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Abstract

To investigate how fertility rates interrelate with the modern economy, we construct a simple model in which variety expansion of consumption goods reduces fertility rates. In our model, variety expansion reduces the relative price of a composite of differentiated goods compared to child-rearing costs. Thus, parents raise the expenditure share for differentiated goods and lower the number of children. We show that this model can be applied to a growth model in which economic growth progresses with variety expansion of consumption goods and fertility rates decrease with economic growth. Thus, we show a new mechanism for fertility decline, and this mechanism can be applied to a growth model.

JEL classification: J13; O10; F12

Kew words: Consumerism; Fertility rates; Variety expansion; Economic growth

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1 Introduction

In the modern world, fertility rates have decreased in developed countries. A low fertility rate is an important feature that identifies a modern developed economy. For example, from 1960 to 2000, the total fertility rate respectively decreased from 2.00 to 1.36 in Japan, from 2.72 to 1.64 in the U. K., from 2.37 to 1.36 in Germany, from 2.73 to 1.88 in France, from 2.20 to 1.54 in Sweden, and from 3.64 to 2.06 in the U. S. (Cabinet Office, Government of Japan (2004)).

Some researchers, e.g., Lutz (1996), have stated that, in modern industrialized countries, parents prefer to consume goods rather than to bear children. He pointed out that "consumerism" is the basis of the decline in fertility rates in modern developed countries. Lutz (1996) states:

Commentators often mention the increase in consumerism as a basic underlying cause for the recent fertility decline. The argument is that people would rather invest in pleasures for themselves than in children; they would rather buy a new car than have another child; they would rather spend their time watching TV than changing diapers. (pp. 273)

Our purpose in this paper is to present a model in which "consumerism" is a cause of fertility decline. [1] We present a model in which "consumerism" is induced by the "variety expansion" of consumption goods. In our model, we assume that parents receive utility from the consumption of differentiated goods and the number of their children.[2] Parents allocate their fixed time to working or rearing children. Thus, there is trade-off between nominal income and children. In our model, variety expansion lowers relative price of composite of differentiated goods. Then, variety expansion induces parents to extend the expenditure share for differentiated goods. With this mechanism, parents have fewer children when the variety of differentiated goods that they can consume increases. We show that this mechanism can be used to explain the decline in fertility in the process of economic growth.

As we wrote above, "consumerism" is induced by the variety expansion of consumption goods. In developed countries, there is a large variety of consumption goods, which enhances "consumerism". The variety of consumption goods has increased with economic growth. In modern developed countries, the economy has grown continuously. The average GDP growth rate from 1981 to 2005 was 2.3% in Japan, 2.5% in the U. K., 1.9% in Germany, 2.0% in France, 2.2% in Sweden, and 3.1% in the U. S. As Grossman and Helpman (1991) and Romer (1990) have shown, the variety of consumption goods increases with economic growth in developed countries. Therefore, in those countries, the variety of consumption goods increases with economic growth.

With economic growth, a large variety of consumption goods becomes an

[1] In this paper, agents engaging in "consumerism" spend a larger share of their income (time) on consumption than on rearing children.

[2] In much of the literature, e.g., Becker, Murphy, and Tamura (1991), Eckstein and Wolpin (1985), and Galor and Weil (1996) (2000), it is assumed that parents receive utility from the number of their children. We have followed their lead and assume that parents obtain utility from the number of their offspring.

important feature of a developed economy. Given these important features of a developed economy, in this paper, we construct a model in which variety expansion affects the fertility rates and variety expansion is induced by economic growth.

Theoretically, our purpose is to present a tractable model in which the variety of consumption goods increases with economic growth, thereby reducing the fertility rates. We show that important examples of variety expansion-type growth models, such as those produced by Grossman and Helpman (1991) can be applied to the endogenous fertility model. With this model, we present a new mechanism through which fertility rates are related with economic activities.

Our model of economic growth integrates the endogenous fertility model and the Grossman-Helpman-Romer (GHR) type of the variety expansion growth model. The GHR growth model has a large influence on the study of economic growth. However, there are not any studies on endogenous fertility in the context of the GHR model. Our model shows that the GHR model can be applied to the model of an endogenous fertility rate.

