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Abstract

This paper explores the role of consumption externalities in an overlapping generations economy with capital accumulation. If consumers in each generation are concerned with other agents' consumption behaviors, there exist intergenerational as well as intragenerational consumption externalities. It is the presence of intergenerational consumption externalities that may produce fundamental effects both on equilibrium dynamics and on steady-state characterization of the economy. This paper demonstrates this fact in the context of a simple model of endogenously growing, overlapping-generations economy with or without asset bubbles.

JEL Classification: E32, J24, O40

Keywords: Consumption Externalities, Overlapping Generations, Long-run Growth, Asset Bubbles.

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1 Introduction

Recently, there has been a renewed interest in the role of consumption externalities in macroeconomics. Earlier studies on this issue such as Abel (1990) and Galí (1994) introduce consumption external effects into the asset pricing models in order to resolve the discrepancies between theoretical outcomes and empirical findings. Those studies, therefore, focus on the external effects of consumption on the individual decision making or on the behavior of asset markets. In contrast, the recent investigations on macroeconomic implications of consumption externalities examine the issue in the dynamic, general equilibrium models with capital accumulation and discuss a wider range of topics than those considered by the earlier studies. For example, the recent studies have explored the effect of consumption externalities on optimal taxation (Ljungqvist and Uhlig 2000), equilibrium efficiency (Alonso-Carrera et al. 2003 and Liu and Turnovsky 2005), indeterminacy and sunspots (Weder 2000), and on the relationship between savings and long-term economic growth (Carroll et al. 1997 and 2000, and Harbaugh 1996).[1]

A common feature of the existing investigations on consumption externalities in the macroeconomics literature is that most of them employ the representative agent models. In general, introduction of consumption externalities into the standard representative-agent models of growth and business cycles does not produce significant qualitative effects on dynamic behavior of the model economy: see Liu and Turnovsky (2005) for a detailed discussion on equilibrium dynamics of the standard representative agent model with consumption externalities. Although consumption externalities may yield large quantitative effects that would be relevant for welfare implications and policy making decisions, the dynamic properties and the steady-state characterization of the model economies are usually the same as those of models without consumption external effects. Such a conclusion

[1] Some of the existing studies such as Ljunavust and Uhlig (2000) and Carroll et al. (1997 and 2000) assume the external habit formation in which the benchmark consumption is given by a weighted average of past levels of the average consumption in the economy. Unlike the internal habit formation, consumers consider that the benchmark consumption is not affected by their own consumption behavior under the external habit formation hypothesis. Thus this assumption represents consumption externalities with time delay rather than (internal) habit formation under which each agent takes its past consumption into account when deciding its optimal saving plans.

is in contrast to the effect of production externalities, which may alter the behavior of the economy in fundamental ways.

Departing from the mainstream studies mentioned above, we examine the role of consumption external effects in an overlapping generations (OLG) economy. We extend the standard two-period-lived OLG model of capital accumulation by introducing external effects among consumption activities of the agents. A relevant difference between the representative agent and the OLG settings is that the contemporaneous external effects of consumption involve the intergenerational as well as intragenerational externalities in the OLG economy. Unlike the representative agent economy, heterogeneity of agents inevitably exists in the OLG economy, and hence contemporaneous interactions among consumption activities of the agents would be more complex in the OLG economy than those in the representative-agent economy. This suggests that the presence of consumption externalities generates more fundamental effects both on equilibrium dynamics and on the long-run equilibrium in the OLG setting than in the representative-agent counterpart. The central purpose of this paper is to confirm this prediction by using a simple growth model of an OLG economy.

More specifically, we develop an endogenous-growth version of Diamond's (1965) model with consumption externalities. We first examine the real economy in the absence of asset bubbles. It is shown that the balanced-growth equilibrium and transitional dynamics depend heavily on the preference structure that characterizes forms of consumption external effects. For example, if consumers have jealousy as to other agents' consumption so that there are negative consumption externalities and if consumers are conformists, then there is a unique balanced-growth path that satisfies global determinacy. However, if the consumers are conformists but they have admiration as to others' consumption, then there may exist dual balanced-growth equilibria. Similarly, if consumers are anti-conformists and if intragenerationanl external effect is sufficiently strong, then dual balanced-growth paths may exist as well. It is shown that in these cases of dual steady states, the long-run equilibrium with a higher growth rate is locally indeterminate, while the steady state with a lower growth rate exhibits local determinacy.

We then investigate the equilibrium dynamics of the model economy in the presence of asset bubbles. As shown by Grossman and Yanagawa (1993), an OLG economy with endogenous growth may sustain the balanced-growth path with bubbles, if and only if the balanced-growth rate of bubble-free economy exceeds the real interest rate.[2] Thus the presence of bubble lowers the long-term growth rate, because the balanced-growth rate in the presence of bubbles is fixed at the rate of return to capital. It is also known that the steady state with bubbles is locally determinate and the bubble-free steady state is locally indeterminate. We show that those results established in the standard framework still hold, if there are only intragenerational consumption externalities. However, if there also exist intergenerational consumption externalities, we may obtain different outcomes. In particular, the growth effect of bubbles and the conditions for sustaining bubbles are considerably modified in the presence of intergenerational consumption externalities. Since we will assume that supply of nominal asset grows at a constant rate, the balanced-growth path with bubbles is uniquely determined. Therefore, even if the corresponding bubble-free economy involves dual steady states, introducing bubbles yields a unique long-term equilibrium. It is shown that in the presence of consumption externalities the growth rate in the steady state with bubbles may or may not exceed the growth rate attained in the bubble-free equilibrium. We also confirm that the economy exhibits global indeterminacy in the sense that every balanced-growth path (with or without bubbles) and a transitional trajectory leading to one of the stable balanced-growth equilibria can be a perfect-foresight competitive equilibrium.

It is to be noted that several authors have examined the roles of consumption externalities in OLG models. Among others, Abel (2005) introduces both intergenerational and intragenerational consumption externalities into the Diamond model of overlapping generations. The basic model structure of his article is thus close to ours. The central concern of Abel (2005) is to characterize the optimal income taxation in the steady-state equilibrium. Alonso-Carrera et al. (2005) also use a model similar to Abel's (2005) in order to investigate how the presence of consumption externalities affects the bequest motives of altruistic agents. In addition, de la Croix (1996) examines an OLG model where the parents' consumption behavior is inherited by their children and thus there is one-way

[2] In the context of an infinitely-lived overlapping generations model, Futagami and Shibata (1999) pointed out that this result may not hold, if supply of useless asset is not constant. In Section 4.1 we reconfirm this fact in our setting.

intergenerational external effect.[3] Since neither equilibrium dynamics with endogenous growth nor asset bubbles are out of touch in the existing studies mentioned above, the contribution of this paper is to present a new insight on the role of consumption externalities in macroeconomic dynamics that has not been fully explored in the literature.

The rest of the paper is organized as follows. Section 2 sets up the analytical framework. Section 3 discusses equilibrium dynamics of the model economy in the absence of asset bubbles. Section 4 introduces asset bubbles into the base model and considers the role of consumption externalities in characterizing long-run equilibrium that may sustain bubbles. Section 5 concludes the paper.

2 The Base Model

In this section, we construct the base model that depicts growth dynamics without bubbles. The analytical framework we use is an endogenous-growth version of Diamond's (1965) model. We extend the baseline model by assuming that felicity of an individual consumers in each generation depends not only on her own consumption but also on the benchmark consumption represented by the average level of consumption in the economy at large.

