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Abstract

This paper characterizes a stationary Markov-perfect political equilibrium where agents vote over income taxation that distorts educational investment. Agents become rich or poor through educational investment, and the poor have a second chance at success. The results show the following concerning the cost of a second chance. First, when the cost is low, the economy is characterized by high levels of upward mobility and inequality, and a low tax burden supported by the poor with prospects for upward mobility. Second, when the cost is high, there are multiple equilibria with various patterns of upward mobility, inequality and redistribution. Numerical examples show that the shift from a high-cost economy to a low-cost economy may reduce social welfare.

Key words: Second chance; Political economy; Inequality; Upward mobility; Intragenerational mobility.

JEL Classification: D30; D72; H55.

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1 Introduction

Why do Western countries have different welfare programs, even though they share a similar background? Who supports redistribution in these countries? How is income inequality related to redistributive politics? Hassler et al. (2003) and Hassler, Storesletten and Zilibotti (2007) developed a model that responds to these questions based on a feedback mechanism between individual decisions and redistributive politics (Glomm and Ravikumar, 1995; Saint-Paul and Verdier, 1997; Benabou, 2000). In this, the feedback mechanism creates the joint determination of inequality and redistribution and results in multiple equilibria: a pro-welfare state and an anti-welfare state. In the pro-welfare (anti-welfare) state, expectations of higher (lower) redistribution lead to lower (higher) educational investment, and thus a majority of the poor (rich) support higher (lower) redistribution. The state realized then depends on the expectations of agents.

The model in Hassler et al. (2003) and Hassler, Storesletten and Zilibotti (2007) presents cross-country differences in welfare programs among Western countries sharing similar backgrounds. However, they abstract from intragenerational mobility in their model and hence preclude the predictions of the POUM (prospect of upward mobility) hypothesis supported by US data (Benabou and Ok, 2001; Alesina and La Ferrara, 2005): namely, the poor do not support higher redistribution because of a hope for upward mobility in the future. In order to explain the differences in welfare programs among Western countries, there is a need to develop a model that includes an equilibrium that supports the POUM hypothesis as well as multiple equilibria with different patterns of redistribution, mobility, and inequality.

Independently of the above-mentioned studies, Quadrini (1999) succeeded in providing a model, including the POUM hypothesis, based on an endogenous growth model that produces multiple equilibria. This model encompasses a growing equilibrium with high mobility, high inequality and low redistribution (supporting the POUM hypothesis), and a stagnant economy with low mobility, low inequality and high redistribution (supporting the case of a pro-welfare state). A key to this result is the assumption that the growth rate of the economy affects agents' preferences over redistribution policies and thus changes the ability of the agents to know their position in the future distribution of income.

The multiple equilibria in Quadrini (1999) predict a negative correlation between redistribution and inequality, and a positive correlation between mobility and inequality. However, while the former is supported by empirical evidence (Kristov, Lindert and McClelland, 1992; Rodriguez, 2004), the latter is not necessarily supported by observable data. Figure 1 illustrates the scatter plot of upward mobility (OECD, 1996) and the Gini index (Föster and Mira d'Ercole, 2005), which appears to show a positive correlation between mobility and inequality among sample countries.[1] For example, the United States features high mobility and inequality and Finland features low mobility and inequality. The difference between these could be explained by the multiple equilibria in Quadrini (1999). However, other European countries display various patterns of upward mobility and inequality. For example, for workers aged 35-49 years, inequality is greater in the United States than in Denmark and Sweden, but the pattern of mobility is similar.[2] We cannot explain this property using the theory in Quadrini (1999).

[Figure 1 about here.]

Motivated by the above-mentioned discrepancy between theory and evidence, we develop a model that includes the equilibrium supporting the POUM hypothesis as well as multiple equilibria describing different patterns of mobility and inequality among European countries. For this purpose, the paper utilizes a politicoeconomic framework that incorporates the endogenous determination of income distribution by individuals' educational investment (Hassler, Storesletten and Zilibotti, 2007) based on the concept of a stationary Markov-perfect equilibrium.[3] We extend their model by introducing a second chance at success. Agents who live for two periods, youth and old age, can become rich or poor by undertaking costly investment in their youth. Successful agents can retain this status over their life cycle. Unsuccessful agents, however, have second chances and thus can become rich in old age through reinvestment in education. Within this extended framework, we explore: (i) the relation between upward mobility and inequality and (ii) the welfare implications of upward mobility in society.

We obtain the following two results not shown by previous research. First, we consider majority voting over redistribution policy and obtain the following result regarding the equilibrium properties. When the costs of a second chance are low, there is a unique, poor-majority equilibrium characterized by a low tax burden on the decisive voter and high levels of upward mobility and inequality: this supports the POUM hypothesis. Although the majority are poor, they support a low tax burden for redistribution because of the hope for upward mobility in the future. In contrast, when the costs of a second chance are high, there are multiple equilibria. The first is a poor-majority equilibrium featuring lower

[1]We include limited, old data here because the available data on upward mobility are restricted to the eight countries provided by the OECD (1996) for the period 1986–1991.

[2]The observation for Denmark, Sweden, and the United States is supported by empirical work by Aaberge et al. (2002), who showed that inequality is greater in the United States than in Scandinavian countries (Denmark, Norway and Sweden), even though the pattern of earnings mobility is remarkably similar.

[3]Examples of studies utilizing a stationary Markov-perfect equilibrium include Krusell, Quadrini and Ríos-Rull (1997), Grossman and Helpman (1998), Azariadis and Galasso (2002), Hassler et al. (2003), Forni (2005), Hassler et al. (2005), Song (2005), Gonzalez-Eiras and Niepelt (2008), Hassler, Storesletten and Zilibotti (2007), and Bassetto (2008).

levels of mobility and inequality; the second is a rich-majority equilibrium featuring higher levels of mobility and inequality. In the rich-majority equilibrium, mobility depends on the costs of a second chance, and the tax rate on the young is indeterminate in that it depends on the expectations of agents. The multiplicity of equilibria and the differences in the costs and expectations jointly provide an explanation for different patterns of mobility and inequality in European countries.

Second, we show the welfare implications of policies enhancing upward mobility. Mobility may be a good thing because higher upward mobility means that the current status of the poor is less persistent. However, mobility may also be viewed as a bad thing because higher mobility means larger income fluctuations and thus economic insecurity (Atkinson, Bourguignon and Morrisson, 1992). Our framework captures the former aspect by modeling upward mobility and characterizing an equilibrium that supports the POUM hypothesis, but abstracts from the latter aspect by assuming linear utility functions and no downward mobility. Therefore, an intuitive prediction is that upward mobility is good for the economy. However, our analysis shows that a low-cost economy characterized by high upward mobility may be inferior to a high-cost economy featuring low upward mobility, at least in terms of social welfare.

The key to the second result is the education disincentive effect. The lower cost of a second chance implies a higher probability of being successful in old age. This gives agents a disincentive to invest in education in their youth and results in a lower number of successful young agents and thus a smaller tax base. To keep tax revenue from the young, the decisive voter imposes a higher tax rate on successful young. The smaller number of successful young agents combined with a higher tax rate on the young jointly reduces the expected utility of the young and thus social welfare.

The organization of this paper is as follows. Section 2 develops the model. Section 3 characterizes the political equilibrium based on the concept of a stationary Markov-perfect equilibrium. Sections 4 and 5 undertake the numerical analysis. Section 4 analyzes the correlation between inequality and upward mobility, while Section 5 investigates the welfare implications of policies that reduce the costs of a second chance. Section 6 shows how the analysis and the results are changed when alternative assumptions are employed. Section 7 provides some concluding remarks. All proofs are given in the Appendix.

2 The Model

The model is based on Hassler, Storesletten and Zilibotti (2007). Time is discrete and denoted by $t = 1, 2, \cdots$. The economy consists of a continuum of agents living for two periods, youth and old age. Each generation has a unit mass. Agents are identical at birth. Young agents can affect their prospects in life with educational investment. In particular, they either become rich or poor, and by undertaking costly investment, can increase the probability e^y of becoming rich in youth. The cost of investment, which is measured in terms of disutility, is given by $(e^y)^2$.

At the beginning of each period, there are two types of old agents: those who were successful in their youth and those who were not. Successful agents can retain this status throughout their lifetime; unsuccessful agents have a second chance at success. A proportion $e^0 \in (0,1)$ of those who were unsuccessful in youth become rich in old age by undertaking costly investment e^0. The cost of investment is given by $\eta_o \cdot (e^o)^2, \ \eta_o > 0$. The parameter η_o represents the cost of getting a second chance. A lower η_o leads to a lower cost of reinvestment and thus a higher probability of being successful in old age. Rich agents earn a high wage, normalized to unity, whereas poor agents earn a low wage, normalized to zero. Figure 2 illustrates the timing of events and the distribution of the rich and poor.

[Figure 2 about here.]

Following conventional terminology, we refer to the first period of life as youth and the second period as old age. Educational investment in youth, e^y, is interpreted as study at tertiary education institutions such as universities and college; educational investment in old age, e^0, is interpreted as study at professional schools for career development or efforts at skill accumulation in on-the-job training. An individual may start in a low-paying job, but with continued opportunity for training, he/she will be able to advance into a higher-paying position or move to a different, higher-paying job.

There is no storage technology in this economy. Each individual uses his/her endowments within the period. The government provides lump-sum transfers, s, financed by taxes levied on the rich. The tax rates are age dependent, τ^o for the old and τ^y for the young. The tax rates are determined before the young agents decide on their investment. Therefore, the expected utility functions of agents alive at time t are given as follows:

$$V_t^{os} = (1-\tau_t^o) + s_t, \tag{1}$$

$$V_t^{ou} = e_t^o \cdot (1-\tau_t^o) - \eta_o \cdot (e_t^o)^2 + s_t, \tag{2}$$

$$\begin{aligned} V_t^y = {} & e_t^y \cdot (1-\tau_t^y) - (e_t^y)^2 + s_t \\ & + \beta \left[e_t^y \cdot (1-\tau_{t+1}^o) + (1-e_t^y) \left\{ e_{t+1}^o \cdot (1-\tau_{t+1}^o) - \eta_o \cdot (e_{t+1}^o)^2 \right\} + s_{t+1} \right], \end{aligned} \tag{3}$$

where V_t^{os}, V_t^{ou} and V_t^y denote the utility of the old who were successful in youth, the utility of the old who were unsuccessful in youth, and the utility of the young. The utility levels of V_t^{ou} and V_t^y are computed prior to individual success or failure. The parameter $\beta \in (0,1)$ is a discount factor.

Given these preferences, an old agent who was unsuccessful in youth has a second chance at success, and thus chooses e_t^o to maximize V_t^{ou}; and a young agent in period t chooses e_t^y to maximize V_t^y by taking account of the optimal investment in his/her old age. Therefore, optimal investments by the old and the young are given by, respectively:

$$
\begin{aligned}
e^{o*}(\tau_t^o) &= \frac{(1-\tau_t^o)}{2\eta_o}, \\
e^{y*}(\tau_t^y, \tau_{t+1}^o) &= \frac{1}{2}\cdot\left[(1-\tau_t^y)+\beta\left\{(1-\tau_{t+1}^o)-\frac{1}{4\eta_o}\cdot(1-\tau_{t+1}^o)^2\right\}\right].
\end{aligned}
$$

We make the following assumption.

Assumption 1: $\eta_o > 1$.

This assumption means that the cost of education in old age is higher than in youth: this reflects the fact that it is harder for agents to get educated as they grow older. The assumption is sufficient to ensure that $e^{o*}(\tau_t^o)$ and $e^{y*}(\tau_t^y, \tau_{t+1}^o)$ are set within the range $(0, 1)$.[4]

Because young agents are ex ante identical, agents of the same cohort choose the same investment, implying that the proportion of the old who were unsuccessful in youth is given by:

$$
u_{t+1} \equiv 1 - e^{y*}(\tau_t^y, \tau_{t+1}^o) = 1 - \frac{1}{2}\cdot\left[(1-\tau_t^y)+\beta\left\{(1-\tau_{t+1}^o)-\frac{1}{4\eta_o}\cdot(1-\tau_{t+1}^o)^2\right\}\right].
$$

Thus, the proportion of the old who were unsuccessful in youth, u_{t+1}, depends on the tax levied on the successful young agents in period t, τ_t^y, and the tax levied on the successful old agents in period $t+1$, τ_{t+1}^o.

The tax revenues from successful agents are transferred to every agent in a lump-sum fashion. The government budget is balanced in each period so that it can be expressed as:

$$
\begin{aligned}
2s_t &= [(1-u_t) + u_t \cdot e^{o*}(\tau_t^o)] \cdot \tau_t^o + e^{y*}(\tau_t^y, \tau_{t+1}^o) \cdot \tau_t^y \\
&= W(\tau_t^o, u_t) + Z(\tau_t^y, \tau_{t+1}^o),
\end{aligned}
$$

where $W(\tau_t^o, u_t) \equiv [(1-u_t) + u_t \cdot e^{o*}(\tau_t^o)]\cdot\tau_t^o$ is the revenue financed by the rich old agents, and $Z(\tau_t^y, \tau_{t+1}^o) \equiv e^{y*}(\tau_t^y, \tau_{t+1}^o)\cdot\tau_t^y$ is the tax revenue financed by rich young agents.[5]

[4] Expanding the range of η_o from $(1, \infty)$ to $(1/2, \infty)$ does not qualitatively affect the result shown below. However, we obtain a new result when η_o is set below $1/2$: the decisive voter will always prefer no taxation on the old: $\tau_t^o = 0$. All of the old agents can become rich after all because $e^{o*}(0) = 1/2\eta_o \geq 1$ if $\eta_o \leq 1/2$. Because this result is trivial, we limit the range of η_o to $(1, \infty)$.

[5] An alternative method of transfer is a targeted transfer to old or poor agents. In addition, we can

3 Political Equilibrium

This section characterizes a political equilibrium where agents vote on taxation period by period. Section 3.1 provides the definition of a political equilibrium based on the concept of a stationary Markov-perfect equilibrium with majority voting. Sections 3.2 and 3.3 provide the characterization of the political equilibrium classified according to the pattern of taxation on the old and the distribution of income.

3.1 Definition of Political Equilibrium

Following Hassler, Storesletten and Zilibotti (2007), we assume that agents vote on current taxes at the beginning of each period but that only the old vote. At the time of voting, young agents are identical, while old agents are heterogeneous; there are two types, those who were successful in their youth and those who were unsuccessful. Therefore, our assumption focuses on the conflict between the two types of old agents. For more discussion about the role of this assumption, see Section IIB in Hassler et al. (2003) and Footnote 6 in Hassler, Storesletten and Zilibotti (2007).

With the optimal investments $e^{o*}(\tau_t^o)$ and $e^{y*}(\tau_t^y, \tau_{t+1}^o)$ and the government budget constraint, the indirect utility of the old who were successful and unsuccessful in youth are given by, respectively:

$$\begin{aligned} V_t^{os} &= (1-\tau_t^o) + \frac{1}{2}\cdot\left(W(\tau_t^o, u_t) + Z(\tau_t^y, \tau_{t+1}^o)\right), \\ V_t^{ou} &= \frac{(1-\tau_t^o)^2}{4\eta_o} + \frac{1}{2}\cdot\left(W(\tau_t^o, u_t) + Z(\tau_t^y, \tau_{t+1}^o)\right), \end{aligned}$$

where the term in the first line, $(1-\tau_t^o)$, is the after-tax income of the old who were successful in youth, the term in the second line, $(1-\tau_t^o)^2/4\eta_o$, is the expected net benefit from the second challenge for the old who were unsuccessful in youth, and the term $(W+Z)/2$ is the lump-sum transfer.

