower) economic growth across regimes with and without risk sharing. The intuition behind this result is as follows: households have a strictly concave utility function, motivating them to smooth their lifetime consumption across their young and old age. A lower (higher) share of type B agents means lower (higher) transfers from their children, leading to a rise (decline) in their savings to provide support into their old age.

Proposition 6 *A lower (higher) share of type $G(B)$ agents implies higher growth under any regime; that is, for each $t \geq 1$:*

$$(a)\frac{d\bar{g}^n_{t+1}}{d\alpha} < 0; \quad (b)\frac{d\bar{g}^a_{t+1}}{d\alpha} < 0; \quad (c)\frac{d\bar{g}^f_{t+1}}{d\alpha} < 0.$$

Proof. Parts (a) and (c) follow immediately from Lemma 2. To show part (b), it suffices to prove that $\frac{d\bar{g}^a_{t+1}}{d\alpha} < 0$. Differentiating both sides of $k^a_{t+1} = k_1(A\xi)^t - c[(A\xi)^t - 1]$ with respect to α yields:

$$\frac{dk_{t+1}^a}{d\alpha} = k_1 t(A\xi)^{t-1}\frac{d(A\xi)}{d\alpha} - ct(A\xi)^{t-1}\frac{d(A\xi)}{d\alpha} - \frac{dc}{d\alpha}[(A\xi)^t - 1].$$

As $d(A\xi)/d\alpha < 0$ (Lemma 2), $c < 0$ (Proposition 2), $A\xi > 1$ (Lemma 2), and $dc/d\alpha > 0$ (see the Appendix C), the results follow. ■

6 Welfare Analysis

In Section 5, we have shown that when agents can share the risk, economic growth decelerates. In this section, we will analyze whether risk sharing is desirable for society.

6.1 The individual's welfare

To begin, consider the members of generation 0, the old households at date $t = 1$. Each member of generation 0 lives only one period (his or her old period of life), starts his or her life with k_1 units of capital, which he or she rents to the firm to earn an income of $(1 + r_1)k_1$. If, in addition, a child of type G is born into his or her family, the old household has a gift of q_1 from his or her child. In the absence of risk sharing, therefore, each old household's expected utility at the beginning of date $t = 1$ is given by:

$$U_0^n = E_1(\ln c_1^0) = \alpha \ln[(1 + r_1)k_1 + q_1] + (1 - \alpha)\ln(1 + r_1)k_1.$$

As $r_1 = \theta A - 1$ and $q_1 = w_t - s_t = (1 - \theta)Ak_1 - k_2 = (1 - \theta)Ak_1 - k_1 A\gamma$ in equilibrium, it follows that:

$$U_0^n = \ln Ak_1 + \alpha \ln(1 - \gamma) + (1 - \alpha)\ln\theta. \tag{46}$$

With risk sharing, on the other hand, the expected utility of each household of generation 0 is:

$$U_0^a = E_1(\ln c_1^0) = \ln[(1 + r_1)k_1 + \alpha q_1 - (1 - \alpha)e],$$

so that, in equilibrium:

$$U_0^a = \ln Ak_1 + \ln\left[\alpha(1 - \xi) + (1 - \alpha)\theta - \frac{(1 - \alpha)e}{Ak_1}(1 + \alpha Q)\right]. \tag{47}$$

In deriving (47), we have used the fact that $k_2 = (k_1 - c)A\xi + c$ along the equilibrium path with risk sharing (see Proposition 3). To determine how risk sharing benefits (or hurts) the initial old household, we subtract (46) from (47):

$$U_0^a - U_0^n = \ln\left[\alpha(1 - \xi) + (1 - \alpha)\theta - \frac{(1 - \alpha)e}{Ak_1}(1 + \alpha Q)\right] - [\alpha \ln(1 - \gamma) + (1 - \alpha)\ln\theta]. \tag{48}$$

As $\xi < \gamma$ (see Lemma 2), the right-hand side of (48) is positive if the cost of monitoring is sufficiently small relative to the initial capital stock. A necessary and sufficient condition for the right-hand side of (48) to be positive is given by:

$$\frac{(1-\alpha)e}{Ak_1} < \frac{\alpha(1-\xi)+(1-\alpha)\theta-(1-\gamma)^{\alpha}\theta^{1-\alpha}}{1+\alpha Q}. \tag{49}$$

Provided condition (49) holds, each member of generation 0 will be better off with risk sharing. As is evident from (48), the welfare gain from risk sharing is greater for the member of generation 0 if the cost of monitoring is lower and the initial capital stock is higher, and it is greatest when e is zero (i.e., under full information).