Many studies have explained the mechanism of fertility decline in the process of economic development. Becker, Murphy, and Tamura (1991) wrote a seminal study that presents a model in which fertility is closely related with human capital accumulation. In their model, parents obtain utility not only from consumption but also from the quantity and quality of their children. Each parent allocates his/her fixed time to working, parenting, and educating children. Hence, there is a quantity-quality trade-off of children. They show that, in the process of economic development, the value of education increases. Then, parents lower the number of children and allocate much of their time to educating their children. Galor and Weil (2000) and Tamura (2002) showed that, with this quantity-quality trade-off, fertility decline is induced by technological progress. Kalemli-Ozcan et al. (2000) and Kalemli-Ozcan (2002, 2003) pointed out that, with a quantity-quality trade-off, fertility decline progresses by a decline in the infant mortality rates.

In the literature cited above, it is commonly assumed that human capital accumulation plays an important role in the decline of the fertility rate. Other explanations have also been given. Galor and Weil (1996) reported that a decline in the fertility rate is induced by a rise in relative wages earned by women. They showed that the relative wage earned by women increases with capital accumulation. Increasing the relative wages of women reduces the fertility rate by causing the cost of child-rearing to exceed the household income. Sato and Yamamoto (2005) constructed a model in which urbanization induces agglomeration economy and congestion diseconomies and the fertility rate decreases with urbanization. In their model, the agglomeration economy raises parents' income, which raises the fertility rate by the income effect and reduces the fertility rate by the substitution effect, and congestion diseconomies lower parents' income, which reduces the fertility rate. They showed that the substitution effect and congestion diseconomies overcome the income effect of an agglomeration economy and urbanization and economic growth reduce the fertility rates. These studies examine important causes of the fertility decline in the modern developed economy. However, as Lutz (1996) stated, "consumerism" is an important aspect of the modern developed economy which reduces the fertility rates. Thus, our paper presents another important mechanism of fertility decline.

The organization of this paper is as follows. In Section 2, we present a simple benchmark partial equilibrium model in which variety expansion lowers fertility rates. In Section 3, we present a growth model that uses the mechanism of the benchmark model. Section 4 is the conclusion.

2 A simple basic model

In this section, we present a basic model in which an agent reduces the number of children with the variety expansion of differentiated goods. In the following section, we apply this basic model to the growth model.

We assume that agents obtain utility from the consumption of differentiated goods and number of children:

$$U = u(C, m), \tag{1}$$

where

$$C = \left(\int_0^n x(i)^\rho di\right)^{\frac{1}{\rho}},$$

where n is the measure of variety of differentiated goods, C is the composite of consumption goods, $x(i)$ is the consumption of differentiated goods indexed i, and m is the number of children. We assume that $0 < \rho < 1$. $\sigma = \frac{1}{1-\rho} > 1$ represents the elasticity of substitution among differentiated goods.

We assume that agents have one amount of time that can be used for working or rearing children. Following Becker (1965) and others, we assume that, if they have a child, they have to use time τ to rear a child. Their budget constraint becomes

$$\begin{gathered} w(1-\tau m) = \int_0^n p(i)x(i)di, \\ w = \int_0^n p(i)x(i)di + w\tau m. \end{gathered} \tag{2}$$

We follow Dixit and Stiglitz (1977) and solve the utility maximization problem in two steps. First, regardless of the value of the composite of consumption goods, C, each $x(i)$ needs to be chosen so as to minimize the cost of attaining C. This means solving the following minimization problem:

$$\begin{gathered} \min \int_0^n p(i)x(i)di, \\ s.t. \left(\int_0^n x(i)^\rho di\right)^{\frac{1}{\rho}} = C. \end{gathered} \tag{3}$$

The first-order condition to this expenditure minimization problem gives equality of marginal rates of substitution to price ratios,

$$\frac{x(i)^{\rho-1}}{x(j)^{\rho-1}} = \frac{p(i)}{p(j)}. \tag{4}$$

We substitute (4) to $\left(\int_0^n x(i)^\rho di\right)^{\frac{1}{\rho}} = C$, and get

$$x(j) = \frac{p(j)^{1/(\rho-1)}}{\left(\int_0^n p(i)^{\rho/(\rho-1)} di\right)^{1/\rho}} C. \tag{5}$$