2.1 Households

We consider a two-period-lived overlapping generations economy where in each period only two types of agents are alive: young and old. Agents are identical within the generation. Population is constant over time and the number of agents in each generation is normalized to one. The utility function of agents in cohort born at the beginning of period t is

$$U_t = u\left(c_t, E_t\right) + \beta u\left(x_{t+1}, E_{t+1}\right), \qquad 0 < \beta < 1,$$

where c_t denotes consumption when the agents are young, x_{t+1} is consumption when they are old, and β denotes a given discount factor. In the above, E_t and E_{t+1} denote the benchmark levels of consumption that express external effects on the felicities in period

[3] See also de la Croix and Michel (2002). Using the term, 'external habit', de la Croix's (1996) formulation represents itntergenerational, external habit formation. Lahiri and Puhakka (1998) examine a two-period-lived overlapping generations economy with internal habit formation.

t and $t+1$, respectively. We assume that the benchmark level of consumption in each period depends on the average consumption levels of existing generations:

$$E_t = E^0\left(\bar{c}_t, \bar{x}_t\right), \qquad E_{t+1} = E^1\left(\bar{x}_{t+1}, \bar{c}_{t+1}\right),$$

where $\bar{c}_{t+s}$ and $\bar{x}_{t+s}$ $(s = 0, 1)$ respectively denote the average consumption of young and old agents in period $t+s$. Notice that since we have assumed that the number of agent in each cohort is normalized to one, in equilibrium the average level of consumption of each agent equals its private level:

$$\bar{c}_t = c_t, \quad \bar{x}_t = x_t \ \text{ for all } t \geq 0. \tag{1}$$

For analytical simplicity, we specify the instantaneous utility function in each period as follows:

$$u\left(c_t, E_t\right) = \frac{\left(c_t E_t^{-\theta}\right)^{1-\sigma}}{1-\sigma}, \quad u\left(x_{t+1}, E_{t+1}\right) = \frac{\left(x_{t+1} E_{t+1}^{-\theta}\right)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \ \sigma \neq 1, \ \theta < 1. \tag{2}$$

We also specify the reference levels of consumption in such a way that

$$E^0\left(\bar{c}_t, \bar{x}_t\right) = \bar{c}_t^{\gamma} \bar{x}_t^{1-\gamma}, \quad E^1\left(\bar{x}_{t+1}, \bar{c}_{t+1}\right) = \bar{x}_{t+1}^{\gamma} \bar{c}_{t+1}^{1-\gamma}, \qquad 0 < \gamma \leq 1. \tag{3}$$

Namely, we assume that the felicity of each agent depends on the weighted geometric mean of consumption levels of all consumers. In this specification, γ denotes the relative strength between the intragenerational and intergenerational external effects.[4] Note that when $\gamma = 1$, the external effects are generated by the average consumption of the same

[4] An alternative specification of the utility function that has been frequently used in the literature is

$$U_t = \frac{\left(c_t - \theta E_t\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(x_{t+1} - \theta E_{t+1}\right)^{1-\sigma}}{1-\sigma}$$

where

$$E_t = c_t + \gamma x_t, \quad E_{t+1} = x_{t+1} + \gamma c_{t+1}.$$

Mino (2004) use this specification to analyze equilibrium dynamics of an OLG model with the neoclassical production technology. Alonso-Carrera et al. (2005) use a slightly more general form such that

$$U_t = \frac{\left(c_t - \theta E_t^{\delta}\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(x_{t+1} - \theta E_{t+1}^{\delta}\right)^{1-\sigma}}{1-\sigma}.$$

cohort alone. Given (3) , the instantaneous utility function in each period is written as

$$
\begin{aligned}
u\left(c_t, E^0\left(\bar{c}_t, \bar{x}_t\right)\right) &= \frac{\left[c_t^{1-\theta}\left(\dfrac{c_t}{\bar{c}_t}\right)^{\theta\gamma}\left(\dfrac{c_t}{\bar{x}_t}\right)^{\theta(1-\gamma)}\right]^{1-\sigma}}{1-\sigma}, \\
u\left(x_{t+1}, E^1\left(\bar{x}_{t+1}, \bar{c}_{t+1}\right)\right) &= \frac{\left[x_{t+1}^{1-\theta}\left(\dfrac{x_{t+1}}{\bar{x}_{t+1}}\right)^{\theta\gamma}\left(\dfrac{x_{t+1}}{\bar{c}_{t+1}}\right)^{\theta(1-\gamma)}\right]^{1-\sigma}}{1-\sigma}.
\end{aligned}
$$

These expressions show that the felicity of t-th generation in each period depends on the intragenerational and intergenerational relative consumption levels as well as on the absolute level of its own consumption.

If external effects are internalized so that the conditions in (1) holds, the social level of marginal utility of private consumption is given by

$$
u_c\left(c_t, E^0\left(c_t, x_t\right)\right) = \left(1-\theta\gamma\right) c_t^{-\theta\gamma-\sigma(1-\theta\gamma)} x_t^{-\theta(1-\gamma)(1-\sigma)}.
$$

In addition, we see that

$$
\text{sign } u_{cc} = \text{sign } \left\{-\left(1-\theta\gamma\right)\left(\sigma+\gamma\theta\left(1-\sigma\right)\right)\right\}.
$$

The same conditions hold for $u\left(x_{t+1}, E^1\left(x_{t+1}, c_{t+1}\right)\right)$. In order to make the social marginal utility of consumption is positive and decreasing, we assume that

$$
1-\theta\gamma > 0 \;\text{ and } \sigma+\gamma\theta\left(1-\sigma\right) > 0. \tag{4}
$$

If $0 < \theta < 1$, the above conditions are satisfied. If $\theta < 0$,we need to assume that $\sigma + \theta\gamma\left(1-\sigma\right) > 0$.

Following the taxonomy given by Dupor and Liu (2003), if an agent's felicity depends both on the average consumption in the economy at large and on her own consumption, the presence of negative external effect means that the consumer has jealousy as to other agents' consumption, while the consumer has admiration for consumption of others if her felicity is positively related to the social level of consumption. Moreover, if the marginal utility of private consumption increases with the social level of consumption, consumers prefer being similar to others (Keeping Up with the Joneses: KUJ). In contrast, if the marginal utility of private consumption decreases with the social level of consumption, consumers want to be different from others (Running Away from the Joneses: RAJ). Using our notation, $\partial u/\partial E < 0$ means jealousy and $\partial u/\partial E > 0$ expresses admiration. Similarly, $\partial^2 u/\partial c\partial E > 0$ indicates KUJ, while consumers' preference satisfies RAJ, if $\partial^2 u/\partial c\partial E < 0$. Given the utility function (2), we see that

$$\text{sign } u_E \left(\equiv \frac{\partial u}{\partial E}\right) = \text{sign } (-\theta), \quad \text{sign } u_{cE} \left(\equiv \frac{\partial^2 u}{\partial c \partial E}\right) = \text{sign } \{-\theta (1-\sigma)\}.$$

Thus the consumer's preference satisfies KUJ if $\theta(1-\sigma) < 0$, while her preference exhibits RAJ if $\theta(1-\sigma) > 0$.

An alternative expression for KUJ (resp. RAJ) is conformism (resp. non-conformism). Collier (2004) defines the 'degree of conformism' in the following manner:

$$\Lambda(c, E) = \left.\frac{dc}{dE}\right|_{u_c = \text{ const}} = -\frac{u_{cE}}{u_{cc}} = \theta\left(\frac{\sigma-1}{\sigma}\right)\frac{c}{E}.$$

This gives the relation between the benchmark and private levels of consumption with keeping the marginal utility of private consumption constant. If $\Lambda(c, E)$ has a negative value, then a higher benchmark consumption reduces private consumption under a given marginal utility of private consumption. This is equivalent to the RAJ condition. Conversely, if $\Lambda(c, E) > 0$, conformism dominates the consumers' behavior. In particular, if $\Lambda(c, E) > 1$, then consumers are over-conformists.[5] It is worth pointing out that in the representative agent economy where $E = c$ in equilibrium, the degree of conformism is determined by the preferences parameters, σ and θ, alone. In the overlapping generations economy where the equilibrium level of benchmark consumption, E, may diverge from c, the degree of conformism is also affected by the relative consumption, c/E. Since conformism and non (anti)-conformism are more straightforward expressions than KUJ and RAJ, in what follows we use the former terms.

[5] By definition, $\Lambda(c, E) > 1$ means that $u_{cc} + u_{cE} > 0$.If this is the case, conformism is so strong to cancel decreasing marginal utility of private consumption. This situation has been excluded in most of the existing studies on consumption externalities in macroeconomics.

