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Osaka University, Toyonaka, Osaka 560-0043, JAPAN

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Abstract

The sample-specificity and path-dependence of the data envelopment analysis (DEA) based technical change index as a component of Malmquist indexes prevent us from obtaining overall and systematic information on technical change. This paper develops a path-independent method to estimate technical change using a systematized set of controlled input–output vectors and visualization of the DEA frontiers. The application to the panel datasets of agricultural production in the Brazil Amazon in 1975–1995 indicates non-Hicks-neutral technical change, with crossings of frontiers in both the 1975–1985 and the 1985–1995 periods. The alternative measure of overall technical change shows that moderate technological progress may have occurred on the whole in 1975–1995. The results also show heterogeneous trends across products. The mean of the sample-specific technical change scores are found to be quite different from the overall technical change measure.

JEL classification: D24、O30
Key words: Data envelopment analysis, Path dependence, Agricultural intensification

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†Corresponding author: Tsunehiro Otsuki, Osaka School of International Public Policy, Osaka University, 1-31 Machikaneyama, Toyonaka, Osaka, 560-0043 JAPAN. E-mail : otsuki@osipp.osaka-u.ac.jp

I. Introduction

Data envelopment analysis (DEA) has been a widely used nonparametric method to estimate productivity. The popularity of this DEA-based Malmquist productivity change index rests in its decomposition into measures of technical change and change in technical efficiency in addition to the common advantage of DEA to accommodate multiple-input and multiple-output technology. As the DEA-based method calculates the components of Malmquist indexes at the level of individual decision-making units (DMUs) (Färe *et al*. (FGNZ), 1994), the contribution of each component can be investigated for each DMU. When overall technical change, such as how the production frontier shifts over time, is of interest, however, the DMU-specific measures provide little information because information on each DMU is mutually-isolated. Also, in the case of non-Hicks-neutral technical change, the DMU-specific technical change indexes encounter the problem of path dependence, or non-transitivity, that stems from the path dependence of Malmquist indexes, which causes the estimate of technical change to take different values depending on the input-output levels at which technical change is measured.

The DMU-specificity of DEA-based indexes prevents estimation of the production frontier as an integrated set of best-practice DMUs and, hence, estimation of overall technical change. While technical efficiency is attributable to individual DMU's ability, technical change is a shift of production frontier that occurs exogenously and indiscriminately to all DMUs over the economy. Accordingly, the difference in technical change scores across DMUs is simply accounted for by the difference in their input-output bundles. Therefore, a method to estimate overall technical change would need to allow for the flexibility to measure technical change at arbitrary input-output bundles without being constrained to existing input-output bundles.

Furthermore, path dependence of the technical change index makes it difficult to estimate overall technical change using actual data, when technical change is non-Hicks-neutral. Färe *et al*. (FGR) (1998) showed that the Malmquist indexes are path dependent except when technical change is Hicks-neutral. The DMU-specific technical change index can take different values depending on which input-output levels are to be used between two different points of time for the same overall technical change. Färe *et al*. (FGGL) (1997) suggested further a decomposition of the technical change index into input and output bias indexes. Managi and Karameri (2004) is among recent empirical studies that estimate FGGL's bias indexes, and they demonstrated using the US agricultural production data in 1960-1996 that the size of the biases is large enough to attract attention. Chen and Ali (2004) indicated the possibility of having an opposite sign of technical change score for the same DMU due to the path dependence of the technical change index when frontiers of two periods cross each other.

This paper develops a DEA-based method to estimate overall technical change using controlled input–output vectors while avoiding the DMU-specificity and the path-dependence problems. An application of the method is undertaken to assess technical change in the agricultural sector in the Brazil Amazon in 1975–1995.

