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Discussion Paper 08-11

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Seigniorage Revenue or Consumer Revenue? Theoretical and Empirical Evidences

By
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Abstract
The purpose of this paper is to propose a seigniorage model including the contributions of Bailey (1956) and Marty (1976), using a different framework to Mankiw (1987), to test whether their results are supported, and use a numerical example to estimate the seigniorage model. The government decides the money growth rate to maximize the social welfare function for each of seigniorage revenue aversion, loving and neutrality. The numerical example using Japanese data shows that the social welfare function supports seigniorage revenue aversion, supporting the results of Bailey and Marty, and that the degree of seigniorage revenue aversion is stronger in the 2000s than in the 1990s.

Keywords: Bailey and Marty; Social welfare function; Mankiw model; Numerical example

JEL Classification Number: E40; E50; C40; C50

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1. Introduction

Money growth theory has two strands. First, the optimal money growth rate aims to control inflation, GDP, the exchange rate and so on. Second, the optimal growth rate aims to maximize seigniorage revenue. We consider the second strand in this paper. To date, the so-called seigniorage model in which the government decides the money growth rate to maximize seigniorage revenue was developed by Bailey (1956) and extended by Blanchard and Fischer (1989, p. 188–194). Honohan (1996), Loviscek (1996), Turner and Benavides (2001) and Tekin-Koru and Özmen (2003) have applied this model to empirical studies. However, prior to Blanchard and Fischer (1989), Bailey (1956) and Marty (1976) proposed a seigniorage model where the government maximizes the seigniorage revenue together with consumer revenue. This is because seigniorage revenue reduces consumer revenue and induces a welfare loss. Then, based on the Bailey and Marty models, Mankiw (1987) proposed a seigniorage model where the government chooses the rates of tax and money growth to minimize the social cost of raising revenue from tax and seigniorage. Amano (1998) and Ho (2003) provided support for Mankiw's model.

The purpose of this paper is to propose a seigniorage model including the contributions of Bailey and Marty, using a different framework from Mankiw (1987), to test whether their results are supported, and use a numerical example to estimate the seigniorage model. The government decides the money growth rate to maximize the social welfare function involving seigniorage revenue aversion, loving and neutrality. The numerical example using Japanese data supports the existence of a social welfare function consistent with seigniorage revenue aversion, supporting the results of Bailey and Marty, and shows that the degree of seigniorage revenue aversion is stronger in the 2000s than in the 1990s. This paper is organized as follows. In Section 2, we propose a seigniorage model. In Section 3, we propose a statistical methodology of estimating and testing this model. In Section 4, we present a numerical example to evaluate whether the social welfare function supports seigniorage revenue aversion (Bailey and Marty's result) and the magnitude of the degree of aversion. Section 5 concludes the paper.

2. The Model

2.1. Market equilibrium

We briefly present the seigniorage model of Blanchard and Fischer (1989, p. 188–194), which is composed of two equations dealing with the relationship between money and price, and transform their model to a discrete type from a continuous one for the sake of empirical analysis:

$$m_t \equiv \frac{M_t}{P_t} = A\exp(-aR_t)\bar{y},\ A > 0,\ a > 0,\ R_t \geq 0, \tag{1}$$

$$\pi_{t+1}^e - \pi_t^e = b(\pi_t - \pi_t^e),\ b > 0,\ ab \neq 1, \tag{2}$$

where $R_t = \bar{r} + \pi^e_{t+1}$ ($\bar{r}$ is the real interest rate and $\pi_{t+1}{}^e \equiv (P^e{}_{t+1} - P_t)/P_t$ is the expected inflation rate formed at period t) is the nominal interest rate, $\bar{y}$ is real output, M_t is the money supply, P_t is the price level and A, a, b are positive constant terms. Furthermore, $\pi_t \equiv (P_t - P_{t-1})/P_{t-1}$ is the actual inflation rate observed at period t. The real variables $\bar{r}$, $\bar{y}$ are assumed to be constant over all periods. Equation (2) shows the expected inflation $\pi_{t+1}{}^e$ (price $P^e{}_{t+1}$) formed at period t, given the actual inflation π_t at period t and expected inflation $\pi^e{}_t$ at period t–1. Equation (1) shows that the actual inflation π_t (price P_t) clears the money market. The government controls only the constant money growth rate θ over time, $\theta \equiv (M_t - M_{t-1}) / M_{t-1}$.[1]

How is the inflation rate π_t decided by the growth rate θ ? Take logarithms of (1) and rearrange, considering period t and t–1 in equation (1) to obtain:

$$\theta - \pi_t = -a(\pi^e_{t+1} - \pi^e_t) \ . \tag{3}$$

Moreover, we insert equation (2) into (3):

$$\theta - \pi_t = -ab(\pi_t - \pi^e_t) . \tag{4}$$

Thus, we can get the actual inflation rate π_t (i.e., P_t) at period t, given the expected inflation rate π_t^e at period t–1 and the money growth θ (i.e., M_t). However, given π_{t-1} and $\pi_{t-1}{}^e$, the expectation π_t^e is formed by (2). What are the dynamics of the expectation π_t^e? Furthermore, what are the dynamics of the actual inflation rate π_t? The detailed derivation and the stability condition are given in Appendix 1. Here, we focus on the steady state. The steady state of $\pi_t^e = \pi_{t+1}{}^e$ lead to $\pi_t^e = \pi_t$ because of (2), which leads to $\pi_t = \theta$ because of (4) and then $\pi_t^e = \theta$. The following is true at this steady state: $\pi_{t+1}{}^e = \theta$ and then $R_t = \bar{r} + \pi^e_{t+1} = \bar{r} + \theta$.

2.2. On seigniorage revenue and consumer revenue

The seigniorage at t, i.e., the revenue raised from the creation of money, is defined as:

$$S_t \equiv \frac{M_{t+1} - M_t}{P_t} = \frac{M_{t+1} - M_t}{M_t} \bullet \frac{M_t}{P_t} = \theta m_t \, . \tag{5}$$

The seigniorage revenue S at the steady state (where $\theta = \pi_t^e = \pi_t$, and money demand (1) equals supply) is, considering (1):

$$S = \theta A \bar{y} \exp(-a(\bar{r} + \theta)) \equiv g(\theta) , \tag{6}$$

$$dS / d\theta = A\bar{y}(1 - a\theta)\exp(-a(\theta + \bar{r})) \, .$$

[1] The validity of these assumptions was explained by the model of Sidrauski (1967) and further developed by Blanchard and Fischer (1989, p. 188–198).

