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Abstract

This paper provides a dynamic general equilibrium framework to investigate how organizations change the modes to govern transactions over time. We show that the agency problem becomes less serious when the economy is developed well so that large market size favors decentralized organizations having more specialization. We then show that different organizational modes endogenously emerge even in the same economy and cause endogenous process of economic development.

Keywords: Economic Development, Incentive Contracts, Moral Hazard, Specialization
JEL Classification Numbers: D82, G30, L22, O11

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1 Introduction

In his classical book of economics *Wealth of Nations* Adam Smith discussed that specialization or division of labor is one of the most important engines to enhance the productivity of labor but the extent of specialization is limited by market size (See Smith (1776, Chapter 1 and 3)). In this paper we will elaborate these Adam Smith's views further in the following three directions and investigate how dynamical changes of organizational modes to manage the division of labor cause the endogenous process of economic development.

First, the effects of market size on specialization are not in one way. Rather the market size affects not only the degree of specialization but also is affected by how specialization proceeds in the economy. This is because specialization enhances the labor productivity and hence raises labor demand. This in turn results in higher wage income to make consumption level higher and thus make market size actually larger. Thus the degree of specialization and market size interact with each other through such general equilibrium effect.

Second, specialization (division of labor) is not only technological but also organizational issue such as how total production process is divided into specialized tasks, how these tasks are delegated to different agents and how they are motivated to work hard for their specialized tasks. More decentralized organizations involve more specialization so that different tasks are delegated to different and specialized agents. Such specialization contributes to expanding the technological productivity of organizations. However, agents who are specialized on some tasks may obtain the powers and discretion to control their tasks in their own interests and hence may not work in the interests of the principal of an organization. In particular their efforts exerted for tasks may not be observable to the principal. This sort of asymmetric information between the principal (capitalist) and delegated agents limits specialization and decentralization because the principal faces the trade–off between incentives and rent extraction: Some *rent* must be left to delegated agent over his reservation payoff which could be otherwise obtained by his outside opportunity in the economy.

Third, the general equilibrium interactions between the market size and specialization are not static but rather dynamic through the change of organizational modes. When the market size changes over time, this may change the trade–off between specialization and agency problem which then causes the change of optimal organizational modes to balance such trade–off. This change of organizational modes has the dynamic feedback effects on the market size and hence the process of economic development.

We will integrate these issues in a unified framework and show that optimal organizational mode (degree of decentralization) depends on the development stages of the economy: When the economy is developed well so that the market wage is high enough, agents who work for the principal of an organization face better outside opportunity which could be otherwise obtained if they rejected the contracts offered by the current principals and worked elsewhere in the economy. Then, although the agents have informational advantage over the principal, they may obtain smaller rent when the economy is developed well. This makes the trade–off between incentives and rent extraction less serious so that it becomes more attractive for the principals to choose more decentralized organizations in which more tasks are specialized and delegated to agents.

However, the story does not end here because the general equilibrium feedback comes in: When the principals choose more decentralized organizations, higher degree of specialization will contribute to expanding the production level and hence labor demand in the economy wide, which then pushes up wage income and actually makes the outside opportunity of agents better.

This general equilibrium effect through the choice of organizational modes causes the diversion of the processes of economic development. If the principals choose decentralized organizations, they expect that specialization will proceed and hence result in higher economic development, which then actually makes decentralized organization attractive for them. On the other hand, if the principals choose centralized organizations, they expect that specialization will be limited and hence the economy will be less developed, which then actually makes centralized organization attractive for them. Hence multiple equilibrium paths arise even in the same economy: Some of them take the advantage of specialization and involve decentralized organizations but others lose the gains from specialization and involve centralized organizations. These different development paths emerge even when the economy starts from the same initial condition. [1]

These results might explain why there are significant differences in organizational modes to govern transactions across countries. Casual empiricism suggests the diversity of organizational structures across countries. For example it has been argued that the Japanese automobile industry is less vertically integrated than the U.S. counterpart.[2] It was also reported that

[1]Some papers focus on the related issues such as how contract (or organizational) designs affect and are affected by economic development. Esfahani and Mookherjee (1995) consider the Shapiro–Stiglitz type shirking model and show that both high powered and low powered contracts may emerge even in the same economy. Legros, Newman and Proto (2006) consider the endogenous growth model with organizational design in which a division of labor plays two different roles; enhancing innovation and monitoring workers by managers. Acemoglu, Aghion and Zilibotti (2002, 2006) investigate the endogenous growth model in which investment technology choice is endogenous and related to contract design and organizational choice such as vertical integration. See also Aghion, Griffith and Howitt (2006).

[2]See Aoki (1988) for a comparative study on the Japanese firms. See also Abraham and Taylor (1996) and Helper (1991) for the evidence about supplier systems and outsourcing decisions in U.S. industries.

in 1980s the Japanese electronics subcontractors had focused on fewer customers than the U.K. firms.[3] Furthermore the U.S. electronics companies like Apple and Dell recently have limited their main businesses to marketing and sales by outsourcing their production decisions to independent contract manufacturers while Asian electronics companies like Sony, Toshiba and Samsung have maintained the strategy of in–house productions (See Berger (2005)).

Second, diversity of organizational modes is not only the phenomenon observed across countries but also even in the same country and industry. [4] To account for this stylized fact, we show that there exist the *mixed steady states* in which different organizational modes, centralized and decentralized organizations, *co–exist* even in the same economy. We then show that there exists equilibrium paths converging to such mixed steady states, and hence co–existence of different organizations persists even in the long run. This result will be also useful for understanding why different patterns to organize productions co–exist even in the same country and same industry.

Third, we show whether or not decentralized organization becomes dominant in the long run depends on the exogenous costs of starting up projects. Such costs may be higher not only when the investment technology of entrepreneurs is inefficient but also when severe regulations are imposed by government, when contracting costs are large and when credit markets are imperfect. We then show that a long run equilibrium is uniquely characterized by decentralized organization when such staring up cost is sufficiently small. However, when the staring up cost is large, the economy is more likely to be stuck in centralized organization equilibrium. Thus our result suggests that the severity and difficulty of staring up projects make organizational modes biased toward more centralization. This view is consistent with the recent empirical study by Acemoglu, Johnson and Mitton (2005, 2007) which focus on the determinants of vertical integration across countries by using a large data set. They found that countries with higher contracting costs and tighter regulation are more likely to be concentrated in vertically integrated industries. [5]

Furthermore our results shed lights on the dynamic relationship between business practices and economic development. In fact we show that the steady state with centralized organization corresponds to smaller economic development level than the steady state with decentralized organization.

[3] See Nishiguchi (Chapter 5 and 6, 1994) for this evidence.

[4] For example Berger (2005) reported that different companies in the U.S. microchip industry have adopted different strategies of organizing their productions.

[5] Acemoglu, Johnson and Mittion (2005) also reported the result that countries with less developed financial markets are more likely to be vertically integrated in the technology intensive or human capital intensive industries. Acemoglu, Johnson and Mittion (2007) emphasized the interactive effects of financial development and contracting costs on the propensity of vertical integration.

This suggests the testable implication that firms are more likely to rely on the business practices in the centralized and integrated manners when the economy is less developed. Khanna and Palepu (1997, 2000) provide the related evidence that group affiliated firms, which are based on common ownership and control, outperform focused, unaffiliated firms in emerging markets such as India. Moreover, as shown by Chandler (1962)'s famous analysis about U.S. business history, in the late 19th century U.S. firms became bigger through vertical integration and re–organized their corporate structures in the centralized way known as U–form (unitary form). However, these U.S. big firms shifted their corporate structures from U–from to more decentralized form known as M–form (multidivisional form) in the early 20th century as their focused businesses were expanded. [6] This U.S. business history might be consistent with our result that organizational modes become more decentralized as the market size becomes bigger.

The remaining sections are organized as follows: In Section 2 we will set up the basic model of the overlapping generations economy. In Section 3 we will characterize the equilibrium organizations and in Section 4 we will derive the dynamical system to govern the change of organizational modes. In Section 5 we will discuss the implications about contracting choices of intermediate inputs such as outsourcing and vertical integration. Section 6 concludes the paper.

2 Model

2.1 Overlapping Generations Economy

We consider the overlapping generations economy in which a continuum of new generation is born with a constant population size in each period. Time is discrete and extends over infinity ($t = 0, 1, 2, ...$). There is a single final good which is used both for consumption and investment. A newly born individual lives for two periods, say "young" and "old." In each generation there are two types of individuals, called *principals* and *agents*, with one unit measure and θ unit measure respectively. Each agent is endowed with one unit labor in each period of his life while each principal is endowed with no labor inputs. In the model there are three perfectly competitive markets, i.e., credit market, labor market and good market. Each individual is risk neutral and maximizes his or her utility when old (he or she can obtain utility only from old consumption).

There are two types of workers, *unskilled* and *skilled workers*. All young agents start as unskilled workers because they do not have any specific skills when young. Thus in period t each young agent inelastically supplies one

[6] In the early 20th century the U.S. market became geographically bigger via expanding railway networks. Such expansion of the market size pushed the U.S. big firms toward the change of organizational forms.

unit labor and earns the competitive market wage w_t. Since each agent obtains no utilities from consumption when young, he simply saves all his income w_t. Let ρ_{t+1} denote the (gross) interest rate paid in credit market in period $t+1$. Then each young agent who was born in period t will earn the interest income $\rho_{t+1}w_t$ when old (period $t+1$). There are two occupations for each old agent in period $t+1$: One is to supply one unit labor and earn the competitive market wage w_{t+1} as an unskilled worker again. The other option is to work as a *skilled worker* for managing a project, which we will explain more in details below. To be a skilled worker, each old agent needs to spend his labor endowment in adulthood for learning and acquiring the skills which are necessary and specific for the implementation of the project. Thus old agents consist of unskilled workers and skilled workers.

Each young principal can create projects by investing $\lambda > 0$ goods per project in one period advance. Here λ captures the parameter value affecting the costs of starting up a project. These costs become higher not only when the principal's investment technology is inefficient but also when institutions underlying the economy are not well developed so that more administrative costs are needed to start up a project. The latter may be due to severe regulations imposed by government and higher contracting costs in the economy (We will give further justification for λ later). Since each young principal has no incomes, she needs to access to credit market in order to finance these projects. If a young principal raises a project in period t, she will obtain stochastic returns from that project in the next period $(t+1)$. To implement a project in period $t+1$, each old principal needs to hire one skilled worker for managing the project as well as employ competitive unskilled workers for production at the market wage w_{t+1}.

In period $t+1$ each project which was financed in period t will yield stochastic returns. If the outcome is failure, then the project yields nothing. If the outcome is success, then the project yields the output Y by employing unskilled workers L from the competitive labor market according to the production function $Y = F(L)$. We will make the standard assumption that F is continuously differentiable, monotone increasing, $F' > 0$, and strictly concave, $F'' < 0$, with $F(0) = 0$ and Inada conditions $F'(0) = \infty$ and $F'(\infty) = 0$.

Since the labor market is perfectly competitive, the maximum profit of each successful project becomes

$$\pi(w) \equiv \max_{L \geq 0} F(L) - wL. \tag{1}$$

Here π is decreasing and convex, $\pi' < 0$ and $\pi'' > 0$. Let also $g(w)$ denote the corresponding labor demand, i.e., $F'(g(w)) \equiv w$.

2.2 Matching

At the beginning of each period the projects created in the previous period and old agents (potential skilled workers) are randomly matched with each other. Let K_{t+1} denote the number of total projects created by young principals in period t and hence used for production in period $t+1$. Then K_{t+1} projects and θ old agents are randomly matched with each other according to the matching function $m(K_{t+1}, \theta)$. Here we assume that m is increasing in its first argument and satisfies the following condition: $m(K, \theta) \leq \gamma\theta$ for all $K \geq 0$ for some $\gamma \in (0, 1)$. Also we assume that $m(0, \theta) = 0$. Then each project will succeed in matching an old agent with probability $\alpha(K_{t+1}) \equiv m(K_{t+1}, \theta)/K_{t+1}$ while each old agent will succeed in matching a project with probability $\beta(K_{t+1}) \equiv m(K_{t+1}, \theta)/\theta$.

The old agent who succeeded in matching a project can be a potential skilled worker who will handle the project he matched. Alternatively he can always reject to work as a skilled worker even when he matched a project. In that case such old agent will work as an unskilled worker to earn the market wage in the competitive labor market. On the other hand, any old agent who failed to match a project has nothing but to work as an unskilled worker.

2.3 Specialization (Division of Labor)

Each project has two tasks, called *task 1* and *task 2*, and costly effort is needed for each task to be completed. We assume that by some technological reasons task 1 of each project can be handled only by old principal. However, task 2 of a project can be delegated to a matched old agent, called *skilled worker*. Each project's outcomes, success and failure, depend on the efforts exerted for two tasks of the project. Effort choice is assumed to be binary, high effort (h) or low effort (l), for any task. We will denote by $a \in \{h, l\}$ the principal's effort exerted for task 1 of a project. Let also denote by $e_p \in \{h, l\}$ (resp. $e_m \in \{h, l\}$) the principal's (resp. the skilled worker's) effort for task 2 of a project when the principal (resp. the skilled worker) handles that task. Then we assume that each matched project will succeed with probability $P(a, e_i) \in (0, 1)$ but fail with probability $1 - P(a, e_i)$ respectively. We will here assume that $p_{hh} \equiv P(h, h) > p_{hl} = p_{lh} \equiv P(h, l) = P(l, h) > p_{ll} \equiv P(l, l) > 0$. Let also $\Delta p \equiv p_{hh} - p_{hl}$ and $\Delta q \equiv p_{hl} - p_{ll}$. We will then assume the diminishing returns: $\Delta q > \Delta p$.

When an old principal handles two tasks of a project together at a time, she will personally incur her effort cost as $c_p(a, e_p)$ where $c_p(h, h) = \tilde{c} > c_p(h, l) = c_p(l, h) = c > c_p(l, l) = 0$. On the other hand, when tasks 2 of a project is carried out by a matched old agent (skilled worker) (but task 1 must be handled by the principal herself), the skilled worker will personally incur his effort cost as $c_m(e_m)$ for task 2 of the project where

$c_m(h) = c > c_m(l) = 0$ while the principal will incur her effort cost as $c_p(e_p)$ for task 1 of the project where $c_p(h) = c > c_p(l) = 0$. [7] Here to simplify notation we are assuming that skilled worker's effort cost $c > 0$ is same as the principal's one. [8] We also assume that each skilled worker has necessary skills to implement one project and hence he is needed for implementation of one project even when he is not delegated task 2 of the project.

