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Abstract

This paper studies stabilization effects of fiscal and monetary policy rules in the context of a standard real business cycle model with money. We assume that the fiscal authority adjusts the rate of income tax subject to the balanced-budget constraint, while the monetary authority controls the nominal interest rate by observing inflation. Inspecting macroeconomic stability of the steady state equilibrium of the model economy, we demonstrate that whether or not policy rules eliminate the possibility of sunspot-driven fluctuations critically depends upon the appropriate combination of progressiveness of taxation and activeness of interest-rate control.

JEL classification code E52, E62, E63

Keywords: balanced budget, interest rate control, determinacy of equilibrium

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1 Introduction

Income taxation under balanced-budget rule and interest-rate control have been considered most effective tools for establishing macroeconomic stability. If it is appropriately selected, each policy rule may stabilize the economy by mitigating income fluctuations. It is, however, rather unclear whether or not those fiscal and monetary policy rules strengthen their stabilizing effects each other, if the fiscal authority and the central bank adopt specific actions simultaneously. Although stabilization effects of policy rules have been discussed extensively, the main stream literature has investigated the stabilization roles of income taxation and interest control rules separately. Therefore, these studies fail to answer the relevant question mentioned above.

The purpose of this paper is to explore the interactions between income taxation and interest rate control rules under the balanced-budget discipline in a prototype model of real business cycle theory. Unlike most of the foregoing studies, this paper treats both fiscal and monetary policy rules in a single model. More specifically, we introduce money into the baseline real business cycle model with flexible price via a cash-in-advance constraint. We assume that the fiscal authority adjusts income tax endogenously in each moment subject to the balanced-budget rule. In the main part of the paper, we follow the taxation scheme assumed by Guo and Lansing (1998) in which the rate of income tax depends on the individual income relative to the average income in the economy at large. The key distinction in this policy rule is whether taxation on individual income relative to the average income is progressive or regressive. Given such a fiscal action, the monetary authority adopts an interest-control rule under which the nominal interest rate responds to the current rate of inflation relative to the target level of inflation. As usual, the effect of interest rate control on macroeconomic stability depends on the sensitivity of interest rate to a change in inflation. Introducing those fiscal and monetary actions into the baseline model, we examine the dynamic behavior of the model economy.

Our study presents three main findings. First, if progressive income taxation is combined with active interest rate control rule (i.e. nominal interest rate responds to inflation more than one for one), then the economy exhibits equilibrium determinacy, so that we will not observe expectations-driven fluctuations. Second, if the interest rate control is passive, equilibrium indeterminacy could emerge even under progressive income taxation. Third, if income taxation is regressive, then the interest rate control rule may play a pivotal role for establishing macroeconomic stability. In this case, indeterminacy may emerge under both active and passive interest rate control rules. However, if the interest rate is relatively insensitive to inflation, then equilibrium indeterminacy can be eliminated even in the presence of strong regressiveness of income taxation. Those findings claim that in the general equilibrium settings with money and capital, it is critically relevant to find appropriate combinations of fiscal and monetary policy rules. Even though the balanced budget rule and progressive income taxation may contribute to establishing aggregate stability, it is still important to select a suitable monetary policy rule to avoid depressing stabilization power of fiscal actions.

In the existing literature, Guo and Lansing (1998) show that progressive tax may eliminate the possibility of equilibrium indeterminacy even in the presence of strong degree of external increasing returns. Schmitt-Grohé and Uribe (1997) and Guo and Harrison (2004) examine the interrelationship between balanced-budget rule and determinacy of equilibrium. While Schmitt-Grohé and Uribe (1997) emphasize that the balanced-budget with a fixed government spending and endogenous taxation may generate sunspot-driven fluctuations, Guo and Harrison (2004) claim that such an unstable behavior can be eliminated if the balanced budget is maintained by adjusting government expenditure under fixed rates of income tax. These studies utilize the baseline real business cycle models without money. As for monetary policy rules, there has been a large body of literature that investigates stabilization effect of interest-rate control rule à la Taylor (1993). Although many authors (e.g. Benhabib et al. 2001) point out that the interest control rule may easily produce expectations driven fluctuations in the economies without capital, more recent studies show that the role of interest rate rules for aggregate stability is less relevant in an economy with capital formation: see, for example, Carlstrom and Fuerst (2005) and Meng and Yip (2004). These studies, however, ignore the role of fiscal policy. The present paper integrates these two lines of research on the stabilization roles of taxation and interest control.

It is to be noted that several authors have examined interactions between fiscal and monetary policy rules in the context of new Keynesian models with sticky price adjustment.

Following Leeper's (1991) modelling, Kurozumi (2005), Linnenmann (2006) and Lubik (2005) consider the effects of interest rate rule when the fiscal authority adjusts the rate of income tax to maintain a target level of the government debt. Those studies, therefore, do not assume the balanced-budget rule in its strict sense. Edge and Rudd (2007) explore how the presence of distortionary taxation on interest income affects the sensitivity of interest rate control to inflation and income necessary for avoiding equilibrium indeterminacy. Although Edge and Rudd (2007) utilize a sticky price model with fixed rates of income tax, the primary concern of their study is close to ours.

The next section constructs the base model. Section 3 presents our main discussion on the stability property of the model economy in the presence of interaction between fiscal and monetary policy rules. Section 4 briefly considers alternative formulations of policy rules. Section 5 concludes.

2 The Base Model

2.1 Households

There is a continuum of identical, infinitely lived households with a unit mass. The flow budget constraint for the household is

$$\dot{M} = (1-\tau)\, py + pT - pc - pv,$$

where M nominal stocks of money, p price level, y real income per capita, c consumption, v gross investment for capital, τ rate of factor income tax, and T is the real transfer from the government (or lump-sum tax if it has a negative value). Since we have normalized the number of household to unity, M, y, T. c and v represent their aggregate values as well. Real income consists of rent from capital and wage revenue:

$$y = rk + wl,$$

where r is real rate of return to capital, w is real wage rate and l denotes labor supply. Denoting real money balances $m \equiv M/p$ and the rate if inflation $\pi \equiv \dot{p}/p$, we rewrite the household's flow budget constraint as

$$\dot{m} = (1-\tau)\,(rk + wl) + T - c - v - \pi m. \tag{1}$$

The stock of capital changes according to

$$\dot{k} = v - \delta k, \tag{2}$$

where $\delta \in (0, 1)$ denotes the rate of capital depreciation. In addition, a cash-in-advance constraint applies to consumption spending so that $pc \leq M$ or

$$c \leq m \tag{3}$$

in each moment of time. In this paper we assume that investment spending is not subject to the cash-in-advance constraint.

The instantaneous utility of the representative family depends on consumption and labor supply. Following the standard specification, we assume that the objective function of the household is

$$U = \int_0^{\infty} e^{-\rho t} \left(\log c - B \frac{l^{1+\gamma}}{1+\gamma} \right) dt, \quad \gamma > 0, \ \rho > 0, \quad B > 0.$$

Given the initial holdings of k_0 and m_0, the household maximizes U subject to (1), (2) and (3) under given sequences of $\{r_t, w_t, \tau_t, T_t\}_{t=0}^{\infty}$.

To derive the optimization conditions for the household, we set up the current-value Hamiltonian function:

$$\mathcal{H} = \log c - B \frac{l^{1+\gamma}}{1+\gamma} + \lambda \left[(1-\tau)(rk + wl) + T - c - v - \pi m \right] + \mu (v - \delta k) + \zeta (m - c),$$

where λ and μ respectively denote the costate variables of m and k, and ζ is a Lagrange multiplier corresponding to the cash-in-advance constraint on consumption spending. In what follows, we assume that the rate of tax, τ, is depends the level of individual income. The rate of income tax is thus given by

$$\tau = \tau(y) = \tau(rk + wl).$$

Considering such a taxation rule, we find that the necessary conditions for an optimum involve the following:

$$\partial \mathcal{H} / \partial c = 1/c - (\lambda + \zeta) = 0, \tag{4}$$

$$\partial\mathcal{H}/\partial l = -Bl^{\gamma} + \lambda\left[1-\tau\left(y\right)-\tau'\left(y\right)y\right]w = 0, \tag{5}$$

$$\partial\mathcal{H}/\partial v = -\lambda + \mu = 0, \tag{6}$$

$$\zeta\left(m-c\right) = 0,\ \ m-c \geq 0,\ \ \zeta \geq 0, \tag{7}$$

$$\dot{\lambda} = \lambda\left(\rho+\pi\right) - \zeta, \tag{8}$$

$$\dot{\mu} = \left(\rho+\delta\right)\mu - \lambda\left[1-\tau\left(y\right)-\tau'\left(y\right)y)\right]r, \tag{9}$$

together with the transversality conditions, $\lim_{t\to\infty} b_t\lambda_t e^{-\rho t} = 0$ and $\lim_{t\to\infty} m_t\mu_t e^{-\rho t} = 0$ as well as the initial conditions on m and k. In conditions (4) and (5), $\tau\left(y\right)+\tau'\left(y\right)y$ represents the marginal tax rate perceived by the household. As in Guo and Lansing (1998), we assume that each household takes the proportional tax rule into account when deciding their optimal consumption plan.

