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Abstract

This paper develops an overlapping generations model that involves the endogenous determination of fertility and an explicit city structure in order to analyze fully the social and natural changes in city populations. We provide conditions under which the model exhibits the spatial features of demography observed in large Japanese cities. We also show by calibration that the low cost of obtaining human capital in Tokyo metropolitan area played a significant role in establishing its urban primacy in Japan.

Keywords: urbanization, demography, migration, monocentric city

JEL Classification: J11, R11, R14, R23

1 Introduction

It has long been recognized that the structure of a city bears a relationship with the demography within its areas. Statements on this issue are easily accessible: for example, the National Institute of Population and Social Security Research of Japan [9] conduted a survey on the number of children that couples are actually going to have and the number of children they would have wished to have under ideal conditions. Table 78 of this survey reports on the reasons why couples are going to have fewer children than the ideal number. The table shows that whereas 16.8 percent of couples who live in Densely Inhabited Districts (DIDs) chose the unaffordability of having a sufficiently spacious house as one of the reasons, only 5.4 percent of the couples who live in non-DIDs cited this reason. These figures indicate that the city structure, through high land rents, may have a significant impact on fertility within its areas.

It is also well known that the growth of cities can be traced to two sources − social and natural population changes. The social changes represent the flows of populations into/out of the city

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whereas the natural changes describe the difference between the numbers of births and deaths within the city. In order to understand what factors are important in making cities grow to be megalopolises, it is necessary to obtain a tool that can deal with the city structure and its social and natural population changes in a unified framework.

However, full-fledged analyses on the relationship between cities and their demographies from the viewpoint of economics have been scarce until recently. An noticeable exception is Shultz [13], which provides empirical results that support the interdependence of city structure and demography by showing that advances in urbanization reduce the national total fertility rate.

Very recently, several studies uncovered possible nonnegligible interactions between city structure and demography. As for empirical evidences, Galor [5] presented stylized facts that imply (i) that urbanization and economic development started simultaneously, and (ii) that in the early phases of urbanization and economic development, the population growth rate had increased but had then declined over succeeding years. Simon and Tamura [14] showed the existence of a negative cross-sectional correlation between the price of living spaces as measured by rent per room and the fertility rate for the Consolidated Metropolitan Statistical Areas in the United States over the period 1940–2000. The results described in these studies imply that city structure − including the land and housing markets − can play a major role in determining the demographic features of cities.

Theoretically, Zhang [16], Sato [10], Sato and Yamamoto [11], and Sato and Yamamoto [12] used reduced form models to analyze the demographic impacts of urbanization .[1] Zhang [16] and Sato and Yamamoto [12] examined the impact of urbanization, caused by better opportunities for earnings and education in the cities, on demography. Sato [10] and Sato and Yamamoto [11] investigated how urbanization and demographic transition interrelate with each other *via* the merits of population concentration (agglomeration economies) and its demerits (congestion diseconomies). All these studies showed that urbanization is accompanied by a decline in fertility, which is consistent with the stylized facts described in Galor [5]. However, since the models developed in these studies have no explicit spatial structure, their analysis does not shed any light on the spatial features of the demography within cities.

In this paper, we contribute to this body of literature by developing an overlapping generations model of endogenous fertility that involves explicit spatial structure. In particular, we focus on the relationship between the city size, the spatial patterns of fertility within the city, and the patterns of land consumption. For this purpose, we construct an overlapping generations model of endogenous fertility that involves the monocentric city structure à la Alonso [1]. In considering fertility decisions, we follow the views of Becker [2], which regards having children as consumption, and not as investment. Each household obtains utility from numéraire consumption, land consumption, and

[1] Eckstein et al [3] developed an overlapping generations growth model that involves land as a production input, and examined the impact of the limitation of land availability on economic growth. However, their model does not deal with land consumption or city structure.

the number of its children. The key assumption here is the complementarity between land consumption and the number of children: one needs a certain amount of land in order to rear a child, and obtains utility from land consumption over the required level for child rearing. Population changes arise not only from changes in the number of children (natural changes) but also from migration into/out of the city (social changes). In such a model, the land rent is higher in the central part of the city, leading to lower land consumption and fewer numbers of children. Moreover, as the city grows, the land rent gets higher, and land consumption and fertility diminish. These features are consistent with the stylized facts that we shall describe in the next section and those shown in Simon and Tamura [14].

By introducing human capital investment, we extend the basic model so that it can describe the growth of a city. We calibrate this extended model to replicate changes in population and demographic characteristics of three of Japan's largest metropolitan areas (Tokyo, Osaka and Nagoya, where Tokyo, Osaka and Nagoya are the primary, secondary and tertiary cities, respectively) during the last half century. Based on these calibrated models, we explore the factors that have been significant in establishing the urban primacy of Tokyo. This is done by executing counterfactuals on Osaka and Nagoya: we investigate what would happen if the value of one of the parameters of the Osaka or Nagoya model took on that of the Tokyo model. Such counterfactuals reveal that Osaka and Nagoya could have grown as much as (or even more than) Tokyo had human capital investment in these cities costed as little as it did in the case of Tokyo. Moreover, we find that the growth of Osaka has been seriously limited by shortage of land supply, whereas Nagoya has the potential to grow more provided that its intra-city commuting infrastructure improves.

The remainder of this paper is organized as follows. Section 2 presents stylized facts on population changes and demographic characteristics of the three largest cities in Japan. Section 3 develops the basic model and describes its properties. Section 4 extends the basic model to incorporate growth factors and provide numerical analysis. Section 5 concludes.

2 Some stylized facts

This section provides an overview of basic stylized facts on the demography of metropolitan areas in Japan. We use data on the three largest Metropolitan Areas (MAs), which are Tokyo MA, Osaka MA, and Nagoya MA.[2] Tokyo, Osaka, and Nagoya are the first, second, and third largest cities in

[2] Here, each MA consists of several prefectures: Tokyo MA=Tokyo+Kanagawa+Saitama+Chiba, Osaka MA=Osaka+Kyoto+Hyogo+Nara, and Nagoya MA=Aichi+Gifu+Mie. These definitions are often used in analysis by the Ministry of Internal Affairs and Communications, Japan. Although these are very coarse definitions, they are superior to other definitions, taking the availability of various data into account.

Even if we were to use another definition of MAs, these three cities would always emerge as largest though we would have different sizes. For instance, by the definition of Urban Employment Area (UEA), the population size of Tokyo UEA is 33.4 million UEA, that of Osaka UEA is12.1 million, and that of Nagoya UEA is 5.2 million (see Kanemoto

Japan. Figure 1 shows their sizes in terms of population from 1950 to 2007.

[Insert Figure 1 around here]

The three cities have grown steadily, while keeping the order of their sizes unaltered. In 2007, the size of Tokyo's population was 34.8 million, Osaka's was 18.4 million, while Nagoya had a population of 11.3 million.

While the three cities have grown steadily during the past half century, their population sizes have undergone some fluctuations, which is confirmed by the decomposition into their trend and cyclical components. We use the Hodrick-Prescott filter for this decomposition.[3]

[Insert Figure 2 around here]

Figure 2 shows the trend (Figure 2-(a)) and cyclical (Figure 2-(b)) components of ln(population). By eliminating the trend components, we can observe that each of three MAs has undegone a slight fluctuation in its size. Moreover, we know that Tokyo and Osaka have similar levels of cyclical fluctuation whereas Nagoya has them to a lesser extent.

Such changes in population may originate from two kinds of sources: social and natural changes. The former represents the to and fro migration of people and the latter represents the fertility decisions of city residents. In this figure, we disregard the effects of longevity, which we discuss in detail later. Figure 3 describes the social changes in the three MAs.

[Insert Figure 3 around here]

We know that these cities experienced enormous in-migration from other regions in the period around 1950-1970. After that, Tokyo has had a slight but steady rate of in-migration, Osaka has experienced a slight net out-migration, and Nagoya has had almost zero net migration.

The total fertility rate is considered to be the primary source of the natural changes. Table 1 shows the total fertility rate for all prefectures that constitute our three MAs over the past half century.

[Insert Table 1 around here]

and Tokuoka [7]).

[3] We set the multiplier $\lambda = 400$, which is often used for annual data.

This table shows that the total fertility rate in all three MAs has steadily declined and that it is lower for a larger city. Moreover, the three cities have a common spatial feature in that fertility is lower in prefectures that are considered to be the central cities than for those that are considered as suburbs. As is well known, the land rent/price is higher in a larger city, and land in the center of a city has a higher rent/price than in the suburbs (see Table 2, for example).

[Insert Table 2 around here]

Put differently, the level of land rent is negatively associated with fertility both across cities and within each city. Since the level of land consumption is positively associated with the level of land rent, it implies that land consumption and fertility have a positive correlation.

3 Basic model

3.1 Individuals

Consider a linear space on which there is one Central Business District (CBD), that is, we assume a linear monocentric city. We approximate the CBD by a point and assume that all workers commute to the CBD. Without loss of generality, we assume that the residential area spreads only on the right hand side of the CBD.[4] We index the location of the CBD as 0, and describe each location by the distance x between it and the location of the CBD.

Time is discrete and each individual lives for two periods; a childhood and a parenthood. Each individual has a single parent. In the parenthood period, each individual is endowed with one unit of time, which she spends on working and on child rearing. At the beginning of period t, she decides on goods and land consumption (c_t and d_t) and the number of her children (n_t). She exits the economy at the end of period t. N_t individuals in the parenthood live in the city in period t. This implies that $n_t N_t$ children are born in period t and grow to be parents in period $t+1$. In this model, n_t represents the total fertility rate.

We assume that individuals have an identical utility function of the Cobb-Douglas form and the utility of each individual depends on the level of goods and land consumption and on the number of children:

$$U_t = \alpha \ln c_t + \beta \ln n_t \left(d_t - \varepsilon n_t\right), \tag{1}$$

where α, β and ε are positive constants and satisfy $\alpha + 2\beta = 1$. There is only one kind of goods in this economy, which we treat as a numéraire. In order to rear a child, one needs a certain amount of land and we represent it as ε. An individual obtains utility from land consumption that is over the

[4] Even if we suppose that the residential area spreads on both sides of the CBD, the results remain unaltered.

level required for child rearing. This complementarity between land consumption and the number of children brings forth the spatial characteristics of demography in a city.

In order to have n_t children, each individual must spend bn_t units of time, where b is a positive constant. This assumption requires that n_t must satisfy $0 \leq n_t \leq 1/b$. Because each working individual is endowed with one unit of time, she spends $1 - bn_t$ units of time on working. The budget constraint for a working individual who resides at x distance from the CBD is given by

$$(1 - bn_t)I - \tau x = c_t + r_t(x)d_t, \tag{2}$$

where I denotes the wage income per unit of time and is a positive constant.[5] τ represents the commuting cost per unit distance and $r_t(x)$ is the market land rent at x distance from the CBD. Although individuals are assumed to be price takers and take $r_t(x)$ as given, $r_t(x)$ is endogenously determined later. We assume that land is owned by absentee landlords.[6]

The utility maximization gives the following demand functions:

$$\begin{aligned} c_t(x) &= \alpha\,(I - \tau x)\,, \\ n_t(x) &= \frac{\beta}{bI + \varepsilon r_t(x)}\,(I - \tau x)\,, \\ d_t(x) &= \beta\left(\frac{1}{r_t(x)} + \frac{\varepsilon}{bI + \varepsilon r_t(x)}\right)(I - \tau x)\,. \end{aligned} \tag{3}$$

This leads to the indirect utility as follows:

$$V_t(x) = A + \ln(I - \tau x) - \beta \ln r_t\,(x)\,(bI + \varepsilon r_t\,(x))\,. \tag{4}$$

where A is defined as $A \equiv \alpha \ln \alpha + 2\beta \ln \beta$.

We can see from (3) that a rise in wage income has two effects on the total fertility rate n_t. One is the positive income effect that is represented in the numerator of the right hand side. The other is the negative substitution effect that raises the opportunity cost of rearing children. This is described by the denominator of the right hand side. More importantly, a higher land rent leads to a smaller number of children and to smaller land consumption. This is because when $r_t(x)$ is high, the land requirement for child rearing forces individuals to have less children.

3.2 City structure and location equilibrium

We assume that children live with their parents, and that a working individual can move freely within a city. The location equilibrium is attained in each period, and it requires that the indirect utility be the same for all locations in a city:

$$V_t(x) = \overline{V}_t, \qquad \forall x \in (0, \overline{x}_t], \tag{5}$$

[5] In a later section, we explicitly endogenize the income by introducing a production structure and human capital investment.

[6] The assumption of absentee landlords is very standard in urban economics. See Fujita [4], for example.

where $\overline{x}_t$ denotes the city fringe in period t and represents the spatial size of the city.