This simply represents the compensated demand function for the j th variety of a consumption product. We can also derive an expression for the minimum cost of attaining C. From (5), we can derive

$$p(j)x(j) = \frac{p(j)^{\rho/(\rho-1)}}{\left(\int_0^n p(i)^{\rho/(\rho-1)} di\right)^{1/\rho}} C.$$

We integrate the above equation over j and obtain

$$\int_0^n p(j)x(j)dj = \left(\int_0^n p(i)^{\rho/(\rho-1)} di\right)^{(\rho-1)/\rho} C.$$

We define the term multiplying C on the right-hand side (RHS) of the above equation as a price index:

$$P \equiv \left(\int_0^n p(i)^{\rho/(\rho-1)} di\right)^{(\rho-1)/\rho} = \left(\int_0^n p(i)^{1-\sigma} di\right)^{1/(1-\sigma)}. \tag{6}$$

With (6), the price index times the quantity composite is equal to the expenditure:

$$\int_0^n p(i)x(i)di = PC. \tag{7}$$

In our setting, the consumer has a preference for the variety of differentiated goods. We can derive (use Leibniz rule)[3]

$$\frac{\partial P}{\partial n} = \frac{\rho-1}{\rho}\left(\int_0^n p(i)^{\frac{\rho}{\rho-1}} di\right)^{\frac{-1}{\rho}} p(n)^{\frac{\rho}{\rho-1}} < 0. \tag{8}$$

Thus, if the variety of differentiated goods increases, the expenditure which enables a consumer to obtain a given amount of C decreases.

Demand for $x(j)$ is derived as follows:

$$x(j) = \left(\frac{p(j)}{P}\right)^{\frac{1}{\rho-1}} C. \tag{9}$$

[3] In this section, we ignore the production side of the economy and assume that each price of differentiated goods is constant. In Section 3, we consider the production side of the economy.

The next step of the consumer's problem is to determine the number of children and the expenditure for C. We substitute (7) into the budget constraint, and the second step problem is defined as

$$\max U = u(C, m),$$
$$s.t. w = PC + w\tau m.$$

We define the solutions of the above problem as follows:

$$C = C(P, w\tau, w), \tag{10}$$

and

$$m = m(P, w\tau, w). \tag{11}$$

These two are demand functions for a composite of consumption goods and children. We substitute (10) and (11) into the budget constraint and differentiate it with P:

$$C + P\frac{\partial C(P, w\tau, w)}{\partial P} + w\tau\frac{\partial m(P, w\tau, w)}{\partial P} = 0. \tag{12}$$

Equation (12) can be rewritten as:

$$\frac{w\tau}{C}\frac{\partial m(P, w\tau, w)}{\partial P} = -1 - \frac{\partial C(P, w\tau, w)}{\partial P}\frac{P}{C} = -1 + \eta. \tag{13}$$

Equation (13) shows that, if $\eta > 1$, $\frac{\partial m(P,w\tau,w)}{\partial P} > 0$. $\eta \equiv -\frac{\partial C(P,w\tau,w)}{\partial P}\frac{P}{C}$ is the price elasticity of the demand for the composite of consumption goods.

From (8), P is a decreasing function of consumption variety n. Therefore, if $\frac{\partial m(P,w\tau,w)}{\partial P} > 0$, $\frac{\partial m(P,w\tau,w)}{\partial P}\frac{\partial P}{\partial n} < 0$: the number of children decreases with variety expansion. The discussion above leads us to the next proposition:

Proposition 1 *If $\eta > 1$, the number of children decreases with the variety expansion.*

We show that the variety expansion reduces the fertility rate, if $\eta > 1$. In our setting, the consumer has a preference for a variety of differentiated goods. When the variety of differentiated goods increases, the price index of differentiated goods decreases (see (8)). Thus, the variety expansion lowers the relative price of the composite of differentiated goods and raises the expenditure share for differentiated goods. Therefore, the variety expansion raises the relative price of children and reduces the expenditure share for children. With this mechanism, the variety expansion reduces the fertility rate.

Here, we specify the utility function as a CES function:

$$U = u(C, m) = \left[C^{\psi} + m^{\psi}\right]^{\frac{1}{\psi}}. \tag{14}$$

We assume that $0 < \psi < 1$. $\theta = \frac{1}{1-\psi} > 1$ represents the elasticity of substitution between children and the composite of differentiated goods.

