



# **Discussion Papers In Economics And Business**

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Discussion Paper 08-36

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## **Abstract**

Based on the investment theory of Abel and Eberly (1994), we develop an analytical model of adjustment costs, which produces a sigmoidal investment function. We also estimate the piecewise linear investment function, which includes as special cases linear models, models with one threshold, the original model of Abel and Eberly, which has two thresholds, and sigmoidal models. Empirical evidence clearly supports the sigmoidal model. The threshold estimate of Tobin's  $q$  is 0.91. The investment ratio does not respond at value of Tobin's  $q$  below 0.91, but begins to react sensitively as Tobin's  $q$  passes 0.91.

**Keywords:** Tobin's  $q$ , financial constraints, irreversibility of investment, unlisted Japanese firms, piecewise linear function

**JEL classification:** E22, G31

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## THE SIGMOIDAL INVESTMENT FUNCTION\*

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Investment is one of the most influential factors for explaining macroeconomic booms and slumps because of its volatile nature. However the models derived from neoclassical theory have not performed well when compared with ad hoc accelerator models of investment (see, e.g., Bernanke et al., 1988).

There are at least five reasons for the poor empirical performance of neoclassical models. First, firms might face the financial constraints, which neoclassical theory ignores. According to neoclassical theory, Tobin's  $q$  is a sufficient statistic for the investment ratio. However, many empirical studies have found the cash flow variable has a statistically significant effect on investment. Firms facing financial constraints may be unable to invest even if they wish to. Fazzari et.al (1998), having divided their sample into groups of firms with high and low dividends, find that severely financially constrained firms react more sensitively to cash flow. They also demonstrate that firms are not indifferent between using internal and external funds for investment. Having divided their sample into firms with main banks and those without, Hoshi et al. (1991) estimate separate investment equations for these groups. Having found that the

estimated coefficient of the cash flow variable is significantly smaller for firms with main banks, Hoshi et al. (1991) argue that main banks mitigate the asymmetric information problem and reduce the agency cost of lending. From this viewpoint, cash flow is an important component of internal funds, and the agency cost of internal funds is lower than that of external funds. Thus, the investment behaviour of firms is sensitive to the volume of internal funds.

Second, the fundamentals that drive investment might be mismeasured. In particular, most studies use average  $q$  as a measure of fundamentals. Average  $q$  in turn is defined as the ratio of the firm's intrinsic value to the replacement cost of its assets. Because the firm's intrinsic value is unobservable, most authors have used its market value as a proxy. However, people might make mistakes in evaluating the firm's intrinsic value and/or may speculate on the stock market. Thus, the stock market value may be misleading, and might be a poor proxy for the intrinsic value. Instead of using stock market values, Cummins et al. (2006) propose using analysts' forecasts to measure average  $q$ .

Third, in practice, fixed investment is often infrequent and lumpy. Among others, Abel and Eberly (1994, 2002) formalise this idea in constructing neoclassical non-linear investment functions. The derived investment functions incorporate an inactive region in which investment does not respond to Tobin's  $q$ . In these models, the adjustment costs of investment incorporate not only standard convex costs, but also lumpy fixed costs and/or those related to the irreversibility of investment. Some authors assert that adjustment costs are asymmetric because firms might incur additional costs above a certain threshold (e.g., when their investments exceed replacement investment). The asymmetric adjustment costs with a certain threshold could also produce an inactive

region in the investment function.

Fourth, some researchers argue that because the lumpy nature of investment originates at the plant level, aggregation over plants distorts the shape of the investment function. To understand the relationship between adjustment costs and the non-linearity in investment, among others, Caballero et al. (1995, 1997), Goolsbee and Gross (2000) and Cooper and Haltiwanger (2006) use plant-level data.

Fifth, Cooper and Ejarque (2001) argue that allowing the profit function at the firm level to be strictly concave to reflect its market power is sufficient to replicate regression results based on q-theory when profits significantly affect investment.

Empirical findings largely support the non-linear investment function instead of the linear one, but there is no consensus on the shape of the non-linearity. For example, the investment theory of Abel and Eberly (1994) suggests the non-linear investment function illustrated in Figure 1. The investment function in Figure 1 incorporates a part in which investment is insensitive to Tobin's  $q$ . However Barnett and Sakellaris (1998) suggest a different shape based on their empirical work. Making use of grid methods, they suggest that the investment ratio is a convex function of average  $q$  for small values of  $q$ , and is a concave function for larger values of  $q$ . The purpose of this paper is to provide estimates of a non-linear investment function, taking into account the five problems mentioned above.

[Figure 1 around here]

Our empirical findings indicate that the firm's investment function is non-linear and has three parts and two thresholds. According to our estimated investment function, investment is insensitive to Tobin's  $q$  for small values of  $q$ , is sensitive above the first threshold value of  $q$ , and is relatively insensitive above the second threshold value of  $q$ .

Although we used piecewise linear functions, incorporating the assumption of smoothness gives the estimated function a sigmoidal shape.

The outline of the paper is as follows. In Section 1, we explain the relationship between various kinds of adjustment costs and the corresponding types of non-linearity in investment functions. In Section 2, we describe our approach to estimation. In Section 3, we explain how to estimate a piecewise linear model with two thresholds, choose the best model among these models with two thresholds and compare this model with linear models and non-linear models that have one threshold. In Section 4, we report our empirical results and compare them with those from other studies. In Section 5, we discuss the implications of our findings and directions future research.

## **1. Adjustment Costs and Types of Non-linear Investment Functions**

In this section, we review the relationship between adjustment costs and the type of non-linearity in investment functions.

### **1.1. Standard Convex Adjustment Costs**

The standard neoclassical investment literature assumes convex adjustment costs. The simplest form is represented by the quadratic adjustment cost function. In this case, one can derive the investment ratio as a linear function of Tobin's  $q$  (as shown in Appendix 1).

### **1.2. The Irreversibility of Investment in Abel and Eberly (1994)**

Arrow (1968) argued that gross investment cannot take a negative value because of large disposal costs. Instead of following Arrow (1968) in assuming that the adjustment cost becomes infinite, Abel and Eberly (1994) quantify the irreversibility of investment as the difference between the purchase price ( $p_{it}^+$ ) and the resale price ( $p_{it}^-$ )

of investment goods at time  $t$ , assuming that  $p_{it}^+ > p_{it}^-$ . The subscript  $i$  denotes the price of investment goods.

Following Barnett and Sakellaris (1998), we briefly explain how incorporating this type of investment irreversibility into the adjustment cost generates an investment function that has an inactive component.

The augmented adjustment cost function  $G(I_t, K_{t-1})$  takes the form of

$$\begin{aligned} G(I_t, K_{t-1}) &= C(I_t, K_{t-1}) + p_{it}^+ I_t D(I_t > 0) + p_{it}^- I_t D(I_t < 0) \quad \text{if } I_t \neq 0, \\ &= C(I_t, K_{t-1}) \quad \text{if } I_t = 0, \end{aligned} \quad (1-1)$$

where  $I_t$ ,  $C(I_t, K_{t-1})$  and  $D$  denote gross investment, the standard convex adjustment cost function, and an indicator function, respectively. The indicator function  $D$  takes a value of unity when the condition in parentheses is satisfied and is zero otherwise. Note that the augmented adjustment cost  $G$  is discontinuous at the value of  $I_t = 0$ .

The first-order condition for investment is given by:

$$p_{it}/p_t + \partial G(I_t, K_{t-1})/\partial I_t = q_t/p_t \quad \text{for } I_t \neq 0, \quad (1-2)$$

where  $p_t$  and  $q_t$  denote the output price and the shadow price of capital at time  $t$ , respectively (see Appendix 1 for details). Denoting the right-hand neighbourhood and the left-hand neighbourhood of zero by  $0^+$  and  $0^-$ , respectively, we have:

$$\begin{aligned} p_{it}^+/p_t + \partial C(0^+, K_{t-1})/\partial I_t &= q_t/p_t \quad \text{at } I_t = 0^+, \\ \text{and} \\ p_{it}^-/p_t + \partial C(0^-, K_{t-1})/\partial I_t &= q_t/p_t \quad \text{at } I_t = 0^-. \end{aligned} \quad (1-3)$$

Dividing (1-3) by  $p_{it}^+/p_t$  throughout yields

$$\begin{aligned} 1 + C_I^+ / (p_{it}^+/p_t) &= q_t/p_{it}^+ \quad \text{at } I_t = 0^+, \\ \text{and} \\ p_{it}^-/p_{it}^+ + C_I^- / (p_{it}^+/p_t) &= q_t/p_{it}^+ \quad \text{at } I_t = 0^-, \end{aligned} \quad (1-4)$$

where  $C_I^+ = \partial C(0^+, K_{t-1}) / \partial I_t$  and  $C_I^- = \partial C(0^-, K_{t-1}) / \partial I_t$ , respectively. Equations (1-4) show that investment is a discontinuous function of  $q$ ; this is illustrated in Figure 1.

