# Discussion Papers In Economics And Business

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# Market Efficiency, Asymmetric Price Adjustment and Over-Evaluation: Linking Investor Behaviors to EGARCH*


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*Abstract*:

The paper incorporates a partial asymmetric price adjustment model for individual investors action into an EGARCH model, clarifies the relationship between the price adjustment speed, the market efficiency and asymmetric price adjustment, and measures over (under)-evaluation of stock value. The stock price does not fully adjust to the market value of stocks hoped by investors and does not adjust symmetrically in upturn and downturn, if and only if the market is not efficient, and moreover the market value generally diverts from the fundamental value of stocks even though the market is efficient. As an operational example, the Tokyo stock market is found to be inefficient during 1980-2005. The speed of price adjustment is asymmetric in the 80s but symmetric in the 90s and 2000s. The over-evaluation of the market value is remarkably observed in the 80s but not in the 90s and 2000s.

## 1. Introduction.

Figure 1 shows that that during 1980 and 1990 the TOPIX (Tokyo Stock Exchange Price Index) goes up and down around the particular trend, and so we are induced to think that (i) the stock prices in upturn adjust to its trend more quickly than the price in downturn and (ii) sometime approaching to its trend instantaneously. However, even after adjustment toward a trend, (iii) the market value of stock is still evaluated over its fundamental value. These facts seem to support three propositions: (i) the asymmetric stock price adjustment by Koutmos (1998, 1999), the market efficiency by Fama (1976) and the over (under)- evaluation of stock value by Lee (1999). Few previous papers explain three propositions, being based on the investor behaviors. That is, how does the investor behave and as a result, how are the market price, the market value (or intrinsic value), and the fundamental value decided? Can the previous model answer this question?

The understanding of the investor behaviors related to three propositions can lead to the better interpretation for them. However, previous papers for the stock price formation are silent about the market processes that might deliver the hypothesis on investor behavior. However, there are, so far, some hypotheses on the investor behaviors. The first idea was formalized by Grossman and Stiglitz (1980). In their theoretical model, the market price does not fully incorporate all knowable information because informed investors make returns by exploiting deviations of prices from security values.[1] Second, without explicitly characterizing investor behaviors, Busse and Green (2002) empirically find that news reports about individual stocks on the financial television network CNBC are incorporated into stock prices within few minutes. They shed light on the degree of efficiency and conclude that the market is efficient enough that a trader cannot generate profits based on widely disseminated news unless he acts almost immediately. Third, Amihud and Mendelson (1987) and Koutmos(1998, 1999) developed and linked the partial adjustment price model for investor behavior to an ARCH-type model. The stock price is adjusted to 'market values' (they use 'intrinsic value' for it) by portfolio mangers and the adjustment speed is different depending on whether the stock price is over or under the market value.[2]

[1] Later, Busse and Green (2002) found that the small profits available to very short horizon traders (i.e., informed investors) are consistent with compensation for continuously monitoring information sources, supporting the theory by Grossman and Stiglitz (1980).

[2] Koutmos (1998, 1999) also offered an empirical new finding that the adjustment speed is faster when the stock price is over the intrinsic value. Motivated by his new finding,

The purpose of this paper is twofold. First, we provide the investor behaviors with three propositions like the market efficiency in the framework of a partial adjustment price model, the asymmetric stock price adjustment, the over (under)-evaluation of market value of stock, and finally link this partial price adjustment model for investor behavior to an EGARCH model to test these propositions. Second, we show an operational example of the model, by examining whether the Tokyo Stock Market is efficient, the stock price in this market adjusts asymmetrically, and the market value of stock is over (under)-evaluated against the fundamental value.

Where is our paper located, compared with the previous papers? First, the definition of market efficiency by Fama (1976) is based on informational concept. It is characterized as one in which security prices fully reflect all available information. In the framework of empirical researches, it is defined that the expectation of the returns conditional on the previous information is constant, i.e. $E\{R_t \mid \mathrm{I}_{t-1}\} =$ constant, which is a so-called weak-form of the market efficiency. Different from our framework, the concept of Fama's market efficiency was silent and slippery, however, about the market processes that might deliver the hypothesis on investor behavior. Then, the concept of the market efficiency should tell about the investor behavior to lead well-understanding. Second, Koutmos (1998, 1999) offered an empirical new finding that the adjustment speed is faster when the stock price is over the market value. Motivated by his new finding of the asymmetric stock price adjustment, several papers including Pagan and Soydemir (2001), Bang and Shin (2003), and Nam et al. (2003, 2005) have got the same finding as Koutmos (1998, 1999).However, Koutmos's empirical framework is based on drastic approximation more than our framework and then should be developed to confirm the empirical findings. Third, the market value (the stock price) is over(under)-valued against the fundamental value. The fundamental value is determined by the fundamental factors underlying the asset. Cheung and Ng (1998) and Gjerde and Saetterm (1999) related its value to the macroeconomic variables. Hess and Lee (1999) and Lee (1999) related its value to other financial variables such as rates of return, dividends, and earnings, while there is no explicit theoretical explanation how the fundamental value is decided. In our approach its value is exogenously given by the Dividend Discount Model with a constant growth (which equals the growth rate of GDP).

The organization of the paper is as follows. Section 2 sketches a partial adjustment price model by Amihud and Mendelson (1987) and Koutmos (1998, 1999) ,

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Pagan and Soydemir (2001), Bang and Shin (2003), and Nam et al. (2003, 2005) have applied Koutmos model and got the same finding as Koutmos (1998, 1999).

provides the mechanism how the fundamental value is decided exogenously, defines the asymmetric price adjustment, the market efficiency, over (under)-evaluated market value, and links the model to an EGARCH. Section 3 provides the estimation results of the Tokyo Stock Market during 1980-2005 as an operational example. Section 4 gives concluding remarks. Appendix gives proofs of propositions.