With this specification, the demand function for the composite of differentiated goods and children become

$$C = \frac{w}{P + (w\tau)^{\frac{-\psi}{1-\psi}} P^{\frac{1}{1-\psi}}}, \tag{15}$$

$$m = \frac{w}{P^{\frac{-\psi}{1-\psi}} (w\tau)^{\frac{1}{1-\psi}} + w\tau}. \tag{16}$$

The price elasticity of the demand for a composite of differentiated goods under this demand function is derived as follows:

$$\eta_{CES} = \frac{P + \frac{1}{1-\psi}(w\tau)^{\frac{-\psi}{1-\psi}} P^{\frac{1}{1-\psi}}}{P + (w\tau)^{\frac{-\psi}{1-\psi}} P^{\frac{1}{1-\psi}}}. \tag{17}$$

Equation (17) shows that $\eta_{CES} > 1$, if $0 < \psi < 1$. Thus, under the CES utility function, variety expansion reduces the number of children.

The simple partial equilibrium model in this section shows that variety expansion reduces the number of children of a consumer. In Sections 3, we show that this simple mechanism can be used in the growth model. For the analytical simplicity, we assume that the utility function is a CES form in next section. In that section, we show that the fertility rate decreases with economic growth.

3 A growth model

In this section, we apply a basic model in the above section to a growth model. Our model in this section is an overlapping generation model in which agents live two periods, childhood and adulthood. In childhood, agents do not perform any economic activities; they are reared by their parents. In adulthood, the agents consume differentiated goods and determine the number of children they will have. Thus, the consumption behavior of agents is the same as that in the model in Section 1. We assume that labor is the numeraire, $w = 1$. There is a continuum of population, L_t, at period t. The utility function of adult agents at period t is

$$U_t = \left[C_t^{\psi} + m_t^{\psi}\right]^{\frac{1}{\psi}}, \tag{18}$$

where

$$C_t = \left(\int_0^{n_t} x_t(i)^{\rho} di\right)^{\frac{1}{\rho}},$$

and their budget constraints are[4]

$$1 - \tau m_t = \int_0^{n_t} p_t(i) x_t(i) di.$$

[4]In our model, we assume that there is no capital. As we show later, the firms obtain zero profits. Thus, the income of agents is composed only with their wages.

The same procedure that derives (15) and (16) leads us to the demand functions for the composite of differentiated goods and children:

$$C_t = \frac{1}{(P_t + P_t^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})}, \tag{19}$$

$$m_t = \frac{1}{P_t^{\frac{-\psi}{1-\psi}} \tau^{\frac{1}{1-\psi}} + \tau}. \tag{20}$$

Substituting (19) into (9), we can derive a consumer's demand function for each differentiated good:

$$x_t(j) = \frac{P_t^{\frac{1}{1-\rho}} p_t(j)^{\frac{1}{\rho-1}}}{(P_t + P_t^{\frac{1}{1-\psi}} \tau^{\frac{-\psi}{1-\psi}})}. \tag{21}$$

In this growth model, we follow Grossman and Helpman (1991) and assume that the variety of differentiated goods increases with innovation activities. In our model, we assume that the innovation activity produces a patent. With this patent, a firm produces a variety of differentiated goods with a constant return to scale technology: $x(i)$ units of labor can produce $x(i)$ units of differentiated goods. To produce a patent in period t requires I_t units of labor. Here, we assume that $I_t = \frac{1}{n_{t-1}^{\gamma}}$, $\gamma \geq 1$. $I_t = \frac{1}{n_{t-1}^{\gamma}}$ means that there is knowledge accumulation in this economy. As in standard endogenous growth models, such as those developed by Grossman and Helpman (1991) and Romer (1990), for each innovation firm, this knowledge accumulation is externality. The term of the validity of a patent is one period. After the term of the validity of a patent, differentiated goods are produced by perfectly competitive firms. The price of those goods is $p = 1$. At the validity term of the patent, firms set the standard mark-up price. Each firm sets the price $p_t(i) = p_t(j) = \frac{\sigma}{\sigma-1} = \frac{1}{\rho}$, which is not affected by n.