In sum, we have four possibilities displayed in Table 1 :

	$\sigma > 1$	$\sigma < 1$
$\theta > 0$	jealousy, conformist	jealousy, non-conformist
$\theta < 0$	admiration, non-conformist	admiration, conformist

Table 1: Classification of the preference structure

In the existing literature on consumption externalities in macroeconomic dynamics, it has been common to assume that the consumer's preference exhibits jealousy and conformism. Although jealousy and conformism may be frequently observed in reality, we also examine other cases in order to consider how the analytical conclusions are related to the specifications of preference structure.

The consumer's intertemporal decision is basically the same as that in Diamond (1965). We assume that agents in each cohort work only when they are young. Hence, their flow budget constraint in their young and old ages are respectively given by

$$w_t = c_t + s_t \quad \text{and} \quad x_{t+1} = R_{t+1} s_t,$$

where w_t is the real wage rate, s_t is saving of the young agent and R_{t+1} denotes the gross rate of interest in period $t+1$. The intertemporal budget constraint for the household is written as

$$c_t + \frac{x_{t+1}}{R_{t+1}} = w_t. \tag{5}$$

The agents born at the beginning of period t select c_t and x_{t+1} to maximize U_t subject to the life-time budget constraint of (5). When solving their optimization problem, the agents take the benchmark levels of consumption, E_t and E_{t+1}, as given. The first-order conditions for an optimum yields

$$\left(\frac{c_t}{x_{t+1}}\right)^{-\sigma} = \beta R_{t+1} \left(\frac{E_t}{E_{t+1}}\right)^{\theta(1-\sigma)}. \tag{6}$$

Therefore, unless $\sigma = 1$, the marginal rate of substitution between consumption when young and old is determined by the relative magnitudes of external effects, E_t/E_{t+1}, and by the rate of interest, R_{t+1}. Using thd consistency conditions in (1) and substituting (3) and (6) into (6), we obtain the following:

$$\left(\frac{c_t}{x_{t+1}}\right)^{-[\sigma+\gamma\theta(1-\sigma)]} = \beta R_{t+1} \left(\frac{x_t}{c_{t+1}}\right)^{\theta(1-\gamma)(1-\sigma)}. \tag{7}$$

This equation shows that the marginal rate of substitution between young and old-age consumption from the social perspective is related to the rate of return to capital in period $t+1$ and the relative consumption, x_t/c_{t+1}, which represents the intergenerational external effects. Note that, given the external effects, the relative consumption c_t/x_{t+1} from the private perspective decreases with the rate of return, R_{t+1}. Similarly, under the restrictions in (4), the relation between the relative consumption from the social perspective is also negatively related to the real interest rate.

2.2 Production

We assume that the economy has an Ak technology that can sustain endogenous growth. Firms produce a single commodity in a competitive market. The private technology of production satisfies constant returns to scale with respect to capital and labor. Suppose that there are many identical firms. The number of firms is normalized to one. The production function of each firm is given by

$$y_t = Ak_t^{\alpha} \left(l_t \bar{k}_t\right)^{1-\alpha}, \quad 0 < \alpha < 1,$$

where y_t, l_t and k_t respectively denote output, labor and capital of an individual firm. Additionally, $\bar{k}_t$ is the aggregate capital which equals k_t in the symmetric equilibrium. If we assume that each young agent supplies one unit of labor so that $l_t = 1$, the aggregate production function is expressed as $y_t = Ak_t^{\alpha}\bar{k}_t^{1-\alpha}$.[6]

[6] Alternatively, we may assume that the production technology is given by

$$y_t = Ak_t^{\alpha} h_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where h_t denotes human capital. If we assume that physical and human capital earn the same rate of return in a competitive environment, it should hold that

$$\frac{\partial y_t}{\partial k_t} = \frac{\partial y_t}{\partial h_t},$$

which means that the optimal factor choice gives $h = (1-\alpha)k/\alpha$. Therefore, the production function is written as

$$y_t = \hat{A}k,$$

where $\hat{A} = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}$. The analytical results based on this formulation is the same as those obtained in our setting. Only difference is that the above model does not involve production externalities so that there is no inefficiency in the production side of the economy.

The final good market is assumed to be competitive. Thus, the net rate of return to capital, r_t, and the real wage rate, w_t, are respectively given by $r_t = \alpha y_t / k_t - \delta$ and $w_t = (1-\alpha) y_t$, where δ denotes the rate of depreciation of capital. Due to the arbitrage condition, the gross rate of interest satisfies $R_t = r_t + 1$. In the following, we assume that capital fully depreciates in one period, that is, $\delta = 1$ so that $R_t = \alpha y_t / k_t$. (This is a plausible assumption in the two-period lived OLG economy in which one period may be 25 years long.).

The symmetric equilibrium requires that $\bar{k}_t = k_t$. Therefore, the social technology that internalizes the external effect is written as follows:

$$y_t = Ak_t. \tag{8}$$

Similarly, the competitive levels of gross rate of return to capital and the real wage rate are respectively given by

$$R_t = \alpha A \equiv R, \tag{9}$$

$$w_t = (1-\alpha) Ak_t. \tag{10}$$

2.3 Commodity Market Equilibrium Condition

In each period the final goods are used for consumption and investment, so that

$$y_t = c_t + x_t + k_{t+1}. \tag{11}$$

Since we have assumed that only young agents save, capital formation is determined by

$$k_{t+1} = s_t = w_t - c_t. \tag{12}$$

3 Growth without Bubbles

3.1 The Dynamic System

In view of the life-time budget constraint (5) and the saving equation (12), we see that $x_{t+1} = R_{t+1}(w_t - c_t) = R_{t+1}k_{t+1}$. Thus from (9) it holds that

$$x_t = Rk_t. \tag{13}$$

By use of (9) , (10), (12) and (13), we may rewrite (7) as follows:

$$c_t = (\beta R)^{-\frac{1}{\sigma+\gamma\theta(1-\sigma)}} \left(\frac{Rk_t}{c_{t+1}}\right)^{-\frac{\theta(1-\sigma)(1-\gamma)}{\sigma+\gamma\theta(1-\sigma)}} Rk_{t+1}.$$

Hence, denoting $z_t = c_t/k_t$ and $G_t = k_{t+1}/k_t$, we can rewrite the above equation as

$$z_t = B(z_{t+1})^{\phi} G_t^{1+\phi}, \tag{14}$$

where

$$B \equiv \beta^{-\frac{1}{\sigma+\gamma\theta(1-\sigma)}} R^{\frac{(\sigma-1)(1+\theta-2\gamma\theta)}{\sigma+\gamma\theta(1-\sigma)}}, \quad \phi \equiv \frac{\theta(1-\sigma)(1-\gamma)}{\gamma\theta + (1-\gamma\theta)\sigma}.$$

Equation (12) presents $k_{t+1}/k_t \equiv G_t = (1-\alpha)A - z_t$, and hence from (14) we obtain

$$z_{t+1} = B^{-\frac{1}{\phi}} z_t^{\frac{1}{\phi}} \left[(1-\alpha)A - z_t\right]^{-\left(1+\frac{1}{\phi}\right)}. \tag{15}$$

This equation depicts the dynamic behavior of ratio between young agents' consumption and capital stock, $z_t\,(\equiv c_t/k_t)$.