2.4 Organizational Modes

After the created projects matched old agents, each old principal decides what organizational modes she should choose for implementing matched projects and what contracts should be offered to the matched old agents who will work as skilled workers for managing the projects. In other words each old principal needs to decide whether or not task 2 of a matched project should be delegated to a matched old agent who will work as an skilled worker to choose costly effort $e_m \in \{h, l\}$ for that task.

We will call the organizational mode in which task 2 of a project is delegated to a skilled worker *decentralized organization* or simply *D–mode*. In D–mode old principal and skilled worker choose efforts $a \in \{h, l\}$ for task 1 and $e_m \in \{h, l\}$ for task 2 respectively. On the other hand we call the organizational mode in which both task 1 and 2 of a project are handled by the old principal herself *centralized organization* or simply *C–mode*. In C–mode only old principal chooses both efforts $a \in \{h, l\}$ for task 1 and $e_p \in \{h, l\}$ for task 2 of the project together.

We assume that the verifiable signals are only the realized outcomes of each project, i.e., success and failure. Then a contract offered to skilled worker should specify the payments to him contingent on the outcomes of the project which he is working on. Let $C_t^D = \{R_t^D, r_t^D\}$ denote the incentive contract used for a project under D–mode in period t where R_t^D (resp. r_t^D) denotes the payment made from old principal to skilled worker in period t when the project succeeds (resp. fails) in period t. Also let $C_t^C = \{R_t^C, r_t^C\}$ denote the contract used for a project under C–mode in period t where R_t^C (resp. r_t^C) is the payment made from old principal to skilled worker when the project's outcome is realized as success (resp. failure). Recall that we are assuming that one skilled worker is needed for implementing one project even when he is not delegated task 2. Thus even in C–mode the contract

[7] One interpretation for effort cost $c_i(\cdot)$ is as follows: Each task of a project gives player $i = m, p$ who handled it some private returns in terms of the good after the project is completed. For example, task 2 of a project yields $B - c_i(e_i)$ goods to the player who handled it where $B > 0$. However, it is not verifiable whether or not each player obtained the private returns.

[8] This assumption is not so essential: Even when we allow the principal's effort cost $c_p(e_p)$ to differ from the skilled worker's one $c_m(e_m)$, our results will not be substantially changed as long as these costs do not differ too much.

C_t^C must induce the skilled worker to accept for working on the project.

The timing of events is summarized as follows:
In period t:

1. Each young agent earns and saves the market wage w_t.

2. Each young principal creates projects by investing λ goods per project.

In period $t+1$:

1. The projects created in period t, denoted by K_{t+1}, and θ old agents randomly match with each other according to the matching function $m(K_{t+1}, \theta)$.

2. Each old principal chooses an organizational mode (D–mode or C–mode) and offers a contract (C_{t+1}^C or C_{t+1}^D) to a matched old agent for each matched project.

3. Each old agent who matched a project decides whether or not to accept the offered contract. If he rejects it, he will work as an unskilled worker (thus he will supply one unit labor to labor market). If he accepts it, he will spend one unit labor for acquiring necessary skills for the project and will work as a skilled worker.

4. In D–mode the skilled workers who accepted the contracts and old principals will simultaneously choose their efforts $e_m \in \{h, l\}$ for task 2 and $a \in \{h, l\}$ for task 1 for matched projects. In C–mode old principals will choose her efforts $a \in \{h, l\}$ and $e_p \in \{h, l\}$ for both tasks of a matched project.

5. The projects' returns are realized and payments are made according to binding contracts.

In initial period ($t = 0$) there are a unit mass of old principals who totally hold K_0 projects, which are historically given in the model, and θ old agents who have no initial wealth.

3 Market Equilibrium and Organizational Choice

3.1 Optimal Contracts

We will first characterize the optimal contract under each organizational mode in period $t \geq 1$, by taking as given the relevant market prices such as the interest and wage rates. The optimal contracts offered in initial period $t = 0$ will be separately examined later.

In each period each old principal chooses optimal organizational mode, centralized or decentralized organization, as well as the associated optimal contract offered to skilled workers. In that stage the principals take the market prices (w_t and ρ_t) as exogenously given because the markets are perfectly competitive.

First we will start with the optimal contract under C–mode. Suppose first that old principal wants to implement high effort only for one task of a matched project. Since the probability of the project's outcomes and effort cost function are symmetric with respect to effort levels, it does not matter for which task the principal exerts high effort ($a = h$ or $e_p = h$). In either case the principal incurs effort cost $c_p(h, l) = c > 0$. Then the success probability of the project is given by $p_{hl} \in (0, 1)$. Given w_t, ρ_{t+1} and w_{t+1}, period $t + 1$ contract C^C_{t+1} under C–mode is offered to maximize the principal's payoff subject to the set of relevant constraints, i.e., it should solve the following problem:

Problem C

$$\max\ p_{hl}\pi(w_{t+1}) - c - \{p_{hl}R^C_{t+1} + (1 - p_{hl})r^C_{t+1}\}$$

subject to

$$(h, l) \in \arg\max_{i,j=h,l} p_{ij}\pi(w_{t+1}) - c_p(i, j) - \{p_{ij}R^C_{t+1} + (1 - p_{ij})r^C_{t+1}\} \quad \text{(ICPC)}$$

$$p_{hl}R^C_{t+1} + (1 - p_{hl})r^C_{t+1} + \rho_{t+1}w_t \geq w_{t+1} + \rho_{t+1}w_t \quad \text{(IRC)}$$

$$R^C_{t+1} \geq -\rho_{t+1}w_t,\ \ r^C_{t+1} \geq -\rho_{t+1}w_t \quad \text{(LLC)}$$

Here (ICPC) says the incentive compatibility constraint for the principal that she optimally chooses high effort only for one task of the project, given the contract C^C_{t+1}. Note here that, since the principal's efforts are not verifiable, she cannot commit herself to her effort levels when offering the contract. Thus (ICPC) must be satisfied.

(IRC) says the individual rationality constraint that the expected payoff of skilled worker by accepting the contract C^C_{t+1} (the left hand side) must not be less than the expected payoff by rejecting it (the right hand side). By our assumption one old agent (skilled worker) is needed for implementing one project. Thus (IRC) must be satisfied for skilled worker to accept for working on the project. Each old agent can obtain his interest income $\rho_{t+1}w_t$ which comes from his saving, irrespective of accepting the contract. In addition to this income each old agent can obtain the market wage w_{t+1} if he will work as an unskilled worker but not skilled worker. Thus at least the total income $\rho_{t+1}w_t + w_{t+1}$ must be guaranteed for ensuring that matched old agent will accept the contract and work as a skilled worker.

(LLC) says the limited liability constraint that, since each old agent has only the income $\rho_{t+1}w_t$, the principal cannot make lower payments than this amount. [9]

To solve Problem C, we will first omit (ICPC) from the problem and check this later. Then, (IRC) must be binding because otherwise R^C_{t+1} or/and r^C_{t+1} can be slightly reduced and the principal's payoff can be increased. In particular we take $R^C_{t+1} = w_{t+1}/p_{hl}$ and $r^C_{t+1} = 0$. Then (ICPC) will be satisfied when

$$\pi(w_{t+1}) \geq w_{t+1}/p_{hl} + (c/\Delta q) \tag{ICPC$'$}$$

and

$$\pi(w_{t+1}) \leq w_{t+1}/p_{hl} + (\tilde{c} - c)/\Delta p. \tag{ICPC$''$}$$

The former says that the principal has no incentives to choose low efforts for both tasks and the latter ensures that she has no incentives to choose high efforts for both tasks.

We will make the following assumption to ensure that it is too costly for old principal to exert high efforts for both tasks of any matched project at a time:

Assumption 1. (i) Minimum wage ($v > 0$): $w_t \geq v$ and (ii) $\tilde{c} - c > \pi(v)$.

Assumption 1(i) says that some minimum wage requirement exists so that the market wage w_t must not be less than the minimum wage $v > 0$. However, in what follows we will assume that v is so small and hence the minimum wage constraint is never binding in any equilibrium. The reason why we introduced Assumption 1(i) is merely to state Assumption 1(ii) in the simple manner. Assumption 1(ii) can then guarantee that $\tilde{c} - c > \pi(w_t)$ for all w_t in any equilibrium. Then, given Assumption 1, (ICPC$''$) is satisfied in any equilibrium. In Appendix B we will also verify that (ICPC$'$) is satisfied in any equilibrium.

Then the *gross* payoff the principal can attain in Problem C (gross of investment cost $\lambda\rho_{t+1}$) is given by

$$\Pi^C(w_{t+1}) \equiv p_{hl}\pi(w_{t+1}) - w_{t+1} - c. \tag{2}$$

[9] In addition to (LLC) the limited liability constraints on the side of the principal must be also satisfied. The strongest version for these constraints is that the principal's payments R^C_{t+1} and r^C_{t+1} must not be greater than the realized revenues $\pi(w_{t+1})$ and zero for *each project* respectively. Then these constraints are given by $\pi(w_{t+1}) \geq R^C_{t+1}$ and $0 \geq r^C_{t+1}$. However, we can verify that the optimal contract to solve Problem C satisfies these constraints. One might also think that since the principal handles many projects which matched old agents, it is sufficient to consider weaker version for the principal's limited liability constraints such that *total* payments cannot be larger than the total revenues for *total matched projects* but not per project. However, since the principal's limited liability constraints are satisfied even as the strongest version per project, they are also satisfied by weaker version for total matched projects as well.

Second, suppose that under C–mode some old principal wants to implement low efforts for both tasks of some project. Then, since the principal must pay at least the market wage w_{t+1} to the skilled worker who accepts the contract C_{t+1}^C, the maximum gross payoff the principal can obtain by implementing low efforts for both tasks is given by

$$p_{ll}\pi(w_{t+1}) - w_{t+1}.$$

Thus each old principal never implements low efforts for both tasks when $\Pi^C(w_{t+1}) \geq p_{ll}\pi(w_{t+1}) - w_{t+1}$ which can be written by

$$\Delta q \pi(w_{t+1}) \geq c.$$

We will assume that this inequality is satisfied in any equilibrium, although we will give sufficient conditions for this in Appendix B.

Third, it is never optimal for any old principal to implement high efforts from both tasks of any matched project under Assumption 1.

We will also check later that each project under C–mode can cover the credit repayment $\lambda\rho_{t+1}$: $\Pi^C(w_{t+1}) \geq \lambda\rho_{t+1}$ (See (BEC–C) introduced below).

Next we will consider the optimal contract C_{t+1}^D under D–mode offered in period $t+1$.

Suppose first that each old principal wants to implement high efforts $a = h$ and $e_m = h$ from both tasks of a matched project, one high effort from the matched skilled worker and the other from herself. Then the success probability of the project is given by $p_{hh} \in (0, 1)$. Also the principal and skilled worker incur their effort costs $c_p(h) = c > 0$ and $c_m(h) = c > 0$ respectively. Then C_{t+1}^D should solve the following problem:

Problem D

$$\max\ p_{hh}\pi(w_{t+1}) - c - \{p_{hh}R_{t+1}^D + (1 - p_{hh})r_{t+1}^D\}$$

subject to

$$p_{hh}\{\pi(w_{t+1}) - R_{t+1}^D + r_{t+1}^D\} - c \geq p_{hl}\{\pi(w_{t+1}) - R_{t+1}^D + r_{t+1}^D\} \quad \text{(ICPD)}$$

$$p_{hh}R_{t+1}^D + (1-p_{hh})r_{t+1}^D - c + \rho_{t+1}w_t \geq p_{hl}R_{t+1}^D + (1-p_{hl})r_{t+1}^D + \rho_{t+1}w_t \quad \text{(ICM)}$$

$$p_{hh}R_{t+1}^D + (1 - p_{hh})r_{t+1}^D - c + \rho_{t+1}w_t \geq \rho_{t+1}w_t + w_{t+1} \quad \text{(IRD)}$$

$$R_{t+1}^D \geq -\rho_{t+1}w_t, \ \ r_{t+1}^D \geq -\rho_{t+1}w_t \quad \text{(LLD)}$$

Here (IRD) and (LLD) mean the individual rationality and limited liability constraints of skilled worker again. [10] Here (ICM) and (ICPD) mean

[10] As we have discussed in footnote 9, the limited liability constraints on the side of the principal are ignored here because they are satisfied at the optimal solution.

the incentive compatibility constraints on the side of skilled worker and the old principal respectively: Both of them can ensure that high effort pair $(a, e_m) = (h, h)$ can be a Nash equilibrium in the game in which the skilled worker and old principal simultaneously choose their efforts, given the contract C^D_{t+1}.

As in the case of C–mode, we will first omit the incentive compatibility constraint for the principal (ICPD) and check this later. Then, if (IRD) is not binding at the optimum, both (ICM) and the second (LLD) must be binding: $R^D_{t+1} - r^D_{t+1} = c/\Delta p$ and $r^D_{t+1} = -\rho_{t+1}w_t$ where recall that $\Delta p \equiv p_{hh} - p_{hl} > 0$. This will be actually the case if $p_{hh}(c/\Delta p) - c > \rho_{t+1}w_t + w_{t+1}$, which means that skilled worker obtains some *rent* over the payoff of his outside opportunity due to limited liability. Otherwise, (IRD) is binding at the optimum.

This argument leads to the following lemma:

Lemma 1. *Suppose that (ICPD) is not binding in Problem D. Then the optimal contract C^D_{t+1} to solve Problem D is characterized as follows: (i) $R^D_{t+1} = c/\Delta p - \rho_{t+1}w_t$ and $r^D_{t+1} = -\rho_{t+1}w_t$ if $p_{hh}(c/\Delta p) - c > \rho_{t+1}w_t + w_{t+1}$ and (ii) $p_{hh}R^D_{t+1} + (1 - p_{hh})r^D_{t+1} = w_{t+1} + c$ otherwise.*

In what follows we will ignore (ICPD), although in Appendix B we will give the conditions for (ICPD) to be actually satisfied.