Here, $P_t = (\int_0^{n_t} p(i)^{\frac{\rho}{\rho-1}} di)^{\frac{\rho-1}{\rho}} = (n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-1}{\rho}}$, where $n_t = n_{t-1} + k_t$ and k_t represents the number of varieties that are created at period t. Using this and (21), the demand q_t faced by a firm that is created at period t is

$$q_t = \frac{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{-1}{\rho}} p^{\frac{1}{\rho-1}}}{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-1}{\rho}} + (n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-1}{\rho(1-\psi)}} \tau^{\frac{-\psi}{1-\psi}}} L_t. \tag{22}$$

Then, the instantaneous profits of a firm are

$$\pi_t = (\frac{1}{\sigma-1}) \frac{p^{\frac{1}{\rho-1}}}{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}}) + (n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-\psi}{\rho(1-\psi)}} \tau^{\frac{-\psi}{1-\psi}}} L_t. \tag{23}$$

The free-entry condition in the innovation market means that $I_t = \pi_t$. From (23), the free-entry condition is written as

$$(\frac{1}{n_{t-1}})^{\gamma} = (\frac{1}{\sigma - 1})\frac{p^{\frac{1}{\rho-1}}}{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}}) + (n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\rho-\psi}{\rho(1-\psi)}} \tau^{\frac{-\psi}{1-\psi}}} L_t. \tag{24}$$

The right-hand side (RHS) of (24) is the profit of a firm, and the left-hand side (LHS) is the cost for an innovation to produce a patent. If $\rho \geq \psi$, the RHS is a decreasing function of n_t. This shows that intensive competition reduces the operating profits of a firm. In addition, the RHS is an increasing function of the population, L_t. This is because a large population results in a large market that brings a firm large profits. If

$$(\frac{1}{n_{t-1}})^{\gamma} < (\frac{1}{\sigma - 1})\frac{p^{\frac{1}{\rho-1}}}{n_{t-1} + n_{t-1}^{\frac{\rho-\psi}{\rho(1-\psi)}} \tau^{\frac{-\psi}{1-\psi}}} L_t, \tag{25}$$

innovation occurs at period t, since the profit of a firm when $k_t = 0$ is larger than the cost of an innovation. If $\rho \geq \psi$, the right-hand side of (25) is a decreasing function of n_{t-1}. From (20), if innovation occurs at period t, the fertility rate in period t becomes

$$m_t = \frac{1}{(n_{t-1} + k_t p^{\frac{\rho}{\rho-1}})^{\frac{\psi(\rho-1)}{\rho(\psi-1)}} \tau^{\frac{1}{1-\psi}} + \tau}. \tag{26}$$

Since $\psi < 1$ and $\rho < 1$, $\frac{\partial m_t}{\partial k_t} < 0$. [5] Therefore, (26) shows that the fertility rate decreases with innovation. Innovation activities raise the variety of differentiated goods, which lowers (raises) the relative price of the composite of differentiated goods (children). Therefore, when innovations progress, fertility rates decrease.

Proposition 2 *Fertility rates decrease with innovation.*

On the other hand, if

$$(\frac{1}{n_{t-1}})^{\gamma} \geq (\frac{1}{\sigma - 1})\frac{p^{\frac{1}{\rho-1}}}{n_{t-1} + n_{t-1}^{\frac{\rho-\psi}{\rho(1-\psi)}} \tau^{\frac{-\psi}{1-\psi}}} L_t, \tag{27}$$

innovation does not occur in period t. If (27) is satisfied and

$$m_t = \frac{1}{n_{t-1}^{\frac{(\rho-1)\psi}{\rho(\psi-1)}} \tau^{\frac{1}{1-\psi}} + \tau} \leq 1, \tag{28}$$

innovation never occurs in period t or the following periods. In this case, the fertility rate is constant at $m_t = \frac{1}{n_{t-1}^{\frac{(\rho-1)\psi}{\rho(\psi-1)}} \tau^{\frac{1}{1-\psi}} + \tau}$ after period t.