II. The Model

Let $\boldsymbol{x} = (x_1,...,x_N) \in \Re_+^N$ and $\boldsymbol{y} = (y_1,...,y_M) \in \Re_+^M$ be the vectors of inputs and outputs, respectively. Let $Y^t(\boldsymbol{x})$ be the output set in period *t* that is producible by $\boldsymbol{x}$. The output set is assumed to be closed, bounded, and convex and to exhibit strong disposability of all outputs. The output-oriented Malmquist index under the Caves *et al*. (1982) (CCD) definition is a ratio of the following two distance functions:

$$D^t(\boldsymbol{y}^t,\boldsymbol{x}^t)=\inf\{\theta:\boldsymbol{y}^t/\theta\in Y^t(\boldsymbol{x}^t)\} \tag{1}$$

and

$$D^t(\boldsymbol{y}^{t+1},\boldsymbol{x}^{t+1})=\inf\{\theta:\boldsymbol{y}^{t+1}/\theta\in Y^t(\boldsymbol{x}^{t+1})\}. \tag{2}$$

Assuming constant returns to scale (CRS), the solution of DEA for the distance function in (1) is obtained for the *k*th DMU using the following linear programming:

$$\begin{aligned}&\{D^t(\boldsymbol{y}_k{}^t,\boldsymbol{x}_k{}^t)^{-1}\}=\sup_{z_j,\theta}\{\theta: s.t.\sum_{j=1}^{J} z_j y^t{}_{mj}\geq\theta y^t{}_{mk},\sum_{j=1}^{J} z^t_j x^t_{nj}\leq x^t{}_{nk},\\&m=1,\ldots,M,n=1,\ldots,N,z_j\geq 0,k=1,\ldots,J\}\end{aligned} \tag{3}$$

The value of $D^t(\boldsymbol{y}^{t+1},\boldsymbol{x}^{t+1})$ is calculated by replacing $y^t{}_{mk}$ and $x^t{}_{nk}$ with $y^{t+1}{}_{mk}$ and $x^{t+1}{}_{nk}$ from (3), respectively. Thus, the Malmquist index is given as $M_{CCD1}=\dfrac{D^t(\boldsymbol{y}^{t+1},\boldsymbol{x}^{t+1})}{D^t(\boldsymbol{y}^t,\boldsymbol{x}^t)}$. It is constructed based on the period *t* reference technology. Alternatively, the Malmquist index can adopt the reference technology in period *t*+1, $M_{CCD2}=\dfrac{D^{t+1}(\boldsymbol{y}^{t+1},\boldsymbol{x}^{t+1})}{D^{t+1}(\boldsymbol{y}^t,\boldsymbol{x}^t)}$.

The CCD Malmquist index can be decomposed into technical change (*TC*) and efficiency change (*EC*) components:

$$M_{CCD1}=TC_{CCD1}\cdot EC_{CCD1}=\left[\frac{D^t(\boldsymbol{y}^{t+1},\boldsymbol{x}^{t+1})}{D^{t+1}(\boldsymbol{y}^{t+1},\boldsymbol{x}^{t+1})}\right]\left[\frac{D^{t+1}(\boldsymbol{y}^{t+1},\boldsymbol{x}^{t+1})}{D^t(\boldsymbol{y}^t,\boldsymbol{x}^t)}\right], \tag{4}$$

Thus, technical change can be defined as a ratio of values of distance functions at periods *t* and *t*+1 evaluated at the same input–output bundle. When the values of distance functions are based on the input–output bundle in period *t*, then $TC_{CCD2}=\dfrac{D^t(\boldsymbol{y}^t,\boldsymbol{x}^t)}{D^{t+1}(\boldsymbol{y}^t,\boldsymbol{x}^t)}$. Likewise, the FGNZ

Malmquist index can be decomposed into TC and EC while TC is a geometric mean of TC_{CCD1} and TC_{CCD2}.

