Market Size and Entrepreneurship

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Discussion Paper 09-07

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Yasuhiro Sato*   Takatoshi Tabuchi†   Kazuhiro Yamamoto ‡

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Abstract

In order to examine the impacts of market size on entrepreneurship, we estimate a monopolistic competition model that involves entrepreneurial decision by using data on Japanese prefectures. Our results show that a larger market size measured by the population density leads to higher incentive of people to become entrepreneurs. A 10 percent increase in the population density increases the share of people who wish to become entrepreneurs by 2 percent. In contrast, the self-employment ratio is lower in prefectures with higher population density, which suggests that the market size has different impacts on the entrepreneurship in different stages.

JEL Classification: J62, L26, R12.

Keywords: market size, entrepreneurship, density economies, market expansion

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1 Introduction

We often observe that the distribution of entrepreneurial activities is not equally distributed across space, and big cities, such as New York, London, and Tokyo, look to have higher entrepreneurship than other cities. What makes the difference in entrepreneurship across space? In answering this question, we focus on the regional market size and the entrepreneurship.

More precisely, the aim of this paper is to investigate empirically how the size of a region (the size of a home market) is associated with entrepreneurship in determining the spatial distribution of entrepreneurship in Japan. For this purpose, building on new economic geography pioneered by Krugman [12], we first develop a monopolistic competition model involving entrepreneurship decision by workers. We then estimate the relationship.

Under imperfect competition, an increase in the home market size has two opposing effects on firms’ profits. Both positive and negative effects affect the decision of (potential) entrepreneurs whether or not to start a new business. Positive effects come from density economies as evidenced by Ciccone and Hall [4] and Ciccone [3], and market expansion as shown by Krugman [11] as the home market effect. On the other hand, a negative effect stems from competition among firms. If the positive effects dominate the negative one, entrepreneurs would establish a new firm in a large market.

In order to examine which is the dominant effect, we estimate the relationship between the market size and entrepreneurship, that are derived from the developed model. In estimation, we use two indices of entrepreneurship: the share of potential entrepreneurs and the share of self-employment. The former is the share of people who wish to start up a new business in those who are either employed but want to change careers or not engaged in work but want to find a job. We consider this to represent the ex ante entrepreneurship and show the degree of incentive of people to start up a new business. The latter is the share of self-employed workers in the total employed workers, and we interpret this as the ex post entrepreneurship.

We use the population density as the index of the market size, and investigate how it is associated with the ex ante/ex post entrepreneurship. Our results show that the population density is positively related to the ex ante entrepreneurship. For large prefectures, such as Tokyo and Osaka, a 10 percent increase in the population density increases the share of ex
ante entrepreneurs by around 2 percent. In contrast, the population density is negatively related to the ex post entrepreneurship. Thus, the market size has opposite impacts on entrepreneurship in different phases.

There are several studies that showed the relationship between the market size and entrepreneurship. According to Berry and Reiss [2], empirical studies based on micro data showed that there are thresholds of regional population size that enables firms to establish themselves as going concerns, and firm entry is high in regions with large population size. Similar results on the relationship between regional population size and firm entry are obtained by Reynolds et al. [15], who showed that higher growth of regional population leads to higher birth rate of firms. Harada [9] used data on workers who wish to start up a business and showed that workers are more likely to become entrepreneurs in regions with larger gross prefectural domestic product. Rosenthal and Strange [16] showed that entrepreneurship is higher in districts with higher employment density using data on new establishments in the New York Metropolitan Area. Glaeser and Kerr [8] measured entrepreneurship by new establishments that are independent from existing firms and showed that new entrants are likely to be drawn to areas with high overall levels of customers and suppliers, and strongly drawn to areas with many smaller suppliers. However, in the results of Glaeser [7] who adopted the self-employment rate and average firm size as indices of entrepreneurship, employment density affects entrepreneurship only insignificantly.

The novelty of our paper lies in the following two points. First, we measure entrepreneurship by the share of potential entrepreneurs as well as the share of self-employment. This enables us to see if the market size is associated with the entrepreneurship in different stages of entrepreneurship, i.e., ex ante and ex post. Second, we consider the possibility of non-linearity and non-monotonicity in the relationship between the market size and entrepreneurship, which allows flexibility in capturing this relationship. This is in contrast to the existing studies mentioned above, most of which assume the linearity and monotonicity.

