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Note on the Determinants of Wage Dispersion

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Discussion Paper 08-37

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Abstract

The purpose of this note is to explain theoretically the observed entire density of wages which is hump-shaped and right-skewed. I extend the model brought up by Halko *et al*. (2008) to introduce heterogeneity of firm's productivity. It causes a difference in the support of wage offers, a wider (narrower) range for high (low) productivity firms. The different support roughly results in the observed wage dispersion because low wage offers are made by all firms (right-skewed), whereas high wage offers are made by only high producivity firms.

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1 Introduction

Many studies have attempted to explain the shape of empirically observed wage density which is hump-shaped and right-skewed. The aim of this note is also to construct a model which can explain empirically observed wage density which is hump-shaped and right-skewed, and suggest an intuitive explanation why such a shape is formed. Bontemps *et al*. (2000) assumes a continuous distribution of firm productivity types and derives hump-shaped wage density. Julien *et al*. (2006) makes firm's productivity endogenous by selecting either high or low levels. In the latter model, workers' wage can differ even if they are employed by the same job according to the number of offer they are made. Halko *et al*. (2008), which is closed to this note, considers a model that there is a market in which only firms make a wage offer (wage offer market), in which only workers make a wage offer (wage demand market), and in which the two markets coexist. In their model, which situation occurs depends on the vacancy-unemployment ratio. In particular, hump-shaped is derived due to wage offer market whereas right-skewed is mainly due to wage demand market. In the actual economy, however, most wage level would be determined by firms (with a few exceptions, for example, labor union's wage negotiation).

The main contribution of this note is that I exhibit a quite simple model that shows the observed wage distribution and intuitive explanation using by " wage offer market" as in Halko *et al*. (2008), so the wage level in this note is determined by firms only. In the market firms randomize over the workers and offer a wage using a mixed strategy with a cumulative distribution function. Unlike Halko *et al*. (2008), I assume that, as in Julien *et al*. (2006), firm's productivity is two types but it is exogenous in this note. This productivity difference leads to long-tail in the wage offer market. Introducting the difference in productivity to the model constructed by Halko *et al*. (2008), the variant support of wage offer between high and low productivity firms is led since high productivity firms can make a higher wage offer than low productivity firms. Given the fact, both high and low productivity firms can make a low wage offer but only high productivity firms can offer a high wage. Hence the upper bound of the support for high productivity firms is wider than for low productivity firms, which leads to high density within low wage offers and low density within high wage offers. By using the theoretical model, I show numerical results and intuitive exlpanations of empirically observed wage density. The model predicts that, as the fraction of high productivity firm increases, the skewedness is enhanced whereas as the productivity difference decreases, the skewedness is weakened.

The rest of note is organized as follows. Section 2 constructs the theoretical model. Numerical analysis of the model and intuitive explanations are shown in section 3 and section 4 concludes.

2 The Model

Consider a static labor market in which firms with a vacancy measured by v send a wage offer and unemployed workers measured by u receive the offer(s). Each vacancy can hire at most one worker and offers a wage using a mixed strategy. Each worker chooses the highest offer and can not make any counter offer. Firm's productivity is either high p_H or low p_L, ($p_H > p_L > 0$), and the fraction of high productivity in aggregate vacancy is indicated by $\alpha \in (0, 1)$. As in Halko *et al.* (2008), the Poisson rate under which firms randomly make an offer to workers is $\theta \equiv v/u$, the vacancy-unemployment rate. Throughout this note, I take this value as given.