3.2 Balanced-Growth Equilibrium

The balanced-growth equilibrium holds when c_t and k_t change at the same rate, so that z_t stays constant over time. First, note that if there is no consumption external effect, i.e. $\phi = 0$, (14) is written as $z_t = \beta^{-1/\sigma} R^{(\sigma-1)/\sigma}\left[(1-\alpha)A - z_t\right]$. Consequently, the gross rate of capital accumulation in the absence of consumption externalities is given by

$$\hat{G} = \frac{(1-\alpha)A.}{1 + R^{(\sigma-1)/\sigma}\beta^{-1/\sigma}}. \tag{16}$$

It is also to be noted that if there are only intragenerational external effects, i.e. $\theta \neq 0$ and $\gamma = 1$, then $\phi = 0$ and $B \equiv \beta^{\frac{1}{\sigma+\theta(1-\sigma)}} R^{\frac{(1-\theta)(\sigma-1)}{\sigma+\theta(1-\sigma)}}$. Thus the dynamic behavior of the economy is the same as that without consumption externalities. The only difference is that the balanced-growth rate in the presence of externalities is given by

$$G^E = \frac{(1-\alpha)A.}{1 + R^{\frac{(1-\theta)(\sigma-1)}{\sigma+\theta(1-\sigma)}} \beta^{-\frac{1}{\sigma+\theta(1-\sigma)}}}. \tag{17}$$

Comparing (17) with (16) , we find that the growth effect of intragenerational consumption externalities depends on the parameter values. In the standard case where jealousy and conformism prevail (that is $\theta > 0$ and $\sigma > 1$), as long as $R \geq 1$ and $0 < \beta < 1$, we find the following[7]:

$$R^{\frac{(1-\theta)(\sigma-1)}{\sigma+\theta(1-\sigma)}} \beta^{-\frac{1}{\sigma+\theta(1-\sigma)}} < R^{\frac{\sigma-1}{\sigma}} \beta^{-\frac{1}{\sigma}}, \tag{18}$$

implying that $G_E > \hat{G}$. Similarly, if consumers have admiration as well as conformism (i.e. $\theta < 0$ and $\sigma < 1$), we also obtain (18). As a result, in the absence of intertegenerational externalities, introducing the consumers' conformism yields the same effect as that produced by an increase in the elasticity of intergenerational substitution in consumption, $1/\sigma$. In contrast, if consumers are anti-conformists, i.e. $\theta(1-\sigma) > 0$, then (18) fails to hold and thus the economy with consumption externalities grows slower than the economy without consumption externalities.[8]

In the general case where there are intergenerational consumption externalities, on the balanced-growth path (15) gives

$$B^{-\frac{1}{1+\theta}-} z^{\frac{1-\phi}{1+\phi}} = (1-\alpha) A - z. \tag{19}$$

It is easy to see that if $(1-\phi)/(1+\phi) > 0$, then (19) has a unique solution for $z \in$

[7] Note that $\frac{(1-\theta)(\sigma-1)}{\sigma+\theta(1-\sigma)} - \frac{\sigma-1}{\sigma} = \frac{\theta(1-\sigma)}{\sigma[\sigma+\theta(1-\sigma)]}$ and $\frac{1}{\sigma+\theta(1-\sigma)} - \frac{1}{\sigma} = -\frac{\theta(1-\sigma)}{\sigma[\sigma+\theta(1-\sigma)]}$. Hence, given the assumptions (4), $0 < \gamma < 1$, $R > 1$ and $0 < \beta < 1$, we confirn that

$$\text{sign}\left\{R^{\frac{(1-\theta)(\sigma-1)}{\sigma+\theta(1-\sigma)}} \beta^{-\frac{1}{\sigma+\theta(1-\sigma)}} - R^{\frac{\sigma-1}{\sigma}} \beta^{-\frac{1}{\sigma}}\right\} = \text{sign}\, \theta(1-\sigma).$$

[8] This result also holds for the case of representative agent economy. In the representative agent economy, the household's objective functional is given by

$$U = \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t E_t^{-\theta}\right)^{1-\sigma}}{1-\sigma},$$

where $E_t = c_t$ holds in equilibrium. Given the same production structure, it is easy to see that the balanced-growth rate in the representative economy with consumption externalities is

$$\frac{k_{t+1}}{k_t} = \frac{c_{t+1}}{c_t} = (\alpha A \beta)^{\frac{1}{\alpha+\theta(1-\sigma)}},$$

in which it is assumed that $\alpha A \beta > 1$. Therefore, if the agents are conformists, i.e. $\theta(1-\sigma) < 0$, then the balanced-growth rate in the presence of consumption externalities is higher than that without externalities. It is also to be noted that, as pointed out by Liu and Turnovsky (2005) and others, in the representative agent economy, the balanced-growth path with externalities satisfies the social optimum condition. This is because if the planner internalized the external effect so that it maximizes $\sum_{t=0}^{\infty} \beta^t \frac{c_t^{(1-\sigma)(1-\theta)}}{1-\sigma}$, the balanced-growth rate is the same as the above with external effects.

$(0, (1-\alpha) A)$. Conversely, if $(1-\phi)/(1+\phi) < 0$ and if (19) has a solution, then there are two feasible solutions at most: see Figures 1 (a) and (b). Hence, we conclude:

Proposition 1 *There exists a unique, feasible balanced growth path if $-1 < \phi < 1$. Otherwise, there are two balanced-growth equilibria at most.*

Given the restrictions in (4), it is easy to confirm that if consumers' preferences exhibit jealousy and conformism (KUJ), i.e. $\theta > 0$ and $\sigma > 1$, then it holds that $-1 < \phi < 0$. Therefore, under the standard assumptions on external effects, the balanced-growth path is uniquely determined. If conformism is associated with admiration ($\theta < 0$ and $\sigma < 1$), then ϕ can be strictly less than -1, so that dual balanced-growth equilibria may exist. If consumers are anti-conformists (i.e. $\theta(1-\sigma) > 0$), ϕ has a positive value. In order to hold $\phi < 1$, we should assume that $\theta(1-\sigma)(1-2\gamma) - \sigma < 0$. Thus if $\gamma > 1/2$, then $\phi < 1$. If the intergenerational consumption external effects dominate so that $\gamma < 1/2$, we may have $\phi > 1$.

From (15) and (19) we see that

$$\begin{aligned} \left.\frac{dz_{t+1}}{dz_t}\right|_{z_t=z} &= \frac{1}{\phi} + \left(1+\frac{1}{\phi}\right)\frac{z}{(1-\alpha)A - z} \\ &= \frac{1}{\phi} + \left(1+\frac{1}{\phi}\right) B^{\frac{1}{1+\phi}} z^{\frac{2\phi}{1+\phi}}. \end{aligned}$$

Thus if $0 < \phi < 1$, then $dz_{t+1}/dz_t > 1$. If $-1 < \phi < 0$, then $dz_{t+1}/dz_t < -1$. This means that in both cases the dynamic system is unstable around the steady state. Since $z_t \, (= c_t/k_t)$ is not a predetermined variable, local instability and uniqueness of the steady state mean that the balanced-growth equilibrium is globally determinate and hence the economy always stays on the balanced-growth path. Figures 2 (a) and (b) depict the cases of $0 < \phi < 1$ and $-1 < \phi < 0$, respectively. The figures show that the dynamic system exhibits global determinacy for $\phi \in (-1, 1)$.

When $\phi > 1$, there may exist two steady states. As Figure 2 (c) shows, the steady state with a lower z is stable and the other with a higher z is unstable. Hence, the balanced growth path with a higher growth rate is locally indeterminate, while one with a lower growth rate is locally determinate. If $\phi < -1$, the graphical representation of (15) is given by Figure 2 (d). This figure reveals that the steady state with a higher z again satisfies local determinacy, because it holds that $-1 < 1/\phi + (1 + 1/\phi)\, z/\left[(1-\alpha) A - z\right]$. In this situation, both balanced-growth paths can be realized for any initial value of z_t.