The remainder of the paper is organized as follows. Section 2 presents a model of new economic geography with entrepreneurship and characterizes entrepreneur formation. Section 3 analyzes the entrepreneur formation and investigates the positive effects arising from density economies and market expansion. It provides a set of empirical results by using Japanese
prefectural-base data. Section 4 checks robustness of these results by different methods of estimations. Section 5 concludes and suggests future research directions.

2 Basic Model

In this section, we present a simple new economic geography model involving entrepreneurship in order to examine how the market size affects entrepreneurship.

There are two goods in a region. The first good is homogeneous. Consumers have a positive initial endowment of this good which is also produced using labor as the only input under constant returns to scale and perfect competition. This good can be traded freely and is chosen as the numéraire. The other good is a horizontally differentiated product, which is supplied by using labor under increasing returns to scale and monopolistic competition.\(^1\)

There is a mass of individuals, \(L\), who move between the two sectors according to the wage differential given later. As in Ottaviano, Tabuchi, and Thisse \(\cite{14}\), consumers’ preferences are identical across individuals and are described by the following utility function of the quasi-linear form:

\[
U = \alpha \int_0^n q(v)dv - \beta \frac{1}{2} \int_0^n (q(v))^2 dv - \gamma \left( \int_0^n q(v)dv \right)^2 + q_0,
\]

where \(q(v)\) is the quantity of variety \(v\), \(n\) is the mass of varieties, \(q_0\) is the quantity of the numéraire, and \(\alpha\), \(\beta\), and \(\gamma\) are positive parameters. The budget constraint of an individual is given by

\[
\int_0^n p(v)q(v)dv + q_0 = \overline{q}_0 + w,
\]

where \(p(v)\) is the price of variety \(v\), \(w\) is the individual’s wage, and \(\overline{q}_0\) is the initial endowment, which is supposed to be sufficiently large to ensure the positive demand of the numéraire.

Maximizing the utility subject to the budget constraint yields the individual demand

\[
q(v) = \frac{\alpha \beta + \gamma P}{\beta (\beta + \gamma n)} - \frac{1}{\beta} p(v),
\]

where \(P = \int_0^n p(v)dv\) is the price index.

\(^1\)This good may be services which consist of a continuum of varieties without export to the rest of the world.
Turning to the supply side, technology in the homogeneous good sector requires one unit of labor in order to produce one unit of output. Due to this numéraire with costless trade, the equilibrium wage of the sector is always one.

In the differentiated good sector, each firm supplies a single variety under increasing returns to scale so that $n$ is also regarded as the number of firms. The fixed requirement of labor is one and the marginal requirement of labor is normalized to zero without much loss of generality.

Each firm chooses a price $p(v)$ that maximizes its profit:

$$\pi(v) = [p(v) - \tau(D)] q(v) L - w, \quad (1)$$

where $\tau(D)$ is the marginal costs of production, which include the distribution costs involving the retail and wholesale costs within the region. A higher density would lead to lower marginal costs because of greater Marshallian externalities, such as knowledge spillover and labor pooling or because of lower distribution costs. We assume $\tau$ is decreasing in the population density $D$ in order to capture such density economies. It is well known in urban economics (Alonso [1]) that the average population density in a city is positively associated with city size, and hence, the density economies may be regarded as agglomeration economies accruing from the high density or large size of a city. This implies that the population density is high in regions involving large cities, where agglomeration economies as well as density economies are large.

Because firms are symmetric with respect to production technologies, they charge the same price in equilibrium, and thus, we drop $v$ hereafter. This is given by

$$p = \frac{\alpha \beta + \tau(D)(\beta + \gamma n)}{2\beta + \gamma n}.$$

Plugging the price into (1), the profit made by a firm is rewritten as follows:

$$\pi = \frac{[\alpha - \tau(D)]^2 \beta L}{(2\beta + \gamma n)^2} - w.$$

Under free entry of firms, this profit equals zero so that the equilibrium wage in the differentiated good sector is determined as

$$w = \frac{[\alpha - \tau(D)]^2 \beta L}{(2\beta + \gamma n)^2}. \quad (2)$$
We follow existing microeconomic studies of the formation of entrepreneurship by assuming that individuals choose to become either a *worker* employed by an entrepreneur or an *entrepreneur* employing workers. Because the equilibrium wage in the homogeneous good sector is one, the arbitrage between the two sectors by individuals requires that

\[ w = 1. \]  