In this paper we focus on the situation where the cash-in-advance constraint is always effective, so that $c = m$ holds for all $t \geq 0$. First, (6) means that $\mu = \lambda$ so that from (8) and (9) we obtain:

$$\zeta = \lambda\left\{\left[1-\tau\left(y\right)-\tau'\left(y\right)y\right]r + \pi\right\}. \tag{10}$$

Thus (4) is written as

$$\frac{1}{c} = \lambda\left\{1 + \left[1-\tau\left(y\right)-\tau'\left(y\right)y\right]r + \pi - \delta\right\}. \tag{11}$$

As a result, (5) and (10) yields:

$$cl^{\gamma}B = \frac{\left[1-\tau\left(y\right)-\tau'\left(y\right)y\right]w}{1+\left[1-\tau\left(y\right)-\tau'\left(y\right)y\right]r+\pi}. \tag{12}$$

The left-hand side of the above is the marginal rate of substitution between consumption and the labor and the right-hand side expresses the effective, after-tax rare of real wage rate. Since we assume that the cash-in-advance constraint always binds, an additional consumption generates an additional opportunity cost of holding money, which is given by the after-tax, net rate of return to capital plus the rate of inflation. Thus the right-hand side of (12) expresses the real wage rate in terms of the effective price including the cost of money holding.

2.2 Firms

The production side of the model economy follows the standard formulation. There are identical, infinitely many firms and the total number of firms is normalized to one. The production function of an individual firms is given by

$$y = Ak^{\alpha} l^{1-\alpha}, \quad 0 < \alpha < 1, \quad A > 0. \tag{13}$$

In in a competitive economy we will consider, α represents the income share of capital. tWe focus on the case where α has an empirically plausible value, so that in what follows, we assume that α is less than 0.5. The commodity market is assumed to be competitive and thus the rate of return to capital and the real wage equal the marginal products of capital and labor, respectively:

$$r = \alpha A k f^{\alpha-1} l^{1-\alpha} = \alpha \frac{y}{k}, \tag{14}$$

$$w = (1-\alpha) A k^{\alpha} l^{-\alpha} = (1-\alpha) \frac{y}{l}. \tag{15}$$

2.3 Policy Rules

The fiscal and monetary authorities respectively control the rate of income tax, τ, and the nominal interest rate, R, according to their own policy rules. As assumed by Schmitt-Grohé and Uribe (1997) and Guo and Lansing (1998), the fiscal authority follows the balanced budget discipline. To emphasize this assumption, we assume away government debt. The flow budget constraint for the government is thus given by

$$\tau y + \dot{m} + \pi m = g + T,$$

where g denotes the government's consumption spending. A key assumption of our analysis is that under the balanced-budget rule the fiscal authority cannot use seigniorage income to finance the government consumption.[1] This means that

$$g = \tau y \tag{16}$$

holds in each moment. As a consequence, the real seigniorage income, $\dot{M}/p$, is transferred back to the households, so that $\dot{m} + \pi m = T$.

[1]Hence, fiscal policy is 'passive' in the sense of Leeper (1991).

Given the general principle mentioned above, the monetary authority is assumed to follow an interest rate control rule such that

$$R(\pi) = \pi + r^* \left(\frac{\pi}{\pi^*}\right)^\eta, \quad r^* > 0, \quad \pi^* \geq 0, \tag{17}$$

where $r^* > 0$ is the steady-state level of net rate of return to capital and π^* expresses the target rate of inflation. We assume that the target rate of inflation is positive so that π^* is a positive constant set by the monetary authority. Under given r^* and π^*, we see that $R'(\pi) > 1$ (resp. $R'(\pi) < 1$) according to $\eta > 0$ (resp. $\eta < 0$). Hence, if $\eta > 0$, then the monetary authority adopts an active control rule under which it adjusts the nominal interest rate more than one for one with inflation. Conversely, when when $\eta < 0$, the interest rate control is passive in the sense that the monetary authority changes the nominal interest rate less than one for one with inflation. When $\eta = 0$, the monetary authority controls the nominal interest rate to keep the real interest rate at the rate of r^*. Notice that the Fisher equation gives the relation between the nominal and real interest in such a way that

$$R = r + \pi. \tag{18}$$

Therefore, (17) and (18) yield:

$$\pi = \left(\frac{r}{r^*}\right)^{\frac{1}{\eta}} \pi^*, \tag{19}$$

which gives the relation between the equilibrium rate of inflation and the real rate of return to capital. This means that in our setting the nominal interest rate control is to adjust the rate of inflation tax accoroding to a specified rule.[2]

As for the fiscal rule under balanced budget, we consider two alternative regimes. One is taxation rule use the formulation by Guo and Lansing (1998). In this regime, the government consumption is adjusted to keep the balanced budget and the rate of income

[2] In the presence of distortional income taxation, the Fisher equation may be modified: see, for example, Feldstein (1976). For example, the non-arbitrage condition (18) may be replaced with $(1-\tau)R = (1-\tau)r + \pi$ so that $R = r + \pi/(1-\tau)$. If this is the case, we assume that the central bank adopts an interest control rule such that

$$R = \frac{\pi}{1-\tau} + r^* \left(\frac{\pi}{\pi^*}\right)^\eta,$$

which is compatible with the modified Fisher equation in the long-run equilibrium where $\pi = \pi^*$. As a result, we obtain () even in the case of modified Fisher condition.

is determined by the following taxation rule:

$$\tau(y) = 1 - (1-\tau_0)\left(\frac{y^*}{y}\right)^{\phi}, \quad -\frac{1-\alpha}{\alpha} < \phi < 1, \quad 0 < \tau_0 < 1, \tag{20}$$

where y^* denotes the steady-state level of per capita income.[3] Given this taxation rule, the after-tax income is written as

$$[1-\tau(y)]\, y = (1-\tau_0)\, y^{*\phi} y^{1-\phi}.$$

As a result, if we denote the after-tax real income by $I(y) \equiv [1-\tau(y)]\, y$, we obtain

$$\frac{I'(y)}{I(y)/y} = 1-\phi.$$

Since $I(0) = 0$, the above equation means that if $0 < \phi < 1$, then $I'(y) < I(y)/y$ so that the after-tax income is strictly concave in taxable income y, that is, income taxation is progressive. Conversely, when $\phi < 0$, function $I(y)$ is strictly convex and hence, income taxation is regressive. When $\phi = 0$, we obtain the liner taxation rule under which the rate of income tax is fixed as τ_0. It is also to be noted that in the steady state where $y = y^*$, the rate of tax is also fixed at τ_0.[4]

The alternative fiscal rule, which is assumed by Schmitt-Grohé and Uribe (1997), is to keep the government spending fixed and the rate of income tax is adjusted to balance the budget. In this case, the rate of income tax is determined by

$$\tau = \frac{g}{y},$$

where g is fixed at a certain level. Obviously, income taxation in this regime is strongly regressive, because a higher income reduces the tax rate. While this paper mostly focus on the first rule, we briefly discuss this second rule in Section 4.

[3] As shown in Section 3.2. the restriction $\phi > -(1-\alpha)/\alpha$ ensures that the steady state level of consumption has a positive value.

[4] Individual tax payment is $T(y) = \tau(y)\, y$. Given (20), we have $T'(y) = 1-(1-\tau_0)(1-\phi)\, y^{*\phi} y^{-\theta}$. In the steady state where $y = y^*$, we see that $T'(y) = \phi(1-\tau_0) + \tau_0$. Hence, if

$$\phi > -\frac{\tau_0}{1-\tau_0},$$

then the total tax payment increases with income y. Since income share of capital, α, is less than 0.5 in reality, when ϕ satisfies $\phi > -\alpha/(1-\alpha)$, it in general holds that $\phi > -\tau_0/(1-\tau_0)$.