We follow the well-established tradition of urban economics in determining the market land rent in a city by using the concept of "bid rent."[7] The bid rent is the maximum land rent at location x that each individual is willing to pay in order to reach her equilibrium utility level. We normalize the land rent outside of the city to one. This implies that the land rent at the city fringe $r_t(\overline{x}_t)$ is equal to 1. From (4) and (5), we obtain the bid rent at x by solving $A+\ln(I-\tau x)-\beta\ln R(bI+\varepsilon R)=V_t(\overline{x}_t)$ with respect to R, which yields

$$R=\frac{1}{2\varepsilon}\left[-bI+\sqrt{(bI)^2+4\varepsilon(bI+\varepsilon)\left(\frac{I-\tau x}{I-\tau\overline{x}_t}\right)^{1/\beta}}\right].$$

The market land rent is then given by

$$\begin{aligned} r_t(x) &= \max[R,1] \\ &= \begin{cases} R & \text{if} \quad x\in(0,\overline{x}_t] \\ 1 & \text{if} \quad x>\overline{x}_t \end{cases}. \end{aligned} \tag{6}$$

We readily obtain $r_t'(x)<0$ and $\partial r_t/\partial\overline{x}_t>0$ for $x\in(0,\overline{x}_t]$, which we summarize in the following lemma.

Lemma 1 *Within the city, the land rent is lower for a location more distant from the CBD ($r_t'(x)<0$). As the city becomes larger, the land rent in the city rises ($\partial r_t/\partial\overline{x}_t>0$).*

These are very standard results in the literature on monocentric models (Fujita [4]).

As shown in (3), differences in the land rent yield differences in the land demand and the fertility rate. Substituting (6) into (3) and differentiating them with respect to x, we have that for $x\in(0,\overline{x}_t]$,

$$\begin{aligned} d_t'(x) &= \frac{\tau(bI)^2(1-\beta)+2\alpha\varepsilon\tau(bI+\varepsilon)\left(\frac{I-\tau x}{I-\tau\overline{x}_t}\right)^{1/\beta}}{(bI+\varepsilon)\left(\frac{I-\tau x}{I-\tau\overline{x}_t}\right)^{1/\beta}(bI+2\varepsilon r_t(x))}>0, \\ n_t'(x) &= \frac{\varepsilon\tau(1-2\beta)r_t(x)-\beta\tau bI}{(bI+\varepsilon r_t(x))(bI+2\varepsilon r_t(x))}. \end{aligned} \tag{7}$$

The latter equation in (7) yields

$$n_t'(x)>0 \qquad \Leftrightarrow \qquad r_t(x)>\frac{\beta\tau bI}{\varepsilon\tau(1-2\beta)}.$$

From the fact that $r_t(x)\geq 1$, this leads to

$$n_t'(x)>0 \qquad \Leftarrow \qquad 1>\frac{\beta\tau bI}{\varepsilon\tau(1-2\beta)} \qquad \Leftrightarrow \qquad \varepsilon>\frac{\beta bI}{\alpha}. \tag{8}$$

From (7) and (8), we obtain the following proposition.

[7] The usage of the bid rent is very standard in urban economics. See Kanemoto [6] and Fujita [4] for a comprehensive discussion on the bid rent in monocentric city models.

Proposition 1 *An individual residing more distant from the CBD consumes more land. If land requirement ε for child rearing is sufficiently large, she has more children.*

The location of an individual has two effects on her land consumption and on the number of her children. On the one hand, when an individual lives more distant from the CBD, she must bear higher commuting costs, which reduces the income net of commuting costs. This has an effect of reducing land consumption and the number of children. On the other hand, the more distant from the CBD the location is, the lower the land rent is. A lower land rent enables an individual to consume more land. This also induces her to have more children because a lower land rent implies lower payments for the land needed for child rearing. With respect to land consumption, the latter effect dominates the former, and one who lives farther away from the CBD always consumes more land. This also holds true with regard to the fertility rate if the land requirement for child rearing is sufficiently large. These arguments provide one possible explanation for the stylized facts described in Table 1.

From (6), we obtain the level of utility in the city as

$$\overline{V}_t = A + \ln\left(I - \tau\overline{x}_t\right) - \beta\ln\left(bI + \varepsilon\right). \tag{9}$$

Therefore, for a given number of individuals N_t in parenthood, we can determine all other endogenous variables once the city fringe $\overline{x}_t$ is determined. The city fringe $\overline{x}_t$ is given by the land market clearing condition for a given N_t:

$$\int_0^{\overline{x}_t} \frac{D}{d_t(x)} dx = N_t. \tag{10}$$

D represents the land supply for each location that is exogenous in our model.

Letting $\overline{x}_t(N_t)$ denote $\overline{x}_t$ that is determined by (10), we can examine $\overline{x}_t'(N_t)$, that is, how the city population affects the city fringe. Let $\Omega(\overline{x}_t)$ denote the left hand side of (10). $\Omega(\overline{x}_t)$ has the following properties:

$$\begin{aligned} \Omega(0) &= 0, \qquad (11)\\ \lim_{\overline{x}_t \to I/\tau} \Omega(\overline{x}_t) &= \infty, \\ \Omega'(\overline{x}_t) &= \frac{D}{d_t(\overline{x}_t)} + \int_0^{\overline{x}_t} \frac{D}{d_t(x)^2}\left(-\frac{\partial d_t}{\partial r_t}\right)\frac{\partial r_t}{\partial \overline{x}_t} dx > 0. \end{aligned}$$

As the city fringe $\overline{x}_t$ increases, the market land rent r_t at each location rises, which leads to a lower land demand at each location and a larger $\Omega(\overline{x}_t)$. Hence, $\Omega(\overline{x}_t)$ is represented by an upward sloping curve in the $\overline{x}_t - N_t$ plane, as described in Figure 2.

[Figure 2 around here]

Since the right hand side of (10) is described by a horizontal line, we can see that $\overline{x}_t$ is uniquely determined once N_t is given.

Now consider increases in the city population from N_t to N_t' where $N_t' > N_t$. Upward shifts of the horizontal line lead to a higher $\overline{x}_t$. Moreover, we know from (11) the values of $\overline{x}_t$ when N_t takes extreme values (i.e., N_t is equal to 0 or converges to ∞).

$$\begin{aligned} \overline{x}_t'(N_t) &> 0, \\ \overline{x}_t(0) &= 0, \\ \lim_{N_t \to \infty} \overline{x}_t(N_t) &= \frac{I}{\tau}. \end{aligned} \tag{12}$$

The following lemma summarizes the results shown in (12).

Lemma 2 *An increase in the number of individuals in the city enlarges the city area. There is no city area if no one is in the city, and the city fringe can at most reach I/τ even if the city population explodes.*

Equation (3) shows that the city population size N_t affects the land demand d_t and the fertility rate n_t only through changes in the land rent r_t, and that $\partial d_t/\partial r_t < 0$ and $\partial n_t/\partial r_t < 0$, which, combined with Lemma 2, prove the following proposition.

Proposition 2 *The land rent is higher in a larger city where an individual consumes less land and has less children ($\partial r_t/\partial N_t > 0$, $\partial d_t/\partial N_t < 0$ and $\partial n_t/\partial N_t < 0$).*

As seen in a standard monocentric city model, an increase in city population raises the land rent, which depresses the land demand. Due to the land requirement for child rearing, this leads to a lower fertility rate. Such relationships are consistent with the stylized facts presented in Section 2 (tables 2 and 3) and in Simon and Tamura [14].

3.3 Population dynamics

We assume that there is migration into or out of the city depending on the difference in utility inside and outside the city. Such migration happens just before each period starts. More specifically, $M\left(\overline{V}_t - \overline{v}\right)$ individuals who are ready to become adults flow into the city just before period $t+1$ starts, where $\overline{v}$ (> 0) is the utility level of people outside of the city and M represents the adjustment speed of migration.[8] We assume that $\overline{v}$ and M are positive constants. This migration function represents that the city attracts people if the people there enjoyed higher utility than the people outside the city in the previous period; otherwise, the city loses people. We further assume that

[8] For a discussion on the stability of spatial equilibrium under this type of migration function, see Tabuchi and Zeng [15].

$\overline{V}_t\big|_{\overline{x}_t=0} > \overline{v}$. This implies that the land rent becomes sufficiently low for people to flow into the city if there are few people in the city.[9] [10]

In period t, an individual residing at x has $n_t(x)$ children, who grow up to be parents in the period $t+1$. Therefore, the law of motion of population is given by

$$N_{t+1} = M\left(\overline{V}_t - \overline{v}\right) + \int_0^{\overline{x}_t} \frac{Dn_t(x)}{d_t(x)} dx. \tag{13}$$

The first term represents the flow into/out of the city and hence the social changes in the city population. The second term is the total number of children in the previous period, which represents the natural changes in the city population. In this setting, the city has inflow of people if and only if $\overline{V}_t > \overline{v}$, which is equivalent to $\overline{x}_t < [I - (bI+\varepsilon)^{\beta} \exp[\overline{v} - A]]/\tau$. This requires that the distance from the CBD to the city fringe (the city area) is sufficiently small compared to the real income denominated by the commuting cost.

Both terms are functions of N_t and hence (13) can be described as

$$\begin{aligned} N_{t+1} &= \Lambda(N_t), \\ \Lambda(N_t) &\equiv M\left(\overline{V}_t - \overline{v}\right) + \int_0^{\overline{x}_t} \frac{Dn_t(x)}{d_t(x)} dx. \end{aligned} \tag{14}$$

Since we can determine all the other variables in period t once we fix N_t and the law of motion of population (14) determines N_{t+1} for a given N_t, we have a steady state equilibrium if there exists a steady state value of N_t. It is, if any, given by

$$N^* = \Lambda(N^*) \tag{15}$$

Let's define Φ and Ψ as

$$\begin{aligned} \Phi &\equiv \frac{\beta}{bI+\varepsilon}\left\{I + (bI+2\varepsilon)\left[\frac{bI^2}{\beta(4\varepsilon)^{3/2}(bI+\varepsilon)^{1/2}} - \frac{\tau M}{D}\right]\right\}, \\ \Psi &\equiv -\frac{\beta\tau M(bI+2\varepsilon)}{D(bI+\varepsilon)}. \end{aligned} \tag{16}$$

The following proposition establishes the existence of N^* and hence the steady state equilibrium, and provides the sufficient condition of its uniqueness and stability.

Proposition 3 *There exists a steady state equilibrium of the model. It is unique and stable when* $\Psi > -1$ and $\Phi < 1$. *These inequalities are satisfied when, for example,* b *and* β *are sufficiently small.*

[9] Remember that $\overline{x}_t(0) = 0$.

[10] These assumptions are equivalent to $A + \ln(I) - \beta\ln(bI+\varepsilon) > \overline{v}$, which is satisfied when the income I is sufficiently high.

Proof. See Appendix A. ■

Figure 2-(b) shows that the city population size fluctuates around the trends. We can obtain sufficient conditions under which such fluctuations are possible in our model as a converging path to the steady state. Let's define Ξ as

$$\Xi \equiv \frac{1}{\tau}\left(\frac{DI}{bI+2\varepsilon}+\frac{bDI^2}{\beta(4\varepsilon)^{3/2}(bI+\varepsilon)^{1/2}}\right).$$

Then we have the following proposition.

Proposition 4 *Suppose that* $\Psi > -1$ and $\Phi < 1$. *If the adjustment speed of migration is sufficiently high(i.e.,* $M > \Xi$*), then the converging path to the steady state shows fluctuations around the steady state.*

Proof. See Appendix B. ■

This proposition implies that the population size fluctuates when the level of responses in terms of the inflow/outflow of city population to changes in the utility differential is sufficiently high.

4 Model with human capital

4.1 Introduction of human capital

In order to capture the continuous growth of metropolitan areas, we now introduce human capital investment into our basic model. There are various ways of introducing human capital into our model, and we adopt the most simple way. Each individual, in her parenthood, decides the level h_t of her human capital in addition to goods and land consumption (c_t and d_t) and the number of her children (n_t). We assume that she needs to spend Sh_t^{σ} in terms of numéraire in order to obtain human capital of level h_t, where $S > 0$ and $\sigma > 1$, and that a worker whose level of human capital is h_t obtains the wage income $w_t h_t$. w_t is the wage rate in the efficiency unit. In our setting, the migration decision is taken before one determines the level of human capital. Of course, there are some people who migrate after obtaining human capital, which implies the possibility of heterogeneity of human capital among city residents. Further, the average income level within a city depends on whether people with high human capital flow into the city (See Mori and Turrini [8]). However, the model becomes highly intractable once we introduce both the human capital accumulation within the city and a sorting possibility into the model. We assume the former only because it is beyond the scope of our paper to develop a model that includes everything at a time.