### 1.3. Linear Adjustment Cost in Barnett and Sakellaris (1998)

Instead of assuming that the purchase price exceeds the resale price of investment goods, Barnett and Sakellaris (1998) postulate that the firm incurs an incremental linear adjustment cost  $\theta$  when its investment exceeds replacement investment. They assume an asymmetric adjustment cost,

$$\begin{aligned} G(I_t, K_{t-1}) &= C(I_t, K_{t-1}) && \text{if } (I_t / K_{t-1}) \leq \delta \\ &= C(I_t, K_{t-1}) + \theta(I_t - \delta K_{t-1}) && \text{if } (I_t / K_{t-1}) > \delta. \end{aligned} \quad (1-5)$$

The investment function based on the augmented adjustment cost (1-5) is illustrated in Figure 2. Under this assumption, investment is inactive at the value of the investment ratio  $I_t / K_{t-1} = \delta$  in Figure 2.

### 1.4. An Alternative Specification of Irreversibility

Alternatively, taking into account the irreversibility of investment, one could specify the augmented adjustment cost as:

$$\begin{aligned} G(I_t, K_{t-1}) &= \frac{\alpha_1}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} && \text{if } (I_t / K_{t-1}) \leq \delta \\ &= \frac{\alpha_2}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} + \theta(I_t - \delta K_{t-1}) && \text{if } (I_t / K_{t-1}) > \delta, \end{aligned} \quad (1-6)$$

where  $\alpha_1 > \alpha_2 > 0$ . The augmented adjustment cost is composed of two quadratic functions. It is asymmetric and thus not differentiable at  $(I_t / K_{t-1}) = \delta$ . Equation (1-6) is illustrated in Figure 3.

[Figure 3 around here]

As in Barnett and Sakellaris (1998), equation (1-6) includes the incremental linear cost

$\theta(I_t - \delta K_{t-1})$ , which implies that the firm incurs an additional linear cost as well as the standard convex cost when its investment exceeds replacement investment. However, given the assumption that  $\alpha_1 > \alpha_2 > 0$ , the firm also incurs increasingly large costs when making negative net investments (when  $I_t - \delta K_{t-1} < 0$ ) because of the irreversibility of investment. As argued by Arrow (1968), the firm incurs prohibitive costs as  $\alpha_1$  goes to infinity with the value of  $\alpha_2$  held constant.

The specification of the augmented adjustment cost in (1-6) together with the first-order condition for investment (1-2) yields the non-linear investment function illustrated in Figure 4.

[Figure 4 around here]

The slope of the third part of the non-linear investment function in Figure 4 is much steeper than that of the first part. This is because the slopes of the first and third parts of the investment function in Figure 4 depend on  $(1/\alpha)$  (see Appendix 1), and because of the assumption that  $\alpha_1 > \alpha_2 > 0$ . The larger the value of  $\alpha$ , the less steep is the slope of the investment function in Figure 4. Indeed, as  $\alpha_1$  tends to infinity, the slope of the first part approaches the horizontal line at the height of  $\delta$ , and the investment function looks like the one illustrated in Figure 5.

[Figure 5 around here]

### **1.5. Potentially Prohibitive Adjustment Costs for Extremely Large Investments**

The assumption of standard convex adjustment costs implies that the second derivative of the adjustment cost function is positive. There are usually two reasons for this. First, as the firm expands its investment, the costs of retraining labour, and management and other associated costs become increasingly high. Second, investment goods prices might rise as the firm purchases more investment goods.

In this sub-section, we postulate that the cost becomes prohibitive as the firm's investment ratio exceeds a certain threshold. We believe that there is some limitation to expanding investment, partly because, as the firm significantly expands its investment, management cannot consider an infinite number of investment projects at the same time and partly because supply shortages of investment goods eventually make investment goods prices prohibitively high.

Suppose that the augmented adjustment cost is given by:

$$\begin{aligned}
G(I_t, K_{t-1}) &= G_1 = \frac{\alpha_1}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} && \text{if } (I_t / K_{t-1}) \leq \delta \\
&= G_2 = \frac{\alpha_2}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} + \theta(I_t - \delta K_{t-1}) && \text{if } \delta < (I_t / K_{t-1}) \leq \lambda \\
&= G_3 = \frac{\alpha_3}{2} \left[ \frac{I_t}{K_{t-1}} - \left\{ \lambda - \frac{\alpha_2}{\alpha_3} (\lambda - \delta) \right\} \right]^2 K_{t-1} \\
&\quad + \theta(I_t - \delta K_{t-1}) + \frac{\alpha_2}{2} \left( 1 - \frac{\alpha_2}{\alpha_3} \right) (\lambda - \delta)^2 K_{t-1} && \text{if } (I_t / K_{t-1}) > \lambda, \tag{1-7}
\end{aligned}$$

where  $\alpha_3$  and  $\lambda$  are positive constant parameters such that  $\alpha_3 > \alpha_2$  and  $\lambda > \delta$ . The augmented adjustment cost function (1-7) is illustrated in Figure 6.

[Figure 6 around here]

The augmented adjustment cost function in (1-7), together with the first-order condition for investment (1-2) and the assumption of  $\alpha_1$  being infinite, yields the non-linear investment function illustrated in Figure 7.

[Figure 7 around here]

The non-linear investment function now has two kinks. Above the first threshold, the firm increases its investment ratio dramatically, as  $q$  increases. However, there is some saturation point for the investment ratio,  $\lambda$ , beyond which the investment ratio

increases only moderately as  $q$  increases. The investment function in Figure 7 is S-shaped or ‘sigmoidal’. If we assume a ‘smooth’ investment function, then a logistic curve would be a good approximation.

Empirical studies such as those of Eberly (1997) and Barnett and Sakellaris (1998) suggest this shape. Abel and Eberly (2002) argue that aggregation over heterogeneous capital goods might yield an S-shaped investment function. To choose between a linear model, a logistic model and a hybrid of the two, Honda and Suzuki (2000) use the Akaike Information Criterion and the Schwartz Information Criterion. They also suggest that their investment function takes the logistic form for a sample of large Japanese manufacturers.

## 2. The Basic Model

Taking into account the relationship between adjustment costs and the type of non-linearity of investment functions, and following Tachibana’s (2007) analysis of piecewise linear models, we estimate a piecewise linear investment function for the  $i$  th firm,

$$\begin{aligned}
\frac{I_t}{K_{t-1}} &= \beta_0^1 + \beta_1^1 q_{t-1} + \gamma_1 L_{t-1} + \gamma_2 L_{t-1} \cdot D_t \quad \text{if } q_{t-1} \leq q_1 \\
&= \beta_0^2 + \beta_1^2 q_{t-1} + \gamma_1 L_{t-1} + \gamma_2 L_{t-1} \cdot D_t \quad \text{if } q_1 < q_{t-1} \leq q_2 \\
&= \beta_0^3 + \beta_1^3 q_{t-1} + \gamma_1 L_{t-1} + \gamma_2 L_{t-1} \cdot D_t \quad \text{if } q_{t-1} > q_2,
\end{aligned} \tag{2-1}$$

where  $L_{t-1}$  and  $D_t$  denote the log of total fixed assets, and a dummy variable that is unity if the  $i$  th firm has never issued corporate bonds and zero otherwise, respectively. The parameters  $\beta$  and  $\gamma$  are to be estimated. The first and second threshold values of Tobin’s  $q$  are denoted by  $q_1$  and  $q_2$ , respectively, and  $q_t$  denotes Tobin’s  $q$  at time  $t$ .

To avoid simultaneity bias, we use one-period lagged values for  $q_t$  in equation (2-1). Hansen (1999), Chida (2003), and Bo et al. (2006) carefully analyse threshold regression models. However, none of them uses Tobin's  $q$  as a right-hand-side threshold variable, and their focus differs from ours. Because we are interested in testing the validity of Tobin's  $q$  theory, the right-hand-side threshold variable should be Tobin's  $q$ . We have explicitly clarified the relationship between the types of adjustment costs and the shape of non-linearity of the investment function in the previous section to extend Tobin's  $q$  theory in such a way that the investment function has an inactive part.

### 2.1. Financial Constraints

Based on a similar sample to ours, Honda and Suzuki (2006) find that financial constraints significantly affect the investment ratio. To incorporate financial constraints, equation (2-1) includes the terms  $L_{t-1}$  and  $L_{t-1} \cdot D_t$  as right-hand-side variables. When a firm has a large amount of capital stock at time  $t-1$ ,  $L_{t-1}$  becomes large. Given a sufficient stock of capital, such a firm is expected to make less investment at time  $t$ . Hence, the expected sign of  $\gamma_1$  is negative. On the other hand, we expect  $\gamma_2$  to be positive. Firms often use fixed capital as collateral when they borrow money, and fixed capital is a rough indicator of 'creditworthiness', which is particularly important for smaller firms that have never issued corporate bonds. The larger the value of collateral, the less likely is the financial constraint to be binding, and thus the larger is the investment ratio. Note that we include the interaction dummy  $L_{t-1} \cdot D_t$  rather than the simple dummy variable  $D_t$  in equation (2-1). This is because including the simple dummy variable  $D_t$  together with individual dummy variables in a panel data regression causes perfect multi-collinearity.