The path dependence of the technical change index implies the difference between TC_{CCD1} and TC_{CCD2}, and vise versa, when the technology does not exhibit the Hicks-neutrality. FGGL (1997) showed that the ratio of TC_{CCD1} to TC_{CCD2} can be expressed as the product of an input and an output bias indexes (*IB* and *OB*, respectively), where *OB* (*IB*) measures the departure from the Hicks-output(input)-neutrality of technical change. When the technology does not exhibit the Hicks-output-neutrality, their Proposition 1 implies that *OB* equals unity (i.e. there is no output bias) only if $\boldsymbol{y}^{t+1} = \lambda \boldsymbol{y}^t$, $\lambda > 0$ (temporal constancy of the output mix). When the technology does not exhibit the Hicks-input-neutrality, their Proposition 2 implies that *IB* equals unity (i.e. no input bias) only if $\boldsymbol{x}^{t+1} = \boldsymbol{x}^t$ under the CRS assumption. Thus, path-dependence problem is inevitable in practice as the above conditions on input-output bundle are seldom satisfied.

This path-dependence problem can be avoided by using common input–output bundles at which technical change is measured. Such input-output bundles need not to coincide with the sample ones, and they can be controlled by the researcher. Thus, the input–output bundles can be set systematically such that technical change is measured in overall directions. Let us denote a controlled input–output vector $(\boldsymbol{u}, \boldsymbol{v})$, where $\boldsymbol{u} \in \Re_+^M$, $\boldsymbol{u} = \{u_1, ..., u_M\}$ and $\boldsymbol{v} \in \Re_+^N$, $\boldsymbol{v} = \{v_1, ..., v_N\}$. Distance functions at t and t+1 can be defined as $D^t(\boldsymbol{u}, \boldsymbol{v}) = \inf\{\theta : \boldsymbol{u} / \theta \in Y^t(\boldsymbol{v})\}$ and $D^{t+1}(\boldsymbol{u}, \boldsymbol{v}) = \inf\{\theta : \boldsymbol{u} / \theta \in Y^{t+1}(\boldsymbol{v})\}$. Then, accordingly:

$$TC = \frac{D^{t+1}(\boldsymbol{u}, \boldsymbol{v})}{D^t(\boldsymbol{u}, \boldsymbol{v})}. \qquad (5)$$

The value of the distance function for lth controlled vector $D^t(\boldsymbol{u}_l, \boldsymbol{v}_l)$, where $l = 1,..., L$, can be calculated by

$$\{D^t(\boldsymbol{u}_l, v_l)^{-1}\} = \sup_{z_j, \theta} \{\theta : s.t. \sum_{j=1}^{J} z_j y^t{}_{mj} \geq \theta u_{ml}, \sum_{j=1}^{J} z_j^t x_{nj}^t \leq v_{nl}, \\ m = 1,..., M, n = 1,..., N, z_j \geq 0, l = 1,..., L\}. \tag{6}$$

The value for $D^{t+1}(\boldsymbol{u}_l, \boldsymbol{v}_l)$ can be calculated by replacing $y^t{}_{mj}$ and $x^t{}_{nj}$ with $y^{t+1}{}_{mj}$ and $x^{t+1}{}_{nj}$, respectively.

By definition, the point on the period t frontier is given by:

$$\boldsymbol{y}_l *^t = \frac{\boldsymbol{u}_l}{D^t(\boldsymbol{u}_l, \boldsymbol{v}_l)} \text{ and } \boldsymbol{y}_l *^{t+1} = \frac{\boldsymbol{u}_l}{D^t(\boldsymbol{u}_l, \boldsymbol{v}_l)}. \tag{7}$$

This projection is repeated for all $l = 1,..., L$ to obtain sufficient points to recover the frontiers for all time points. The input vector $\boldsymbol{v}$ is kept constant to normalize the output set and to rule out the presence of input bias.

Once $\boldsymbol{y}*^t$ and $\boldsymbol{y}*^{t+1}$ are obtained for each controlled vector, those points are used to construct a convex hull for the each year's frontier, and the frontiers are visualized using 2-D or 3-D graphs as long as $m \leq 3$. Aoki *et al*.'s (2005) introduced a method of visualizing DEA frontiers while their method used only the existing data points in a single time period. This paper extends their method using controlled vectors to allow for comparison of frontiers in multiple time points.