Then, we derive the *gross* maximum payoff the principal can attain in Problem D (gross of investment cost $\lambda\rho_{t+1}$) as follows:

$$\Pi^D(w_t, w_{t+1}, \rho_{t+1}) \equiv \begin{cases} p_{hh}\pi(w_{t+1}) - c - p_{hh}(c/\Delta p) + \rho_{t+1}w_t & \text{if } p_{hh}(c/\Delta p) - c > \rho_{t+1}w_t + w_{t+1} \\ \\ p_{hh}\pi(w_{t+1}) - w_{t+1} - 2c & \text{otherwise.} \end{cases} \tag{3}$$

Next suppose that some old principal wants to implement other effort pairs than the high effort one for some matched project. Then, by similar argument to the case of C–mode, the maximum payoff the principal can obtain by implementing other effort pairs than $(a, e_m) = (h, h)$ is given by

$$\Pi^*(w_{t+1}) \equiv \max\{p_{ll}\pi(w_{t+1}) - w_{t+1}, p_{hl}\pi(w_{t+1}) - w_{t+1} - c\}.$$

Thus it is optimal for each old principal to implement high effort pair $(a, e_m) = (h, h)$ when

$$\Pi^D(w_t, w_{t+1}, \rho_{t+1}) \geq \Pi^*(w_{t+1}). \tag{4}$$

Again we will assume throughout the main text that this inequality holds in any equilibrium, although we will verify this in Appendix B.

We will also check later that each project under D–mode can cover the credit repayment $\lambda\rho_{t+1}$: $\Pi^D(w_t, w_{t+1}, \rho_{t+1}) \geq \lambda\rho_{t+1}$ (See condition (BEC–D) introduced below).

Each old principal compares the payoffs (2) and (3) under different organizational modes. Then each old principal will choose D–mode only if

$$\Pi^D(w_t, w_{t+1}, \rho_{t+1}) \geq \Pi^C(w_{t+1}), \tag{5}$$

but C–mode only if

$$\Pi^C(w_{t+1}) \geq \Pi^D(w_t, w_{t+1}, \rho_{t+1}), \tag{6}$$

given the market prices w_t, w_{t+1} and ρ_{t+1}. Furthermore, when $\Pi^D(w_t, w_{t+1}, \rho_{t+1}) = \Pi^C(w_{t+1})$, old principals will be indifferent for choosing D–mode and C–mode.

We will finally derive the optimal contracts in initial period $t = 0$. In initial period $t = 0$ old agents are assumed to be have fixed wealth, normalized to zero. Again we assume that each old principal wants to implement high efforts for both tasks of any matched project. Thus the optimal contract $C_0^D = \{R_0^D, r_0^D\}$ under D–mode in period 0 should solve the following:

Problem D0

$$\max \; p_{hh}[\pi(w_0) - R_0^D] + (1 - p_{hh})[-r_0^D] - c$$

subject to (ICPD), (ICM) and

$$p_{hh}R_0^D + (1 - p_{hh})r_0^D - c \geq w_0, \tag{IRD0}$$

$$R_0^D \geq 0, \;\; r_0^D \geq 0. \tag{LLD0}$$

This problem differs from those in other periods only in that old agents have no wealth initial period and thus they can obtain only the market wage w_0 when they reject the contract. By this reason, the individual rationality constraint (IRD0) and the limited liability constraint (LLD0) were changed.

Then the optimal contract to solve Problem D0 should be given as follows: $R_0 = c/\Delta p$ and $r_0 = 0$ if $p_{hh}(c/\Delta p) - c > w_0$ and $p_{hh}R_0 + (1 - p_{hh})r_0 = w_0 + c$ otherwise. Thus, in initial period ($t = 0$) the gross (per project) payoff of old principal under D–mode becomes

$$\Pi_0^D(w_0) \equiv \begin{cases} p_{hh}\pi(w_0) - c - p_{hh}(c/\Delta p) & \text{if } p_{hh}(c/\Delta p) - c > w_0 \\ \\ p_{hh}\pi(w_0) - w_0 - 2c & \text{otherwise.} \end{cases} \tag{7}$$

Next we consider the optimal C–mode contract in initial period $t = 0$. By Assumption 1, any old principal never exerts high efforts for both tasks of any matched project. Also we assume that it is optimal for each old principal to implement high effort only for one task of any matched project as in the case of other periods ($t \geq 1$). Then, the gross payoff each old principal can obtain per project under C–mode is given by

$$\Pi_0^C(w_0) \equiv p_{hl}\pi(w_0) - w_0 - c. \tag{8}$$

Thus each old principal will choose D–mode in period 0 only if $\Pi_0^D(w_0) \geq \Pi_0^C(w_0)$. In contrast to other periods only the market wage w_0 determines the equilibrium organizational mode in initial period ($t = 0$).

3.2 Labor Market Equilibrium

The labor demand in period t is totally given by $g(w_t)p_{hh}m(K_t, \theta)$ when all old principals choose D–mode in period t. This is because $m(K_t, \theta)$ projects are matched with θ old agents and p_{hh} fraction of them succeeds, each of which requires labor demand $g(w_t)$. When all old principals choose C–mode, total labor demand is given by $g(w_t)p_{hl}m(K_t, \theta)$ because each project owned by the principals will succeed with probability $p_{hl} \in (0, 1)$. Of course, when D–mode yields strictly higher (resp. lower) payoff $\Pi^D(w_t, w_{t+1}, \rho_{t+1})$ than C–mode $\Pi^C(w_{t+1})$ does, all old principals will choose D–mode (resp. C–mode) with certainty. When old principals obtain the same payoff under both C–mode and D–mode, they will be indifferent between these modes, and hence in this case we suppose that $x_t \in [0, 1]$ fraction of them will choose D–mode.

On the other hand, labor supply comes from all θ young agents and the old agents who failed to match the projects, $\theta(1 - \beta(K_t))$. All of them, $\theta + \theta(1 - \beta(K))$, supply their labor inputs to work as unskilled workers. Thus, recalling that we denoted by $x_t \in [0, 1]$ the fraction of the old principals choosing D–mode in period t, the labor market will clear in period t when

$$\{x_t p_{hh} + (1 - x_t)p_{hl}\}g(w_t)m(K_t, \theta) = \theta + \theta(1 - \beta(K_t)). \tag{LMC}$$

Since $g' < 0$, we can derive a unique market clearing wage to solve the above equation. In particular we will denote by $w_t = w_D(K_t)$ the market clearing wage for $x_t = 1$ (when all old principals choose D–mode) and by $w_t = w_C(K_t)$ the market clearing wage for $x_t = 0$ (when all old principals choose C–mode) respectively. Since β is increasing in K_t, the market wages w_D and w_C are both increasing in K_t. Since $p_{hh} > p_{hl}$, we also have $w_D(K) > w_C(K)$ for all K.

Note here that, since $m(K, \theta) \leq \theta$ for all $K \geq 0$, and $p_{hh} > p_{hl}$, (LMC) implies that $g(w_t) \geq 1/p_{hh}$ which then shows $w_t \leq g^{-1}(1/p_{hh})$ where g^{-1} is the inverse function of g. This shows that the market wage must be bounded above by $g^{-1}(1/p_{hh})$ in any equilibrium.

3.3 Credit Market Equilibrium

Suppose that all old principals in period $t+1$ will choose D–mode. Then in period t the following break even condition for each young principal must be satisfied:

$$\alpha(K_{t+1})\Pi^D(w_t, w_{t+1}, \rho_{t+1}) = \lambda\rho_{t+1}. \tag{BEC–D}$$

Here the left hand side means the expected payoff of a young principal per project in period t because she can match an old agent with probability $\alpha(K_{t+1})$ and obtain the gross payoff $\Pi^D(w_t, w_{t+1}, \rho_{t+1})$ in the next period $t+1$. The right hand side means the cost of raising one project (recall that λ goods are needed to create one project). If the left (resp. right) hand side of the above inequality is greater than the right (resp. left) hand side, each young principal will require infinite (resp. zero) credit demand, which will not make the credit market clear. Thus the above equality must hold in credit market equilibrium.

Similarly, when all old principals choose C–mode, the following break even condition must hold:

$$\alpha(K_{t+1})\Pi^C(w_{t+1}) = \lambda\rho_{t+1}. \tag{BEC–C}$$

When each old principal is indifferent for choosing between D–mode and C–mode, she must obtain the same expected payoff $\Pi^D(w_t, w_{t+1}, \rho_{t+1}) = \Pi^C(w_{t+1})$ under both modes. This case can be thus included in the above break even condition (BEC–D) or (BEC–C).

In credit market equilibrium the total credit demand (total investment) must be also equal to the total credit supply (total saving):

$$\theta w_t = \lambda K_{t+1}. \tag{9}$$

Here the total saving comes from θ young agents who will save all their wage incomes w_t. The total investment comes from one unit mass of young principals who need λ goods per project to be financed.

Note that (BEC–D) and $\alpha(K_{t+1}) < 1$ imply $\Pi^D(w_t, w_{t+1}, \rho_{t+1}) > \lambda\rho_{t+1}$. This shows that each project under D–mode can cover the credit repayment $\lambda\rho_{t+1}$ after matching but before projects' outcomes are realized.[11] The same argument can be applied to C–mode.

3.4 Equilibrium Organizations

Given the market wage w_t determined in the previous period, say t, we define the *static equilibrium* in the current period ($t+1$) which consists of the

[11] Since each old principal handles many projects, she can perfectly diversify the risks of the projects' failure due to the law of large numbers. Thus her expected payoff per project can be viewed as her average one and hence the actual revenues of all her projects can cover her total credit repayments.

number of total projects K_{t+1}, the market wage rate w_{t+1}, the organizational mode, and the interest rate ρ_{t+1}. In the static equilibrium D–mode emerges in period $t+1$ when (5), (LMC) with $x_{t+1} = 1$, (BEC–D) and (9) are all satisfied. We will call this *D–mode equilibrium.* On the other hand, C–mode emerges in period $t+1$ when (6), (LMC) with $x_{t+1} = 0$, (BEC–C) and (9) are all satisfied. We will call this *C–mode equilibrium.* Furthermore, when $\Pi^D(w_t, w_{t+1}, \rho_{t+1}) = \Pi^C(w_{t+1})$, (LMC) with some $x_{t+1} \in (0, 1)$, (BEC–D) (or (BEC–C)) and (9) will give an equilibrium in which both C–mode and D–mode co–exist. We will call this *mixed mode equilibrium.*

Then, in order to see which type of the static equilibrium arises, we define the following functions:

$$\hat{\rho}^D(w_{t+1}, K_{t+1}) \equiv \frac{\alpha(K_{t+1})}{\lambda(1 - \beta(K_{t+1}))}\{p_{hh}\pi(w_{t+1}) - c - p_{hh}(c/\Delta p)\}, \tag{10}$$

$$\tilde{\rho}^D(w_{t+1}, K_{t+1}) \equiv \frac{\alpha(K_{t+1})}{\lambda}\{p_{hh}\pi(w_{t+1}) - w_{t+1} - 2c\}, \tag{11}$$

$$\rho^C(w_{t+1}, K_{t+1}) \equiv \frac{\alpha(K_{t+1})}{\lambda}\{p_{hl}\pi(w_{t+1}) - w_{t+1} - c\}. \tag{12}$$

By using (9) and $\alpha(K_{t+1})K_{t+1}/\theta = \beta(K_{t+1})$, we can readily verify that $\hat{\rho}^D$ means the interest rate to ensure the break even condition (BEC–D) of each young principal in period t, provided D–mode will be chosen and (IRD) will not be binding in period $t+1$. $\tilde{\rho}^D$ is the interest rate to ensure the break even condition (BEC–D), provided D–mode will be chosen and (IRD) will be binding in period $t+1$. $\tilde{\rho}^C$ means the interest rate which ensures the break even condition (BEC–C), provided C–mode will be chosen in period $t+1$. Then, in order to ensure that these interest rates are all positive, we must have $p_{hh}\pi(w_{t+1}) - p_{hh}(c/\Delta p) - c > 0$, $p_{hl}\pi(w_{t+1}) - w_{t+1} - c > 0$ and $p_{hh}\pi(w_{t+1}) - w_{t+1} - 2c > 0$. To this end we will make the following assumption:

Assumption 2. $\pi(g^{-1}(1/p_{hh})) > \max\{(g^{-1}(1/p_{hh}) + 2c)/p_{hh}, c/\Delta p + c/p_{hh}\}$.

Since the market wage w_{t+1} is bounded above by $g^{-1}(1/p_{hh})$ in any equilibrium as we have noted before, Assumption 2 can guarantee the following: First, $\pi(w) > c/\Delta p$ and hence $\tilde{\rho}^D(w, K) > \rho^C(w, K)$ in any equilibrium. Second, all $\rho^C(w, K)$, $\tilde{\rho}^D(w, K)$ and $\hat{\rho}^D(w, K)$ are positive in any equilibrium.

By defining the following functions:

$$\Gamma(w, K) \equiv \hat{\rho}^D(w, K) - \rho^C(w, K), \tag{13}$$

and

$$\Phi(w, K) \equiv \hat{\rho}^D(w, K) - \tilde{\rho}^D(w, K), \tag{14}$$

we can show the following lemma:

Lemma 2. *Given K, if $\Gamma(w, K) = 0$ has solutions for w, its number is generically two. Also, given K, if $\Phi(w, K) = 0$ has solutions for w, its number is generically two.*

Proof. See Appendix A.

Since $\Gamma(w, K) > \Phi(w, K)$ for all w and all K by Assumption 2, $\Phi(w, K)$ has solutions as long as $\Gamma(w, K)$ does so. Let $\overline{w}(K)$ and $\underline{w}(K)$ denote these solutions to $\Gamma(w, K) = 0$ for w, given K, where $\overline{w}(\cdot) > \underline{w}(\cdot)$. Then, we can verify that $\overline{w}$ (resp. $\underline{w}$) is decreasing (resp. increasing) in K. Let also denote by $w^*(K)$ and $w^{**}(K)$ the two solutions of w to the equation $\Phi(w, K) = 0$ for a given K, where $w^*(K) < w^{**}(K)$ for all K. Again, w^* is increasing but w^{**} is decreasing. Since $\Gamma(w, K) > \Phi(w, K)$ for all w, we also have $w^*(K) < \underline{w}(K) < \overline{w}(K) < w^{**}(K)$ for all $K \geq 0$.

Now we can show the following result:

Proposition 1. *Suppose that w_t is given ($t \geq 0$). Suppose also that $\Gamma(w_{t+1}, K_{t+1}) = 0$ has the solutions of w_{t+1} for a given $K_{t+1} = (\theta/\lambda)w_t$. Then the static equilibrium in period $t+1$ is characterized as follows:*
(i) C–mode equilibrium emerges if and only if w_{t+1} and K_{t+1} satisfy

$$K_{t+1} = (\theta/\lambda)w_t, \tag{15}$$

$$w_{t+1} \in [\underline{w}(K_{t+1}), \overline{w}(K_{t+1})], \tag{16}$$

$$w_{t+1} = w_C(K_{t+1}) \tag{17}$$

(ii) D–mode equilibrium emerges if and only if w_{t+1} and K_{t+1} satisfy

$$K_{t+1} = (\theta/\lambda)w_t, \tag{18}$$

$$w_{t+1} \in (0, \underline{w}(K_{t+1})] \cup [\overline{w}(K_{t+1}), \infty) \tag{19}$$

$$w_{t+1} = w_D(K_{t+1}) \tag{20}$$

(iii) Mixed mode equilibrium emerges if and only if

$$K_{t+1} = (\theta/\lambda)w_t, \tag{21}$$

$$w_{t+1} \in \{\underline{w}(K_{t+1}), \overline{w}(K_{t+1})\} \tag{22}$$

respectively.