Summing up the above discussion, we obtain:

Proposition 2 *If $-1 < \phi < 1$, then the economy exhibits global determinacy. If $\phi > 1$, the balanced-growth equilibrium with a higher growth rate is locally in determinate, while that with a lower growth rate is determinate. In the case of $\phi < -1$, the balanced-growth path with a lower growth rate is either locally determinate or indeterminate.*

4 Growth and Bubbles

4.1 Equilibrium Dynamics with Bubbles

We now assume that the government issues non-interest-bearing asset. This asset is intrinsically worthless in the sense that it serves neither for consumption nor for production activities. For convenience of exposition, we call this asset money. Since 'money' in our economy does not present any intratemporal transaction service, it may be called 'pure asset bubbles' as well.[9] We assume that the initial old receives a given stock of money, M_0. Then they sell M_0 to the young in period 1. We also assume that the government changes money supply at a constant rate of $\mu - 1$ in each period. Thus the nominal stock of money supply changes according to

$$M_{t+1} = \mu M_t.$$

Note that if $\mu < 1$, the government contracts money supply. We assume that the newly created money is distributed to the young agents as lump-sum transfers. This means that the flow budget constraint for the government is given by

$$M_{t+1} - M_t = p_t \tau_t,$$

[9] If there is no capital, money can serve as a medium of intertemporal exchange. Since capital plays a role of store of value, 'money' in our economy may has a positive value only when the rate of return dominance fails to hold, that is, the declining rate of price of money in terms of final goods equals the rate of return to capital.

where p_t is the price of final goods in terms of money and τ_t is the real transfer for the young households (if τ_t is negative, it represents a lump-sum tax levied on the young generation). Letting $m_t = M_t/k_t p_t$, the government's budget constraint is expressed as

$$\tau_t = (\mu - 1)\, m_t k_t. \tag{20}$$

We have assumed that the newly created money is distributed to the young agents, so that the budget constraint for the young is now replaced with

$$c_t + s_t = w_t + \tau_t. \tag{21}$$

The saving of the young, s_t, is spent for physical capital investment and money holding, and hence we obtain:[10]

$$s_t = k_{t+1} + \frac{M_{t+1}}{p_t}. \tag{22}$$

Since holding money does not bear interest income, the gross rate of return to money holding, p_t/p_{t+1}, should be equal to the gross rate of return to capital:

$$\frac{p_t}{p_{t+1}} = R. \tag{23}$$

The real rate of return to capital is fixed under our assumption of Ak technology and thus the rate of change in monetary price of goods stays constant over time. As a consequence, from (23) the flow budget constraint for the t-th generation in their old age is $x_{t+1} = Rs_t = R\left(k_{t+1} + M_{t+1}/p_t\right)$. Again, the equilibrium condition for the final good market is $y_t = c_t + x_t + k_{t+1}$. Substituting $y_t = w_t + Rk_t$, $c_t = w_t + \tau_t - s_t$, $x_t = Rs_{t-1}$ and (22) into the commodity-market equilibrium condition, we obtain

$$w_t + Rk_t = w_t + \tau_t - \left(k_{t+1} + \frac{M_{t+1}}{p_t}\right) + R\left(k_t + \frac{M_t}{p_{t-1}}\right) + k_{t+1}.$$

Hence, by use of (20), we find that the above equation yields $m_t k_t = \mu R m_{t-1} k_{t-1}$, which

[10] The young agents' savings are used for investment and money holding. Since the young genration's money holding consists of the existing stock of money M_t plus newly distributed one, $M_{t+1} - M_t$, it holds that

$$s_t = k_{t+1} + \frac{M_t}{p_t} + \frac{M_{t+1} - M_t}{p_t} = k_{t+1} + \frac{M_{t+1}}{p_t}.$$

gives the following:[11]

$$m_{t+1} = \frac{\mu R}{G_t} m_t. \tag{24}$$

Difference equation (24) describes dynamics of the ratio of real money balances and capital.

In order to derive the motion of consumption-capital ratio, z_t, we use (7) to derive

$$c_t = (\beta R)^{-\frac{1}{\sigma+\gamma\theta(1-\sigma)}} \left(\frac{x_t}{c_{t+1}}\right)^{-\phi} x_{t+1}. \tag{25}$$

Substituting $x_{t+1} = R(k_{t+1} + M_{t+1}/p_t)$ and $x_t = R\left(k_t + M_t/p_{t-1}\right)$ into the above and arranging the terms, we obtain

$$\frac{c_t}{k_t} \equiv z_t = B\left(\frac{k_t + M_t/p_{t-1}}{c_{t+1}}\right)^{-\phi} \left(G_t + \mu m_t\right),$$

where $\left(k_t + M_t/p_{t-1}\right)/c_{t+1}$ can be expressed as

$$\frac{k_t + M_t/p_{t-1}}{c_{t+1}} = \frac{1}{z_{t+1}G_t} + m_t \frac{p_t}{p_{t-1}} \frac{k_t}{k_{t+1}} \frac{k_{t+1}}{c_{t+1}} = \frac{1}{z_{t+1}G_t}\left(1 + \frac{m_t}{R}\right).$$

Hence, we express (25) as follows:

$$z_t = B\left[\frac{1}{z_{t+1}G_t}\left(1 + \frac{m_t}{R}\right)\right]^{-\phi} \left(G_t + \mu m_t\right). \tag{26}$$

From (20), (21) and (22), the gross rate of capital accumulation is given by

$$\frac{k_{t+1}}{k_t} \equiv G_t = (1-\alpha) A - m_t - z_t. \tag{27}$$

Using (26) and (27), we obtain:

$$z_{t+1} = B^{-\frac{1}{\phi}} z_t^{\frac{1}{\phi}} \left(1 + \frac{m_t}{R}\right) \frac{\left[(1-\alpha) A + (\mu - 1) m_t - z_t\right]^{-\frac{1}{\phi}}}{\left[(1-\alpha) A - m_t - z_t\right]}, \tag{28}$$

which describes the motion of $z_t \left(= c_t/k_t\right)$.

The dynamic equations (24) and (27) yield:

$$m_{t+1} = \frac{\mu R m_t}{(1-\alpha) A - m_t - z_t}. \tag{29}$$

To sum up, a complete dynamic system with consumption externalities is presented by a pair of difference equations (28) and (29). Notice that if $m_t = 0$ for all $t \geq 0$, our dynamic system reduces to (15) which depicts the dynamics of the bubble-free economy.

[11] Dynamic equation (24) can be directly derived by manipulating (23) and using the definition of m_t, However, it does not come from the definition of variable but from the equilibrium and optimization conditions as shown in the main text.

4.2 Balanced-Growth Characterization in the Absence of Consumption Externalities

First, let us review the main results obtained in the model without consumption externalities. Since it holds that $\phi = 0$ in the absence of consumption external effects, (28) reduces to

$$z_t = \frac{\beta^{-1/\sigma} R^{(\sigma-1)/\sigma}}{1+\beta^{-1/\sigma} R^{(\sigma-1)/\sigma}} [A(1-\alpha) - m_t].$$

Substituting the above into (27) presents

$$G_t = \frac{A(1-\alpha)}{1+\beta^{-1/\sigma} R^{(\sigma-1)/\sigma}} - m_t. \tag{30}$$

By use of the above and (24), we obtain:

$$m_{t+1} = \frac{\mu R m_t}{\left[\frac{A(1-\alpha)}{1+\beta^{-1/\sigma} R^{(\sigma-1)/\sigma}} - m_t\right]}.$$

Since the balanced-growth rate in the absence of bubble is given by (16), the motion of m_t is described by

$$m_{t+1} = \frac{\mu R m_t}{\hat{G} - m_t}. \tag{31}$$

The dynamic equation (31) shows that there may exist two steady states: the non-monetary steady state with $m = 0$ and the monetary steady state with a positive level of m.

If money supply is constant ($\mu = 1$), then (31) becomes

$$m_{t+1} = \frac{R m_t}{\hat{G} - m_t}.$$

As claimed by Grossman and Yanagawa (1993), the existence of balanced-growth equilibrium with a positive, steady-state value of m_t requires that

$$\left.\frac{dm_{t+1}}{dm_t}\right|_{m_t=0} = \frac{R}{\hat{G}} < 1. \tag{32}$$

Namely, the economy has a balanced-growth path that sustains bubbles if $\hat{G} > R\,(= \alpha A)$, i.e. the balanced-growth rate of the bubble-free economy exceeds the real interest rate. In addition, equation (30) means that if $\mu = 1$, then $G_t = \hat{G} - m$ and $G_t = R$. Therefore, the balanced-growth path with bubbles attains a lower steady-growth rate than that realized in the economy without bubbles.