Here, the subscript t denoting the period is deleted as we consider the steady state hereafter. The seigniorage revenue *S* is illustrated as the shaded part in Figure 1. In the seigniorage model of Blanchard and Fischer (1989), the maximized seigniorage revenue *S** of (6) at the steady state is obtained at:

$$\theta^* = 1/a.: S^* = S(\theta^*). \tag{7}$$

We assume that a consumer decides money demand m to maximize the consumer profit *V*, given *R*:

$$V \equiv U(m) - mR, \tag{8}$$

where $U(m)$ is a consumer's utility measured in money *m*, the so-called consumer revenue, and *mR* is the cost of interest payments, the so-called consumer cost. The consumer's maximum profit *V* is obtained at m^d where $U'(m^d) = R$. Its maximum value is:

$$\int_0^{m^d} U'(x)dx - m^d R. \tag{9}$$

On the other hand, the bank profit is assumed to be the interest payment Rm^d. Then, the private sector profit is the total profit of the consumer and the bank, which is always equal to the consumer revenue as long as the consumer behaves so that:

$$\int_0^{m^d} U'(x)dx - m^d R + m^d R = \int_0^{m^d} U'(x)dx. \tag{10}$$

Using the necessary condition for the maximum, $U'(m^d) = R$, we can rewrite (10) as $\int_0^{m^d} U'(x)dx = \int_0^{m^d} R(x)dx$. Moreover, the consumer revenue $U(m^d)$ is expressed as $U(m)$, considering that m^d equals the money supply m^S, which is exogenously controlled by the government, through the adjustment of *R* because of the money market equilibrium condition (1): $m^d = m^S = m$. Thus, given the exogenously determined money supply ($m^S = m$) controlled by the government, the maximized private sector profit (which equals consumer revenue) is as follows:

$$U(m) \equiv \int_0^m \left[\frac{\ln(A\bar{y})}{a} - \frac{1}{a}\ln(x) \right] dx = \frac{m(\ln(A\bar{y}) + 1 - \ln(m))}{a}$$

$$= \frac{A\bar{y}(a(\theta+\bar{r})+1)\exp(-a(\theta+\bar{r}))}{a} \equiv f(\theta), \tag{11}$$

$$dU/d\theta = A\bar{y}(-a(\theta+\bar{r}))\exp(-a(\theta+\bar{r}))$$

where $R = \frac{1}{a}\ln(A\bar{y}) - \frac{1}{a}\ln(m)$ is resolved in terms of *R* after taking the logarithm of (1).

Next, we consider how the government maximizes the private sector profit by adjusting the exogenously given money supply *m*. By using (11), the maximization of total private profit is obtained at:

$$\theta^{**} = -\bar{r}\text{, that is, } R^{**} = \theta^{**} + \bar{r} = 0 : U^{**} = U(\theta^{**})\,. \tag{12}$$

[INSERT Figure 1]

2.3. Government behavior

The government chooses the money growth rate θ to maximize the following social welfare function *W*, which is expressed in terms of the exogenously given money supply *m* (θ) controlled by the government:

$$\begin{aligned} &\underset{\theta}{Max}\, W = (S+\bar{S})^{\alpha} U^{1-\alpha} : \\ &S = g(\theta),\, U = f(\theta),\, S+\bar{S} > 0, U > 0,\, 0 \le \alpha \le 1,\; \bar{S} > 0\text{'} \end{aligned} \tag{13}$$

where the social welfare function consists of seigniorage *S* and consumer revenues (private sector profit) *U*, considering the viewpoint of Bailey (1956) and Marty (1976). The seigniorage revenue *S* may be negative if the growth rate θ is negative, as is obvious in (6). The minimum of *S* is $-\bar{S}$ at $\theta = -\bar{r}$, *i.e.*, $R = \theta + \bar{r} = 0$. Then, $\bar{S} = (-1)(-\bar{r})A\bar{y}\exp(-a(-\bar{r}+\bar{r})) = \bar{r}A\bar{y}$. This social welfare function *W* has the following features, as shown in Figure 2. In particular, when the function displays seigniorage revenue loving, that is $\alpha = 1$, it is flat against the *U* axis and social welfare increases only through an increase in seigniorage revenue. However, when the function displays seigniorage revenue neutrality, that is $\alpha = 0$, it is flat against the *S* axis and social welfare increases only through increases in consumer revenue. Finally, when the function displays seigniorage revenue aversion, that is $0 < \alpha < 1$, it is convex against the origin. The decrease in α produces an increase in seigniorage revenue aversion.

[INSERT Figure 2]

We rewrite the maximizing problem (13) as follows, by deleting the money growth rate θ:

$$\underset{S,\,U}{Max}\, W = (S+\bar{S})^{\alpha} U^{1-\alpha} \textit{ subject to } S = g(f^{-1}(U))\,. \tag{14}$$

$S = g(f^{-1}(U))$ shows a locus of realizable values of *S* and *U* for any θ. We choose *S* and *U* through θ. However, in (14), we seem to be choosing *S* and *U* directly. The shape of its locus is as follows. The maximized *S* is obtained at $U = U(\theta^*)$ where $\theta^* = 1/a$ as seen in (7), while the maximized U is obtained at $S = S(\theta^{**}) = \overline{S}$ where $\theta^{**} = -\overline{r}$ as seen in (12), as shown in Figure 3. When θ increases towards $\theta^* = 1/a$ beyond $\theta^{**} = -\overline{r}$, U decreases from $U^{**} = U(\theta^{**})$ because of (12) and S increases towards $S(\theta^*)$ because of (7). When θ increases beyond $\theta^* = 1/a$, U decreases because of (12) and S decreases from $S(\theta^*)$ because of (7), as shown in Figure 3. Then, U is not a one-to-one correspondence in S, but U in the region of $[\theta^{**}, \theta^*]$ is a one-to-one correspondence in S. Thus, $S = g(f^{-1}(U))$ can be shown in Figure 3.

