Discussion Papers In Economics And Business

Precautionary Demand for Labor in Search Equilibrium

Noritaka Kudoh Masaru Sasaki

Discussion Paper 07-34

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Precautionary Demand for Labor in Search Equilibrium

Noritaka Kudoh　Masaru Sasaki

Discussion Paper 07-34

October 2007

この研究は「大学院経済学研究科・経済学部記念事業」
基金より援助を受けた、記して感謝する。

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

Precautionary Demand for Labor in Search Equilibrium*

Noritaka Kudoh† Masaru Sasaki‡

October 1, 2007

Abstract

This paper studies firms' job creation decisions in a labor market with search frictions. A simple labor market search model is developed in which a firm can search for a *second employee* while producing with a first worker. A firm expands employment even if the instantaneous payoff to a large firm is less than that of staying small—a firm has a *precautionary motive* to expand its size. In addition, this motive is enhanced by a greater market tightness. Because of this effect, firms' decisions become interdependent—a firm creates a vacancy if it expects other firms to do the same, creating strategic complementarity among firms and thereby self-fulfilling multiple equilibria.

JEL classification: E24, J23

Keywords: labor demand, firm size distribution.

*We would like to thank Ryoichi Imai, Takashi Shimizu and the participants of the Japanese Economic Association Meetings at Chuo University and the DC conference at Keio University for their helpful comments and discussions. Part of this research is financially supported by the Japan Society for the Promotion of Science.

†Department of Economics, Hokkaido University; E-mail: kudoh@econ.hokudai.ac.jp

‡Department of Economics, Osaka University; E-mail: sasaki@econ.osaka-u.ac.jp

1 Introduction

Why do firms want to be larger at some points in time and smaller at different points in time? This paper extends the standard search-matching model of the labor market to investigate the firms' job creation decision regarding whether to expand employment or to stay small, and looks for factors that influence the decision. This paper demonstrates the existence of precautionary labor demand in search equilibrium.

The standard search-matching model, which is summarized by Pissarides (2000) and Rogerson *et al.* (2005), has received much attention as the framework for studying various labor market issues. In the standard model, a match, not a firm, is the production unit. Each job and worker engages time-consuming search in the labor market, and each matched pair produces. This is novel in itself because researchers can focus on the number of jobs created in the market instead of studying the employment policy of each firm. In the basic model, the number of jobs is determined by free entry. However, such a job creation process ignores the decision of each incumbent firm regarding whether to expand employment or not. We find this issue very important and worth exploring.

Papers that are most related to this study are those of Bertola and Caballero (1994) and Smith (1999), who studied models with endogenous firm size. Bertola and Caballero (1994) considered a search-matching model in which firms can open many vacancies at once and job destruction is endogenous. The business conditions shock causes labor demand to fluctuate, and firms may have an incentive to fire some of their employees. However, Bertola and Caballero (1994) found that firms *hoard* labor during recessions because the good time will come sooner or later and hiring takes time. Smith (1999) employed a simple representative model of a firm and demonstrated that, with concave production functions, firms overemploy because the wage rate declines as more workers are employed.

In contrast, we stay as close to the standard Pissarides (2000) economy as possible, but modify it by assuming that a firm can employ up to *two* workers. We focus on a firm's *discrete choice* problem regarding whether to expand employment or to stay small. In a frictionless static economy, the answer to the question would be evident. A firm expands employment if the instantaneous payoff of being large exceeds that of being small.

The answer becomes less evident when we address the question in a dynamic economy with search frictions. The main finding of this paper is that a firm of size one creates a vacancy and tries to expand employment even if the instantaneous payoff to a large firm is less than that of staying small. We call this result *precautionary demand for labor.* The mechanism that drives the result is similar to that found in Pissarides (1994), in which a worker can engage in on-the-job search. If on-the-job search is possible, an unemployed worker's reservation wage can be below the unemployment benefit because the value of employment includes the option of finding a better job in the future. A similar mechanism works in this paper, but in the opposite direction. Even if the instantaneous payoff to be a size-one firm is high, the value of this state is not very good because the firm is exposed to the risk of losing the only employee and the opportunity to produce.

The notion of precautionary labor demand slightly differs from that of labor hoarding found in Bertola and Caballero (1994). labor hoarding occurs when firms decide not to cut employment during a recession, waiting for a better aggregate economic condition to come. Firms do so because hiring takes time due to the search friction. In contrast, the precautionary demand for labor arises when firms expand employment in anticipation that employees will leave sooner or later. The key is that firms are exposed to the risk of losing *productive workers.* With this mechanism in mind, we conjecture that precautionary motives for under-employment are also possible, if firms are exposed to the risk of (unwillingly) retaining unproductive workers. The literature on firing restriction deals with this issue.

We also find that this precautionary motive is enhanced by an increase in market tightness. In other words, a firm is more willing to expand employment when the market is tighter. Such a mechanism is absent in the standard model. With this effect, firms' decisions become interdependent: a firm's employment policy depends on the degree of market tightness and at the same time it influences the tightness of the market. It is shown that the firms' decisions are connected in that they strategically complement each other. As a result, there is scope for self-fulfilling multiple equilibria, and the precautionary labor demand plays a crucial role for the outcome. Suppose that a firm of size one expects that other firms will expand employment. This expectation leads to the belief that there will be greater congestion and higher market tightness. Thus, it will take longer to find a replacement once it has lost the employee. Since all matches are productive, the opportunity lost by being vacant is significant. Thus, the firm wishes to reduce the risk of being vacant, and therefore prefers to be large. The same reasoning will imply that if a firm believes other firms will stay small, then it will prefer to remain small.