2.4 Capital Accumulation

Combining the flow budget constraints for the household and the government yields the commodity-market equilibrium condition: $y = \dot{k} + \delta k + c + g$. Under the first fiscal rule the government consumption is endogenously determined, and thereby the market equilibrium is written as

$$\dot{k} = (1 - \tau)\, y - c - \delta k. \tag{21}$$

3 Policy Rules and Macroeconomic Stability

In this section we assume that the fiscal authority uses the taxation rule given by (20). We first derive the dynamical system that describes the equilibrium dynamics of the model economy and explore the stability condition around the steady state equilibrium.

3.1 Dynamic System

In order to derive a complete dynamic system that summarizes the model displayed above, we focus on the behaviors of capita stock, k, and the shadow value of real money balances, λ. First observe that (20) gives

$$1 - \tau\,(y) - \tau'\,(y)\, y = (1 - \tau_0)\,(1 - \phi) \left(\frac{y^*}{y}\right)^{\phi}.$$

Using the above equation, together with (5) and (15), we may express the equilibrium level of employment in the following way:

$$l = \left[\frac{(1 - \tau_0)\,(1 - \phi)\,(1 - \alpha)}{B}\right]^{\frac{1}{1+\gamma}} y^{*\frac{\phi}{1+\gamma}} y^{\frac{1-\phi}{1+\gamma}} \lambda^{\frac{1}{1+\gamma}}.$$

Inserting the above into the production function (13) and solving it with respect to y, we obtain

$$y = \hat{A} k^{\frac{\alpha(1+\gamma)}{\Delta}} \lambda^{\frac{1-\alpha}{\Delta}} \equiv y\,(k, \lambda)\,, \tag{22}$$

where

$$\Delta = \alpha + \gamma + \phi(1 - \alpha),$$

$$\hat{A} = A^{\frac{1+\gamma}{\Delta}} \left[\frac{(1 - \tau_0)(1 - \phi)(1 - \alpha)}{B}\right]^{\frac{1-\alpha}{\Delta}} y^{*\frac{\phi(1-\alpha)}{\Delta}\phi}.$$

Equation (22) represents the short-run production function under a given level of y^*. Similarly, the real interest rate is expressed as

$$r = \alpha \frac{y}{k} = \alpha \hat{A} k^{-\frac{(1-\alpha)(\gamma+\phi)}{\Delta}} \lambda^{\frac{1-\alpha}{\Delta}},$$

implying that the after-tax marginal rate of return to capital is

$$\begin{aligned}\left(1-\tau-\tau' y\right) r &= \alpha\left(1-\tau_0\right)\left(1-\phi\right) y^{*\phi} \hat{A}^{1-\phi} k^{\frac{\alpha\gamma(1-\phi)-(\gamma+\phi)}{\Delta}} \lambda^{\frac{(1-\phi)(1-\alpha)}{\Delta}} \\ &\equiv \hat{r}\left(k, \lambda\right).\end{aligned}$$

For determining the equilibrium rate of inflation, in view of (14), (19) and (22), we may express π as a function of k and λ in such a way that

$$\pi = \pi^* \left(\frac{\alpha \hat{A}}{r^*}\right)^{\frac{1}{\eta}} k^{-\frac{(1-\alpha)(\gamma+\phi)}{\eta\Delta}} \lambda^{\frac{1-\alpha}{\eta\Delta}} \equiv \pi\left(k, \lambda\right). \tag{23}$$

Hence, using (23), we see that the optimal consumption depends on k and λ in the following manner:

$$c = \frac{1}{\lambda[1+\hat{r}\left(k, \lambda\right)+\pi\left(k, \lambda\right)-\delta]} \equiv c\left(k, \lambda\right). \tag{24}$$

Summing up the above manipulation, we find that the dynamic equation of capital stock is expressed as

$$\dot{k} = (1-\tau_0) y^{*\phi} y(k, \lambda)^{1-\phi} - c\left(k, \lambda\right) - \delta k, \tag{25}$$

and the shadow value of capital (real money balances) changes according to

$$\dot{\lambda} = \lambda\left[\rho + \delta - \hat{r}\left(k, \lambda\right)\right]. \tag{26}$$

A pair of differential equations, (25) and (26), constitute a complete dynamic system under the interest rate control and the taxation rule with endogenous government expenditure. Note that $y\left(k, \lambda\right)$ and $\hat{r}\left(k, \lambda\right)$ satisfy

$$\hat{r}\left(k, \lambda\right) = \alpha\left(1-\tau_0\right)\left(1-\phi\right)\frac{y\left(k, \lambda\right)}{k}, \tag{27}$$

$$\pi\left(k, \lambda\right) = \pi^* \left(\frac{r}{r^*}\right)^{\frac{1}{\eta}} = \pi^* r^{*-\frac{1}{\eta}} \left(\frac{y\left(k, \lambda\right)}{k}\right)^{\frac{1}{\eta}}. \tag{28}$$

3.2 Steady-State Equilibrium

In the steady state where k and λ stay constant over time, it should hold that $\pi = \pi^*$, $r = r^*$ and $y = y^*$. It is to be noted that in the steady state, we obtain:

$$1 - \tau(y^*) - \tau'(y^*)\, y^* = (1 - \tau_0)(1 - \phi).$$

We should also notice that from (22) the production function in the steady state is given by

$$y^* = \hat{A}^{\frac{\Delta}{\alpha+\gamma}} k^{*\frac{\alpha(1+\gamma)}{\alpha+\gamma}} \lambda^{*\frac{1-\alpha}{\alpha+\gamma}} \tag{29}$$

and the values of k, c and λ satisfy the following conditions:

$$\frac{y^*}{k^*} = \frac{\rho+\delta}{\alpha(1-\tau_0)(1-\phi)}, \tag{30}$$

$$\frac{c^*}{k^*} = (1-\tau_0)\frac{y^*}{k^*} - \delta = \frac{\rho + \delta[1-\alpha(1-\phi)]}{\alpha(1-\phi)}, \tag{31}$$

$$\lambda^* k^* = \frac{k^*}{c^*(1+\rho+\pi^*)}, \tag{32}$$

In the above, k^*, c^* and λ^* denote their steady-state values. Equation (30) is the modified Golden-rule condition corresponding to $\dot{\lambda} = 0$, while (31) comes from the long-rum market equilibrium condition: $\dot{k} = 0$. The modified golden-rule condition (30) determines the income-capital ratio, y^*/k^*, which gives the consumption-capital ratio, c^*/k^* by (31). Then the steady-state implicit value of capital, $k^*\lambda^*$, is given by (32). The last condition yields

$$\lambda^* = \frac{\alpha(1-\phi)}{(1+\rho+\pi^*)[\rho+\delta(1-\alpha(1-\phi))]\, k^*} = \frac{\beta}{k^*},$$

where β denotes the coefficient of $1/k^*$. Using the above relation, together (29) and (30), we find that the steady-state level of capital is uniquely determined such that:

$$k^* = A^{\frac{1}{1-\alpha}} \left[\frac{\alpha(1-\tau_0)(1-\phi)}{\rho+\delta}\right]^{\frac{(1-\alpha)(1+\gamma)}{\alpha+\gamma}} \left[\frac{(1-\alpha)(1-\tau_0)(1-\phi)}{B}\right]^{\frac{1}{1-\alpha}} \beta^{\frac{1}{1+\gamma}}. \tag{33}$$

Therefore, the steady-state value of k and λ are uniquely expressed by all the parameters involved in the model. Once k^* and λ^* are given, the steady state levels of $c\,(= m)$ and l are determined uniquely as well.

The steady-state value of capital given by (33) demonstrates that policy parameters, τ_0, ϕ, η and π^* affect the long-run levels of capital, income, employment and consumption in a complex manner. However, it is rather easy to derive intuitive implications of the effects of a change in policy parameters. First, observe that the degree of activeness of interest-rate control, η, fails to affect the steady state levels of capital, employment and income. Second, regardless of progressiveness of income tax (i.e. the sign of ϕ), the steady-state capital decreases with τ_0, ϕ and π^*. Third, (30) and (31) show that a change in π^* will not affect y^*/k^* and c^*/k^*, so that it alters k^*, y^* and c^* proportionally. Additionally, (31) also shows that a rise in ϕ increases c^*/k^*, while τ_0 does not affect c^*/k^*.

Finally, by use of (12), (14), (15), (30) and (31), the steady-state rete of employment satisfies the following relation:

$$l^{*\gamma+1} = \frac{(1-\alpha)(\rho+\delta)(1-\phi)}{(1+\rho+\pi^*)\{\rho+\delta[1-\alpha(1-\phi)]\}},$$

implying that l^* decreases with ϕ and π^*, while τ_0 does not affect l^*.