The derivatives of $S = g(f^{-1}(U))$ in terms of U are, by using (6) and (11):

$$\frac{dS}{dU} = \frac{dS}{d\theta} \bullet \frac{1}{\frac{dU}{d\theta}} = \frac{A\overline{y}(1-a\theta)\exp(-a(\theta+\overline{r}))}{A\overline{y}(-a(\theta+\overline{r}))\exp(-a(\theta+\overline{r}))} = \frac{1-a\theta}{-a(\theta+\overline{r})}, \tag{15}$$

$$\frac{dU}{dS} = \frac{dU}{d\theta} \bullet \frac{1}{\frac{dS}{d\theta}} = \frac{-a(\theta+\overline{r})}{1-a\theta}, \tag{16}$$

where $R = \overline{r} + \theta > 0$ and then $\theta > -\overline{r}$: the derivative is undefined for $\theta = -\overline{r}$. What is the shape of $S = g(f^{-1}(U))$ in terms of U? Is it strictly concave?

$$d\left(\frac{dS}{dU}\right)/dU = \frac{d\left(\frac{dS}{dU}\right)}{d\theta}\frac{1}{\frac{dU}{d\theta}} = \frac{a^2\overline{r}+a}{a^2(\theta+\overline{r})^2}\frac{1}{A\overline{y}(-a(\theta+\overline{r}))\exp(-a(\theta+\overline{r}))} < 0 \tag{17}$$

Then, $S = g(f^{-1}(U))$ is strictly concave in U.

Among realizable couples of (S, U) on $S = g(f^{-1}(U))$ for a given θ, we must choose the particular couple of (S, U) to maximize the social welfare function, $W = (S+\overline{S})^{\alpha}U^{1-\alpha}$. When this function displays seigniorage revenue loving ($\alpha = 1$), the maximized social welfare W is uniquely obtained at point A $(S(\theta^*), U(\theta^*))$ because of the strict concavity of $S = g(f^{-1}(U))$ and the convexity of social welfare function $\overline{W}$, where $\theta^* = 1/a$, as shown in Figure 3. However, when the function displays seigniorage revenue neutrality ($\alpha = 0$), the maximized social welfare W is obtained at point B $(S(\theta^{**}), U(\theta^{**}))$ uniquely, where $\theta^{**} = -\overline{r}$: *i.e.*, $R = \theta^{**} + \overline{r} = 0$. When the function displays seigniorage revenue aversion ($0 < \alpha < 1$), the maximized social welfare W is obtained uniquely at some point C $(S(\theta^{***}), U(\theta^{***}))$ between A and B.

Proposition 1: *The optimal θ^{***} is expressed as follows when $0 < \alpha < 1$.*

$$\alpha = \frac{a^2\theta^{***}(\theta^{***}+\overline{r})}{1+a\overline{r}} \tag{18}$$

The proof is given in Appendix 2. Equation (18) can be interpreted as a solution for the optimal money growth θ^{***}, given the other parameters. Alternatively, the unknown preference parameter α is expressed by the observed (optimal) money growth θ, observed real interest rate $\bar{r}$ and the estimated money demand parameter a. We have already shown that the decrease in α (which means that the indifference curve of a social welfare function in Figure 2 approaches the vertical line) is an increase in seigniorage revenue aversion.

The actual seigniorage revenue and consumer revenue data, dependent on the money growth rate θ, are observed in Figure 4. We investigate whether these data are observed around point A, point B or point C. That is, whether the government has a social welfare function W displaying seigniorage revenue loving, neutrality or aversion, or, how is the seigniorage model?

[INSERT Figure 3 and Figure 4]

3. Statistical Methodology

We show a testing and estimation method to evaluate whether the social welfare function displays seigniorage revenue aversion, as suggested by Bailey and Marty, and estimate the degree of aversion. To do this, we proceed as follows. First, we estimate the money demand function (1). By using the estimated parameter a, we calculate $\theta^* = 1/a$, seigniorage revenue and consumer revenue, i.e., point A ($S(\theta^*)$, $U(\theta^*)$). However, we calculate point B ($S(\theta^{**})$, $U(\theta^{**})$) where $\theta^{**} = -\bar{r}$, by using $R = \theta^{**} + \bar{r} = 0$. Second, we test whether the observed data have originated from point A, B or C. However, because the observed data consists of the growth rate of money θ and nominal interest rate R, we test whether the observed money growth rate θ was obtained from $\theta^* = 1/a$ and whether the observed nominal interest rate R was obtained from $R = 0$. If the data originated from C, this supports the results of Bailey and Marty that the government considers both revenues. Moreover, by estimating the preference parameter α in (18) based on the observed data, we measure and compare the degree of aversion on seigniorage revenue in two different periods.

We estimate the parameter a for money demand function (1) in the logarithm form:

$$\ln(m_t) = C - aR_t, \quad C \equiv \ln(A\bar{y}). \tag{19}$$

This money demand function is a function at the steady state for not only real, but also monetary variables. We then consider the function to be a cointegration term in a two-dimensional VAR model with a nominal interest rate R and a real money supply m. We have to estimate a in (19) as a cointegrating term in the following equation (21). The following statistical methodology is used. Let $X_t = (\ln(m_t), R_t)'$ and assume that this vector is generated from a vector autoregression VAR(k) model with a constant term ψ and Gaussian errors ε_t:

$$X_t = \Pi_1 X_{t-1} + \Pi_1 X_{t-2} + \ldots + \Pi_k X_{t-k} + \psi + \varepsilon_t . \quad (20)$$

We write the model in error correction form as:

$$\begin{aligned} &\Delta X_t = \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \psi + \varepsilon_t : \\ &\Pi \equiv \mathrm{I} - \Pi_1 - \Pi_2 - \ldots - \Pi_k \end{aligned} . \quad (21)$$

The parameters ($\Gamma_1, \ldots, \Gamma_{k-1}$) define the short-run adjustment to changes in the process, whereas $\Pi = \alpha\beta'$ defines the short-run adjustment (α) and the long-run relations (β).Johansen and Juselius(1990,1992) shows that if $X_t \sim I(1)$, Π ($p \times p$ matrix) has the reduced rank of r (< p) and can be represented as $\Pi = \alpha\beta'$. The parameterization in $\Pi = \alpha\beta'$ facilitates the investigation of the r linearly independent stationary relations between the levels of the variables, and the p–r linearly independent nonstationary relations. Thus, the representation of $\Pi = \alpha\beta'$ implies that the process ΔX_t is stationary, X_{t-1} is nonstationary, but also that $\beta' X_{t-1}$ is stationary. Thus, we can interpret the relation $\beta' X_{t-1}$ as the stationary relations among nonstationary variables, i.e., as cointegrating relations. Johansen and Juselius (1990, 1992) developed the likelihood procedure for estimating the parameters, and testing the order of cointegration rank and the various hypotheses on the restrictions of parameters.