The current paper focuses on stationary Markov-perfect equilibria with majority voting. The proportion of the old who were unsuccessful in youth (u_t) summarizes the state of the economy; the identity of a decisive voter depends on this proportion. An office-seeking politician elected by voters sets policies to maximize the utility of the larger group. Given these features, we now provide the definition of the political equilibrium as follows.

Definition: A (*stationary Markov-perfect*) *political equilibrium* is defined as a triplet

consider alternative means of government spending such as the provision of public goods. However, we abstract from these alternatives and adopt a lump-sum transfer to all agents in order to keep the analysis as simple as possible and to focus our attention on the political determination of taxes affected by the presence of a second chance.

of functions $\{T^o, T^y, U\}$, where $T^o : [0,1] \to [0,1]$ and T^y are two public policy rules, $\tau_t^o = T^o(u_t)$ and $\tau_t^y = T^y$, and $U : [0,1] \to [0,1]$ is a private decision rule, $u_{t+1} = U(\tau_t^y)$, such that given u_0, the following functional equations hold.

1. $T^o(u_t) = \arg\max_{\tau_t^o \in [0,1]} W^{dec}(\tau_t^o, u_t)$ $(dec = os, ou)$, where:

$$W^{dec}(\tau_t^o, u_t) = \begin{cases} W^{os} \equiv (1-\tau_t^o) + \frac{1}{2} \cdot W(\tau_t^o, u_t) & \text{if } u_t \leq 1/2, \\ W^{ou} \equiv \frac{(1-\tau_t^o)^2}{4\eta_o} + \frac{1}{2} \cdot W(\tau_t^o, u_t) & \text{if } u_t > 1/2. \end{cases}$$

2. $U(\tau_t^y) = 1 - e^{y*}(\tau_t^y, \tau_{t+1}^o)$, with $\tau_{t+1}^o = T^o(U(\tau_t^y))$.

3. $T^y = \arg\max_{\tau_t^o \in [0,1]} Z(\tau_t^y, \tau_{t+1}^o)$ subject to $\tau_{t+1}^o = T^o(U(\tau_t^y))$.

The first equilibrium condition requires the decisive voter to choose τ_t^o to maximize the utility of the old who were successful (if $u_t < 1/2$) or unsuccessful (if $u_t > 1/2$) in youth. In the case of an equal number of successful and unsuccessful agents (i.e., $u_t = 1/2$), the old who were successful in youth are assumed to be decisive. The second equilibrium condition implies that all young individuals choose their investment optimally, given τ_t^y and τ_{t+1}^o, under rational expectations about future taxes and distributions of types. The third equilibrium condition requires the decisive old voter to choose τ_t^y to maximize revenue from the young. Rational voters understand that their choice over current redistribution affects future redistribution via the private decision rule and public policy.

3.2 The Determination of T^o and U

We now solve the equilibrium conditions recursively. Condition 1 defines a one-to-one mapping from the state variable to the equilibrium choice of taxation of the old: $\tau_t^o = T^o(u_t)$. Suppose that the majority are the old who were successful in youth: $u_t \leq 1/2$. The objective function of the majority is given by $W^{os}(\tau_t^o, u_t) = (1-\tau_t^o) + W(\tau_t^o, u_t)/2$ that is strictly decreasing in $\tau_t^o : \partial W^{dec}(\tau_t^o, u_t)/\partial \tau_t^o < 0$.[6]. This implies that the old who were successful in youth pay more than they receive because the unsuccessful agents pay no tax, but the revenue is distributed equally between the successful and unsuccessful agents. Therefore, the old who were successful in youth prefer $\tau_t^o = 0$.

Alternatively, suppose that the majority are the old who were unsuccessful in youth: $u_t > 1/2$. The objective function of the majority is given by $W^{ou}(\tau_t^o, u_t) = (1-\tau_t^o)^2/4\eta_o + W(\tau_t^o, u_t)/2$ that is strictly convex in $\tau_t^o : \partial^2 W^{dec}(\tau_t^o, u_t)/\partial \tau_t^{o2} = (1-u_t)/2\eta_o > 0$. This

[6]When $u_t \leq 1/2$, the differentiation of W^{dec} with respect to τ_t^o is:

$$\partial W^{dec}/\partial \tau_t^o = (-1) \cdot [1 - \{(1-u_t) + u_t(1-\tau_t^o)/2\eta_o\}/2] - u_t \tau_t^o/4\eta_o,$$

where $(1-u_t) + u_t(1-\tau_t^o)/2\eta_o < 1$ under Assumption 1. Therefore, we have $\partial W^{dec}/\partial \tau_t^o < 0$ if $u_t \leq 1/2$.

implies that $T^o(u_t) = 0$ if $W^{ou}(0, u_t) \geq W^{ou}(1, u_t)$, i.e., if $u_t \geq 1 - 1/2\eta_o$; and $T^o(u_t) = 1$ if $W^{ou}(0, u_t) \leq W^{ou}(1, u_t)$, i.e., if $u_t \leq 1 - 1/2\eta_o$. In order to understand the result, let us focus on the condition that derives $T^o(u_t) = 0$. This condition means that the size of the tax base $(1 - u_t)$ is small and the cost of education aimed at a second chance (η_o) is low. The unsuccessful agents expect a low return from taxation on the old and a high probability of being taxed in old age. Therefore, they prefer $\tau_t^o = 0$ if $u_t \geq 1 - 1/2\eta_o$.

The mapping that satisfies equilibrium condition 1 is summarized as follows:

$$T^o(u_t) = \begin{cases} 0 & \text{if } u_t \leq 1/2 \text{ or } 1 - 1/2\eta_o \leq u_t \leq 1, \\ 1 & \text{if } 1/2 < u_t \leq 1 - 1/2\eta_o. \end{cases} \tag{4}$$

[Figure 3 about here.]

Figure 3 illustrates (4) that satisfies equilibrium condition 1. Suppose that the decisive voter is an old agent who was successful in youth: $u_t \leq 1/2$. He/she sets $\tau_t^o = 0$ because his/her marginal cost of taxation is greater than his/her marginal benefit of taxation, $\partial(W/2)/\partial\tau_t^o$. Alternatively, suppose that the decisive voter is an old agent who was unsuccessful in youth: $u_t > 1/2$. He/she sets either $\tau_t^o = 0$ or 1 depending on the magnitude of the correlation between the marginal benefit of taxation, $\partial(W/2)/\partial\tau_t^o$, and the marginal cost of taxation, $(1 - \tau_t^o)/2\eta_o$. The marginal benefit is smaller than the marginal cost for $u_t \in [1 - 1/2\eta_o, 1]$; and the marginal benefit is larger than the marginal cost for $u_t \in (1/2, 1 - 1/2\eta_o]$.[7]

Next, we rewrite equilibrium condition 2 by substituting in the optimal investment $e^{y*}(\tau_t^y, \tau_{t+1}^o)$. This yields the following functional equation:

$$U(\tau_t^y) = 1 - \frac{1}{2} \cdot \left[(1 - \tau_t^y) + \beta \cdot \left\{ (1 - T^o(U(\tau_t^y))) - \frac{1}{4\eta_o} \cdot (1 - T^o(U(\tau_t^y)))^2 \right\} \right], \tag{5}$$

where $T^o(\cdot) \in \{0, 1\}$ is given by (4). Because $T^o(\cdot) \in \{0, 1\}$, any solution of (5) must be a combination of the two linear functions: $U(\tau_t^y) = 1 - (1 - \tau_t^y + \beta \cdot (1 - 1/4\eta_o))/2$ and $U(\tau_t^y) = 1 - (1 - \tau_t^y)/2$.

We derive the solution to the functional equation (5) by assuming rational expecta-

[7] Our model approximates Hassler, Storesletten and Zilibotti (2007) by assuming the prohibitively high cost of education, $\eta_o \to \infty$. Under this approximation, the critical level of u_t illustrated in Figure 3, $1 - 1/2\eta_o$, approaches one; the unsuccessful agents no longer prefer $\tau_t^o = 0$ if $\eta_o \to \infty$. Therefore, Hassler, Storesletten and Zilibotti (2007) abstracted from the case where unsuccessful agents prefer no taxation on the old. In contrast, we produce this case by introducing a second chance into their model. It is shown below that the case supports the POUM hypothesis proposed by Benabou and Ok (2001).

tions:

$$U(\tau_t^y) = \begin{cases} 1-\frac{1}{2}\cdot\left((1-\tau_t^y)+\beta\cdot\left(1-\frac{1}{4\eta_o}\right)\right) & \text{if } \tau_t^y \in \left[0, \beta\left(1-\frac{1}{4\eta_o}\right)\right], \text{ or} \\ & \tau_t^y \in \left[\max\left\{\beta\cdot\left(1-\frac{1}{4\eta_o}\right)+\left(1-\frac{1}{\eta_o}\right), 1\right\}, 1\right], \\ 1-\frac{1}{2}\cdot(1-\tau_t^y) & \text{if } \tau_t^y \in \left[0, 1-\frac{1}{\eta_o}\right]. \end{cases}$$

Interpretation of the solution is as follows. Suppose that young agents in period t expect $\tau_{t+1}^o = 0$. Under this expectation, young agents choose their investment as $e^{y*}(\tau_t^y, 0) = (1-\tau_t^y+\beta\cdot(1-1/4\eta_o))/2$. By (4), this expectation is rational if $1 - e^{y*}(\tau_t^y, 0) \leq 1/2$ or $1-1/2\eta_o \leq 1-e^{y*}(\tau_t^y, 0) \leq 1$, that is, if $\tau_t^y \leq \beta\cdot(1-1/4\eta_o)$ or $\beta\cdot(1-1/4\eta_o)+(1-1/\eta_o) \leq \tau_t^y$. Suppose, instead, that young agents in period t expect $\tau_{t+1} = 1$. Under this expectation, young agents choose their investment as $e^{y*}(\tau_t^y, 1) = (1-\tau_t^y)/2$. By (4), their expectation is rational if $1/2 < 1-e^{y*}(\tau_t^y, 1) \leq 1-1/2\eta_o$, that is, if $\tau_t^y \leq 1-1/\eta_o$.

The solution depends on the expectations of agents. Figure 4 illustrates four possible cases of U that satisfy the second equilibrium condition. These cases are classified according to the magnitude of the correlation between $\beta\cdot(1-1/4\eta_o)+(1-1/\eta_o)$ and 1, and between $\beta\cdot(1-1/4\eta_o)$ and $(1-1/\eta_o)$.

[Figure 4 about here.]

As depicted in Figure 4, there are multiple, self-fulfilling expectations of U for a certain set of τ_t^y. Which U arises in equilibrium depends on the expectations of agents. To illustrate U in equilibrium, we introduce the critical rate of $\tau_t^y : \theta \leq \min\{1-1/\eta_0, \beta\cdot(1-1/4\eta_o)\}$. The rate θ, which depends on the expectations of agents, is the highest tax rate that yields a majority of the old who were successful in youth for the range of τ_t^y that yields multiple expectations. For $\tau_t^y > \theta$, the majority is the old who were unsuccessful in youth. However, for $\tau_t^y \in (0, \theta)$, the majority are either successful or unsuccessful depending on agents' expectations. The function is thus reduced as follows:

$$U(\tau_t^y) = \begin{cases} \left\{1-\frac{1}{2}\cdot\left((1-\tau_t^y)+\beta\cdot\left(1-\frac{1}{4\eta_o}\right)\right), 1-\frac{1}{2}\cdot(1-\tau_t^y)\right\} & \text{for } \tau_t^y \in (0, \theta], \\ 1-\frac{1}{2}\cdot(1-\tau_t^y) & \text{for } \tau_t^y \in \left(\theta, 1-\frac{1}{\eta_o}\right], \\ 1-\frac{1}{2}\cdot\left((1-\tau_t^y)+\beta\cdot\left(1-\frac{1}{4\eta_o}\right)\right) & \text{for } \tau_t^y \in \left(1-\frac{1}{\eta_o}, \beta\cdot\left(1-\frac{1}{4\eta_o}\right)\right), \text{ or} \\ & \tau_t^y \in \left[\beta\cdot\left(1-\frac{1}{4\eta_o}\right)+\left(1-\frac{1}{\eta_o}\right), 1\right]. \end{cases} \tag{6}$$

The third solution, $U(\tau_t^y) = 1 - ((1-\tau_t^y) + \beta \cdot (1 - 1/4\eta_o))/2$, can be given provided that the set $(1 - 1/\eta_o, \beta \cdot (1 - 1/4\eta_o))$ or $[\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o), 1]$ is nonempty.

Figure 4 reveals the following two features. First, if $\tau_t^y \leq \theta$, there can be multiple, self-fulfilling expectations. If agents expect not to be taxed when old, they choose high investment, and this leads to a majority of agents who were successful in youth; they will set $\tau_{t+1}^o = 0$. If, instead, they expect high future taxes, they choose low investment, and this leads to a majority of the agents who were unsuccessful in youth; they will set $\tau_{t+1}^o = 1$.

Second, suppose that the decisive voter chooses a sufficiently large τ_t^y such that $\tau_t^y \geq \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ (see Panels (a) and (c)). In this case, the majority in the next period will be unsuccessful agents, but they will choose no taxation on the old: $\tau_{t+1}^o = 0$. The intuition of this result is as follows. Suppose that the proportion of agents who were unsuccessful in youth is high such that $u_t \in [1 - 1/2\eta_0, 1]$. By (4), the decisive voter chooses no taxation on the old: $\tau_t^o = 0$. Given $\tau_t^o = 0$, if the decisive old voter chooses a high tax rate on the young, this choice gives young individuals a disincentive to invest in education, thereby resulting in a higher proportion of unsuccessful agents: $u_{t+1} \in [1 - 1/2\eta_0, 1]$. By (4), the tax rate on the old in the next period is given by $\tau_{t+1}^o = 0$: this satisfies the stationary property of the function T^o.

3.3 The Determination of T^y and the Characterization of the Political Equilibria

Given the characterization of T^o and U satisfying equilibrium conditions 1 and 2, respectively, we now consider the political determination of τ_t^y that satisfies equilibrium condition 3. Because there are two possible cases of a majority, we introduce corresponding definitions of the political equilibria: a *poor-majority equilibrium* and a *rich-majority equilibrium*. When the majority are poor (i.e., the old who were unsuccessful in youth), there are two sorts of equilibria: a poor-majority equilibrium where agents expect $\tau_{t+1}^o = 0$ and choose τ_t^y to induce a majority of poor at time $t+1$, and a poor-majority equilibrium where agents expect $\tau_{t+1}^o = 1$ and choose τ_t^y to induce a majority of poor at time $t+1$. When the majority are rich (i.e., the old who were successful in their youth), there is a rich-majority equilibrium where agents expect $\tau_{t+1}^o = 0$ and choose τ_t^y to induce a majority of rich at time $t+1$.

We first characterize a poor-majority equilibrium with no taxation on the old.

Proposition 1

(i) *Suppose that $\beta \leq (2/\eta_o - 1)/(1 - 1/4\eta_o)$ holds. There exists a set of poor-majority equilibria with no taxation on the old such that $\forall t$, T^o is given by (4), $U(\tau_t^y)$ is given* *by* (6)*, and* $T^y = (1+\beta\cdot(1-1/4\eta_o))/2$*. The equilibrium outcome is unique, such that* $\forall t$*,* $\tau_t^y = (1+\beta\cdot(1-1/4\eta_o))/2$*,* $\tau_t^o = 0$*,* $u_t = 1-(1+\beta\cdot(1-1/4\eta_o))/4$*, and* $2s_t = \{1+\beta\cdot(1-1/4\eta_o)\}^2/8$*.*

(ii) *Suppose that* $\beta > (2/\eta_o - 1)/(1-1/4\eta_o)$*,* $\beta \geq \{\eta_o/4 - (1-1/\eta_o)\}/(1-1/4\eta_o)$*, and* $\beta \leq (1/\eta_o)/(1-1/4\eta_o)$ *hold. There exists a set of poor-majority equilibria with no taxation on the old such that* $\forall t$*,* T^o *is given by* (4)*,* $U(\tau_t^y)$ *is given by* (6)*, and* $T^y = \beta\cdot(1-1/4\eta_o)+(1-1/\eta_o)$*. The equilibrium outcome is unique, such that* $\forall t$*,* $\tau_t^y = \beta\cdot(1-1/4\eta_o)+(1-1/\eta_o)$*,* $\tau_t^o = 0$*,* $u_t = 1-1/2\eta_o$*, and* $2s_t = \{\beta\cdot(1-1/4\eta_o)+(1-1/\eta_o)\}/2\eta_o$*.*

Proof. See the Appendix.