Whereas the utility function is still given by (1), the budget constraint is now

$$(1-bn_t)w_t h_t - \tau x = c_t + r_t(x)d_t + Sh_t^{\sigma}. \tag{17}$$

The resulting demand functions are determined by

$$
\begin{aligned}
c_t(x) &= \alpha(w_t h_t(x) - \tau x - S h_t(x)^\sigma), \qquad (18)\\
d_t(x) &= \frac{\beta(b w_t h_t(x) + 2\epsilon r_t(x))}{r_t(x)(b w_t h_t(x) + \epsilon r_t(x))}(w_t h_t(x) - \tau x - S h_t(x)^\sigma),\\
n_t(x) &= \frac{\beta}{b w_t h_t(x) + \epsilon r_t(x)}(w_t h_t(x) - \tau x - S h_t(x)^\sigma),\\
S\sigma h_t(x)^{\sigma-1} &= (1 - b n_t(x)) w_t.
\end{aligned}
$$

We know from the first three equations that the other demands are determined once $h_t(x)$ is determined. The last equation of (18) shows that an individual who invests more (i.e., has higher h_t) has fewer children (i.e., smaller n_t), which implies that our model has the standard "quantity-quality trade off" of children (see, e.g., Becker [2]).

Substituting the third equation of (18) into the last one, we have

$$
\frac{w_t - S\sigma h_t(x)^{\sigma-1}}{w_t h_t(x) - \tau x - S h_t(x)^\sigma} = \frac{\beta b w_t}{\varepsilon r_t(x) + b w_t h_t(x)}.
$$

This implicitly determines the level of human capital as $h_t(x) = h_t^*(x; w_t, r_t(x))$, where we explicitly write that h_t depends on w_t and $r_t(x)$ for later reference. Substituting this into (18), we have demand functions that lead to the following indirect utility:

$$
V = A + \ln(w_t h_t^*(x; w_t, r_t(x)) - \tau x - S h_t^*(x; w_t, r_t(x))^\sigma) - \beta \ln r_t(x)(b w_t h_t^*(x; w_t, r_t(x)) + \epsilon r_t(x)).
$$

Location equilibrium again requires (5), which we solve to obtain the equation that implicitly determines the bid rent function:

$$
\begin{aligned}
R \;=\; & \frac{1}{2\epsilon}\Bigg\{ - b w_t h_t^*(x; w_t, R) + \Bigg[(b w_t h_t^*(x; w_t, R))^2 + 4\epsilon(b w_t h_t^*(\bar{x}_t; w_t, 1) + \epsilon) \qquad (19)\\
& \times \left(\frac{w_t h_t^*(x; w_t, R) - \tau x - S h_t^*(x; w_t, R)^\sigma}{w_t h_t^*(\bar{x}_t; w_t, 1) - \tau \bar{x}_t - S h_t^*(\bar{x}_t; w_t, 1)^\sigma} \right)^{1/\beta} \Bigg]^{1/2} \Bigg\}.
\end{aligned}
$$

The market land rent function is given by (6).

In the basic model, we ignored the production side of the economy and assumed a fixed level of income in the city. Here, we endogenize the production decision, in which the production function is given by

$$
y_t = \gamma(1 - e^{-\delta H_{t-1}}) l_{ht}^\lambda k_t^{1-\lambda}, \qquad (20)
$$

where γ, δ, and λ are positive constants. λ satisfies $0 < \lambda < 1$. H_{t-1}, l_{ht} and k_t represent the aggregate level of human capital in the previous period, the human capital input and the physical capital input, respectively. In our framework, the total factor productivity depends on the result of the past human capital accumulation. We assume that the physical capital market is global whereas the human capital market (i.e., the labor market in our model) is local. Without loss of generality, we normalize the rental price of physical capital to one. The cost function is then $w_t l_{ht} + k_t$. The first order conditions for profit maximization are

$$\begin{aligned}\lambda\gamma(1-e^{-\delta H_{t-1}})\left(\frac{k_t}{l_{ht}}\right)^{1-\lambda} &= w_t, \\ (1-\lambda)\gamma(1-e^{-\delta H_{t-1}})\left(\frac{l_{ht}}{k_t}\right)^{\lambda} &= 1.\end{aligned}$$

Eliminating the human capital to physical capital ratio from these equations, we obtain the wage rate in the efficiency unit.

$$w_t = \lambda(1-\lambda)^{(1-\lambda)/\lambda}[\gamma(1-e^{-\delta H_{t-1}})]^{1/\lambda}. \tag{21}$$

As the level of aggregate human capital in the city goes up, the wage rate also rises although it has upper bound of $\lambda(1-\lambda)^{(1-\lambda)/\lambda}\gamma^{1/\lambda}$.

The city fringe $\overline{x}_t$ is determined by the land market clearing condition (10) for a given N_t. The law of motion of population is given by (13) and the law of motion of human capital is given by

$$H_{t+1} = \zeta H_t + \int_0^{\bar{x}_t} \frac{D h_t(x)}{d_t(x)} dx, \tag{22}$$

which implies that human capital depreciates at the rate of $1-\zeta$.

The level of population and aggregate level of human capital in the city is determined by (13) and (22), respectively. Once these values are determined, the other variables are well determined: (21) determines the wage rate, the market land rent is determined by (19) and (6), and the level of human capital is given by $h_t(x) = h_t^*(x; w_t, r_t(x))$, which determines the demands (18). Finally, the city fringe is determined by (10).

4.2 Numerical analysis

Using the model developed in the previous subsection, we explore numerically the factors that make a difference to the features of growth in the three largest metropolitan areas of Japan. We proceed by employing two steps: first, we seek a parameter set for each MA under which the basic model developed in Section 3 behaves in a consistent manner with the cyclical components of the population changes during the last five decades for that particular MA. We determine the parameters not related to the growth components in this step. Second, we extend this parameter set so that the urban growth model developed in Section 4 can replicate the total, social and natural changes of population for each MA.

The first step The expenditure share α of the numéraire consumption in the disposable income is set to be the share of consumption expenditure net of transportation and education costs in the disposable income for each MA (Annual Report on the Family Income and Expenditure Survey, Ministry of Internal Affairs and Communications, Japan). We take the average of this figure from 1980 to 2008 for each MA.

We set the level of real income I as follows. First, we obtain the per household nominal income of each MA from 1963 to 2008 for every 5 years by using the figures for the Gross Prefectural Domestic Income (Prefectural Accounts, Cabinet Office, Japan) and the number of households (Population Census, Ministry of Internal Affairs and Communications, Japan). The real income is derived by dividing the nominal income by the GDP deflator. I is then calculated as the geometric mean of the series of the per household real income.

The transport cost τ is determined so that the average share of commuting expenditure in disposable income at the steady state becomes the observed average share of transport related expenditure in the disposable income from 1980 to 2008 for each MA (Annual Report on the Family Income and Expenditure Survey, Ministry of Internal Affairs and Communications, Japan).

We set the land supply D to the average areas of inhabitable land in 1970,80,90,2000 and 2005, and normalize D for Tokyo MA to be one. This leads to $D = 0.67$ for Osaka MA and $D = 0.784$ for Nagoya MA.

The time cost b of child rearing is determined based on the total fertility rate in the central city of each MA. We set it so that the steady state value of the number of children in the CBD ($n_t(0)$) is equal to the average total fertility rate of the central city from 1950 to 2005 for every five years.

The land requirement ε for child rearing is set so that the model can satisfy $\epsilon \geq \beta b I/\alpha$, under which ${n_t}'(x) > 0$ for all $x < \overline{x}_t$. This ensures that the total fertility rate is lower in the central city than in the suburbs in each MA as seen in Table 1.

Because we observe the cyclical fluctuations around the trend components for all MAs, we choose the adjustment speed M of migration such that the following two criteria are satisfied; (i) $\Psi > -1$ and $\Phi < 1$, which ensure the existence and stability of the steady state, and (ii) $M > \Xi$, which leads to the fluctuations in the converging path to the steady state.

The utility level outside the city, $\overline{v}$, determines the overall degree of social change in the city population. We assume that each generation spans ten years, and determine $\overline{v}$ so that the series of simulated populations for six generations can replicate the observed variance in the cyclical components of ln(population) of each MA from 1950 to 2008.

Finally, we set the initial number of adult population N_0 as the number of people over the age of 14 in 1950 for each MA divided by 2. Appendix C reports the parameter values for the basic model.

The second step When setting the parameters of the urban growth model, we use the values determined in the first step for all parameters other than $\overline{v}$. Note here that the first step ignores the social changes arising from income growth in the city, which is relevant in the observed population changes and in the urban growth model. Since $\overline{v}$ controls the overall degree of social population change, the value of it determined in the first step must be modified in the urban growth model.

The additional parameters are determined as follows. The share of labor λ in the production function comes from the average share of labor income for each MA from 1955 to 2006 (Prefectural Accounts, Cabinet Office, Japan).

The parameters related to the human capital accumulation S, σ, and ζ are determined so that $2N_t + \int_0^{\overline{x}_t} \frac{Dn_t(x)}{d_t(x)}dx$ can replicate the total population figures from 1950 to 2000. One parameter is used to adjust the initial total population $2N_0 + \int_0^{\overline{x}_0} \frac{Dn_0(x)}{d_0(x)}dx$, one is used to determine the overall movement of the total population, and the last parameter determines the pace of changes in the total population.

The utility level $\overline{v}$ outside the city, the production parameters γ and δ, and the initial level of aggregate human capital H_0 are set as follows. We first set the same value of γ for all MAs at a moderate level. In the production function $y_t = \gamma(1 - e^{-\delta H_{t-1}})l_{ht}^{\lambda}k_t^{1-\lambda}$, the term $(1 - e^{-\delta H_{t-1}})$ represents the productivity related to the human capital and γ describes the productivity with origins other than human capital. We assume that the productivity difference comes only from the productivity related to the human capital, which reflects the difference in access to information and education. We then determine $\overline{v}$, δ, and H_0 to produce the observed net in-migration (social population change) from 1950 to 2000. Here, because we assume that each generation spans ten years, we calibrate the net in-migration for each ten-year period. Again, one parameter is used to adjust the initial level of net in-migration, one is used to determine its overall movement, and the last parameter determines the pace of changes.

Table 3 reports the resulting parameter values for the urban growth model.

[Insert Table 3 around here]

A few comments are in order. First, the commuting cost (τ) takes a similar value in Tokyo and Osaka, and a smaller value in Nagoya, which is considered to reflect the differences in the development of public transportation such as the subway system. Second, Tokyo has the largest land supply (D) and Osaka has the most severe land shortage. Third, the time and land requirements for child rearing (b and ε) are the highest in Tokyo and the lowest in Nagoya, whereas the costs of human capital investment (S and σ) are the highest in Nagoya and the lowest in Tokyo. Hence, our calibrated models imply that one is most likely to have less children and invest more in human capital in Tokyo and is least likely to do the same in Nagoya. Finally, the adjustment speed of migration (M) is the highest in Tokyo and the lowest in Nagoya, which is considered to reflect the fact that Tokyo is most well connected to all regions in Japan.

The results of a numerical realization of the model are presented in Figures 5 to 7.

[Insert Figures 5 to 7 around here]

These figures prove that our model can replicate the observed total, social and natural changes in the populations of the three MAs qualitatively and, often, quantitatively. (a) and (b) of each figure show that our model can replicate well the rapid increase in city populations between the years 1950 and 1970, followed by the slowdown in population growth, and the corresponding net in-migration. In (c) of each figure, the calibrated series of total fertility rates in the CBD decline over time but less so than the observed values. This is because our model does not have a component representing gains of longevity during the past five decades. Without longevity effects, in order to support the observed population increases, we must have smaller declines in the total fertility rate. (d) of each figure shows that the total fertility rate is lower for locations closer to the CBD. Moreover, as a city grows ($\overline{x}_t$ increases), the overall level of the total fertility rate declines.

Counterfactual analysis By using the calibrated models, we examine the factors that are significant in making a difference to the growth patterns of the three MAs. We do this by employing a counterfactual analysis, as follows. We explore how Osaka MA or Nagoya MA would behave if one of the parameters took on the same value as that of Tokyo MA. For example, we simulate how Osaka MA would look like if it had the same inhabitable area as Tokyo MA. In such a case, we use the parameter set of Osaka MA and change only D from 0.67 to 1, and simulate it for six generations. Figures 8 and 9 describe the results of our counterfactual analysis.

[Insert Figures 8 and 9 around here]

Figures 8 and 9 show the counterfactual analysis on Osaka MA and Nagoya MA, respectively. Osaka MA would have been as large as Tokyo MA if it were possible to obtain human capital in Osaka for as little cost as in Tokyo (if Osaka had had the same value of S or σ). Furthermore, the result with respect to the land supply D proves that the growth of Osaka MA might have been limited because of the shortage of land endowment. As for Nagoya MA, the cost of obtaining human capital (S or σ) again has a great effect on the growth of the city. Here, whereas D does not have a significant impact, the commuting cost τ does. Given the cost structure of obtaining human capital, the production structure of Tokyo MA may be harmful to the growth of Osaka and Nagoya. Under δ, λ, and ξ of Tokyo, Osaka and Nagoya grow less than they have in reality. Other parameters including initial population (N_0) and human capital (H_0) have little of no impact on city growth. Put differently, Osaka and Nagoya would have grown as large as Tokyo if human capital could have been obtained as easily as in Tokyo. Smaller land endowments and larger commuting costs have also limited the growth of Osaka and Nagoya, respectively.