Proof. See Appendix A.

Although the formal proof for Proposition 1 is relegated to Appendix A, the intuition behind this result can be understood as follows: Roughly speaking, *the reservation payoff* of skilled worker $\rho_{t+1}w_t + w_{t+1}$, which he could obtain if he rejected the contract offered by matched principal and worked as an unskilled worker, is U–shaped with respect to the market wage w_{t+1} in period $t+1$, given the previous period wage w_t (See Figure 1). When w_{t+1} is large, this directly raises the opportunity cost of working as a skilled worker instead of an unskilled worker. When w_{t+1} is small, such opportunity cost also rises because the profit of successful project $\pi(w_{t+1})$ becomes larger and hence the equilibrium interest rate ρ_{t+1} must go up in order to satisfy the credit market equilibrium. Hence, by recalling that (IRD) is binding under D–mode when $p_{hh}(c/\Delta p) - c \leq \rho_{t+1}w_t + w_{t+1}$, (IRD) is more likely to be binding when the market wage w_{t+1} becomes larg or small. Since the trade–off between incentive and rent extraction becomes less serious when (IRD) is more likely to be binding, D–mode becomes more profitable for each old principal than C–mode when w_{t+1} is either large or small. This means that D–mode emerges as an equilibrium when the market wage w_{t+1} in period $t+1$ is large or small, given the previous period wage w_t.

In initial period $t = 0$ the argument should be modified as follows: Recall that the maximum gross payoffs of initial old principals under D–mode and C–mode are given by $\Pi^D(w_0)$ and $\Pi^C(w_0)$ respectively. The only difference from any other period $t \geq 1$ is that the maximum payoff of old principal under D–mode is changed from $p_{hh}\pi(w_t) - c - p_{hh}(c/\Delta p) + \rho_{t+1}w_t$ to $p_{hh}\pi(w_0) - c - p_{hh}(c/\Delta p)$ when (IRD0) is not binding in Problem D0. Thus, when (IRD0) is not binding, the payoff difference $\Pi^D(w_0) - \Pi^C(w_0)$ between D–mode and C–mode becomes

$$(p_{hh} - p_{hl})\pi(w_0) - p_{hh}(c/\Delta p) + w_0,$$

which is the same as the function Γ we have defined for other periods ($t \geq 1$) once we set $\beta(K_t) \equiv 0$ or equivalently $K_t \equiv 0$. Thus we can show that for all $w_0 \in (\underline{w}(0), \overline{w}(0))$ we have $\Pi^C(w_0) > \Pi^D(w_0)$ but $\Pi^D(w_0) \geq \Pi^C(w_0)$ for all $w_0 \in (0, \underline{w}(0)] \cup [\overline{w}(0), \infty)$. Then D–mode equilibrium emerges in initial period if $w_0 \in (0, \underline{w}(0)] \cup [\overline{w}(0), \infty)$ and the labor market clears, i.e., $p_{hh}g(w_0)m(K_0, \theta) = \theta + (1 - \beta(K_0))\theta$. Here the initial value K_0 is historically given. On the other hand, C–mode equilibrium emerges in initial period if $w_0 \in (\underline{w}(0), \overline{w}(0))$ holds and the labor market clears, i.e., $p_{hl}g(w_0)m(K_0, \theta) = \theta + (1 - \beta(K_0))\theta$. Also, mixed mode equilibrium arises in initial period when $w_0 = \underline{w}(0)$ or $w_0 = \overline{w}(0)$.

4 Equilibrium Dynamics

As we have shown, if they exist, the two cutoff market wages $\overline{w}(K)$ and $\underline{w}(K)$, which determine the equilibrium organizational mode, are decreasing and increasing in K respectively. In Figure 2 we depicted these two curves and the different regions, labeled by D1, C and D2. The shaded area, labeled by C, means the set of pairs (w, K) such that C–mode equilibrium emerges while in all other areas, labeled by D1 and D2, D–mode equilibrium emerges. Furthermore, on the boundaries of the curves $\underline{w}(K)$ and $\overline{w}(K)$, mixed mode equilibria arise where old principals are indifferent between C–mode and D–mode as we have shown in Proposition 1.

A can be seen in Figure 2, both D–mode and C–mode static equilibria may arise for the same number of projects K because the market wage $w_D(K_t)$ under D–mode belongs to region D1 as well as the wage $w_C(K_t)$ under C–mode belongs to region C even when the economy has the same number of projects K_t. Also, mixed mode equilibrium can co–exist together with C–mode and D–mode static equilibria: Given the same K_t, the market wage $\overline{w}(K_t)$ satisfies the labor market clearing condition (LMC) and thus some $x_t \in (0, 1)$ fraction of old principals choose D–mode while $1-x_t$ fraction of them choose C–mode. This fraction x_t can be determined by (LMC) as well. Such mixed mode equilibrium is depicted as point M in Figure 2. Thus for the same value of K_t there may exist three possible static equilibria, i.e., C–mode, D–mode and mixed mode equilibria.

Note that the market wage $w_C(K)$ (resp. $w_D(K)$) must belong to the corresponding region, C (resp. D1 or D2) when C–mode (resp. D–mode) equilibrium arises. Note also that in a mixed mode equilibrium the market wage must be $\underline{w}(K)$ or $\overline{w}(K)$. Then we can derive the entire equilibrium market wage which is a function of the number of total projects K_t as follows:

$$w^*(K_t) \equiv \begin{cases} w_C(K_t) & \text{if } w_C(K_t) \in (\underline{w}(K_t), \overline{w}(K_t)), \\ w_D(K_t) & \text{if } w_D(K_t) \in [0, \underline{w}(K_t)) \cup (\overline{w}(K_t), \infty) \\ \underline{w}(K_t) & \text{if } \underline{w}(K_t) \in [w_C(K_t), w_D(K_t)], \\ \overline{w}(K_t) & \text{if } \overline{w}(K_t) \in [w_C(K_t), w_D(K_t)]. \end{cases} \tag{23}$$

Given this wage function $w^*(K)$, we can characterize the equilibrium dynamics which governs the path of the number of created projects $\{K_t\}_{t=0}^{\infty}$ which satisfies the credit market clearing condition (9) as well as (23):

$$K_{t+1} = (\theta/\lambda)w^*(K_t). \tag{24}$$

We will here make the following assumption:

Assumption 3. $w_i'(0) = +\infty$, $i = C, D$.

Assumption 3 will be satisfied in the following example: $F(L) = L^\eta$, where

$\eta \in (0,1)$, and $\beta'(0) > 0$. In this example we can show that $dw_i/dK|_{K=0} = \eta(1-\eta)2^{\eta-1}z^{1-\eta}\beta'(0)\beta(0)^{-\eta} = +\infty$ for $z = p_{hh}$ when $i = D$ (resp. $z = p_{hl}$ when $i = C$).

Then we can define the steady state value of K in D–mode equilibrium, called the D–mode steady state and denoted K^D, as

$$\theta w_D(K^D) = \lambda K^D$$

and the steady state value of K in C–mode equilibrium, called C–mode steady state and denoted K^C, as

$$\theta w_C(K^C) = \lambda K^C.$$

By Assumption 3 and the fact that $w_i(K) \leq g^{-1}(1/p_{hh})$ for all K, there exist the steady state values K^D and K^C for all $\lambda > 0$. To avoid complication, we will assume that the steady state K^i uniquely exists for each $i = D, C$. Note here that $K^D > K^C$ holds because $w_D(K) > w_C(K)$ for all $K > 0$.

The properties of equilibrium dynamics (24) depend on the parameter value λ which captures the costs of investing and starting up a project. As we have mentioned, λ may measure both technological and institutional inefficiencies underlying the economy when starting up a project. We can also give further interpretation of λ as the measure of contracting costs of finance in credit market by extending the basic model to allow default possibility (See Appendix C for more details).

We will first consider the case that the investment cost λ is so small. Then the economy eventually converges to the D–mode steady state K^D.

Proposition 2. *Suppose that Assumption 1–3 hold. Suppose also that the investment cost λ is sufficiently small. Then the economy eventually converges to the D–mode steady state, i.e., $K_t \to K^D$ as $t \to \infty$.*

Proof. See Appendix A.

When the investment cost λ is small enough, the number of created projects K_t becomes much larger. Then the market equilibrium wage $w_i(K_t)$ is pushed up under both D–mode and C–mode ($i = D, C$). Thus, as the economy grows, it must enter the region in which the market wage is large enough and hence the rent of skilled workers becomes small enough as well. This means that D–mode becomes optimal choice forever from some period onward. Thus, in this case there are no diversities of organizational modes to govern transactions in the long run.

One implication of this result is as follows: When the institutions underlying the economy are developed well so that it becomes cheaper for young principals to raise the fund and start up projects, λ becomes smaller. Then Proposition 2 suggests that decentralization is more likely to be dominant form and the economy is more likely to enjoy the benefits from specialization when well developed institutions emerge to reduce the costs of starting up the projects.

Next we will turn to more interesting case that the investment cost λ is moderately large. To make the analysis interesting, we will impose the following assumption:

Assumption 4. (i) $\min_{w \geq 0} \Gamma(w, \infty) < 0$, *and* (ii) $g^{-1}(\sigma/p_{hl}) > \overline{w}(\infty)$ *where* $\sigma \equiv (2-\gamma)/\gamma$.

Assumption 4(i) ensures that the two cutoff wages $\overline{w}(K)$ and $\underline{w}(K)$ actually exist for all $K \geq 0$. This is because $\Gamma(w, K)$ is increasing in K and hence $\min_w \Gamma(w, \infty) < 0$ implies $\min_w \Gamma(w, K) < 0$ for all $K \geq 0$. Then, by definition of $\Gamma(w, K)$, we have two solutions of w to the equation $\Gamma(w, K) = 0$ for a given K, due to Lemma 2. Thus there exists the region in which C–mode becomes a static equilibrium (labeled by C in Figure 2). Assumption 4(i) also implies that $\overline{w}(K)$ and $\underline{w}(K)$ exist for all $K \geq 0$ and hence $\underline{w}(\infty) = \overline{w}(\infty)$ exists. Assumption 4(ii) then guarantees that these cutoff curves $\overline{w}(K)$ and $\underline{w}(K)$ intersect with the market wage curves $w_C(K)$ and $w_C(K)$ corresponding to D–mode and C–mode respectively (as depicted in Figure 2). This is because $w_D(K) > w_C(K)$ for all K and $\lim_{K \to \infty} w_C(K) = g^{-1}(\sigma/p_{hl})$ by $\lim_{K \to \infty} m(K, \theta) = \gamma\theta$. In Appendix B we will give the parametric conditions for Assumption 4 to be satisfied and consistent with other assumptions we have made.

Given Assumption 4, we will consider the range of the investment cost parameter values λ such that $w_C(K^C) \in (\underline{w}(K^C), \overline{w}(K^C))$ and $w_D(K^D) > \overline{w}(K^D)$ are satisfied. Then, as depicted in Figure 4, there exist multiple equilibrium paths some of which converge to the C–mode steady state K^C but others of which converge to the D–mode steady state K^D *even when the economy starts from the same initial value* K_0. To see that this is actually the case, we define the values of K, denoted K' and K'', such that $w_C(K') = \overline{w}(K')$ and $K'' \equiv \max\{K > 0 \mid w_C(K'') = \underline{w}(K'')\}$. [12]

Then we can show the following result:

Proposition 3. *Suppose that Assumption 1–4 hold. Then there exists some range of the investment cost parameter* λ*, denoted* $(\underline{\lambda}, \overline{\lambda})$*, such that for all* $\lambda \in (\underline{\lambda}, \overline{\lambda})$ *the following statements hold: (i) If* $K_0 \in (K'', K')$*, then multiple equilibrium paths arise, some of which converge to the D–mode steady state* K^D *and others of which converge to the C–mode steady state* K^C*. (ii) If* $K_0 \geq K'$*, then there exist no equilibria which converge to the C–mode steady state* K^C*.*

[12] Since $\underline{w}(0) > w_C(0) = 0$ and Assumption 4, such K'' is well–defined. In Figure 2 and 3 we assume that such K'' is unique to simplify argument.

Proof. See Appendix A.

Proposition 3 states that the economy may be diverted into different development phases, which are characterized by different organizational modes, even when it starts from the same economic condition. If all the principals expect higher economic growth (and hence higher market wage), D–mode becomes more profitable for them than C–mode as we have seen in Proposition 1. Since D–mode is more productive than C–mode in the sense that it can exploit the benefits from specialization of tasks (note that $p_{hh} > p_{hl}$), production and hence employment levels are expanded when all the principals choose D–mode. This in turn pushes up the labor demand and thus the market equilibrium wage, which actually fulfills the principals' original expectation. Thus D–mode can be an equilibrium. On the other hand, if all the principals expect lower economic growth (and hence lower market wage), C–mode becomes more profitable for them than D–mode, which actually makes the market wage lower. Thus the principals' original expectation is fulfilled again and thus C–mode becomes an equilibrium as well. Which development path, the path converging to the D–mode steady state or the C–mode steady state, the economy will eventually reach depends on the expectation held by the principals.

The equilibrium paths we have shown above eventually converge to either the D–mode steady state or the C–mode steady state in the long run. Thus, although equilibrium paths are not unique, *each* long run steady state corresponds to a *unique* organizational mode, one for D–mode and the other for C–mode. This result will be useful for understanding why different countries are sometimes characterized by different organizational modes: Different countries might be on different equilibrium paths converging to different steady states. However, diversity of organizational modes to govern transactions is observed not only across countries but also even in the same country and same industry (See Berger (2005)). To address this issue, we show that there exists the *mixed steady state* in which both C–mode and D–mode *co–exist* together. These steady states are defined as

$$\theta\overline{w}(K^M) = \lambda K^M,$$

and

$$\theta\underline{w}(K^m) = \lambda K^m.$$

Here $K^M > K^m$ holds (See Figure 5). Since $\underline{w}(0) > 0$ and $\underline{w}(K)$ is bounded above, the steady state K^m exists. Also, since $\overline{w}$ is decreasing in K, there also exists the steady state K^M as well.