Technical change in individual sectors can be visually compared in 2-D or 3-D space. Also, we can quantify technical change on the whole by comparing the space bounded by the frontier and the three planes that are formed by the coordinate axes. Let us denote the volume of the bounded space by $Vol(Y(\cdot))$ in the case $m = 3$. The value of the ratio

$\delta = \frac{Vol(Y^{t+1}(\boldsymbol{v}))}{Vol(Y^{t}(\boldsymbol{v}))}$ greater (less) than one means positive (negative) technical change. Assuming that technical change had occurred in a Hicks-neutral fashion, a homothetic ratio that is given by $\delta^{\frac{1}{3}}$ can be considered as an alternative measure of technical change.

III. Application to the Agricultural Sector in the Brazil Amazon

I apply the above method to estimating technical change in the agricultural sector in the Brazil Amazon using a cross-county dataset for 255 counties in 1975, 1985 and 1995 that are available from the online data that were compiled by Instituto de Pesquisa Econômica Aplicada (IPEA). The counties in this region are reasonably homogeneous in geoclimatic conditions and production profiles, which allows us to assume that they face the same technology at a given point of time. Agricultural intensification through technological progress is thought to contribute to both economic development and environmental conservation.[*] Furthermore, this is one of the few datasets in developing countries that are available for a large number of cross-section units for multiple-year periods.[†]

For analytical tractability, I aggregate four annual crops (rice, cassava, beans and maize) into one category, "annual crop," measured in 2000 US dollar. "Perennial crop" includes only bananas, measured in tons. "Cattle" measures the number of head of cattle. The two inputs comprise labor and land.[‡]

The input levels of the controlled vectors are fixed at the regions' average in 1985. Figures 1 and 2 show the visualized frontiers in 1975 and 1985, and in 1985 and 1995,

[*] See Lee *et al*. (2006) for extensive discussion on definitions for agricultural intensification.

[†] IPEA established a geographical definition for the analytical units of the dataset that remained consistent throughout the period despite the rearrangements of counties that occasionally took place.

[‡] While it would be ideal to have capital data, they are not available. The major products in the region are not capital intensive and, hence, it is unlikely that the omission of capital data will lead to large biases.

respectively, where levels of outputs are normalized such that the intercepts of the 1975 frontier are unity.

Both figures suggest non-Hicks-neutrality in technical change. From 1975 to 1985, crossing of the two frontiers is observed around the middle of the frontiers in Figure 1. The 1985 frontier dominates the 1975 frontier as we approach the cattle (CT) and the annual crop (AC) axes, which is likely to be the result of positive technical change in the CT and AC sectors. On the other hand, the dominance of the 1975 frontier around the perennial crop (PC) axis implies a technological regress in that product sector. The first three columns of Table 1 show the degree of frontier shifts along each axis. The shift of the frontier along the CT, AC and PC axes from 1975 to 1985 is found to be 33.4%, 21.3% and –14.6%, respectively.

From 1985 to 1995, the overall dominance of the frontier is not obvious as, again, the frontiers cross in the middle. The frontier shift is –6.9%, 6.7% and –14.3% along the cattle, annual crop and perennial crop axes, respectively. The Hicks-neutral equivalent of frontier shift is calculated by $\delta^{\frac{1}{3}}$. The growth rates are found to be –3.1% (technological regress) in 1975–1985 and 4.8% (technological progress) in 1985–1995.

For comparison purposes, CCD and FGNZ's DMU-specific technical change measures are calculated and the descriptive statistics are presented in Table 2. The mean value of TC_{FGLZ} is 7.7 (28) percentage points smaller (greater) than the Hicks-neutral equivalent measure in 1975–1985 (1985–1995). Potentially, this may be because the DMU-specific measures tend to be more heavily weighted by the sectors with higher sample concentrations—perennial crops for the former period and annual crops for the latter period. Further, the last column of Table 2 shows that only 32.9% of the counties benefited from technological progress, whereas 90.6% benefited from technological progress in 1985–1995 based on the FGNZ measure.