Inspecting the steady state conditions derived above, we may summarize the comparative statics results in the steady state equilibrium as follows:

Proposition 1 *The degree of activeness of interest rate control, η, does not affect the steady state levels of capital, income, consumption and employment. The impacts of changes in policy parameters τ_0, ϕ and π^* are shown as the following table.*

	k^*	l^*	y^*	y^*/k^*	c^*/k^*	c^*/y^*
τ_0	$-$	0	$-$	$+$	0	$-$
ϕ	$-$	$-$	$-$	$+$	$+$	$+$
π^*	0	$-$	$-$	0	0	0

Table 1: Policy impacts on the steady state values of key variables.

3.3 Equilibrium Determinacy

In order to examine the equilibrium dynamics near the steady state, let us conduct linear approximation of (25) and (26) at the steady-state equilibrium. The coefficient matrix of the approximated system is given by

$$J = \begin{bmatrix} (1-\tau_0)(1-\phi)\, y_k(k^*,\lambda^*) - c_k(k^*,\lambda^*) - \delta & (1-\tau_0)(1-\phi)\, y_\lambda(k^*,\lambda^*) - c_\lambda(k^*,\lambda^*) \\ -\lambda^* \hat{r}_k(k^*,\lambda^*) & -\lambda^* \hat{r}_\lambda(k^*,\lambda^*) \end{bmatrix}.$$

Since the shadow value of capital, λ, is an unpredetermined variable, if J has one stable root, the converging path under perfect foresight is at least locally unique. Thus determinacy of equilibrium is established when the determinacy of J has a negative value. In contrast, when $\det J > 0$ and the trace of J is negative, there exists a continuum of equilibria around the steady state. As shown in Appendix 1, using the steady-state conditions, we find that the partial derivatives appeared in J can be expressed by the given parameter values. The trace and determinant of J are respectively written as:

$$\text{trace } J = \rho - \frac{1}{(1+\pi^*+\rho)\Delta}\frac{\rho+\delta[1-\alpha(1-\phi)]}{\alpha(1-\phi)} \times \left\{[\phi(\alpha\gamma+1)+\gamma(1-\alpha)](\rho+\delta)+(1-\alpha)(\gamma+\phi)\frac{\pi^*}{\eta}\right\}, \tag{34}$$

$$\det J = -\frac{\rho+\delta\left[1-\alpha\left(1-\phi\right)\right]}{\alpha\left(1-\phi\right)}\frac{\rho+\delta}{\Delta}\left[(\gamma+1)(1-\alpha(1-\phi))+\frac{\phi(1-\alpha)\pi^*}{\eta(1+\pi^*+\rho)}\right]. \tag{35}$$

where $\Delta = \alpha+\gamma+\phi\left(1-\alpha\right)$.

First of all, it is easy to see that if the target rate of inflation, π^*, is non negative, income taxation is progressive $(\phi > 0)$ and the interest rate control is active $(\eta > 0)$, then $\det J$ has a negative value, so that the steady-state equilibrium is locally determinate. Similarly, if $-\frac{\alpha+\gamma}{1-\alpha} < \phi < 0$ (so that $\Delta > 0$) and $\eta < 0$, then $\det J < 0$. Thus in this case indeterminacy of equilibrium will not emerge either. In addition, if $\phi = 0$ and the rate of tax is fixed at τ_0, then

$$\det J = -\frac{\rho+\delta(1-\alpha)}{\alpha^2}\left(\rho+\delta\right)(\gamma+1)(1-\alpha) < 0,$$

implying that, regardless of the monetary policy rules, the dynamic system exhibits equilibrium determinacy. To sum up, a set of sufficient conditions for equilibrium determinacy are the following:

Proposition 2 *(i) Given a positive rate of target inflation, either if income taxation is progressive and interest rate control is active or if income taxation is regressive to satisfy* $-\frac{\alpha+\gamma}{1-\alpha} < \phi < 0$ *and interest rate control is passive, then the steady-state equilibrium is locally determinate. (ii) If income tax is flat* $(\phi = 0)$, *local determinacy holds regardless of monetary policy rules.*

To focus on the other possibilities of equilibrium (in)determinacy,as clear as possible, let us assume that the elasticity of labor supply is zero: $\gamma = 0$. This case corresponds to the real business cycle model with indivisible labor analyzed by Hansen (1985). Given this assumption, we obtain

$$\text{trace } J = \rho - \frac{\rho + \delta\left[1 - \alpha(1-\phi)\right]}{\alpha(1-\phi)(1+\pi^* + \rho)}\left[\rho + \delta + (1-\alpha)\frac{\pi^*}{\eta}\right]\frac{\phi}{\Delta}, \tag{36}$$

$$\det J = -\frac{\left\{\rho + \delta\left[1 - \alpha\left(1-\phi\right)\right]\right\}\left(\rho+\delta\right)}{\alpha\left(1-\phi\right)}\left[1 - \alpha\left(1-\phi\right) + \frac{\phi(1-\alpha)\pi^*}{\eta(1+\pi^*+\rho)}\right]\frac{1}{\Delta}. \tag{37}$$

where

$$\Delta \equiv \alpha + \phi\left(1-\alpha\right).$$

First, assume that income taxation is progressive $(\phi > 0)$. In this case $\Delta > 0$ and, hence, the necessary and sufficient condition for determinacy is

$$1 - \alpha\left(1-\phi\right) + \frac{\phi(1-\alpha)\pi^*}{\eta(1+\pi^*+\rho)} > 0.$$

The above conditiuon implies that if $\pi^* \geq 0$, equilibrium determinacy is established under the following conditions:

$$\eta > 0 \quad \text{or} \quad \eta < -\frac{(1-\alpha)\pi^*\phi}{\left[1-\alpha\left(1-\phi\right)\right]\left(1+\rho+\pi^*\right)}. \tag{38}$$

If η satisfies

$$-\frac{(1-\alpha)\pi^*\phi}{\left[1-\alpha\left(1-\phi\right)\right]\left(1+\rho+\pi^*\right)} < \eta < 0, \tag{39}$$

then we see that $\det J > 0$. It is easy to see that in this case iwe obtain $\rho+\delta+(1-\alpha)\frac{\pi^*}{\eta} < 0$, and, hence, from (36) the trace of J has a positive value. Therefore, if η satisfies (39), the steady state is a source and there is no converging path around it.

Table 2 summarizes the patterns of dynamics under progressive tax.

	$\phi > 0$
$\eta > 0$	D
$\hat{\eta} < \eta < 0$	I or U
$\eta < \hat{\eta}$	D

Table 2: Stability properties under progressive taxation

D :determinacy, I :indeterminacy, U :unstable

$\hat{\eta} = -\frac{(1-\alpha)\pi^*\phi}{[1-\alpha(1-\phi)](1+\rho+\pi^*)}\ (< 0)$

It is to be pointed out that, as shown by numerical examples presented in Section 3.5, when $0 < \phi < 1$, condition (39) may not be satisfies for plauseible parameter values. Therefore, the steady state is mostly unstable for the case of $-\hat{\eta} < \eta < 0$. To sum up, in the case of progressive taxation we obtain:

Proposition 3 *If income taxation is progressive and the target rate of inflation is non-negative, the perfect-foresight competitive equilibrium is locally determinate, either if the interest-rate control is active or it is sufficiently passive. Equilibrium indeterminacy may not emerge in this regime.*

Next, consider the case of regeressive taxation $(\phi < 0)$. In this case the necessary and sufficient condition for local determinacy is

$$\left[1 - \alpha\left(1 - \phi\right) + \frac{(1-\alpha)\pi^{*}\phi}{\eta(1+\pi^{*}+\rho)}\right]\frac{1}{\Delta} > 0. \tag{40}$$

This condition is fulfilled, either if

$$-\frac{\alpha}{1-\alpha} < \phi < 0 \quad (\Longleftrightarrow \Delta > 0) \text{ and } \eta > -\frac{(1-\alpha)\pi^{*}\phi}{\left[1-\alpha\left(1-\phi\right)\right]\left(1+\rho+\pi^{*}\right)} \equiv \hat{\eta}\,(>0)$$

or if

$$\phi < -\frac{\alpha}{1-\alpha} \quad (\Longleftrightarrow \Delta < 0) \quad \text{and } 0 < \eta < -\frac{(1-\alpha)\pi^{*}\phi}{\left[1-\alpha\left(1-\phi\right)\right]\left(1+\rho+\pi^{*}\right)} \equiv \hat{\eta}\,(>0)\,.$$

In words, if a relatively low degree of regressiveness taxaion, coupled with a high degree of passive interest-rate control, may produce indeterminacy.