## 2. The Model

### 2. 1 Partial Asymmetric Adjustment Price Model and Market Efficiency

We follow a partial adjustment price model by Amihud and Mendelson (1987) and Koutmos (1998, 1999) with some modifications. The model distinguishes the unobserved market value of stock ($V_t$) from the observed stock price ($P_t$), both are expressed in natural logarithms. [3] The process of market value follows a random walk process with drift:

$$V_t = a + V_{t-1} + u_t, \quad u_t \mid I_{t-1} \sim N\left(0, \sigma_{ut}^2\right), \ t = 1, \ldots, T, \tag{1}$$

where $a$ is constant and $I_{t-1}$ denotes the information set of the time t-1. The market value of stock is a value which investors in the market hope. This market value is different from the so-called 'fundamental value' of stock. We assume that the disturbance term ($u_t$) has the EGARCH process proposed by Nelson (1991):

$$\begin{aligned} \log \sigma_{ut}^2 \ &= \ \alpha_0 + \alpha_1 z_{t-1} + \alpha_2 (|z_{t-1}| - E(|z_{t-1}|)) + \alpha_3 \log \sigma_{ut-1}^2 \\ \text{where} & \\ z_t &= u_t / \sigma_{ut} \ \sim N(0,\ 1)\ . \end{aligned} \tag{2}$$

The partial asymmetric adjustment price process of $P_t$ represents that adjustment costs ($\theta^+$, $\theta^-$) are asymmetric in upturn and downturn markets:

$$P_t - P_{t-1} = (1-\theta^+)(V_t - P_{t-1})^+ + (1-\theta^-)(V_t - P_{t-1})^-, \ -1 < \theta^+, \theta^- < 1, \tag{3}$$

where $(V_t - P_{t-1})^+ = \max\{V_t - P_{t-1}, 0\}$, and $(V_t - P_{t-1})^- = \min\{V_t - P_{t-1}, 0\}$. If $\theta^+ = \theta^-$ (= $\theta$), equation (3) reverts to the basic partial adjustment price process proposed by Amihud and Mendelson (1987;p.536). Koutmos (1998; p.280, 1999; p.86) formulated the asymmetric adjustment to market value in (3). After the market value $V_t$ is recognized at t, the stock price $P_{t-1}$ is adjusted by (3) to $P_t$. Our adjustment process is based on the idea that price approaches its market value in competitive markets, so that the value can be estimated

[3] Based on an idea by Black (1986;p.533), Amihud and Mendelson (1987;p.536) and Koutmos (1998; p.280, 1999; p.86) call this market value to be the intrinsic value.

from price directly.

The model consisting of (1),(2) and (3) is called a partial asymmetric adjustment price model. As explained Koutmos(1998;p278), the partial price adjustment, $\theta^{+} \neq 0,\ \theta^{-} \neq 0$ *in (3),* is attributed to the following costs which slow down the adjustment of a security's price toward its market values.: 1) the cost of acquiring and processing information; 2) the attempts of market specialists to create orderly markets and assure price continuity; 3) the particular institutional market mechanism by which securities are traded. Moreover, the possibility of the asymmetric adjustment ($\theta^{+} \neq \theta^{-}$) is that investors have a higher aversion to downside risk, so they react faster to bad news. The use of stop-loss orders is an example of such aversion. Also, portfolio managers feel they are penalized more if they underperform in a falling market than in a rising market. As such, they are quicker to react to bad news. Similarly, for market makers, i.e., dealers and floor specialists, the cost of not adjusting prices downward is higher than the cost of not adjusting prices upward. Market specialists, who are required to maintain price continuity, will find it easier and less costly to do so in a rising market than in a falling market, as the latter involves building up inventory with overpriced securities.

Thus, Amihud and Mendelson (1987) and Koutmos (1998, 1999) provide the investor behaviors for the asymmetric partial adjustment price process, while in this process they investigated neither the market efficiency, the asymmetric partial adjustment ( which are analyzed in Section 2.1) and nor the over (under) –evaluation of stock, which is given in Section 2.2. Also, they could not link this model to EGARCH model to test the validity of the model in empirical sense, which is done in Section 2.3.

First, we define the market efficiency in the process as follows:

***Definition 1****: (i) The market is said to be efficient if* $\theta^{+} = \theta^{-} = 0$ *in (3) and inefficient otherwise. (ii) The stock price adjustment is said to be asymmetric* if $\theta^{+} \neq \theta^{-}$ *and* symmetric *otherwise.*

This definition of market efficiency indicates that the speeds ($1-\theta^{+}$) and ($1-\theta^{-}$) of price adjustment of both positive and negative discrepancy for the market value are equal to unity in (3), namely the stock price instantaneously and fully adjusts to the market value due to no adjustment costs ($\theta^{+} = \theta^{-} = 0$):

$$P_t - P_{t-1} = (V_t - P_{t-1})^{+} + (V_t - P_{t-1})^{-} = V_t - P_{t-1}, \text{ and hence } P_t = V_t. \qquad (4)$$

When $0 < \theta^{+}, \theta^{-} < 1$, the stock price partially adjusts for the market value. However, when $\theta^{+}, \theta^{-} < 0$, the stock price overshoots the market value. We test the null hypothesis of market efficiency against the inefficiency of market as shown in (3):

$$H_0 : \theta^+ = \theta^- = 0 \quad \text{vs} \quad H_1 : \theta^+ \neq 0 \text{ or } \theta^- \neq 0. \tag{5}$$

The symmetric adjustment speed can be tested as

$$H_0 : \theta^+ = \theta^- \quad \text{vs} \quad H_1 : \theta^+ \neq \theta^- . \tag{6}$$

Second, the digit of the asymmetric partial adjustment price model can be understood by the following proposition.