The welfare gain as displayed on the right-hand side of (48) may be referred to as a direct or "short-run" effect of risk sharing.

Consider now the member of generation $t \geq 1$. The welfare of each young household is measured by his or her expected lifetime utility given by (7). As $\ln c_t^t = \ln q_t = \ln(w_t - s_t) = \ln Ak_t(1-\theta-\gamma)$ and: $E_t \ln c_{t+1}^t = \alpha \ln[(1+r_{t+1})s_t + q_{t+1}] + (1-\alpha)\ln[(1+r_{t+1})s_t] = \ln Ak_{t+1} + \alpha\ln(1-\gamma) + (1-\alpha)\ln\theta$ in equilibrium without risk sharing, substituting these values into (7) yields:

$$U_t^n = \ln Ak_t^n(1-\theta-\gamma) + \ln Ak_{t+1}^n + \alpha\ln(1-\gamma) + (1-\alpha)\ln\theta. \tag{50}$$

Similarly, the equilibrium level of welfare of each household of generation $t \geq 1$ with risk sharing can be obtained (30) by noting that $\ln c_t^t = \ln q_t = \ln Ak_t\left[(1-\theta-\xi) - \frac{(1-\alpha)e}{Ak_t}Q\right]$ and $E_t \ln c_{t+1}^t = \ln[(1+r_{t+1})s_t + \alpha q_{t+1} - (1-\alpha)e] = \ln Ak_{t+1}\left[\alpha(1-\xi)+(1-\alpha)\theta - \frac{(1-\alpha)e}{(Ak_{t+1}^a)}(1+\alpha Q)\right]$:

$$U_t^a = \ln Ak_t^a\left[(1-\theta-\xi) - \frac{(1-\alpha)e}{Ak_t^a}Q\right] + \ln Ak_{t+1}^a\left[\alpha(1-\xi)+(1-\alpha)\theta - \frac{(1-\alpha)e}{Ak_{t+1}^a}(1+\alpha Q)\right]. \tag{51}$$

The welfare gain (or loss) from risk sharing can then be obtained by subtracting(50) from (51):

$$\begin{aligned} U_t^a - U_t^n &= [(t-1)(\ln\bar{g}_t^a - \ln\bar{g}_t^n) + t(\ln\bar{g}_{t+1}^a - \ln\bar{g}_{t+1}^n)] \\ &+ \left\{\ln\left[(1-\theta-\xi) - \frac{(1-\alpha)e}{Ak_t^a}Q\right] - \ln(1-\theta-\gamma)\right\} \\ &+ \left\{\ln\left[\alpha(1-\xi)+(1-\alpha)\theta - \frac{(1-\alpha)e}{Ak_{t+1}^a}(1+\alpha Q)\right] - [\alpha\ln(1-\xi)+(1-\alpha)\ln\theta]\right\}. \end{aligned} \tag{52}$$