In contrast, the necessary conditions for equilibrium indeterminacy are the following:

$$\left[1 - \alpha\left(1 - \phi\right) + \frac{(1-\alpha)\pi^{*}\phi}{\eta(1+\pi^{*}+\rho)}\right]\frac{1}{\Delta} < 0, \tag{41}$$

$$\left[\rho + \delta + (1-\alpha)\frac{\pi^{*}}{\eta}\right]\frac{\phi}{\Delta} > 0. \tag{42}$$

When $-\alpha/\left(1-\alpha\right) < \phi < 0$ (so $\Delta > 0$), then both (41) and (42) are satisfied, if and only if

$$-\frac{\left(1-\alpha\right)\pi^{*}}{\rho+\delta} < \eta < -\frac{\left(1-\alpha\right)\pi^{*}\phi}{1+\pi^{*}+\rho}.$$

Note that the above condition is necessary but not sufficient for establishing equilibrium indeternacy: if trace $J > 0$ in (36), the steady state is a source (unstable). In contrast, if $-\frac{1-\alpha}{\alpha} < \phi < -\frac{\alpha}{1-\alpha}$ (so $\Delta < 0$), we find that indeterminacy may emerge more easily. Table 3 gives a classification of dynamic patterns in the case of regressive taxation.

	$-\frac{\alpha}{1-\alpha} < \phi < 0$	$-\frac{1-\alpha}{\alpha} < \phi < -\frac{\alpha}{1-\alpha}$
$-\hat{\eta} < \eta$	D	I or U
$0 < \eta < -\hat{\eta}$	I or U	D
$\bar{\eta} < \eta < 0$	D	I or U
$\eta < \bar{\eta}$	D	I

Tanle 3: Stability properties under regressive taxation

D: determinate, I: indeterminate, U: unstable

$\hat{\eta} \equiv -\frac{(1-\alpha)\pi^*\phi}{[1-\alpha(1-\phi)](1+\rho+\pi^*)} (> 0)$, $\bar{\eta} = -\frac{(1-\alpha)\pi^*}{\rho+\delta} (< 0)$,

The following propoisition summarizes our finding.

Proposition 4 *(i) Suppose that income taxation is mildly regressive and the target rate of inflation is positive. Then the steady state is locally determinate, either if interest rate control is sufficiently active or if it is passive. (ii) Suppose that income taxation is sufficiently regressiveness and the target rate of inflation is positive. Then the steady state holds equilibrium determinacy only when interest rate control is mildly active.*

3.4 Discussion

To obtain intuitive implication of determinacy/indeterminacy conditions displayed in Propositions 2, 3 and 4, let us inspect the optimization condition (12) in detail. Using $w = \alpha y/l$, this condition is rewritten as

$$cl^{\gamma}B = \frac{(1-\tau_0)(1-\phi)(1-\alpha)\, y^{*\phi} A^{1-\phi} k^{\alpha(1-\phi)} l^{(1-\alpha)(1-\phi)-1}}{1+\hat{r}+\pi}. \tag{43}$$

Under a given level of consumption, c, the left-hand side of (43) represents the labor supply curve and the right hand side is considered the labor demand curve. Given c, $\hat{r}$ and π. If we assume that $\gamma = 0$ for expositional simplicity, the labor supply curve becomes a horizontal line. Hence, if $(1-\phi)(1-\alpha) - 1 = -\Delta < 0$, the labor demand curve has a negative slope and thus less steeper than the labor supply curve. In contrast, if $\Delta < 0$, then the labor demand has a positive slope and steeper than the labor supply curve.

Now suppose that the economy initially stays at the steady state equilibrium. Suppose further that a sunspot-deriven shock makes agents optimistic and households anticipate that the output and employment will expand. This raises consumption demand and the labor supply curve shifts upward and, hence, the equilibrium employment decreases, as long as the labor demand curve does not shift. Remember that the after-tax rate of return, $\hat{r}$, and the rate of inflation, π, are respectively expressed in the following manner:

$$\hat{r} = \alpha (1 - \tau_0)(1 - \phi) y^{*\phi} A^{1-\phi} k^{\alpha(1-\phi)-1} l^{(1-\alpha)(1-\phi)},$$

$$\pi = r^{*-\frac{1}{\eta}} \pi^* y^{*\phi} A^{\frac{1}{\eta}} k^{\frac{\alpha}{\eta}-1} l^{\frac{1-\alpha}{\eta}}.$$

These expressions show that an expected increase in l raises $\hat{r}$. It also increases inflation, if $\eta > 0$. As a result, the anticipated increase in employment shifts the labor demand curve downwards. Then the equilibrium employment decrease further, implying that the initial, optimistic expectation will not be self-fulfilled, because product will contract rather than expand. Consequently, such an expectation driven fluctuations cannot be realized so that equilibrium is determinate.

Notice that if η is negative and its absolute value is small, a rise in employment yields a sufficiently large decrease in inflation. This may reduces the after-tax nominal interest rate, $\hat{r} + \pi$, and therefore, the labor demand curve may shift up. If such a shift large enough to enhance the equilibrium level of employment, the initial optimistic expectations can be self-fulfilled. Hence, as Table 2 shows, if the interest rate control is mildly passive, there may exist multiple converging paths. In contrast, if the interest rate control is strongly passive so that the absolute value of η is large enough, a decrease in the after-tax nominal interest rate, $\hat{r} + \pi$, is small. As a result, an upward shift of the labor demand curve cannot cancel a reduction of employment due to an upward shift of labor supply curve. Consequently, if η is small enough to fulfill $\eta < -\hat{\eta}\phi$, the possibility of equilibrium indeterminacy is eliminated.

Next, assume that $\Delta < 0$ so that the labor demand curve has a positive slope. If the after-tax nominal interest rate, $\hat{r} + \pi$, is constant, then the initial shift of labor supply curve due to an increase in consumption raises the equilibrium level of employment. If we consider the effective real wage, labor demand curve may have negative slope even if $\Delta < 0$. In particular, when η has a small positive value, a rise in l yields a large increase in π so that the labor demand curve may have a negative slope. In this case the initial expectation cannot be self fulfilled. At the same time, if $\eta > 0$, both $\hat{r}$ and π are increased by an expansion of employment, which yields a downward shift of the labor demand curve. Since the labor demand curve is steeper than the labor supply curve, this shift produces a further enhancement of the employment level. As a consequence, the initial expectation can be self-fulfilled and equilibrium indeterminacy emerges. In contrast, if $\eta < 0$ and η is close to zero, a higher employment may lower the after-tax nominal interest rate, implying that the labor demand curve shifts upward. If this is the case, the equilibrium level of employment may not increase, which means that the initial expected change in economic condition cannot be realized. Thus indeterminacy may not emerge in this situation.

3.5 A Numerical Example

To focus on the roles of key policy parameters, ϕ and η, more clearly, let us inspect a numerical example. In so doing, we first depict the graphs of the conditions for $\det J = 0$ and trace $J = 0$ in (ϕ, η) space. In what follows, we still focus on the case of indivisible labor ($\gamma = 0$).

Remembering that $\Delta \equiv \alpha + \phi\,(1-\alpha)$ and defining

$$\tilde{\phi} \equiv -\frac{\alpha}{1-\alpha},$$

we see that sign $\Delta =$ sign $\left(\phi - \tilde{\phi}\right)$. Note that

$$\text{sign } \det J = \text{sign}\left\{(1-\alpha+\alpha\phi)\eta + \frac{\phi(1-\alpha)\pi^*}{(1+\pi^*+\rho)}\right\} \quad \text{if } \Delta\eta > 0,$$
$$\text{sign } \det J = -\text{sign}\left\{(1-\alpha+\alpha\phi)\eta + \frac{\phi(1-\alpha)\pi^*}{(1+\pi^*+\rho)}\right\} \quad \text{if } \Delta\eta < 0.$$

and that $\det J = 0$ holds when the folloiwing condition is fulfilled:

$$\eta = -\frac{\phi(1-\alpha)\pi^*}{(1+\pi^*+\rho)(1-\alpha+\alpha\phi)} \equiv \eta_d\,(\phi;\ \pi^*)\,. \tag{44}$$

Eauation (44) shows that, given a positive rate of the target inflation, π^* (> 0), η deareases as ϕ rises. In addition η decreases (resp. increases) with π^*, if ϕ is positive (resp. negative).