Let $G_i(\cdot)$ be a cumulative distribution function of wage offer by type i firm with support $w_i \in [\underline{w}_i, \overline{w}_i]$ and V_i an expected value of type i vacancy. Since workers accept the highest wage offer, the expected value of type L vacancy that makes a wage offer

w is given by

$$
\begin{aligned}
V_L &= e^{-(1-\alpha)\theta}(p_L - w) + (1-\alpha)\theta e^{-(1-\alpha)\theta}(p_L - w)G_L(w) + \cdots \\
&\quad + \frac{[(1-\alpha)\theta]^k e^{-(1-\alpha)\theta}}{k!}(p_L - w)[G_L(w)]^k + \cdots \\
&= (p_L - w)\sum_{k=0}^{\infty}\frac{[(1-\alpha)\theta]^k e^{-(1-\alpha)\theta}}{k!}[G_L(w)]^k \\
&= (p_L - w)e^{-(1-\alpha)\theta(1-G_L(w))}. \qquad (1)
\end{aligned}
$$

Interpretation of this expression is as follows: when a firm offering w meets a certain worker and no other firms make an offer to the worker, then this firm hires this worker and obtains the gain $p_L - w$. When the firm competes against another firm for a certain worker, this firm hires the worker and obtain the gain if the competitor bids a smaller wage offer than w ($G_L(w)$). In general, when the firm competes with k firms, this firm obtains the worker if it offers the highest wage ($[G_L(w)]^k$). Analogously, the expected value of type H vacancy that bids a wage w is

$$
V_H = (p_H - w)e^{-\alpha\theta(1-G_H(w))}. \qquad (2)
$$

Since firm's offer is determined by a mixed strategy, it must be indifference between the highest offer and the lowset offer. For type H, the expected value offering the lowest offer $\underline{w}_H$, $(1-\underline{w}_H)e^{-\alpha\theta(1-G_H(\underline{w}_H))}$, equals to the expected value with the highest offer $\overline{w}_H$, $(1-\overline{w}_H)e^{-\alpha\theta(1-G_H(\overline{w}_H))}$. Note that $\underline{w}_H = 0$ since if no other firms make an offer to a certain worker, this firm can hire the worker with no payment. Given $G_H(\underline{w}_H) = 0$, $G_H(\overline{w}_H) = 1$ and $\underline{w}_H = 0$, the highest offer for type H firm is $\overline{w}_H = p_H(1-e^{-\alpha\theta})$. By similar procedure, I obtain the highest offer for type L firm, $\overline{w}_L = p_L[1-e^{-(1-\alpha)\theta}]$. Note that the support of type H wage offer, $w_H \in [0, \overline{w}_H]$, is wider than that of type L, $w_L \in [0, \overline{w}_L]$, if the difference of productivity is sufficiently large, $\frac{p_H}{p_L} > \frac{1-e^{-(1-\alpha)\theta}}{1-e^{-\alpha\theta}}$ [1]. Hereafter, I focus on this case. From (1) and (2), the wage distribution function

[1] It can be easily shown that $\lim_{\alpha\to 0}\frac{1-e^{-(1-\alpha)\theta}}{1-e^{-\alpha\theta}} \to +\infty$ and $\lim_{\alpha\to 1}\frac{1-e^{-(1-\alpha)\theta}}{1-e^{-\alpha\theta}} \to 0$. So the condition is satisfied for almost all values of p_H/p_L.

of type L is $G_L(w_L) = \frac{1}{(1-\alpha)\theta} \ln \frac{p_L}{p_L - w}$ with support $w_L \in [0, \overline{w}_L]$ and that of type H is $G_H(w_H) = \frac{1}{\alpha\theta} \ln \frac{p_H}{p_H - w_H}$ with support $w_H \in [0, \overline{w}_H]$, respectively. Note that, as in Halko *et al.* (2008), the density functions are increasing in w ($G_i'(w_i) > 0$ for $i = H, L$).