***Proposition 1:*** (i) *When the previous stock price $P_{t-1}$ is under (over) the market value $V_t$ at the present period t, the stock price $P_t$ at the present period is adjusted to increase (decrease):*

$$V_t - P_{t-1} \gtreqless 0 \Leftrightarrow R_t \gtreqless 0 \tag{7}$$

*(ii) The market value $V_t$ is computed by using the stock price $P_t$, return $R_t$ and the estimated adjustment speed $(1-\theta^+)$ or $(1-\theta^-)$:*

$$P_t - V_t = R_t\left(1-\frac{1}{\xi_t}\right) \tag{8}$$

*where* $\xi_t \equiv (1-\theta^-) + (\theta^- - \theta^+)D_t^+$, $D_t^+ = 1$ for $V_t - P_{t-1} \geq 0$, and $D_t^+ = 0$ otherwise.

*(iii) The market value $\hat{V}_t$ in level can be expressed by stock price $\hat{P}_t$ in level, return and the estimated adjustment speed as follow:*

$$\hat{V}_t = \frac{\hat{P}_t}{\exp\{R_t(1-1/\xi_t)\}}, \text{ where } \hat{V}_t \equiv \exp(V_t) \text{ and } \hat{P}_t \equiv \exp(P_t) \tag{9}$$

***Proof of Proposition 1:*** Rewrite the adjustment process (3) as

$$R_t = (1-\theta^+)(V_t - P_{t-1})D_t^+ + (1-\theta^-)(V_t - P_{t-1})(1-D_t^+)$$

$$= \left\{(1-\theta^-) + (\theta^- - \theta^+)D_t^+\right\}(V_t - P_{t-1}) \tag{10}$$

$$= \xi_t (V_t - P_{t-1})$$

where $D_t^+ = 1$ for $V_t - P_{t-1} \geq 0$, and $D_t^+ = 0$ otherwise, and $\xi_t \equiv (1-\theta^-) + (\theta^- - \theta^+) D_t^+$.

Then, the relation of (7) in (i) holds. On the other hand, (7) yields to $P_{t-1} + R_t / \xi_t = V_t$. By using this relation, we get the (8) in (ii): $P_t - V_t = P_t - P_{t-1} - \frac{R_t}{\xi_t} = R_t(1 - \frac{1}{\xi_t})$. We can express the fundamental value by using (8) and then express its value in level by the stock price, return and the estimated $\xi_t$ as shown in (9) of (iii). ■

After the market value $V_t$ hoped by investors is found at the beginning of period t and compared with the previous stock price $P_{t-1}$ (i.e., $V_t - P_{t-1}$), the stock price $P_t$ at period t is adjusted to increase when $V_t - P_{t-1} > 0$ and then $R_t (= P_t - P_{t-1}) > 0$, as shown in (3) and (7). However, the adjustment speed, $0 < (1-\theta^+)$, is less than one (i.e., partially), so that $P_t$ is still $P_t < V_t$ shown in (3) and (8) after adjustment. Moreover, the market price $V_t$ is computed by using stock price $P_t$, return $R_t$ and parameters $\theta^+$ and $\theta^-$. The eq.(9) is a level format of $V_t$.

## 2.2. Over (under) –evaluation of the market value

Is the market value of stock over- (under-) evaluated, compared with the fundamental value? We provide the fundamental value of stock by using a dividend discounted model with a constant growth rate λ, which decides the fundamental value $S_t$ of stock at period t and t-1 as follows:

$$S_t \equiv \sum_{j=1}^{\infty} \frac{D_1(1+\lambda)^j}{(1+\rho)^j} = (\frac{1+\lambda}{\rho-\lambda}) D_1, \quad S_{t-1} \equiv \sum_{j=1}^{\infty} \frac{D_0(1+\lambda)^j}{(1+\rho)^j} = (\frac{1+\lambda}{\rho-\lambda}) D_0 \qquad (11)$$

where ρ is a discount rate and $D_1$, $D_0$ are a dividend at t and t-1, then the dividend grows at the rate λ (ρ>λ) and $D_1 = D_0(1+\lambda)$. A dividend discounted model determines the fundamental value to equal the discounted value of dividend stream with a constant growth rate λ. We assume that λ equals the growth rate g of GDP per day (which is a converted rate from per year basis to per day basis). Therefore, the growth rate η of the fundamental value is expressed by the growth rate λ of dividends and then the GDP growth rate g as follows:

$$\frac{S_t}{S_{t-1}} - 1 = \frac{D_1}{D_0} - 1 \ = \lambda = g \qquad (12)$$

Then, given $S_0$, we can compute the fundamental value $S_t$ as :

$$S_t = S_0 (1+g)^t \qquad (13)$$

Here, we can define the over (under)-evaluation of market value.

***Definition 2****: The market value of stock is said to be over-evaluated if* $\hat{V}_t > S_t$ *in (9)* and *(13), under-evaluated if* $\hat{V}_t < S_t$ *, and normal otherwise.*

This definition evaluates the value of stock in level as well as usual. This definition is first characterized by the comparison between market value and fundamental value. Second, since the stock price includes the noise like the adjustment delay to the market value, we did not use the stock price in the definition. Hess and Lee (1993) and Lee (1999) related its value to other financial variables such as rates of return, dividends, and earnings. However, in our idea, the fundamental value is exogenously given by the dividend discount model.