(3)

Let \( e \) be the share of entrepreneurs in the differentiated good sector. Since the mass of varieties, \( n \), is equal to the number of firms in the region, the labor market clearing implies \( eL = n \). Substituting this into (3) with (2), we can uniquely determine the entrepreneur share by the following equation

\[ 1 = \frac{[\alpha - \tau (D)] \sqrt{\beta L}}{2\beta + \gamma eL}. \]  

(4)

Let \( h \) be the area of inhabitable land so that \( L = hD \) holds. Then, we can rewrite (4) as a function of the population density. Taking the logarithm of it, we have

\[ \ln (e) = \ln \left\{ \left[ \alpha - \tau (D) \right] \sqrt{\beta hD} - 2\beta \right\} - \ln (\gamma hD). \]  

(5)

The first term of the RHS in (5) is increasing in the population density \( D \), whereas the second term is decreasing in \( D \). Hence, as \( D \) increases, the first term has positive effects on the entrepreneur share \( e \), whereas the second term has a negative effect on \( e \).

The positive effect comes from two sources. One is the *density economies*, which are described by changes in \( \tau (D) \): increasing density reduces the marginal costs, enhances firms’ profits, and then encourages to be entrepreneurs. The other is the effect of *market expansion*, which is represented by changes in \( \sqrt{\beta hD} \). Since increasing market size enlarges the amounts of sales for a given cost-price margin \( \alpha - \tau (D) \), raises profits, and augments the number of entrepreneurs. The negative effect is given by the second term of the RHS in (5). This term expresses *competition among firms* because it involves the substitutability parameter \( \gamma \). Concentration of firms reduces firms’ revenues and profits due to the substitutability between varieties, and thus discourages the incentive to become entrepreneurs in the differentiated good sector. Which effect dominates the other is not clear a priori by theory. We therefore resort to empirical analysis in order to reveal the overall effect of market size on the formation of entrepreneurship.
3 Empirical analysis

3.1 Model specification

In order to obtain an explicit equation that can be estimated, we have to specify the functional form of the distribution costs. We linearly approximates the distribution costs $\tau(D)$ as

$$\tau(D) = t_0 - t_1 D,$$

where $t_0$ and $t_1$ are positive constants. We assume $\alpha > t_0$ so that the differentiated good is produced. Substituting this $\tau(D)$ into (5), we have the equation to be estimated as:

$$\ln(e) = a_0 - \ln(D) - \frac{1}{2} \ln(h) + \ln \left(-2h^{-1/2} + a_1 D^{1/2} + a_2 D^{3/2}\right),$$

where $a_0$, $a_1$, and $a_2$ are defined as

$$a_0 \equiv \ln \left(\frac{\beta}{\gamma}\right), \quad a_1 \equiv \frac{\alpha - t_0}{\sqrt{\beta}}, \quad a_2 \equiv \frac{t_1}{\sqrt{\beta}}.$$

As we saw in the previous section, the population density as a surrogate for the home market size has the positive and negative effects on entrepreneurship. By estimating equation (6) using Japanese data, we investigate which effect dominates.

3.2 Data and preliminary results

We use the data for years 1992, 1997 and 2002 on Japanese prefectures in order to estimate (6). For the data of entrepreneurship, we use two indices. One is the share $e_a$ of people who wish to start up a new business in those who are either employed but want to change careers or not engaged in work but want to find a job. The other is the share $e_p$ of self-employment, which is calculated by the ratio of the number of self-employed people to the number of people in all jobs. All these data are taken from Employment Status Survey published by Ministry of International Affairs and Communications of Japan. $e_a$ can be interpreted as the share of ex ante (potentially new) entrepreneurs, whereas $e_p$ is considered as the share of ex post

\footnote{We can also obtain this estimation equation by considering Cournot competition instead of monopolistic competition, which implies that our results are robust with respect to the type of imperfect competition. See Appendix for this point.}
entrepreneurs. The population density $D_{pop}$ is computed by the prefectural population $L$ (Population Estimates published by Ministry of International Affairs and Communications) divided by the inhabitable area $H$ (Social Indicators by Prefecture published by Ministry of International Affairs and Communications). Table 1 lists the summary statistics of these variables and Table 2 describes and sources of all variables.3