4. A Numerical Example

4.1. Data

The monthly data for this study are taken from International Monetary Fund's International Financial Statistics CD-ROM July 2006, and covers the period 1990:1–2005:12. The prices *P* are line 63 (consumer price, 2000 = 100) or line 64 (wholesale price, 2000 = 100), and the nominal interest rate *R* is line 60P (lending rate %). An exception is the money supply, which is high-powered money compiled from the Bank of Japan (monetary base, seasonally adjusted, 100 million yen, average outstanding): HmoneyC (money/consumer price) and HmoneyW (money/wholesale price) are in logarithm form. The choice of variables followed previous papers including Phylaktis and Taylor (1993), Loviscek (1996) and Turner and Benavides (2001). Figure 5 shows real output *y*, line 66..czf (industrial production seasonally adjusted 2000 = 100), which shows that output seems to be fixed or stationary as assumed in Section 2. Finally, we also use deposit money (line 59MBFZF (M2+CDs, seasonally adjusted, 100 million yen)): DmoneyC (money/consumer price) and DmoneyW (money/wholesale price) are in logarithm form. All data are shown in Figure 5.

We now discuss the difference between the two kinds of money supply, deposit money and high-powered money. The former is related to consumer revenue *U*, the latter to seigniorage revenue *S*. When the money multiplier between deposit money and high-powered money is constant, the growth rate of both measures is equal to each other. Then, (14) can be theoretically expressed by the same growth rate θ. However, as shown in Figure 5, the growth rate of both measures is actually different. Moreover, the money demand function of consumers is for deposit money. Therefore, we use deposit money and high-powered money as the money supply, as done by previous studies including Phylaktis and Taylor (1993), Honohan (1996), Loviscek (1996), Turner and Benavides (2001) and Tekin-Koru and Özmen (2003).

[INSERT Figure 5]

4.2. Estimation for the money demand function

We have to estimate the money demand function in (19) as a cointegrating term in (21). From the estimated parameter a in all pairs of cointegration relations, we seek an optimal money growth $\theta^* = 1/a$. The first step is to implement the augmented Dickey–Fuller (ADF) unit root test (using an autoregression (AR)) for R and the money supply, and the Johansen–Juselius cointegration test (using the error correction model (ECM) of (21)). A lag length must be chosen for the AR and the ECM following Phillips (1987) and Gonzalo (1994).[2] For the AR, the optimal lag lengths together with the t-statistics of the ADF tests are reported in Table 1 and all are I(1).[3] For the ECM of (21) in Table 2, we found that the two-order to six-order lags satisfied their criteria. The resulting cointegration rank between R and money is one at the 1% significance level, according to the λ-max test and the trace test in Table 3. We conclude that all pairs of time series variables have one cointegrating vector. We have estimated a money demand function in the cointegrating term, which is shown in Table 4. The optimal money growth rate θ^* to maximize the seigniorage revenue S is $1/a$ = 3.08% for HmoneyW, 2.26% for HmoneyC, 11.09% for DmoneyW and 9.17% for DmoneyC. Thus, we have estimated two types of money demand function and optimal money growth, based on consumer and wholesale prices. The difference between deposit and high-powered money causes the difference between optimal money growth for seigniorage revenue. The computations have been performed with the computer package CATS in RATS by Hansen and Juselius (1995).

[Insert Table 1, Table 2, Table 3 and Table 4]

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[2] Our procedure for choosing the optimal lag length was to test between a two- and an 18-order lag for the AR and between a two- and a six-order lag for the ECM, by using the minimum value of Schwarz's Bayesian Information Criterion (SBIC). The residuals from the chosen AR or ECM were then checked for whiteness. Phillips (1987) and Gonzalo (1994) suggest the robustness of the Johansen–Juselius procedure to heterogeneity and nonnormality. Here, whiteness is checked only by Ljung–Box (LB) Q tests for absence of correlation for all 18 lags (for AR) and six lags (ECM) at the 5% significance level. If the residuals in any equation proved to be nonwhite, we sequentially chose a higher lag structure until they were whitened.

[3] For some data based on visual inspection, we implement Perron's (1989) unit root test with a break at 1995:7 for the lending rate R, and at 2003:12 for high-powered money m. The former is I(0), but the latter is I(1). Not all variables are necessarily I(0). Then, we assume all data are nonstationary even with a break.

4.3. Testing whether the observed data have occurred around point A, B or C

If the observed money growth rate θ, which is assumed to follow the normal distribution, occurs around the optimal rates $\theta^* = 1/a$, the observed data have occurred at point A and then the social welfare function displays seigniorage revenue loving. We test this hypothesis as follows. If the null hypothesis is H_0: $\theta = \theta^*$ (in Table 5), the test statistics follows the t-distribution: $t = (\theta^A - \theta^*)/\hat{\sigma} \sim t(n-1)$, where n is the number of observations in the sample, θ^A is the sample mean and $\hat{\sigma}$ is the standard deviation of the sample. As shown in Figure 5, the seven high-powered money growth rate data overshoot 4% in magnitude. We omit these abnormal data prior to testing. The null hypothesis is rejected for each money demand function at the 5 or 10% significance level. When we estimate the money demand function by using HmoneyC, θ^*=2.26% and we have to test this null hypothesis by using the average of the nominal high-powered money growth rate θ^A=0.46, as shown in Table 5.

Similar logic is applied to whether the observed data occurs at point B. The null hypothesis is H_0: R = 0 (in Table 6). This hypothesis is rejected at the 10% significance level. As a result, the social welfare function displays neither seigniorage revenue loving by suggesting point A nor seigniorage revenue neutrality by point B. That is, the social welfare function displays seigniorage revenue aversion by point C. This result shows that the Japanese government has a social welfare function W with a view of seigniorage revenue and consumer revenue that supports Mankiw (1987), where the government chooses the rates of tax and money growth to minimize the social cost of raising revenue from tax and seigniorage.