The last column of Table 2 also indicates that there is a large difference between TC_{CCD1} and TC_{CCD2}. This implies that one can reach quite different conclusions on technical change depending on the choice of the input–output bundle between two points in time.

IV. Conclusions

This paper develops a path-independent method to estimate technical change using a systematized set of controlled input–output vectors and visualization of the DEA frontiers. The application to the panel datasets of agricultural production in the Brazil Amazon in 1975–1995 indicates non-Hicks-neutral technical change, with crossings of frontiers in both the 1975–1985 and the 1985–1995 periods. The alternative measure of overall technical change shows that moderate technological progress may have occurred in 1975–1995; thereby suggesting that agricultural intensification was in progress. The results also show heterogeneous trends across products—moderate progress in cattle and annual crops, and a decline in perennial crops (bananas). The mean DMU-specific technical change scores are found to be quite different from the overall technical change measure and, thus, one should not rely solely on the DMU-specific measures.

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Table 1. Technical change estimates along coordinate axes and volume-based indexes

	Cattle	**Annual crop**	**Perennial crop**	**Hicks-neutral equivalent growth rate (based on homothetic ratio)**
1985 level (1975=1)	1.334	1.213	0.854	
Growth in 1975–1985	+33.4%	+21.3%	–14.6%	–3.1%
1995 level (1975=1)	1.242	1.294	0.732	
Growth in 1985–1995	–6.9%	+6.7%	–14.3%	+4.8%

Table 2. FGNZ- and CCD-based technical change estimates

		Obs	**Mean**	**SD**	**Min**	**Max**	**Median**	**Number of counties with non-negative growth ($TC \geq 0$)**
1975–1985	TC_{FGLZ}	255	0.892	0.227	0.432	1.986	0.930	84(32.9%)
	TC_{CCD1}	255	0.858	0.252	0.383	1.922	0.900	73(28.6%)
	TC_{CCD2}	255	0.944	0.271	0.402	2.421	0.959	102(40.0%)
1985–1995	TC_{FGLZ}	255	1.328	0.288	0.875	2.429	1.263	231(90.6%)
	TC_{CCD1}	255	1.300	0.299	0.576	2.055	1.250	220(86.3%)
	TC_{CCD2}	255	1.380	0.425	0.839	4.348	1.290	221(86.7%)

Figure 1. Visualized production frontiers in 1975 and 1985

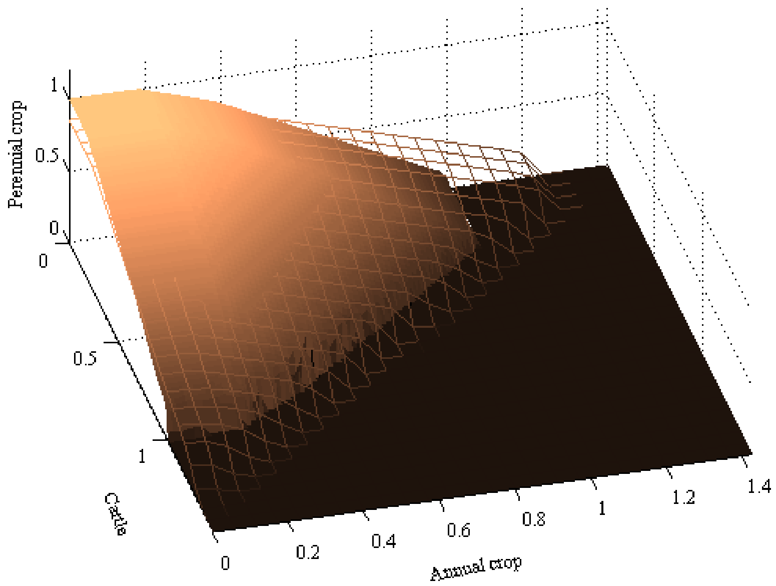

Figure 2. Visualized production frontiers in 1985 and 1995