Among others, Abel and Eberly (1994, 2002), Eberly (1997) and Barnett and Sakellaris (1998) exclude financial constraints from their non-linear estimated equations. However, Fazzari et al. (1998), Hoshi et al. (1991) and Honda and Suzuki (2006), among others, argue that financial constraints are important determinants of investment ratios. Thus, omitting these financial constraint variables from the non-linear regressions might seriously bias the estimates. Thus, we include  $L_t$  and  $L_t \cdot D_t$  on the right-hand side of equation (2-1).

## **2.2. Sample Selection**

We use data on unlisted Japanese auto parts suppliers for a number of reasons. First, data on listed large corporations might distort the true investment behaviour of firms because large corporations tend to produce multiple products in different fields. When there are multiple products, the relationship between the investment ratio and Tobin's  $q$  is less clear.

At the other extreme, some authors use plant-level data. Doms and Dunne (1998) find that most fixed investments are infrequent but lumpy at the plant level. Incorporating the finding of Doms and Dunne (1998), Caballero et al. (1995) develop a model in which fixed investments are not made gradually but are undertaken only when the difference between the desired and the existing capital stock exceeds a certain threshold.

We do not use plant-level data because it is difficult to measure Tobin's  $q$  at the plant level. In addition, firms examine all potential projects when making decisions on fixed investments. They do not investment in one plant without considering the situations in other plants.

There is another advantage using data on unlisted Japanese auto parts suppliers. Cooper and Ejarque (2003), among others, point out that the  $q$ -theory of investment

does not apply to firms that have market power. Unlisted Japanese auto parts suppliers supply auto parts to giant auto manufacturers such as Toyota, and are less likely to have market power. Thus, sampling unlisted smaller firms that have limited market power is appropriate for our purposes.

### 2.3. Data on Tobin's q

Because we use data on unlisted firms, stock price data are not available. Therefore, we have constructed our measure of Tobin's q based on the balance sheet and the profit and loss account of each firm. (See Appendix 2 for details.)

## 3. Piecewise Linear Models

As was explained in Section 1, the investment theory on Tobin's q suggests that there are at most two thresholds in a piecewise linear model. We first explain how we estimate a piecewise linear model with two thresholds, and then discuss the procedure for choosing between models that contain zero, one, or two thresholds.

### 3.1. Estimation of a Model with Two Thresholds

In this sub-section, we explain how to estimate a piecewise linear model with two thresholds. Rewriting equation (2-1) as an econometric model yields:

$$\begin{aligned}
 y_{it} = & \left( \beta_0^1 + \beta_1^1 q_{it-1} \right) I(q_{it-1} \leq q_1) \\
 & + \left( \beta_0^2 + \beta_1^2 q_{it-1} \right) I(q_1 < q_{it-1} \leq q_2) \\
 & + \left( \beta_0^3 + \beta_1^3 q_{it-1} \right) I(q_{it-1} > q_2) \\
 & + \gamma_1 L_{it-1} + \gamma_2 L_{it-1} \cdot D_{it} + \phi W_i + u_{it}, \tag{3-1}
 \end{aligned}$$

where we define  $y_{it} = I_{it} / K_{it-1}$ . Subscript  $i$  denotes the  $i$ th firm.  $I(\cdot)$  and  $u_{it}$  denote the indicator function and the disturbance term, respectively.  $W_i$  denotes the individual dummy variable, which is unity for the  $i$ th firm and zero otherwise, and  $\phi$

is the corresponding coefficient parameter. The disturbance terms are assumed to be independently and normally distributed. Given the values of  $L_{it-1}$ ,  $D_{it}$ ,  $W_i$ , and  $u_{it}$ , equation (3-1) is illustrated in Figure 8. Our estimation method is based on grid methods. We first provide a combination of respective numbers for the values of  $q_1$  and  $q_2$ , and then estimate equation (3-1) by using the fixed-effects model.

[Figure 8 around here]

To estimate equation (3-1), given the values of  $q_1$  and  $q_2$ , we transform equation (3-1). The conditions for our piecewise linear model to be continuous at the respective points  $q_1$  and  $q_2$  are given by:

$$\begin{aligned} \beta_0^1 + \beta_1^1 q_1 &= \beta_0^2 + \beta_1^2 q_1 \\ \text{and} \\ \beta_0^2 + \beta_1^2 q_2 &= \beta_0^3 + \beta_1^3 q_2. \end{aligned} \tag{3-2}$$

Solving for  $\beta_0^1$  and  $\beta_0^2$  in equation (3-2), expressing these two parameters in terms of  $\beta_0^3$ ,  $\beta_1^1$ ,  $\beta_1^2$ , and  $\beta_1^3$  and then substituting the resulting equations into equation (3-1) yields:

$$y_{it} = \beta_0^3 + \beta_1^1 G_{it} + \beta_1^2 J_{it} + \beta_1^3 M_{it} + \gamma_1 L_{it-1} + \gamma_2 L_{it-1} \cdot D_{it} + u_{it},$$

where

$$G_{it} = (q_{it-1} - q_1)I(q_{it-1} \leq q_1),$$

$$J_{it} = (q_1 - q_2)I(q_{it-1} \leq q_1) + (q_{it-1} - q_2)I(q_1 \leq q_{it-1} < q_2),$$

and

$$M_{it} = q_{it-1} [1 - I(q_{it-1} \leq q_1) - I(q_1 < q_{it-1} \leq q_2)] + q_2 [I(q_{it-1} \leq q_1) + I(q_1 < q_{it-1} \leq q_2)]. \tag{3-3}$$

One can estimate equation (3-3) using standard computer software.

Our sample is an unbalanced panel. There are 1,553 observations on 104 firms for the sample period from 1977 to 2006. However, after excluding outliers, our final number

of observations is 1,527. (See A2-1 in Appendix 2.) Table 1 provides summary statistics for our sample of 1,527 observations. Figure 9 plots these observations with the investment ratio on the vertical axis and the lagged Tobin's  $q$  on the horizontal axis. This shows that the ratio of investment to capital stock jumps upward near the value of one of the lagged Tobin's  $q$  values.

[Table 1 around here]

[Figure 9 around here]

### **3.2. The Best Model with Two Thresholds**

We first choose the best model among those models with two thresholds. Our procedure comprises two steps. First, we select our benchmark model by using 'likelihood' as our criterion. Then, in the second step, we compare this benchmark model with all other models, and choose the best model by using as our criterion the 'encompassing principle' (based on non-nested J-tests).

#### **3.2.1. Likelihood**

We first provide a combination of respective numbers for the values of  $q_1$  and  $q_2$ , and estimate equation (3-3). Similarly, we repeat the same procedure for all combinations of the respective values of  $q_1$  and  $q_2$ , and choose the best model, which is the one that produces the largest likelihood. We define this model as our benchmark model.

More concretely, we vary the values of  $q_2$  from 1.0 to 2.5 by 0.01 and vary the values of  $q_2 - q_1$  from 0.0 to 2.5 by 0.01 respectively, and calculate the log likelihood for each case. It turns out that the pair  $q_1 = 0.91$  and  $q_2 = 1.20$  yields the largest log likelihood of 384.21. Table 2 reports the estimation results of this benchmark model.

[Table 2 around here]

### 3.2.2. J-Tests

To select the best estimated model with two thresholds, we use Davidson and Mackinnon's (1981) non-nested testing procedure based on J-Tests. The model selection procedure proceeds as follows. Suppose that we wish to test the above benchmark model B, with  $q_1 = 0.91$  and  $q_2 = 1.20$ , against an alternative model A with  $q_1 = q_1^A$  and  $q_2 = q_2^A$ , where  $q_1^A$  and  $q_2^A$  are exogenously given values. We first estimate the alternative model A with  $q_1 = q_1^A$  and  $q_2 = q_2^A$  in equation (3-3), and obtain the predicted values of  $y_{it}$ , which we denote by  $\hat{y}_{it}^A$ . Superscript A indicates model A. In the second step, we add these predicted values  $\hat{y}_{it}^A$  to the right-hand side of the benchmark model B, in which  $q_1 = 0.91$  and  $q_2 = 1.20$ , and apply an artificial regression to the resulting equation. We are interested in the significance of the coefficient estimate of the predicted values  $\hat{y}_{it}^A$ . If it is statistically significant, the implication is that the alternative model A has some additional information that our benchmark model B does not have. Otherwise, model A adds no information to our benchmark model B. Table 3 reports the results of these tests. Because none of the t-values indicates significance, all alternative models have no additional information. (All t-statistics reported in the paper are based on heteroskedasticity-consistent standard errors.)