[Figure 5 about here.]

The area P.1(i) in Figure 5 indicates the set of parameters (β, η_o) satisfying the equilibrium condition in Proposition 1(i). The assumption of Proposition 1(i), $\beta \leq (2/\eta_o - 1)/(1-1/4\eta_o)$, implies that it is rational for the decisive voter to choose an interior solution $\tau_t^y = \arg\max_{\tau_t^y\in[0,1]} Z(\tau_t^y, 0) = (1+\beta\cdot(1-1/4\eta_o))/2$ under the expectation of $\tau_{t+1}^o = 0$. The young are taxed at the top of the Laffer curve, conditional on their expectation of no taxation when old (see Panel (a) of Figure 6).

[Figure 6 about here.]

The area P.1(ii) in Figure 5 illustrates the set of parameters (β, η_o) satisfying the equilibrium condition in Proposition 1(ii). The first assumption of Proposition 1(ii), $\beta > (2/\eta_o - 1)/(1-1/4\eta_o)$, implies that the interior solution is irrational: the decisive voter is unable to set the interior solution but is able to set a corner solution $\tau_t^y = \beta\cdot(1-1/4\eta_o)+(1-1/\eta_o)$. Under this situation, there are (at most) two alternative options that may dominate the corner solution: voting that induces a future majority of poor (unsuccessful) by setting $\tau_t^y = \arg\max Z(\tau_t^y, 1) = 1/2$; and voting that induces a future majority of rich (successful) by setting $\tau_t^y = \beta\cdot(1-1/4\eta_o)$ or $\tau_t^y = \theta(\leq \beta\cdot(1-1/4\eta_o))$. These alternatives cannot dominate the corner solution if they provide less tax revenue from the young; that is, the second and third assumptions in Proposition 1(ii) hold.

As illustrated in Figure 5, given the discount factor β, the poor-majority equilibrium with no taxation on the old requires a low η_o, i.e., a low cost of a second chance. A low η_o implies a high probability of being successful in old age, which gives individuals a disincentive to invest in education in youth. The unsuccessful agents form a majority, but they do not support a high tax rate (i.e., 100% taxation) on the old because of the hope for upward mobility through a second chance. The unsuccessful agents take into account the fact that with a high probability they can move up the income distribution and therefore be harmed by the high tax rate. Hence, given the low cost of a second chance, there is a unique political equilibrium outcome that supports the POUM hypothesis.

Next, we provide a characterization of a poor-majority equilibrium with 100% taxation on the old. For this characterization, we introduce a critical level of $\theta, \tilde{\theta}$, such that $Z(1/2, 1) \gtreqless Z(\theta, 0)$ if and only if $\theta \lesseqgtr \tilde{\theta}$, where:

$$\widetilde{\theta}(\beta, \eta_o) \equiv \frac{1 + \beta \cdot \left(1 - \frac{1}{4\eta_o}\right) - \sqrt{\beta \cdot \left(1 - \frac{1}{4\eta_o}\right) \cdot \left\{\beta \cdot \left(1 - \frac{1}{4\eta_o}\right) + 2\right\}}}{2}.$$

Proposition 2

Suppose that (i) $(1/\eta_o)/(1 - 1/4\eta_o) < \beta \leq (1 - 1/\eta_o)/(1 - 1/4\eta_o)$ *or* (ii) $\beta \leq (1/\eta_o)/(1 - 1/4\eta_o)$ *and* $\beta < \{\eta_o/4 - (1 - 1/\eta_o)\}/(1 - 1/4\eta_o)$. *Given the expectation of* $\theta \in (0, \tilde{\theta})$, *there exists a set of poor-majority equilibria with 100% taxation on the old such that* $\forall t$, T^o *is given by* (4), $U(\tau_t^y)$ *is given by* (6), *and* $T^y = 1/2$. *The equilibrium outcome is unique and such that* $\forall t$, $\tau_t^y = 1/2$, $\tau_t^o = 1$, $u_t = 3/4$, *and* $2s_t = 3/8$.

Proof. See the Appendix.

The area P.2 in Figure 5 indicates the set of parameters that attains a poor-majority equilibrium with 100% taxation on the old. From this figure, we can see that $\beta \leq (1 - 1/\eta_o)/(1 - 1/4\eta_o)$ is a necessary condition for the existence of a poor-majority equilibrium with 100% taxation on the old. In order to understand the role of this necessary condition, suppose alternatively that $\beta > (1 - 1/\eta_o)/(1 - 1/4\eta_o)$ holds. In this case, it is rational for the decisive voter to set $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$. Because the condition $\beta > (1 - 1/\eta_o)/(1 - 1/4\eta_o)$ is rewritten as $Z(\beta \cdot (1 - 1/4\eta_o), 0) > Z(1/2, 1)$, setting $\tau_t^y = 1/2$ under the expectation of $\tau_{t+1}^o = 0$ is not sustained as an equilibrium.

Based on this argument, we suppose that $\beta \leq (1 - 1/\eta_o)/(1 - 1/4\eta_o)$ holds, and we seek additional conditions that ensure the existence of a poor-majority equilibrium with 100% taxation on the old. Under the current situation, there are at most two alternative options that may dominate the choice of $\tau_t^y = 1/2$: setting $\tau_t^y = \theta(\leq \beta \cdot (1 - 1/4\eta_o))$ under the expectation of $\tau_{t+1}^o = 0$ and setting $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$ (see Panel (b) of Figure 6). The condition $\theta \in (0, \tilde{\theta})$ implies that the former alternative provides less tax revenue, condition (i) in Proposition 2 implies that the latter alternative is irrational, and condition (ii) implies that the latter alternative is rational but provides less tax revenue. Therefore, given $\theta \in (0, \tilde{\theta})$, the choice of $\tau_t^y = 1/2$ under the expectation of $\tau_{t+1}^o = 1$ is sustained as an equilibrium when the one of conditions (i) and (ii) hold.

As illustrated in Figure 5, the condition given in Proposition 2 requires a low value of β and a high value of η_o. First, a low value of β implies that agents attach a low weight to the utility in their old age, thereby having a weak incentive to invest in education in their youth. This results in a larger number of the old who were unsuccessful in youth, thereby leading to a majority of the poor. Second, a high value of η_o implies a low probability of being successful in old age for the old who were unsuccessful in their youth. This provides them with a low prospect of upward mobility through a second chance and gives them an incentive to prefer 100% taxation on the rich old. Therefore, given a set of a low β and a high η_o satisfying the conditions in Proposition 2, there exists a unique poor-majority equilibrium featuring 100% taxation on the old.

Finally, we provide the condition for the existence of a rich-majority equilibrium.

Proposition 3

(i) *Suppose that* $\beta > \max\{(1/\eta_o)/(1-1/4\eta_o), (1-1/\eta_o)/(1-1/4\eta_o)\}$ *holds. There exists a set of rich-majority equilibria such that* $\forall t$, T^o *is given by* (4), $U(\tau_t^y)$ *is given by* (6), *and* $T^y = \beta \cdot (1-1/4\eta_o)$. *The equilibrium outcome is unique and such that* $\forall t$, $\tau_t^y = \beta \cdot (1-1/4\eta_o), \tau_t^o = 0$, $u_t = 1/2$, *and* $2s_t = \beta \cdot (1-1/4\eta_o)/2$.

(ii) *Suppose that* $\beta > (1/\eta_o)/(1-1/4\eta_o), \beta \geq (1/4)/(1-1/4\eta_o)$ *and* $\beta \leq (1-1/\eta_o)/(1-1/4\eta_o)$ *holds. There exists a set of rich-majority equilibria such that* $\forall t$, T^o *is given by* (4), $U(\tau_t^y)$ *is given by* (6), *and* $T^y = \theta \in [\tilde{\theta}, \beta \cdot (1-1/4\eta_o)]$. *The equilibrium outcome is indeterminate, such that* $\forall t$, $\tau_t^y = \theta, \tau_t^o = 0$, $u_t = 1-(1-\theta+\beta \cdot (1-1/4\eta_o))/2$, *and* $2s_t = (1-\theta+\beta \cdot (1-1/4\eta_o))\theta/2$.

Proof. See the Appendix.

The area P.3(i) in Figure 5 indicates the set of parameters (β, η_o) satisfying the equilibrium condition in Proposition 3(i). The assumption $\beta > (1-1/\eta_o)/(1-1/4\eta_o)$ implies that it is rational for voters to set $\tau_t^y = \beta \cdot (1-1/4\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$. When this choice is available, there are at most two alternatives that may dominate the current choice: setting $\tau_t^y = 1/2$ under the expectation of $\tau_{t+1}^o = 1$ and setting $\tau_t^y = (1+\beta \cdot (1-1/4\eta_o))/2$ or $\tau_t^y = \beta \cdot (1-1/4\eta_o) + (1-1/\eta_o)$ under the expectation $\tau_{t+1}^o = 0$. The first alternative always provides less tax revenue; the second alternative is irrational under the assumption of $\beta > (1/\eta_o)/(1-1/4\eta_o)$. Therefore, the choice of $\tau_t^y = \beta \cdot (1-1/4\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$ is sustained as an equilibrium when the condition in Proposition 3(i) holds.

The area P.3(ii) in Figure 5 indicates the set of parameters (β, η_o) satisfying the equilibrium condition in Proposition 3(ii). The third assumption $\beta \leq (1-1/\eta_o)/(1-1/4\eta_o)$ implies that it is irrational for voters to set $\tau_t^y = \beta \cdot (1-1/4\eta_o)$ under the expectation of $\tau^o_{t+1} = 0$; the first assumption $\beta > (1/\eta_o)/(1 - 1/4\eta_o)$ implies that it is irrational for voters to set $\tau^y_t = (1 + \beta \cdot (1 - 1/4\eta_o))/2$ or $\tau^y_t = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ under the expectation $\tau^o_{t+1} = 0$. The remaining alternative is to set $\tau^y_t = 1/2$ under the expectation of $\tau^o_{t+1} = 1$. This alternative provides less tax revenue than the current choice if $Z(\theta, 0) \geq Z(1/2, 1)$, i.e., if $\theta \in [\tilde{\theta}, \beta \cdot (1 - 1/4\eta_o)]$. This set is nonempty if the second assumption, $\beta \geq (1/4)/(1 - 1/4\eta_o)$, holds. Therefore, given $\theta \in [\tilde{\theta}, \beta \cdot (1 - 1/4\eta_o)]$, the choice of $\tau^y_t = \theta$ under the expectation of $\tau^o_{t+1} = 0$ is sustained as an equilibrium when the condition in Proposition 3(ii) holds.

The next proposition summarizes the results established thus far. The proof of this proposition is immediately obvious from Figure 5.

Proposition 4

(i) *If* $\beta \in [\{\eta_o/4 - (1 - 1/\eta_o)\}/(1 - 1/4\eta_o), (1/\eta_o)/(1 - 1/4\eta_o)]$, *there exists a unique poor-majority equilibrium with no taxation on the old as in Proposition 1.*

(ii) *If* $\beta > \max\{1/\eta_o/(1 - 1/4\eta_o), (1 - 1/\eta_o)/(1 - 1/4\eta_o)\}$, *there exists a unique rich-majority equilibrium with no taxation on the old as in Proposition 3(i).*

(iii) *If* $\beta > (1/\eta_o)/(1 - 1/4\eta_o), \beta \geq (1/4)/(1 - 1/4\eta_o)$ *and* $\beta < (1 - 1/\eta_o)/(1 - 1/4\eta_o)$, *the equilibrium is indeterminate. There exists both a poor-majority equilibrium with 100% taxation on the old as in Proposition 2 and a set of rich-majority equilibria as in Proposition 3(ii).*

(iv) *If* $\beta < \{\eta_o/4 - (1 - 1/\eta_o)\}/(1 - 1/4\eta_o)$ *and* $\beta \leq (1/4)/(1 - 1/4\eta_o)$, *there exists a unique poor-majority equilibrium with 100% taxation on the old as in Proposition 2.*

By focusing on the parameter η_o, we now provide an interpretation of the result established in Proposition 4. To facilitate understanding, we assume the following economic environment. We take a generation to be 20 years in length because the current model assumes that agents work for both periods of their life. The first and the second periods correspond to, for example, ages 21–40 and 41–60 years, respectively. Our selection of β is based on the single period discount rate provided by Kydland and Prescott (1982) of 0.96. Because the agents in our model plan over generations that span 20 years, we discount the future by $(0.96)^{20}$.

Figure 5 illustrates the equilibrium properties under the current economic environment. There is a threshold level of η_o, 2.5124, which determines the equilibrium properties. When η_o is below the threshold level, the unsuccessful agents can succeed in a second chance with a high probability. This implies that unsuccessful agents have a high prospect of upward mobility in the future, which gives the young a weak incentive to invest in education. Therefore, a low η_o leads to a large number of agents who were unsuccessful in youth. They prefer no taxation on the old because given a high probability of being successful in old age, they wish to avoid being taxed. The POUM hypothesis is supported in this case.

When η_o is above the threshold level, there is no equilibrium that supports the POUM hypothesis. This is because given a high cost of a second chance, there is no possibility of changing status from poor to rich by attempting a second chance. The agents who were unsuccessful in youth are expected to remain poor in old age. Therefore, for $\eta_o > 2.5124$, there is a poor-majority equilibrium with 100% taxation on rich old agents. However, there is also a rich-majority equilibrium because of the multiple self-fulfilling expectations. Which particular equilibrium is realized depends on the expectations of agents. Expectations of 100% (0%) taxation on the old lead to low (high) educational investment and thus a majority of the poor (rich); the majority of the poor (rich) then support 100% (0%) taxation and thus high (low) redistribution.

4 Upward Mobility and Inequality

This section examines the correlation between upward mobility and inequality. Section 4.1 undertakes a numerical analysis of upward mobility and inequality. Section 4.2 explores the empirical implications of the numerical results.

4.1 Numerical Analysis

We define the Gini coefficient as a measure of inequality in the current framework and then undertake a numerical analysis under the economic environment of $\beta = (0.96)^{20}$. We calculate the Gini coefficient in terms of after tax-and-transfer income (see the Appendix for the method of calculation). Figure 7 illustrates the numerical results of the upward mobility rate (i.e., the probability of being successful in old age, e^o) and the Gini coefficient.

The result illustrated in Figure 7 provides the following prediction for upward mobility and inequality. When the cost is low, such that $\eta_o < 2.5124$, the economy attains a unique poor-majority equilibrium featuring high levels of upward mobility and inequality. The low cost of a second chance then enhances the upward mobility of unsuccessful agents. However, it gives young individuals a disincentive to invest in education, which results in a greater number of unsuccessful young agents and thus a larger level of inequality among them. Therefore, the lower cost of a second chance results in higher levels of upward mobility and inequality.

[Figure 7 about here.]