In order to see the overall distributions of $e_a$ and $e_p$ across regions, we estimate their distribution by use of kernel density estimation. The estimated distributions of $e_a$ and $e_p$ are drawn in Figures 1 and 2, respectively, where the horizontal axis represents the share of entrepreneurs and the vertical one shows its density.4

Note first that the entrepreneurship significantly varies across regions in each year: 5 to 11 percent with respect to ex ante entrepreneurs and 7 to 19 percent with respect to ex post entrepreneurs. On average, the share of ex post entrepreneurs is higher than the share of ex ante entrepreneurs. This would reflect the fact that the latter does not include those inherit business from their parents. Second, both figures show that the distribution has shifted from year to year, which indicates that the entrepreneurship in all regions is affected by changes in the national economy as a whole. The decline of the Japanese economy during the 1990s is thought to be the reason for this steady fall in the entrepreneurship.

Before estimating the equation (6), we provide some preliminary estimation of the relationship between the population density and the entrepreneurship. Table 3 shows the regression results in which the dependent variables are $\ln(e_a)$ and $\ln(e_p)$, and the independent variables are $\ln(D_{pop})$ and $\ln(G_d)$, where $G_d$ is the per capita GDP (National Accounts

3 The Hokkaido prefecture is the largest in terms of inhabitable area and contains one big city (Sapporo) and many small cities. Since the average density of Hokkaido does not represent its regional characteristics, we have eliminated it from our sample, which leads to our sample size of forty-six prefectures times three years.

4 We used the Silverman’s default bandwidth in the kernel estimation. See Härdle [10] for more details.
published by Department of National Accounts in Cabinet Office) that controls for the year effect.

[Insert Table 3 around here]

In Table 3, (1) and (4) represent the results estimated by OLS regression, regressions (2) and (5) by the fixed effect model, and (3) and (6) by the random effect model. From the Hausman test, we can see that the random effect model is shown to be more appropriate than the fixed effect one. These estimated results show that the ex ante entrepreneurship is positively related to the population density, whereas the ex post entrepreneurship is negatively related to the population density. In what follows, we estimate (6) in order to analyze in more detail on the relationship between the population density and the entrepreneurship.

### 3.3 Model for estimation

In the above preliminary regressions, we ignore several variables that are considered to affect the entry decision of entrepreneurs based on the literature of empirical studies of firm entry in the tradition a la Orr [13]. We therefore include the following control variables. First, we append the per capita gross prefectural domestic product (GPDP) ($G_p$), the year to year comparison of the per capita GPDP ($g_p = G_{pt}/G_{pt-1}$), and the price-cost margin ($M$), which describe the overall economic conditions of the corresponding region. $G_p$ represents the current condition and $g_p$ indicates the future prospect. $M$ is defined as $M = (\text{GPDP} - \text{intermediate output} - \text{indirect tax less subsidies} - \text{compensation of employees})/\text{GPDP}$, which is a proxy for profitability of business in the region. Second, in order to control the difference in industry structures, we add the shares of agriculture, manufacturing, and public sectors in the GPDP ($S_{ha}$, $S_{hm}$, and $S_{hp}$). All the data for $G_p$, $g_p$, $M$, $S_{ha}$, $S_{hm}$, and $S_{hp}$ are taken from Prefectural Accounts published by Department of National Accounts in Cabinet Office. Third, we also control the effective job opening to job applicant ratio ($V$), which is available in Monthly Report of Public Employment Security Statistics published by Ministry of Health, Labour and Welfare. Table 1 provides the basic statistics of these variables, and Table 2 describes and sources of all variables.

---

Introducing all these variables into (6), the equation to be estimated becomes

\[
\ln(e_{it}) = a_0 - \ln(D_{it}) - \frac{1}{2} \ln(h_{it}) + \ln \left( -2h_{it}^{-1/2} + a_1 D_{it}^{1/2} + a_2 D_{it}^{3/2} \right) + b_1 \ln(G_{pit}) + b_2 \ln(g_{pit}) + b_3 \ln(M_{it}) + b_4 \ln(S_{hit}) + b_5 \ln(S_{hmmit}) + b_6 \ln(S_{hpit}) + b_7 \ln(V_{it}) + b_8 \ln(G_{dt}) + u_{it},
\]

where \( u_{it} \) is the standard error term. We estimate this equation by the nonlinear least squares method.