We demonstrate these results using a model that modifies the standard labor market search model. Although we try to keep the model simple and stay as close to the standard model as possible, some additional assumptions are necessary. Some of them are made to simplify the exposition while others play crucial roles in generating the results. We present a section for clarifying the importance of the way we modify the standard framework, and we also present a section for discussing the results in detail.

2 The Model

2.1 Environment

The model is a straightforward extension of the standard labor market search model presented in Pissarides (2000). Thus, we follow Pissarides (2000) as much as possible. In order to investigate the incumbent firms' decision regarding whether to expand employment or not, we make the following modifications to the standard model:

- There is a fixed number of firms.
- A firm can employ up to two workers.
- The instantaneous payoffs to the firms are exogenous.

There is a fixed number of firms and workers, K firms and L workers, in the economy. It is assumed that L is sufficiently greater than K. In particular, L cannot be below $2K$. The firm size in this paper is determined by the number of employees in the firm, and a firm with n workers is called a firm of size n. The firm size in this model can be either 0, 1, or 2. Note that we are not investigating the optimal firm size here. Thus, the presence of an upper bound to firm size does not influence the issue we are investigating. In order to focus on a size-one firm's decision, we assume that no firm can post two vacancies at once.

Time is continuous.[1] In addition, we focus on a steady state throughout this paper. Let us start with describing the matching technology. As is standard, the total number of matches in the labor market, denoted by M, is determined by a constant-returns-to-scale matching function:

$$M = m\left(uL, vL\right), \tag{1}$$

where u is the unemployment rate while v is the number of vacant jobs as a fraction of the labor force. The rate at which a firm with a vacancy meets an unemployed worker during a unit of time is $M/vL = m\left(uL/vL, 1\right) \equiv q\left(\theta\right)$, where $\theta \equiv v/u$. It is easy to check that $q\left(\theta\right)$ is decreasing in θ. In addition, we assume that $\lim_{\theta \to \infty} q(\theta) = 0$ and $\lim_{\theta \to 0} q(\theta) = \infty$. Similarly, the rate at which a job seeker meets a vacancy during a unit of time is $M/uL = \theta q\left(\theta\right)$. In addition, $\lim_{\theta \to \infty} \theta q(\theta) = \infty$ and $\lim_{\theta \to 0} \theta q(\theta) = 0$.

Define k_n as the proportion of firms of size n. Then, $k_0 + k_1 + k_2 = 1$ must hold. A firm of size one is either searching for additional worker or not searching. Thus, $k_1 = k_1^s + k_1^{ns}$, where k_1^s is the measure of size-one firms that engage in search activity while k_1^{ns} is the measure of size-one firms that are not searching. Thus:

$$k_0 + k_1^s + k_1^{ns} + k_2 = 1. \tag{2}$$

The total number of employees in this economy, E, is given by

$$E = [k_1^s + k_1^{ns} + 2k_2]K. \tag{3}$$

The unemployment rate is defined by

$$u = \frac{L - E}{L}. \tag{4}$$

[1] In a previous version of this paper, we have used a discrete-time model. The main messages of the two versions are the same, while the exposition in this paper is much simpler.

The total number of vacancies in the market is given by

$$vL = k_0 K + k_1^s K. \tag{5}$$

That is, the total number of vacancies equals the number of firms of size zero plus the number of firms of size one seeking a second worker. From (3), (4) and (5), we obtain

$$\theta \equiv \frac{v}{u} = \frac{k_0 K + k_1^s K}{L - [k_1^s + k_1^{ns} + 2k_2]K} = \frac{k_0 + k_1^s}{\frac{L}{K} - k_1^s - k_1^{ns} - 2k_2}, \tag{6}$$

where L/K is constant. This suggests that the degree of labor market tightness θ is determined jointly with the distribution of firms.

Let λ be the Poisson arrival rate at which a match exogenously dissolves. Then, in any steady state, the flows into and out of the unemployment pool must be the same. Thus, $\lambda(1-u) = \theta q(\theta) u$ must hold, from which it is easy to show that $u = \lambda/[\lambda + \theta q(\theta)]$. Thus, the equilibrium unemployment rate is determined if market tightness θ is determined. In addition, the conventional negative relationship between u and θ holds.

2.2 Stationary Distribution of Firms

This paper focuses on the steady-state equilibrium in which the distribution of firms is constant over time. In any steady state, the number of firms that flow into and out of a particular state must be balanced. Let $\bar{\sigma}$ denote the fraction of size-one firms that engage in search activity while σ is a firm's choice variable; $\sigma = 1$ if a particular firm of size one opens a vacancy and $\sigma = 0$ if it does not. Then:

$$\lambda k_1^s + \lambda k_1^{ns} = q(\theta) k_0, \tag{7}$$

$$\bar{\sigma} q(\theta) k_0 + \bar{\sigma}\lambda k_2 = \lambda k_1^s + q(\theta) k_1^s, \tag{8}$$

$$(1-\bar{\sigma}) q(\theta) k_0 + (1-\bar{\sigma})\lambda k_2 = \lambda k_1^{ns}, \tag{9}$$

$$q(\theta) k_1^s = \lambda k_2. \tag{10}$$

According to (7), the measures of firms that flow into and out of the state of size zero must be balanced. The firms that flow into this state are those of size-one firms that have just lost the employee, which amount to $\lambda k_1^s + \lambda k_1^{ns}$. The outflow equals $q(\theta)k_0$ because a firm in this state finds an employee with the rate $q(\theta)$. Two comments are in order. First, a size-two firm does not lose two employees at once because in a continuous-time environment, the probability of two independent events occurring is negligible. Second, we assume that no firm can post two vacancies at once. This precludes the flow of size-zero firms jump into the state of size two.