The above discussion demonstrates that there are two cases in the model: innovation and decline in fertility rates and no innovation and constant fertility

[5] $\rho < 1$ means that the elasticity of substitution among differentiated goods is larger than 1. $\psi < 1$ represents that the elasticity of substitution between composite of differentiated goods and children is larger than 1.

rates. To see them explicitly, we study the dynamic property of the economy. There are two state variables, L_t and n_t, which follow dynamic equations. Population, L_t, follows $L_{t+1} = m_t L_t$. We substitute (26) and $k_t = n_t - n_{t-1}$ into $L_{t+1} = m_t L_t$ and derive

$$L_{t+1} = \frac{1}{(n_{t-1} + (n_t - n_{t-1})p^{\frac{\rho}{\rho-1}})^{\frac{\psi(\rho-1)}{\rho(\psi-1)}}\tau^{\frac{1}{1-\psi}} + \tau} L_t. \tag{29}$$

Next, we derive the dynamic equation of variety n_t. n_t increases with innovation activities. Thus, the dynamic equation of n_t is a free-entry condition of the innovation market. We substitute $k_t = n_t - n_{t-1}$ into (24) and obtain

$$(\frac{1}{n_{t-1}})^{\gamma} = (\frac{1}{\sigma-1})\frac{p^{\frac{1}{\rho-1}}}{(n_{t-1} + (n_t - n_{t-1})p^{\frac{\rho}{\rho-1}}) + (n_{t-1} + (n_t - n_{t-1})p^{\frac{\rho}{\rho-1}})^{\frac{\rho-\psi}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}} L_t. \tag{30}$$

(29) and (30) express the dynamic property of the model. To analyze the dynamic property, we create a phase diagram.

The phase diagram is depicted in Figure 1. In Appendix 1, we show that the locus of the constant population, $\frac{L_{t+1}}{L_t} = 1$, requires $n_t = (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}$ when innovation is not carried out and the locus of constant population becomes a downward-sloping curve when innovation activities are carried out. In Appendix 2, we show that, under the assumption of $\rho \geq \psi$, we can depict a downward-sloping curve of $\frac{n_t}{n_{t-1}} = 1$ as in Figure 1.

In Figure 1, we can observe four regions. In Region 1, population increases, and innovation does not occur. In this region, the population is small. Since the population is small, the market for differentiated consumption goods is small. The small markets result in low profits. Thus, innovation does not occur in this region. In Region 1, the variety of differentiated goods is also small. The relative price of the composite of differentiated goods is therefore high. Thus, the fertility rate is high, $m_t > 1$. Since the fertility rate is high, the population increases with time.

In Region 2, the population increases, while innovation activities are carried out. In Region 2, the population becomes large. Therefore, the markets for differentiated consumption goods are large enough to induce innovation activities. Thus, the variety of differentiated goods increases with time. In addition, in this region, the variety of differentiated goods is relatively small. Thus, fertility rates are high, $m_t > 1$. Then, in Region 2, both the population and the variety of differentiated goods increase with time. However, fertility rates decrease with innovation. Therefore, in Region 2, fertility rates decrease with time. Galor (2005) reported that fertility rates start to decrease when economic development takes off. Region 2 of our model is consistent with this phenomenon. Our model shows that a large population enables economic development, and economic development increases as a result of variety expansion. Variety expansion reduces fertility rates.

In Region 3, the population decreases, and innovation activities are carried out. In Region 3, the population is large, and the market for differentiated goods is large. Since the market for differentiated goods is large, innovation progresses. Thus, the variety of differentiated goods increases with time in this region. However, since the variety of differentiated goods is large, fertility rates decrease, $m_t < 1$. The population then decreases with time in this region. In addition, the fertility rate decreases with innovation activities. Thus, the fertility rate decreases with time in Region 3. As noted in the Introduction, the total fertility rates in developed countries are low, and population decreases with time. In addition, economic growth continues in developed countries. Region 3 of our model is consistent with these phenomena.

In Region 4, the population decreases, and innovation stops. In Region 4, the population decreases. Since the market for differentiated goods decreases, it is impossible to obtain profits with innovation. Thus, innovation activities are not conducted, and the variety of differentiated goods is constant in this region. In Region 4, there is a large variety of differentiated goods, which lowers fertility rates, $m_t < 1$. Thus, the population decreases with time in this region. Since innovation does not occur, the fertility rate is constant in Region 4. [6]