### 3.4 Results

Estimation results of (7) are reported in Table 4. Columns 1 and 2 lists the parameter estimates for ex ante entrepreneurs and ex post entrepreneurs, respectively, when the above-mentioned conditions are controlled for.

[Insert Table 4 around here]

[interpretations on the estimated coefficients]

Using these estimated regression equations, we can simulate the elasticity \((D_{pop}/e)(\partial e/\partial D_{pop})\) of the entrepreneur share \( e \) with respect to the population density \( D_{pop} \). Figure 3 provides the simulation results for the relevant range of \( D_{pop} \), when the elasticity is evaluated at the mean of the inhabitable area \( h \).

[Insert Figure 3 around here]

The horizontal axis represents the logarithm of \( D_{pop} \), whereas the vertical axis describes the elasticity. The solid curve shows the results for the ex ante entrepreneurs \( e_a \), and the dashed curve describes the results for the ex post entrepreneurs \( e_p \). We observe from this figure that the population density has a positive impact on the ex ante entrepreneurship. For large prefectures, such as Tokyo and Osaka, a 10 percent increase in the population density increases the share of ex ante entrepreneurs by around 2 percent. In contrast, the population density is likely to have a negative impact on the ex post entrepreneurship. The ex ante positive impact may suggest that high density is considered to be an entrepreneurial chance.
to start business. High density would mean large and diversified cities, where innovation is fostered and new products are developed (Duranton and Puga [5]). However, the ex post negative impact may indicate that high density turns out to be keen competition between firms so that quite a few entrepreneurs get out of business.

### 3.5 Density economies or market expansion?

The population density has a positive effect on the ex ante entrepreneurship. It also has a positive effect on the ex post entrepreneurship although the range of a negative effect dominates that of a positive effect. As we discussed in section 2, such a positive effect comes from two sources: density economies and market expansion. It is interesting to focus on the positive effect and investigate which of these two is the main source.

The positive effect comes from changes in \([\alpha - \tau(D)] \sqrt{\beta h D}\) in (5). Therefore, we can check which of the two sources dominate by computing the derivative of this term with respect to the population density \(D\). That is, the positive effect can be decomposed as follows:

\[
\frac{d}{dD} \left\{ [\alpha - \tau(D)] \sqrt{\beta h D}\right\} = t_1 \sqrt{\beta h D} \equiv A(D) + \frac{[\alpha - \tau(D)] \sqrt{\beta h}}{2\sqrt{D}} \equiv B(D).
\]

The relative strength of these two is given by

\[
\frac{A(D)}{B(D)} = \frac{2D}{(\alpha - t_0)/t_1 + D}.
\]

Because \((\alpha - t_0)/t_1 = a_1/a_2\) from (6), the estimates of this ratio can be obtained by Table 4. We then depict the ratio of \(A(D_{\text{pop}})/B(D_{\text{pop}})\) for ex ante and ex post entrepreneurship in Figure 4, respectively.

[Insert Figure 4 around here]

We observe that the market expansion is more important than the density economies with respect to ex post entrepreneurship for a wide range of the population density. This is also true for ex ante entrepreneurship when the population density is low. This may
imply that the market expansion is crucial for regions with low density involving small cities, while the density economies are insufficient to start up new business in small cities for ex ante entrepreneurship. When the population density is high, the density economies are more important than the market expansion for entrepreneurship. This may suggest that the density economies are at work for entrepreneurship in regions involving large cities, whereas the market expansion is exhausted in such cities.

4 Robustness checks

In what follows, we conduct two different estimations in order to check robustness of our results.

4.1 Nonparametric estimation

The analysis in the previous section may rely on the functional form of (6). This restriction, on the one hand, enables us to simulate the responses of entrepreneurship to changes in the population density. However, on the other hand, it may be too simple to capture the true relationship between the entrepreneurship and population density. We therefore do not assume the explicit functional form in estimating the relationship between the entrepreneurship and population density. This can be done by estimating the mean of the entrepreneur share conditioned on prefectural population density by Kernel estimation. More specifically, we estimate the values of \( E(e_t | \ln(D_{pop})) \).\(^6\) Estimated conditional means are shown in Figure 5 with the ex ante entrepreneurs \( e_a \) and in Figure 6 with the ex post entrepreneurs \( e_p \). Figures 5 and 6 verifies the findings obtained in the previous section.