Koutmos (1998, 1999) only incorporates the asymmetric effects to the partial price adjustment model of Amihud and Mendelson (1987), though he did deal with neither market efficiency nor over (under) –evaluation of stock prices based on the investor behavior.

If we can get the parameters $\theta^+$ and $\theta^-$ ( i.e., $\xi_t$ ) in (5),(6), (8),(9) and (13), we can investigate whether the market is efficient, the stock price adjustment is asymmetric, the market value of stock is over(under)-evaluated. Let us estimate the parameters $\theta^+$ and $\theta^-$ in the next section

## 2. 3 The Reduced Model

We have an autoregressive process for the returns.

***Proposition 2:*** (i) *The return process consisting of (1) and (3) has the following* *expression*

$$R_t^* = \mathrm{a} + \theta^+ \mathrm{R}_{t-1}^{*+} + \theta^- \mathrm{R}_{t-1}^{*-} + u_t, \tag{14}$$

*where* $R_t^* = \xi_t^{-1} R_t$ , $R_{t-1}^{*+} = \xi_{t-1}^{-1} R_{t-1}^+$ $(R_{t-1}^+ \equiv Max\{R_{t-1}, 0\})$, $R_{t-1}^{*-} = \xi_{t-1}^{-1} R_{t-1}^- (R_{t-1}^- \equiv Min\{R_{t-1}, 0\})$,

and $\xi_t = 1 - \theta^+ (\equiv \xi^+$ for all t) for $\mathrm{R}_t^+$, and $\xi_t = 1 - \theta^- (\equiv \xi^-$ for all t) otherwise.

***Proof of Proposition 2:*** Noting the facts $\mathrm{V}_t = \xi_t^{-1} R_t + \mathrm{P}_{t-1}$ from (8), $0 < \xi_t < 1$ from (3),

hence $\mathrm{V}_t - \mathrm{V}_{t-1} = \xi_t^{-1} R_t - \xi_{t-1}^{-1} R_{t-1} + \mathrm{P}_{t-1} - \mathrm{P}_{t-2} = \mathrm{a} + \mathrm{u}_t$. Considering the last equation, we have

$$\xi_t^{-1} R_t = \mathrm{a} + (1 - \xi_{t-1}) \xi_{t-1}^{-1} (R_{t-1}^+ + R_{t-1}^-) + \mathrm{u}_t \tag{15}$$

where $\mathrm{P}_{t-1} - \mathrm{P}_{t-2} = \mathrm{R}_{t-1} = \mathrm{R}_{t-1}^+ + \mathrm{R}_{t-1}^-$. Considering (7), $\mathrm{R}_{t-1}^+ = \mathrm{R}_{t-1} \geq 0$ in (15) corresponds uniquely to $\mathrm{V}_{t-1} - \mathrm{P}_{t-2} \geq 0$ in (10) which decide uniquely the $\xi_{t-1}$ $(\equiv \left(1 - \theta^-\right) + \left(\theta^- - \theta^+\right) \mathrm{D}_{t-1}^+)$.

Then, the $\mathrm{R}_{t-1}^+ = \mathrm{R}_{t-1} \geq 0$ can decide the $\xi_{t-1}$. As a symmetrical to this, the $\mathrm{R}_{t-1}^- = \mathrm{R}_{t-1} \leq 0$ can decide the $\xi_{t-1}$. The return process has the expression in (14). ■

The model (14) with (2) is an EGARCH model. Equation (14) is alternatively expressed as

$$R_t = \xi_t (\mathrm{a} + \beta^+ \mathrm{R}_{t-1}^+ + \beta^- \mathrm{R}_{t-1}^-) + \varepsilon_t , \tag{16}$$

where $\beta^+ = (\xi^+)^{-1} \theta^+$, $\beta^- = (\xi^-)^{-1} \theta^-$ and $\varepsilon_t = \xi_t u_t$.[4] The process of $R_t$ in (16) is apparently similar to that of $R_t^*$ in (14). However, except for the case of $\theta^+ = \theta^-$, the conditional expectation of $\varepsilon_t$ is not zero and the process of $\{\varepsilon_t\}$ is serially dependent: see

[4] If $\theta^+ = \theta^-$ $(= \theta)$ and then $\xi_t = 1 - \theta$, equation (16) reduces to

$$R_t = \mathrm{a}(1 - \theta) + \theta \mathrm{R}_{t-1} + (1 - \theta) u_t .$$

Lemma 1 of Appendix in Tsukuda, Miyakoshi and Shimada (2006). The conditional variance of $\varepsilon_t$ does not follow an EGARCH process unlike to that of $u_t$. Then, the joint density function based on equation (16) and (2) is expressed as follows.