5 Concluding remarks

This paper provided a model with which we could analyze the interaction between city structures and demographic factors. This enabled us to treat the social and natural changes in city populations within a unified framework. In the developed model, we supposed the monocentric city structure and a complementarity between land consumption and having children. We showed conditions under which our model could replicate the observed demographic characteristics in the three largest metropolitan areas of Japan. A counterfactual analysis revealed that the low cost of obtaining human capital in Tokyo has been the major factor in establishing its urban primacy, and that other factors such as initial levels of population and human capital have little effect on urban growth.

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Appendix A: Proof of Proposition 3.

Remind here that we can determine all the other variables in period t once we fix N_t and that the law of motion of population (13) determines N_{t+1} for a given N_t. Therefore, we have a steady state equilibrium if there exists N^* that satisfies (15).

$\Lambda(N_t)$ is rewritten as

$$\begin{aligned}\Lambda(N_t) &= M\left(\overline{V}_t - \overline{v}\right) + \int_0^{\overline{x}_t} \frac{Dn_t(x)}{d_t(x)}dx \\ &= M\left(\overline{V}_t - \overline{v}\right) + \int_0^{\overline{x}_t} \frac{Dr_t(x)}{bI + 2\varepsilon r_t(x)}dx.\end{aligned}$$

From (6), (9) and the results that $\overline{x}_t(0) = 0$ and $\lim_{N_t\to\infty} \overline{x}_t(N_t) = I/\tau$, we have

$$\begin{aligned}\Lambda(0) &= M\left(\overline{V}_t\big|_{\overline{x}_t=0} - \overline{v}\right) > 0, \\ \lim_{N_t\to\infty} \Lambda(N_t) &= -\infty.\end{aligned}$$

These establish the existence of at least one N^* that satisfies (15).

Moreover, the uniqueness and stability is ensured if $-1 < \Lambda'(N_t) < 1$. We readily obtain

$$\begin{aligned}\Lambda'(N_t) &= \Delta\overline{x}_t'(N_t), \qquad \text{(A1)} \\ \Delta &\equiv \frac{D}{bI + 2\varepsilon} - \frac{\tau M}{I - \tau\overline{x}_t} + \int_0^{\overline{x}_t} \Omega dx, \\ \Omega &\equiv \frac{bDI}{(bI + 2\varepsilon r_t(x))^2}\frac{\partial r_t(x)}{\partial \overline{x}_t}.\end{aligned}$$

Note that

$$\begin{aligned} 0 &\le \Omega \\ &= \frac{\tau bDI\left(bI+\varepsilon\right)\frac{(I-\tau x)^{1/\beta}}{(I-\tau\overline{x}_t)^{1+1/\beta}}}{\beta\left[(bI)^2+4\varepsilon\left(bI+\varepsilon\right)\left(\frac{I-\tau x}{I-\tau\overline{x}_t}\right)^{1/\beta}\right]^{3/2}} \\ &< \frac{\tau bDI\left(I-\tau\overline{x}_t\right)^{1/(2\beta)-1}}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}\left(I-\tau x\right)^{1/(2\beta)}} \\ &\le \frac{\tau bDI}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}\left(I-\tau\overline{x}_t\right)}. \end{aligned} \tag{A2}$$

Note next that (10) yields

$$\begin{aligned} 0 &< \overline{x}_t'(N_t) \\ &= \left\{\frac{D\left(bI+\varepsilon\right)}{\beta\left(I-\tau\overline{x}_t\right)\left(bI+2\varepsilon\right)}+\int_0^{\overline{x}_t}\frac{2\varepsilon^2D\left(r_t(x)\right)^2+bDI\left(bI+2\varepsilon r_t(x)\right)}{\beta\left(I-\tau x\right)\left(bI+2\varepsilon r_t(x)\right)^2}\frac{\partial r_t(x)}{\partial\overline{x}_t}dx\right\}^{-1} \\ &< \frac{\beta\left(I-\tau\overline{x}_t\right)\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)}. \end{aligned}$$

Consider the case in which $\Delta \ge 0$. From Lemma 2, we have

$$\begin{aligned} 0 &\le \Lambda'(N_t) \\ &< \left[-\frac{\tau M}{I-\tau\overline{x}_t}+\frac{D}{bI+2\varepsilon}+\int_0^{\overline{x}_t}\frac{\tau bDI}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}\left(I-\tau\overline{x}_t\right)}dx\right]\overline{x}_t'(N_t) \\ &< \left[-\frac{\tau M}{I-\tau\overline{x}_t}+\frac{D}{bI+2\varepsilon}+\frac{\tau bDI\overline{x}_t}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}\left(I-\tau\overline{x}_t\right)}\right]\frac{\beta\left(I-\tau\overline{x}_t\right)\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)} \\ &= \frac{\beta}{bI+\varepsilon}\left\{I+\left(bI+2\varepsilon\right)\left[\frac{bI^2}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}}-\frac{\tau M}{D}\right]\right\}. \end{aligned}$$

Consider next the case in which $\Delta < 0$. In this case, we obtain

$$\begin{aligned} 0 &> \Lambda'(N_t) \\ &> \left(\frac{D}{bI+2\varepsilon}-\frac{\tau M}{I-\tau\overline{x}_t}\right)\frac{\beta\left(I-\tau\overline{x}_t\right)\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)} \\ &= \frac{\beta}{bI+\varepsilon}\left[I-\tau\overline{x}_t-\frac{\tau M\left(bI+2\varepsilon\right)}{D}\right] \\ &\ge -\frac{\beta\tau M\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)}. \end{aligned}$$

Let Φ and Ψ denote

$$\begin{aligned} \Phi &\equiv \frac{\beta}{bI+\varepsilon}\left\{I+\left(bI+2\varepsilon\right)\left[\frac{bI^2}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}}-\frac{\tau M}{D}\right]\right\}, \\ \Psi &\equiv -\frac{\beta\tau M\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)}. \end{aligned}$$

We have $-1 < \Lambda'(N_t) < 1$ if $\Psi > -1$ and $\Phi < 1$. From the facts that $\lim_{(b,\beta)\to(0,0)} \Phi = \lim_{(b,\beta)\to(0,0)} \Psi = 0$, we have Proposition 3.

Appendix B: Proof of Proposition 4.

We know from (A1) that $\Lambda'(N_t) = \Delta \overline{x}_t'(N_t)$. Because $\overline{x}_t'(N_t) > 0$, we observe fluctuations in the converging path if $\Delta < 0$. Equations (A1) and (A1) show that

$$\begin{aligned}
\Delta &< \frac{D}{bI+2\varepsilon} - \frac{\tau M}{I-\tau\overline{x}_t} + \int_0^{\overline{x}_t} \frac{\tau b D I}{\beta\,(4\varepsilon)^{3/2}\,(bI+\varepsilon)^{1/2}\,(I-\tau\overline{x}_t)} dx \\
&= \frac{1}{I-\tau\overline{x}_t}\left(\frac{D(I-\tau\overline{x}_t)}{bI+2\varepsilon} - \tau M + \frac{\tau b D I \overline{x}_t}{\beta\,(4\varepsilon)^{3/2}\,(bI+\varepsilon)^{1/2}}\right) \\
&< \frac{\tau}{I-\tau\overline{x}_t}\left[\frac{1}{\tau}\left(\frac{DI}{bI+2\varepsilon} + \frac{bDI^2}{\beta\,(4\varepsilon)^{3/2}\,(bI+\varepsilon)^{1/2}}\right) - M\right] \\
&= \frac{\tau}{I-\tau\overline{x}_t}\left(\Xi - \tau M\right).
\end{aligned}$$

Therefore, we have $\Delta < 0$ if

$$M > \Xi.$$

Appendix C: Parameter values for the basic model.

[Insert Table C around here]

Tokyo MA				
	Central city	Suburbs		
year	Tokyo	Saitama	Chiba	Kanagawa
1930	3.51	5.33	5.05	4.34
1950	2.73	3.92	3.59	3.25
1960	1.7	2.16	2.13	1.89
1970	1.96	2.35	2.28	2.23
1980	1.44	1.73	1.74	1.7
1990	1.23	1.5	1.47	1.45
2000	1.07	1.3	1.3	1.28
Osaka MA				
	Central city	Suburbs		
year	Osaka	Kyoto	Hyogo	Nara
1930	3.21	3.59	3.94	4.39
1950	2.87	2.8	3.08	3.08
1960	1.81	1.72	1.9	1.87
1970	2.17	2.02	2.12	2.08
1980	1.67	1.67	1.76	1.7
1990	1.46	1.48	1.53	1.49
2000	1.31	1.28	1.38	1.3
Nagoya MA				
	Central city	Suburbs		
year	Aichi	Mie	Gifu	
1930	4.6	5.01	5.47	
1950	3.27	3.33	3.55	
1960	1.9	1.95	2.04	
1970	2.19	2.04	2.12	
1980	1.81	1.82	1.8	
1990	1.57	1.61	1.57	
2000	1.44	1.48	1.47	

Table 1: Total fertility rate in three largest metropolitan areas in Japan.

	Central city	Suburbs		
Tokyo MA	Tokyo	Saitama	Chiba	Kanagawa
	1081.7	525.3	417.3	796.9
Osaka MA	Osaka	Kyoto	Hyogo	Nara
	749.4	538.2	487.5	422.4
Nagoya MA	Aichi	Mie	Gifu	
	427.4	227	246.6	

Table 2: Value of land for housing per 3.3 ㎡ (in thousand yen) in three largest metropolitan areas in Japan for the year 2000.

	Tokyo MA	Osaka MA	Nagoya MA
α	0.64	0.66	0.65
τ	0.00011	0.000095	0.00021
D	1	0.67	0.78
b	0.01	0.001	0.0001
ϵ	740	670	640
$\bar{v}$	6.62	6.56	6.46
M	26647	20935	10731
$2N_0$	4466.4	3388.7	2145.4
λ	0.68	0.66	0.61
S	44	50	68
σ	1.78	1.92	2.25
ς	0.75	0.9	0.94
γ	185	185	185
δ	0.00002	0.000029	0.000029
H_0	175000	140000	120000

Table 3. Parameter values for the urban growth model.

	Tokyo MA	Osaka MA	Nagoya MA
α	0.64	0.66	0.65
I	4876.5	4613.0	4528.9
τ	0.00011	0.000095	0.00021
D	1	0.67	0.78
b	0.01	0.001	0.0001
ϵ	740	670	640
$\bar{v}$	6.2592	6.2885	6.249
M	26647	20935	10731
N_0	4466.4	3388.7	2145.4

Table C. Parameter values for the basic model.

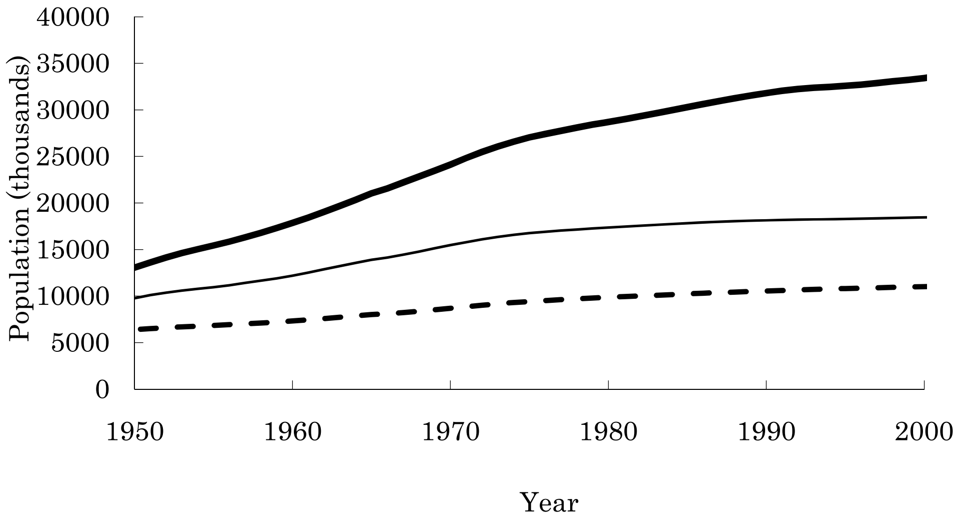
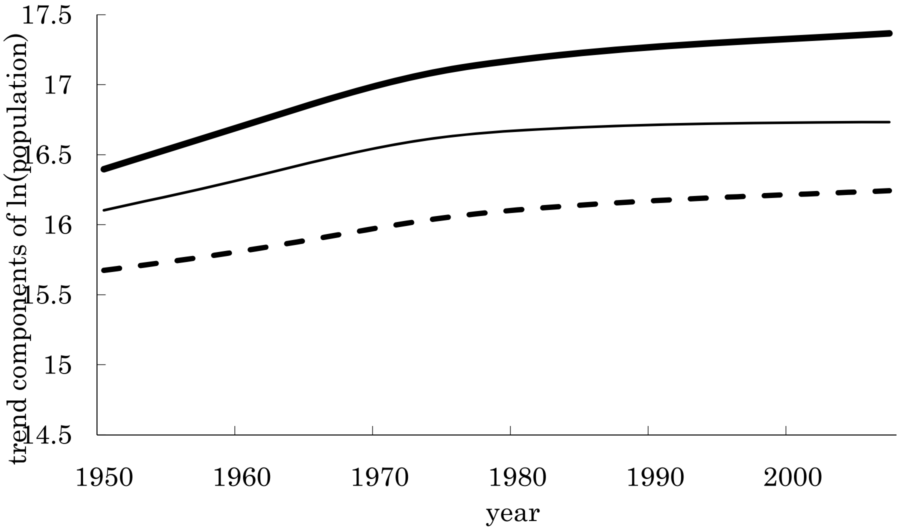
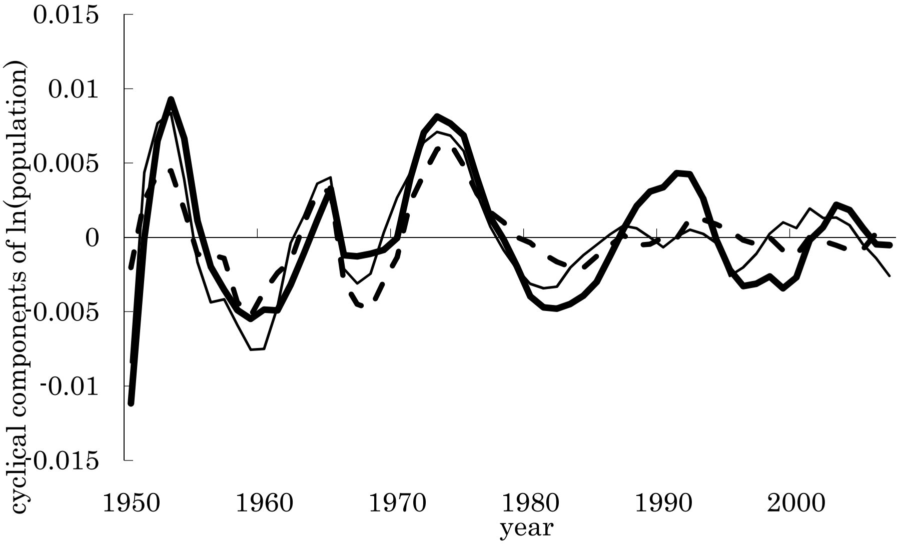