To consider the sign of trace J, define

$$\mathcal{C}(\phi;\pi^*) \equiv \frac{\rho\alpha(1-\phi)(1+\pi^*+\rho)\Delta}{\rho+\delta[1-\alpha(1-\phi)]} - \phi(\rho+\delta).$$

Then it hodls that

$$\text{sign }\{\text{trace } J\} = \text{sign}\left\{\frac{\mathcal{C}(\phi;\pi^*)\eta - (1-\alpha)\phi\pi^*}{\Delta\eta}\right\}.$$

This condition is rewritten as

$$\begin{aligned}\text{sign }\{\text{trace } J\} &= \text{sign }\{\eta - \eta_{tr}(\phi;\pi^*)\} \quad \text{if } \frac{\mathcal{C}(\phi;\pi^*)}{\Delta\eta} > 0,\\ \text{sign }\{\text{trace } J\} &= -\text{sign }\{\eta - \eta_{tr}(\phi;\pi^*)\} \quad \text{if } \frac{\mathcal{C}(\phi;\pi^*)}{\Delta\eta} < 0,\end{aligned}$$

where the locus of trace $J = 0$ is given by

$$\eta_{tr}(\phi;\pi^*) = \frac{(1-\alpha)\phi\pi^*}{\mathcal{C}(\phi;\pi^*)}.$$

We set the convetional magnitude for each parameter:

$$\text{time discount rate } (\rho) = 0.04, \quad \text{income share of capita } (\alpha) = 0.4,$$
$$\text{capital depreciation rate } (\delta) = 0.05, {}^{5}.$$

Given those paprameter values, $\phi \in \left(-\dfrac{1-\alpha}{\alpha}, 1\right) = (-1.5, 1)$. Assuming that the target rate of inflation is $\pi^* = 0.03$, we obtain the following:

$$\tilde{\phi} = -0.67,$$

$$\eta_d(\phi; 0.03) = -\frac{18\phi}{107(6+4\phi)}.$$

$$\eta_{tr}(\phi; 0.03) = \frac{180\phi(7+2\phi)}{-12072\phi^2 - 2876\phi + 6848}.$$

Using the numerical results displayed above, we depict the graphical results in Figure 1.[6] This figure first reveals that in our numerical example equilibrium indeterminacy may not emerge as long as ϕ exceeds -0.67 and, hence, progressive taxation ensures determinacy regardless of interest control. Second, in the case of progressive taxation ($\phi > 0$), even though there is a region in which the steady state is totally unstable (so

[6] When depicting graphs in Figure 1, we use the following facts. First, note that $\mathcal{C}(\cdot) > 0$ for $\phi \in (\phi_L, \phi_H)$, where ϕ_L and ϕ_H respectively satisfy $-\dfrac{1-\alpha}{\alpha} < \phi_L < \tilde{\phi}$ and $0 < \phi_H < 1$. Second, $\eta_{tr}(\cdot)$ is an increasing function of ϕ with a positive value of $\pi^* > 0$. Moreover, $\eta_{tr}(\cdot)$ may move around the origin in the (ϕ, η) plane in the clockwise (resp. counterclockwise) as π^* rises when $-\dfrac{1-\alpha}{\alpha} < \phi < \phi_L$ (resp. $\phi_L < \phi < 1$).

the equilibrium path is nonstationary), such a region in (ϕ, η) space is considerably small. Third, the steady state would be totally unstable if η is positive and sufficiently small for the case of $\phi \in [-0.67, 0]$. Finally when income taxation is regressive enough to satisfy $\phi < -0.67$, the interest control, i.e. the magnitude of η, critically affects the stability property of the economy.

4 Alternative Policy Rules

In this section we briefly discuss alternative fiscal and monetary policy rules that would modify our main findings shown in the previous sections.

4.1 Fixed Government Spending

So far, we have assumed that the government consumption is endogenously determined to satisfy the balanced-budget rule. The second scheme of fiscal rule is that the fiscal authority fixes the government expenditure, g, by adjusting the rate of tax, τ, to balance its budget. Schmitt-Grohe and Uribe (1997) assume such a balanced-budget rule. If this is the case, the rate of average tax is determined as

$$\tau(y) = \frac{g}{y}. \tag{45}$$

Unlike the first rule, when deciding its optimal plan, the household takes the tax rate τ as given, because y in (45) represents the aggregate income rather than an individual income. In equilibrium, the after tax income is simply given by $I(y) = [1 - \tau(y)]\, y = y - g$ and

$$\frac{I'(y)}{I(y)/y} = \frac{y}{y-g} > 1,$$

implying that income tax is regressive. In this case the after-tax factor prices are given by

$$\hat{r} = (1 - \tau(y))\, \alpha \frac{y}{k} = \alpha \frac{y-g}{k}, \tag{46}$$

$$\hat{w} = (1 - \tau(y))(1 - \alpha) \frac{y}{l} = (1 - \alpha) \frac{y-g}{l}. \tag{47}$$

Since the household considers that τ is exogenously determined, two of the the first-order conditions for an optimum shown in Section 2.1 are replaced with the following:

$$\partial \mathcal{H} / \partial l = -B l^{\gamma} + \lambda [1 - \tau(y)]\, w = 0, \tag{48}$$

$$\dot{\mu} = (\rho + \delta)\,\mu - \lambda\,[1 - \tau\,(y)]\,r, \tag{49}$$

where $\tau\,(y) = g/y$ and $g\ (> 0)$ is given. From (47) and (48) ,the instantaneous equilibrium level of employment satisfies

$$\frac{Bl^{\gamma+1}}{(1-\alpha)\,\lambda} + g = Ak^{\alpha}l^{1-\alpha}.$$

If exists, there are at most two values of l satisfying the above. In the following we ignore the smaller level of l because it produces unconventional results (for example, a higher government consumption increases employment). The higher equilibrium level of l can be written as

$$l = l\,(k, \lambda; g)\,, \quad l_k > 0, \quad l_\lambda > 0, \quad l_g < 0. \tag{50}$$

Using (50) , we find that the equilibrium level of output, the after-tax rate of return are written as

$$\begin{aligned} Ak^{\alpha}l\,(k, \lambda; g)^{1-\alpha} &= y\,(k, \lambda; g)\,, \quad y_k > 0, \quad y_\lambda > 0, \quad y_g < 0, \\ \left(1 - \frac{g}{y}\right)\alpha\frac{y}{k} = \alpha\left(\frac{y\,(k, \lambda, g) - g}{k}\right) &= \hat{r}\,(k, \lambda; g)\,, \quad \hat{r}_\lambda > 0, \quad \hat{r}_g < 0. \end{aligned}$$

Note that the after-tax rate of return to capital may increase with capital if a higher k sufficiently reduces g/k. Consequently, the reduced dynamic system is given by

$$\begin{aligned} \dot{k} &= y\,(k, \lambda; g) - c\,(k, \lambda; g) - \delta k - g, \\ \dot{\lambda} &= \lambda\,[\rho + \delta - \hat{r}\,(k, \lambda; g)]\,, \end{aligned}$$

where

$$\begin{aligned} c\,(k, \lambda; g) &= \frac{1}{\lambda\,[1 + \hat{r}\,(k, \lambda; g) + \pi\,(k, \lambda; g)]}, \\ \pi\,(k, \lambda; g) &= \pi^{*}r^{*-\frac{1}{\eta}}\left(\frac{y\,(k, \lambda; g)}{k}\right)^{\frac{1}{\eta}}. \end{aligned}$$

The following discussion is essentially the same as that in Sections 3.2 and 3,3. The key for the analysis is the behavior of the after-tax levels of rate of return and real wage. In this fiscal policy regime, equation (12) is written as

$$cl^{\gamma}B = \frac{(1-\alpha)\,(Ak^{\alpha}l^{-\alpha} - g/l)}{1 + \hat{r} + \pi}, \tag{51}$$

where

$$\hat{r} = \alpha\left(Ak^{\alpha-1}l^{1-\alpha} - \frac{g}{k}\right),$$
$$\pi = \pi^{*}r^{*-\frac{1}{\eta}}\left(\alpha Ak^{\alpha-1}l^{1-\alpha}\right)^{\frac{1}{\eta}}.$$

Notice that

$$\frac{\partial\left(\left(1-\tau\right)w\right)}{\partial l} = l^{2}\left(g - \alpha y\right)$$

so that the government consumption is large enough to satisfy $g > \alpha y$, the labor demand function represented by the right-hand side of (51) increases with l. Again, assume that $\gamma = 0$. According to the discussion in Section 4.4, if $g > \alpha y$, then indeterminacy of equilibrium is easy to be observed. In addition, if $\eta > 0$, a higher employment caused by a sunspot driven disturbance increases the after-tax nominal interest rate, $\hat{r} + \pi$. Hence, the labor demand curve shifts downward, which enhances the possibility of indeterminacy. If $g < \alpha y$, then the labor demand curve is negatively sloped. Even in this case, if η is negative and its absolute value is small, a higher employment reduces the after-tax nominal interest rate. As a consequence, the labor demand curve shifts up, under which emergence of multiple equilibrium can remain.