[Insert Figures 5 and 6 around here]

4.2 Employment density

Thus far, we focus on the population density. Since the employment density may be more important for entrepreneurs, we pursue this possibility. Replacing the population density

\(^6\)Again, we used the Silverman’s default bandwidth in the kernel estimation.
with the employment density, we have conducted the estimation of (7), and obtained the results shown in Table 5. As before, the first and second columns describe the parameter estimates using data of ex ante entrepreneurs $e_a$ and ex post entrepreneurs $e_p$, respectively. We can then compute the elasticities $(D/e)(\partial e/\partial D)$ of the entrepreneur share with respect to the market size by use of the parameter estimates. They are drawn in Figure 7.

[Insert Table 5 around here]

[Insert Figure 7 around here]

We confirm from Figure 7 that the employment density is positively (resp. negatively) associated with the ex ante (resp. ex post) entrepreneurship. Since this result is similar to that by Figure 3, we conclude that the impact of the employment density on entrepreneurship does not differ from that of the population density.

5 Concluding remarks

This paper explored how the market size affects entrepreneurship. Our results show that a larger market stimulates ex ante entrepreneurship and gives workers stronger incentives to start up a new business, but leads to lower ratio of self-employment, i.e., lower ex post entrepreneurs. Moreover, the effect of market density economies is important in encouraging people to be entrepreneurs ex ante especially in large cities. In contrast, with respect to the self-employment ratio, the effect of market expansion dominates the effect of density economies in general and these two positive effects are smaller than the negative effect. This may be due to the fierce competition among firms in the large market ex post.

The current analysis cannot provide the reason why the market size has different impacts on entrepreneurship in different stages. By exploiting micro-data on firms, we may be able to find the reason. In particular, it would be useful to work on data regarding firm turnovers. This is an important direction of future research.
References


Appendix: A Cournot competition model with entrepreneurship

In this appendix, we show a different model with Cournot competition yields the same estimation equation.

The utility function is quasi-linear with respect to the homogeneous good $x$

$$U = ax - \frac{\beta}{2}x^2 + q_0,$$

where $a$ and $\beta$ are positive parameters. Given the budget constraint $w + q_0 = px + q_0$, where $p$ is the price of $x$, the inverted demand function is

$$p = a - \beta \frac{nx}{L}.$$

Firms are competing a la Cournot. The good market clearing requires $Lx = \sum_{v=1}^{n} x_v$, where $x_v$ is the good supplied by firm $v$, and hence, the inverted demand function that firms face is given by

$$p = a - \beta \left( \frac{\sum_{v=1}^{n} x_v}{L} \right).$$

In producing good $x$, the marginal labor requirement is given by $\tau(D) = t_0 - t_1 D$ so that there are density or agglomeration economies. The profit of firm $v$ is described as

$$\pi_v = (p - \tau(D)w)x_v.$$  (A1)
Solving the first-order condition for the profit maximization, we get the equilibrium output and profit as

\[ x = \frac{L(\alpha - t_0 + t_1D)}{\beta(n + 1)} \quad \text{and} \quad \pi = \frac{L(\alpha - t_0 + t_1D)^2}{\beta(n + 1)^2}, \quad (A2) \]

where subscript \( v \) is dropped due to the identical production technologies. Because the number of firms is equal to the number of entrepreneurs, \( n = eL \) holds. Plugging this relation and (A2) into (A1) with \( w = 1 \) and \( \pi(v) = 0 \), we have

\[ \ln(e) = -\ln(D) - \frac{1}{2} \ln(h) + \ln \left( -\frac{1}{h^{1/2}} + a_1D^{1/2} + a_2D^{3/2} \right), \]

which is the same estimation equation as (6) except the constant terms. We therefore conclude that all the results obtained in this paper are valid for the Cournot competition model.
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<tr>
<td>$G_d$</td>
<td>3973.0</td>
<td>109.0</td>
<td>3887.7</td>
<td>4126.4</td>
</tr>
</tbody>
</table>