In Figure 1, we depict an example case of a dynamic path that starts from Region 1. In this path, in the early periods, the population increases as long as there are not any innovations (Region 1). An increase in population enlarges the market for differentiated goods. When the economy switches from Region 1 to 2, innovation activities start. In this region, the population increases ($m_t > 0$), while the fertility rates decrease with innovation (see Proposition 1). When the economy reaches Region 3, the population starts to decrease, and fertility rates continuously decrease with innovation ($m_t < 1$). The important features of developed countries in the modern world are declines in fertility rates and continuous economic growth. Therefore, developed countries in the modern world are in Region 3. If the economy reaches Region 4, innovation activities stop, and population decreases at a constant rate. Region 4 may not be realized in the modern world. Our model shows the only one possibility for the future economy: continuous decline in fertility rates and a curtailment of economic growth. In this dynamic path, fertility rates decrease with economic growth (Regions 2 and 3). This result is consistent with experiences of economic development (See Galor (2004)). [7]

[6] Region 4 of our model may not be realized in the real world. Our model shows the possibility of decline in population and the curtailment of economic growth. In much of the literature devoted to the analysis of endogenous fertility rates, such as that by Becker, Murphy, and Tamura (1991) and Galor and Weil (2000), the fertility rates and population continued to decrease. Lutz (1996) pointed out the possibility of a continuous decline in population in the future.

[7] In our model, there are many possibilities of dynamic paths. For example, if the economy starts from Region 4, the variety is constant, and population decreases with time.

4 Conclusion

We presented a simple model in which variety expansion reduces fertility rates. We showed that this simple model can be applied to the model of economic growth. In the model of economic growth, fertility rates decrease with innovation activities, which are the engine of economic growth. Thus, our model presented a new mechanism of fertility decline.

In the modern world, fertility rates in developed countries are low and decrease with time. Our model clarified that low fertility rates in developed countries are to be attributed to a large variety of consumption goods. Developed countries have experienced variety expansion by innovation activities, which reduce fertility rates.

Our model is very simple. Thus, we can extend the model in some directions. In our growth model, parents solve a static problem: when they decide number of children, they do not consider their future consumption. If we assume that agents live three period, they must consider their old time consumption when they decide number of children. Under this extended model, suppose that individuals expect that price index falls in the near future. In this case, they may decrease the number of children, since they optimally substitute current consumption for future consumption. The timing matter. Further, the scenario that the expectations for future innovation decreases current fertility is itself interesting. This extension is one important problem. We can study the effects of public policy, such as subsidies for rearing children. We can extend the growth model and study human capital accumulation, as in Becker, Murphy, and Tamura (1991). This extension analyzes the quantity-quality trade-off that is an important mechanism of the fertility decline in developed countries. Finally, we consider the relationship between the fertility rate and urbanization with the mechanism used in this model. Urbanization is an important aspect of economic development. Sato and Yamamoto (2005) showed that demographic transition and urbanization are closely related. These extensions are important for future studies.

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Appendix 1

Here, we define $z_t = n_{t-1} + (n_t - n_{t-1})p^{\frac{\rho}{\rho-1}}$. With this definition, we can write (29) and (30) as follows:

$$\frac{L_{t+1}}{L_t} = \frac{1}{z_t^{\frac{\psi(1-\rho)}{\rho(1-\psi)}} \tau^{\frac{1}{1-\psi}} + \tau}, \tag{31}$$

$$(\frac{1}{n_{t-1}})^{\gamma} = (\frac{1}{\sigma-1}) \frac{p^{\frac{1}{\rho-1}}}{z_t + z_t^{\frac{\rho-\psi}{\rho(1-\psi)}} \tau^{\frac{-\psi}{1-\psi}}} L_t. \tag{32}$$

Assuming that $\rho \geq \psi$, Equation (32) has a unique solution,

$$z_t^* = z(n_{t-1}, L_t),$$

where $\frac{\partial z(n_{t-1},L_t)}{\partial n_{t-1}} > 0$ and $\frac{\partial z(n_{t-1},L_t)}{\partial L_t} > 0$. From (31), the locus of constant population is given by

$$z(n_{t-1}, L_t) = (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}. \tag{33}$$

Obviously, the locus of (33) is a downward sloped curve in the space of (n_{t-1}, L_t).

On the other hand, when innovation activities are not carried out, $\frac{L_{t+1}}{L_t} = 1$ requires

$$n^* = (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}. \tag{34}$$

(33) and (34) are connected with each other when $z(n_{t-1}, L_t) = n^* = (\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}$.