The second equation, (8), balances the inflow and the outflow regarding the state in which a firm is of size one and opens a vacancy. The inflow equals $\bar{\sigma}q(\theta)k_0 + \bar{\sigma}\lambda k_2$, which equals the measure of size-zero firms that have found an employee *and* opens another vacancy plus the measure of size-two firms that have lost an employee *and* opens another vacancy. The outflow equals $\lambda k_1^s + q(\theta)k_1^s$, which is the sum of measures of size-one firms that have lost an employee and found a second employee.

The third equation, (9), balances the inflow and the outflow regarding the state in which a firm is of size one and does not open a vacancy. The inflow equals $(1-\bar{\sigma})q(\theta)k_0 + (1-\bar{\sigma})\lambda k_2$, which equals the measure of size-zero firms that have found an employee but do not open another vacancy plus the measure of size-two firms that have lost an employee but do not open another vacancy. Since they do not search, the outflow is simplyλk_1^{ns}. Finally, (10) states that the inflow into the state of size two is $q(\theta)k_1^s$ and the outflow is λk_2.

2.3 Firms

In what follows, we describe each firm's behavior. There are four states to consider, each of which is assigned a value function. Let V denote the value to an empty firm of searching for an employee. J_1^s is the value to a size-one firm of engaging in search activity, while J_1^{ns} is the value of a size-one firm that does not engage in search activity. Finally, J_2 is the value of being a size-two firm.

Let r be the discount rate. The value of a firm without an employee, V, is given by

$$rV = -c + q(\theta) \max_{\sigma \in [0,1]} \left[\sigma \left(J_1^s - V\right) + (1-\sigma)\left(J_1^{ns} - V\right)\right], \tag{11}$$

where $c \geq 0$ is the cost of maintaining a vacancy. Equation (11) is interpreted as follows. As long as a firm is empty, it has to engage in search activity. With the Poisson arrival rate $q(\theta)$, the firm finds an employee and becomes a size-one firm. The firm must choose whether or not it will engage in search activity to employ a second worker.

A firm of size one is in either two states, searching or not searching. Consider the value to a size-one firm of engaging search activity while producing. The value is given by

$$rJ_1^s = R_1 - c + \lambda[V - J_1^s] + q(\theta)\,[J_2 - J_1^s]\,, \tag{12}$$

where R_1 is the *net* instantaneous payoff to the firm of producing with one employee. Thus, R_1 equals the total revenue minus the cost of employing one worker, including wage bills and other benefits. We do not follow the routine of determining the share of revenue with Nash bargaining. We merely postulate that the total revenue is *somehow* split between the firm and the worker, and R_1 is the payoff the firm gets.

Equation (12) may be interpreted as follows. A firm of size one receives R_1 and pays c as the cost of posting a vacancy. With Poisson arrival rate λ, the employee leaves the firm. With arrival rate $q(\theta)$, the firm finds a second worker.

The value to a size-one firm of not engaging in search activity is given by

$$rJ_1^{ns} = R_1 + \lambda[V - J_1^{ns}]. \tag{13}$$

The value of a firm that employs two employees, J_2, is given by

$$rJ_2 = R_2 + \lambda \max_{\sigma\in[0,1]} \left[\sigma\left(J_1^s - J_2\right) + (1-\sigma)\left(J_1^{ns} - J_2\right)\right], \tag{14}$$

where R_2 is the net instantaneous payoff to the firm. Again, R_2 is exogenous. With Poisson arrival rate λ, one of the employees leaves the firm, in which case the firm needs to choose whether it will seek a second employee or not.

It is assumed that the firm size cannot exceed two. Of course, one could follow Bertola and Caballero (1994) and Smith (1999) to allow firms to choose any size. As emphasized earlier, we are interested in the very choice that a firm faces regarding whether it should expand its size or it should stay small. We believe that focusing on the choice faced by a size-one firm is enough to give us important insights regarding the employment policy of a firm.

From (12) and (13), $(r+\lambda)(J_1^s - J_1^{ns}) = -c + q(\theta)[J_2 - J_1^s]$ holds. Thus, $J_1^s > J_1^{ns}$ holds if and only if $J_2 - J_1^s > c/q(\theta)$. This condition compares the net gain from becoming a size-two firm with the expected total fixed cost of vacancy. Solve (11)-(14) for $J_2 - J_1^s$ as a function of θ and substitute it into the above inequality. After some algebra, it is easy to show that $J_1^s > J_1^{ns}$ holds if and only if

$$R_2 - R_1 + \frac{(R_1 + c)\,\lambda}{r + \lambda + q(\theta)} > \frac{(r+\lambda)\,c}{q(\theta)}. \tag{15}$$

An interesting special case is obtained by letting $c = 0$. In the conventional Pissarides (2000) model, $c > 0$ is necessary for determinacy of equilibrium because of the free entry of jobs. Here, the presence of the cost of maintaining a vacancy is not an essential part of the model. With $c = 0$, it is evident that $J_1^s > J_1^{ns} \Longleftrightarrow J_2 > J_1^s$, implying that a firm tries to expand employment if and only if the value of being a size-two firm is greater than that of being a size one that engages in search activity. After some algebra, this condition is expressed by

$$\begin{aligned} R_2 &> \phi(\theta) R_1, \qquad (16) \\ \phi(\theta) &\equiv \frac{r + q(\theta)}{r + \lambda + q(\theta)} \in (0,1). \end{aligned}$$