Figure 1. Population of three largest metropolitan areas in Japan. Bold line: Tokyo, solid line: Osaka, dotted line: Nagoya.



(a) Trend components of *ln(population)*.



(b) Cyclical components of *ln(population)*.

Figure 2. Trend and cyclical components of *ln(population)* of three largest metropolitan areas in Japan. Bold line: Tokyo, solid line: Osaka, dotted line: Nagoya.

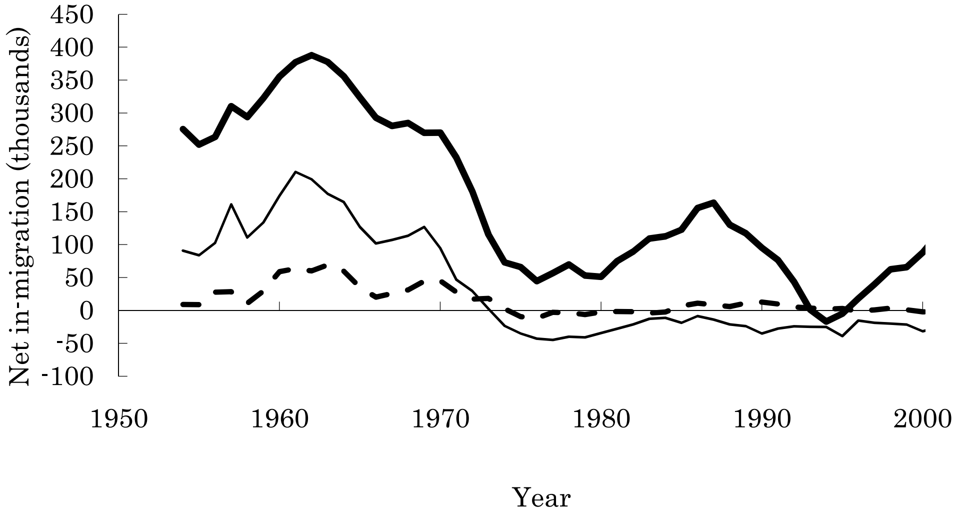


Figure 2. Net in-migration of three largest metropolitan areas in Japan. Bold line: Tokyo, solid line: Osaka, dotted line: Nagoya.

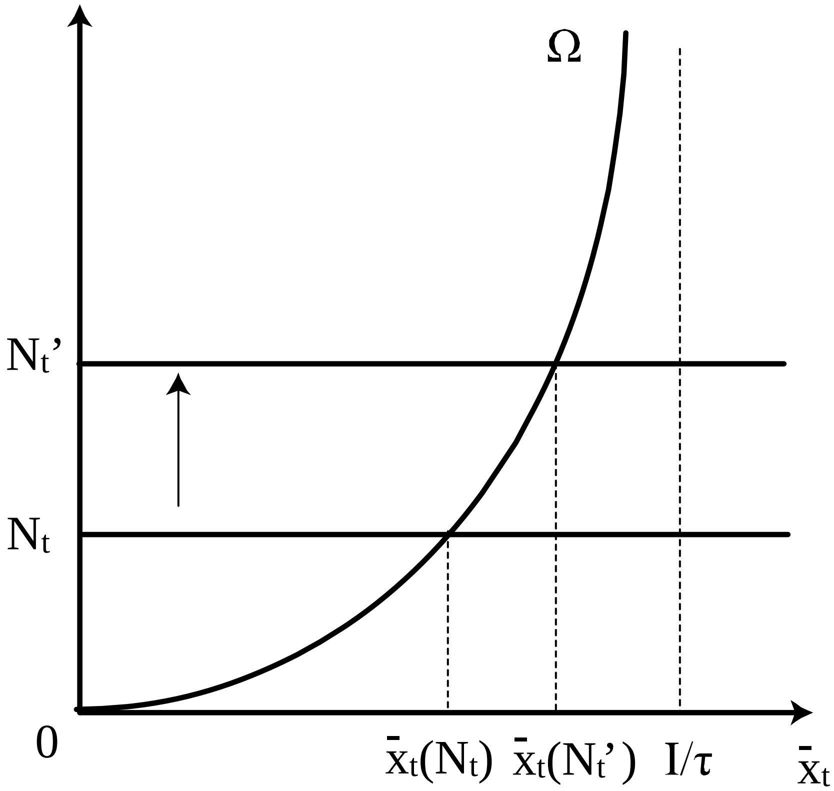
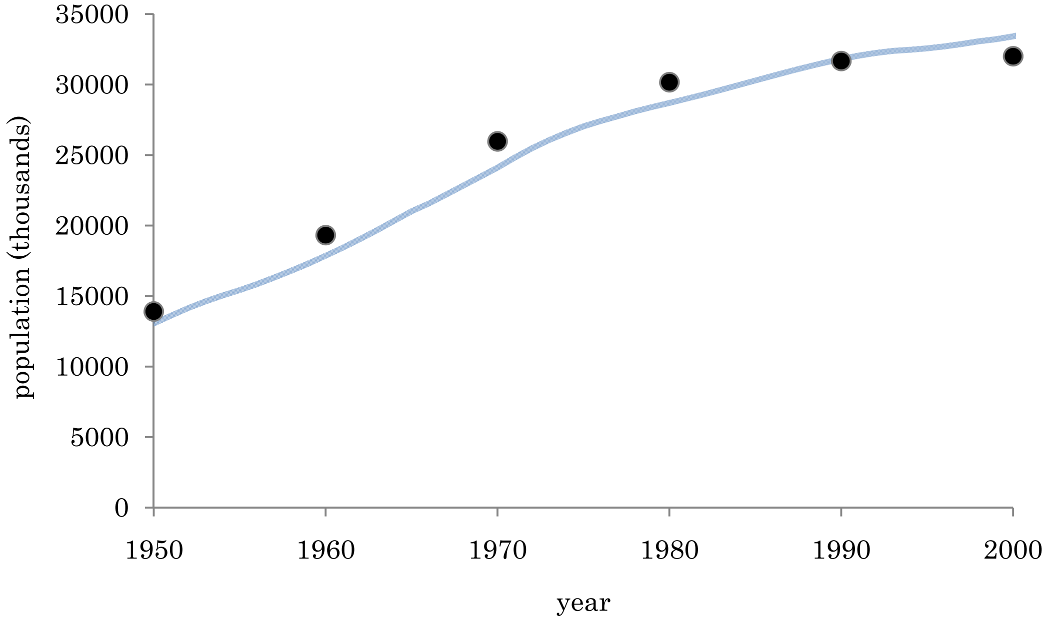
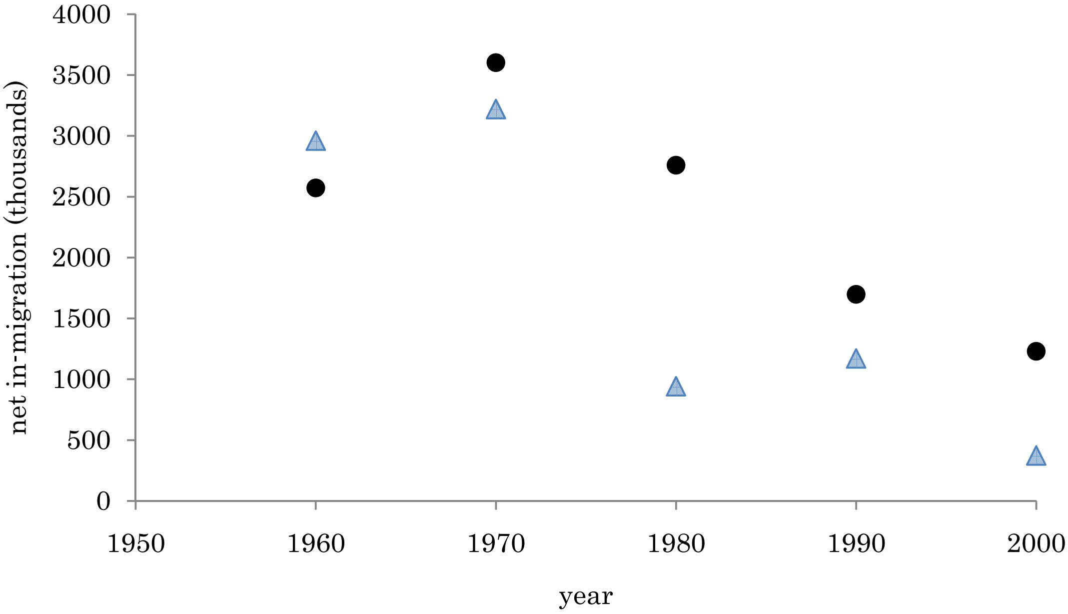


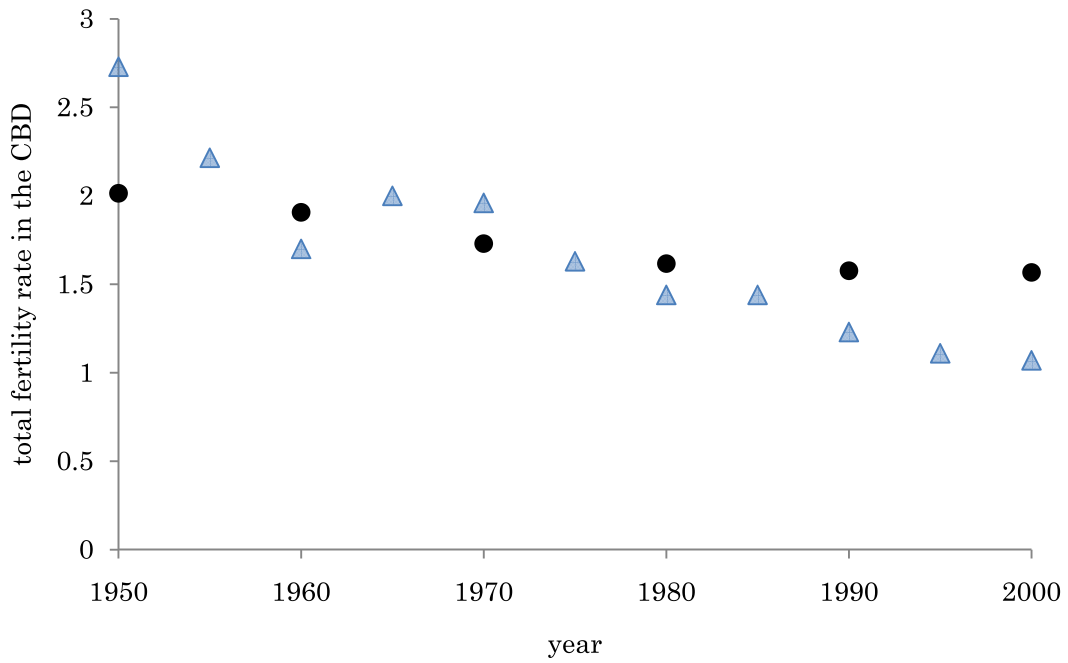
Figure 4. Determination of the city fringe $\overline{x}_t$ for a given number of individuals N_t .