When the cost of a second chance is high, such that $\eta_o \geq 2.5124$, the economy attains multiple equilibria depending on the expectations of agents: one is a poor-majority equilibrium featuring lower levels of upward mobility and inequality; the other is a rich-majority equilibrium featuring higher levels of upward mobility and inequality. In the poor-majority equilibrium, agents expect higher redistribution in the future, and this expectation is realized as 100% taxation on the old. The upward mobility rate becomes zero, and thus old agents are equal in the sense that all of them obtain s_t units of after tax-and-transfer income. In contrast, in the rich-majority equilibrium, agents expect lower redistribution in the future, and this expectation is realized as no taxation on the old. The upward mobility rate is given by $e_t^o = 1/2\eta_o$, and thus there is inequality among old agents. Therefore, the rich-majority equilibrium attains higher levels of upward mobility and inequality compared with the poor-majority equilibrium.

A noteworthy feature of the result in Figure 7 is that a decrease in η_o around the threshold level of $\eta_o = 2.5124$ leads to a sharp increase in inequality. The mechanism behind this is as follows. The objective of the decisive voter is to maximize tax revenue from the young. This objective is achieved by: (i) the larger size of the tax base associated with a lower tax rate that enhances educational investment by the young, or (ii) the smaller size of the tax base associated with a higher tax rate that gives the young a disincentive to invest in education. When η_o is above the threshold level, the young have an incentive to invest in education because they expect a lower probability of being successful in old age. The decisive voter can expect a large number of successful young agents and thus attempts to maximize the tax revenue from the young based on the first option. However, when η_o is below the threshold level, the young have a disincentive to invest in education because they expect upward mobility in the future with a high probability. The decisive voter then maximizes revenue from the young based on the second option. Therefore, at $\eta_o = 2.5124$, a reduction in η_o leads to a discontinuous increase in the tax rate on the young and a discontinuous decrease in the number of successful young agents (see Figure 8), both of which cause a sharp increase in inequality.

[Figure 8 about here.]

Another noteworthy feature is that countries included in the rich-majority equilibria could present different patterns of mobility and inequality. To understand this, let us consider the effects of θ and η_o on mobility and inequality in the rich-majority equilibria. The parameter θ, representing the expectations of agents, has no effect on mobility but has a critical effect on inequality via taxation on the young. The parameter η_o, representing the cost of a second chance, has a primary effect on mobility but has a minor effect on inequality. These effects imply that depending on θ and η_o, the rich-majority equilibrium demonstrates a positive or a negative relation between mobility and inequality. For example, a country with a low η_o features a high level of mobility but either a high or a low level of inequality depending on the expectations of agents.

4.2 Empirical Implications

The result in Figure 7 shows that the political economy tends to generate a positive correlation between mobility and inequality as in Quadrini (1999) but may present various patterns of mobility and inequality in the rich-majority equilibria. In this section, we consider the scatter plot of upward mobility and inequality illustrated in Figure 1, and show that the correlation in Figure 1 is in line with the model predictions.[8]

As shown in Figure 1, among the eight countries presented, the United States displays the highest upward mobility for workers aged 25–34 years, the second-highest upward mobility for workers aged 35–49 years, and the highest inequality. Finland shows the lowest upward mobility for workers aged 25–34 years, the second-lowest upward mobility for workers aged 35–49 years, and the lowest inequality. The remaining six countries generally show lower levels of upward mobility and inequality when compared with the United States, and higher levels of upward mobility and inequality when compared with Finland. These observations suggest a positive correlation between mobility and inequality.

However, there are some exceptions to this view: for instance, high mobility and low inequality are observed in Sweden for workers aged 25–34 years, and in Denmark and Sweden for workers aged 35–49 years. Empirical work by Aaberge et al. (2002) supports this observation by showing that inequality is greater in the United States than in Scandinavia, even though the pattern of mobility is remarkably similar. One possible interpretation of this finding is that Scandinavia is represented by the rich-majority equilibrium featuring high mobility and low inequality caused by low levels of θ and η_o. Therefore, the various patterns of mobility and inequality in the rich-majority equilibria, which were not shown in Quadrini (1999), are key factors in explaining the relation between the United States and Scandinavia.

From Figures 1 and 7, we can draw some further inferences about the relative η_o and θ across countries included in the rich-majority equilibria. For example, Sweden and the United Kingdom feature low levels of η_o because they show high levels of upward mobility. However, they might have different expectations of future taxation because they

[8] An alternative way of connecting the model predictions to empirical evidence is to focus on tax burden. However, this is not necessarily appropriate because the current model has the following limitations. First, the binary choice of the tax rate on the old makes the model analytically tractable but fails to demonstrate realistic tax rates. Second, the preferences of the young are not reflected in the political determination of the tax rate on the young because the model assumes that the young do not participate in voting. These limitations imply that the tax rates analytically derived from the model do not fit the empirical data well. Therefore, we have decided to observe the inequality in terms of after-tax income instead of directly focusing on tax rates.

show substantial differences in inequality. As another example, Germany and Italy may feature high levels of η_o because they show low levels of upward mobility. However, they might also have different expectations of θ as the data in Figure 1 shows that inequality is lower in Germany than in Italy.

Our analysis of inequality and intragenerational mobility supplements the analysis of inequality and intergenerational mobility in Hassler, Rodriguez Mora and Zeira (2007). They developed a model including intergenerational mobility from parents to children through altruistic investment in education, and showed that a reduction in the cost of upward mobility has the primary effect of enhancing upward mobility. This is in common with Hassler, Rodriguez Mora and Zeira (2007) and the current analysis. However, the secondary effect and its implications differ completely from those in the current paper. In Hassler, Rodriguez Mora and Zeira (2007), the secondary effect is a decrease in the gain from education. This effect decreases upward mobility. They showed that the primary effect generally overrides the secondary effect so that a reduction in the cost of education results in improvements in both upward mobility and equality (see Proposition 3 in Hassler, Rodriguez Mora and Zeira, 2007). In contrast, in our framework, the secondary effect provides young individuals with a disincentive to invest in education, and this overrides the primary effect. Therefore, the correlation between inequality and upward mobility depends on whether the focus is on intragenerational or intergenerational mobility.

5 Welfare Analysis

Based on the analysis given in Section 3, we consider the welfare implications of a second chance by focusing on the parameter representing the cost of a second chance, η_o. While policies to decrease η_o benefit old agents who were unsuccessful in youth, they will not be welfare improving if the agents who were successful in their youth or young agents are made worse off. To examine the total welfare effect of these policies, we introduce a Benthamite social welfare function Γ defined by:

$$\Gamma(\tau^y, \tau^o) = V^y(\tau^y, \tau^o) + (1 - u(\tau^y, \tau^o)) \cdot V^{os}(\tau^y, \tau^o) + u(\tau^y, \tau^o) \cdot V^{ou}(\tau^y, \tau^o),$$

where V^{os}, V^{ou} and V^y are given by (1), (2) and (3), respectively. Using this function, we undertake comparative statics analysis of η_o via numerical examples under the assumption of $\beta = (0.96)^{20}$. We examine the social welfare implications of polices that reduce η_o to enhance upward mobility, and consider whether a low-cost economy is superior to a high-cost economy in terms of social welfare.

[Figure 9 about here.]

We calculate the educational investment by the young, the tax rates on the young and old, and the utility of each type of agent and social welfare by taking the value of η_o from one to infinity. We have already presented educational investment by the old in Figure 7, and the tax rate on the young and the educational investment by the young in Figure 8. Figure 9 illustrates the expected utility of young agents (Panel (a)), and social welfare (Panel (b)). We provide an interpretation of the simulation results by focusing on the parameter η_o. In particular, we consider a larger set of η_o to a smaller set of η_o in turn and provide the welfare implications of policies that reduce η_o to enhance upward mobility.

First, consider the case of $\eta_o \in [2.5124, \infty)$: there are multiple equilibria. If the expectation θ is larger than the critical level $\tilde{\theta}$, there is an equilibrium where the majority of voters are the successful agents who prefer $\tau_t^y = \theta$ and $\tau_{t+1}^o = 0$. However, if the expectation θ is lower than the critical level $\tilde{\theta}$, there is an equilibrium where the majority of voters are the unsuccessful agents who prefer $\tau_t^y = 1/2$ and $\tau_{t+1}^o = 1$. The former equilibrium is superior to the latter in terms of social welfare because in the former, the successful agents benefit from no taxation in their old age.

Second, consider the case of $\eta_o \in (1, 2.5124)$. There is a unique equilibrium where the majority of voters are the unsuccessful agents who prefer $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ or $(1 + \beta \cdot (1 - 1/4\eta_o))/2$ and $\tau_{t+1}^o = 0$. A noteworthy feature is that the current case may show a lower level of social welfare compared with the case of $\eta_o \in [2.5124, \infty)$. In particular, a decrease in η_o around the threshold level of $\eta_o = 2.5124$ leads to a sharp decrease in social welfare. The mechanism behind this result is as follows. At $\eta_o = 2.5124$, a decrease in η_o produces a discontinuous increase in the tax rate on the young and a discontinuous decrease in the number of successful young agents. Both of these effects were explained in Section 4.1. These effects result in a sharp decrease in the expected utility of the young and thus in social welfare. This result implies that the United States (as represented by the low-cost economy) may be inferior to some European countries (as represented by the high-cost economy), at least in terms of social welfare.

The empirical validity of numerical analysis so far can be checked by comparing the model prediction of the number of successful young agents with the empirical data. We take the graduation rate in tertiary education (especially tertiary type A programs including universities and college) as a proxy variable for the proportion of successful young agents because we regard education in youth as study at tertiary education institutions. The graduation rates in 2005 of the countries concerned are as follows: 45.5% for Denmark 47.3% for Finland, 19.9% for Germany, 41.0% for Italy, 37.7% for Sweden, 39.4% for the UK, and 34.2% for the US (OECD, 2007).[9] With the second-lowest graduation rate,

[9]The data for France are not available.

the US has a lower graduation rate than European countries such as Iceland, Ireland, the Netherlands and Norway (OECD, 2007). These results imply that it is reasonable to consider that the US is represented by the low-cost economy featuring a lower graduation rate whereas some European countries are represented by the high-cost economy featuring higher graduation rates.

6 Extensions and Further Analysis

The analysis so far has assumed that: (i) the cost function in youth is specified by $(e^y)^2$, and (ii) the same wages are paid to individuals who were successful in youth and individuals who were unsuccessful in youth but became successful in old age. This section briefly considers how the analysis and the results are changed when each of these assumptions is relaxed.

6.1 The Generalization of the Investment Cost Function in Youth

The analysis in the previous sections assumed that the investment cost functions in youth and old age are given by $(e^y)^2$ and $\eta_o(e^o)^2$, respectively. Given these specifications, we focused on the parameter η_o representing the cost in old age, and analyzed the role of η_o for the determination of political equilibria. In this subsection, we generalize the investment cost function in youth by assuming $\eta_y(e^y)^2$, where $\eta_y(> 0)$ is a parameter representing the cost in youth, and we consider the role of η_y for the determination of political equilibria.[10]

Given the cost functions $\eta_y(e^y)^2$ and $\eta_o(e^o)^2$, educational investments in youth and old age are given by, respectively:

$$\begin{aligned} e_t^{y*}(\tau_t^y, \tau_{t+1}^o) &= \frac{1}{2\eta_y}\left[(1-\tau_t^y) + \beta\left\{(1-\tau_{t+1}^o) - \frac{1}{4\eta_o}(1-\tau_{t+1}^o)^2\right\}\right], \\ e_t^{0*}(\tau_t^o) &= \frac{1}{2\eta_o}(1-\tau_t^o). \end{aligned}$$

A lower value of η_y implies a lower cost in youth, thereby yielding a higher probability of being successful in youth. However, the value of η_y does not affect the probability of being successful in old age because the educational cost in youth has no relevance to the investment in old age.

The equations presented imply that the impact of the cost in youth is limited to the incentive of educational investment in youth and thus is limited to the determination of the majority. For cases of high values of η_y, with $\eta_y = 1$ as a special case, young indi-

[10] The authors would like to thank an anonymous referee for suggesting the important role of education costs in youth.

viduals have a weak incentive to invest in education; this results in a similar equilibrium characterization to that presented in Section 3. However, for cases of low values of η_y, young individuals have a strong incentive to invest in education even if they expect high tax rates. The economy attains a unique rich-majority equilibrium.[11] Therefore, the generalization of the cost function in youth has no impact on the mobility but has an effect on the characterization of political equilibria in that a rich-majority equilibrium is more likely to hold when the cost in youth is lower. Because the focus of this paper is on upward mobility and its impact on the welfare, the previous sections assumed $\eta_y = 1$ and concentrated on the parameter η_o that affects mobility.

6.2 Experience Premiums

One of the main results in this paper is that an equilibrium exists that supports the POUM hypothesis: the majority are poor but they support low redistribution because of the hope of upward mobility in the future. A key to this result is the education disincentive effect produced by low costs of a second chance in old age. Low costs of a second chance provide young individuals with a disincentive for educational investment in youth because they expect that they will become successful in old age with a high probability, even if they fail in education in their youth.

The education disincentive effect is mitigated when experience premiums are considered. The analysis so far has assumed that the same wages are paid to individuals who were successful in youth and individuals who were unsuccessful in youth but became successful in old age. However, in the real world, the former would be more experienced than the latter and thus would receive experience premiums in old age. The experience premiums give young individuals an incentive to invest in education in youth, which mitigates the education disincentive effect created by low costs of a second chance.[12] This raises the following question: does the model still predict an equilibrium that supports the POUM hypothesis when experience premiums are considered? In other words, how robust is the equilibrium that supports the POUM hypothesis in the presence of experience premiums?

In order to answer these questions, we assume here that the old who were successful in youth receive $w(>1)$ units of wages in old age: $w-1$ units of wages represent experience premiums. Under this setting, the educational investment in youth is given by:

$$e_t^{y*}(\tau_t^y, \tau_{t+1}^o) = \frac{1}{2}\left[(1-\tau_t^y) + \beta\left\{w(1-\tau_{t+1}^o) - \frac{1}{4\eta_o}(1-\tau_{t+1}^o)^2\right\}\right].$$

This equation implies that a higher level of experience premiums results in a larger amount

[11] Ono and Arawatari (2008) show this result in a model without second chances.

[12] The authors would like to thank an anonymous referee for pointing this out.

of educational investment in youth. Given this equation, we obtain the following result that provides the conditions for the existence of an equilibrium that supports the POUM hypothesis.

Proposition 5

(i) *If* $\beta \leq (2/\eta_o w - 1)/(w - 1/4\eta_o)$, *there exists a poor-majority equilibrium with the outcome of* $\tau_t^y = [1+\beta(w-1/4\eta_o)]/2, \tau_{t+1}^o = 0$ *and* $u_{t+1} = 1-[1+\beta(w-1/4\eta_o)]/4(> 1/2)$.

(ii) *If* $\beta > (2/\eta_o w - 1)/(w - 1/4\eta_o), \beta \geq \{\eta_o w/4 - (1 - 1/\eta_o w)\}/(w - 1/4\eta_o)$ *and* $\beta \leq (1/\eta_o w)/(w - 1/4\eta_o)$, *there exists a poor-majority equilibrium with the outcome of* $\tau_t^y = \beta(w - 1/4\eta_o) + (1 - 1/\eta_o w), \tau_{t+1}^o = 0$ *and* $u_{t+1} = 1 - 1/2\eta_o w (> 1/2)$.

Proof. See the supplementary explanation in Section 9.

The first and second statements of Proposition 5 correspond to the first and second statements of Proposition 1, respectively. Proposition 5 implies that when β is given by $(0.96)^{20}$ as in the previous sections, there exists an equilibrium that supports the POUM hypothesis if η_o satisfies:[13]

$$\beta = (0.96)^{20} \leq \frac{1/\eta_o w}{w - 1/4\eta_o}, \text{ i.e., } \eta_o \in \left(\frac{1}{w}, \frac{w + (0.96)^{20}/4}{(0.96)^{20} w} \right].$$

The set is nonempty if and only if $w < 4/3 \cdot (0.96)^{20} \approx 1.589$. Therefore, the existence of the equilibrium that supports the POUM hypothesis is ensured as long as the experience premiums are below 58% of the wage in youth.