Proposition 1 *In the limiting case with $c = 0$, a size-one firm creates a vacancy if and only if $R_2 > \phi(\theta)\, R_1$ where $\phi(\theta) \in (0,1)$.*

Corollary 2 *A firm expands its size even if $R_2 < R_1$.*

Proposition 1 asserts that firms have *precautionary motives* to hire extra workers because firms are willing to create vacancies even if the instantaneous payoff to a size-two firm is below the current instantaneous payoff. The mechanism is better understood by recalling the labor market search model with on-the-job-search, in which an unemployed worker's reservation wage is *below* the unemployment benefit.[2] This is because the worker foresees that there is a chance of getting a better job in the future; even though the wage is low, the value of the job can be very high. Quite similarly, a firm of size one is willing to create a vacancy to be of size two because even if

[2] A simple exposition of this result appears in Rogerson *et al.* (2005).

$R_2 < R_1$, the value of being a size-one firm can be very small. Here, the value of being a size-two firm comes from the fact that it faces no risk of being empty in the next instant. Remember that in this economy, separations are exogenous and firms do not want to lose their employees because all matches are productive. Thus, we can conclude that in the real world, we should expect to observe precautionary labor demand in an economy in which workers quit while firms do not wish to lose them.

Proposition 3 $\phi'(\theta) < 0$.

Figure 1 shows the determination of σ under $c = 0$. Let $\hat{\theta}$ solve $R_2/R_1 = \phi(\theta)$. Propositions 3 and 1 suggest that a firm expands employment if and only if $\theta \geq \hat{\theta}$. When θ is large, a firm finds it hard to fill its vacancy. In other words, the expected duration of vacancies is relatively long. In this case, the firm wishes to be large so that it is not exposed to the risk of being empty from the stochastic separation. The importance of Proposition 3 is that it works in the direction that *enhances* the precautionary labor demand.

When θ is small, on the other hand, a firm finds it easy to fill its vacancy. Thus, there is no need to avoid being empty. It is important to point out that firms are worse off with a higher θ. In fact, an increase in θ reduces the value of being a size-two firm. However, the value of being a size-one firm is even more significantly reduced because size-one firms are more exposed to the risk of devoid of employees.[3]

That a firm's decision depends on the tightness of the market has an important implication. Since market tightness is influenced by the decision itself, the decision depends on what *other firms* do. As described later, this causes strategic complementarity among firms and opens up the possibility of self-fulfilling multiple equilibria.

Consider the general case in which $c \geq 0$. In this case, a size-one firm creates a vacancy if and only if (15) holds. Rewrite the condition as

$$R_2 > \phi(\theta)R_1 + \frac{[r+\lambda]\,\lambda + [r+\lambda+q(\theta)]\,r}{[r+\lambda+q(\theta)]\,q(\theta)}c.$$

[3] After some algebra, one can show that $J_2 - J_1^s$ is increasing in θ while J_2 is decreasing with θ.

It is evident that as the vacancy cost increases, a size-one firm becomes more reluctant to engage in search activity. Therefore, the precautionary demand for labor exists only for sufficiently small values of c.

2.4 Discussion of the Key Assumptions

One vacancy at a time: Related models such as Bertola and Caballero (1994) and Smith (1999) allow firms to create an arbitrary amount of vacancies at a time. Of course, one could consider an alternative version of the model in which a firm can post two vacancies at once. Such a model would be interesting in itself. Using such a model, one could ask, for example, whether a firm opens one vacancy at a time or tries to fill two vacancies at once. In this paper, we are simply asking whether firms want to be large or not, rather than how fast they want to be large.

Upper bound of firm size: That a firm cannot employ more than two workers is too restrictive an assumption. However, note that we are not investigating the optimal firm size. We are focusing on the binary choice that a firm faces regarding whether to expand employment or not. In the standard search-matching model, the size of a firm is, by construction, one. By allowing the firm size to be two, the model of this paper generates the simplest possible endogenous size distribution of firms.

Exogenous separations: An important implication of exogenous separation is that a match dissolves while it is productive. In other words, firms lose employees even though they do not want to. We suspect that the precautionary labor demand would not arise if job destruction were a choice variable to the firms. Thus, an appropriate interpretation of separation in this model is that workers engage in on-the-job search. In this case, it is possible that a firm loses its (productive) employee. Alternatively, one could simply assume that workers retire. How plausible is the assumption of exogenous separation? Hall (2005) finds that fluctuations in unemployment are not a result of massive layoffs. Hall's (2005) finding at least partially supports the use of a framework that does not incorporate endogenous job destruction.

No entry/exit of firms: Most of the standard labor market search models assume free entry of jobs.[4] Models with endogenous firm size commonly employ a framework in which the number of firms is predetermined. On behalf of these models, we emphasize that there is an important distinction between free entry of *jobs* and free entry of *firms*. Although there is no entry or exit of *firms*, the number of jobs can change as σ changes. In addition, job creation is actually *free* when $c = 0$. In the standard search-matching model, the free entry assumption seems the most appropriate choice, as the economy is viewed as a collection of jobs. Free entry of firms, on the other hand, is less obvious because firms face start-up costs (Fonseca *et al.* (2001)), capital market imperfection (Acemoglu (2001)), and regulation (Blanchard and Giavazzi (2003)), to name a few.