[Table 3 around here]

We then switch the respective roles of models A and B, and test the significance of the coefficient estimate of the predicted values  $\hat{y}_{it}^B$  in another artificial regression model with  $q_1 = q_1^A$  and  $q_2 = q_2^A$ . If it is significant, our benchmark model has some additional information that model A does not have. Otherwise, the benchmark model adds no information to model A. Table 4 reports the results of these tests. We reject the

null hypothesis that our benchmark model has no additional information at least at the 10% significance level in 77 out of 83 cases. We cannot reject the null hypothesis in only six cases. The respective values of  $q_1$  and  $q_2$  in these six alternative models cluster around 0.91 and 1.2 in our benchmark model. This suggests that these six alternative models are so similar to our benchmark model that the tests are not sufficiently powerful to discriminate between them.

[Table 4 around here]

In summary, we cannot reject the null hypothesis that an alternative model has no additional information in all the 83 cases reported in Table 3. In Table 4, we reject the null hypothesis that our benchmark model has no additional information at least at the 10% significance level in 77 cases. These results imply that our benchmark model dominates these 77 alternative models. We cannot reject the null hypothesis that our benchmark model has no additional information in six cases in Table 4. However, the respective values of  $q_1$  and  $q_2$  in these six alternative models are so close to those in our benchmark model that our tests do not have sufficient power to discriminate between them. These results indicate that our benchmark model is the best model among those non-linear models with two thresholds.

### **3.3. Linear Models and Models with One Threshold**

In the previous sub-section we found that our benchmark model is the best model among those models with two thresholds. In this sub-section we compare our benchmark model with linear models and with non-linear models with one threshold. For this, we use both standard F-tests and J-tests.

#### **3.3.1. F-Tests**

We have found that model (3-3) with  $q_1 = 0.91$  and  $q_2 = 1.20$  is the best model among models with two thresholds. To compare this model with the equivalent model with one threshold, we use a standard F-test to test the null hypothesis that  $\beta_1^1 = \beta_1^2$  in model (3-3) with  $q_1 = 0.91$  and  $q_2 = 1.20$ .

Substituting  $\beta_1^1 = \beta_1^2$  into equation (3-3) yields the restricted model:

$$y_{it} = \beta_0^3 + \beta_1^1(G_{it} + J_{it}) + \beta_1^3 M_{it} + \gamma_1 L_{it-1} + \gamma_2 L_{it-1} \cdot D_{it} + u_{it} . \quad (3-4)$$

We estimate (3-3) with  $q_1 = 0.91$  and  $q_2 = 1.20$ , estimate (3-4) with  $q_2 = 1.20$ , and then calculate the F-value, which turns out to be 6.24. Under the null hypothesis, this statistic has an F-distribution with (1, 1418) degrees of freedom. We reject the null hypothesis that  $\beta_1^1 = \beta_1^2$  at the 5% significance level. Therefore, we reject the model with one threshold.

Similarly, we compare our best model with two thresholds with a linear model, and test the null hypothesis that  $\beta_1^1 = \beta_1^2 = \beta_1^3$ . Substituting these two restrictions into equation (3-3) yields the restricted equation:

$$y_{it} = \beta_0^3 + \beta_1^3(G_{it} + J_{it} + M_{it}) + \gamma_1 L_{it-1} + \gamma_2 L_{it-1} \cdot D_{it} + u_{it} . \quad (3-5)$$

Having estimated (3-3) with  $q_1 = 0.91$  and  $q_2 = 1.20$ , and (3-5), we test the null hypothesis that  $\beta_1^1 = \beta_1^2 = \beta_1^3$ . The F-value turns out to be 7.48. This statistic has F-distribution with (2, 1418) degrees of freedom under the null. We reject the null hypothesis at the 1% significance level. Therefore, we reject the linear model.

### 3.3.2. Non-nested J-Tests

In sub-section 3.3.1, we confined our attention to model (3-3) with  $q_1 = 0.91$  and  $q_2 = 1.20$ , and tested restrictions on the coefficients in this model. In this sub-section, we first search for the best model among those with one threshold. Then, we compare

this model with the best model with two thresholds, which we validated in sub-section 3.2.

The procedure used to search for the best model with one threshold is the same as that used to search for the best model with two thresholds. In the first step, we find the benchmark model based on the likelihood; i.e., we find the one-threshold mode with the largest likelihood. In the second step, we use grid methods to compare the best model with two thresholds with all alternative models with one threshold, including the benchmark model with one threshold.

In the first step, we vary the value of the threshold  $q_1$  from 0.0 to 2.5 by 0.01, and find that the model with  $q_1 = 1.34$  gives the largest log likelihood of 381.8763. Our estimated benchmark model with  $q_1 = 1.34$  is reported in Table 5.

[Table 5 around here]

In the second step, we vary the value of threshold  $q_1$  from 0.5 to 2.4 by 0.1, which produces 20 alternative models with one threshold. We also include the benchmark model with one threshold (with  $q_1 = 1.34$ ) as another alternative model. Therefore, we compare the best model with two thresholds ( $q_1 = 0.91$  and  $q_2 = 1.20$ ) with these 21 alternative models with one threshold. Tables 6 and 7 report the results of the non-nested J-tests.

[Tables 6 and 7 around here]

The number in each cell in Table 6 is the t-value of the coefficient of the fitted variable  $\hat{y}_{it}^A$  in the corresponding alternative model with one threshold. Because none of these indicates statistical significance, the alternative models with one threshold have no additional information.

On the other hand, all the t-values in Table 7 indicate significance at least at the 5% level. This implies that the best model with two thresholds ( $q_1 = 0.91$  and  $q_2 = 1.20$ ) has some additional information that is not incorporated in any of the alternative models.

The exceptional case in Table 6 is that of  $q_1 = 1.20$ . In this case, the estimated coefficient of the fitted variable  $\hat{y}_{it}^A$  is unusually large. We suggest that this is the result of multi-collinearity. Recall that our best model with two thresholds has  $q_1 = 0.91$  and  $q_2 = 1.20$ .

The results of this section indicate that our best model with two thresholds ( $q_1 = 0.91$  and  $q_2 = 1.20$ ) is the best model among linear models, models with one threshold, and models with two thresholds.

#### **4. Results and Discussion**

In this section, we summarise our findings and discuss their relationship to those of existing studies.

##### **4.1. Non-linearity**

We apply the neoclassical theory of investment that specifies investment as a function of Tobin's  $q$ . With the introduction of non-convex adjustment costs into the model, Abel and Eberly (1994) show theoretically that the investment function incorporates an inactive portion in which investment does not respond to Tobin's  $q$ . Although the consensus seems to be that the empirical literature supports this theory and the notion at investment is a non-linear function of Tobin's  $q$ , there is no consensus about the type of non-linearity exhibited by the investment function.

For example, although Bo et al. (2006) rely on real-options theory and adopt a threshold variable that differs from Tobin's  $q$ , they essentially estimate an investment function with one threshold. Based on our evidence the investment function is non-linear and sigmoidal with two thresholds. This empirical finding confirms those of Eberly (1997), Barnett and Sakellaris (1998), and Honda and Suzuki (2000).

#### 4.2. Irreversibility

Our preferred estimated investment function is reported in Table 2. The estimate of the slope coefficient in the first regime,  $\beta_1^1$ , is 0.0134 and its t-value is 0.2362. This shows that the null hypothesis that the slope in the first regime is zero cannot be rejected. This suggests the existence of an inactive portion in which investment is insensitive to Tobin's  $q$ , because of the irreversibility of investment. This finding confirms the theory of Abel and Eberly (1994), and is also consistent with the recent empirical findings of Chirinko and Schaller (2008), who estimate a Euler equation and obtain empirical evidence of an irreversibility premium. Our estimates also indicate that this inactive part holds for the domain  $q \leq 0.91$ . When Tobin's  $q$  exceeds this threshold value of 0.91, the investment ratio starts to respond sharply to changes in Tobin's  $q$ .

What is the best estimate of the height of the investment ratio  $I/K$  for the domain  $q \leq 0.91$ ? The floor of the investment ratio in the first regime is given by:

$$\beta_0^1 + \gamma_1 L_{it-1} + \gamma_2 L_{it-1} \cdot D_{it}. \quad (4-1)$$

(See equation (2-1).)

Substituting our preferred estimates from Table 2 with  $q_1 = 0.91$  and  $q_2 = 1.20$  into equation (3-2) yields our preferred estimate for  $\beta_0^1$ . Making use of this estimate of  $\beta_0^1$  together with the estimates of  $\gamma_1$  and  $\gamma_2$  from Table 2 enables us to estimate

equation (4-1) for each firm. We obtain such estimates for 104 firms. The simple average of equation (4-1) over these 104 firms turns out to be 0.1992. This represents the estimated depreciation rate  $\delta$  in Figure 7.