7 Conclusion

This paper provides a framework for the analysis of the dynamic political economy of redistribution when agents who failed in education early in life have a second chance for upward mobility later in life. In this framework, the analysis considers repeated voting over redistribution policy and provides an analytical characterization of the stationary Markov-perfect political equilibrium by focusing on the parameter representing the cost of a second chance. In addition, we undertake a comparative static analysis of this parameter via numerical example and present how the parameter affects (i) the correlation between inequality and upward mobility, and (ii) the determination of social welfare.

The results established in this paper are as follows. When the cost of a second chance is low, there is a unique, poor-majority equilibrium characterized by high levels of upward mobility and inequality with a low tax burden on the decisive voter. Although the

[13] We assume $\eta_o > 1/w$. This assumption corresponds to Assumption 1 with the specification of $w = 1$.

majority are the poor who benefit from redistribution, they do not support a high tax rate because of a hope for upward mobility in the future. This prediction is consistent with empirical observations on the US economy by Benabou and Ok (2001) and Alesina and La Ferrara (2005). However, when the cost of a second chance is high, the economy displays multiple equilibria representing many European countries: one is a rich-majority equilibrium characterized by higher levels of upward mobility and inequality; the another is a poor-majority equilibrium characterized by lower levels of upward mobility and inequality. Furthermore, mobility and inequality are lower in the high-cost economy than in the low-cost economy representing the United States. Based on this characterization of political equilibrium, we investigate the welfare implications of upward mobility and show that in terms of social welfare, the low-cost economy featuring high upward mobility may be inferior to the high-cost economy featuring low upward mobility. This implies that enhancing upward mobility is not necessarily good for the economy.

In order to obtain these results, we simplified the analysis by adopting a simple lump-sum transfer scheme. We did not consider alternative policy methods, for example, transfers targeted at the elderly or the poor. In addition, we abstracted from downward mobility in the analysis. Furthermore, some results are based on numerical analysis because of the difficulty in solving the model analytically. However, the current paper does provide a framework for understanding: (i) cross-country differences in inequality and upward mobility from the viewpoint of political economy, and (ii) the role of upward mobility from the viewpoint of social welfare.

8 Appendix

8.1 Proof of Proposition 1

(i) Suppose that under the assumption of $\beta \leq (2/\eta_o - 1)/(1 - 1/4\eta_o)$, agents know that $\tau_t^y = (1 + \beta \cdot (1 - 1/4\eta_o))/2$ and expect $\tau_{t+1}^o = 0$. Then:

$$1 - e^{y^*}\left(\frac{1 + \beta \cdot (1 - 1/4\eta_o)}{2}, 0\right) = u_{t+1} = 1 - \frac{1}{4}\left(1 + \beta \cdot \left(1 - \frac{1}{4\eta_o}\right)\right).$$

Because the assumption $\beta \leq (2/\eta_o - 1)/(1 - 1/4\eta_o)$ is rewritten as $1 - (1 + \beta \cdot (1 - 1/4\eta_o))/4 \geq 1 - 1/2\eta_o$, $u_{t+1} \geq 1 - 1/2\eta_o$ holds for all t. By (4), this implies that $\tau_{t+1}^o = 0$, fulfilling initial expectations. Therefore, under the assumption of $\beta \leq (2/\eta_o - 1)/(1 - 1/4\eta_o)$, there exists a poor-majority equilibrium with no taxation on the old if the decisive voter finds it optimal to set $\tau_t^y = (1 + \beta \cdot (1 - 1/4\eta_o))/2$.

To establish that setting $\tau_t^y = (1 + \beta \cdot (1 - 1/4\eta_o))/2$ is optimal for the decisive voter, we note the following properties of the function Z : (i) $Z(\tau_t^y, 0)$ is concave in τ_t^y and is maximized at $\tau_t^y = \arg\max_{\tau_t^y \in [0,1]} Z(\tau_t^y, 0) = (1 + \beta \cdot (1 - 1/4\eta_o))/2$; (ii) $Z(\tau_t^y, 0) > Z(\tau_t^y, 1) \forall \tau_t^y \in [0, 1]$. These properties imply that setting $\tau_t^y = (1 + \beta \cdot (1 - 1/4\eta_o))/2$ is optimal if this setting is rational under the expectation of $\tau_{t+1}^o = 0$, i.e., if $\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o) \leq (1 + \beta \cdot (1 - 1/4\eta_o))/2 \leq 1$. The first inequality is rewritten as $\beta \leq (2/\eta_o - 1)/(1 - 1/4\eta_o)$, which is equivalent to the current assumption; and the second inequality always holds under the assumption of $\beta \in [0, 1]$ and $\eta_o > 1$ (Assumption 1).

(ii) Suppose that at time t, agents know that $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ and expect $\tau_{t+1}^o = 0$. Then:

$$1 - e^{y^*}\left(\beta \cdot \left(1 - \frac{1}{4\eta_o}\right) + \left(1 - \frac{1}{\eta_o}\right), 0\right) = u_{t+1} = 1 - \frac{1}{2\eta_o}.$$

By (4), this implies that $\tau_{t+1}^o = 0$, fulfilling initial expectations. Therefore, a poor-majority equilibrium exists if the decisive voter finds it optimal to set $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$.

To establish that setting $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ is optimal for the decisive voter, we first note that setting $\tau_t^y = (1 + \beta \cdot (1 - 1/4\eta_o))/2$ is not rational under the first assumption:

$$\beta > (2/\eta_o - 1)/(1 - 1/4\eta_o). \tag{7}$$

Given this condition, we consider the following two cases: $\beta > (1 - 1/\eta_o)/(1 - 1/4\eta_o)$ and $\beta \leq (1 - 1/\eta_o)/(1 - 1/4\eta_o)$.

First, suppose that a set of parameters satisfies $\beta \cdot (1 - 1/4\eta_o) > (1 - 1/\eta_o)$, i.e.:

$$\beta > (1 - 1/\eta_o)/(1 - 1/4\eta_o), \tag{8}$$

as illustrated in Panel (a) of Figure 4. Under this situation, the relevant payoff function is $Z(\tau_t^y, 0)$ or $Z(\tau_t^y, 1)$ for $\tau_t^y \in (0, \theta]$, $Z(\tau_t^y, 1)$ for $\tau_t^y \in (\theta, 1 - 1/\eta_o]$, and $Z(\tau_t^y, 0)$ for $\tau_t^y \in (1 - 1/\eta_o, \beta \cdot (1 - 1/4\eta_o)]$. Given the properties of the payoff function demonstrated in the proof of Proposition 1(i), an alternative option is to choose $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$. Thus, under the assumptions of (7) and (8), setting $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ is sustained as an equilibrium if $Z(\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o), 0) \geq Z(\beta \cdot (1 - 1/4\eta_o), 0)$, i.e., if:

$$\beta \leq \frac{1/\eta_o}{1 - 1/4\eta_o}. \tag{9}$$

Second, suppose that a set of parameters satisfies $\beta \cdot (1 - 1/4\eta_o) \leq (1 - 1/\eta_o)$, i.e.:

$$\beta \leq (1 - 1/\eta_o)/(1 - 1/4\eta_o), \tag{10}$$

as illustrated in Panel (c) of Figure 4. Under this situation, the relevant payoff function is $Z(\tau_t^y, 0)$ or $Z(\tau_t^y, 1)$ for $\tau_t^y \in (0, \theta]$ and $Z(\tau_t^y, 1)$ for $\tau_t^y \in (\theta, 1 - 1/\eta_o]$. Given the properties of the payoff function, there are two alternatives: setting $\tau_t^y = \theta(\leq \beta \cdot (1 - 1/4\eta_o))$ under the expectation of $\tau_{t+1}^o = 0$ and setting $\tau_t^y = \arg\max Z(\tau_t^y, 1) = 1/2$ under the expectation of $\tau_{t+1}^o = 1$. Thus, under the assumptions of (7) and (10), setting $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ is sustained as an equilibrium if:

$$Z(\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o), 0) \geq Z(\beta \cdot (1 - 1/4\eta_o), 0),$$

and:

$$Z(\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o), 0) \geq Z(1/2, 1),$$

that is, if:

$$\beta \leq \frac{1/\eta_o}{1 - 1/4\eta_o} \text{ and } \beta \geq \frac{\eta_o/4 - (1 - 1/\eta_o)}{1 - 1/4\eta_o}. \tag{11}$$

In sum, there exists a poor-majority equilibrium featured by $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ and $\tau_{t+1}^o = 0$ if (7), (8) and (9) hold or if (7), (10) and (11) hold. These conditions are reduced to (7) and (11). ■

8.2 Proof of Proposition 2

Under the set of parameters given in Proposition 2, $\eta_o > 2$, i.e., $1/2 < 1 - 1/\eta_o$ holds (see Figure 5). This implies that given the expectations of $\tau_{t+1}^o = 1$ and $\theta < 1/2$, agents can choose τ_t^y to attain the top of the Laffer curve $Z(\tau_t^y, 1) : \arg\max Z(\tau_t^y, 1) = 1/2$.

Suppose that at time t, agents know that $\tau_t^y = 1/2$ and expect that $\tau_{t+1}^o = 1$. Then,

$1 - e^{y*}(1/2, 1) = u_{t+1} = 3/4$. Because the condition in Proposition 2 ensures that $\eta_o > 2$, $u_{t+1} = 3/4 \in (1/2, 1 - 1/2\eta_o]$ holds for all t. By (4), this implies that $\tau^o_{t+1} = 1$, fulfilling initial expectations. Therefore, there exists a poor-majority equilibrium with 100% taxation on the old if the decisive voter finds it optimal to set $\tau^y_t = 1/2$.

To establish that setting $\tau^y_t = 1/2$ is optimal for the decisive voter, we consider the following two cases: $\beta > (1 - 1/\eta_o)/(1 - 1/4\eta_o)$ and $\beta \le (1 - 1/\eta_o)/(1 - 1/4\eta_o)$. First, suppose that a set of parameters satisfies $\beta \cdot (1 - 1/4\eta_o) > (1 - 1/\eta_o)$, i.e., $\beta > (1-1/\eta_o)/(1-1/4\eta_o)$ as illustrated in Panels (a) and (b) of Figure 4. Under this situation, setting $\tau^y_t = \beta\cdot(1-1/4\eta_o)$ is rational under the expectation of $\tau^o_{t+1} = 0$ (see Panels (a) and (b) of Figure 4). It holds that $Z(\beta\cdot(1-1/4\eta_o), 0) \ge Z(1/2, 1) \Leftrightarrow \beta \ge (1-1/\eta_o)/(1-1/4\eta_o)$, implying that setting $\tau^y_t = 1/2$ is not sustained as an equilibrium.

Alternatively, suppose that a set of parameters satisfies $\beta \cdot (1 - 1/4\eta_o) \le (1 - 1/\eta_o)$, i.e.:

$$\beta \le (1 - 1/\eta_o)/(1 - 1/4\eta_o), \tag{12}$$

as illustrated in Panels (c) and (d) of Figure 4. Under this situation, there are two alternative options: (i) setting $\tau^y_t = \theta(\le \beta \cdot (1 - 1/4\eta_o))$ under the expectation of $\tau^o_{t+1} = 0$ (see Panels (c) and (d) of Figure 4); and (ii) setting $\tau^y_t = \beta\cdot(1-1/4\eta_o)+(1-1/\eta_o)$ under the expectation of $\tau^o_{t+1} = 0$ provided that $(1+\beta\cdot(1-1/4\eta_o))/2 < \beta\cdot(1-1/4\eta_o)+(1-1/\eta_o) \le 1$ holds (Panel (c) of Figure 4).[14] Therefore, under the condition of (12), setting $\tau^y_t = 1/2$ is sustained as an equilibrium if:

$$Z(\theta, 0) < Z\left(\frac{1}{2}, 1\right) \text{ and } 1 < \beta \cdot \left(1 - \frac{1}{4\eta_o}\right) + \left(1 - \frac{1}{\eta_o}\right), \tag{13}$$

or:

$$\begin{gathered} Z(\theta, 0) < Z\left(\tfrac{1}{2}, 1\right), \tfrac{1+\beta\cdot(1-1/4\eta_o)}{2} < \beta \cdot \left(1 - \tfrac{1}{4\eta_o}\right) + \left(1 - \tfrac{1}{\eta_o}\right) \le 1, \\ \text{and } Z\left(\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o), 0\right) < Z\left(\tfrac{1}{2}, 1\right). \end{gathered} \tag{14}$$

The conditions in (13) are reduced to:

$$\theta < \tilde{\theta} \text{ and } \beta > (1/\eta_o)/(1 - 1/4\eta_o), \tag{15}$$

and the condition in (14) is reduced to:

$$\theta < \tilde{\theta},\ \beta \le (1/\eta_o)/(1 - 1/4\eta_o) \text{ and } \beta < \{\eta_o/4 - (1 - 1/\eta_o)\}/(1 - 1/4\eta_o), \tag{16}$$

where $\tilde{\theta}$ is defined by:$\widetilde{\theta}(\beta, \eta_o) \equiv \frac{1+\beta\cdot\left(1-\frac{1}{4\eta_o}\right)-\sqrt{\beta\cdot\left(1-\frac{1}{4\eta_o}\right)\cdot\left\{\beta\cdot\left(1-\frac{1}{4\eta_o}\right)+2\right\}}}{2}$. From Figure 5, we can see that the second and third conditions in (16) imply (12). Thus, given $\theta < \tilde{\theta}$, there

[14] If $\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o) \le (1 + \beta \cdot (1 - 1/4\eta_o))/2$ holds, setting $\tau^y_t = (1 + \beta \cdot (1 - 1/4\eta_o))/2$ under the expectation of $\tau^o_{+1} = 0$ dominates all the other options.

exists an equilibrium featured by $\tau_t^y = 1/2$ and $\tau_{t+1}^o = 1$ if (12) and (15) hold; or if (16) holds. ■

8.3 Proof of Proposition 3

(i) Suppose that at time t, agents know that $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$ and expect that $\tau_{t+1}^o = 0$. Then, $1 - e^{y*}(\beta \cdot (1 - 1/4\eta_o), 0) = u_{t+1} = 1/2$. By (4), this implies that $\tau_{t+1}^o = 0$, fulfilling initial expectations. Therefore, there exists a rich-majority equilibrium with $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$ and $\tau_{t+1}^o = 0$ if the decisive voter finds it optimal to set $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$.

To establish that setting $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$ is optimal for the decisive voter, we consider the following two cases: $\beta \cdot (1 - 1/4\eta_o) > (1 - 1/\eta_o)$ and $\beta \cdot (1 - 1/4\eta_o) \leq (1 - 1/\eta_o)$.

First, suppose that a set of parameters satisfies $\beta \cdot (1 - 1/4\eta_o) > (1 - 1/\eta_o)$, i.e.:

$$\beta > (1 - 1/\eta_o)/(1 - 1/4\eta_o), \tag{17}$$

as illustrated in Panels (a) and (b) of Figure 4. If $\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o) \leq 1$, i.e., if $\beta \leq (1/\eta_o)/(1 - 1/4\eta_o)$, there is an alternative option $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$ (see Panel (a) of Figure 4). This dominates the initial choice $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$ because $\beta \leq (1/\eta_o)/(1 - 1/4\eta_o)$ is equivalent to $Z(\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o), 0) \geq Z(\beta \cdot (1 - 1/4\eta_o), 0)$. However, if $\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o) > 1$, i.e., if:

$$\beta > (1/\eta_o)/(1 - 1/4\eta_o), \tag{18}$$

it is irrational for voters to set $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$. An alternative option is to set $\tau_t^y = \arg\max Z(\tau_t^y, 1) = 1/2$ or $\tau_t^y = 1 - 1/\eta_o$ under the expectation of $\tau_{t+1}^o = 1$. Given the properties of the payoff function, this alternative is dominated by the initial choice. Therefore, there exists a rich-majority equilibrium featured by $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$ and $\tau_{t+1}^o = 0$ if (17) and (18) hold.