Zero vacancy cost: It is important to emphasize that letting $c = 0$ does not pose any problem in our analysis because the number of firms is fixed. In the standard labor market search model, there is free entry of jobs, and a positive vacancy cost plays a crucial role in generating a search equilibrium. In fact, letting $c = 0$ does cause a problem, for example, in Pissarides (2000). In particular, an infinite number of vacancies opens, and the value of an occupied job will be zero, implying that the worker takes all the available profit. This is a Walrasian outcome.

3 Equilibrium

An important property of the model is that there is a non-trivial distribution of firms that is driven by each firm's optimal employment policy. As demonstrated earlier, a firm's decision depends on what other firms would do. Such feedback is possible partly because there is a fixed number of firms, so it is impossible for a firm's decision to depend on the behavior of firms that have not yet entered the market. In order to characterize the firms' behavior in full, we need to compute the

[4] One can argue that a model with free entry should be interpreted as a model describing the long-run behavior of the labor market while a model with a fixed number of firms should be interpreted as describing short-run behavior. Although such an interpretation is possible and may be plausible, we do not necessarily regard the model as one describing some transitory state of the economy.

firm distribution as well as the market tightness.[5]

We focus on symmetric equilibria in which all firms in the same state choose the same employment policy. There are potentially two types of equilibria, one in which all size-one firms choose to create a vacancy and another in which all firms choose to be small. First, consider the equilibrium in which all firms of size one wish to expand employment. Such an equilibrium is characterized by $\bar{\sigma} = 1$. From (7)-(10), the firm distribution of this equilibrium is given by $\lambda k_1^s = q(\theta)k_0$, $q(\theta)k_0 + \lambda k_2 = \lambda k_1^s + q(\theta)k_1^s$, $q(\theta)k_1^s = \lambda k_2$, and $k_0 + k_1^s + k_2 = 1$. Solve these equations as functions of θ to obtain

$$k_0 = \frac{\lambda^2}{\lambda^2 + \lambda q(\theta) + [q(\theta)]^2},\ k_1^s = \frac{\lambda q(\theta)}{\lambda^2 + \lambda q(\theta) + [q(\theta)]^2},\ k_2 = \frac{[q(\theta)]^2}{\lambda^2 + \lambda q(\theta) + [q(\theta)]^2}.$$

After some algebra, (6) reduces to

$$\theta = \frac{\lambda^2 + \lambda q(\theta)}{\left[\lambda^2 + \lambda q(\theta) + [q(\theta)]^2\right] L/K - \lambda q(\theta) - 2[q(\theta)]^2} \equiv \Omega_L(\theta). \tag{17}$$

The properties of the function $\Omega_L(\theta)$ are summarized as follows.

Lemma 4 $\lim_{\theta\to 0}\Omega_L(\theta) = 0$; $\lim_{\theta\to\infty}\Omega_L(\theta) = K/L$; $\Omega_L'(\theta) \geq 0$ *holds if and only if* $\theta \leq \bar{\theta}$, *where* $\bar{\theta}$ *is defined below.*

Proof. It is convenient to rewrite (17) as $\Omega_L(\theta) = [\lambda^2/q(\theta) + \lambda]\{[\lambda^2/q(\theta) + \lambda + q(\theta)]L/K - \lambda - 2q(\theta)\}^{-1}$. It is then easy to verify that $\lim_{\theta\to 0}\Omega_L(\theta) = 0$. It is easy, though tedious, to compute the derivative of $\Omega_L(\theta)$ as $\Omega_L'(\theta) = [\lambda^2 - (L/K - 2)(2\lambda + q(\theta))q(\theta)]\lambda q'(\theta)D^{-2}$, where $D \equiv \lambda^2 L/K + \lambda q(\theta)(L/K - 1) + [q(\theta)]^2(L/K - 2)$. Then, it is evident that $\Omega_L'(\theta) \geq 0$ if and only if

$$\lambda^2 \leq (L/K - 2)(2\lambda + q(\theta))\, q(\theta). \tag{18}$$

Such a requirement is satisfied as long as L/K is sufficiently large and λ is not too large. $L/K > 2$ is necessary. Consider the right-hand side of (18). For any $q > 0$, it is strictly increasing. This

[5] In a previous version of this paper, we have also studied the equilibrium with free entry of firms. With free entry, each firm's decision is independent of those of other firms because the degree of market tightness is determined by free entry, not by the employment policy of the firms.

implies that there exists a unique q that satisfies (18) at equality. Let such value denote $\bar{q}$. Then, the condition (18) is satisfied for $q \geq \bar{q}$. Additionally, define $\bar{\theta}$ as the tightness that corresponds to $\bar{q}$. Then, it is evident that (18) holds if and only if $\theta \leq \bar{\theta}$. ■

Figure 2 depicts the typical configuration of the function $\Omega_L(\theta)$. A candidate equilibrium is given by a fixed point of the mapping.

Next, consider the equilibrium in which all firms wish to stay small. Thus, $\bar{\sigma} = 0$. In this equilibrium, the distribution of firms is given by $\lambda k_1^{ns} = q(\theta) k_0$, $q(\theta) k_0 = \lambda k_1^{ns}$, and $k_0 + k_1^{ns} = 1$. Solve these to obtain

$$k_0 = \frac{\lambda}{\lambda + q(\theta)},\ k_1^{ns} = \frac{q(\theta)}{\lambda + q(\theta)}.$$

After some algebra, (6) reduces to

$$\theta = \frac{\lambda}{[\lambda + q(\theta)]\, L/K - q(\theta)} \equiv \Omega_S(\theta)\,. \tag{19}$$

The properties of the function $\Omega_S(\theta)$ are summarized as follows.