If $\mu \neq 1$, the necessary and sufficient condition for the presence of sustainable bubble in the balanced-growth equilibrium is:

$$\left.\frac{dm_{t+1}}{dm_t}\right|_{m_t=0} = \frac{\mu R}{\hat{G}} < 1.$$

This means that the equilibrium with bubble can be sustained in the steady state if and only if

$$\hat{G} > \mu R. \tag{34}$$

As a result, when $\mu < 1$ so that the government contracts the nominal stock of money, there exists a feasible steady state with bubbles even if the balanced-growth rate in the bubble-free economy, $\hat{G}$, is strictly lower than the real interest rate, R. Futagami and Shibata (1999) confirm this result in the context of an infinitely-lived overlapping generations model with population growth. Note that from $R = \alpha A$ and (16) the existence condition (34) is fulfilled if the parameters involved in the model satisfy

$$\frac{1}{\mu} > \frac{\alpha}{1-\alpha}\left[1 + \beta^{-\frac{1}{\sigma}}\left(\alpha A\right)^{\frac{\sigma-1}{\sigma}}\right]. \tag{35}$$

In our simple setting, (35) is the necessary and sufficient condition for the presence of steady state with asset bubbles.

Condition (34) means that the non-monetary steady-state where $m_t = 0$ is locally stable, while the monetary steady state with a positive m is locally unstable. Therefore, the monetary balanced-growth path locally satisfies determinacy of equilibrium and the non-monetary balanced-growth path holds local indeterminacy. Since the initial level of $m_t\ (= M_t/k_t p_t)$ is not predetermined, the economy exhibits global indeterminacy.[12]

4.3 Effects of Consumption Externalities

When discussing the effects of consumption externalities in our setting, we should point out again that if there are no intergenerational externalities, the main conclusion shown in Section 4.2 still holds. As was pointed out in Section 4.2, if asset bubbles emerge in an economy without consumption externalities, the balanced-growth rate should be

[12] The global indeterminacy means that in each moment the economy may stay either at the monetary steady state or at the non-monetary steady state. In addition, the economy can be on a transition path that converges to the non-monetary steady state.

lower than one sustained in the corresponding bubble-free economy. This is because the balanced-growth rate in the bubble economy is μR and the necessary and sufficient condition for sustaining bubbles in the long-run equilibrium is $\hat{G} > \mu R$. In words, a part of saving is used for purchasing money (bubbles) rather than capital so that capital formation is inevitably decreased. Such a conclusion holds in the presence of consumption externalities as well, if the external effects are only intragenerational. If there is no intergenerational externalities ($\gamma = 1$), it is easy to show that the dynamic behavior of the economy is summarized as

$$m_{t+1} = \frac{\mu R m_t}{G^E - m_t},$$

where G^E is given by (17). Therefore, the necessary and sufficient condition for sustaining a steady state with bubbles is $G^E > \mu R$, implying that the bubbly steady state attains a lower rate of balanced growth than that in the bubble-free economy.

In the general case where both intragenerational and intergenerational externalities exist, from (28) and (29) the balanced-growth path in the presence of bubbles is characterized by the following conditions:

$$z^{1-\frac{1}{\phi}} = B^{-\frac{1}{\phi}} \left(1 + \frac{m}{R}\right) \frac{\left[(1-\alpha) A + (\mu - 1) m - z\right]^{-\frac{1}{\phi}}}{\left[(1-\alpha) A - m - z\right]}, \tag{36}$$

$$(1-\alpha) A - m - z = \mu R, \tag{37}$$

where z and m respectively denote the steady-state values of z_t and m_t. Equations (36) and (37) can be combined in the following manner:

$$(1-\alpha) A - m - \mu R = B^{-\frac{1}{\phi-1}} (\mu R)^{-\frac{\phi}{\phi-1}} \left(1 + \frac{m}{R}\right)^{\frac{\phi}{\phi-1}} \left[\mu R + \mu m\right]^{-\frac{1}{\phi-1}}.$$

This equation gives the steady-state value of m:

$$m = \frac{(1-\alpha) A - \mu R - B^{-\frac{1}{\phi-1}} (\mu R)^{-\frac{\phi+1}{\phi-1}}}{1 + B^{-\frac{1}{\phi-1}} \mu^{-\frac{\phi+1}{\phi-1}} R^{-\frac{2\phi}{\phi-1}}}. \tag{38}$$

Using (37) and (38), we obtain the steady-state value of z:

$$z = \frac{B^{-\frac{1}{\phi-1}} \mu^{-\frac{\phi+1}{\phi-1}} R^{-\frac{2\phi}{\phi-1}} \left[(1-\alpha) A + R - \mu R\right]}{1 + B^{-\frac{1}{\phi-1}} \mu^{-\frac{\phi+1}{\phi-1}} R^{-\frac{2\phi}{\phi-1}}}. \tag{39}$$

Thus a feasible balanced-growth path with a positive m exists if the numerator of the right hand side of the above equation has a positive value, which is given by

$$\mu a A + B^{-\frac{1}{\phi-1}} (\mu \alpha A)^{-\frac{\phi+1}{\phi-1}} < (1-\alpha) A, \tag{40}$$

where $B \equiv \beta^{-\frac{1}{\sigma+\gamma\theta(1-\sigma)}} (\alpha A)^{\frac{(\sigma-1)(1+\theta-2\gamma\theta)}{\sigma+\gamma\theta(1-\sigma)}}$. Equation (39) shows that z has a positive value if $(1-\alpha) A + R - \mu R = A - \mu\alpha A > 0$, which is fulfilled under (40). It is also to be noted that when $\theta = 0$ (so that $\phi = 0$), condition (40) reduces to (35). To sum up, we have shown:

Proposition 3 *If (40) is statisfied, there is a unique, feasible balanced growth equilibrium with bubbles.*

When there are intergenerational external effects in consumption, introducing bubbles does not always lower the balanced-growth rate. To see this, first note that the steady-state condition in the bubble-free economy given by (19) and $G = (1-\alpha) A - z$ present

$$G = (1-\alpha) A - B^{\frac{1}{1-\phi}} G^{\frac{1+\phi}{1-\phi}},$$

which determines the balanced-growth rate of the real economy with intergenerational and intragenerational consumption externalities. Define

$$F(G) = G + B^{\frac{1}{1-\phi}} G^{\frac{1+\phi}{1-\phi}} - (1-\alpha) A. \tag{41}$$

Then the balanced-growth rate without bubbles is a solution of $F(G) = 0$. Proposition 1 states that if $\phi \in (-1,1)$, then $F(G) = 0$ has a unique positive solution. Additionally, $F(G)$ is monotonically increasing in G for $\phi \in (-1,1)$: see Figure 3 (a). Remember that in the case of constant money supply the balanced-growth rate with bubbles is $\mu R (= \mu\alpha A)$. We find that under the existence condition (40) the following holds:

$$F(\mu\alpha A) = \mu\alpha A + B^{-\frac{1}{\phi-1}} (\mu\alpha A)^{\frac{1+\phi}{1-\phi}} - (1-\alpha) A < 0.$$

This means that the balanced-growth rate with bubble is strictly less than that attained in the bubble-free economy.

If $\phi \notin [-1,1]$ and if there are two steady states, the graph of $F(G)$ is like Figure 3 (b). The figure indicates that the balanced-growth rate of the economy with bubbles is in between the two growth rates attained in the long-run equilibrium without bubbles. Therefore, if the economy without bubbles stays on the balanced-growth path with a lower growth rate, the emergence of bubbles may raise the long-term growth rate. Conversely, if the initial position is the steady state with a higher growth rate, as in the standard case, introducing bubble reduces the balanced growth rare. As a consequence, we have confirmed the following:

Proposition 4 *If $\phi \in (-1, 1)$, the balanced-growth rate with bubbles is lower than that attained in the bubble-free economy. If $\phi \notin [-1, 1]$, the balanced-growth rate in the presence of bubbles is higher (resp. lower) than that of the corresponding bubble-free economy staying on the steady state with a lower (resp. higher) growth rate.*

The above proposition indicates that the welfare implication of bubbles may be different from that obtained in the standard models without consumption externalities. In the standard setting the economy with bubbles yields a lower balanced-growth rate than in the bubble-free economy, and hence the emergence of bubbles cannot be Pareto improving. Namely, except for the initial old whose welfare is increased by receiving money which can be sold to the young, all the subsequent generations obtain lower utilities due to a permanent reduction in the growth rate of consumption. By contrast, in our model with intergenerational consumption externalities, in the case of dual steady-states the emergence of bubbles may raise the balanced growth rate, implying that the existence of asset bubbles can be Pareto improving in the sense that all the generation can attain higher welfare when bubbles emerge.