### **4.3. Estimates of the Thresholds**

Our preferred estimate of the first threshold  $q_1$  is 0.91. This implies that a firm starts to increase its investment ratio when Tobin's  $q$  exceeds 0.91. This estimate 0.91 is smaller than those from existing studies. The corresponding benchmark estimates obtained by Barnett and Sakellaris (1998) are 1.95 for their first model and 1.13 for their second model. Honda and Suzuki (2000) report a corresponding benchmark estimate of 1.62.

There are too few empirical studies to draw definitive conclusions on why these estimates differ. However, our estimates may differ from theirs for several reasons. First, unlike Barnett and Sakellaris (1998) and Honda and Suzuki (2000), we control for the influence of financial constraints by adding the variables  $L$  and  $L \cdot D$  to the right-hand side of equation (2-1).

Second, our estimate of the first threshold applies to smaller unlisted firms whereas those of Barnett and Sakellaris (1998) and Honda and Suzuki (2000) apply to larger listed firms.

Third, our estimates are for Japanese firms whereas those of Barnett and Sakellaris (1998) are for US firms. More empirical evidence must be accumulated to explain estimation difference across different studies.

### **4.4. Financial Constraints**

To control for the potential effects of financial constraints, we added  $\gamma_1 L_{it-1}$  and  $\gamma_2 L_{it-1} \cdot D_t$  to the right-hand side of equation (2-1). Our estimate of  $\gamma_1$  is -0.1622 in

Table 2. The negative estimate implies that a firm with sufficient capital stock at the beginning of a period undertakes less investment in that period, as expected. The t-value of -10.2899 indicates that our estimate is highly significant.

$D_1$  in equation (2-1) is a dummy variable that is unity for firms that have never issued corporate bonds. Our estimate of  $\gamma_2$  is 0.0588 in Table 2. This implies that the level of the capital stock is a rough indicator of ‘creditworthiness’, and that relatively large firms can borrow and invest more easily. The result is as expected. The t-value of 2.3592 indicates that the parameter is highly significant. Our results support the notion that financial constraints matter and confirm those of in Honda and Suzuki (2006).

#### **4.5. Market Power**

Cooper and Ejarque (2001) argue that allowing the firm’s profit function to be strictly concave to reflect its market power is sufficient to replicate regression results based on q-theory in which profits significantly affect investment. Partially because of this argument, we have carefully selected our samples. More specifically, we have chosen to use data on auto parts suppliers. The sampled firms are small and unlisted. They are price takers and do not have the power to control their product prices. Therefore, the arguments of Cooper and Ejarque (2001) do not apply to our study.

#### **5. Concluding Remarks**

Based on the investment theory developed by Abel and Eberly (1994), we have provided a simple analytical model of adjustment costs, which produces a sigmoidal investment function. We have also estimated the piecewise linear investment function, which includes the linear model, a model with one threshold, the original model of Abel

and Eberly (1994) with two thresholds, and the sigmoidal model as special cases. Our empirical evidence clearly supports the sigmoidal investment function.

The sigmoidal investment function casts serious doubt on the credibility of existing large macro-econometric models that specify a linear investment function.

Is it possible to aggregate sigmoidal investment functions over firms? If so, under what conditions? Although answering these questions is important for understanding the volatile nature of investment in practice, they are beyond the scope of our research.

What is the threshold value of Tobin's  $q$  above which a firm undertakes new investment? Our estimate is 0.91. Are these threshold values the same across industries and countries? Do they change over time? Answers to these questions require estimated thresholds for different samples, which is a task for future research.

## Appendix 1. Derivation of the Standard Investment Function

Following Abel (1980, 1990), we briefly derive the standard investment function in this appendix. A firm maximises its value  $V$ , which is the sum of future discounted net cash flows, with respect to capital investment  $I$  and other variable inputs, such as labour input,  $X$ . To simplify the exposition, we ignore taxes, which can easily be incorporated if necessary.

$$V_t = \int_t^{\infty} [p_s \{F(K_{s-1}, X_s) - C(I_s, K_{s-1})\} - w_s X_s - p_{is} I_s] R(t, s) ds, \quad (\text{A-1})$$

where  $R(t, s) = \exp\left(-\int_t^s r_v dv\right)$ .

$K_s$ ,  $X_s$ ,  $I_s$ ,  $p_s$ ,  $w_s$ ,  $p_{is}$ ,  $F(K_{s-1}, X_s)$ ,  $R(t, s)$ , and  $r_v$  denote the capital stock at the end of period  $s$ , variable input in period  $s$ , investment input in period  $s$ , the product price in period  $s$ , the variable input price in period  $s$ , the investment goods price in period  $s$ , a homogeneous production function of degree one, the discount factor that discounts the net cash flow of period  $s$  to their value at time  $t$ , and the discount rate at time  $v$ , respectively.  $C(I_s, K_{s-1})$  is the adjustment cost incurred when a firm changes its capital stock  $K$ .  $C$  is assumed to be strictly convex with respect to  $I$ , and homogeneous of degree one with respect to  $I$  and  $K$ .

$C$  represents the standard convex adjustment cost function. A firm maximises its value  $V_t$ , given by (A-1), subject to the dynamic constraint,

$$\dot{K}_t = I_t - \delta K_{t-1}, \quad (\text{A-2})$$

where  $\dot{\cdot}$  denotes the derivate with respect to time.

The current value Hamiltonian of this maximisation problem is given by:

$$H_t = p_t \{F(K_{t-1}, X_t) - C(I_t, K_{t-1})\} - w_t X_t - p_{it} I_t + q_t (I_t - \delta K_{t-1}), \quad (\text{A-3})$$

where  $q_t$  is the shadow price of capital. The above maximisation problem yields the first-order condition:

$$p_{it}/p_t + \partial C(I_t, K_{t-1})/\partial I_t = q_t/p_t. \quad (\text{A-4})$$

The shadow price  $q_t$  can be shown to be the sum of the discounted values of marginal profits that accrue from an additional unit of capital installed at time  $t$ ; see Abel (1980, 1990) for details.

When we assume a quadratic adjustment cost function:

$$C(I_t, K_{t-1}) = \frac{\alpha}{2} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}, \quad (\text{A-5})$$

we have:

$$\partial C_t / \partial I_t = \alpha \left( \frac{I_t}{K_{t-1}} - \delta \right). \quad (\text{A-6})$$

By substituting (A-6) into (A-4), we can derive the investment ratio  $I/K$  as a linear function of  $q$ :

$$\frac{I_t}{K_{t-1}} = \delta + \frac{1}{\alpha} \frac{p_{it}}{p_t} \left( \frac{q_t}{p_{it}} - 1 \right). \quad (\text{A-7})$$

This is the standard investment equation on which empirical studies are based.

## **Appendix 2.Data**

We describe data in this appendix. Because our sample comprises unlisted firms, stock price data are not available. Hence, we obtain our data from the financial statements of unlisted firms.

### **A2.1. Data Sources**

The sample data are selected from the financial statements of 104 unlisted corporations that produce automobile parts. These data are compiled by Tokyo Shoko Research (TSR). The Kaisha-Sokan (Corporation List) is used to identify automobile parts suppliers. Although we attempted to include all suppliers, the TSR database does not cover all firms. Our final sample comprises information on 104 corporations.

The potential sample period is from 1977 to 2006, or 30 years. However, because few firms are represented for the entire sample period, we use an unbalanced panel. The total number of observations is 1,553. We discard as outliers cases with negative values of  $q_{t-1}$ . We also discard those samples for which any value of the gross profit rate,  $q_{t-1}$ , or  $I/K$  is more than five standard deviations from the corresponding mean (see, e.g., Abel and Eberly (2002) for similar treatment of the data). Therefore, our final number of observations is 1,527.

The fact that most of our sample consists of data with missing observations hampers use of the perpetual inventory method as used by Hoshi and Kashyap (1990) and Hayashi and Inoue (1991), and poses difficulties in measuring the replacement cost of capital, which is needed to measure marginal  $q$ . Hence, we follow Kaplan and Zingales (1997), Polk and Sapienza (2008), Almeida and Campello (2007) and others, and use the book value of the nominal capital stock or total assets.

### **A2.2.Tobin's $q$**

We define the gross profit rate as:

$$\frac{(1-\tau)\pi + \delta K_{-1}}{K_{-1}}, \quad (\text{A-8})$$

where  $\tau$ ,  $\pi$ ,  $\delta$ , and  $K_{-1}$  denote the corporate tax rate, operating profit, the depreciation rate, and capital stock at the beginning of the period, respectively. When we discount this value (A-8) by the cost of capital  $(1-\tau)r + \delta$  with the assumption of

static expectations, we have Tobin's  $q$ ,  $q = V/K_{-1}$ , where  $V = \frac{(1-\tau)\pi + \delta K_{-1}}{(1-\tau)r + \delta}$  and  $r$

denotes the interest rate. We use the average loan rate of each firm for  $r$ .