Appendix 2

From (30), $\frac{n_t}{n_{t-1}}=1$ requires that

$$(\frac{1}{n})^{\gamma}=(\frac{1}{\sigma-1})\frac{p^{\frac{1}{\rho-1}}}{n+n^{\frac{\rho-\psi}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}}L_t. \tag{35}$$

(35) is written as

$$L_t=(\frac{\sigma-1}{p^{\frac{1}{\rho-1}}})(n^{1-\gamma}+n^{\frac{\rho-\psi-\gamma\rho(1-\psi)}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}). \tag{36}$$

$\frac{\partial(n^{1-\gamma}+n^{\frac{\rho-\psi-\gamma\rho(1-\psi)}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}})}{\partial n}=(\frac{\psi-\rho}{(\psi-1)\rho\tau^{\frac{\psi}{1-\psi}}}n^{\frac{-\psi(1-\rho)}{\rho(1-\psi)}})\frac{1}{n^{\gamma}}-(\gamma-1)\frac{1}{n^{\gamma}}-(\frac{1}{\tau^{\frac{\psi}{1-\psi}}}n^{\frac{\rho-\psi}{\rho(1-\psi)}})\frac{\gamma}{n^{\gamma+1}}<$ 0, since we assume that $\rho\geq\psi$ and $\gamma\geq 1$. Thus, the line $\frac{n_t}{n_{t-1}}=1$ is a downward-sloping curve as in Figure 1. We define $R(n)=(\frac{\sigma-1}{p^{\frac{1}{\rho-1}}})(n^{1-\gamma}+n^{\frac{\rho-\psi-\gamma\rho(1-\psi)}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}})$. Since $1>\rho\geq\psi$ and $\gamma\geq 1$, $\frac{\rho-\psi}{\rho(1-\psi)}<1\leq\gamma$. Thus, $\frac{\rho-\psi-\gamma\rho(1-\psi)}{\rho(1-\psi)}<0$ is satisfied. Therefore, $\lim_{n\to 0}R(n)=\infty$ and $\lim_{n\to\infty}R(n)=0$.

We substitute (29) into (30) and get

$$L_t=\frac{(\frac{1}{n_{t-1}})^{\gamma}(\sigma-1)}{p^{\frac{1}{\rho-1}}}\left[(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}+(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho-\psi}{\psi(1-\rho)}}\tau^{\frac{-\psi}{1-\psi}}\right]. \tag{37}$$

From (36), we can derive that

$$L_t=(\frac{(\frac{1}{n_t})^{\gamma}(\sigma-1)}{p^{\frac{1}{\rho-1}}})(n_t+n_t^{\frac{\psi(1-\rho)}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}). \tag{38}$$

Equation (37) is the locus of $\frac{L_{t+1}}{L_t}=1$ when $n_t\leq(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}$, while equation (38) is the locus of $\frac{n_{t+1}}{n_t}=1$. Since $n_t\leq(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}$, $(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}+(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho-\psi}{\psi(1-\rho)}}\tau^{\frac{-\psi}{1-\psi}}\geq n_t+n_t^{\frac{\psi(1-\rho)}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}$. Since $n_{t-1}\leq n_t$, $(\frac{1}{n_{t-1}})^{\gamma}\geq(\frac{1}{n_t})^{\gamma}$. Therefore,

$$\frac{(\frac{1}{n_{t-1}})^{\gamma}(\sigma-1)}{p^{\frac{1}{\rho-1}}}\left[(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}+(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho-\psi}{\psi(1-\rho)}}\tau^{\frac{-\psi}{1-\psi}}\right]\geq(\frac{(\frac{1}{n_t})^{\gamma}(\sigma-1)}{p^{\frac{1}{\rho-1}}})(n_t+n_t^{\frac{\psi(1-\rho)}{\rho(1-\psi)}}\tau^{\frac{-\psi}{1-\psi}}).$$

Thus, the RHS of (37) is larger than the RHS of (38), which means that the locus of $\frac{L_{t+1}}{L_t}=1$ always locates above the locus of $\frac{n_{t+1}}{n_t}=1$ when $n_t\leq(\frac{1-\tau}{\tau^{\frac{1}{1-\psi}}})^{\frac{\rho(1-\psi)}{\psi(1-\rho)}}$. Thus, the phase diagram can be depicted as in Figure 1.