Next I will derive the aggregate wage offer distribution. Note that both type L and type H firms make an offer in the range of $w \in [0, \overline{w}_L]$. Let β be the fraction in overall type H. By definition, $\beta = G_H(\overline{w}_L)$ can be easily derived as follows:

$$\beta = (\alpha\theta)^{-1} P, \quad \text{where } P \equiv \ln \frac{p_H}{p_H - p_L[1 - e^{-(1-\alpha)\theta}]}.$$

Since the fraction of type H firm is α, in aggregate, $1 - \alpha$ type L firm and $\alpha\beta$ type H firm make an wage offer in the range of $w \in [0, \overline{w}_L]$ and $\alpha(1 - \beta)$ type H firm does in the range of $w \in [\overline{w}_L, \overline{w}_H]$. Hence the entire wage offer distribution, indexed by $G(w)$, is summarized as

$$G(w) = \begin{cases} \dfrac{1}{\theta}\left[\ln\left(\dfrac{p_L}{p_L - w}\right) + \beta \ln\left(\dfrac{p_H}{p_H - w}\right)\right], & \text{for } w \in [0, \overline{w}_L], \\[2ex] \dfrac{1-\beta}{\theta} \ln\left(\dfrac{p_H}{p_H - w}\right), & \text{for } w \in [\overline{w}_L, \overline{w}_H]. \end{cases}$$

Note that, in general, this distribution does not consist with the "realized" wage distribution since not all the offers are accepted: a worker who receives multiple offers chooses only the highest one. So an offered wage is rejected if a competitor bids a higher wage. Letting $\tilde{G}(\cdot)$ be the realized wage distribution, it is given by $\tilde{G}(w) = \frac{1}{1-e^{-\theta}} \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} [G(w)]^k$, which is the probability that the highest offer, conditional on the worker receiving at least one offer, is at most w. Substituting $G(w)$ for $\tilde{G}(w)$, I obtain the realized wage distribution [2]

$$\tilde{G}(w) = \begin{cases} \frac{e^{-\theta}}{1-e^{-\theta}} \left[\left(\frac{p_L}{p_L - w} \right) \left(\frac{p_H}{p_H - w} \right)^{P/\alpha\theta} - 1 \right], & \text{for } w \in [0, \overline{w}_L], \\ \\ \frac{e^{-\theta}}{1-e^{-\theta}} \left[\left(\frac{p_H}{p_H - w} \right)^{1-P/\alpha\theta} - 1 \right], & \text{for } w \in [\overline{w}_L, \overline{w}_H]. \end{cases} \tag{3}$$

As noted above, since not all offers are accepted, it can be easily confirmed that $\tilde{G}(0) = 0$ but $\tilde{G}(\overline{w}_H) < 1$ [3].

3 Numerical Analysis

In this section I show numerical results of the realized wage distribution (3). Note that parameters must satisfy the condition that the support of high productivity firm is wider than that of low productivity firm, $\frac{p_H}{p_L} > \frac{1-e^{-(1-\alpha)\theta}}{1-e^{-\alpha\theta}}$. As a benchmark, I set the parameters as follows: $p_H = 1.0$ (normalized) and $p_L = 0.1$, $\theta = 1.2$, and $\alpha = 0.1$. Thus there is sufficient productivity difference ($\frac{p_H}{p_L} > 10.0$) and vacancy-unemployment ratio is larger than one ($\theta = 1.2$), which is empirically observed (e.g., Hall, 2005). Figure 1 shows the realized wage distribution under benchmark parameters. As mentioned above, since both type H and type L firms make an wage offer in the range of $w \in [0, \overline{w}_L]$, the density is substantially high than in the range of $w \in [\overline{w}_L, \overline{w}_H]$ where only type H firms offer. Note also since each of the wage density functions are increasing in w, the aggregate density function is increasing within $w \in [0, \overline{w}_L]$ and $w \in [\overline{w}_L, \overline{w}_H]$.