[Insert Table 5 and Table 6]

4.4. Degree of seigniorage revenue aversion

We find that the social welfare function displays seigniorage revenue aversion, supporting Bailey and Marty. The next logical step is to investigate the degree to which the money growth rate policy with seigniorage revenue aversion changed during the period 1990–2005 using the preference parameter α in the social welfare function. Considering (18), the parameter α is computed by using the average level of the observed (optimal) money growth θ, the average level of observed real interest rate $\bar{r}$, and the estimated money demand parameter a. Here, the real interest rate $\bar{r}$ can be computed as $\bar{r} = R^A - \theta^A$, where R^A is the average of the observed nominal interest rate, θ^A is the average of the observed (optimal) money growth rate and the preference parameter α is given in Table 4. Thus, as shown in Table 7, by using HmoneyW, the preference parameter α = 0.083 in the 1990s (1990:1–1999:12) decreases to 0.076 in the 2000s (2000:1–2005:12) by 0.007: that is, a 10% decrease. By using the other money measures, we get the similar decrease in the preference parameter α. This shows that the money growth rate policy in the 2000s has stronger seigniorage revenue aversion compared with the 1990s, as shown in Figure 6.

[Insert Table 7 and Figure 6]

5. Concluding Remarks

The purpose of this paper was to develop a seigniorage model that includes the contributions of Bailey and Marty, using a different framework to Mankiw (1987), to test whether their result is supported, and provide a numerical example to estimate the seigniorage model. The government decides the money growth rate to maximize the social welfare functions that display seigniorage revenue aversion, loving and neutrality. The numerical example using Japanese data showed the existence of seigniorage revenue aversion, supporting Bailey and Marty, and showed that the degree of seigniorage revenue aversion was stronger in the 2000s than the 1990s. However, this paper has only provided a theoretical model and an empirical application of the model. Further empirical studies are needed.

Appendix 1

Derivation of difference equation and stability condition: Inserting (4) into (2), we obtain the difference equation:

$$\pi_{t+1}^{e} - (1+\mu)\pi_t^e = -\mu\theta \quad ; \quad \mu \equiv \frac{b}{ab-1}. \tag{A1}$$

Because 1 + $\mu \neq 1$ because of (1) and (2), the solution is:

$$\pi_t^e = c(1+\mu)^t + \frac{1-(1+\mu)^t}{1-(1+\mu)}(-\mu\theta). \tag{A2}$$

When $|1 + \mu| < 1$, that is, $-2 < \mu < 0$, the solution is stable. First, we analyze the interval of the parameters of a and b for $\mu < 0$. Then, $ab < 1$. Second, for $-2 < \mu$, we find $(2 - b)/2b > a$. We require a close interval of both:

$$\frac{1}{b} - \frac{1}{2} > a. \tag{A3}$$

This is the stability condition. However, we do not check empirically whether this condition holds for observed data. Our model is a difference equation and the stability condition is different from Blanchard and Fischer (1989, p. 188–198): $ab < 1$. The steady state is $\lim_{t\to 0} \pi_t^e = \theta$. Furthermore, the dynamics of π_t are, considering (4):

$$\pi_t = \frac{ab}{ab-1}\pi_t^e - \frac{\theta}{ab-1}. \tag{A4}$$

Because the dynamics of π_t do not include t except for $\pi^e{}_t$, the stability condition is the same as (A3). In addition, the steady state of π_t is obviously θ, considering (A4). ■

Appendix 2

Proof of Proposition 1: Taking the logarithm of W in (13) and considering (6) and (11):

$$\begin{aligned}\ln W &= \alpha \ln(S + \overline{S}) + \beta \ln U \\ &= \alpha \ln\left(\theta A\overline{y}\exp(-x) + \overline{r}A\overline{y}\right) + (1-\alpha)\ln\left(\frac{A\overline{y}(x+1)\exp(-x)}{a}\right) \\ &= A\overline{y} + \alpha\ln[(\frac{x}{a} - \overline{r})\exp(-x) + \overline{r}] + (1-\alpha)(-\ln a) + (1-\alpha)(\ln(x+1) - x),\end{aligned} \tag{A5}$$

$$\textit{where } x \equiv a(\theta + \overline{r}),\ \theta = (\frac{x}{a} - \overline{r}) \textit{ and } \overline{S} = \overline{r}A\overline{y}$$

We maximize ln*W* in terms of *x* (i.e., maximize *W* in terms of *θ*). We take derivatives in terms of x:

$$\frac{d\ln W}{dx}=\alpha\frac{\exp(-x)(\frac{1-x+a\bar{r}}{a})}{(\frac{x}{a}-\bar{r})\exp(-x)+\bar{r}}+(1-\alpha)(\frac{1}{x+1}-1)=0, \tag{A6}$$

$$\frac{d^2\ln W}{dx^2}<0\,. \tag{A7}$$

Then, when *W* displays seigniorage revenue loving ($\alpha = 1$), the optimal money growth rate θ is, considering (A2), $1-x+a\bar{r}=0$, that is, $\theta = 1/a$. Also, when *W* displays seigniorage revenue neutrality ($\alpha = 0$), $\frac{1}{x+1}=1$, *i.e.*, $\theta=-\bar{r}$. When $0 < \alpha < 1$, what is the optimal money growth rate θ? In (A6) , we assume $\exp(-x)\approx 1$. Because nominal interest rate R($=\theta+\bar{r}$) and a are empirically zero point something shown in Table 4 and then x= $a(\theta+\bar{r})$ is very small. Therefore, (A6) can be rewritten as:

$$\frac{d\ln W}{dx}=\alpha\frac{(\frac{1-x+a\bar{r}}{a})}{(\frac{x}{a}-\bar{r})+\bar{r}}+(1-\alpha)(\frac{1}{x+1}-1)=0\,. \tag{A8}$$

$$x^2-xa\bar{r}\alpha-\alpha(1+a\bar{r})=0$$

We can solve for *x* from (4), considering $x > 0$:

$$x=\frac{1}{2}[a\bar{r}\pm\sqrt{a^2\bar{r}^2+4\alpha(1+a\bar{r})}\,]. \tag{A9}$$

$$\begin{aligned}&Due\,to\;\; x=a(\theta+\bar{r})>0,\\ &x=\frac{1}{2}[a\bar{r}+\sqrt{a^2\bar{r}^2+4\alpha(1+a\bar{r})}\,].\end{aligned} \tag{A10}$$