### **A2.3. Corporate Investment**

We use the same definition of corporate investment as in Honda and Suzuki (2006), which improves measuring investment used in existing studies, including that of Hayashi and Inoue (1991). Anyone can download this working paper, Honda and Suzuki (2006), from [http://www2.econ.osaka-u.ac.jp/library/local/e\\_HP/e\\_g\\_shiryō.html](http://www2.econ.osaka-u.ac.jp/library/local/e_HP/e_g_shiryō.html), School of Economics, Osaka University.

### **Footnote**

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Tables

Table 1: Summary Statistics from 1,527 Observations

	Mean	Std Dev	Minimum	Maximum
I/K	0.3192	0.2327	-0.1662	1.5964
q	1.3914	0.4651	0.0328	4.0472
L	14.5747	1.1266	10.7462	17.7081

Table 2: Benchmark Estimates of the Model with Two Thresholds

( $q_1 = 0.91$  and  $q_2 = 1.20$ )

	Estimates
$\beta_0^3$ (intercept of the third regime)	2.5645 (11.0512)
$\beta_1^1$ (slope of the first regime)	0.0134 (0.2362)
$\beta_1^2$ (slope of the second regime)	0.3598 (4.9787)
$\beta_1^3$ (slope of the third regime)	0.0354 (1.8413)
$\gamma_1$ (coefficient of L)	-0.1622 (-10.2899)
$\gamma_2$ (coefficient of L·D)	0.0588 (2.3592)

The log likelihood of the estimated equation is 384.2105. Numbers in parentheses are t-values. Throughout the paper, all reported t-values are based on heteroskedasticity-consistent standard errors.

**Table 3: Testing the Significance of  $\hat{y}_{it}^A$  in the Benchmark Model**

$q_2$	1.0	<b>1.2</b>	1.4	1.6	1.8	2.0	2.2	2.4
$q_1$								
0.2	-0.01	-0.66	0.26	0.60	1.09	0.83	0.91	1.10
0.3	-0.04	-0.43	0.25	0.60	1.08	0.83	0.90	1.09
0.4	0.47	0.36	0.46	0.70	1.15	0.87	0.93	1.12
0.5	0.85	0.85	0.69	0.83	1.24	0.94	1.00	1.17
0.6	0.93	0.96	0.73	0.85	1.25	0.95	1.00	1.18
0.7	0.38	0.27	0.43	0.68	1.13	0.85	0.92	1.10
0.8	0.47	0.36	0.42	0.67	1.13	0.85	0.91	1.09
<b>0.91</b>	0.49		0.35	0.64	1.11	0.84	0.91	1.09
1.0		-0.49	0.30	0.63	1.11	0.86	0.93	1.12
1.1		-0.41	0.32	0.67	1.15	0.90	0.98	1.16
1.2			0.35	0.64	1.11	0.84	0.91	1.09

Note: The number in each cell is the t-value for the coefficient of the fitted variable  $\hat{y}_{it}^A$ .

Because none of these indicates statistical significance, the alternative models convey no additional information.

**Table 4: Testing the Significance of  $\hat{y}_{it}^B$  in the Alternative Model**

$q_2$ $q_1$	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
0.2	3.87***	2.90***	2.31**	2.44**	2.43**	2.85***	3.11***	3.24***
0.3	3.82***	2.68***	2.19**	2.40**	2.41**	2.84***	3.10***	3.24***
0.4	3.67***	2.31**	1.93**	2.28**	2.35**	2.81***	3.08***	3.23***
0.5	3.51***	1.88*	1.70*	2.19**	2.31**	2.79***	3.08***	3.23***
0.6	3.40***	1.51	1.57	2.16**	2.30**	2.80***	3.09***	3.25***
0.7	3.36***	1.29	1.62*	2.23**	2.36**	2.83***	3.11***	3.26***
0.8	3.12***	0.58	1.60	2.28**	2.41**	2.88***	3.15***	3.29***
0.91	2.73***		1.73*	2.37**	2.47**	2.91***	3.16***	3.29***
1.0		1.39	2.02**	2.49**	2.54**	2.93***	3.15***	3.25***
1.1		2.18**	2.40**	2.60***	2.57***	2.85***	2.99***	3.05***
1.2			2.64***	2.70***	2.64***	2.80***	2.86***	2.89***

Note: The number in each cell is the t-value for the coefficient of the fitted variable  $\hat{y}_{it}^B$ . \*\*\*, \*\*, and \* indicate that the fitted variable  $\hat{y}_{it}^B$  is significant at the 1% level, 5% level, and 10% level, respectively, which implies that the benchmark model conveys more information than does each alternative model.

**Table 5: Benchmark Estimates of the Model with One Threshold**

( $q_1 = 1.34$ )

	Estimates
$\beta_0^2$ (intercept of the third regime)	2.5797 (11.0218)
$\beta_1^1$ (slope of the first regime)	0.1742 (5.6493)
$\beta_1^2$ (slope of the second regime)	0.0313 (1.5338)
$\gamma_1$ (coefficient of L)	-0.1627 (-10.1861)
$\gamma_2$ (coefficient of L·D)	0.0577 (2.3046)

The log likelihood of the estimated equation is 381.8763. Numbers in parentheses are t-values.

**Table 6: Testing the Significance of  $\hat{y}_{it}^A$  (the Fitted Values from the Model with One Threshold) in the Best Model with Two Thresholds**

$q_1$	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.34	1.4
<b>t-value</b>	0.85	0.96	-0.27	-0.36	0.22	0.49	0.41	-	0.35	0.39	0.35

$q_1$	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
<b>t-value</b>	0.49	0.64	1.03	1.11	0.94	0.84	0.78	0.91	1.01	1.09

Note: The number in each cell is the t-value for the coefficient of the fitted variable  $\hat{y}_{it}^A$  in an alternative model with one threshold. Because none indicates statistical significance, alternative models with one threshold convey no additional information.

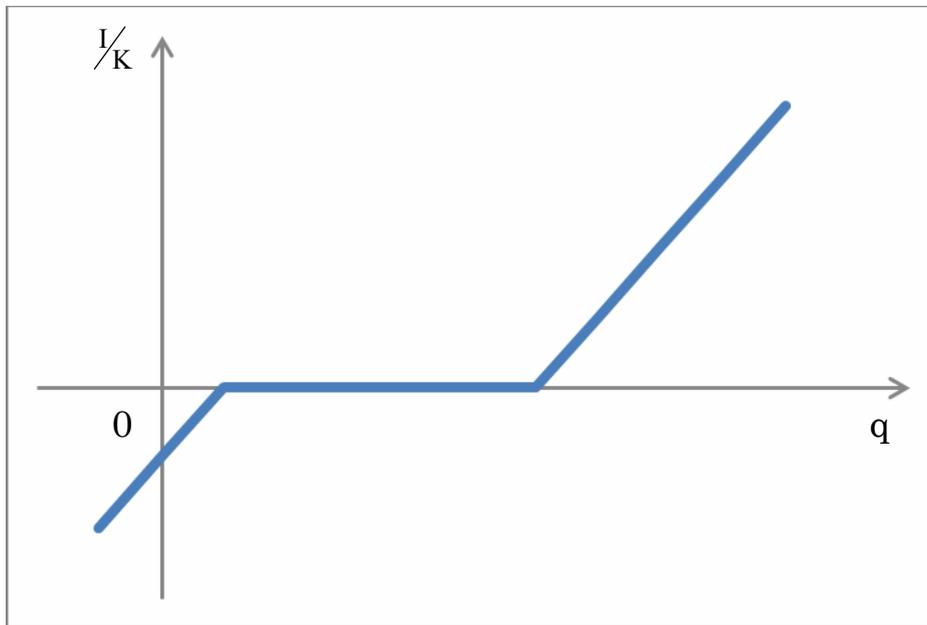
**Table 7: Testing the Significance of  $\hat{y}_{it}^B$  (the Fitted Values from the Model with Two Thresholds) in Alternative Models with One Threshold**

<b>q<sub>1</sub></b>	<b>0.5</b>	<b>0.6</b>	<b>0.7</b>	<b>0.8</b>	<b>0.9</b>	<b>1.0</b>	<b>1.1</b>	<b>1.2</b>	<b>1.3</b>	<b>1.34</b>	<b>1.4</b>
<b>t-value</b>	4.12***	4.13***	4.10***	4.09***	4.05***	3.91***	3.52***	3.14***	2.67***	2.53**	2.49**
<b>q<sub>1</sub></b>	<b>1.5</b>	<b>1.6</b>	<b>1.7</b>	<b>1.8</b>	<b>1.9</b>	<b>2.0</b>	<b>2.1</b>	<b>2.2</b>	<b>2.3</b>	<b>2.4</b>	
<b>t-value</b>	2.45**	2.51**	2.40**	2.47**	2.67***	2.86***	3.04***	3.11***	3.19***	3.24***	