One may claim that degeneration of the aggregate density function is questionable (since empirically observed distribution does not degenerate) and that it does not have

[2] Note that $\tilde{G}(w) = \frac{1}{1-e^{-\theta}} \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} [G(w)]^k = \frac{e^{-\theta}}{1-e^{-\theta}} \left[\sum_{k=0}^{\infty} \frac{\theta^k}{k!} [G(w)]^k - \frac{\theta^0}{0!} [G(w)]^0 \right]$. Since $\sum_{x=0}^{\infty} \frac{m^x}{x!} = e^m$, $\tilde{G}(w) = \frac{e^{-\theta}}{1-e^{-\theta}} \left\{ \exp \left[\ln \left(\frac{p_L}{p_L - w} \right) + \beta \ln \left(\frac{p_H}{p_H - w} \right) \right] - 1 \right\}$ for $w \in [0, \overline{w}_L]$. The arrangement of this expression and analogous calculation for $w \in [\overline{w}_L, \overline{w}_H]$ yields the result.

[3] Substitution $\overline{w}_H = p_H(1 - e^{-\alpha\theta})$ into $\tilde{G}(w)$ for $w \in [\overline{w}_L, \overline{w}_H]$ yields $\tilde{G}(\overline{w}_H) = \frac{e^{-[(1-\alpha)\theta + P]} - e^{-\theta}}{1-e^{-\theta}}$. Since $-[(1-\alpha)\theta + P] < 0$, $\tilde{G}(\overline{w}_H)$ is definitely smaller than one.

a long-tail (since the function is increasing in right-hand edge), however, they would not be problematic. Consider a general case in which there are k different types of productivity $0 < p_1 < p_2 < \cdots < p_k < +\infty$ and the fraction of type i is denoted by α_i ($\sum_{i=1}^{k} \alpha_i = 1$). From discussion in the previous section, a firm with higher productivity has wider support of wage offer. Let s_i be the support of firm with productivity i ($s_1 < s_2 < \cdots < s_k$) and n_i be the number of firms which make an offer within the support s_i. Recalling that the total number of firms (vacancies) is v, n_i is determined by $n_i = v \sum_{j=i}^{k} \alpha_j$ and $n_1 > n_2 > \cdots > n_k$. Assuming that the fraction of high productivity firms is relatively less than low roductivity firms, a similar shape of wage density function would be obtained in the model with k different types of productivity and degeneration would be moderately improved. In the extreme case in which productivity heterogeneity is continuous as in Bontemps *et al.* (2000), the analogous discussion is applied and degeneration would not arise. Analogous discussions justify an increasing tail. The aggregate wage density is weighted with the measure of firm with productivity i, α_i. Consequently, in the k types model, the measure is generally small in right-hand edge of the entire wage density, which would lead to non-increasing (or decreasing) tail in aggregate.

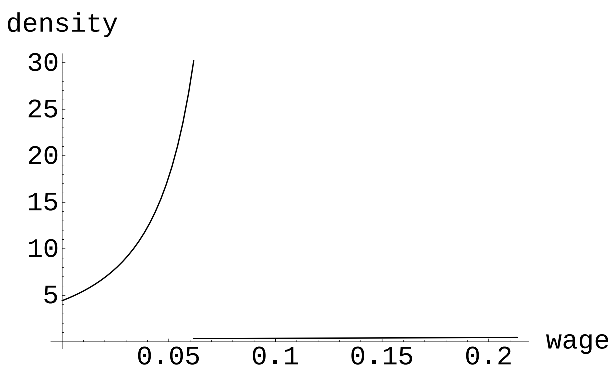

Figure 1: benchmark case

($\alpha = 0.2,\ \theta = 1.2,\ p_H = 1.0,\ p_L = 0.1$)

Next I examine the effect of change in α and productivity difference. Figure 2 shows the results under various values of α (the other parameters are the same as benchmark case). It suggests that as the fraction of type H firm increases, the skewedness is enhanced: that is, the density in low wage offers is decreased and the long tail is lengthened. The intuition is straightforward. Given the productivity difference p_H/p_L, increase in α makes type H firms more apt to offer a high wage because competitors for them increase. To make sure that they hires a worker, they must make a high wage offer. Justification of degeneration and increasing tail is same as the above discussion. Finally the effect of change in productivity difference, under which p_H is held constant and p_L is increased, is in Figure 3. It shows that as the productivity difference decreases, the skewedness is weakened. The intuition is also straightforward. Given the component ratio of productivity α, this is because increase in p_L widens the support of wage offer for type L firms whereas the support for type H firms. Briefly speaking, increase (decrease) in α enhances (weakens) the right skewedness whereas decrease (increase) in productivity difference weakens (enhances) the skewedness [4].