The direct short-run effect of the risk sharing of each member of generation $t \geq 1$, which shows up in the third term in brackets on the right-hand side of (52), is positive for each old household at date 1. The later the generation the household belongs to, the greater is the direct effect from risk sharing as the cost of monitoring becomes less and less important (relative to the capital stock) with economic growth and capital accumulation. Its magnitude tends to monotonically increase to its maximum values as $t \to \infty$, the value that would obtain under full information. However, in addition to its short-run effects, risk sharing has its indirect or "long-run" effects on the welfare of the members of generation $t \geq 1$. As Proposition 5 makes it clear, risk sharing implies lower (average) growth rates, having both positive and negative implications for the welfare of generation $t \geq 1$. Although for a *given* level of the capital stock, lower growth implies a lower level of saving (a higher level of first-period consumption or a gift to his or her parent), leading to a higher level of welfare. It also means a *lower* level of capital stock and income, implying a lower level of welfare. The former effect shows up in the second term in braces on the right-hand side of (52), which is unambiguously positive under the assumption of (49), while the latter effect is associated with the first term. As $\bar{g}^n_{t+1} > \bar{g}^a_{t+1}$ and $\bar{g}^n_{t+1} - \bar{g}^a_{t+1}$ is increasing in t for each $t \geq \bar{t}$, some $\bar{t} \geq 1$ (Proposition 5), the first term in brackets is negative for large t and grows (negatively) without bound as time (and generation) advances. As the second and third terms in brackets are positive and increasing with time but *bounded above*, it follows that there is some generation for which the first term in braces dominates the second and third terms in braces on the right-hand side of (52); the negative long-run growth effects from risk sharing dominates the positive direct effect. From that generation on, the households will be *worse off* under any regimes with risk sharing than under the no-risk-sharing regime.

6.2 Social welfare

In the previous subsection, we only analyzed the effect of an individual's welfare. Therefore, this subsection investigates the social welfare effect by employing the Benthamite social welfare function; that is, the welfare level of period t is measured by the sum of the utility of generation $t-1$ and generation t who live in period t. Now, let us derive a social welfare function as a discounting sum of the welfare level of each generation:

$$W^t = E\sum_{t=1}^{\infty}\beta^{t-1}[\ln c^t_t + \alpha \ln c^{t-1,G}_t + (1-\alpha)\ln c^{t-1,B}_t], \tag{53}$$

where β shows the rate of social time preference across generations.

From the analysis of the previous subsection, because the initial (future) generation enjoys the short-run effect (long-run effect), the initial (future) generation experiences a welfare improvement (deterioration) with risk sharing. Thus, social welfare under risk sharing is anticipated such that when the rate of social time preference is sufficiently large (small), the long-run effect (short-run effect) is dominated by the short-run effect (long-run effect). To see whether this forecast is correct or not, we derive the social welfare function in the same manner as in the previous subsection. Noting the fact that $k_t = k_1(A\gamma)^{t-1}, \bar{g}_t^n \equiv A\gamma, k_t^a = (k_1^c)(A\xi)^{t-1} + c$ and $\bar{g}_t^a \equiv (k_t^a/k_1)^{\frac{1}{t-1}}$, the social welfare function without and with risk sharing is derived as follows:

$$W^n = \frac{1}{1-\beta}N + \frac{2\beta}{(1-\beta)^2}\ln \bar{g}_t^n, \tag{54}$$

where $N \equiv 2\ln Ak_1 + \ln(1 - \theta - \gamma) + \alpha\ln(1-\gamma) + (1-\alpha)\ln\theta$.

$$W^a = \frac{1}{1-\beta}P + \frac{2\beta}{1-\beta}\ln \bar{g}_t^a, \tag{55}$$

where $P \equiv 2\ln Ak_1 + \ln\left[(1-\theta-\xi) - \frac{(1-\alpha)e}{Ak_t^a}\right] + \ln\left[\alpha(1-\xi) + (1-\alpha)\theta - \frac{(1-\alpha)e}{Ak_t^a}(1+\alpha Q)\right]$. Therefore the difference between (54) and (55) is represented as follows:

$$\begin{aligned} W^a &- W^n \\ &= \frac{2\beta}{(1-\beta)^2}(\ln \bar{g}_t^a - \ln \bar{g}_t^n) \\ &+ \frac{\beta}{1-\beta}\left\{\ln\left[(1-\theta-\xi) - \frac{(1-\alpha)e}{Ak_t^a}Q\right] - \ln(1-\theta-\gamma)\right\} \\ &+ \frac{\beta}{1-\beta}\left\{\ln\left[\alpha(1-\xi) + (1-\alpha)\theta - \frac{(1-\alpha)e}{Ak_t^a}(1+\alpha Q)\right] - [\alpha\ln(1-\xi) + (1-\alpha)\ln\theta]\right\}. \end{aligned} \tag{56}$$