Second, suppose that a set of parameters satisfies:

$$\beta \leq (1 - 1/\eta_o)/(1 - 1/4\eta_o),$$

as illustrated in Panels (c) and (d) of Figure 4. In this case, the relevant payoff function is $Z(\tau_t^y, 0)$ or $Z(\tau_t^y, 1)$ for $\tau_t^y \in (0, \theta]$ and $Z(\tau_t^y, 1)$ for $\tau_t^y \in (\theta, 1 - 1/\eta_o]$. Therefore, there is no choice of $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$ in this case.

(ii) First, suppose that a set of parameters satisfies $\beta \cdot (1 - 1/4\eta_o) > (1 - 1/\eta_o)$, i.e., $\beta > (1-1/\eta_o)/(1-1/4\eta_o)$ as illustrated in Panels (a) and (b) of Figure 4. It is possible to set $\tau_t^y = \theta(< 1 - 1/\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$. However, it is also possible to set $\tau_t^y = \beta \cdot (1 - 1/4\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$, and this alternative dominates the initial one because $Z(\beta \cdot (1-1/4\eta_o), 0) > Z(\theta, 0)$ holds for all t. Therefore, there is no rich-majority equilibrium featured by $\tau_t^y = \theta$ and $\tau_{t+1}^o = 0$ if $\beta > (1 - 1/\eta_o)/(1 - 1/4\eta_o)$.

Next, suppose that a set of parameters satisfies $\beta \cdot (1 - 1/4\eta_o) \leq (1 - 1/\eta_o)$, i.e.:

$$\beta \leq (1 - 1/\eta_o)/(1 - 1/4\eta_o), \tag{19}$$

as illustrated in Panels (c) and (d) of Figure 4. It is possible to set $\tau_t^y = \theta (< 1 - 1/\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$. If $\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o) \leq 1$ holds (see Panel (c)), there is an alternative option of setting $\tau_t^y = \beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o)$ under the expectation of $\tau_{t+1}^o = 0$: this dominates the initial option. However, if: $\beta \cdot (1 - 1/4\eta_o) + (1 - 1/\eta_o) > 1$, i.e., if:

$$\beta > (1/\eta_o)/(1 - 1/4\eta_o), \tag{20}$$

as illustrated in Panel (d) of Figure 3, an alternative option is to set $\tau_t^y = \arg\max Z(\tau_t^y, 1) = 1/2$ under the expectation of $\tau_{t+1}^o = 1$. Setting $\tau_t^y = \theta$ is sustained as an equilibrium if $Z(\theta, 0) \geq Z(1/2, 1)$, i.e., if $\theta \geq \tilde{\theta}$. Therefore, under the condition of (19) and (20), there exists a rich-majority equilibrium featured by $\tau_t^y = \theta$ and $\tau_{t+1}^o = 0$ if θ is set within the range $[\tilde{\theta}, \beta \cdot (1 - 1/4\eta_o)]$. This set is nonempty if and only if $\tilde{\theta} \leq \beta \cdot (1 - 1/4\eta_o)$, i.e., if and only if $\beta \geq (1/4)/(1 - 1/4\eta_o)$. ■

8.4 The Gini Index

This section provides an explanation for the calculation of the Gini Index in Section 4. We calculate the Gini index in terms of the after-tax-and-transfer income. In each period, there are three types of agents: young rich agents with per capita income given by $(1 - \tau_t^y) + s_t$, old rich agents with per capita income given by $(1 - \tau_t^o) + s_t$, and young and old poor agents with per capita income given by s_t. Panel (a) of Figure 10 summarizes information about per capita income, the size of the population and the total income for each type of agent. For an equilibrium with $\tau_t^o = 0$, per capita income levels are ranked by $(1 - \tau_t^o) + s_t > (1 - \tau_t^y) + s_t > s_t$. For an equilibrium with $\tau_t^o = 1$, per capita income levels are ranked by $(1 - \tau_t^y) + s_t > (1 - \tau_t^o) + s_t > s_t$. Based on this rank, we can illustrate the Lorenz curve and calculate the corresponding Gini coefficient. Panel (b) of Figure 10 illustrates an example of the Lorenz curve for the equilibrium with $\tau_t^o = 0$. The Gini coefficient is calculated by $A/(A + B)$, where A is the shaded area and B is the non-shaded area in Panel (b). The Gini index is equal to the Gini coefficient multiplied by 100.

[Figure 10 about here.]

9 Supplementary Explanation for Section 6.2

We modify the model presented in Section 2 by assuming that the old who were successful in youth earn experience premiums in old age. Specifically, they earn a wage w, which is greater than one: $w > 1$. Under this setting, the expected utility functions of agents alive at time t are given as follows:

$$\begin{aligned} V_t^{os} &= w(1-\tau_t^o) + s_t, \\ V_t^{ou} &= e_t^o \cdot (1-\tau_t^o) - \eta_o \cdot (e_t^o)^2 + s_t, \\ V_t^y &= e_t^y \cdot (1-\tau_t^y) - (e_t^y)^2 + s_t \\ &\quad + \beta \left[e_t^y \cdot w(1-\tau_{t+1}^o) + (1-e_t^y) \left\{ e_{t+1}^o \cdot (1-\tau_{t+1}^o) - \eta_o \cdot (e_{t+1}^o)^2 \right\} + s_{t+1} \right]. \end{aligned}$$

The optimal investments by the old and the young are given by, respectively:

$$\begin{aligned} e^{o*}(\tau_t^o) &= \frac{(1-\tau_t^o)}{2\eta_o}, \\ e^{y*}(\tau_t^y, \tau_{t+1}^o) &= \frac{1}{2} \cdot \left[(1-\tau_t^y) + \beta \left\{ w(1-\tau_{t+1}^o) - \frac{1}{4\eta_o} \cdot (1-\tau_{t+1}^o)^2 \right\} \right]. \end{aligned}$$

We make the following assumption.

Assumption A: (i) $\eta_o w > 1$; (ii) $\beta w < 1$.

The first assumption includes Assumption 1 as a special case of $w = 1$. The second assumption ensures that $e^{y*}(\tau_t^y, \tau_{t+1}^o)$ is set within the range $(0, 1)$.

The government budget constraint is given by $2s_t = W(\tau_t^o, u_t) + Z(\tau_t^y, \tau_{t+1}^o)$, where $W(\tau_t^o, u_t) \equiv [(1-u_t)w + u_t \cdot e^{o*}(\tau_t^o)] \cdot \tau_t^o$ is the revenue financed by the rich old agents, and $Z(\tau_t^y, \tau_{t+1}^o) \equiv e^{y*}(\tau_t^y, \tau_{t+1}^o) \cdot \tau_t^y$ is the tax revenue financed by the rich young agents.

The first equilibrium condition is $T^o(u_t) = \arg\max_{\tau_t^o \in [0,1]} W^{dec}(\tau_t^o, u_t)$ $(dec = os, ou)$, where:

$$W^{dec}(\tau_t^o, u_t) = \begin{cases} W^{os} \equiv w(1-\tau_t^o) + \frac{1}{2} \cdot W(\tau_t^o, u_t) & \text{if } u_t \leq 1/2, \\ W^{ou} \equiv \frac{(1-\tau_t^o)^2}{4\eta_o} + \frac{1}{2} \cdot W(\tau_t^o, u_t) & \text{if } u_t > 1/2. \end{cases}$$

After some calculation, we obtain the mapping that satisfies equilibrium condition 1 as follows:

$$T^o(u_t) = \begin{cases} 0 & \text{if } u_t \leq 1/2 \text{ or } 1 - 1/2\eta_o w \leq u_t \leq 1, \\ 1 & \text{if } 1/2 < u_t \leq 1 - 1/2\eta_o w. \end{cases} \tag{21}$$

The second equilibrium condition is given by:

$$U(\tau_t^y) = 1 - \frac{1}{2} \cdot \left[(1-\tau_t^y) + \beta \cdot \left\{ w\left(1 - T^o(U(\tau_t^y))\right) - \frac{1}{4\eta_o} \cdot \left(1 - T^o(U(\tau_t^y))\right)^2 \right\} \right],$$

where $T^o(\cdot) \in \{0,1\}$ is given by (21). After some calculation, we obtain the following solution:

$$U(\tau_t^y) = \begin{cases} \left\{1-\frac{1}{2}\cdot\left((1-\tau_t^y)+\beta\cdot\left(w-\frac{1}{4\eta_o}\right)\right), 1-\frac{1}{2}\cdot(1-\tau_t^y)\right\} & \text{for } \tau_t^y \in (0,\theta], \\ 1-\frac{1}{2}\cdot(1-\tau_t^y) & \text{for } \tau_t^y \in \left(\theta, 1-\frac{1}{\eta_o w}\right], \\ 1-\frac{1}{2}\cdot\left((1-\tau_t^y)+\beta\cdot\left(w-\frac{1}{4\eta_o}\right)\right) & \text{for } \tau_t^y \in \left(1-\frac{1}{\eta_o w}, \beta\cdot\left(w-\frac{1}{4\eta_o}\right)\right), \text{ or} \\ & \tau_t^y \in \left[\beta\cdot\left(w-\frac{1}{4\eta_o}\right)+\left(1-\frac{1}{\eta_o w}\right), 1\right]. \end{cases} \tag{22}$$

Given the first and the second equilibrium conditions presented above, we now consider the political determination of τ_t^y that satisfies equilibrium condition 3, and characterize a poor-majority equilibrium with no taxation on the old, that is, an equilibrium that supports the POUM hypothesis.

Proposition A

(i) *Suppose that $\beta \leq (2/\eta_o w - 1)/(w - 1/4\eta_o)$ holds. There exists a set of poor-majority equilibrium with no taxation on the old such that $\forall t$, T^o is given by (21), $U(\tau_t^y)$ is given by (22), and $T^y = (1+\beta\cdot(w-1/4\eta_o))/2$. The equilibrium outcome is unique and such that $\forall t$, $\tau_t^y = (1+\beta\cdot(w-1/4\eta_o))/2$, $\tau_t^o = 0$, and $u_t = 1 - (1+\beta\cdot(w-1/4\eta_o))/4$.*

(ii) *Suppose that $\beta > (2/\eta_o w - 1)/(w - 1/4\eta_o)$, $\beta \geq \{\eta_o w/4 - (1-1/\eta_o w)\}/(w-1/4\eta_o)$, and $\beta \leq (1/\eta_o w)/(w-1/4\eta_o)$ hold. There exists a set of poor-majority equilibrium with no taxation on the old such that $\forall t$, T^o is given by (21), $U(\tau_t^y)$ is given by (22), and $T^y = \beta\cdot(w-1/4\eta_o)+(1-1/\eta_o w)$. The equilibrium outcome is unique and such that $\forall t$, $\tau_t^y = \beta\cdot(w-1/4\eta_o)+(1-1/\eta_o w)$, $\tau_t^o = 0$, and $u_t = 1-1/2\eta_o w (> 1/2)$.*

Proof.
(i) Suppose that under the assumption of $\beta \leq (2/\eta_o w - 1)/(w - 1/4\eta_o)$, agents know that $\tau_t^y = (1+\beta\cdot(w-1/4\eta_o))/2$ and expect $\tau_{t+1}^o = 0$. Then:

$$1 - e^{y^*}\left(\frac{1+\beta\cdot(w-1/4\eta_o)}{2}, 0\right) = u_{t+1} = 1 - \frac{1}{4}\left(1+\beta\cdot\left(w-\frac{1}{4\eta_o}\right)\right) \geq 1 - \frac{1}{2\eta_o w}.$$

By (21), this implies that $\tau_{t+1}^o = 0$, fulfilling initial expectations.

Setting $\tau_t^y = (1+\beta\cdot(w-1/4\eta_o))/2$ is optimal for the decisive voter if this setting is feasible under the expectation of $\tau_{t+1}^o = 0$, i.e., if $\beta\cdot(w-1/4\eta_o)+(1-1/\eta_o w) \leq$

$(1+\beta\cdot(w-1/4\eta_o))/2 \leq 1$. The first inequality is rewritten as $\beta \leq (2/\eta_o w-1)/(w-1/4\eta_o)$, which is equivalent to the current assumption; and the second inequality always holds under the assumption of $\beta w < 1$.

(ii) Suppose that at time t, agents know that $\tau_t^y = \beta \cdot (w - 1/4\eta_o) + (1 - 1/\eta_o w)$ and expect $\tau_{t+1}^o = 0$. Then:

$$1 - e^{y^*}\left(\beta \cdot \left(w - \frac{1}{4\eta_o}\right) + \left(1 - \frac{1}{\eta_o w}\right), 0\right) = u_{t+1} = 1 - \frac{1}{2\eta_o w}.$$

By (21), this implies that $\tau_{t+1}^o = 0$, fulfilling initial expectations. A poor-majority equilibrium exists if the decisive voter finds it optimal to set $\tau_t^y = \beta \cdot (w - 1/4\eta_o) + (1 - 1/\eta_o w)$.

To establish that setting $\tau_t^y = \beta \cdot (w - 1/4\eta_o) + (1 - 1/\eta_o w)$ is optimal for the decisive voter, we first note that setting $\tau_t^y = (1 + \beta \cdot (w - 1/4\eta_o))/2$ is not feasible under the first assumption:

$$\beta > (2/\eta_o w - 1)/(w - 1/4\eta_o). \tag{23}$$

Given this condition, we consider the following two cases: $\beta > (1 - 1/\eta_o w)/(w - 1/4\eta_o)$ and $\beta \leq (1 - 1/\eta_o w)/(w - 1/4\eta_o)$.