Table 1. Summary statistics
<table>
<thead>
<tr>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_a$ share of people who wish to start up a business</td>
<td>Employment Status Survey</td>
</tr>
<tr>
<td>$e_p$ share of self-employment</td>
<td></td>
</tr>
<tr>
<td>$E$ number of employed workers (in thousands)</td>
<td></td>
</tr>
<tr>
<td>$D_{em}$ employment density $= 1000 \times E/H$</td>
<td></td>
</tr>
<tr>
<td>$L$ prefectoral population (in thousands)</td>
<td>Population Estimates</td>
</tr>
<tr>
<td>$D_{pop}$ population density $= 1000 \times L/H$</td>
<td></td>
</tr>
<tr>
<td>$H$ inhabitable area ($km^2$)</td>
<td>Social Indicators by Prefecture</td>
</tr>
<tr>
<td>$G_p$ per capita GPDP (in thousand yen)</td>
<td></td>
</tr>
<tr>
<td>$g_p$ year to year comparison of $G_p$</td>
<td></td>
</tr>
<tr>
<td>$M$ price-cost margin $= (GPDP - intermediate output - indirect tax less subsidies - compensation of employees) / GPDP$</td>
<td></td>
</tr>
<tr>
<td>$S_{ha}$ share of agriculture in the GPDP</td>
<td>Prefectural Accounts</td>
</tr>
<tr>
<td>$S_{hm}$ shares of manufacturing in GPDP</td>
<td></td>
</tr>
<tr>
<td>$S_{hp}$ share of public sectors in the GPDP</td>
<td></td>
</tr>
<tr>
<td>$G_d$ per capita GDP (in thousand yen)</td>
<td>National Accounts</td>
</tr>
</tbody>
</table>

Table 2. Description and sources of variables
<table>
<thead>
<tr>
<th>dependent variable</th>
<th>ln($e_a$)</th>
<th>ln($e_p$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln($D_{pop}$)</td>
<td>0.0951*** (0.0202)</td>
<td>0.0949*** (0.0244)</td>
</tr>
<tr>
<td>ln($G_d$)</td>
<td>0.753 (0.514)</td>
<td>0.753* (0.442)</td>
</tr>
<tr>
<td>const.</td>
<td>-9.43** (4.26)</td>
<td>-9.43*** (3.66)</td>
</tr>
<tr>
<td>fixed effect</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>random effect</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>p-vale of Hausman test</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.15</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table 3. Preliminary estimation
<table>
<thead>
<tr>
<th>dependent variable</th>
<th>$\ln(c_a)$</th>
<th>$\ln(c_p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>$-5.35$</td>
<td>$9.72^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(4.25)$</td>
<td>$(3.25)$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$0.00312^{***}$</td>
<td>$0.00365^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.210 \times 10^{-3})$</td>
<td>$(0.000206)$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>$0.850 \times 10^{-6}^{***}$</td>
<td>$0.526 \times 10^{-6}^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.170 \times 10^{-6})$</td>
<td>$(0.116 \times 10^{-6})$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$-0.114$</td>
<td>$-0.0125$</td>
</tr>
<tr>
<td></td>
<td>$(0.169)$</td>
<td>$(0.123)$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$-0.170$</td>
<td>$0.803$</td>
</tr>
<tr>
<td></td>
<td>$(0.743)$</td>
<td>$(0.565)$</td>
</tr>
<tr>
<td>$b_3$</td>
<td>$0.299^{**}$</td>
<td>$-0.215^{**}$</td>
</tr>
<tr>
<td></td>
<td>$(0.142)$</td>
<td>$(0.109)$</td>
</tr>
<tr>
<td>$b_4$</td>
<td>$0.140^{***}$</td>
<td>$0.165^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0451)$</td>
<td>$(0.0353)$</td>
</tr>
<tr>
<td>$b_5$</td>
<td>$-0.135^{**}$</td>
<td>$-0.117^{**}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0566)$</td>
<td>$(0.0461)$</td>
</tr>
<tr>
<td>$b_6$</td>
<td>$-0.539^{***}$</td>
<td>$-0.140$</td>
</tr>
<tr>
<td></td>
<td>$(0.174)$</td>
<td>$(0.132)$</td>
</tr>
<tr>
<td>$b_7$</td>
<td>$0.173^{***}$</td>
<td>$0.0892^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.0444)$</td>
<td>$(0.0334)$</td>
</tr>
<tr>
<td>$b_8$</td>
<td>$2.00^{***}$</td>
<td>$0.166$</td>
</tr>
<tr>
<td></td>
<td>$(0.522)$</td>
<td>$(0.398)$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>$0.46$</td>
<td>$0.70$</td>
</tr>
<tr>
<td>$N$</td>
<td>138</td>
<td>138</td>
</tr>
</tbody>
</table>