Because of (A10), $x>a\bar{r}$ *and then* $\theta>0$ *considering* $x\equiv a(\theta+\bar{r})>0$ *with* $a>0$. Then, the relationship between the optimal growth rate *θ*, the preference parameter *α*, the real interest rate $\bar{r}$ and the parameter of money demand *a* is as follows:

$$\begin{aligned}&a(\theta+\bar{r})=\frac{1}{2}[a\bar{r}+\sqrt{a^2\bar{r}^2+4\alpha(1+a\bar{r})}\,],\;\; a^2(2\theta+\bar{r})^2=a^2\bar{r}^2+4\alpha(1+a\bar{r}),\\ &\alpha=\frac{a^2\theta(\theta+\bar{r})}{1+a\bar{r}}\end{aligned} \tag{A11}$$

where (A11) can be interpreted as a solution for the optimal money growth θ, given other parameters. Alternatively, it is interpreted that the unknown preference parameter α except for the government is expressed by the observed (optimal) money growth θ, the observed real interest rate $\bar{r}$ and the estimated money demand parameter a. ■

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Table 1. Augmented Dickey–Fuller (ADF) test for a unit root in the data

	Level τ_μ	First difference τ_μ	Level τ_τ	First difference τ_τ
R	–2.33 (4)	–3.14 (5)	–1.47 (4)	–4.93 (3)
HmoneyW	0.77 (3)	–6.01 (2)	–2.61 (3)	–6.19 (2)
HmoneyC	1.58 (1)	–6.29 (2)	–2.22 (1)	–6.72 (2)
DmoneyW	–0.84 (3)	–8.07 (2)	–0.29 (3)	–8.07 (2)
DmoneyC	1.50 (6)	–5.28 (5)	–3.25 (6)	–5.78 (5)

Notes: τ_μ and τ_τ are the test statistics allowing for constant mean and trend in mean, respectively. The reported numbers in the columns are the ADF statistics. Numbers in parenthesis after these statistics indicate the lag length used. The critical value for sample size 250 at the 0.05 significance level is –2.88 for τ_μ and –3.43 for τ_τ.

Table 2. Determination of lag lengths (multivariate LB test and SBIC test)

	LB	d.f.	p-value	Resulting lag
R-HmoneyW	149.75	180	(0.95)	2
R-HmoneyC	138.10	180	(0.99)	2
R-DmoneyW	185.43	172	(0.23)	4
R-DmoneyC	196.53	160	(0.03)	6

Notes: R-HmoneyW is an error correction model consisting of interest rate R and HmoneyW. The other labeling is similar to this. The multivariate LB test statistic under the null hypothesis of the uncorrelatedness of residuals has an asymptotic χ^2 distribution: see Hansen and Juselius (1995, p. 73). The values in parentheses denote p-values.

Table 3. Tests of cointegration ranks

Rank	λ-Max		Trace		Resulting ranks
Null hypothesis: H_0	$r = 0$	$r = 1$	$r = 0$	$r = 1$	
Alternative: H_1	$r = 1$	$r = 2$	$r = 1$	$r = 2$	
R-HmoneyW	32.45	5.52	37.97	5.52	1
R-HmoneyC	27.71	6.55	34.26	6.55	1
R-DmoneyW	29.50	7.79	37.29	7.79	1
R-DmoneyC	43.55	7.53	51.08	7.53	1
Critical values, 5%	15.67	9.24	19.96	9.24	

Note: See the notes of Table 2. The r denotes the number of cointegrating vectors. The 5% critical values of the maximum eigenvalue (λ-max) and the trace statistics are taken from Osterwald-Lenum (1992, pp. 468).

Table 4. Estimates of money demand function

	$\ln(m) = C - aRt$ (19)	An optimal money growth % ($\theta^* = 1/a$) for seigniorage revenue
HmoneyW	$\ln(m) = 10.784 - 0.325Rt$	3.08%
HmoneyC	$\ln(m) = 11.395 - 0.442Rt$	2.26%
DmoneyW	$\ln(m) = 11.463 - 0.084Rt$	11.90%
DmoneyC	$\ln(m) = 11.615 - 0.109Rt$	9.17%

Table 5. Test for H_0: $\theta = \theta^*$: optimal money growth for seigniorage revenue

	H_0: $\theta = \theta^*$	Av. rates θ^A	Std. of θ: σ_θ	t-ratio: t = $(\theta^A - \theta^*)/\sigma_\theta$
HmoneyW	3.08%	0.46	0.94	–2.79**
HmoneyC	2.26%	0.46	0.94	–1.91*
DmoneyW	11.90%	0.23	0.52	–22.44**
HmoneyC	9.17%	0.23	0.52	–17.19**

Notes: ** and * indicate significance at the 5% and 10% level, respectively. Av. θ^A and Std. σ_θ are the mean and the standard deviation, respectively.

Table 6. Test for H_0: $R = R^*$: optimal money growth for consumer revenue

	H_0: $R = R^*$	Av. rates R^A	Std. of R: σ_R	t-ratio: t = $(R^A - R^*)/\sigma_R$
R	0%	3.37	1.92	1.76*

Notes: See notes of Table 5. The zero-truncated normal distribution for R is assumed.

Table 7. Seigniorage revenue aversion α: 1990s and 2000s

		a	θ^A	$\bar{r} = R^A - \theta^A$	α in (18)
HmoneyW	1990s	0.325	0.414	3.882 = 4.296 – 0.414	0.083
	2000s		0.554	1.301 = 1.855 – 0.554	0.076
HmoneyC	1990s	0.442	0.414	3.882 = 4.296 – 0.414	0.128
	2000s		0.554	1.301 = 1.855 – 0.554	0.127
DmoneyW	1990s	0.084	0.265	4.031 = 4.296 – 0.265	0.006
	2000s		0.181	1.674 = 1.855 – 0.181	0.002
DmoneyC	1990s	0.109	0.265	4.031 = 4.296 – 0.265	0.010
	2000s		0.181	1.674 = 1.855 – 0.181	0.003

Notes: See notes of Table 5. The 1990s and 2000s mean 1990:1–1999:12 and 2000:1–2005:12, respectively.