First, when λ is large, there exists a low mixed steady state K^m where $K^m < K^C$ as well as some equilibrium path converging to such steady state (See Figure 5). In this equilibrium path the number of total projects is low and the resulting market wage is low as well. Second, when λ is in the moderate range, a high mixed steady state K^M exists where $K^M > K^C$ (See point M in Figure 5). Since the equilibrium wage curve $\overline{w}(K_t)$ is decreasing and the curve of credit demand λK_t is increasing, there may exist equilibrium path which fluctuates around high mixed steady state K^M and eventually converges to it in the long run (as shown in Figure 6). Then the mixed steady state can be attained and hence co–existence of different organizations persists even in the long run.

Proposition 4. *There exists some range of the investment cost parameter λ for which the mixed steady states exist in which both C–mode and D–mode co–exist. Furthermore, there may exist an equilibrium path which fluctuates around the high mixed steady state K^M and converges to it in the long run.*

Proposition 4 shows the possibility that the economy exhibits endogenous fluctuation due to the endogenous non–monotonic change of the composition of organizational modes: The fractions of D–mode in the economy fluctuate over time. Furthermore, depending on the costs of starting up project λ, there may exist multiple mixed steady states, K^m and K^M, which differ each other in terms of the fractions of the principals choosing D–mode. In Figure 7 we depicted such situation where there also exist multiple equilibrium paths some of which converge to the low mixed steady state K^m but others of which converge to the high mixed steady state K^M even from the same initial condition.

We will finally comment on the welfare property of different steady states. We define the social welfare of this economy in a steady state as the sum of all individuals' payoffs in each generation (the Benthamian welfare). All individuals obtain utilities from their old period consumption. The total consumption of all old principals and old agents in each generation is given by the total output $F(g(w))[xp_{hh} + (1-x)p_{hl}]m(K, \theta)$ they generate from matched projects minus total wage paid to young workers in the next generation θw. Since $\theta w = \lambda K$, this payment is equal to the investment cost incurred by the principals in each generation. Here $x \in [0, 1]$ denotes the fraction of old principals choosing D–mode in a steady state. Also, total effort costs in a generation under C–mode are given by c because only old principal exerts high effort only for one task of each matched project. Under D–mode both old principal and skilled worker exert high efforts for both tasks of each matched project, and hence the total effort costs in a generation is given by $2c$.

Let denote by $w^C \equiv w_C(K^C)$ and $w^D \equiv w_D(K^D)$ the equilibrium wages in the C–mode and D–mode steady states. Then the sum of the utilities of all individuals in each generation is given by

$$V^D \equiv F(g(w^D))p_{hh}m(K^D, \theta) - m(K^D, \theta)2c - \theta w^D \tag{25}$$

in the D–mode steady state and

$$V^C \equiv F(g(w^C))p_{hl}m(K^C, \theta) - m(K^C, \theta)c - \theta w^C \tag{26}$$

in the C–mode steady state respectively.

Which V^D or V^C is larger depends on the two opposite effects: Since $K^D > K^C$, more projects are financed in the D–mode steady state than the C–mode steady state. This gives larger number of successful projects in the D–mode steady state than the C–mode steady state. On the other hand, since $w^D > w^C$, the cost–rising effect arises under D–mode. First, the output per successful project becomes lower in the D–mode steady state than the C–mode steady state. Second, the investment cost θw^D in the D–mode steady state becomes larger than the C–mode steady state, θw^C. When the former project expansion effect dominates the latter cost–rising effect, the D–mode steady state attains higher welfare than the C–mode steady state. However, when the latter dominates the former, the ranking of the long run welfare in these steady states is reversed.

Although it is difficult to derive general conclusions on the welfare comparison, we can give some parametric example to show that more decentralization yields larger steady state welfare than less decentralization:

Example. Suppose that $F(L) = AL^\eta$ where $\eta \in (0, 1)$ and $A > 0$. Let $K = K(m)$ denote the inverse function of $m = m(K, \theta)$. Then we have $K(0) = K'(0) = 0$ and $\lim_{m \to \gamma} K(m) = \infty$ under our assumptions. Under this specification we can derive the steady state welfare under D–mode as follows:

$$V^D(m) \equiv Ap_{hh}^{1-\eta}(2\theta - m)^\eta m^{1-\eta} - 2cm - \lambda K(m)$$

for $m = m^D \equiv m(K^D)$ and the welfare under C–mode as follows:

$$V^C(m) \equiv Ap_{hl}^{1-\eta}(2\theta - m)^\eta m^{1-\eta} - cm - \lambda K(m)$$

for $m = m^C \equiv m(K^C)$ respectively. Then we obtain that $w^D = A\eta(2\theta - m^D/p_h m^D)^{\eta-1}$ and $\theta w^D = \lambda K(m^D)$. Here note that $m^D > m^C$ by $K^D > K^C$. Under our assumptions, the equilibrium value m^D tends to be zero when η goes to zero. Similarly, m^C tends to be zero as η goes to zero. Also, by letting $f(m) \equiv V^D(m) - V^C(m)$, we can show that $f(0) = 0$ and $f'(0) = +\infty$. [13] Then $V^D(m) > V^C(m)$ holds for all small m, and hence we have $V^D = V^D(m^D) > V^C(m^D) > V^C(m^C) = V^C$ when η is small enough

[13] In fact we can derive $f(m) = A[p_{hh}^{1-\eta} - p_{hl}^{1-\eta}](2\theta - m)^\eta m^{1-\eta} - cm$ and hence $f'(0) = +\infty$ due to $p_{hh} > p_{hl}$.

because both m^C and m^D are small and $m^D > m^C$ when η is small. This shows that the steady state welfare V^C under D–mode become higher than that under C–mode V^C when η is small enough.

Furthermore the social welfare in the high mixed steady state ($K = K^M$) is given by

$$V^M(m) \equiv A[xp_{hh} + (1-x)p_{hl}]^{1-\eta}(2\theta - m)m^{1-\eta} - [x2c + (1-x)c]m - \lambda K(m)$$

for $m = m^M$ where m^M is defined by $K(m^M) = K^M$. $V^M(m)$ lies between $V^D(m)$ and $V^C(m)$ for all m. Thus, since $m^C < m^M < m^D$, we also have $V^D(m^D) > V^M(m^M) > V^C(m^C)$ when η is small enough. This shows that the steady states with more decentralization attains higher social welfare than those with less decentralization.

5 Implications

5.1 Outsourcing vs. Vertical Integration

In this subsection we discuss the implications of the model about contracting choices of intermediate inputs, i.e., outsourcing and vertical integration. In contrast to the basic model, we assume that there are two types of goods, one final good and one intermediate input. The principals are the final good manufactures and one unit of the final good can be produced by using one unit of the intermediate input. The final good manufacturers can create the projects when they are young. These projects will yield stochastic returns when they are old by using the intermediate inputs. The output of the intermediate input Y is produced by the production function $Y = F(L)$ by using labor L. Each old agent is the intermediate input producer who initially owns the technology or knowledge to develop the intermediate input.

The projects created by the final good manufacturers are randomly matched with the intermediate input producers. The intermediate input is not traded in the market but its trading and payment decisions are made by bilateral bargaining between the final good manufacturer and the intermediate input producer who matched the project owned by the final good manufacturer.

For each matched project there are two tasks to be completed, one for "input development" and the other for "output development:" To enter the production process of final good and intermediate input, both the input and the final good must be developed by ex ante efforts. These two tasks correspond to task 1 and task 2 in the basic model. We assume that the output development can be carried out only by the final good manufacturers. However the input development can be handled by either the input producer or the final good manufacturer. We here assume that the party who developed the input can access to the production technology of the input. [14]

Then each final good manufacturer has two options for handling the input development: One is to keep the arm's length transaction with the input producer who owns the technology to develop the input and hence produces the input. This is called *outsourcing*. The other option is to buy the technology of the input development from the input producer and produce the intermediate input by herself. This is called *vertical integration*.

Extending the basic model in this way, decentralized organization (resp. centralized organization) which we have defined in the basic model corresponds to the mode of contracting choice as outsourcing (resp. vertical integration). Then Proposition 1 gives a prediction that outsourcing becomes dominant in well developed countries with smaller costs of setting up the projects such as smaller contracting costs and less regulations. This prediction might be consistent with the recent finding by Acemoglu, Johnson and Mitton (2005, 2007). Proposition 3 also suggests that both vertical integration and outsourcing emerge as organizational modes to govern the productions of intermediate inputs even in the similar economies. This might explain the stylized fact that contracting modes of intermediate inputs are different across countries. [15]

5.2 Decentralization and Economic Development

Our result also shows that the steady state with centralized organization corresponds to smaller economic development in terms of the number of total projects K^C than the steady state with decentralized organization K^D. This might suggest the testable implication that organizational modes to govern transactions become more centralized and integrated in less developed countries. Kahnna and Palepu (1997, 2000) provide supportable evidence for this and found that group affiliated firms which are based on common ownership and control outperform focused, unaffiliated firms in emerging markets such as India.

Also, as Chandler (1962) has extensively analyzed in his famous book, in the late 19th century U.S. firms became bigger through vertical integration and changed their corporate structures in the centralized way known as U–form (unitary form). However, as the U.S. market was expanded through increasing connections of railway networks, in the early 20th century the U.S. big firms shifted their corporate structures from U–form to more decentralized form known as M–form (multidivisional form). Since top management had incurred large overload costs under U–form when the market was expanded, the U.S. big firms were forced to be restructured toward more decentralization such as M–form. This U.S. business history might be con-

[14] This may be because the party who developed the input can own some knowledge and skills which are necessary for the production of the input.

[15] See for example Berger (2005) for recent evidence.

sistent with our result that organizational modes become more decentralized to pursue the gains from specialization as the market size is expanded.

6 Conclusion

In this paper we have developed the dynamic general equilibrium framework to investigate how organizations change the modes to govern transactions and how such organizational change leads to endogenous process of economic development. Then we have argued that specialization or division of labor makes decentralized organization advantageous while it is limited by the agency problem caused by asymmetric information in organizations. We have shown how centralized and decentralized organizations emerge in the economy and how they dynamically interact with the market equilibrium conditions. Depending on the financing cost underlying the economy, equilibrium organizational mode becomes centralized and decentralized. Furthermore, there exist multiple equilibrium paths some of which converge to the steady state with decentralized organization and others of which converge to the steady state with centralized organization. We have also shown that such endogenous choice of organizational modes causes endogenous fluctuation of the economy such as business cycles.

7 Appendix A

7.1 Proof of Lemma 2

$\Gamma(w,K)=0$ can be written by

$$\left[\frac{p_{hh}}{1-\beta(K)}-p_{hl}\right]\pi(w)+w-c\left[\frac{1}{1-\beta(K)}\left(\frac{p_{hh}}{\Delta p}+1\right)-1\right]=0.$$

Since π is convex function with $\pi(0)=\infty$ and $\pi(\infty)=0$, the number of the solutions for w to the above equation is generically two, given K.

Similarly $\Phi(w,K)=0$ generically has two solutions of w, given K. Q.E.D.

7.2 Proof of Proposition 1

To save notation, we will denote $K\equiv K_{t+1}$, $\rho\equiv\rho_{t+1}$ and $w\equiv w_{t+1}$. We will also use the fact $w_t=(\lambda/\theta)K_{t+1}$.

Given K, both $\hat{\rho}^D(w,K)$ and $\tilde{\rho}^D(w,K)$ coincide with each other at a point w if and only if

$$p_{hh}(c/\Delta p)-c=\hat{\rho}^D(w,K)(\lambda/\theta)K+w.$$

As we have seen in Lemma 2, there are two such points w, denoted $w^*(K)$ and $w^{**}(K)$, and we obtain $\tilde{\rho}^D(w,K)>\hat{\rho}^D(w,K)$ if and only if $w\in$

$(w^*(K), w^{**}(K))$. Furthermore, since both $\tilde{\rho}^D$ and $\hat{\rho}^D$ are decreasing and convex functions of w, we have

$$p_{hh}(c/\Delta p) - c \geq \rho(\lambda/\theta)K + w$$

for any (w, ρ) such that $\rho = \max\{\hat{\rho}^D(w, K), \tilde{\rho}^D(w, K)\}$ and $w \in (w^*(K), w^{**}(K))$ while

$$p_{hh}(c/\Delta p) - c < \rho(\lambda/\theta)K + w$$

for any (w, ρ) such that $\rho = \min\{\tilde{\rho}^D(w, K), \hat{\rho}^D(w, K)\}$ and $w \in (0, w^*(K)) \cup (w^{**}(K), \infty)$. Thus, if D–mode becomes an equilibrium and (IRD) is not binding, then $w \in (w^*(K), w^{**}(K))$ and $\rho = \hat{\rho}^D(w, K)$ must be satisfied. Also, if D–mode becomes an equilibrium and (IRD) is binding, then $w \in (0, w^*(K)) \cup (w^{**}(K), \infty)$ and $\rho = \tilde{\rho}^D(w, K)$ must be satisfied.

Furthermore, since $\rho^C(w, K) < \tilde{\rho}^D(w, K)$ for all w and K due to Assumption 2, we always have the relation that $w^*(K) < \underline{w}(K) < \overline{w}(K) < w^{**}(K)$ for any K.

Note also that, since $\lambda > \alpha(K)(\lambda/\theta)K = \lambda\beta(K)$, we have $\rho > (<)$ $\hat{\rho}^D(w, K)$ if and only if

$$\alpha(K)\{p_{hh}\pi(w) - c - p_{hh}(c/\Delta p) + \rho(\lambda/\theta)K\} < (>)\lambda\rho. \tag{*}$$

(i) Sufficiency Part: Now take an interest rate $\rho = \rho^C(w, K)$ and the market wage $w = w_C(K)$ such that $w \in [\underline{w}(K), \overline{w}(K)]$, given $K = (\theta/\lambda)w_t$. Then, since $w \in [\underline{w}(K), \overline{w}(K)]$ implies $w \in (w^*(K), w^{**}(K))$, we have

$$p_{hh}(c/\Delta p) - c > \rho^C(w, K)(\lambda/\theta)K + w$$

which implies that (IRD) is not binding under D–mode, given $\rho^C(w, K)$ and w. Then, by using inequality (*) and the fact that $w \in [\underline{w}(K), \overline{w}(K)]$ implies $\hat{\rho}^D(w, K) \leq \rho^C(w, K)$ and (IRD) is not binding for $\rho^C(w, K)$, we obtain

$$\begin{aligned}
\Pi^D(w_t, w, \rho^C(w, K)) &= p_{hh}\pi(w) - c - p_{hh}(c/\Delta p) + \rho^C(w, K)w_t \\
&= p_{hh}\pi(w) - c - p_{hh}(c/\Delta p) + \rho^C(w, K)(\lambda/\theta)K \\
&\leq \rho^C(w, K)(\lambda/\alpha(K)) \\
&= \Pi^C(w).
\end{aligned}$$

Hence each old principal optimally chooses C–mode, given w and $\rho = \rho^C(w, K)$. Also, by definition, all $\rho = \rho^C(w, K)$, $w_t = (\lambda/\theta)K$ and $w = w_C(K)$ satisfy the credit market equilibrium and labor market equilibrium conditions as well. Thus these constitute an equilibrium in period $t + 1$.