First, suppose that a set of parameters satisfies $\beta \cdot (w - 1/4\eta_o) > (1 - 1/\eta_o w)$, i.e.:

$$\beta > (1 - 1/\eta_o w)/(w - 1/4\eta_o). \tag{24}$$

Following the logic in the proof of Proposition 1(i), setting $\tau_t^y = \beta\cdot(w-1/4\eta_o)+(1-1/\eta_o w)$ is sustained as an equilibrium if $Z(\beta\cdot(w-1/4\eta_o)+(1-1/\eta_o w), 0) \geq Z(\beta\cdot(w-1/4\eta_o), 0)$, i.e., if:

$$\beta \leq \frac{1/\eta_o w}{w - 1/4\eta_o}. \tag{25}$$

Second, suppose that a set of parameters satisfies $\beta \cdot (w - 1/4\eta_o) \leq (1 - 1/\eta_o w)$, i.e.:

$$\beta \leq (1 - 1/\eta_o w)/(w - 1/4\eta_o). \tag{26}$$

Following the logic in the proof of Proposition 1(ii), setting $\tau_t^y = \beta\cdot(w-1/4\eta_o)+(1-1/\eta_o w)$ is sustained as an equilibrium if:

$$Z(\beta \cdot (w - 1/4\eta_o) + (1 - 1/\eta_o w), 0) \geq Z(\beta \cdot (w - 1/4\eta_o), 0),$$

and:

$$Z(\beta \cdot (w - 1/4\eta_o) + (1 - 1/\eta_o w), 0) \geq Z(1/2, 1),$$

that is, if:

$$\beta \leq \frac{1/\eta_o w}{w - 1/4\eta_o} \text{ and } \beta \geq \frac{\eta_o w/4 - (1 - 1/\eta_o w)}{w - 1/4\eta_o}. \tag{27}$$

In sum, there exists a poor-majority equilibrium featured by $\tau_t^y = \beta \cdot (w - 1/4\eta_o) + (1 - 1/\eta_o w)$ and $\tau_{t+1}^o = 0$ if (23), (24) and (25) hold or if (23), (26) and (27) hold. These conditions are reduced to (23) and (27). ■

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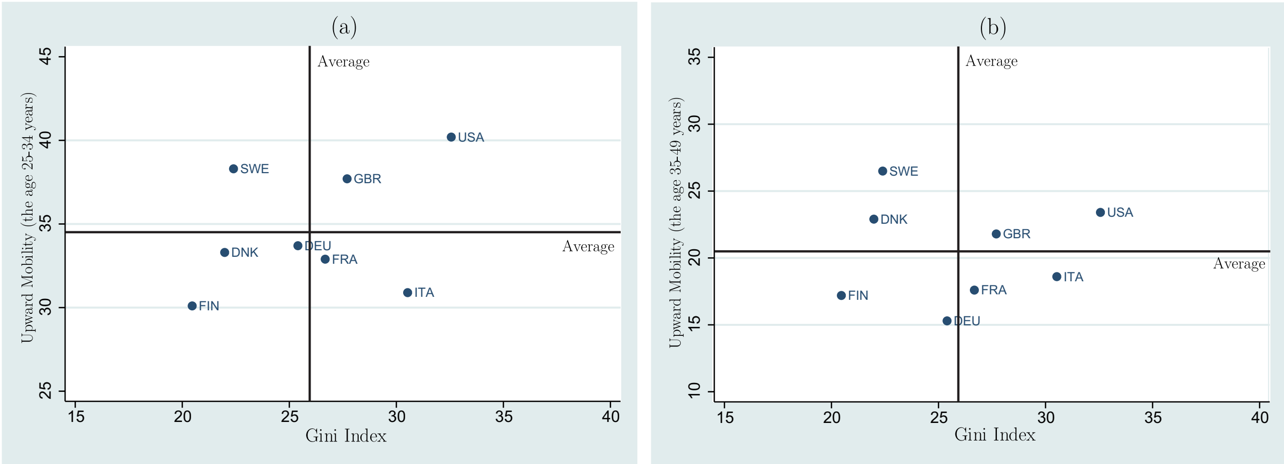

Figure 1: The figure illustrates the scatter plot of the Gini indexes for eight OECD countries (Denmark, Finland, Frace, Germany, Italy, Sweden, the United Kingdom, and the United States). The data of the Gini indexes are from Figure 10 in Förster and Mira d'Ercole (2005). The data of upward mobility are from Table 3.7a in OECD (1996). Panel (a) depicts the mobility rate of aged 25-34 workers; Panel (b) depicts the mobility rate of aged 35-49 workers.

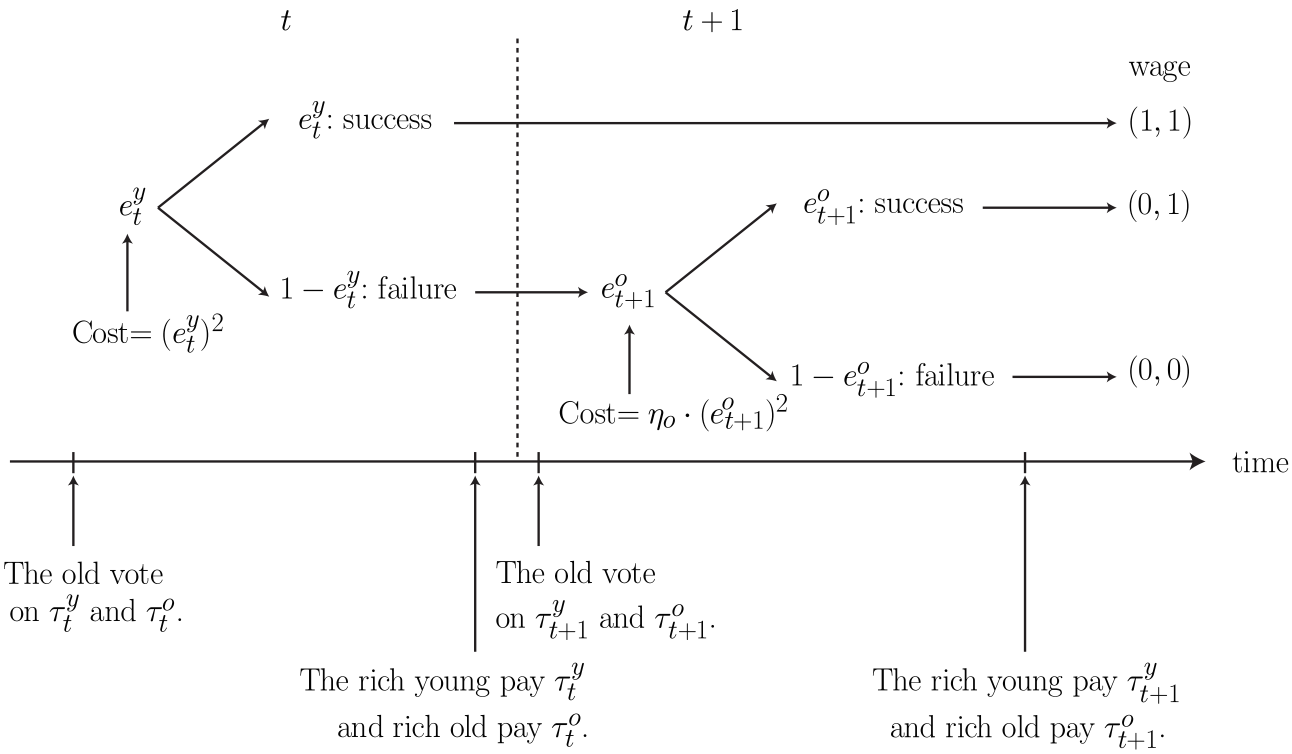

Figure 2: Timing of events and the distribution of the rich and the poor.

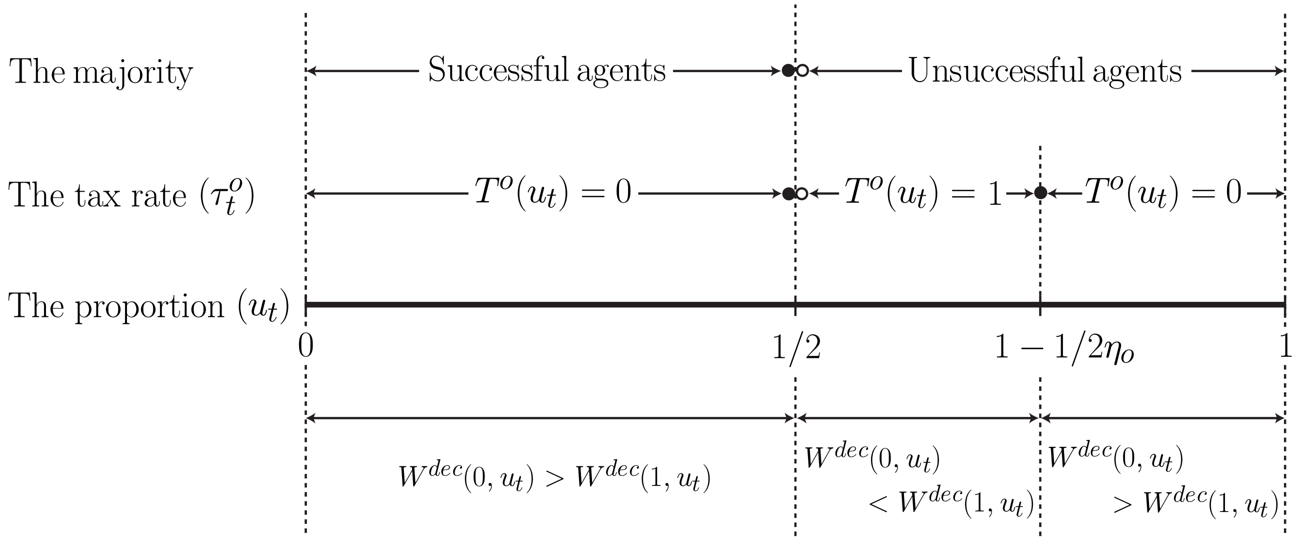

Figure 3: The determination of τ_t^o.

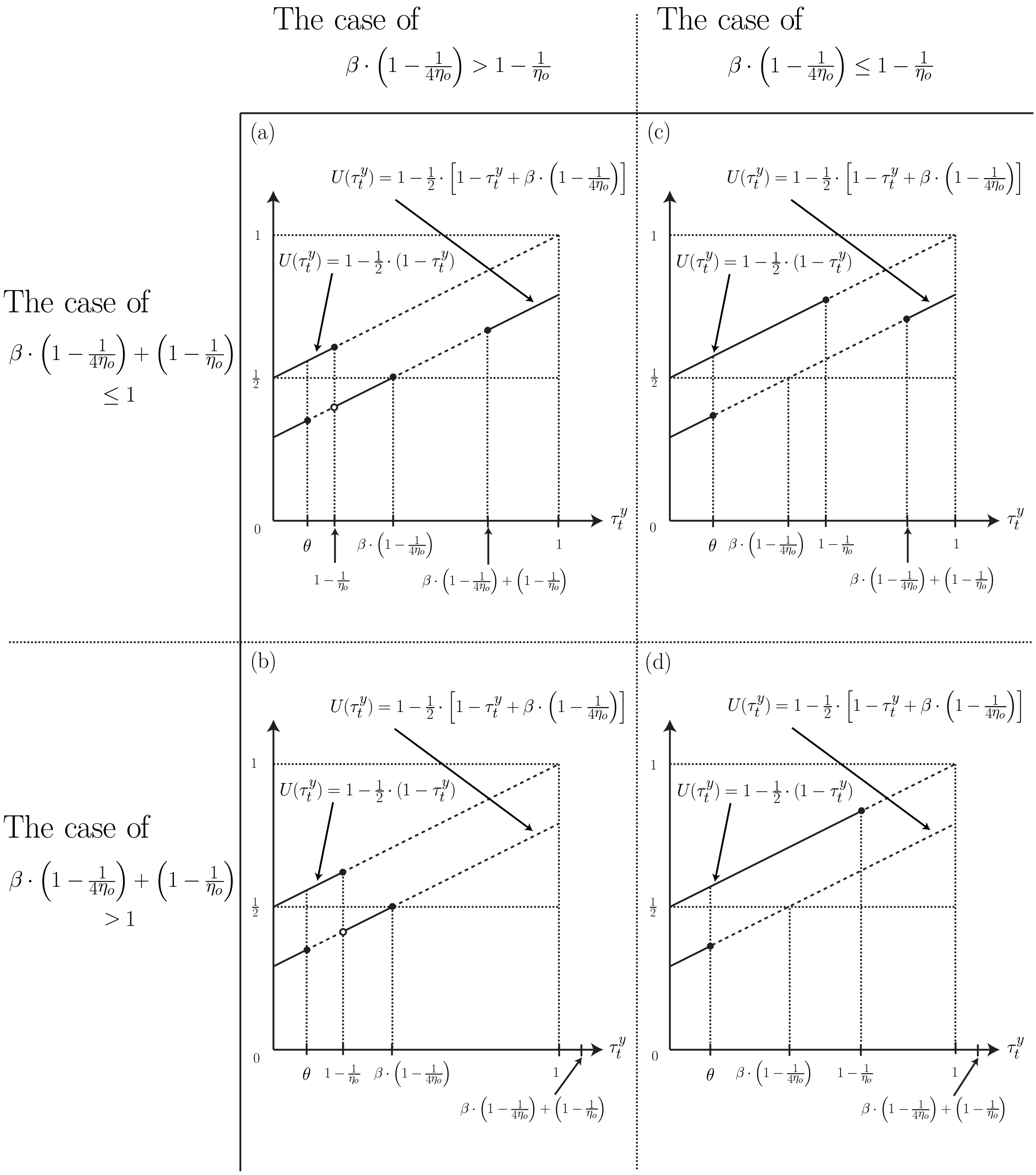

Figure 4: The equilibrium decision rule $u_{t+1} = U(\tau_t^y)$. The solod lines show the graphs of U satisfying equilibrium condition 2.

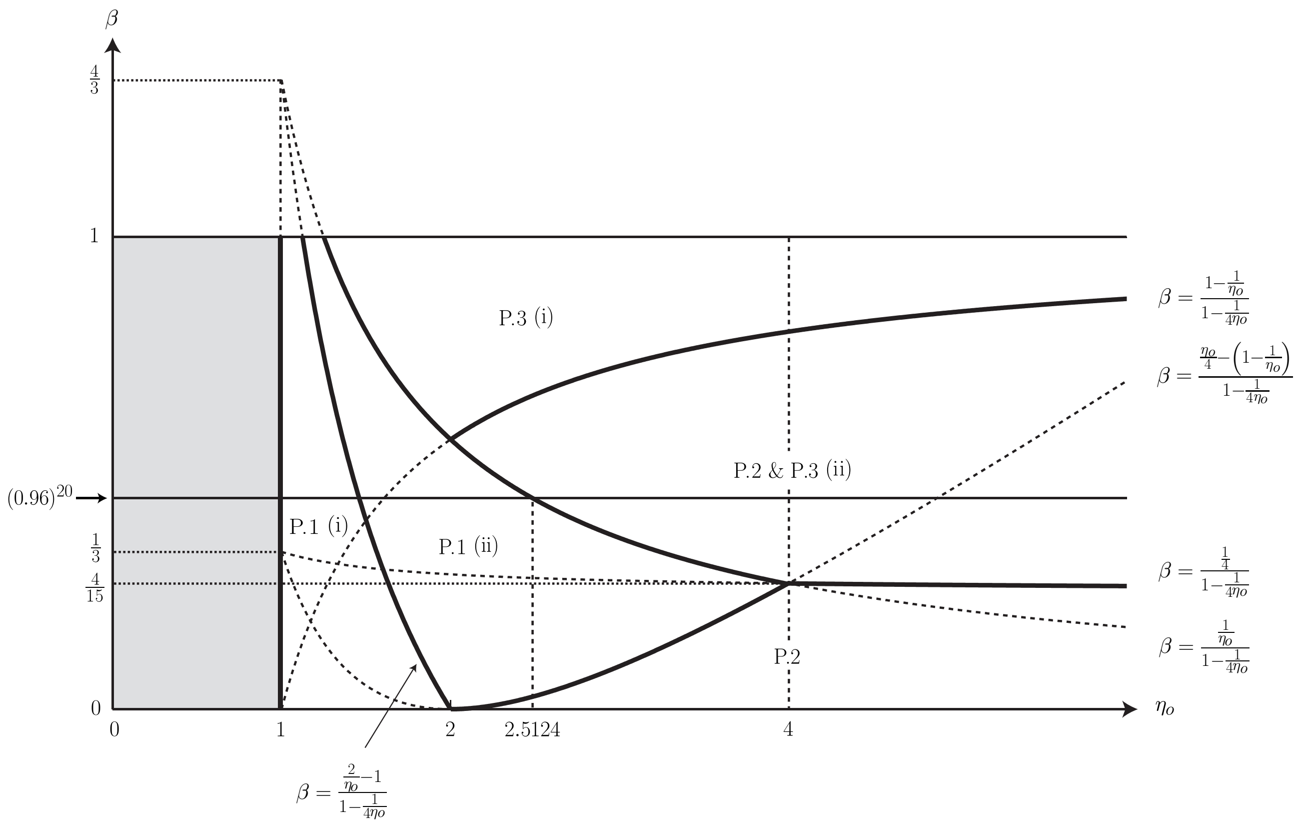

Figure 5: The set of parameters (β, η_o) classified according to the characterization of political equilibria.