Figure 1

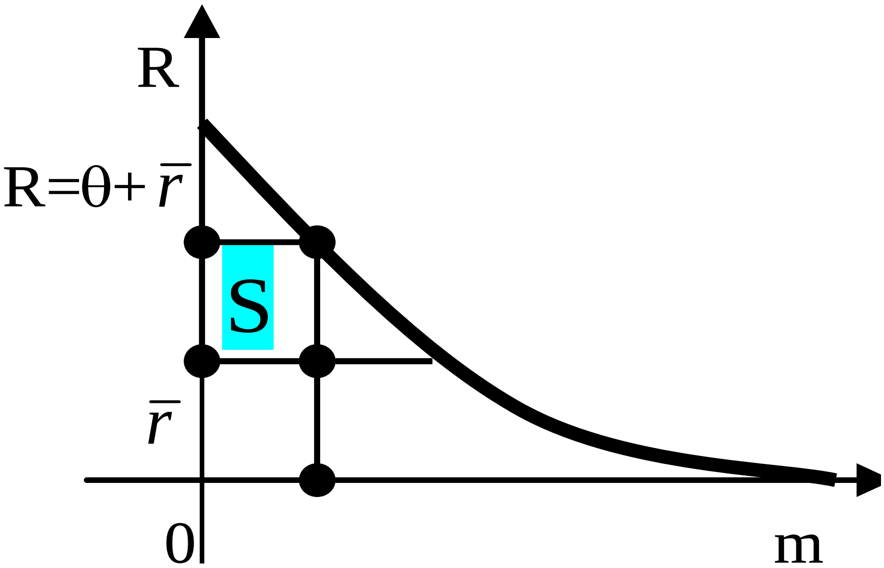


Figure 2

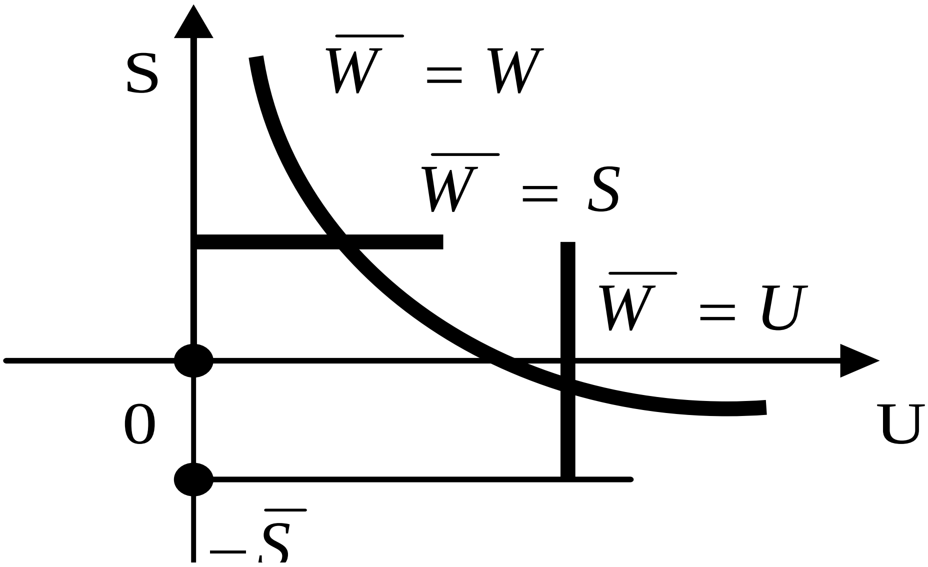


Figure 3

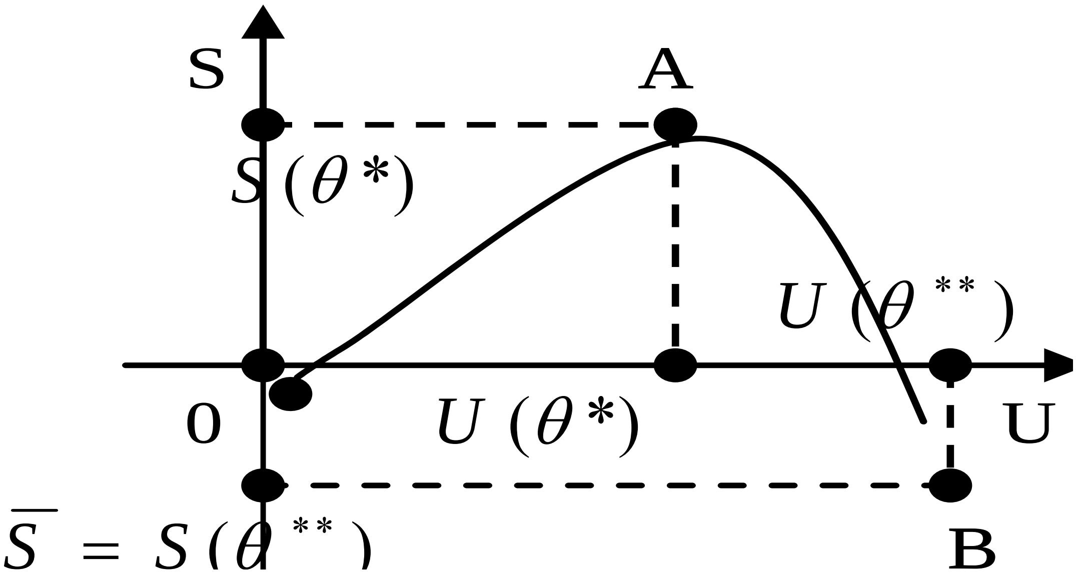


Figure 4

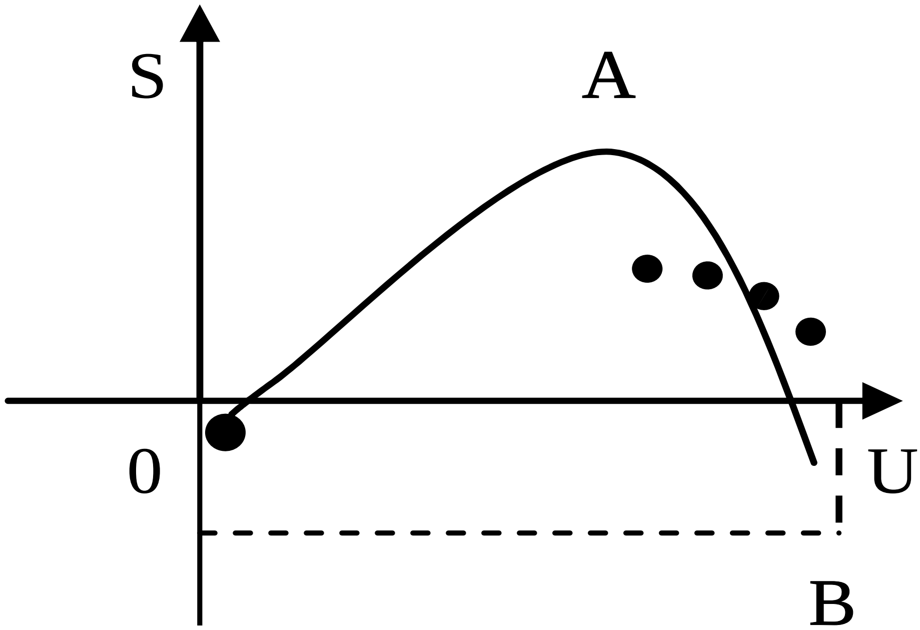
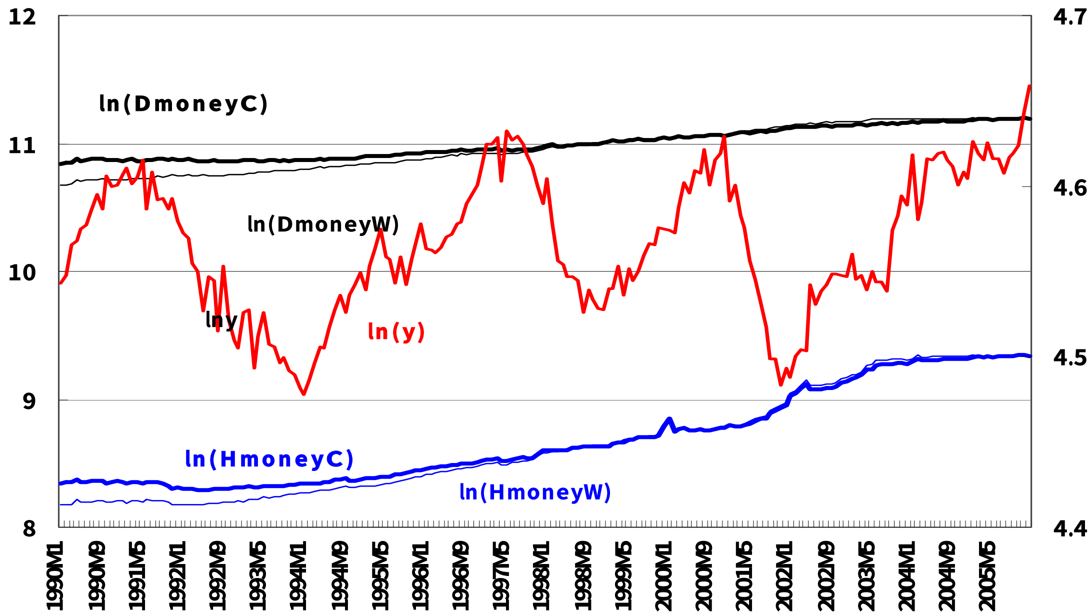


Figure 5. Data:

High-powered real money, real M2 + CD and production index in logarithms



Note: moneys are measured in 100 millions at right left axis and production in no unit at right axis.

Lending rate and growth rates of high-powered nominal money and nominal M2+CD (%)

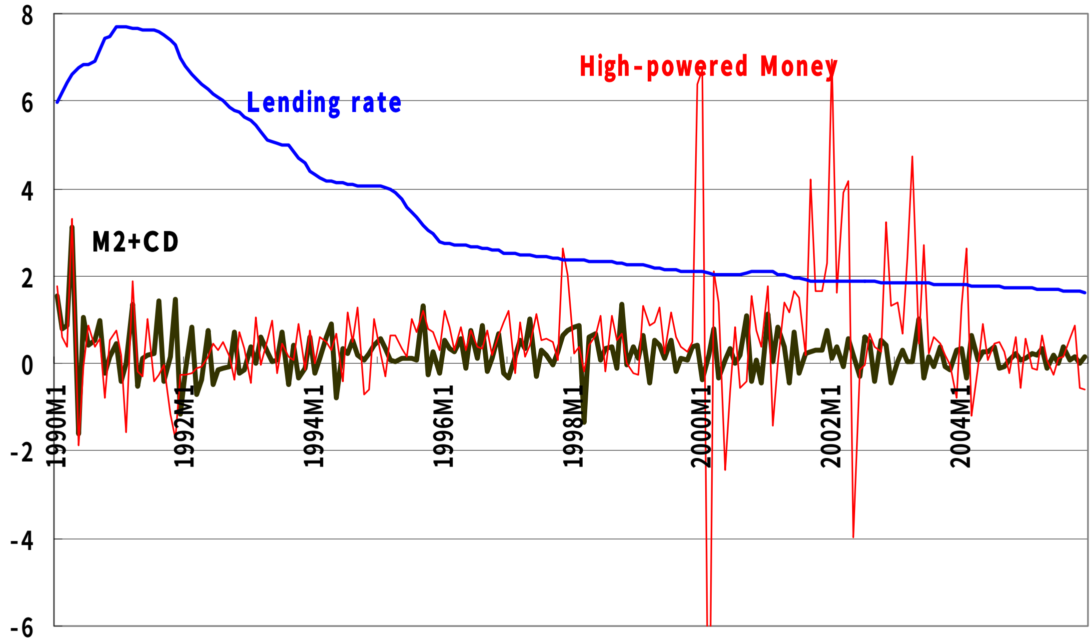


Figure 6