From the numerical results, some predictions are obtained regarding the real economy: (i) there is substantial difference in productivity among firms; (ii) the fraction of high productivity is very small; (iii) many high productivity firms offer a low wage in spite of they can afford to make a high wage due to randomizing over workers.

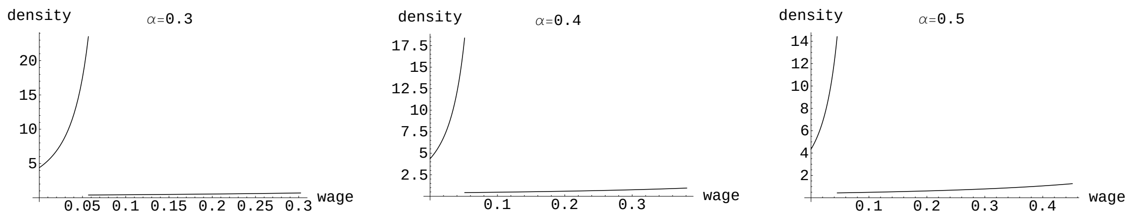

Figure 2: change in α

[4]These results are conditional on $\frac{p_H}{p_L} > \frac{1-e^{-(1-\alpha)\theta}}{1-e^{-\alpha\theta}}$ holds. If the condition does not satisfy, a situation in which type L firms have wider support than type H firms may arise, so the skewedness is not defined (for instance, α is close to 0).

$$(\theta = 1.2,\ p_H = 1.0,\ p_L = 0.1)$$

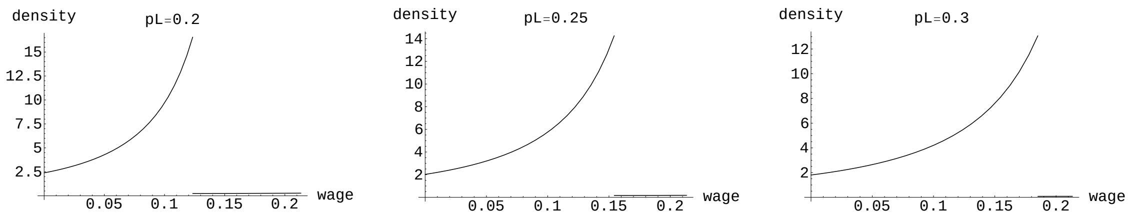

Figure 3: change in productivity difference

$$(\alpha = 0.2,\ \theta = 1.2,\ p_H = 1.0)$$

4 Conclusion

This note have explored the theoretical model explains empirically observed wage distribution. It shows that the right skewedness and long tail can be explained by two reasons: First, both high and low productivity make a low wage offer due to randomizing over workers. Second, only high productivity (can) make a high wage offer. The model predicts that, as the fraction of high productivity firm increases, the skewedness is enhanced whereas as the productivity difference decreases, the skewedness is weakened.

In this note, I have shown a theoretical model that can explain the actual wage distribution but not shown the empirically observed wage distributions. As further tasks, I shall cite the empirical distributions and explain why such a distribution occurs. For instance, following my theoretical model, a contributing factor of a high degree hump-shaped is predicted as a large ratio of low productivity firms. As another example, a longer tail seems to be derived by a large productivity difference. To prove right of my theoretical model, I shall show some empirical facts that exhibit the ratio of high and low productivity firms, the size of productivity difference, and so on.

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