4.2 Factor Specific Taxation

If capital and labor income are taxed separately, we may set the following tax functions:

$$\tau_{k}\left(rk\right) = 1 - \left(1 - \tau_{0}^{k}\right)\left(\frac{r^{*}k^{*}}{rk}\right)^{\phi_{r}}, \qquad \tau_{w}\left(wl\right) = 1 - \left(1 - \tau_{0}^{w}\right)\left(\frac{w^{*}l^{*}}{wl}\right)^{\phi_{w}},$$

where $0 < \tau_{0}^{k},\ \tau_{0}^{r} < 1$. Notice that in the case of Cobb-Douglas production function, we obtain

$$\frac{r^{*}k^{*}}{rk} = \frac{y^{*}}{y}, \quad \frac{w^{*}l^{*}}{wl} = \frac{y^{*}}{y}$$

Again, the equibalnce between labor demand and supply is decribed by

$$cl^{\gamma}B = \frac{\hat{w}}{1 + \hat{r} + \pi},$$

where

$$\hat{w} = \left(1 - \tau_{0}^{w}\right)\left(1 - \phi_{w}\right)\left(1 - \alpha\right)y^{*\phi_{w}}l^{*-\phi_{w}}A^{1-\phi_{w}}k^{\alpha\left(1-\phi_{w}\right)}l^{\left(1-\alpha\right)\left(1-\phi_{w}\right)}$$
$$\hat{r} = \alpha\left(1 - \tau_{0}^{r}\right)\left(1 - \phi_{r}\right)y^{*\phi}A^{1-\phi}k^{\alpha\left(1-\phi\right)-1}l^{\left(1-\alpha\right)\left(1-\phi_{r}\right)}$$

Therefore, it is easy to see that indterminacy tends to emerge more easily, if wage income taxation is regressive ($\phi_w < 0$) and capital income taxation is progressive ($\phi_r > 0$).

4.3 Taylor Rule

Taylor (1993) originally proposes the interet rate control rule under which the nominal interest rate responeds to real income as well as inflation. In our notation, the original Taylor rule can be formulated as

$$R(\pi) = \pi + r^* \left(\frac{\pi}{\pi^*}\right)^{\eta} \left(\frac{y}{y^*}\right)^{\xi}, \quad \xi < 1. \tag{52}$$

In this case, the Fisher equattion, $R = r + \pi$, gives

$$\pi = \pi^* \left(\frac{r}{r^*}\right)^{\frac{1}{\eta}} \left(\frac{y}{y^*}\right)^{-\frac{\xi}{\eta}}.$$

Since $r = \alpha y/k$, the above is rewritten as

$$\pi = \alpha^{\frac{1}{\eta}} A^{\frac{1-\varepsilon}{\eta}} \pi^* r^{*-\frac{1}{\eta}} y^{*\frac{\xi}{\eta}} k^{\frac{\alpha-1-\varepsilon}{\eta}} l^{\frac{(1-\alpha)(1-\varepsilon)}{\eta}}. \tag{53}$$

When $\xi = 0$, the equilibrium rate of inflation is $\pi = \pi = A^{\frac{1}{\eta}} r^{*-\frac{1}{\eta}} \pi^* y^{*\phi} k^{\frac{\alpha}{\eta}-1} l^{\frac{1-\alpha}{\eta}}$. Therefore, if the intrest rate control is active with respective to real income, i.e. ξ has a positive value, then a change in labor employment, l, has a smaller impact on the rate of inflation under (52) than under (17). Hence, when $\eta < 0$, a rise in l yields a smaller decrease in π in the case of $\xi > 0$. Therefore, in view of the discussion in Section 3.4, the Taylor type control rule given by (52) may contribute to reducing the possiblity of equilibrium indeterminacy.

5 Conclusion

We have analyzed the stabilization roles of fiscal and monetary policy rules in a monetary real business cycle model with flexible price adjustment. In this paper, we have assumed that the rate of income tax is endogenously adjusted to balance the government budget, while the monetary authority uses the Taylor type interest rate control scheme. Our investigation reveals that in the context of a simple real business cycle model we use, equilibrium determinacy depends heavily on the taxation rule rather than on monetary policy rule. In particular, as suggested by our numerical example, progressive taxation under balanced budget rule tends to eliminate the possibility of equilibrium indeterminacy regardless of activeness of interest rate control. On the contrary, if income taxation is regressive, whether interest rate rule is active or passive may be pivotal to hold equilibrium determinacy. Since the effects of regressive income tax are close to those generated by increasing returns to scale, our finding suggests that the role of interest rate control would be more relevant in the non-standard situation like regressive taxation or increasing return to scale.

It is, however, to be noticed that our main results emphasized above may partly come from the simplicity of our model. Our conclusion would be modified, if we assume more general settings. Possible generalization of the model includes non-separable utility between consumption and labor as well as a more general form of money demand (for example, distinction between cash goods and credit goods or cash-in-advance constraint on investment),.and more general form of interest rate rule in which the nominal interest rate responds to real income as well as to inflation. Re-examining our discussion in those extended frameworks deserves further research.

Appendix

The coefficient matrix is expressed as

$$J = \begin{bmatrix} (1-\tau_0)(1-\phi)\, y_k(k^*,\lambda^*) - c_k(k^*,\lambda^*) - \delta & (1-\tau_0)(1-\phi)\, y_\lambda(k^*,\lambda^*) - c_\lambda(k^*,\lambda^*) \\ -\lambda^* \hat{r}_k(k^*,\lambda^*) & -\lambda^* \hat{r}_\lambda(k^*,\lambda^*) \end{bmatrix}.$$

Note that from (22), (27) and (28) all of the functions $y(.)$, $\hat{r}(.)$ and $\pi(.)$ are of Cobb-Dougla forms. Thus we find that the partial derivatives in J are respectively given by the following:

$$y_k(k^*,\lambda^*) = \frac{\alpha(1+\gamma)}{\Delta}\frac{y^*}{k^*},$$

$$y_\lambda(k^*,\lambda^*) = \frac{1-\alpha}{\Delta}\frac{y^*}{\lambda^*},$$

$$\hat{r}_k(k^*,\lambda^*) = -\frac{\phi(\alpha\gamma+1)+\gamma(1-\alpha)}{\Delta}\frac{\hat{r}^*}{k^*},$$

$$\pi_k(k^*,\lambda^*) = -\frac{(1-\alpha)(\gamma+\phi)}{\eta\Delta}\frac{\pi^*}{k^*},$$

$$\pi_\lambda(k^*,\lambda^*) = \frac{1-\alpha}{\eta\Delta}\frac{\pi^*}{\lambda^*},$$
$$\hat{r}_\lambda(k^*,\lambda^*) = \frac{(1-\phi)(1-\alpha)}{\Delta}\frac{\hat{r}^*}{\lambda^*},$$
$$c_k(k^*,\lambda^*) = -c^*\frac{\hat{r}_k(k^*,\lambda^*)+\pi_k(k^*,\lambda^*)}{1+\rho+\pi^*},$$
$$c_\lambda(k^*,\lambda^*) = -(c^*)^2[1+\pi^*+\rho+\lambda^*(\hat{r}_\lambda(k^*,\lambda^*)+\pi_k(k^*,\lambda^*))].$$