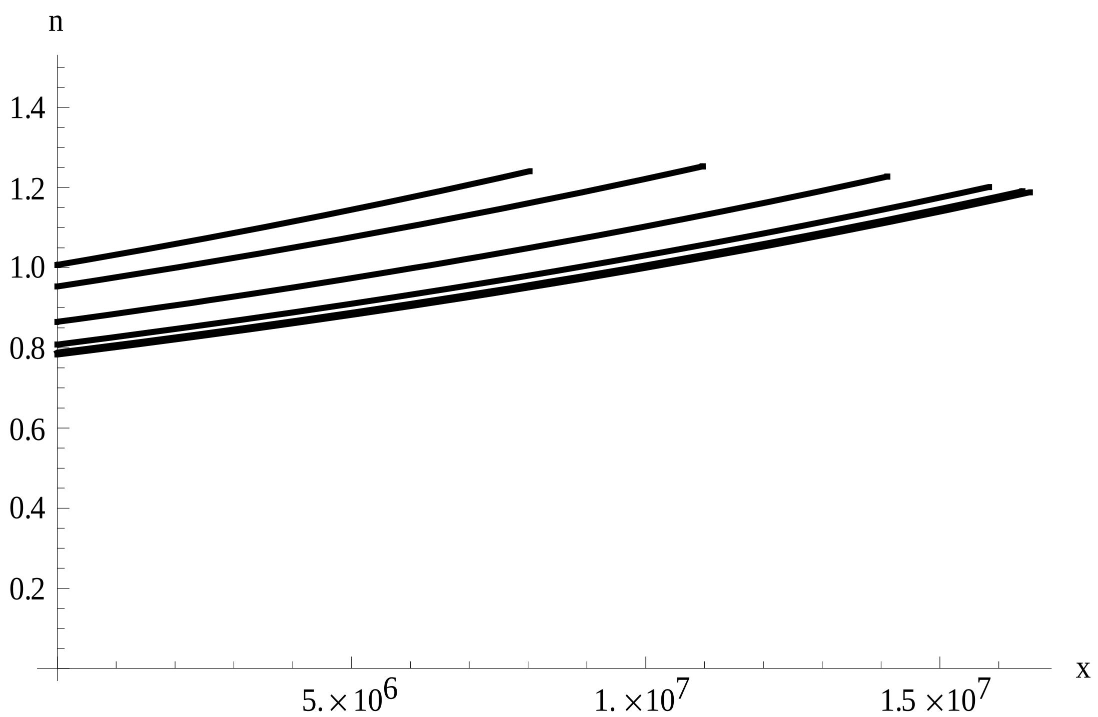
(a) Gray line: observed population of Tokyo MA. Dots: simulated population of Tokyo MA.



(b) Triangles: observed net in-migration of Tokyo MA for each ten years (the first triangle represents the figure for ten years from1950 to 1960). Dots: simulated net in-migration of Tokyo MA.

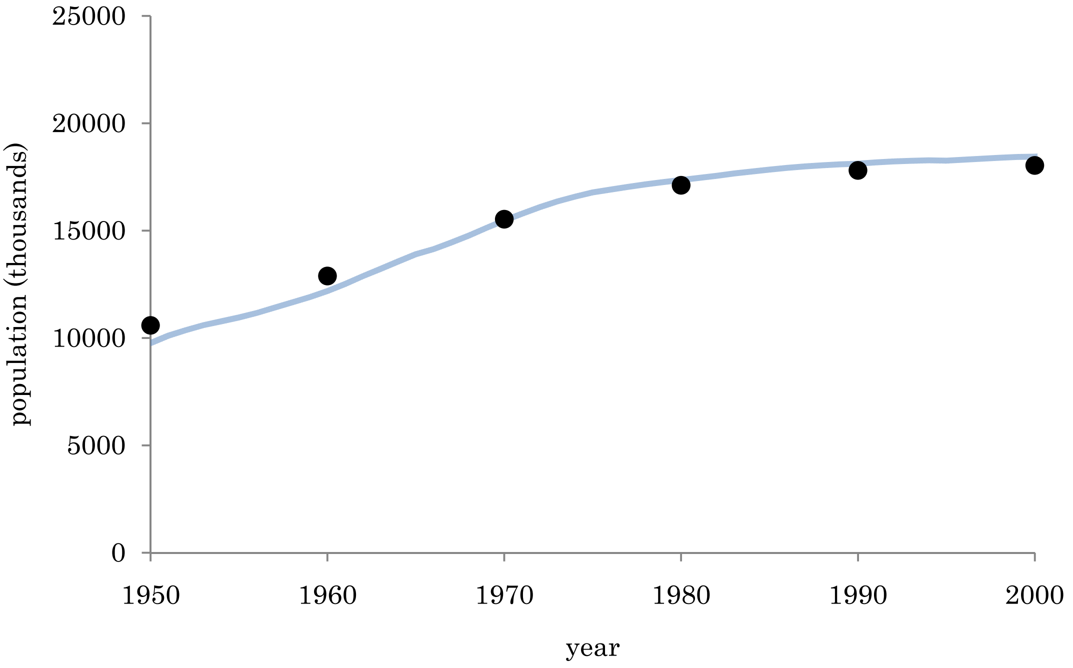


(c) Triangles: observed total fertility rate of the CBD of Tokyo MA (Tokyo prefecture). Dots: simulated total fertility rate of the CBD of Tokyo MA (*x=0*).

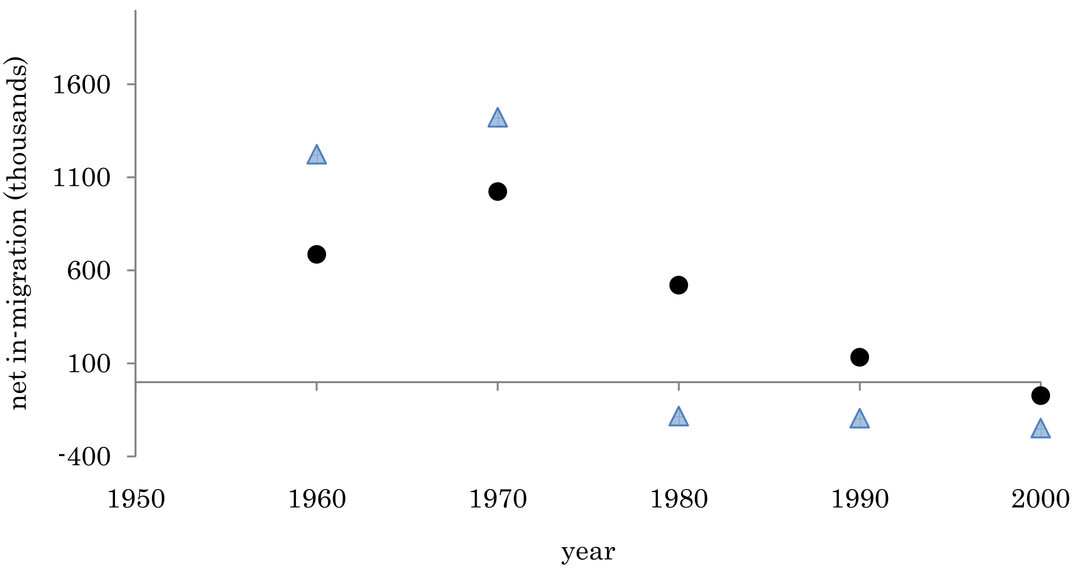


(d) Simulated total fertility rate within Tokyo MA. A line located higher in the graph describes the fertility rate of an earlier generation.

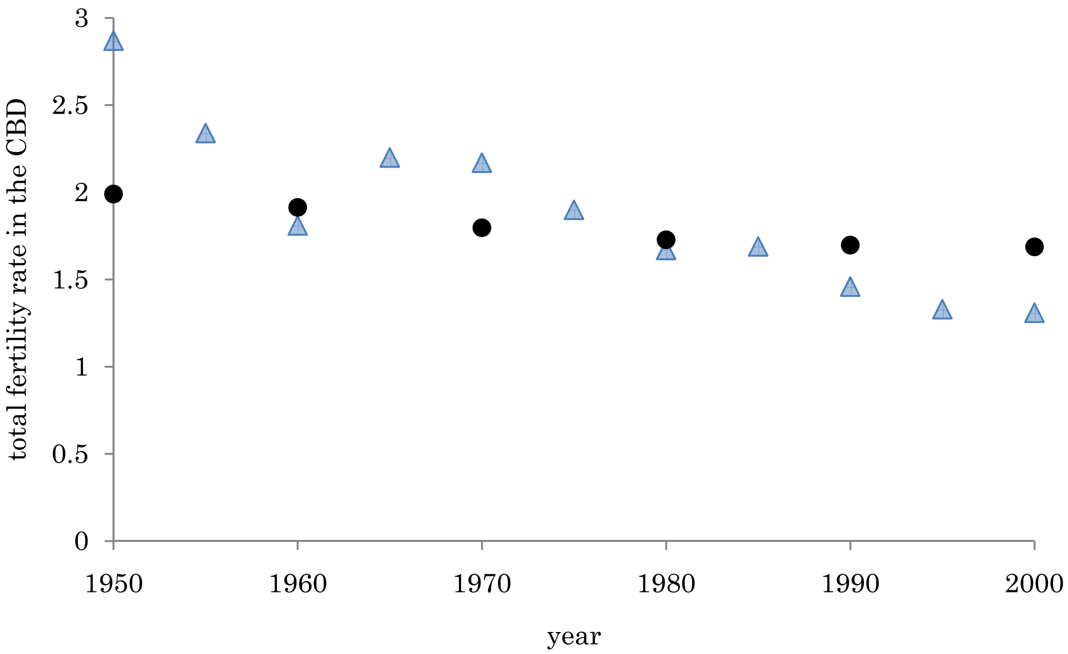
Figure 5. Calibration results of Tokyo MA.



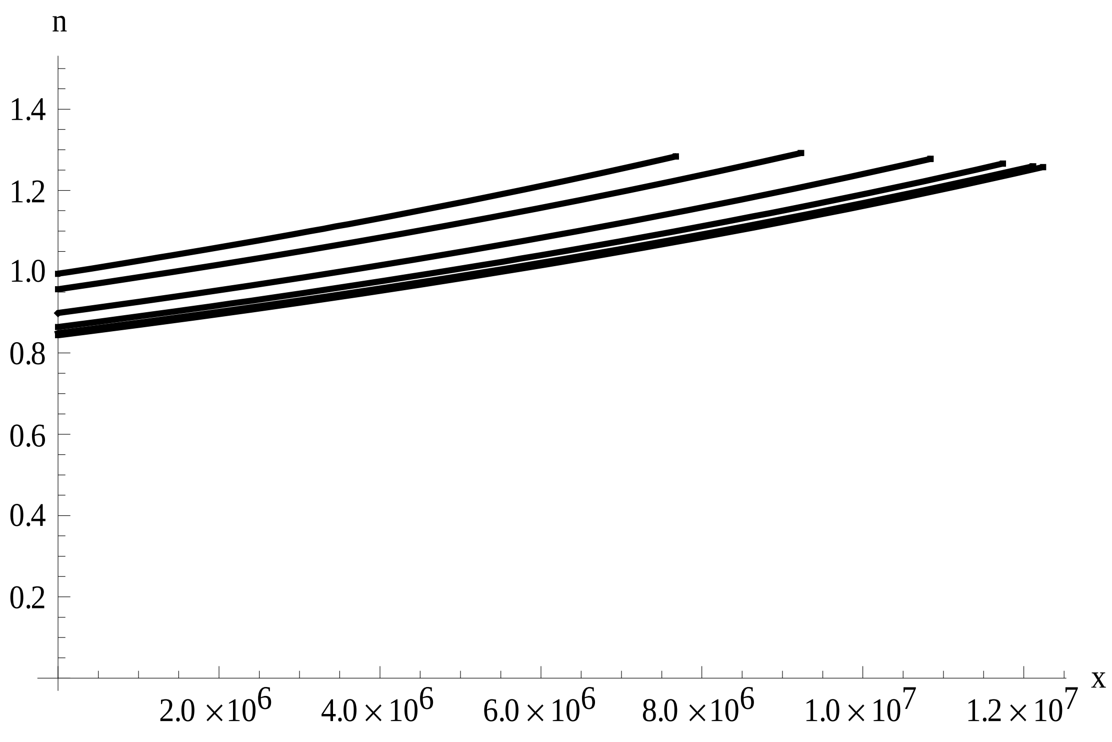
(a) Gray line: observed population of Osaka MA. Dots: simulated population of Osaka MA.



(b) Triangles: observed net in-migration of Osaka MA for each ten years (the first triangle represents the figure for ten years from1950 to 1960). Dots: simulated net in-migration of Osaka MA.