As the second and third terms of (56) are pretty much the same as in (52), then the value of the inside bracket is positive with the same reason as given in our previous subsection. In addition, this, the first term of (56) is negative (Proposition 5). Then, there exists a unique β^* such that $W^a - W^n = 0$ and $W^a > (<) W^n$ if $\beta < (>) \beta^*$, where $\beta^* \equiv \frac{P-N}{2(\ln \bar{g}_t^n - \ln \bar{g}_t^a) - (P-N)}$. These results indicate that if the rate of social time preference is sufficiently large (small), then the schedule of risk sharing accelerates (deteriorates) the social welfare level. Hence, these results follow the above anticipation.

7 Conclusion

This paper has investigated how the risk sharing methods that cover the uncertainty of intergenerational transfers affect aggregate saving and welfare in a two-period overlapping generations model. For this purpose, we investigated the gifts economy and assumed that young agents are divided into two types according to their attitude towards their parents. One group is constituted of agents who are dutiful to their parents and transfer gifts, whereas the other is constituted of agents who are selfish and do not transfer gifts. If agents are risk averse, they have an incentive to share the risk within the same generation. Thus, agents who have the ability to monitor contracts act as insurance companies. We assume that information asymmetry exists between agents and insurance companies about the income realization.

We investigated economies both with and without risk sharing. By analyzing these economies, we found that the rate of economic growth becomes constant in both regimes. Income fluctuations can be completely smoothed under the schedule of risk sharing, and, thus, the precautionary motive for saving decreases, resulting in a lower rate of economic growth. As to welfare, risk sharing can enhance the welfare level of the present generation, but it deteriorates the welfare level of the future generation. Therefore, when the social rate of time preference is sufficiently high (low), risk sharing accelerates (deteriorates) the welfare level.

8 Appendices

Appendix A: Proof of Proposition 1

We define the function $G(g)$ by:

$$G(g) \equiv \frac{2-\alpha(1-\theta)A}{2-\alpha} - \frac{\alpha\theta(1-\theta)A^2}{(2-\alpha)[(2-\alpha)g-(1-\alpha)(1-\theta A]}.$$

Then, (14) can be written as $g_{t+2} = G(g_{t+1})$. Evaluating the derivative of G at b_1 and b_2, we find that $G'(b_1) < 1$, and $G'(b_2) > 1$, where b_1 and b_2 are given by (17) and (18). Thus, any growth path $\{g_{t+1}\}_{t=1}^{\infty}$ that starts with a growth rate other than b_2 at date $t = 1$, covers to b_1 (see Figure 1). As $h[(1-\theta)A] = -(1-\theta)\theta A^2 < 0$, we have $b_2 < (1-\theta)A < b_1$. It follows that the only growth path that is consistent with the resource constraint (15) is the stationary one with $g = b_2$ as its initial growth rate.

Thus, the growth path must be such that $g_{t+1} \equiv k_{t+1}/k_t \equiv b_2$ for each $t \geq 1$, so that the unique equilibrium path is given by $k_t = Zb_2^t$ with Z to be determined by its initial condition. The growth rate is positive if and only if $b_2 > 1$, which in turn holds if and only if (19) holds.

Finally, the equilibrium path must satisfy its initial condition so that $Z = k_1/b_2$, where $k_1 > 0$ is the initial endowment of capital stock of generation 0. Therefore, the equilibrium path is given by $k_t = k_1(b_2)^{t-1} = k_1(A\gamma)^{t-1}$ for each $t \geq 1$.