It is also worth emphasizing that intergenerational consumption externalities may affect the dynamic behavior of the economy. As mentioned in Section 4.2, either if there is no consumption externality or if there is only intragenerational external effect, the bubble-free steady state is locally indeterminate, while the steady state with bubbles satisfies local determinacy. If there are intergenerational consumption externalities, then dynamic system involves two state variables, $m_t \, (= M_t/p_t k_t)$ and $z_t \, (= c_t/k_t)$. Since both price p_t and consumption c_t are unpredetermined variable in the perfect-foresight equilibrium, we cannot specify the initial values of m_t and z_t. In this sense, the dynamic system exhibits global indeterminacy, because every point in the feasible region of m_t- z_t space can be an equilibrium. In addition, if we focus on the local behavior of the economy near the balanced-growth path, the local determinacy of a balanced-growth equilibrium requires that the dynamic equation system consisting of (28) and (29) satisfies total instability near the steady state solution. Otherwise, the steady-equilibrium exhibits either a saddle point or a sink so that there are a continuum of converging path around the bakanced-growth path. Hence, there may exist expectation-driven, sunspot-type fluctuations near these long-run equilibrium with local indeterminacy.

The above argument means that if $\phi \in (-1, 1)$ so that there is a unique bubble-free (i.e. $m = 0$) steady state, the economy can stay at the bubble-free or at the bubbly balanced path or on a transitional path towards the steady state that is either a saddle point or a sink. Similarly, if $\phi \notin [-1, 1]$ and there are two steady states without bubbles, the economy may reach one of three steady states: one with bubbles and other two without bubbles. In any case, the equilibrium path is globally indeterminate. We thus arrive the following:

Proposition 5 *In the case of $\phi \in (-1, 1)$, there are one bubble free and one bubbly balanced-growth paths. If $\phi \notin [-1, 1]$, the economy involves three balanced-growth paths at most: one is associated with bubbles and other two are bubble free. Regardless of the value of ϕ, the economy exhibits global indeterminacy of equilibrium.*

Finally, we should point out that the main results presented so far are established under the assumption of constant money growth. If money supply is endogenously determined, the balanced-grwoth rate in the presence of bubbles is not simply given by the constant value of μR. As for a simple example, suppose that newly created money is not distributed to the young agents but the government consumes final goods by financing printing money. The government budget constraint is now replaced with

$$M_{t+1} - M_t = p_t \gamma Y_t, \quad 0 < \gamma < 1,$$

where γ denotes the government's conumption share of income. In this case the gross growth rate of nomonal money stock is

$$\mu_t = 1 + \frac{\gamma A}{m_t}.$$

Since (23) should be satisfied and since m_t stays constant in the steady state, the balanced grwoth rate is given by $G = \mu R = \left(1 + \frac{\gamma A}{m}\right) \alpha A$. The balanced grwoth rate now depends on the steady-state value of m which may be affected by the presence of consumption externalities. In fact, in this simple money supply regime, we can show that the balanced-growth equilibrum with bubbles may not be uniquely determined and that the balaned growth rate is directly linked to the degree of consumption externalities.[13]

5 Conclusion

In the context of a simple model of endogenous growth, this paper has addressed economic implication of consumption externalities in an overlapping generations economy. In particular, we have focused on the existence of sustainable asset bubbles in the long-run growth equilibrium. The main finding of our study is that the presence of intergenerational consumption externalities may play a relevant role. If the consumption external effects are intragenerational alone, that is, each generation are concerned with other agents' consumption behaviors in the same cohort, then the external effects are only quantitative ones. Thus in this case growth and welfare implications of bubbles are essentially the same as those obtained in the economy without consumption externalities. We have, however, shown that if the intergenertaional external effect in consumption exist, the growth and welfare effects of bubbles could be considerably different from these established in the absence of bubbles. Our discussion has demonstrated that consumption externalities may yield more fundamental effects on the equilibrium dynamics and policy implication in dynamic economy with overlapping generations than in the representative-agent economies.

In this paper, we have analyzed a stylized model with or without non-fundamental asset bubbles. It is interesting to extend our analytical framework to discuss more general situations such that money also serves for intratemporal transactions, labor supply is endogenously determined[14] and that bubbles may be related to fundamentals[15]. Additionally, introducing the interest-bearing government debt into our model would be a useful extension.

[13] In the context of a continuous-time OLG model, Mino and Shibata (2000) show that the simple endogenous money supply rule mentioned above may generate multiple balanced growth paths.

[14] For example, Mino and Shibata (2000) analyze an endogenously growing OLG model in which money is introduced in the form of money-in-the utility function. Itaya and Mino (2005) examine the relation between preference structure and monetary growth. Adding consumption externalities to those frameworks would yield richer results than those obtained in our model of pure bubble.

[15] See, for example, Ventura (2005).

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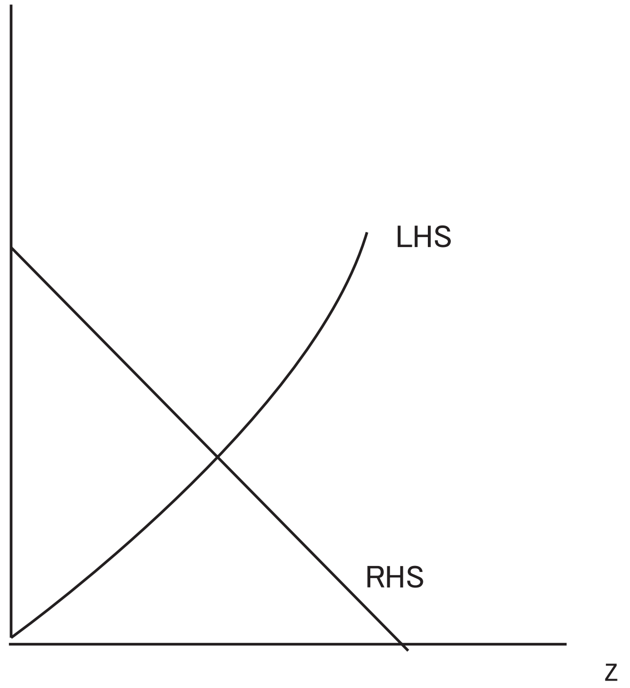
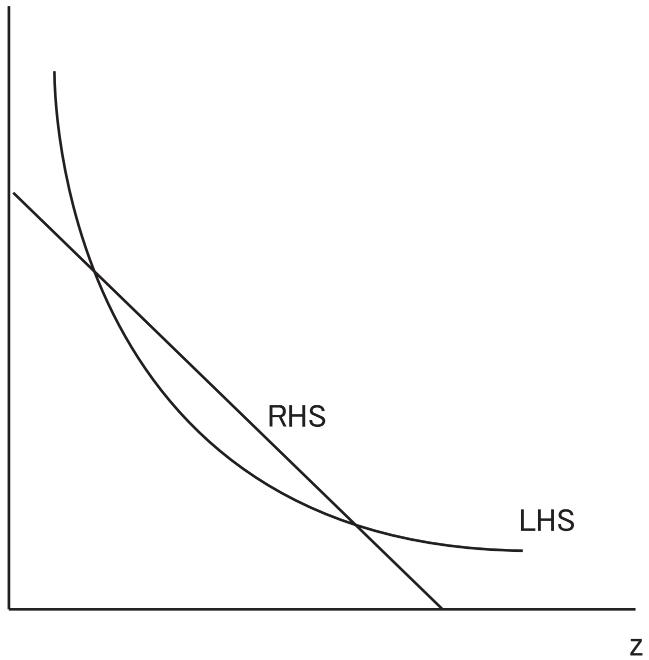