Lemma 5 *$\lim_{\theta \to 0} \Omega_S(\theta) = 0$; $\lim_{\theta \to \infty} \Omega_S(\theta) = K/L$; $\Omega_S'(\theta) > 0$.*

Proof. From (19), it is easy to compute the derivative of $\Omega_S(\theta)$ as $\Omega_S'(\theta) = -\lambda(L/K - 1)q'(\theta)[\lambda L/K + (L/K-1)q(\theta)]^{-2} > 0$. ■

Figure 3 depicts the typical configuration of the function $\Omega_S(\theta)$. A candidate equilibrium is given by a fixed point of the mapping.

Proposition 6 *Let θ_L and θ_S solve $\theta = \Omega_L(\theta)$ and $\theta = \Omega_S(\theta)$, respectively. Then, $\theta_S < \theta_L$ holds.*

Proof. Notice that (17) can be rewritten as

$$\Omega_L(\theta) = \lambda \times \left[\frac{\left[\lambda^2 + \lambda q(\theta) + [q(\theta)]^2\right]}{\lambda + q(\theta)} \frac{L}{K} - \frac{\lambda + 2q(\theta)}{\lambda + q(\theta)} q(\theta) \right]^{-1}.$$

Then, it is easy to see that $\Omega_L(\theta) > \Omega_S(\theta)$ holds if and only if

$$[\lambda + q(\theta)] \frac{L}{K} - q(\theta) > \frac{\left[\lambda^2 + \lambda q(\theta) + [q(\theta)]^2\right]}{\lambda + q(\theta)} \frac{L}{K} - \frac{\lambda + 2q(\theta)}{\lambda + q(\theta)} q(\theta),$$

which reduces to $\lambda L/K > -q(\theta)$ and this is true for any positive q. This proves $\Omega_L(\theta) > \Omega_S(\theta)$ for all θ. ■

Since u is decreasing in θ, the unemployment rate associated with θ_S is higher than the other one. Thus, the result suggests that the unemployment rate is higher in the equilibrium where firms stay small.

Since the general condition for $\bar{\sigma} = 1$, given in (15), is complicated, we focus on the special case in which $c = 0$. Since all size-one firms choose to expand employment if and only if $\theta > \hat{\theta}$, a stationary equilibrium is given by a fixed point of $\theta = \Omega(\theta)$, where

$$\Omega(\theta) = \begin{cases} \Omega_L(\theta) \text{ for } \theta > \hat{\theta} \\ \Omega_S(\theta) \text{ for } \theta \leq \hat{\theta}. \end{cases}$$

Figures 4–6 depict the three possible configurations of $\Omega(\theta)$. Consider Figure 4. This case arises if $\hat{\theta} < \theta_S$, or R is sufficiently large. As shown in the figure, the unique steady-state equilibrium is found from the map $\theta = \Omega_L(\theta)$ and is θ_L. Consider the case with $\hat{\theta} > \theta_L$ (R is sufficicently small). In this case, the steady state is found by a map $\theta = \Omega_S(\theta)$, as depicted in Figure 5. There is a unique equilibrium and is θ_S. In either case, the stationary equilibrium is uniquely determined.

An interesting case arises when $\theta_S < \hat{\theta} < \theta_L$. This case is depicted in Figure 6. Since the maps $\theta = \Omega_L(\theta)$ and $\theta = \Omega_S(\theta)$ are both valid, so are θ_L and θ_S. Thus, we encounter the case of *multiple equilibria*. The multiplicity obtained here is worthy of comment. We have seen that the precautionary labor demand has the important property that a firm's employment policy depends on other firms' employment policies. That is, strategic complementarity resulting from the precautionary labor demand generates the multiplicity of equilibria. Suppose that a firm expects all other firms to expand employment (*i.e.*, $\bar{\sigma} = 1$). It is then reasonable for the firm to expect that the market tightness θ will be high and that it will be harder to fill its vacancy. Because the precautionary demand is enhanced by high θ (Proposition 3), the firm's choice reinforces this reasoning. Therefore the firm creates a vacancy.

Similarly, suppose that the firm expects that all other firms will stay small (*i.e.*, θ will be low). Then the value of being vacant is expected to be high because it will be easier for the firm to find a match. Thus, the firm does not need to reduce the chance of being devoid of employees. Therefore, the precautionary motive for larger size will be weak and the firm is likely to stay small.

4 Discussion of the Results

In the standard search-matching model, there is no precautionary labor demand at work. Since market tightness is determined by free entry, each firm ignores its impact on market tightness when competing for unemployed workers. To put it differently, in the standard model, firms do not take into account the congestion effect they cause. Dropping the free entry assumption changes that. Since market tightness is determined solely by the firms' employment policy, each firm faces the congestion effect. Thus, for precautionary labor demand to arise, the free entry assumption must be dropped.[6]

The key to understanding the precautionary labor demand is that firms might lose their employees when they do not wish to. Remember that job destruction in this model occurs exogenously, so a separation occurs even though a match is productive. The opportunity lost by a separation is greater when labor market tightness is high because it takes more time to fill a vacancy.