Next we take the interest rate $\rho^D \equiv \min\{\hat{\rho}^D(w, K), \tilde{\rho}^D(w, K)\}$ and the market wage $w = w_D(K)$ such that $w \in (0, \underline{w}(K)] \cup [\overline{w}(K), \infty)$. Then, since $w \in (0, \underline{w}(K)] \cup [\overline{w}(K), \infty)$, $\underline{w}(K) > w^*(K)$ and $\overline{w}(K) < w^{**}(K)$, we have

$\rho^D \equiv \min\{\hat{\rho}^D(w,K), \tilde{\rho}^D(w,K)\} \geq \rho^C(w,K)$ for such w and K. Then, we have

$$\begin{aligned}(\lambda/\alpha(K))\rho^D &= \Pi^D(w_t, w, \rho^D) \\ &\geq (\lambda/\alpha(K))\rho^C(w,K) \\ &= \Pi^C(w).\end{aligned}$$

Hence all old principals will choose D–mode, given w and $\rho = \rho^D \equiv \min\{\hat{\rho}^D(w,K), \tilde{\rho}^D(w,K)\}$.

Finally suppose that $w = \overline{w}(K)$. Then we can take $\rho = \rho^C(\overline{w}(K), K) = \hat{\rho}^D(\overline{w}(K), K)$ due to the definition of $\overline{w}(K)$. Given these market prices, we have

$$\begin{aligned}\Pi^D(w_t, w, \rho) &= (\lambda/\alpha(K))\hat{\rho}^D(w,K) \\ &= (\lambda/\alpha(K))\rho^C(w,K) \\ &= \Pi^C(w),\end{aligned}$$

which shows that each old principal is indifferent for choosing C–mdoe and D–mode given such w and ρ.

(ii) Necessity Part: Suppose that C–mode becomes an equilibrium in period $t+1$. Then $w = w_C(K)$ must hold (labor market equilibrium). Also $w \in [\underline{w}(K), \overline{w}(K)]$ must be satisfied. Suppose not. Then we have $\min\{\hat{\rho}^D(w,K), \tilde{\rho}^D(w,K)\} > \rho^C(w,K)$. Thus $\hat{\rho}^D(w,K) > \rho^C(w,K)$ or $\tilde{\rho}^D(w,K) > \rho^C(w,K)$. Then, $\rho = \rho^C(w,K)$ must hold for C–mode to be an equilibrium in period $t+1$ ((BEC–C): $\alpha(K)\Pi^C(w) = \lambda\rho^C(w,K)$). First, when (IRD) is not binding for $\rho = \rho^C(w,K)$, by using inequality (*), we obtain

$$\begin{aligned}\Pi^D(w_t, w, \rho^C(w,K)) &= p_{hh}\pi(w) - c - p_{hh}(c/\Delta p) + \rho^C(w,K)(\lambda/\theta)K \\ &> (\lambda/\alpha(K))\rho^C(w,K) \\ &= \Pi^C(w),\end{aligned}$$

which shows that each old principal should choose D–mode, given w and $\rho = \rho^C(w,K)$. This is a contradiction. Second, when (IRD) is binding for $\rho = \rho^C(w,K)$, we have

$$\begin{aligned}\Pi^D(w_t, w, \rho^C(w,K)) &= p_{hh}\pi(w) - w - 2c \\ &= (\lambda/\alpha(K))\tilde{\rho}^D(w,K) \\ &> (\lambda/\alpha(K))\rho^C(w,K) \\ &= \Pi^C(w)\end{aligned}$$

which shows that each old principal should choose D–mode again. This is a contradiction again.

Next suppose that D–mode becomes an equilibrium in period $t+1$. Then $w = w_D(K)$ must hold (labor market equilibrium). Also $w \in (0, \underline{w}(K)] \cup$

$[\overline{w}(K), \infty)$ must be satisfied. Suppose not. Then we have $\rho^C(w,K) > \hat{\rho}^D(w,K)$ and (IRD) is not binding under D–mode (since $w \in (w^*(K), w^{**}(K))$). Thus $\rho = \hat{\rho}^D(w,K)$ must hold in such D–mode equilibrium. However, then since $\rho^C(w,K) > \hat{\rho}^D(w,K)$, we obtain

$$\begin{aligned}
\Pi^C(w) &= (\lambda/\alpha(K))\rho^C(w,K) \\
&> (\lambda/\alpha(K))\hat{\rho}^D(w,K) \\
&= p_{hh}\pi(w) - c - p_{hh}(c/\Delta p) + \hat{\rho}^D(w,K)(\lambda/\theta)K \\
&= \Pi^D(w_t, w, \hat{\rho}^D(w,K)),
\end{aligned}$$

which shows that each old principal should choose C–mode, given $\rho = \hat{\rho}^D(w,K)$ and w. This is a contradiction.

Finally suppose that there exists a mixed mode equilibrium with the equilibrium prices w and ρ. Then $\rho = \hat{\rho}^D(w,K) = \rho^C(w,K)$ must be satisfied because $\Pi^D(w_t, w, \rho) = \Pi^C(w) = (\lambda/\alpha(K))\rho$ must hold in the mixed mode equilibrium. Solving $\rho^C(w,K) = \hat{\rho}^D(w,K)$ for w gives $w \in \{\underline{w}(K), \overline{w}(K)\}$. Q.E.D.

7.3 Proof of Proposition 2

For sufficiently small $\lambda > 0$, we have $\theta w^*(K) > \lambda K$ for all $K \in (0, K^D)$ because $w^*(K) \geq w_C(K) > 0$ for all $K > 0$. Then, since $K_{t+1} = (\theta/\lambda)w^*(K_t) > K_t$ for all $t \geq 0$ and for all $K_t \in (0, K^D)$, when the initial condition is given by $K_0 \in (0, K^D)$ K_t increases over time at least until it reaches K^D. Also, when λ is small enough, K^D becomes large enough and $w_D(K^D)$ approaches to $g^{-1}(\sigma/p_{hh})$, where recall that $\sigma \equiv (2\gamma)/\gamma$, because (LMC) and $\lim_{K\to\infty} m(K,\theta) = \gamma\theta$ by our assumption. Thus when λ is sufficiently small, we have $K^D \to \infty$ and $w_D(K^D) \to g^{-1}(\sigma/p_{hh})$.

Note that, due to Proposition 1, the region in which C–mode becomes a static equilibrium is given by $Q_C \equiv \{(w,K) \in \Re^2_+ \mid \overline{w}(K) \geq w \geq \underline{w}(K)\}$. Note also that $\overline{w}$ is increasing and $\underline{w}$ is decreasing. Thus, if $\overline{w}(\tilde{K}) = \underline{w}(\tilde{K})$ exists for some finite $\tilde{K}$, we have $(w_D(K^D), K^D) \notin Q_C$ for small enough λ. If $\lim_{K\to\infty} \underline{w}(K) = \lim_{K\to\infty} \overline{w}(K)$ exists, we have $(w_D(K^D), K^D) \notin Q_C$ again except for the knife–edge case that $\lim_{K\to\infty} \overline{w}(K) = \lim_{K\to\infty} \underline{w}(K) = g^{-1}(\sigma/p_{hh})$. Hence, generically the economy eventually converges to the D–mode steady state K^D in the long run when λ is sufficiently small.

When the initial condition K_0 satisfies $K_0 > K^D$, we have $\lambda K_t > \theta w_D(K_t)$ for all $K_t > K^D$. Since $(w_D(K^D), K^D) \notin Q_C$ generically holds when λ is small enough, we have $(w(K_t), K_t) \notin Q_C$ for all $K_t > K_D$ as well when λ is small enough. Thus K_t eventually converges to the D–mode steady state K^D in the long run. Q.E.D.

7.4 Proof of Proposition 3

Note first that λ affects only for determining the steady state values K^D and K^C. Then we can take the values of $\lambda > 0$ such that $K'' < K^C < K'$ and $w_D(K^C) > \overline{w}(K^C)$ are satisfied. This is done by taking the values of λ to ensure that $K^C < K'$ but K^C is close to K' (See Figure 4). Since all the functions $\underline{w}$, $\overline{w}$ and w_i $(i = D, C)$ are independent of λ, we can actually find the range of such parameter values of λ.

Then, given such λ, if the economy starts from $K_0 \in (K'', K')$, multiple equilibrium paths arise where some of them converge to the D–mode steady state K^D but others converge to the C–mode steady state K^C (See Figure 4). To see this, note that $w_C(K^C) \in (\underline{w}(K^C), \overline{w}(K^C))$ and $w_D(K^C) > \overline{w}(K^C)$. Also we have $w_C(K_0) \in (\underline{w}(0), \overline{w}(0))$ for $K_0 \in (K'', K')$. Thus C–mode becomes an equilibrium at initial period ($t = 0$). Then, since $w_C(K^C) \in (\underline{w}(K^C), \overline{w}(K^C))$, the economy can converge to the C–mode steady state K^C. On the other hand, since $w_D(K^C) > \overline{w}(K^C)$, there exists some period, say $T \geq 1$, such that D–mode can be an equilibrium from period T onward and the economy converges to the D–mode steady state K^D as well.

However, if the economy starts at $K_0 \geq K'$, then we have $w_C(K_t) > \overline{w}(K_t)$ for all t. Thus there are no equilibria which converge to the C–mode steady state K^C. Q.E.D.

8 Appendix B

In the main text we have made the following conditions:

- **Assumption 2**. $\pi(g^{-1}(1/p_{hh})) > \max\{(g^{-1}(1/p_{hh})+2c)/p_{hh}, c/\Delta p + c/p_{hh}\}$.
- **Assumption 4**. (i) $\min_{w \geq 0} \Gamma(w, \infty) < 0$, *and* (ii) $g^{-1}(\sigma/p_{hl}) > \overline{w}(\infty)$.
- It is optimal for any old principal to implement high effort pair $(a, e_m) = (h, h)$ from any matched project under D–mode:

  $\Pi(w_t, w_{t+1}, \rho_{t+1}) \geq \Pi^*(w_{t+1}) \equiv \max\{p_{ll}\pi(w_{t+1}) - w_{t+1}, p_{hl}\pi(w_{t+1}) - w_{t+1} - c\}$.
- (ICPD) is satisfied at the optimal contract in Problem D.
- It is optimal for any old principal to implement effort pair $(a, e_p) = (h, l)$ from any matched project under C–mode.
- (ICPC$'$) is satisfied at the optimal contract in Problem C.

In this Appendix we will give some parametric conditions for all these restrictions to be satisfied simultaneously. To this end we will assume that the production function $Y = F(L)$ takes the following form: $Y = AL^{\eta}$ where $A > 0$ and $\eta \in (0, 1)$.

Then we can show the following:

$$\pi(g^{-1}(1/p_{hh})) = (1-\eta)Ap_{hh}^{-\eta},$$

and

$$g^{-1}(1/p_{hh}) = \eta A p_{hh}^{1-\eta}.$$

Thus we have

$$\pi(g^{-1}(1/p_{hh})) > g^{-1}(1/p_{hh})/p_{hl}$$

as long as

$$(1-\eta)/\eta > p_{hh}/p_{hl}. \tag{A1}$$

(i) Given (A1), Assumption 2 will be satisfied when $c > 0$ is small enough. In what follows we will assume (A1). We also define the following function:

$$\begin{aligned} G(A) &\equiv \pi(g^{-1}(1/p_{hh})) - c/\Delta p \\ &= (1-\eta)Ap_{hh}^{\eta} - c/\Delta p. \end{aligned}$$

Next, letting $\overline{\Gamma}(w) \equiv \Gamma(w, \infty)$, we can show that $\overline{\Gamma}(w)$ attains its minimum at $w = w_{min}$ where

$$w_{min} \equiv \eta A \left(\frac{1}{1-\overline{\beta}} p_{hh} - p_{hl} \right)^{1-\eta}$$

where $\overline{\beta} \equiv \lim_{K\to\infty} \beta(K) < \gamma \in (0, 1)$ by our assumption. Then we can show that

$$\min_{w \geq 0} \overline{\Gamma}(w) = H(A) \equiv \frac{1+\eta}{\eta} w_{min} + c \left[1 - \frac{1}{1-\overline{\beta}} \left(\frac{p_{hh}}{\Delta p} + 1 \right) \right].$$

Now, by taking small $\overline{\beta}$ (so small γ) and small Δp (letting p_{hl} close to p_{hh}), then we can verify that $H'(A) > 0$ takes a small value but $G'(A) = (1-\eta)p_{hh}^{\eta}$ is not changed. Thus, in such case there exist some $\underline{A}$ and $\overline{A}$ such that for all $A \in (\underline{A}, \overline{A})$ we have $G(A) > 0 > H(A)$ where $H(\overline{A}) = G(\underline{A}) = 0$. This shows that when $\overline{\beta}$ and Δp are small we obtain

$$\pi(g^{-1}(1/p_{hh})) > c/\Delta p,$$

and

$$\min_{w \geq 0} \overline{\Gamma}(w) < 0.$$

Given such $A \in (\underline{A}, \overline{A})$, we can find some small $c > 0$ such that Assumption 2 is satisfied.[16] In what follows we will assume $A \in (\underline{A}, \overline{A})$.

[16] In fact we can take small $c/\Delta p$ by keeping Δp small as well.

(ii) Next we will verify Assumption 4(ii). Taking A close to $\overline{A}$ defined above, $\lim_{K\to\infty}\overline{w}(K) = \lim_{K\to\infty}\underline{w}(K) = w_{min}$ because $H(\overline{A}) = \min_w \overline{\Gamma}(w) = 0$. Recalling that $w_{min} = \eta A((1/(1-\overline{\beta}))p_{hh} - p_{hl})$, when $A \to \overline{A}$ we have Assumption 4(ii):

$$g^{-1}(\sigma/p_{hl}) > \overline{w}(\infty) \simeq w_{min}$$

as long as

$$(\sigma/p_{hl})^{1-\eta} > \left(\frac{1}{1-\overline{\beta}}p_{hh} - p_{hl}\right) \tag{A3}$$

which we will assume.