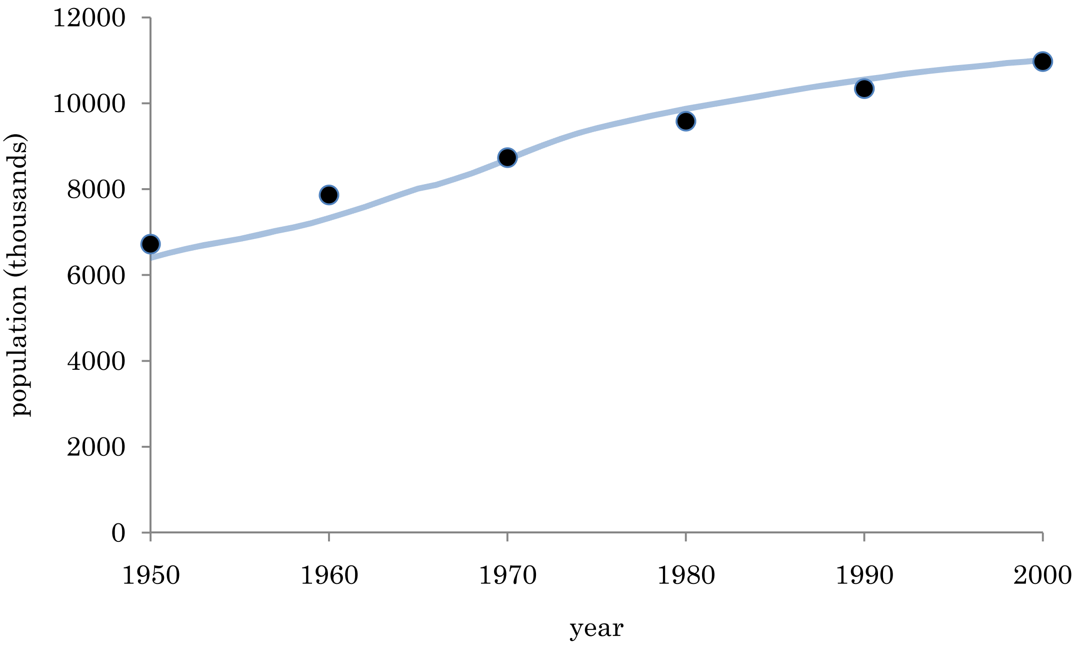


(c) Triangles: observed total fertility rate of the CBD of Osaka MA (Osaka prefecture). Dots: simulated total fertility rate of the CBD of Osaka MA (*x=0*).

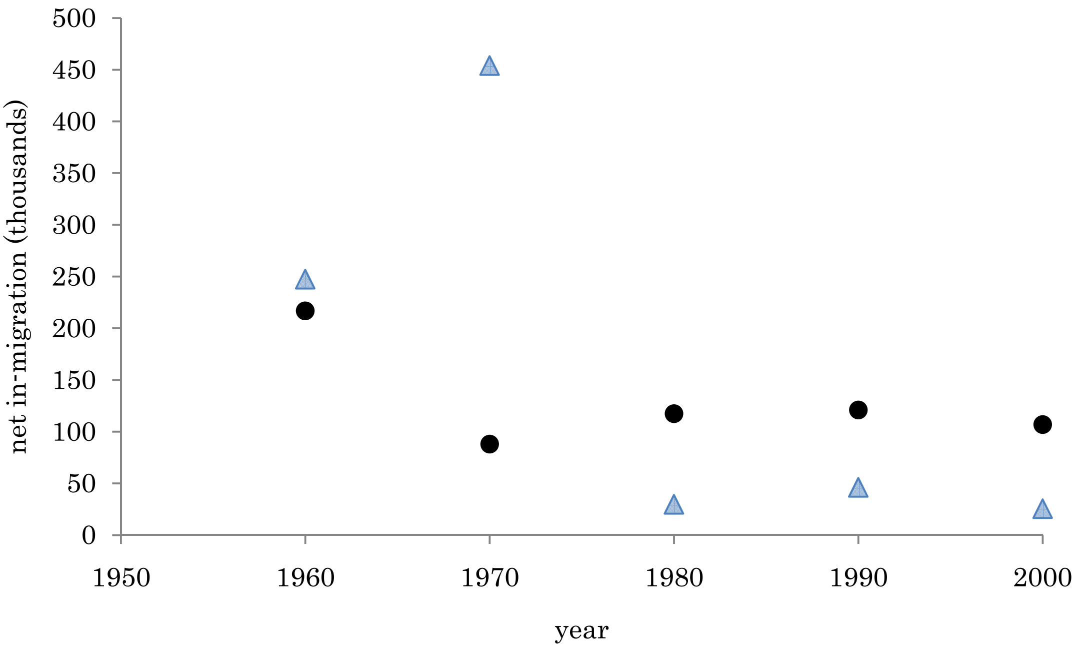


(d) Simulated total fertility rate within Osaka MA. A line located higher in the graph describes the fertility rate of an earlier generation.

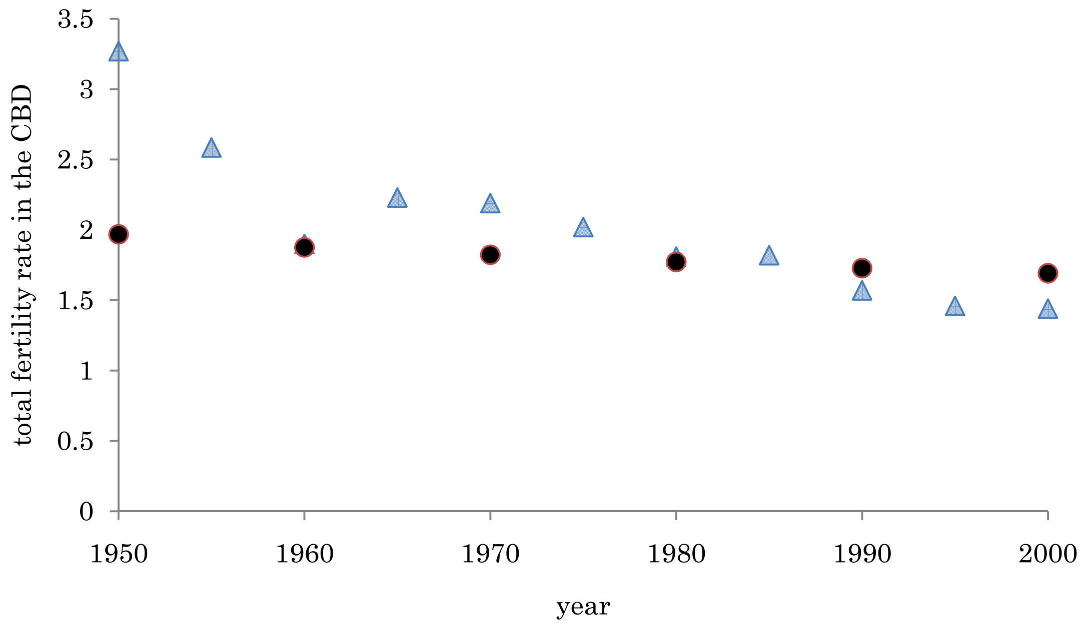
Figure 6. Calibration results of Osaka MA.



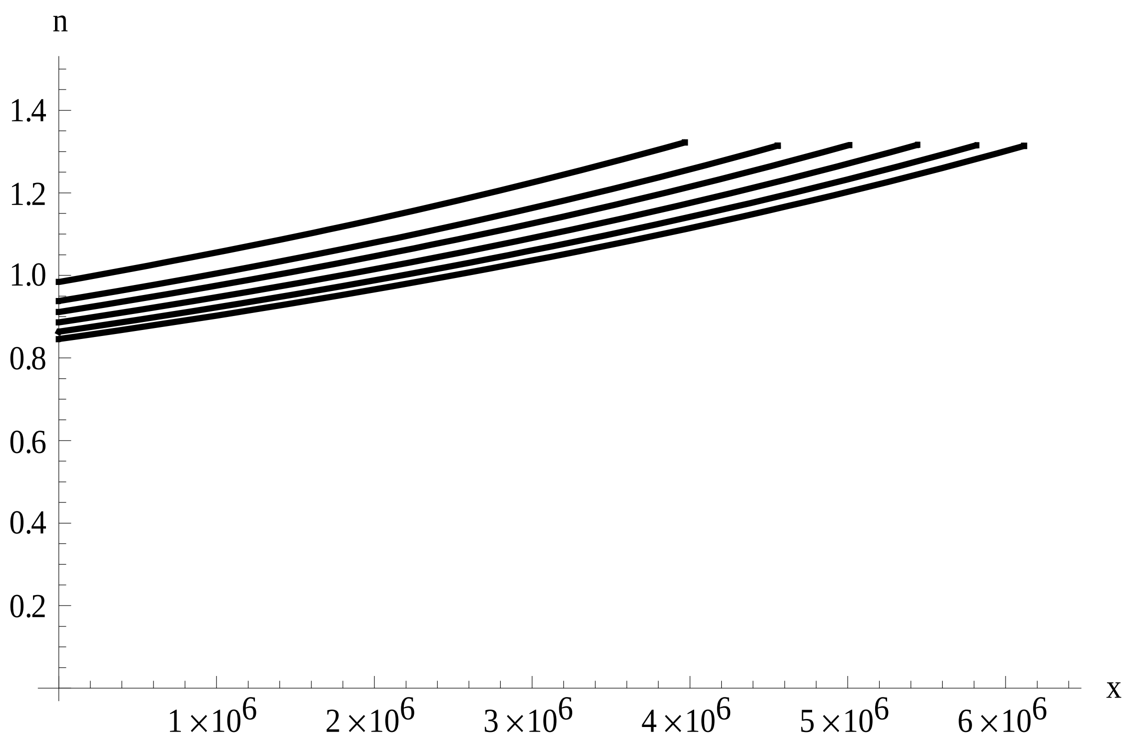
(a) Gray line: observed population of Nagoya MA. Dots: simulated population of Nagoya MA.



(b) Triangles: observed net in-migration of Nagoya MA for each ten years (the first triangle represents the figure for ten years from1950 to 1960). Dots: simulated net in-migration of Nagoya MA.



(c) Triangles: observed total fertility rate of the CBD of Nagoya MA (Aichi prefecture). Dots: simulated total fertility rate of the CBD of Nagoya MA (*x=0*).



(d) Simulated total fertility rate within Nagoya MA. A line located higher in the graph describes the fertility rate of an earlier generation.

Figure 7. Calibration results of Nagoya MA.

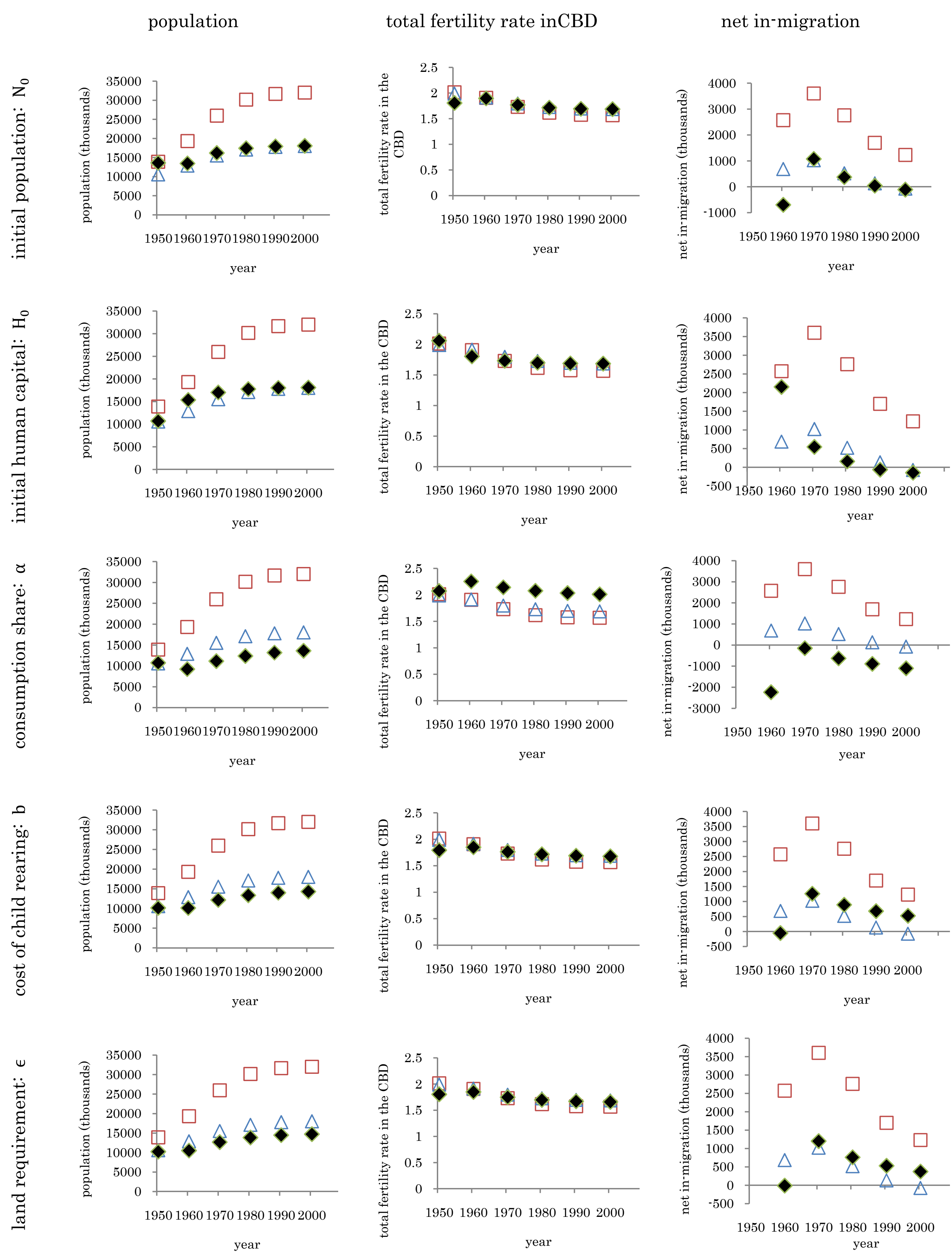


Figure 8. Counterfactuals on Osaka MA.