***Proposition 3:*** *The joint density function of $\{R_1, \ldots, R_T\}$ is given by*

$$\mathrm{pdf}(R_1, \ldots, R_T|\omega) = \prod_{t \in T^+} \left\{ \frac{(\xi^+\sigma_{ut})^{-1}\ \phi((R_t - \xi^+\mu_t)/\xi^+\sigma_{ut})}{\Phi(\mu_t/\sigma_{ut})} \right\} \cdot \prod_{t \in T^-} \left\{ \frac{(\xi^-\sigma_{ut})^{-1}\phi((R_t - \xi^-\mu_t)/\xi^-\sigma_{ut})}{\Phi(-\mu_t/\sigma_{ut})} \right\} \tag{17}$$

*where* $\mu_t = a + \beta^+ R^+_{t-1} + \beta^- R^-_{t-1}$ *,* $T^+ = \{t \mid R_t \geq 0, t \in N\}$*,* $T^- = \{t \mid R_t < 0, t \in N\}$*,* $N = \{1, \ldots, T\}$*, and* $\omega = \{a, \theta^+, \theta^-, \alpha_0, \alpha_1, \alpha_2, \alpha_3\}$ *is a vector of unknown parameters, and* $\Phi$ and $\phi$ *respectively denote the distribution and density functions of the standard normal distribution.*

***Proof of Proposition 3:*** From (16), the return process is rewritten as

$$R_t = \begin{cases} \xi^+\mu_t + \xi^+ u_t & \text{for } R_t \geq 0 \\ \xi^-\mu_t + \xi^- u_t & \text{for } R_t < 0 \end{cases} . \tag{18}$$

The conditional density of $R_t$ given $I_{t-1}$ is written by

$$\mathrm{pdf}(R_t;\omega|I_{t-1}) = \begin{cases} \dfrac{(\xi^+\sigma_{ut})^{-1}\ \phi((R_t - \xi^+\mu_t)/\xi^+\sigma_{ut})}{\Phi(\mu_t/\sigma_{ut})} & \text{for} R_t \geq 0 \\ \\ \dfrac{(\xi^-\sigma_{ut})^{-1}\phi((R_t - \xi^-\mu_t)/\xi^-\sigma_{ut})}{\Phi(-\mu_t/\sigma_{ut})} & \text{for } R_t < 0 \end{cases} \tag{19}$$

Substituting equation (17) into the following relation

$$\mathrm{pdf}(R_1, \ldots, R_T \mid \omega) = \prod_{t \in T} \mathrm{pdf}(R_t; \omega \mid I_{t-1}) , \tag{20}$$

the required joint density in (17) is obtained. ■

The model of (16) is estimated by the maximum likelihood estimation method using the joint density of (17).Thus, we get the parameter set of ω including ($\theta^+$ , $\theta^-$), which provides the market efficiency, asymmetric price adjustment, and over(under)-evaluation of stock value.

Equation (7) in Koutmos (1998, pp. 280) takes too rough approximation and then should be developed as equation (16) in this paper. That is, since the conditional expectation of $\varepsilon_t$ is not zero and $\{\varepsilon_t\}$ is serially dependent, then $\varepsilon_t$ can not follow the GARCH model. Though Koutmos (1998) finds empirically useful facts about the stock market movements, the partial asymmetric adjustment model does not logically induce the Threshhold GARCH model which is used for his empirical studies. Extending Koutmos (1998), equation (4) of Koutmos (1999, pp. 86) introduces an error term in the asymmetric price adjustment process. However, the error term in equation (4) causes discrepancy of stochastic orders between $u_t$ and $\varepsilon_\tau$ in his equations (4) and (5) because $\varepsilon_t$ is expressed as difference of $u_t$ and $u_{t-1}$. In other words, if $\varepsilon_t$ is an I(0) process, then $u_t$ becomes an I(1) process.

## 3. **An Illustrative Example**

We investigate the Tokyo Stock Market for illustrating how our model works. The daily closing stock price data of the Tokyo Stock Exchange Price Index (TOPIX) are purchased from the Data Base of Nomura Research Institute, JAPAN. Figure 1 indicates the data of stock prices in natural logarithms and returns from January 4, 1980 to December 2, 2005. It shows the up-trend to the end of 80s, but the down-trend in 90s and 2000s. The returns move mildly in the former period, while they greatly fluctuate in the latter one. Based on these visual observations, the sample period is divided into the two sub-periods: the first is from January 4, 1980 to the end of 80s, the second is from the beginning of 1990 to December 2, 2005.

The estimation results of the parameters in the model (16) with (2) are shown in Table 1. The estimates of the drift term in equation of (16) are positive in the first sub-sample but zero in the second, supporting the up-trend of stock prices in the 80s and mostly the down-trend in the 90s and after. The estimates of $\alpha_1$ and $\alpha_3$ reveal asymmetric volatility and variance persistence, supporting the stylized facts for stock price movements. The estimates of $\theta^+$ and $\theta^-$ are positive and significant at 1% level for each sub-sample period. By using estimated $\theta^+$ and $\theta^-$, we can estimate $V_t$ due to (8) or Proposition 1-(ii) and can find how much the stock prices (i.e., closing price of the day) are over (under) market value hoped by the investors.

The first main interests of this study are market efficiency and asymmetric price adjustment speeds to market values. Figure 2 provides the visual inspection, showing that $|V_t-P_{t-1}|$ in the first period is larger than that in the second period. The adjustment speed $(1-\theta^+, 1-\theta^-)$ in the first period is slower (smaller) so that the market is less efficiency in the first period than the second period. Figure 3A shows the plotted data of $V_t-P_{t-1}$ and $R_t$ in the first period. The coefficients of both variables imply $(1-\theta^+)$ in the first quadrant and $(1-\theta^-)$ in the third quadrant, which show $(1-\theta^+) < (1-\theta^-)$. That is, the adjustment speed is asymmetric as found out by Koutmos (1998) and others. However, Figure 3B in the second period shows symmetric adjustment speed, $(1-\theta^+) = (1-\theta^-)$. Table 2 shows the statistical test that the null hypothesis of market efficiency (H1: $\theta^+=\theta^-=0$) is rejected in both sub-samples. Thus, the market is inefficient in the sense of adjustment speed. The null hypothesis of symmetric adjustment speeds (H2: $\theta^+ = \theta^-$) in equation (3) is rejected in the first sub-sample but not in the second.