Is it possible to get the opposite result? In other words, is it equally plausible that firms have a precautionary motive to hire *fewer* workers? Our conjecture is yes. Imagine an alternative version of the model in which firms cannot fire workers when they need to. As the recent research on the search model with firing costs suggests, firms are less likely to hire when there is a firing restriction (Blanchard and Summers (1988); Saint-Paul (1995)). However, as Bentolila and Bertola (1990) claim, the firing costs do not always generate under-employment.

Bertola and Caballero (1994) construct a labor market search model with endogenous firm size and endogenous job destruction to find labor hoarding behavior. Firms tend to fire too little during a recession because aggregate business conditions might improve in the future and hiring takes time. The precautionary demand for labor found in this paper can be understood as a counterpart of labor

[6] Indeed, in an earlier version of the paper we have verified that adding the free entry condition ($V = 0$) leads to no precautionary labor demand.

hoarding. To be more precise, labor hoarding arises in a model with endogenous job destruction, and it works at the *firing margin*. Precautionary labor demand arises at the other end: *hiring margin*. They both arise from a single fact: separation is immediate and hiring takes time.

In this paper we have assumed that firms' instantaneous payoffs are constant. How important is this assumption? To take a quick look at the consequence of introducing endogenous wages, suppose that the firms' payoffs can be written as $R_1 = p_1 - w_1(\theta)$ and $R_2 = p_2 - 2w_2(\theta)$, where p_n is the profit to be shared with n employees. The wage rate at a firm of size n is denoted by $w_n(\theta)$, which is assumed to be increasing in θ. That is, the wage rate increases as market tightness increases, which is consistent with the standard model with wage bargaining. Now, if we denote $R(\theta) \equiv [p_2 - 2w_2(\theta)]/[p_1 - w_1(\theta)]$, then, with $c = 0$, (16) suggests that a firm of size one tries to expand employment if and only if $R(\theta) > \phi(\theta)$. The shape of $R(\theta)$ could be anything; there is no guarantee that it is even monotonic. However, quite a lot can be said from this short-cut model. Consider the slope of $R(\theta)$. With exogenous payoffs, $R(\theta)$ is a straight line, as in Figure 1. Suppose $R(\theta)$ is increasing in θ. Such a case occurs if an increase in the market tightness raises w_1 much more than w_2. In this case, the result found in Proposition 1 survives.

Alternatively, suppose that $R(\theta)$ is decreasing. If the slope is relatively flat, then the results would not change. A completely new possibility arises when the slope of $R(\theta)$ is negative and is steeper than that of $\phi(\theta)$. In this case, a firm would expand employment if and only if the market tightness is low instead of high. There is still precautionary labor demand in the sense that a firm expands employment even if $R_2 < R_1$. However, an increase in market tightness does not enhance the precautionary motive. Without this channel, the strategic complementarity among firms ceases to exist and therefore no multiplicity of equilibria can arise. This suggests that no-free-entry is not enough for multiple equilibria; precautionary labor demand and especially its enhancement (Proposition 3) are crucial.

5 Conclusion

This paper has studied the choice of job creation using a stylized model of a labor market with search frictions. Two findings are worthwhile. One is that firms have precautionary motives to expand employment. One of the factors that generate the precautionary labor demand is that firms face the risk of losing (productive) workers. The precautionary demand is enhanced by greater market tightness, implying that firms' employment policies influence each other through impacts on market tightness. This leads to the other main finding of this paper. Self-fulfilling multiple equilibria are possible due to the strategic complementarity among firms. The analysis suggests that precautionary labor demand should be observed in an economy in which (i) entry of firms is to a certain degree limited and (ii) firms unwillingly lose their employees.

We conclude by suggesting some directions for future research. In this paper, we have employed the assumption of exogenous separations. It is worthwhile to explore a model in which workers' choice of separation is introduced along the line of Burdett and Mortensen (1998) to see if there is another source of strategic complementarity. We have found that precautionary labor demand is enhanced by market tightness. It is worth exploring this effect in the context of the business cycle. This framework might help explain the cyclical behavior of market tightness. Finally, it is very important to consider other margins within which labor input may be adjusted. In particular, it is worth investigating a model in which a firm can choose between the extensive margin (number of employment) and intensive margin (hours of work). The model presented in this paper provides a reasonable framework for studying these issues.

References

[1] Acemoglu, Daron. "Credit Market Imperfections and Persistent Unemployment," *European Economic Review* 45 (2001) 665–679.

[2] Bentolila, Samuel, and Giuseppe Bertola. "Firing Costs and labor Demand: How Bad Is Eurosclerosis?" *Review of Economic Studies* 57 (1990) 381–401.

[3] Bertola, Giuseppe, and Ricardo J. Caballero. "Cross-Sectional Efficiency and labor Hoarding in a Matching Model of Unemployment." *Review of Economic Studies* 61 (1994) 435–456.

[4] Blanchard, Olivier, and Francesco Giavazzi. "Macroeconomic Effects of Regulation and Deregulation in Goods and labor Markets," *Quarterly Journal of Economics* 118 (2003) 879–907.

[5] Blanchard, Olivier, and Lawrence Summers. "Beyond the Natural Rate Hypothesis," *American Economic Review* 78 (1988) 182–187.

[6] Burdett, Kenneth, and Dale Mortensen. "Wage Differentials, Employer Size, and Unemployment," *International Economic Review* 39 (1998) 257–237.

[7] Fonseca, Raquel, Paloma Lopez-Garicia, and Christopher Pissarides. "Entrepreneurship, Start-up Costs and Employment," *European Economic Review* 45 (2001) 692–705.

[8] Hall, Robert. "Employment Efficiency and Sticky Wages: Evidence from Flows in the labor Market," *Review of Economics and Statistics* 87 (2005) 397–410.