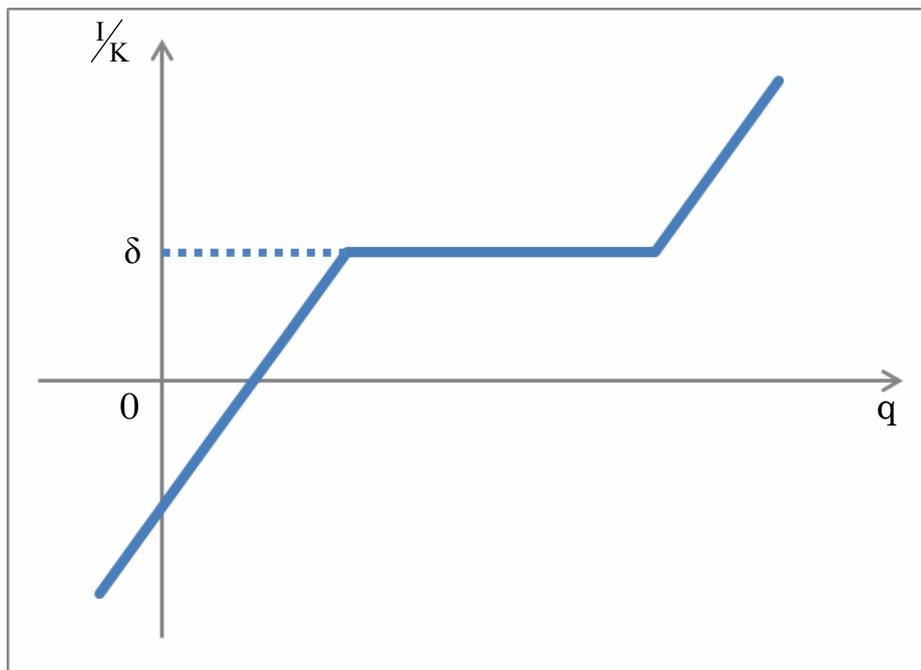
Note: Number in each cell is the t-value for the coefficient of the fitted variable  $\hat{y}_{it}^B$ . \*\*\*, \*\*, and \* indicate that the fitted variable  $\hat{y}_{it}^B$  is significant at the 1% level, 5% level, and 10% level, respectively, which implies that the best model with two thresholds (  $q_1 = 0.91$  and  $q_2 = 1.20$  ) conveys additional information not conveyed by each alternative model.

**Figures**

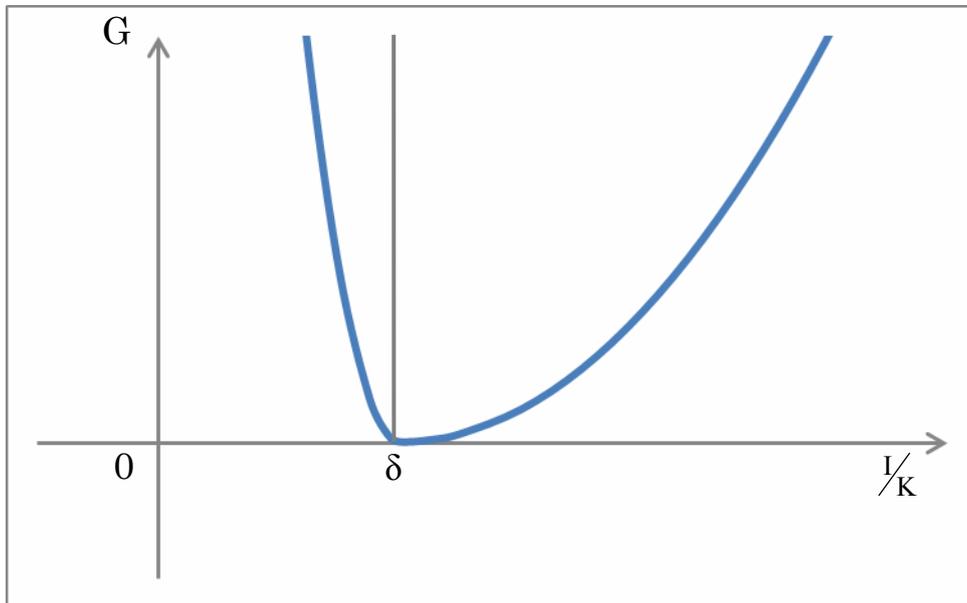
**Fig. 1. The Irreversibility of Investment in Abel and Eberly (1994)**



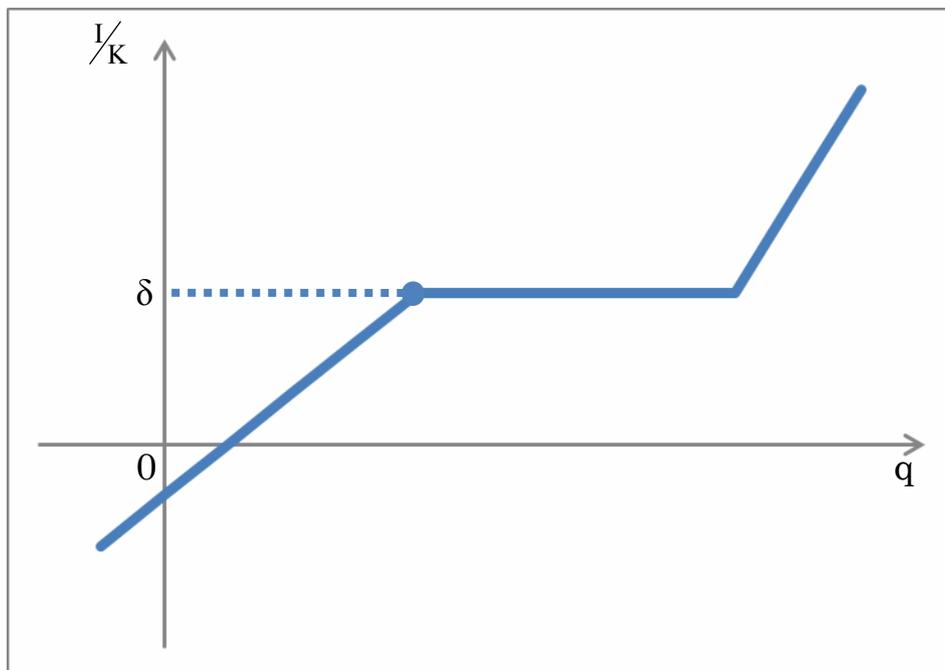
**Fig. 2. Linear Adjustment Costs in Barnett and Sakellaris (1998)**