How do we interpret the findings? In our framework, the observed values of stock price (i.e., closing prices of the day) are adjusted towards the market values. The adjustment speeds are $(1-\theta^+)$ if $V_t - P_{t-1} \geq 0$ (or equivalently $R_t \geq 0$ from Proposition 1-(i)) and $(1-\theta^-)$ if $V_t - P_{t-1} < 0$ ($R_t < 0$). In particular, in the first sub-sample the speed is (1-0.289) for the case of under-market value (then, positive return), and (1-0.216) for over-market value (then, negative return). The adjustment speeds are significantly different in this period. Koutmos (1998, pp. 285) finds the asymmetric adjustment speeds in many stock markets and argues as "….One possibility is that investors have a higher aversion to downside risk, so they react faster to bad news ($V_t - P_{t-1} < 0$). The use of stop-loss orders is an example of such aversion. Also, portfolio managers feel they are penalized more if they under-perform in a falling market than in a rising market…."

**[ INSERT Figures 1, 2, 3, 4 and Tables 1, 2]**

The second main interest of this study is whether the market values $\hat{V}_t$ in the level bases are evaluated over (under) the fundamental values $S_t$. Based on Definition 2, the market value and the fundamental value in level bases are illustrated in Figure 4. Both values are computing by using (9) with the estimates in Table 1 and by using (13) with g=const per daily bases in each year (while it is changeable over the years) and $S_0=\hat{V}_{Jan/5/80}$ equal to the estimated market value 460 at January 5, 1980.[5] As shown in

[5] For example, the operating days of the market is about 280 days per year in Japan. Then, the average growth rate per year of GDP in 80 is 0.075 and the growth rate per

Figure 4, the market value in 80s is obviously evaluated over the fundamental value, while in 2002-2004 it is evaluated mostly under the fundamental value.

## 4. Concluding Remarks

We have defined and analyzed three propositions such as the market efficiency in the framework of an asymmetrical partial price adjustment model of the revised version of Koutmos for investor behavior, the asymmetric stock price adjustment, and the over (under) -evaluation of market value of stock, and finally linked the model to an EGARCH model to test these propositions. The understanding of the investor behavior related to three propositions can lead to the better interpretation for them. In addition, we add new view points of market efficiency and over (under)-evaluation to his model and develop its model. Second, as an operational example of the model, we pick up the Tokyo Stock Market. We find that the Tokyo stock market is inefficient during 1980-2005 in the sense that the adjustment speeds to the market values are less than 1. In the 80s, the stock price adjustment is faster when the price is above the market value than when the price is below it: the adjustment cost is cheaper when the stock price is above the market value and then the adjustment speed is faster. The market value in 80s which investors hope is evaluated over the fundamental value, while it is relatively weekly over-evaluated in 90s and 2000s.

There have never been trials to investigate what kinds of investor behaviors lead to the market efficiency, partial asymmetric adjustment price and over (under)-evaluation of market value, by incorporating it into the ARCH-type model.

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operating day of the market is 0.000258=EXP((1/280)*LN(1.075))-1. For 1990, that is 0.000265 =EXP((1/280)*LN(1.077))-1 where that for GDP per year is 0.0773. Data source for GDP is from the annual estimates of GDP by Cabinet Office of Japan (Base-year=1995; calendar year; nominal GDP; http://www.esri.cao.go.jp/jp/sna/qe052-2/ritu-smcy0522.csv).

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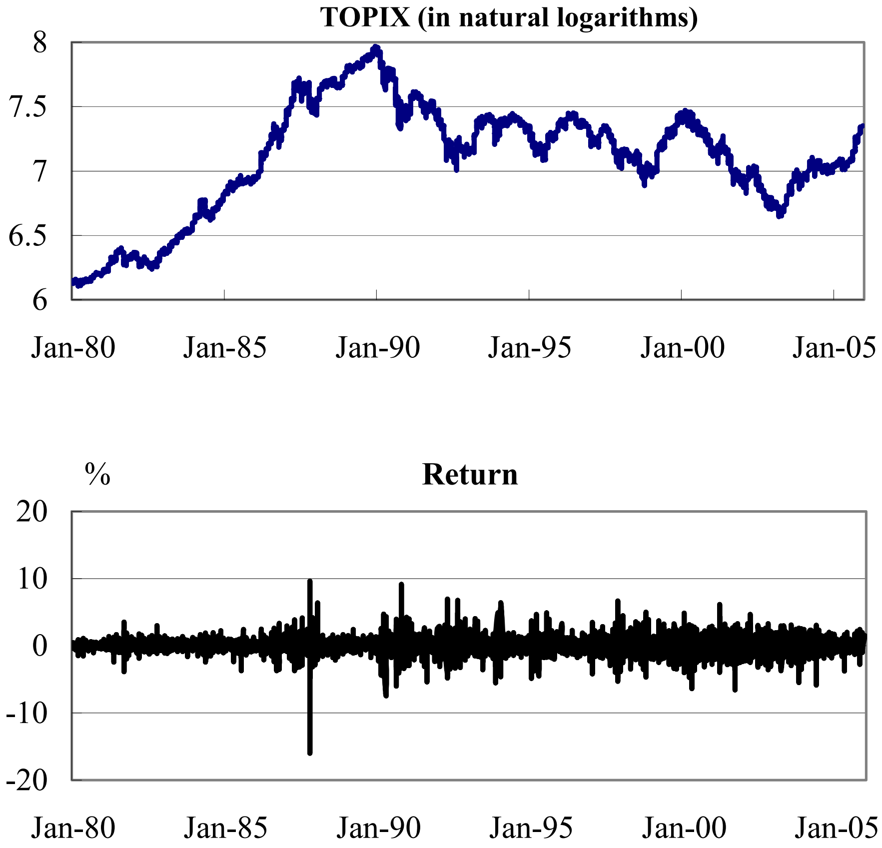
## Figure 1. TOPIX and Its Returns