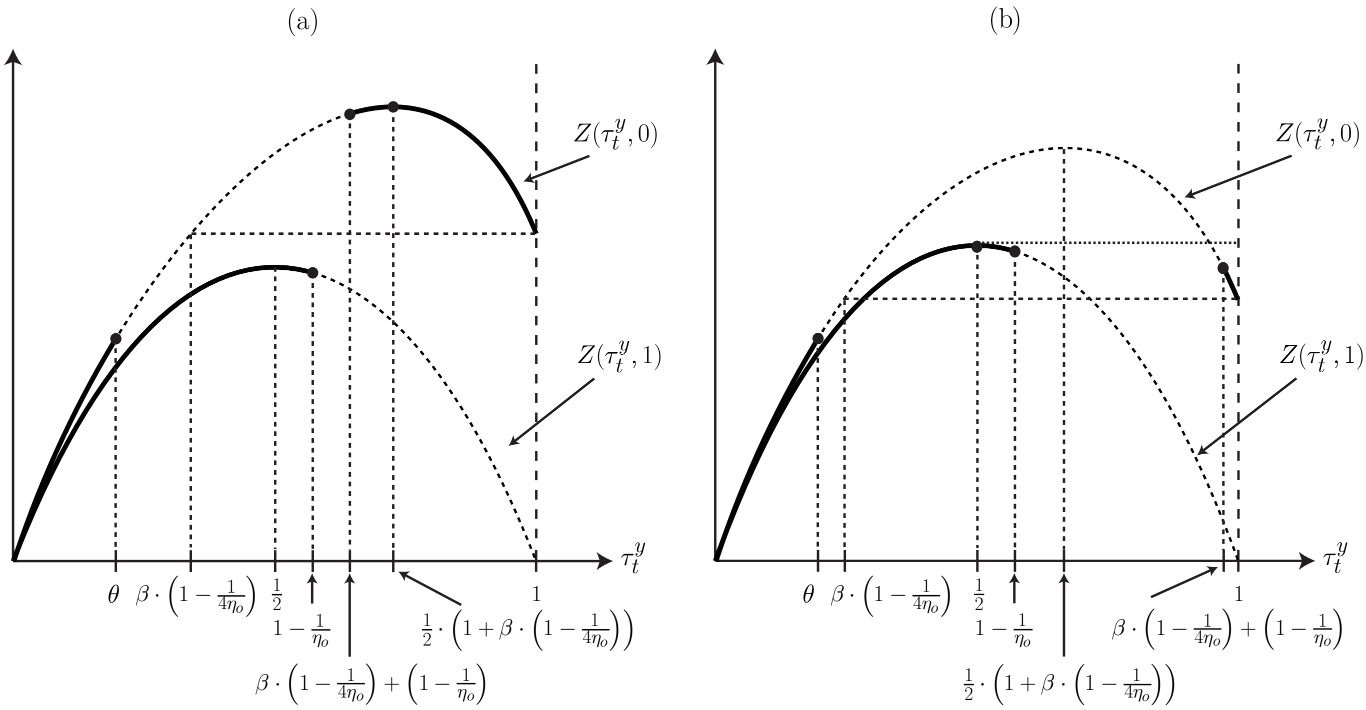

Figure 6: The tax revenue from the young, Z, under the set of parameters provided in Proposition 1. Panel (a) illustrates the case of Proposition 1(i); panel (b) illustrates the case of Proposition 2(ii).

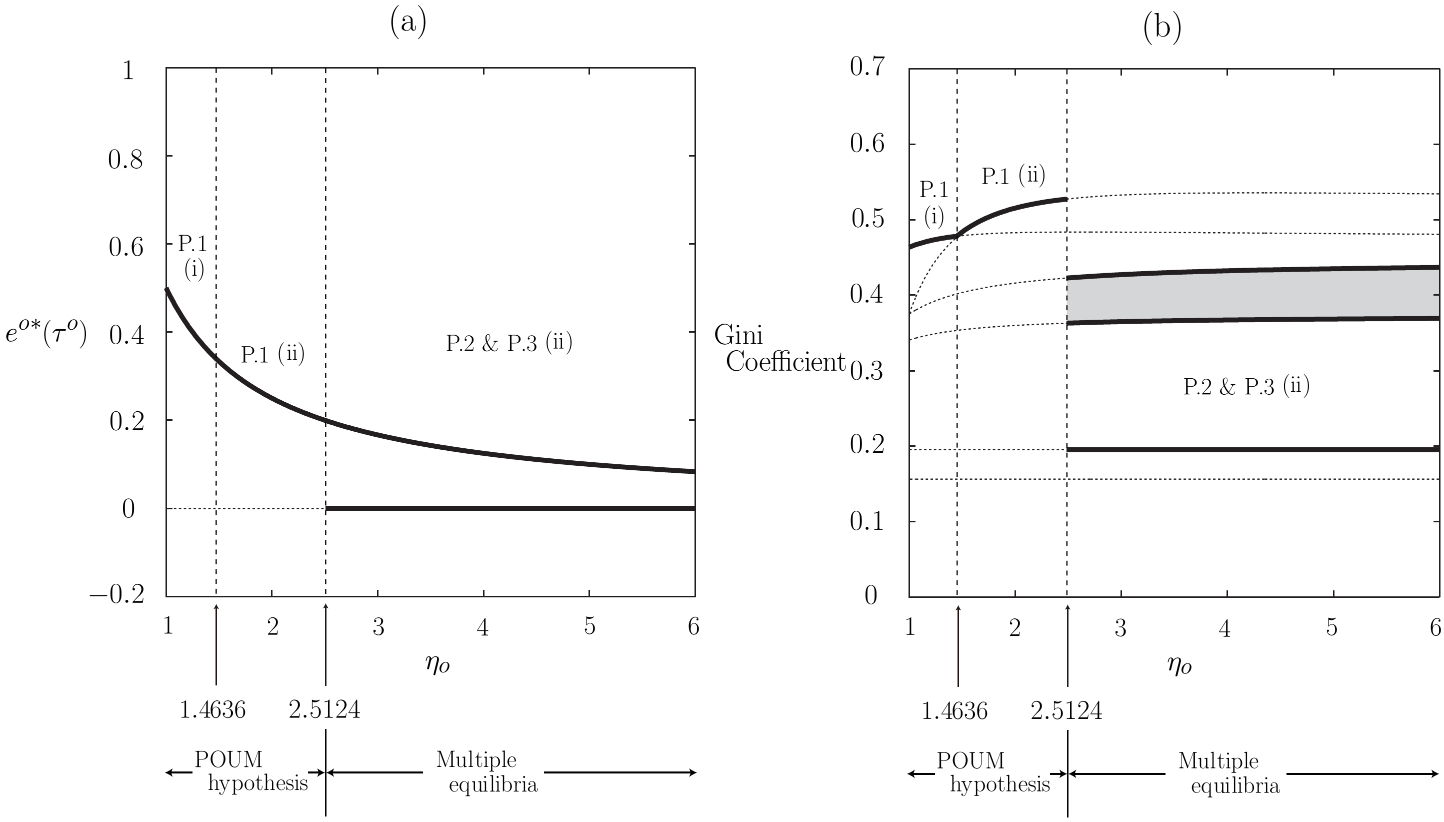

Figure 7: Numerial results for the case of $\beta = (0.96)^{20}$. The horizontal axis indicates the parameter η_o. Panel (a) depicts the upward mobility rate e^o; panel (b) depicts the Gini index.

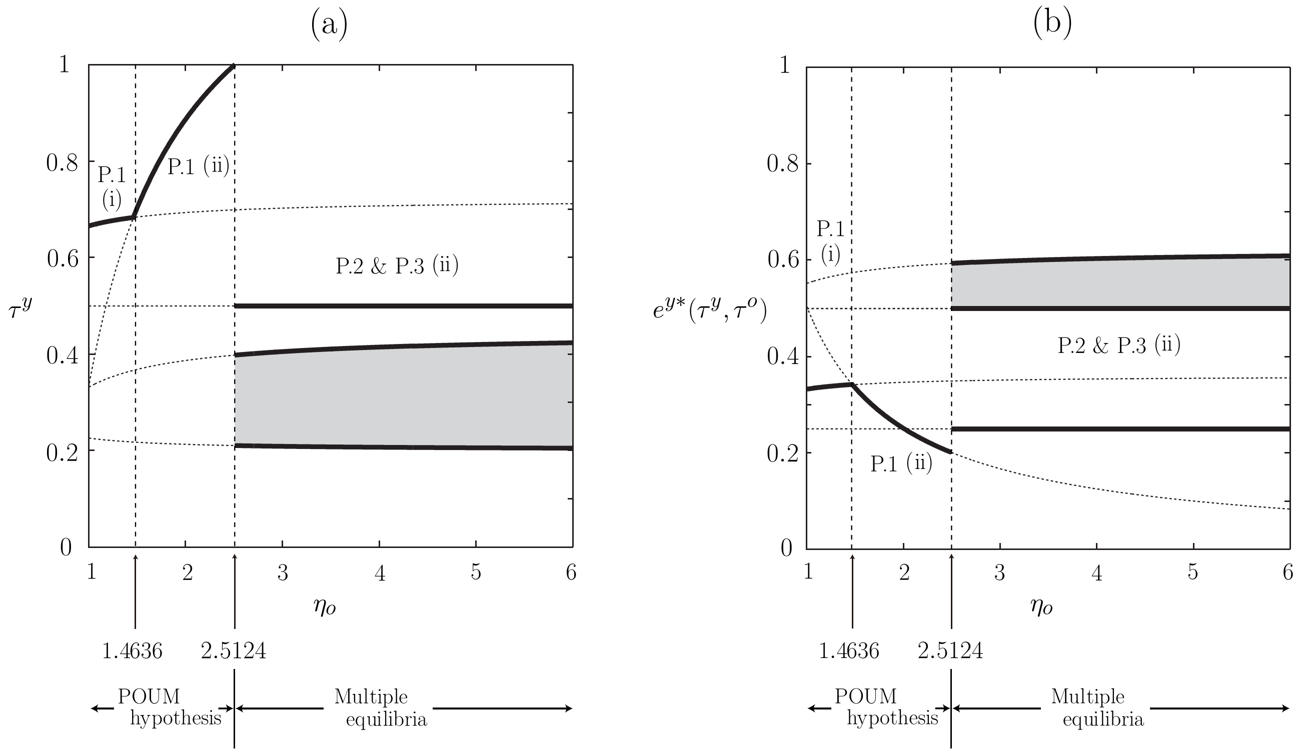

Figure 8: Numerical results for the case of $\beta = (0.96)^{20}$. Panel (a) depicts the tax rate on the young; Panel (b) depicts educational investment by the young.

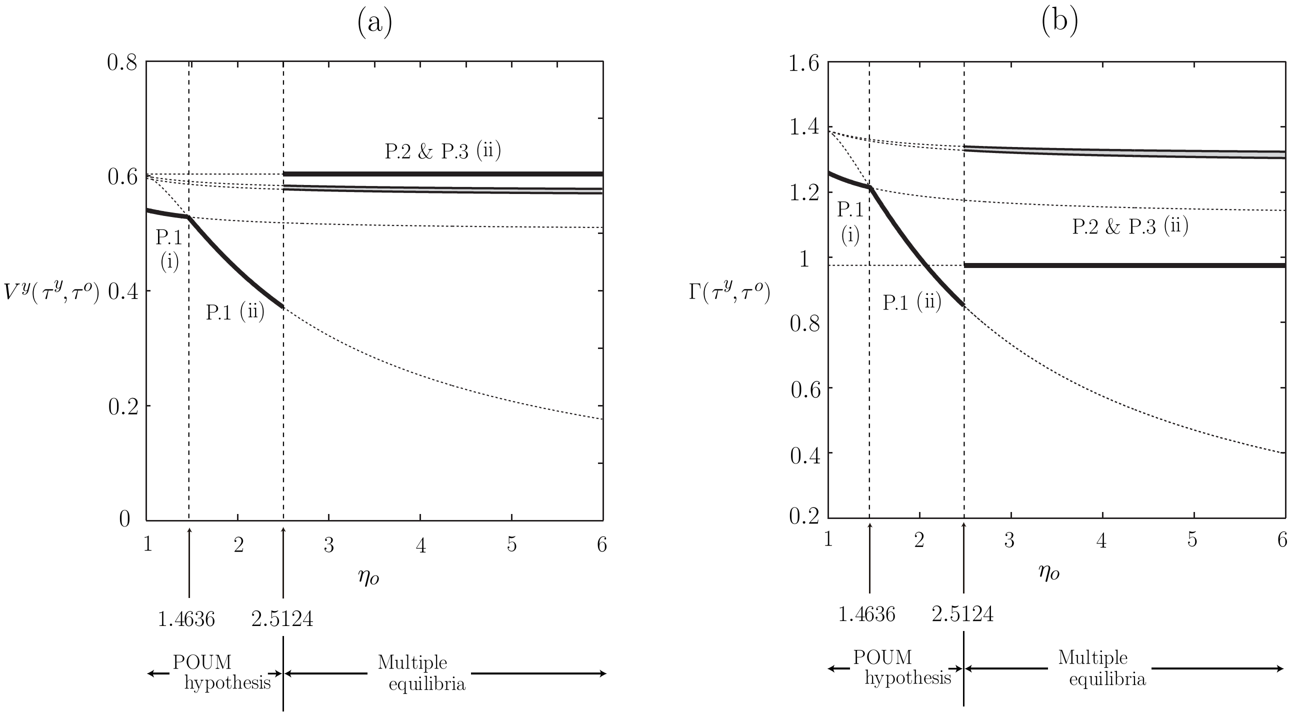

Figure 9: Numerical results for the case of $\beta = (0.96)^{20}$. Panel (a) depicts the expected utility of the young agents; Panel (b) depicts the social welfare.

(a)

	Per capita after tax-and-transfer income	Population	Total after tax-and-transfer income
Young rich agents	$(1-\tau_t^y)+s_t$	e_t^y	$e_t^y \cdot \{(1-\tau_t^y)+s_t\}$
Old rich agents	$(1-\tau_t^o)+s_t$	$(1-e_{t-1}^y)\cdot e_t^o + e_{t-1}^y$	$\left\{(1-e_{t-1}^y)\cdot e_t^o + e_{t-1}^y\right\} \times \{(1-\tau_t^o)+s_t\}$
Poor agents	s_t	$(1-e_{t-1}^y)\cdot(1-e_t^o) + (1-e_t^y)$	$\left\{(1-e_{t-1}^y)\cdot(1-e_t^o) + (1-e_t^y)\right\}\cdot s_t$

(b)

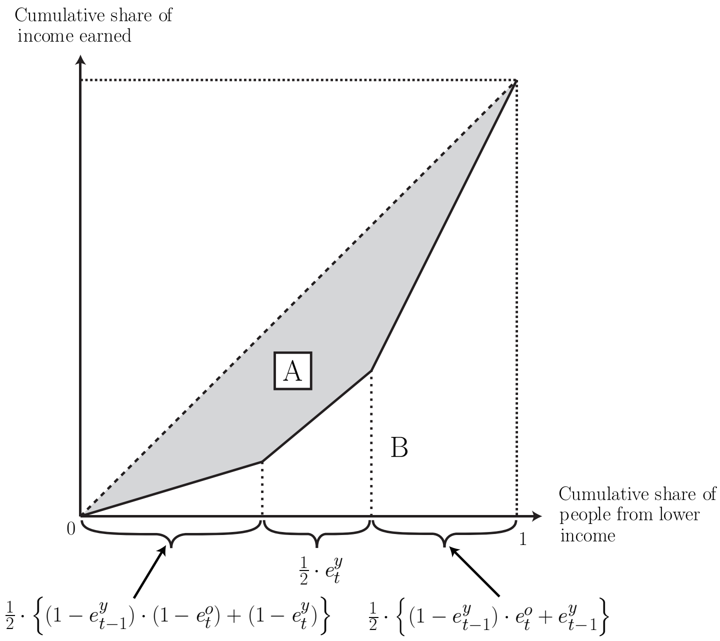

Figure 10: Panel (a) summarizes the information about per capita income, the size of population and total income for each type of agents. Panel (b) illustrates the Lorenz curve for the case in which the tax rate on the old is zero.