Appendix B: Proof of Proposition 2

To show the sufficiency part, suppose that (26) holds. Then:

$$e - \frac{\alpha}{1-\alpha} q_{t+1} \leq \frac{1}{1-\alpha}\{(1+r_{t+1})s_t - [(1+r_{t+1})s_t + q_{t+1}]^{\alpha}[(1+r_{t+1})s_t]^{1-\alpha}\} < 0. \tag{57}$$

We note that (21) is equivalent to:

$$-p^G_{t+1} + \varphi_{t+1} \geq 0. \tag{58}$$

Let $\lambda_1, \lambda_2, \lambda_3$, and λ_5 be the nonnegative multipliers associated with constraints (58), (22), (23), and (25), respectively, and let λ_4 be the multiplier associated with (24). Then, the following Kuhn–Tucker conditions, together with (58), (22), (23), (24), and (25), are necessary and sufficient for the maximization problem in (20)–(25); that is:

$$\lambda_1 - \lambda_3 \leq 0 \quad \text{with equality if } \varphi_{t+1} > 0, \tag{59}$$

$$-\frac{\alpha}{(1+r_{t+1})s_t + q_{t+1} - p^G_{t+1}} - \lambda_1 - \lambda_2 + \alpha\lambda_4 - \frac{\alpha\lambda_5}{(1+r_{t+1})s_t + q_{t+1} - p^G_{t+1}} \leq 0$$
$$\text{with equality if } p^G_{t+1} > 0, \tag{60}$$

$$\frac{1-\alpha}{(1+r_{t+1})s_t + p^B_{t+1}} - (1-\alpha)\lambda_4 + \frac{(1-\alpha)\lambda_5}{(1+r_{t+1})s_t + p^B_{t+1}} \leq 0$$
$$\text{with equality if } p^B_{t+1} > 0, \tag{61}$$

$$\lambda_1(-p^G_{t+1} + \varphi_{t+1}) = 0, \tag{62}$$

$$\lambda_2(q_{t+1} - p^G_{t+1}) = 0, \tag{63}$$

$$\lambda_3(q_{t+1} - \varphi_{t+1}) = 0, \tag{64}$$

$$\lambda_5\{\alpha \ln[(1+r_{t+1})s_t + q_{t+1} - p^G_{t+1}] + (1-\alpha)\ln[(1+r_{t+1})s_t + p^B_{t+1}], \tag{65}$$
$$-\alpha \ln[(1+r_{t+1})s_t + q_{t+1}] - (1-\alpha)\ln[(1+r_{t+1})s_t]\}.$$

As (24) implies that p^G_{t+1} and p^B_{t+1} are both strictly positive, (60) and (61) hold with equality. Then, eliminating λ_4 from the equality versions of (60) and (61) yields:

$$\lambda_1 + \lambda_2 = \frac{\alpha(1+\lambda_5)[q_{t+1} - p^G_{t+1} - p^B_{t+1}]}{[(1+r_{t+1})s_t + p^B_{t+1}][(1+r_{t+1})s_t + q_{t+1} - p^G_{t+1}]}. \tag{66}$$

We want to show that $\lambda_1 = \lambda_2 = 0$. To see that $\lambda_1 = 0$, suppose that, on the contrary, $\lambda_1 > 0$. Then, (59) implies that $\lambda_3 > 0$, and (62) and (64) in turn imply that:

$$p^G_{t+1} = q_{t+1}. \tag{67}$$

Substituting (67) into (24) gives:

$$p_{t+1}^B = \frac{\alpha}{1-\alpha} q_{t+1} - e. \tag{68}$$

Then, (66), (67), and (68) yield:

$$\lambda_1 + \lambda_2 = \frac{\alpha(1+\lambda_5)}{\left[(1+r_{t+1})s_t + \frac{\alpha}{1-\alpha}q_{t+1} - e\right](1+r_{t+1})s_t}\left(e - \frac{\alpha}{1-\alpha}q_{t+1}\right) < 0, \tag{69}$$

resulting in a contradiction, where the strict inequality follows from (57). To see that $\lambda_2 = 0$, suppose now that $\lambda_2 > 0$. Then, (63) implies (67). The same argument as above leads us to a contradiction, which establishes that $\lambda_1 = \lambda_2 = 0$. If $\lambda_1 = \lambda_2 = 0$, then (66) and (24) imply (27) and (28). Then, (22), (58), and (27) lead to (29). Finally, it is easy to see that (27) and (28) satisfy (25) under the condition shown in (26).