[9] Pissarides, Christopher. "Search Unemployment with On-the-Job Search." *Review of Economic Studies* 61 (1994) 457–75.

[10] Pissarides, Christopher. *Equilibrium Unemployment Theory*, 2nd edition, MIT Press, Massachusetts, 2000.

[11] Rogerson, Richard, Robert Shimer, and Randall Wright. "Search-Theoretic Models of the labor Market: A Survey," *Journal of Economic Literature* 43 (2005) 959–988.

[12] Saint-Paul, Gilles. “The High Unemployment Trap,” *Quarterly Journal of Economics* 110 (1995) 527–550.

[13] Smith, Eric. “Search, Concave Production, and Optimal Firm Size.” *Review of Economic Dynamics* 2 (1999) 456–471.

Figure 1: Threshold level of θ

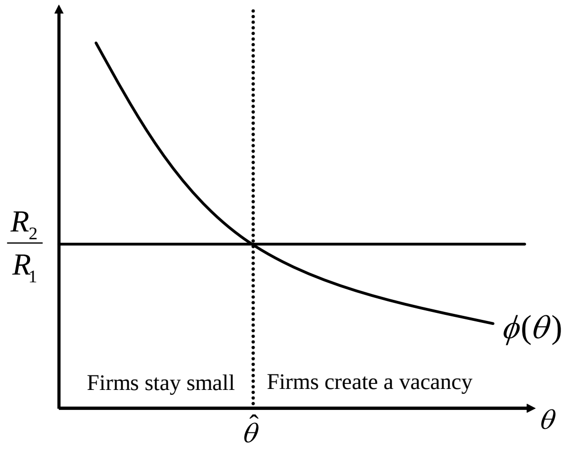

Figure 2: Function Ω_L

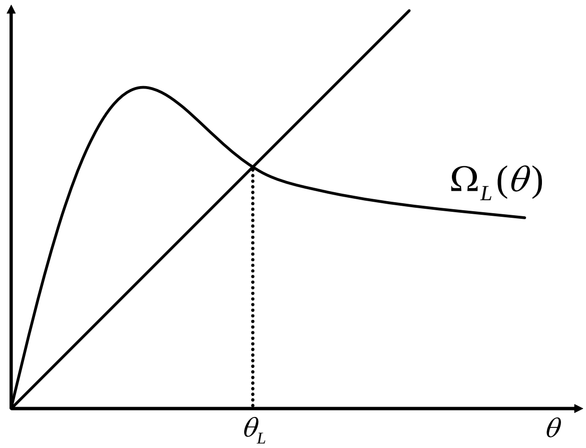

Figure 3: Function Ω_S

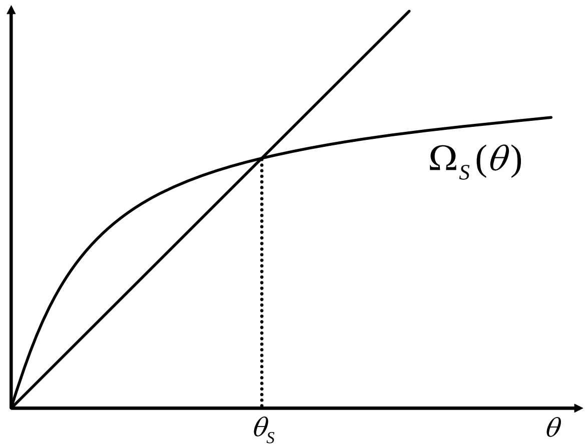

Figure 4: Stationary Equilibrium (Case 1)

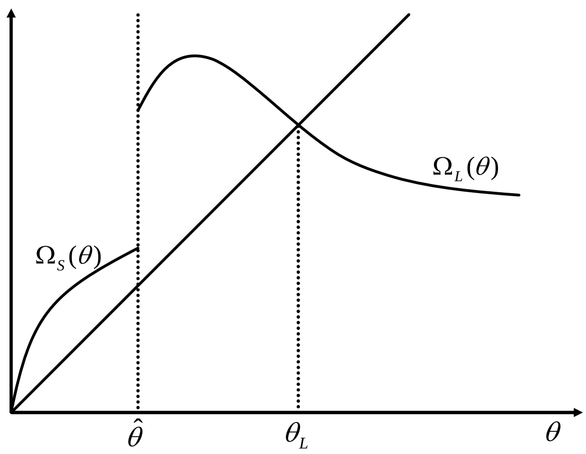

Figure 5: Stationary Equilibrium (Case 2)

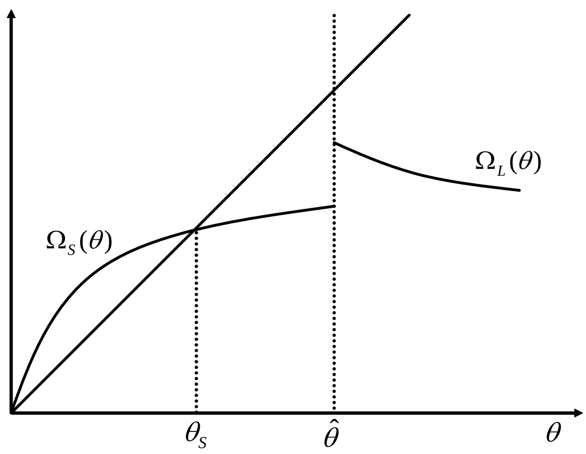

Figure 6: Multiple Equilibria