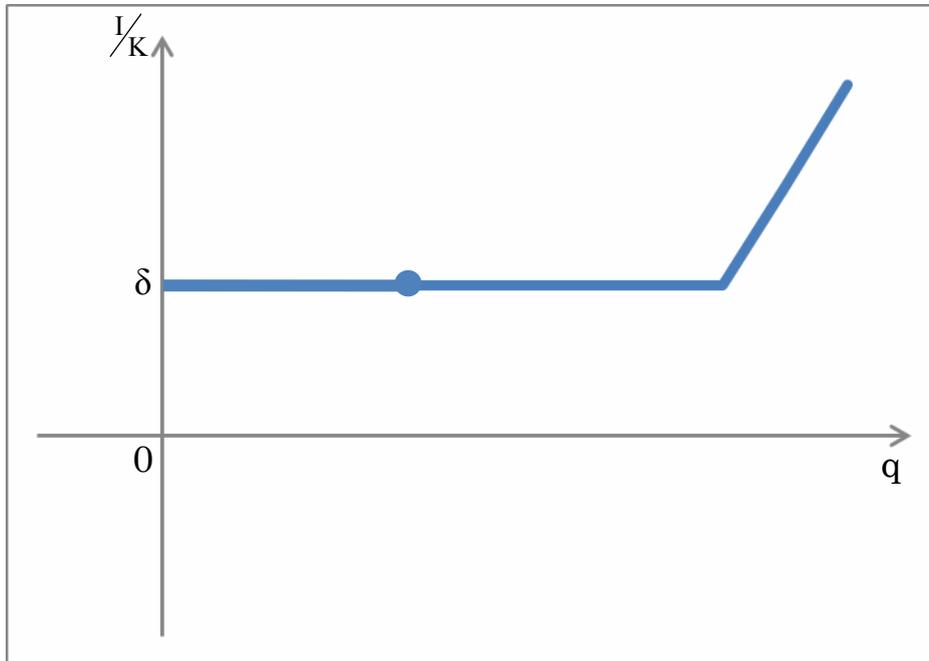
**Fig. 3. Asymmetric Adjustment Costs**



**Fig. 4. An Alternative Specification of Irreversibility**



**Fig. 5.A Horizontal First Part**



**Fig. 6. Prohibitive Adjustment Costs for Extremely Large Investments**

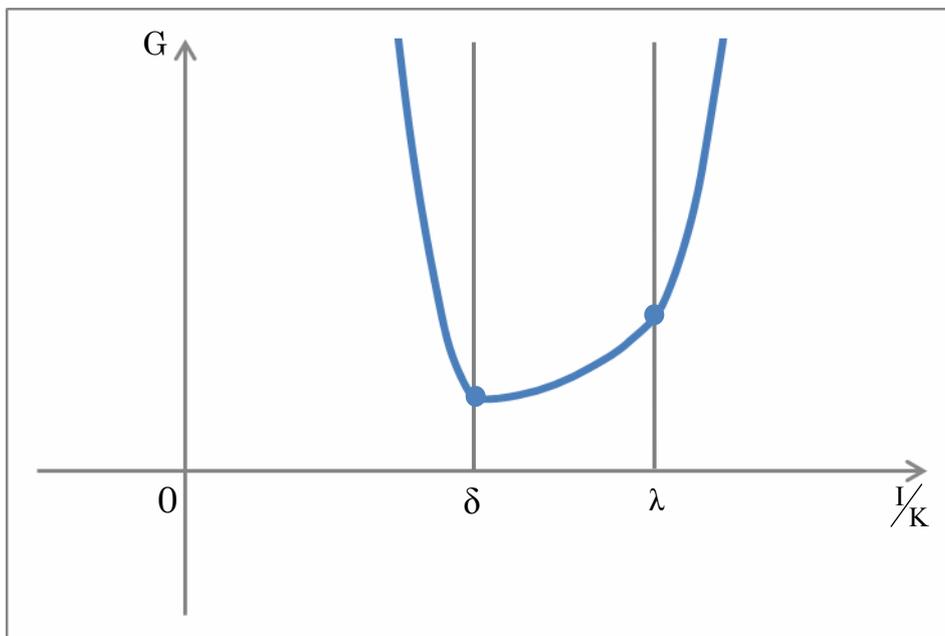


Fig. 7.A Non-linear Investment Function with Two Kinks

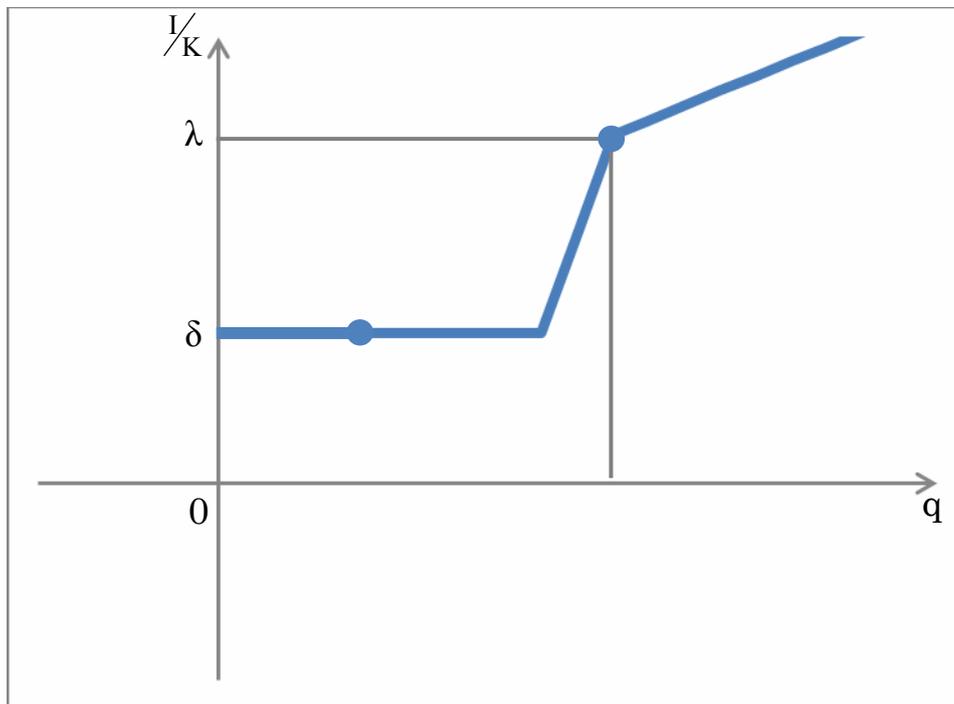
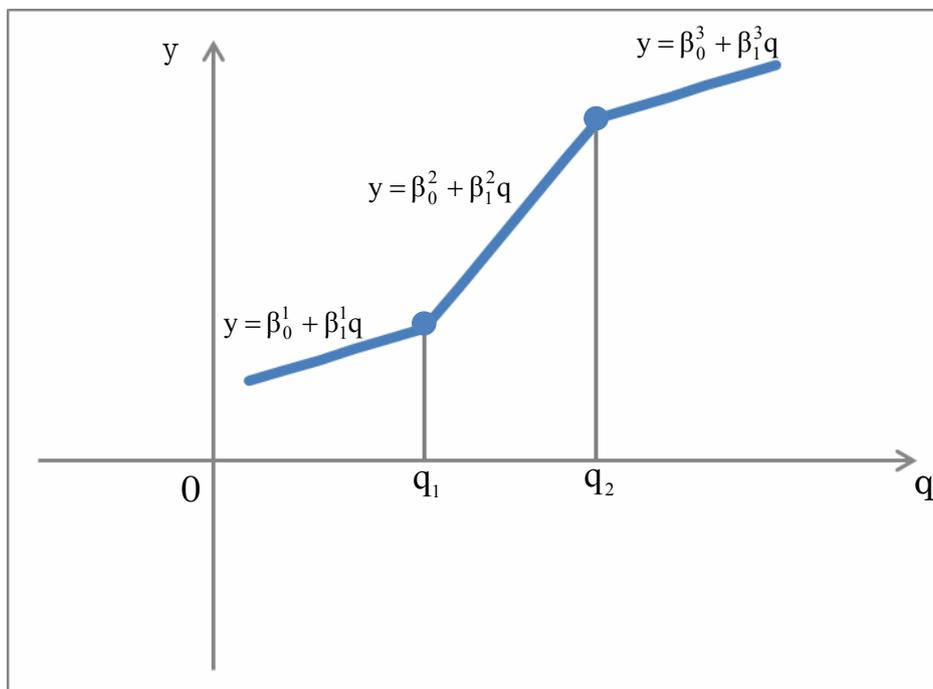
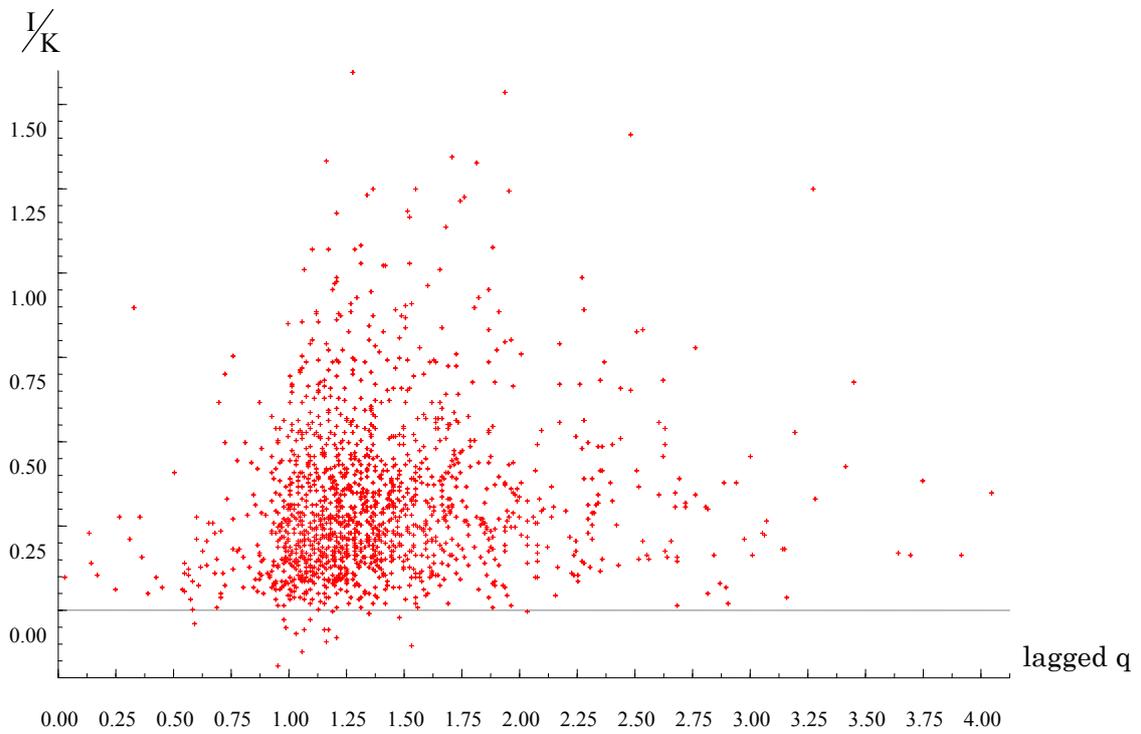


Fig. 8.A Model with Two Thresholds



**Fig. 9. Sample Plot of Investment Ratio against Lagged Tobin's q (N=1,527)**



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