As a result, using

$$\frac{y^*}{k^*} = \frac{\rho+\delta}{\alpha(1-\tau_0)(1-\phi)}, \quad \frac{c^*}{k^*} = \frac{\rho+\delta[1-\alpha(1-\phi)]}{\alpha(1-\phi)} \ \text{ and } \ \hat{r}^* = \rho+\delta,$$

we express the trace and determinant of J in the following way:

$$\begin{aligned}
\text{trace } J &= \frac{\rho+\delta}{\Delta}((1+\gamma)-(1-\alpha)(1-\phi))-\delta \\
&- \frac{c}{1+\pi^*+\rho}\left(\frac{\phi(\alpha\gamma+1)+\gamma(1-\alpha)}{\Delta}\frac{\rho+\delta}{k^*}+\frac{(1-\alpha)(\gamma+\phi)}{\eta\Delta}\frac{\pi^*}{k^*}\right) \\
&= \Bigg[\rho\eta - \frac{1}{(1+\pi^*+\rho)\Delta}\frac{\rho+\delta[1-\alpha(1-\phi)]}{\alpha(1-\phi)} \\
&\times\{[\phi(\alpha\gamma+1)+\gamma(1-\alpha)](\rho+\delta)\eta+(1-\alpha)(\gamma+\phi)\pi^*\}\Bigg]\frac{1}{\eta}.
\end{aligned}$$

$$\begin{aligned}
\det J &= -[(1-\tau_0)(1-\phi)y_k(k^*,\lambda^*)-c_k(k^*,\lambda^*)-\delta][\lambda^*\hat{r}_\lambda(k^*,\lambda^*)] \\
&+[(1-\tau_0)(1-\phi)y_\lambda(k^*,\lambda^*)-c_\lambda(k^*,\lambda^*)][\lambda^*\hat{r}_k(k^*,\lambda^*)] \\
&+\lambda^*[-c^*\frac{\hat{r}_k(k^*,\lambda^*)+\pi_k(k^*,\lambda^*)}{1+\rho+\pi^*}\left(\frac{(1-\phi)(1-\alpha)}{\Delta}\frac{\hat{r}^*}{\lambda^*}\right)+(c^*)^2[1+\pi^*+\rho+\lambda^*(\hat{r}_\lambda(k^*,\lambda^*) \\
&+\pi_k(k^*,\lambda^*)]\left[-\frac{\phi(\alpha\gamma+1)+\gamma(1-\alpha)}{\Delta}\frac{\hat{r}^*}{k^*}\right]-\delta\lambda^*\frac{(1-\phi)(1-\alpha)}{\Delta}\frac{\hat{r}^*}{\lambda^*} \\
&= -\frac{(1-\phi)(1-\alpha)}{\Delta}(\rho+\delta)\frac{c^*}{k^*}-\frac{c^*}{k^*}\frac{\rho+\delta}{\Delta}\left(\frac{\phi(1-\alpha)\pi^*}{(1+\pi^*+\rho)\eta}+\phi(\alpha\gamma+1)+\gamma(1-\alpha)\right) \\
&= -\frac{\rho+\delta[1-\alpha(1-\phi)]}{\alpha(1-\phi)}\frac{\rho+\delta}{\Delta}\left[(\gamma+1)(1-\alpha+\alpha\phi)+\frac{\phi(1-\alpha)\pi^*}{(1+\pi^*+\rho)\eta}\right],
\end{aligned}$$

References

[1] Benhabib, J., and Eusepi, S.(2005), "The Design of Monetary and Fiscal Policy: A Global Perspective", *Journal of Economic Theory* 123, 40-73.

[2] Benhabib, J., Schmitt-Grohé, S., and Uribe, M. (2001), "Monetary Policy and Multiple Equilibria", *American Economic Review* 91, 167-186.

[3] Carlstrom, C., T., and Fuerst, T., S. (2005), "Investment and Interest Rate Policy: a Discrete Time Analysis", *Journal of Economic Theory* 123, 4-20.

[4] Dromel, N., L., and Pintus, P., A. (2006), "Are Progressive Fiscal Rules Stabilizing?", unpublished manuscript.

[5] Dupor, B. (2001), "Investment and Interest Rate Policy", *Journal of Economic Theory* 98, 85-113.

[6] Edge, R., M., and Rudd, J., B. (2007), "Taxation and the Taylor Principle", *Journal of Monetary Economics* 54, 2554-2567.

[7] Guo, J.-T., and Harrison, S., G (2004), "Balanced-Budget Rules and Macroeconomic (In)stability", *Journal of Economic Theory* 119, 357-363.

[8] Guo, J.-T., and Lansing, K., J. (1998), "Indeterminacy and Stabilization Policy", *Journal of Economic Theory* 82, 481-490.

[9] Feldstein, M. (1976), "Inflation, Income Taxes and the Rate of Interest: A Theoretical Analysis," *American Economic Review* 66, 809-820.

[10] Fujisaki, S. (2007), "Monetary Policy Rules and Equilibrium Determinacy with Capital Accumulation", unpublished manuscript.

[11] Fujisaki, S. and Mino, K. (2007), "Generalized Taylor Rule and Determinacy of Growth Equilibrium", *Economics Bulletin* 15, No.11, 1-7.

[12] Hansen, G. D. (1985), "Indivisible Labor and the Business Cycle", *Journal of Monetary Economics* 16, 309-327.

[13] Itaya, J. and Mino, K. (2004), "Inflation, Transaction Costs and Indeterminacy in Monetary Economies with Endogenous Growth", *Economica* 70, 1-20.

[14] Itaya, J. and Mino, K. (2007), "Technology, Preference Structure and the Growth Effect of Momey Supply", *Macroeconomic Dynamics* 11, 589-612.

[15] Kurozumi, T. (2005), "Distortionary Taxation, Interest Rate Policy, and Equilibrium Determinacy", unpublished manuscript.

[16] Leeper, E., M. (1991), "Equilibria under 'Active'and 'Passive' Monetary and Fiscal Policies", *Journal of Monetary Economics* 27, 129-147.

[17] Leitha, C. and von Thaddenb, L. (2007), "Monetary and Fiscal Policy Interactions in a New Keynesian Model with Capital Accumulation and Non-Ricardian Consumers", forthcoming in *Journal of Economic Theory.*

[18] Linnemann, L. (2006), "Interest Rate policy, debt, and indeterminacy with distortionary taxation, *Journal of Economic Dynamics and Control* 30, 487-510.

[19] Linnemann, L. (2008), "Balanced Budget Rules and Macroeconomic Stability with Non-Separable Utility", *Journal of Macroeconomics* 30, 199-215.

[20] Lubik, T., A. (2003), "Investment Spending, Equilibrium Determinacy, and the Interactions of Monetary and Fiscal Policy, Economics", Working Paper Archive 490, Department of Economics, Johns Hopkins University.

[21] Meng, Q. and Yip, C-K. (2004), "Investment, Interest Rate Rules, and Equilibrium Determinacy", *Economic Theory*, 23, 863-878.

[22] Schmitt-Grohé, S., and Uribe, M. (1997), "Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability", *Journal of Political Economy* 105, 976-1000.

[23] Taylor, J., B. (1993), "Discretion versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy* 39, 195-214.

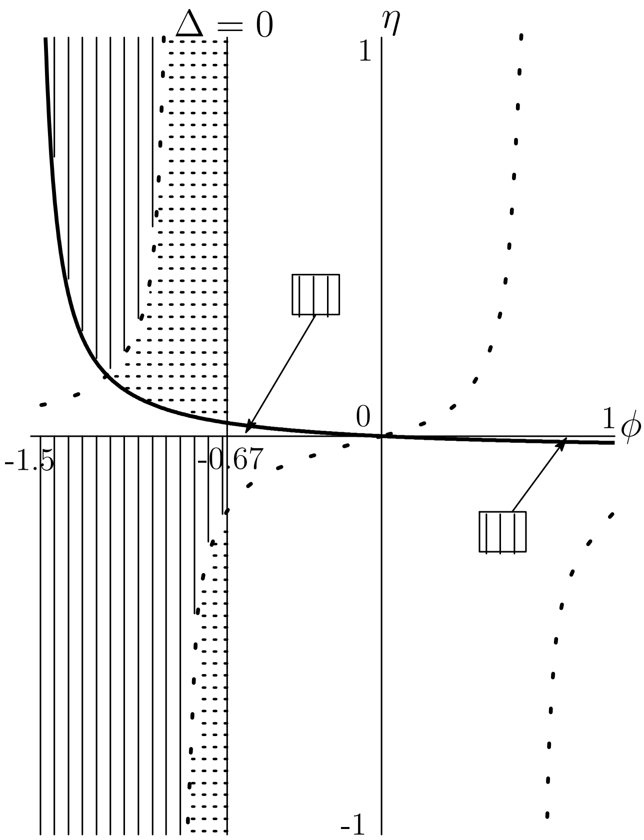

Figure 1. Equilibrium determinacy