Table 4. Results for entrepreneurship
Note: Standard errors are in parentheses. Significance levels: $^* = 10\%$, $^{**} = 5\%$, $^{***} = 1\%$. $^* = 1\%$. 
Table 5. Robustness check: employment density

<table>
<thead>
<tr>
<th>dependent variable</th>
<th>(\ln(e_a))</th>
<th>(\ln(e_p))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0)</td>
<td>-4.63 (4.26)</td>
<td>9.31*** (3.31)</td>
</tr>
<tr>
<td>(a_1)</td>
<td>0.00420*** (0.000275)</td>
<td>0.00504*** (0.000280)</td>
</tr>
<tr>
<td>(a_2)</td>
<td>(0.244 \times 10^{-5})*** ((0.449 \times 10^{-6}))</td>
<td>(0.141 \times 10^{-5})*** ((0.302 \times 10^{-6}))</td>
</tr>
<tr>
<td>(b_1)</td>
<td>-0.0850 (0.162)</td>
<td>-0.0186 (0.117)</td>
</tr>
<tr>
<td>(b_2)</td>
<td>-0.153 (0.730)</td>
<td>0.798 (0.564)</td>
</tr>
<tr>
<td>(b_3)</td>
<td>0.309** (0.141)</td>
<td>-0.215** (0.109)</td>
</tr>
<tr>
<td>(b_4)</td>
<td>0.161*** (0.0438)</td>
<td>0.168*** (0.0345)</td>
</tr>
<tr>
<td>(b_5)</td>
<td>-0.126** (0.0545)</td>
<td>-0.114** (0.0454)</td>
</tr>
<tr>
<td>(b_6)</td>
<td>-0.504*** (0.168)</td>
<td>-0.145 (0.130)</td>
</tr>
<tr>
<td>(b_7)</td>
<td>0.153*** (0.0419)</td>
<td>0.0823** (0.0323)</td>
</tr>
<tr>
<td>(b_8)</td>
<td>1.83*** (0.525)</td>
<td>0.145 (0.405)</td>
</tr>
</tbody>
</table>

\(R^2\) 0.47 0.70
\(N\) 138 138

Note: Standard errors are in parentheses. Significance levels: * = 10%, ** = 5%, *** = 1%.
Figure 1: Kernel density estimation of the share ($\epsilon_a$) of ex ante entrepreneurs
Figure 2: Kernel density estimation of the share ($\epsilon_p$) of ex post entrepreneurs
Figure 3: Simulated elasticity of ex ante/ex post entrepreneurship with respect to the population density. The solid curve represents $(D_{\text{pop}}/e_a)(\partial e_a/\partial D_{\text{pop}})$ and the dashed curve represents $(D_{\text{pop}}/e_p)(\partial e_p/\partial D_{\text{pop}})$. 

\[
\begin{align*}
\frac{D_{\text{pop}}}{e_i} & \frac{\partial e_i}{\partial D_{\text{pop}}} \\
\frac{D_{\text{pop}}}{e_a} & \frac{\partial e_a}{\partial D_{\text{pop}}} \\
\frac{D_{\text{pop}}}{e_p} & \frac{\partial e_p}{\partial D_{\text{pop}}}
\end{align*}
\]
Figure 4: Relative importance of density economies to market expansion. The solid curve represents the case of ex ante entrepreneurship and the dashed curve represents the case of ex post entrepreneurship.
Figure 5: Kernel estimation of the conditional mean of $e_a$ ($= E(e_a|\ln(D_{pop}))$)
Figure 6: Kernel estimation of the conditional mean of $e_p (= E(e_p | \ln(D_{pop}))$
Figure 7: Robustness check. Simulated elasticity of ex ante/ex post entrepreneurship with respect to the employment density. The solid curve represents \((D_{em} / e_a)(\partial e_a / \partial D_{em})\) and the dashed curve represents \((D_{em} / e_p)(\partial e_p / \partial D_{em})\).