## Figure 2. Adjustment of the stock prices Pt towards market value Vt: Market Efficiency

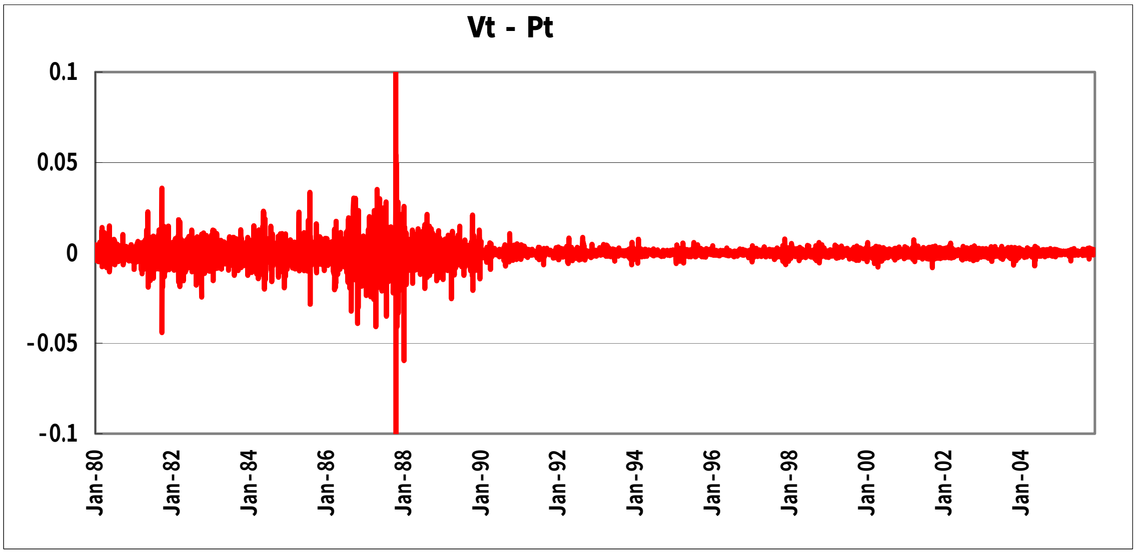
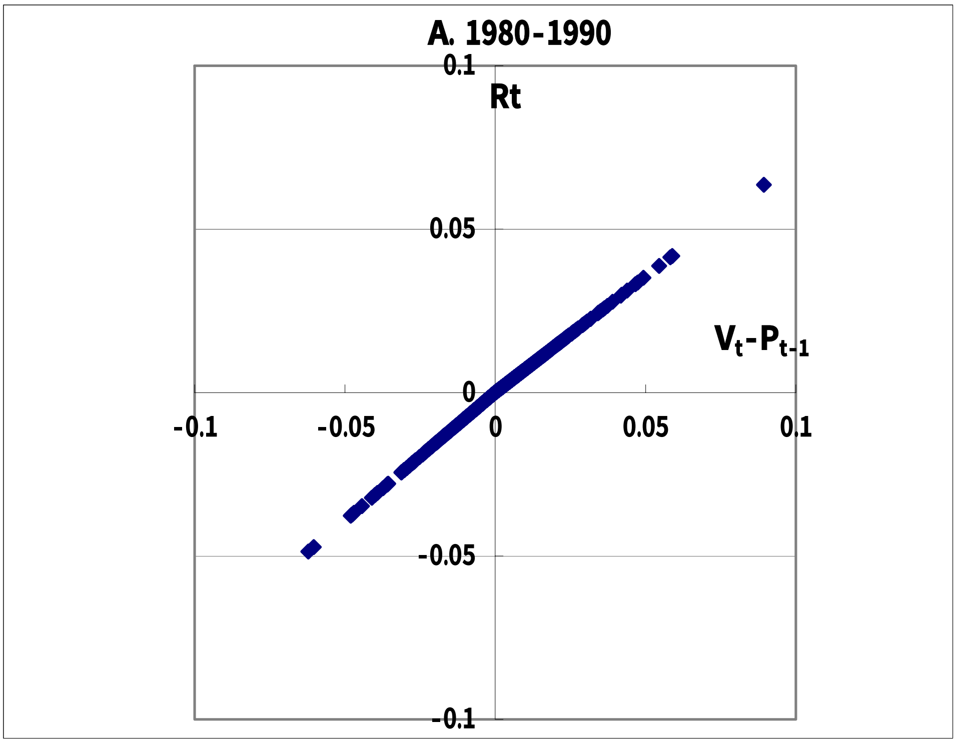