To show that (26) is necessary, suppose that (26) does not hold. Then:

$$e - \frac{\alpha}{1-\alpha}q_{t+1} > \frac{1}{1-\alpha}\{(1+r_{t+1})s_t - [(1+r_{t+1})s_t + q_{t+1}]^\alpha[(1+r_{t+1})s_t]^{1-\alpha}\}.$$

However, as the right-hand side of this inequality is negative, there are two possible cases: (i) $e - \frac{\alpha}{1-\alpha}q_{t+1} \leq 0$ and (ii) $e - \frac{\alpha}{1-\alpha}q_{t+1} > 0$. Suppose that case (i) holds. Then, the above sufficiency argument applies and we are led to (27) and (28). However, we find that (27) and (28) do not satisfy (25) when (26) is violated. Consider now case (ii). We go back to the sufficiency argument to find that p_{t+1}^G and p_{t+1}^B are given by (67) and (68), respectively, recalling that (70) implies that $\lambda_1 > 0$ or $\lambda_2 >$ in this case. However, case (ii) implies that p_{t+1}^G, contradicting (24), which completes the proof.

Appendix C: Proof of Proposition 3

The characteristic roots of (33) are given by (35) and (36). The general solution to (33) is then given by (34). As $f[(1-\theta)\alpha] = -\theta(1-\theta)A^2 < 0$, it must be true that $b_2 < (1-\theta)A < b_1$. As any equilibrium path must satisfy the resource constraint (13), the particular solution to (33) must be such that Z_1 of (34) is zero; that is, the equilibrium path is given by $k_t = Z_2(b_2)^t + c = Z_2(A\xi)^t + c$ for each $t \geq 1$, with Z_2 to be determined by its initial condition. The growth rate is positive if and only if $b_2 = A\xi > 1$, which in turn holds if and only if (41) holds. From (37) and (38) we find that c is given by $-(1-\alpha)/f(1)$. When A meets (41), it must be the case that $f(1) > 0$, and, therefore, that $c < 0$. The equilibrium path must satisfy its initial condition so that $Z_2 = (k_1 - c)/b_2$, which yields $k_t = (k_1 - c)(b_2)^{t-1} + c = (k_1 - c)(A\xi)^{t-1} + c$.

Next, we check that when the condition (42) is met and (26) is met, that (26) in Proposition 2 is indeed satisfied along the equilibrium path in Proposition 3. We note that if the risk sharing regime prevails from date $t+1$ on, then:

$$q_{t+1} = w_{t+1} - s_{t+1} = (1-\theta)Ak_{t+1} - k_{t+2} = (1-\theta)Ak_{t+1} - [(k_{t+1} - c)A\xi + c]. \tag{70}$$

Then, we substitute (6) and (70) as well as $s_t = k_{t+1}$ into (26) to obtain:

$$\ln\left[\alpha(1-\xi)+(1-\alpha)\theta-(1+\alpha Q)\frac{(1-\alpha)}{Ak_{t+1}}e\right]-\alpha\ln\left[(1-\xi)-Q\frac{(1-\alpha)}{Ak_{t+1}}e\right]-(1-\alpha)\ln\theta \geq 0. \tag{71}$$

As $A\xi < A(1-\theta)$ or $1-\xi > \theta$, as shown above, Lemma 1 implies that (71) holds for $t+1 = 1$, provided (41) is met. As $k_{t+1} = (k_1 - c)(A\xi)^t + c > k_1$ for $t+1 = 2, 3, \cdots$ along the path, (49) holds for $t+1 = 2, 3, \cdots$ as well.