(a) $|\phi| < 1$



(b) $|\phi| > 1$

Figure 1: Existence of Balanced-Growth Equilibrium

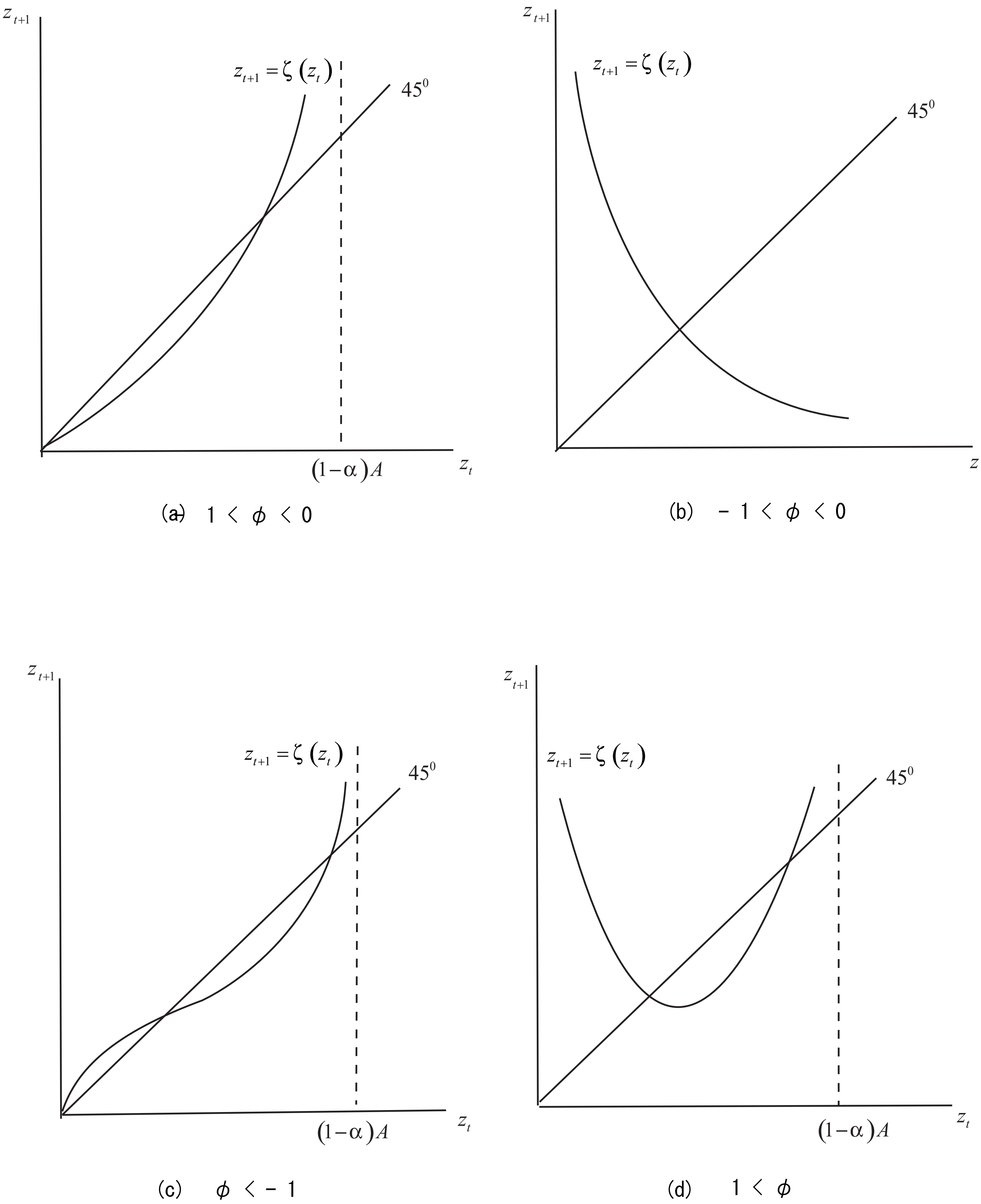


Figure 2 : shapes of $\zeta(z_t)$ function

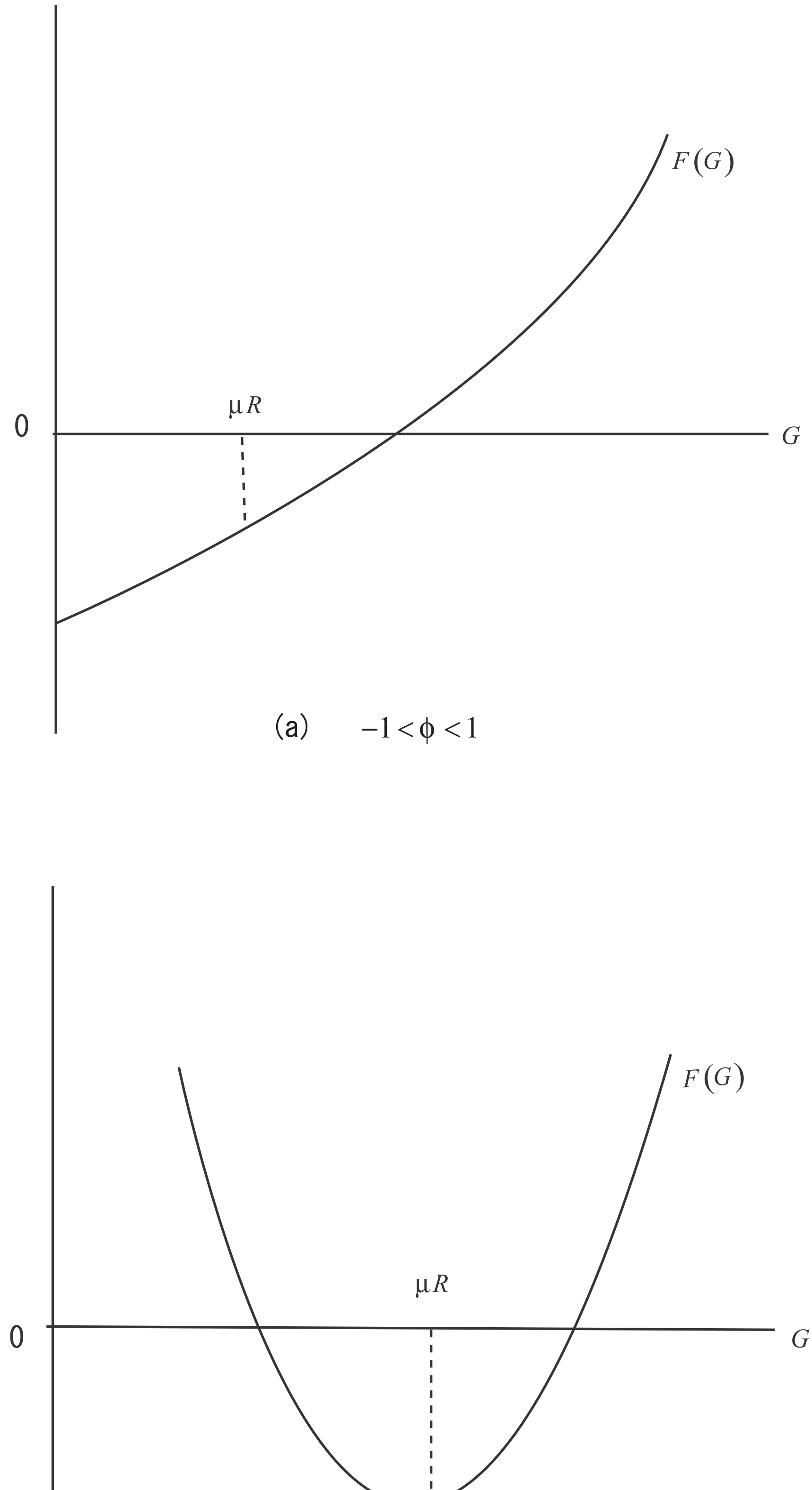


Figure 3: the balanced-growth rates with and without bubbeles