**Figure 3. Adjustment of the stock prices based on $V_t$-$P_{t-1}$: Asymmetric Adjustment Price**



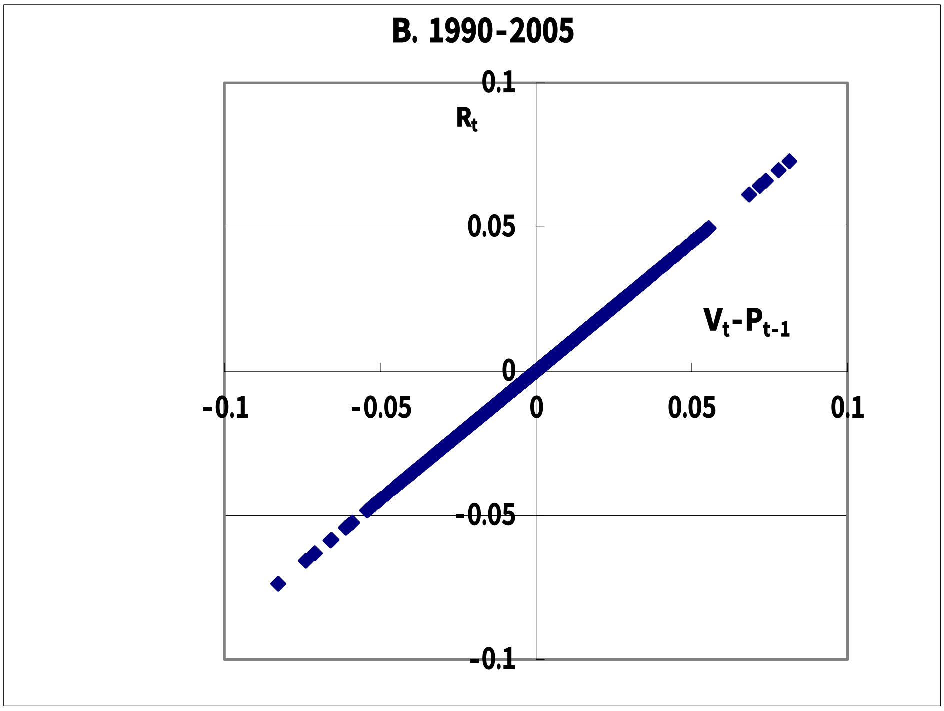
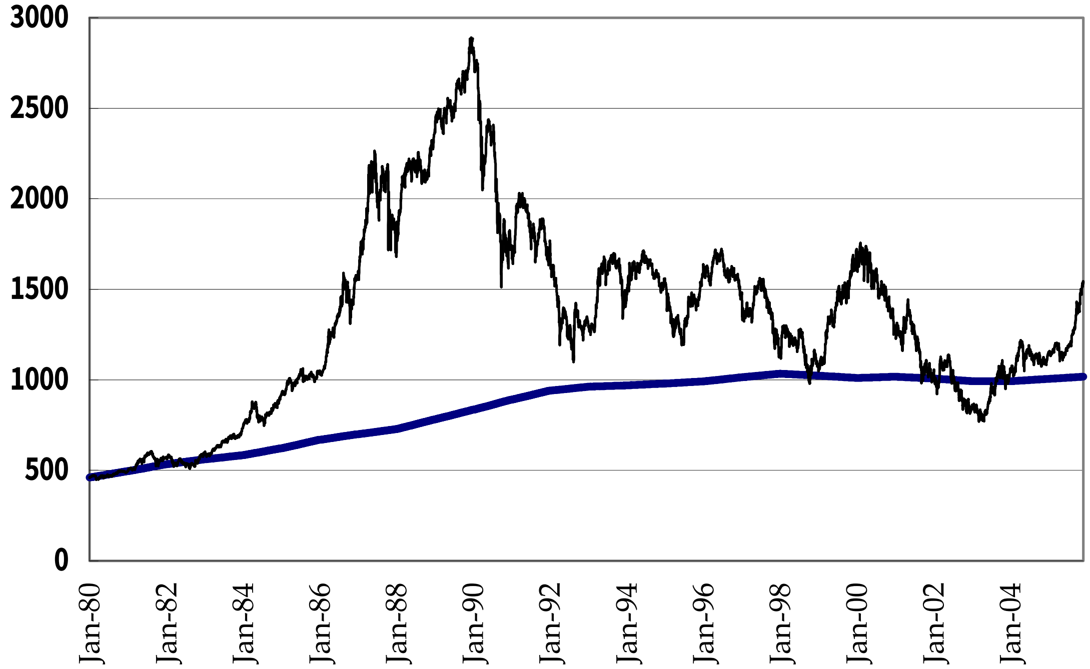



**Figure 4. Over(under)-evaluation of market value in level**




Market Value $\hat{V}_t$ (–) and Fundamental Value $S_t$ (**–**)

Table 1. Estimates of Parameters

| Periods | 1/4/80 –12/ 28/89 | 1/4/90 – 12/2/05 |
|---|---|---|
| a | 0.055* | 0.006 |
| | (6.45) | (0.45) |
| $\theta^+$ | 0.288* | 0.104* |
| | (12.02) | (6.69) |
| $\theta^-$ | 0.216* | 0.110* |
| | (5.94) | (5.27) |
| $\alpha_1$ | -0.144* | -0.092* |
| | (-8.55) | (-9.01) |
| $\alpha_3$ | 0.932* | 0.965* |
| | (94.47) | (163.47) |

Notes: The numbers in parentheses denote t-statistics. The asterisk "*" is significant at 1% level.

**Table 2. Testing Results**

| Null Hypothesis | Distribution | 1/4/80 – 12/ 28/89 | 1/4/90 – 12/2/05 |
|---|---|---|---|
| H1: $\theta^+ = \theta^- = 0$ | $\chi^2(2)$ | 161.36* | 49.86* |
| H2 : $\theta^+ = \theta^-$ | $\chi^2(1)$ | 8.33* | 0.08 |

Note : The asterisk "*" is significant at 1% level. The critical values of $\chi^2(2)$ and $\chi^2(1)$ distributions are respectively 9.21 and 6.34 at 1% level.