Appendix D: Proof of Lemma 2(a)

By Propositions 1 and 3, $A\gamma$ and $A\xi$ are the smaller roots of $h(b) = 0$ and $f(b) = 0$, respectively. We define the function $m(b)$ as $m(b) \equiv h(b) - f(b)$. Then:

$$m(b) = 2(1-\alpha)b^2 - 3(1-\theta)(1-\alpha)Ab + (1-\theta)^2(1-\alpha)A^2.$$

The quadratic equation $m(b) = 0$ has two distinct roots, $(1-\theta)A/2$ and $(1-\theta)A$, which implies that the graphs of $h(b)$ and $f(b)$ cross at $b = (1-\theta)A/2, (1-\theta)A$. This also implies that: $m(b) \equiv h(b) - f(b) > 0$ for each $b < (1-\theta)A/2$. As $h[(1-\theta)A/2] = f[(1-\theta)A/2] = -\frac{\alpha(1-\theta)^2A^2}{4} < 0$ and as $A\gamma$ and $A\xi$ are the smaller roots of $h(b) = 0$ and $f(b) = 0$, respectively, it must be the case that $A\gamma < (1-\theta)A/2$ and $A\xi < (1-\theta)A/2$. However, as $h(b) > f(b)$ for each $b < (1-\theta)A/2$, as shown above, we have:

$$0 = h(A\gamma) > f(A\gamma),$$

$$h(A\xi) > f(A\xi) = 0,$$

which in turn implies that $A\gamma > A\xi$.

Appendix E: Proof of Proposition (c)

Here, we show that $dc/d\alpha > 0$. Differentiating (37) with respect to α and rearranging yields:

$$\frac{dc}{d\alpha} = \frac{e[1 - 2\theta A - (1-\theta)A + (1-\theta)\theta A^2]}{f(1)^2}.$$

To see that the numerator on the right-hand side is positive, observe that $f(1) = \alpha - [2\theta + \alpha(1-\theta)]A + (1-\theta)\theta A^2 > 0$ for any $\alpha \in (0, 1]$ (see the proof of Proposition 3) and that it holds for $\alpha = 1$ in particular.

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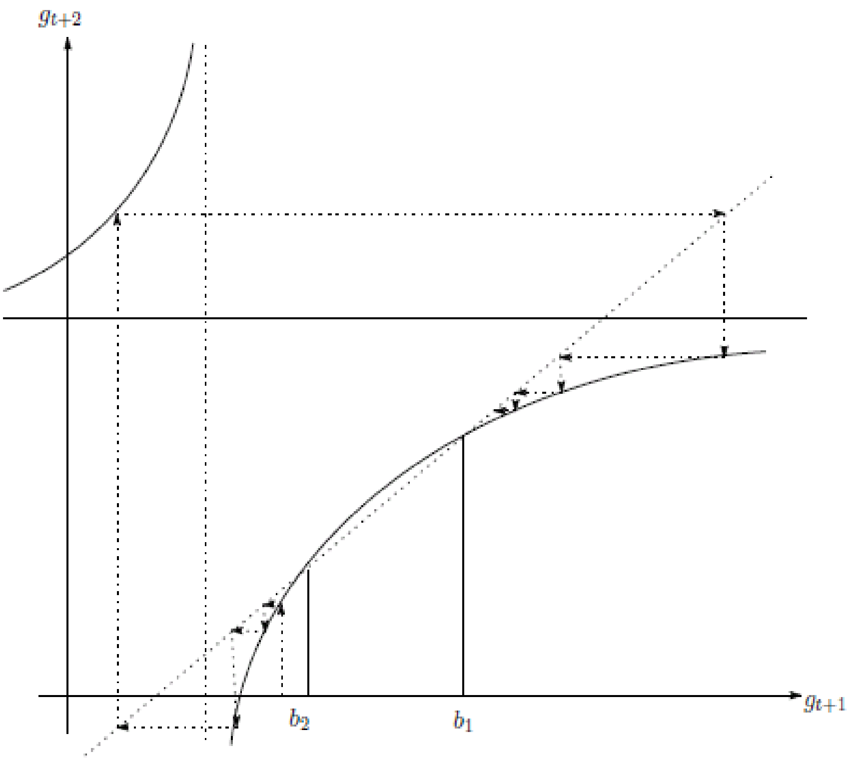

Figure 1: The dynamics of the economy