(iii) Next we will show that it becomes optimal for any old principal to implement high effort pair $(a, e_m) = (h, h)$ from both tasks of any matched project under D–mode. First note that, due to Assumption 2, we have

$$\Pi^*(w_{t+1}) = p_{hl}\pi(w_{t+1}) - w_{t+1} - c.$$

Thus it is sufficient to show that

$$p_{hh}\pi(w_{t+1}) - c - \max\{p_h h(c/\Delta p) - \rho_{t+1}w_t, w_{t+1} + c\} \geq p_{hl}\pi(w_{t+1}) - w_{t+1} - c,$$

which will be satisfied if

$$\pi(w_{t+1}) > p_{hh}(c/\Delta p^2)$$

because $\rho_{t+1}w_t \geq 0$, $w_{t+1} \geq 0$ and Assumption 2. However, since $w_{t+1} \leq g^{-1}(1/p_{hh})$ in any equilibrium, this inequality will be satisfied if

$$\pi(g^{-1}(1/p_{hh})) > p_{hh}(c/\Delta p^2)$$

which can be written by

$$(1-\eta)Ap_{hh}^{-\eta} > (c/\Delta p^2). \tag{A4}$$

Now let $k \equiv c/\Delta p^2$ and keep k constant and small. Then, we can take small Δp and $c/\Delta p$ while keeping k constant and small. Hence (A4) will be satisfied for small k, given $A \in (\underline{A}, \overline{A})$.

(iv) Next we will check that (ICPD) in Problem D will be satisfied at the optimal D–mode contract without (ICPD). To see this, first consider the case that (IRD) is binding at the optimal D–mode contract without (ICPD). In particular let $r^D_{t+1} = 0$ and $R^D_{t+1} = (w_{t+1} + c)/p_{hh}$ which satisfy (IRD) and (LLD). Then (ICPD) can be written by

$$\pi(w_{t+1}) \geq c/\Delta p + (w_{t+1} + c)/p_{hh},$$

which will be satisfied if

$$\pi(g^{-1}(1/p_{hh})) > c/\Delta p + (g^{-1}(1/p_{hh}) + c)/p_{hh}. \tag{A6}$$

However, given (A1), when $c/\Delta p$ and c are small, (A6) will hold as well. Second, suppose that (IRD) is not binding at the optimal contract without (ICPD). Then we have $r^D_{t+1} = -\rho_{t+1}w_t$ and $R^D_{t+1} = p_{hh}(c/\Delta p) - \rho_{t+1}w_t$ by Lemma 1. Then (ICPD) can be written by

$$\pi(w_{t+1}) \geq c/\Delta p + p_{hh}(c/\Delta p)$$

which will be satisfied if

$$\pi(g^{-1}(1/p_{hh})) > (1 + p_{hh})(c/\Delta p). \tag{A7}$$

However, when $(c/\Delta p)$ is small, (A7) will hold as well.

(v) Next we will show that it is optimal for any old principal to implement effort pair $(a, e_p) = (h, l)$ from any matched project under C–mode. As we have seen, it is never optimal to implement high effort pair $(a, e_p) = (h, h)$ due to Assumption 1. Thus the remaining case is that the principal implements low effort pair (l, l): In that case the principal can obtain at most $p_{ll}\pi(w_{t+1}) - w_{t+1}$. Thus it is not optimal to implement low effort pair if $\Delta q\pi(w_{t+1}) \geq c$ which will be satisfied by (A7) due to $\Delta q > \Delta p$.

(vi) Finally (ICPC′) will be satisfied when $\pi(g^{-1}(1/p_{hh})) > g^{-1}(1/p_{hh})/p_{hl} + c/\Delta q$. This will hold when $c > 0$ is small, given (A1).

8.1 Appendix C

We will modify the basic model by introducing the possibility of default by young principals in credit market and show that the parameter λ can be interpreted as the measure of contracting costs in credit market.

Instead of assuming perfectly competitive credit market, we consider the Bertrand competition among young principals who compete each other to attract the fund for project financing by offering interest rates to creditors (young agents). Suppose that each young principal may renege on making repayment and be in default after she borrows from creditors. Suppose also that each young agent (creditor) can access to the storage technology which converts one unit good in the current period into $\varepsilon > 0$ goods in the next period. Thus, each young agent never lends to a young principal unless she can commit herself not to go on default (otherwise the agent will use the storage technology instead of lending to credit market). However, each young principal can develop the verification technology by which she can perfectly reveal the hard evidence about her repayment decision and persuade creditors not to be in default. To develop such verification technology, each young principal needs to raise λ goods per project.

Then we consider the following two–stage game in credit market in each period (say period t): In the first stage young principals simultaneously offer the loan contracts which specify the interest rate to be paid to creditors in period $t+1$ and commitment to the development of the verification technologies. Let denote by ρ_{t+1}^{i} the interest rate offered by young principal i in period t (thus it will be paid in period $t+1$). In the second stage, given the loan contracts offered by young principals, young agents (creditors) decide to which young principals they should lend their wage income w_t. Then only the young principals who developed the verification technologies can attract credit from young agents (because otherwise young agents can use the storage technology to obtain ε unit goods when they are old). Also, by the Bertrand competition among young principals, they will bid up the interest rates to make them break even. Thus they will offer the same interest rate $\rho_{t+1} \equiv \rho_{t+1}^{i}$ in equilibrium, and the equilibrium interest rate ρ_{t+1} is determined by the same break even conditions (BEC–C) and (BEC–D) as we have already derived in the main text. Finally we assume that ε is small enough so that young agents never use the storage technology in equilibrium.

By modifying the model in the above way, we can interpret λ as the parameter value to measure how it is costly to detect default in credit market. Put differently λ captures the contracting costs incurred by the principals (borrowers) to commit themselves not to go on default in credit market.

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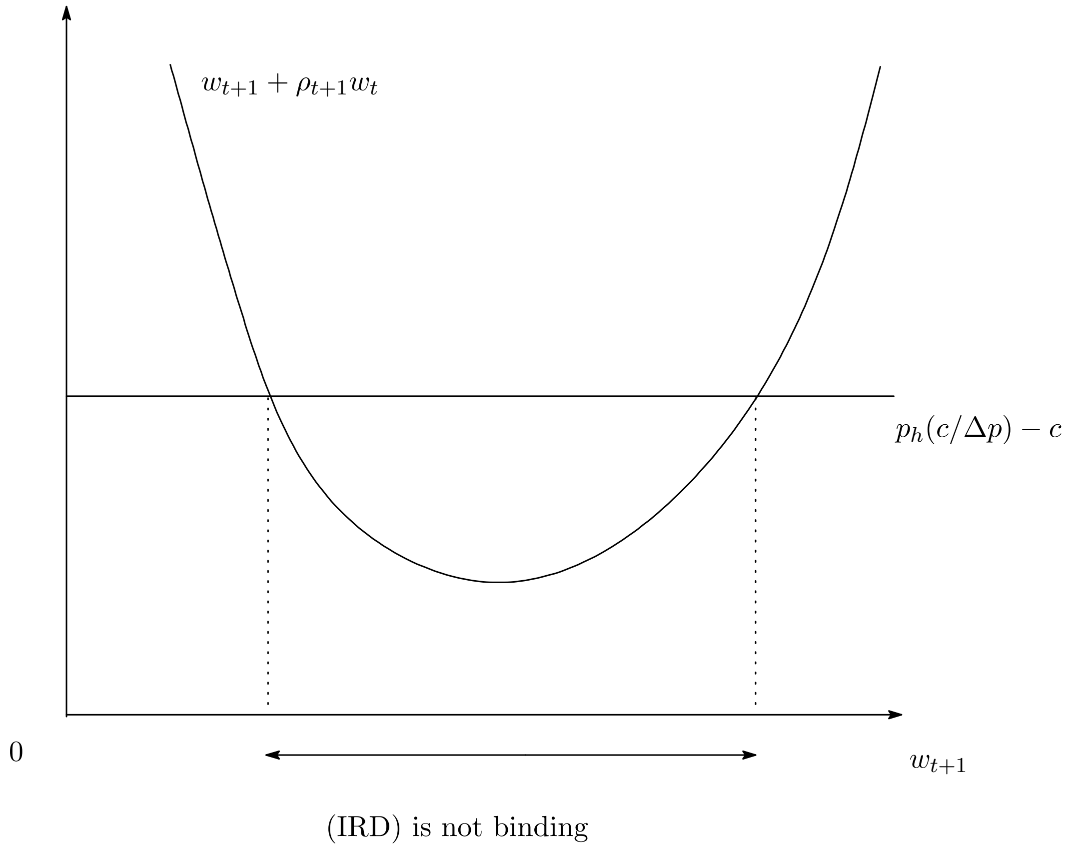

Figure 1: Optimal D–mode Contract

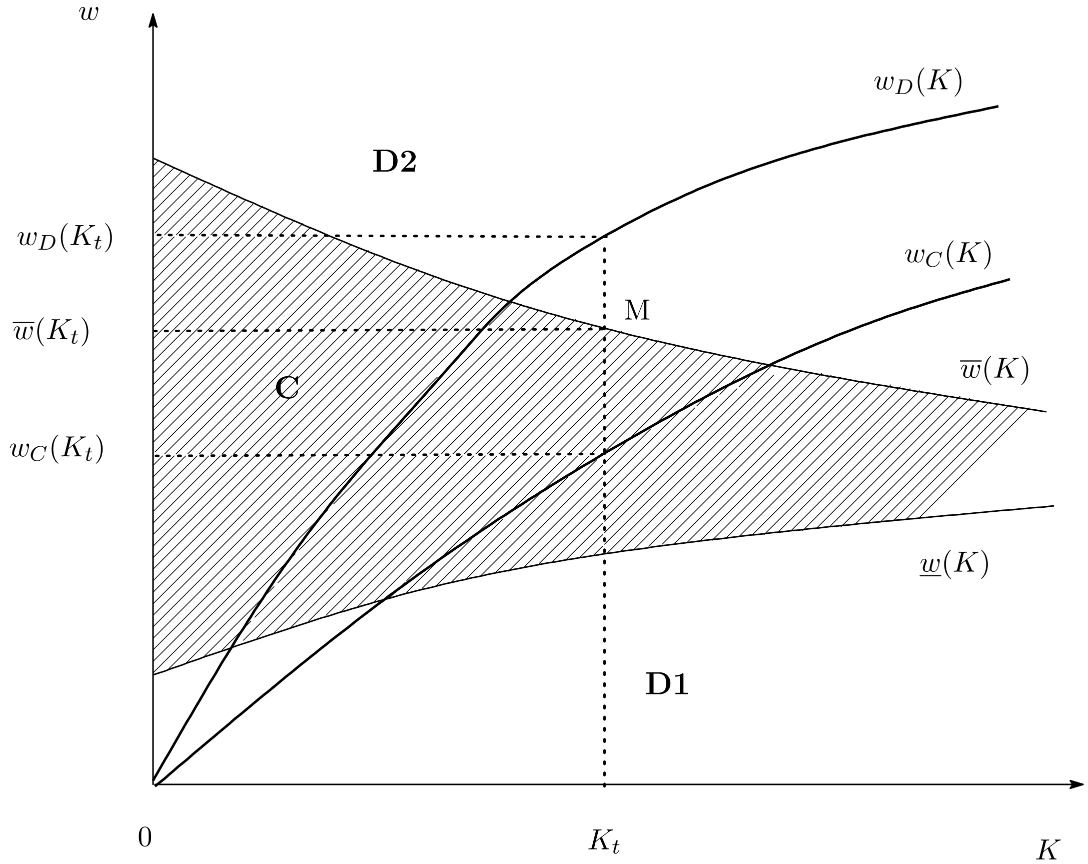

Figure 2

Patterns of Equilibrium Organziations

D1, D2: D–mode. C: C–mode

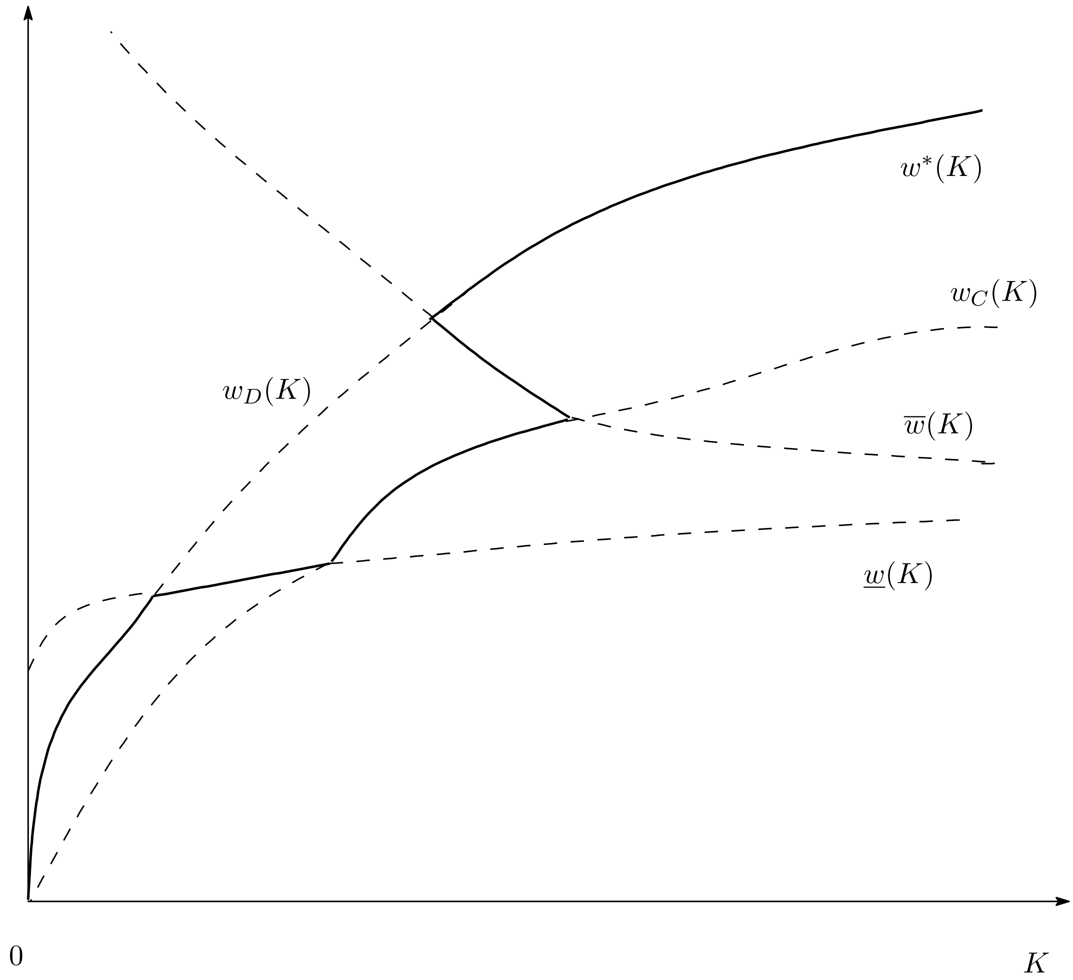

Figure 3: Market Wage Function

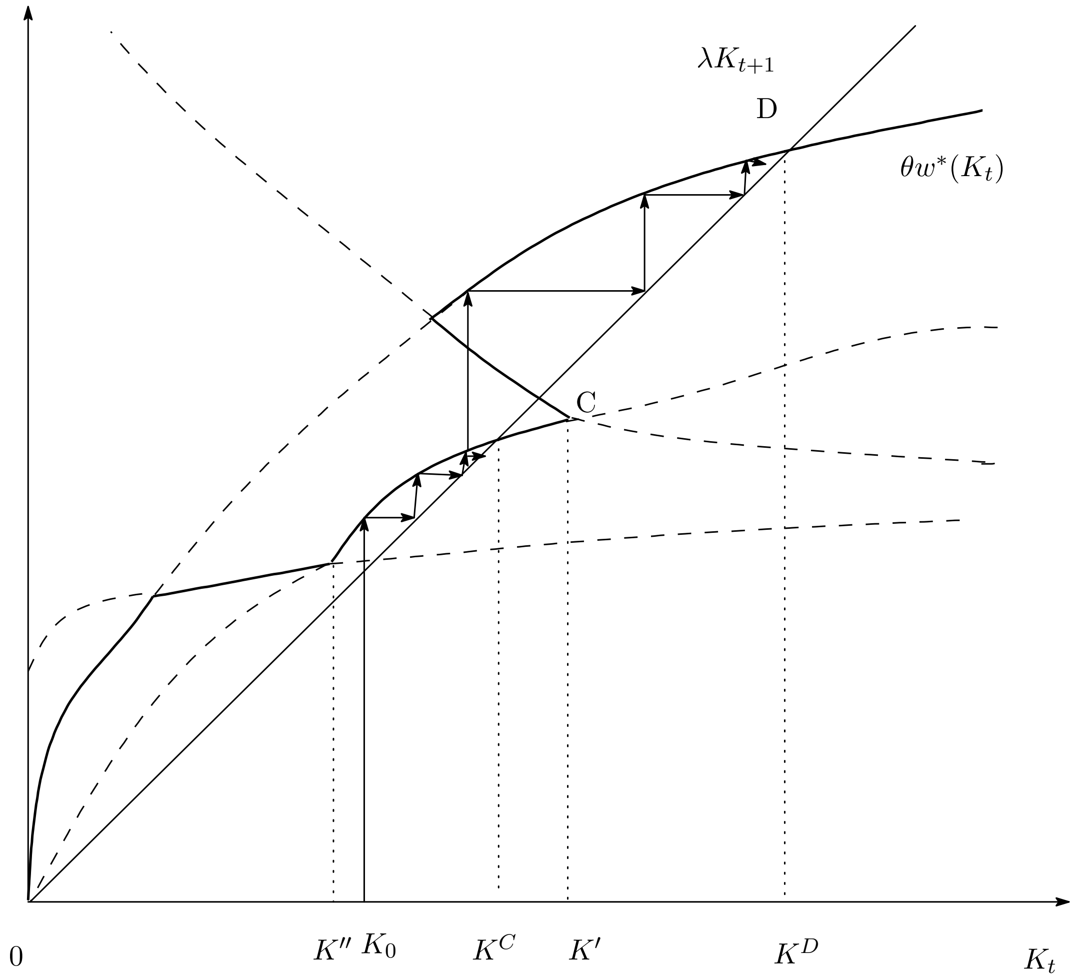

Figure 4: Multiple Equilibrium Paths

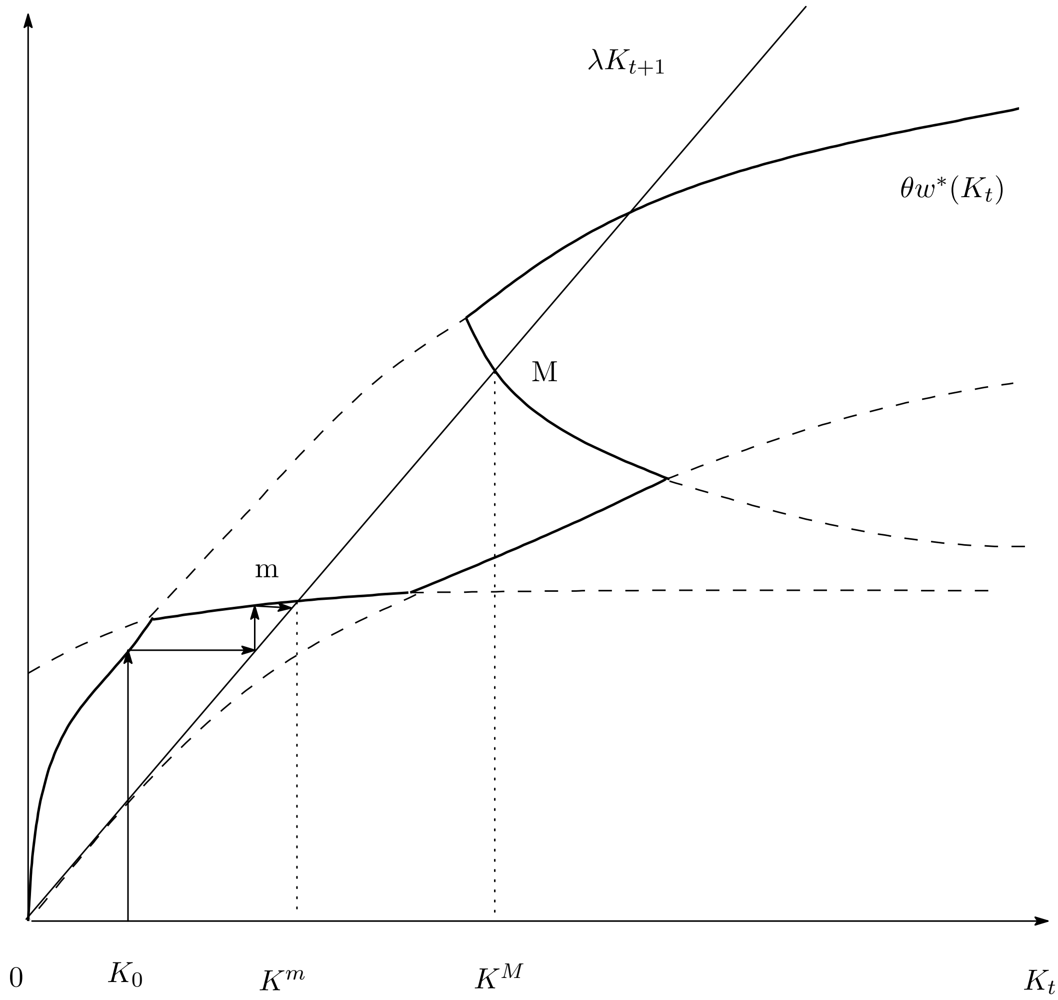

Figure 5: Mixed Steady States

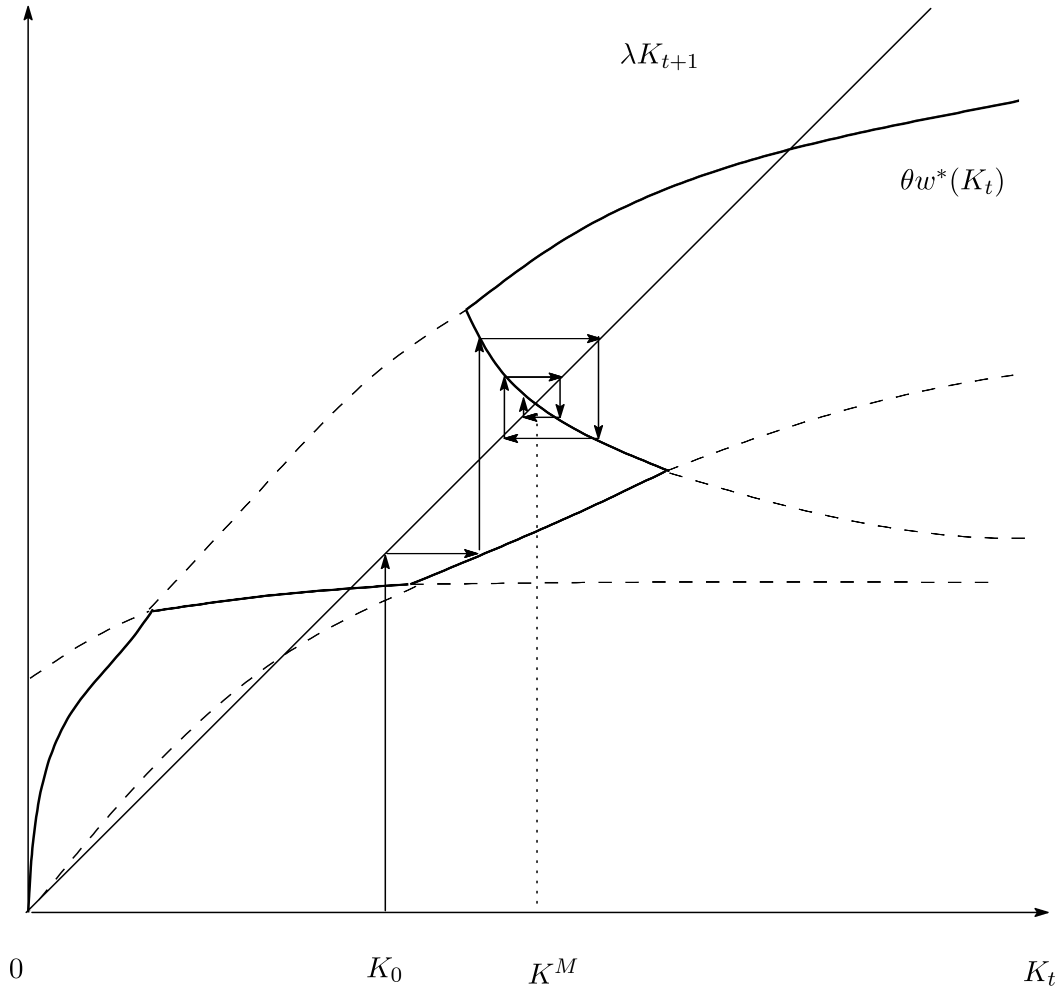

Figure 6: Endogenous Fluctuation

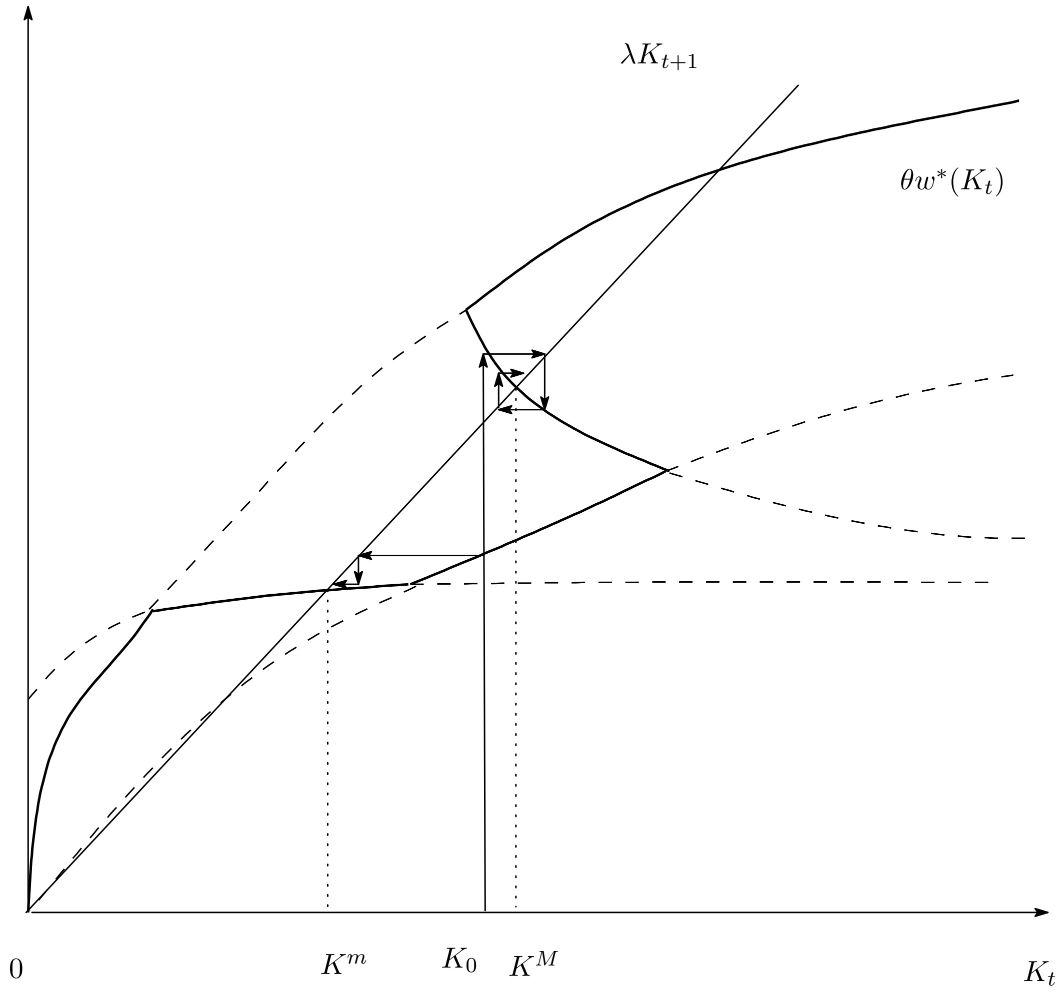

Figure 7: Multiple Mixed Equilibrium Paths