□: Tokyo, △: Osaka, ◆: Counterfactual Osaka

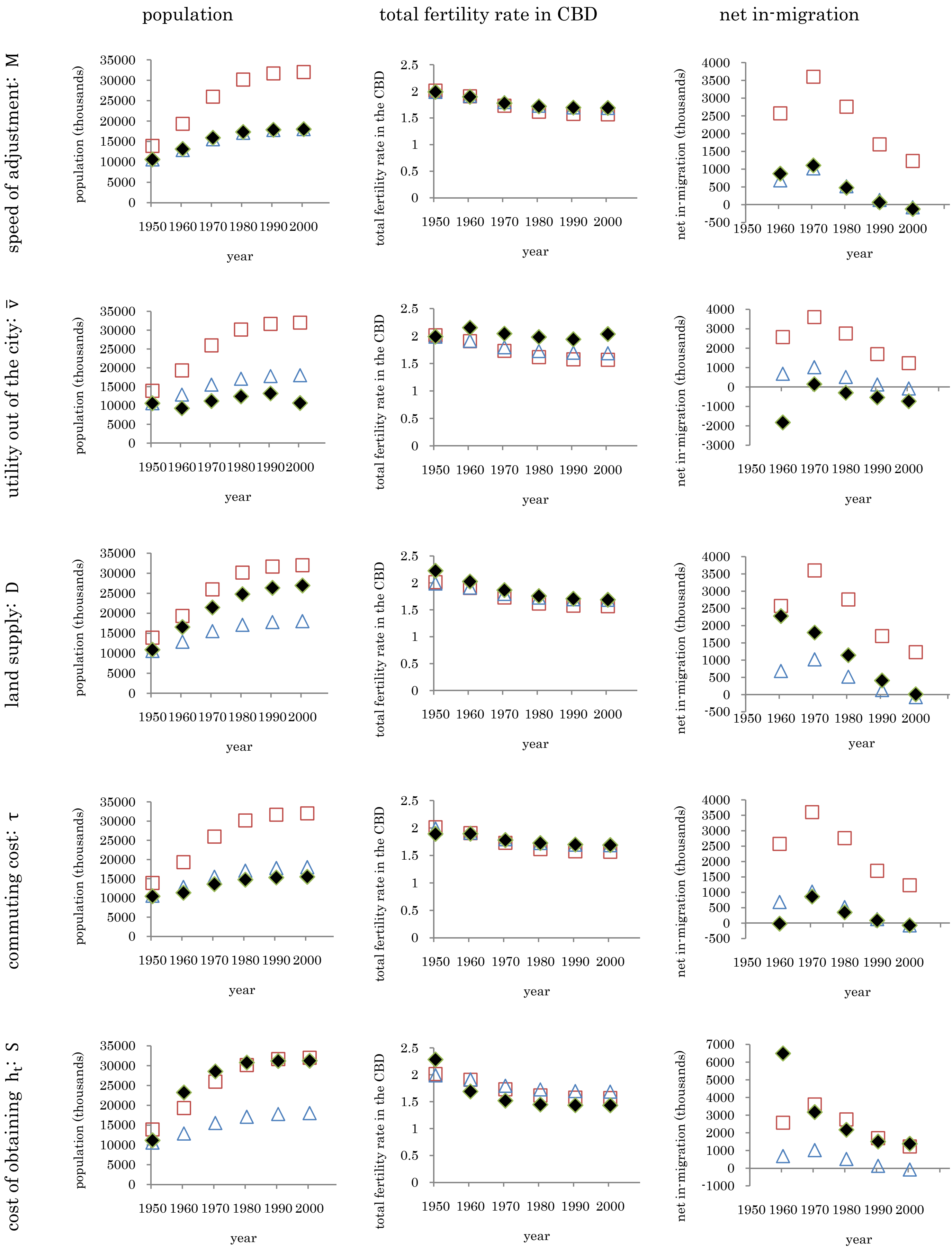


Figure 8. Counterfactuals on Osaka MA (continued).

□: Tokyo, △: Osaka, ◆: Counterfactual Osaka.

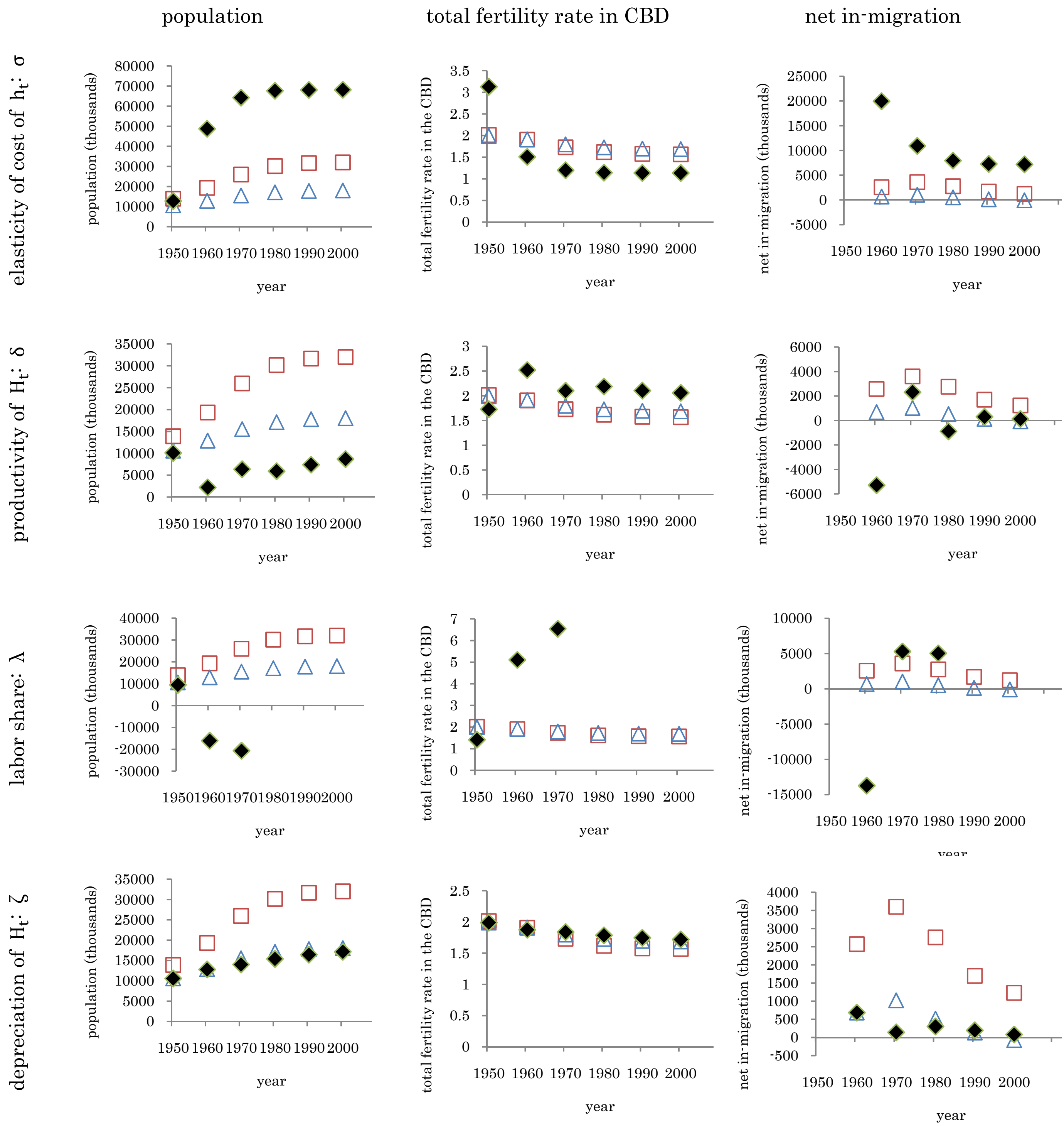


Figure 8. Counterfactuals on Osaka MA (continued).

□: Tokyo, △: Osaka, ◆: Counterfactual Osaka.

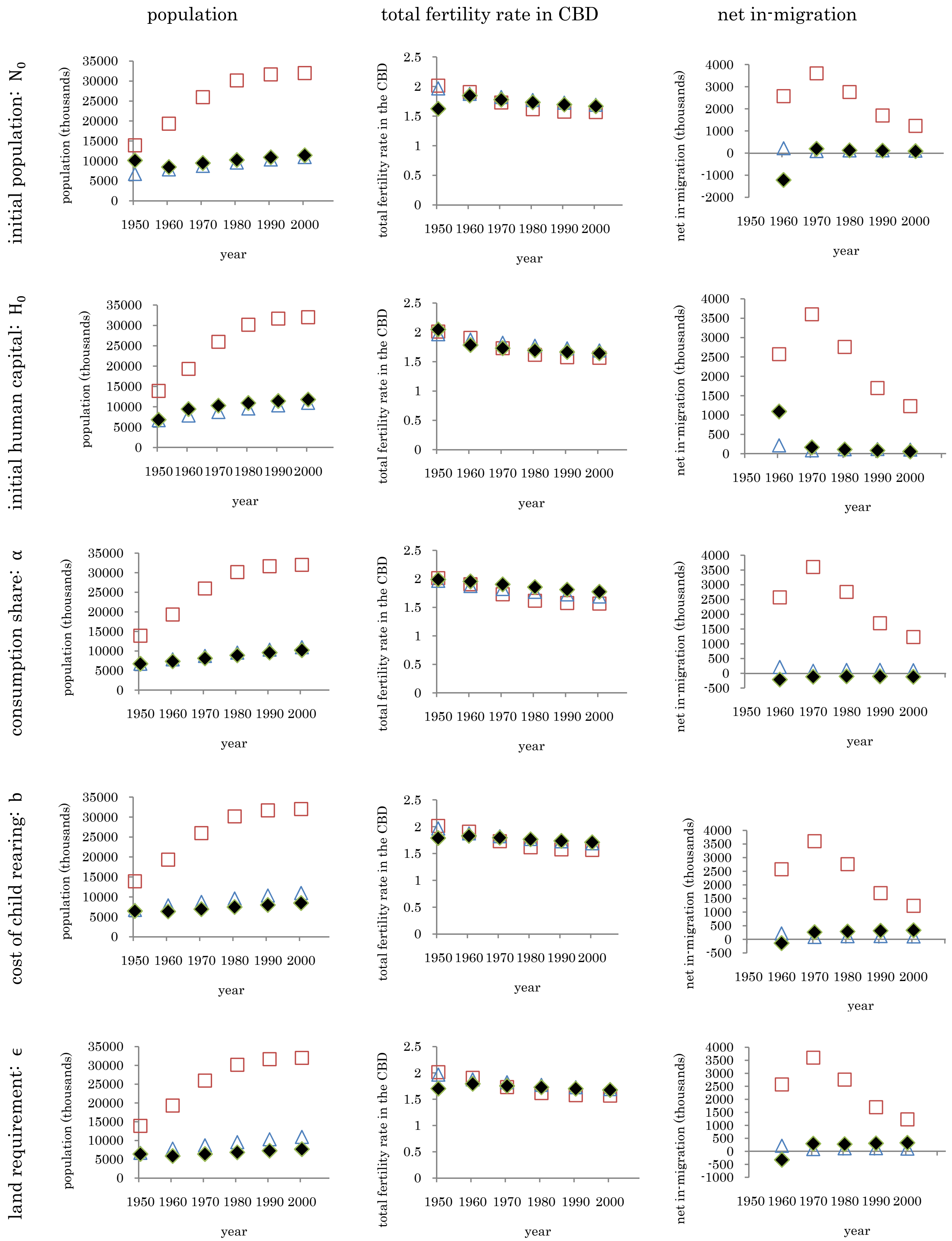


Figure 9. Counterfactuals on Nagoya MA.

□: Tokyo, △: Nagoya, ◆: Counterfactual Nagoya

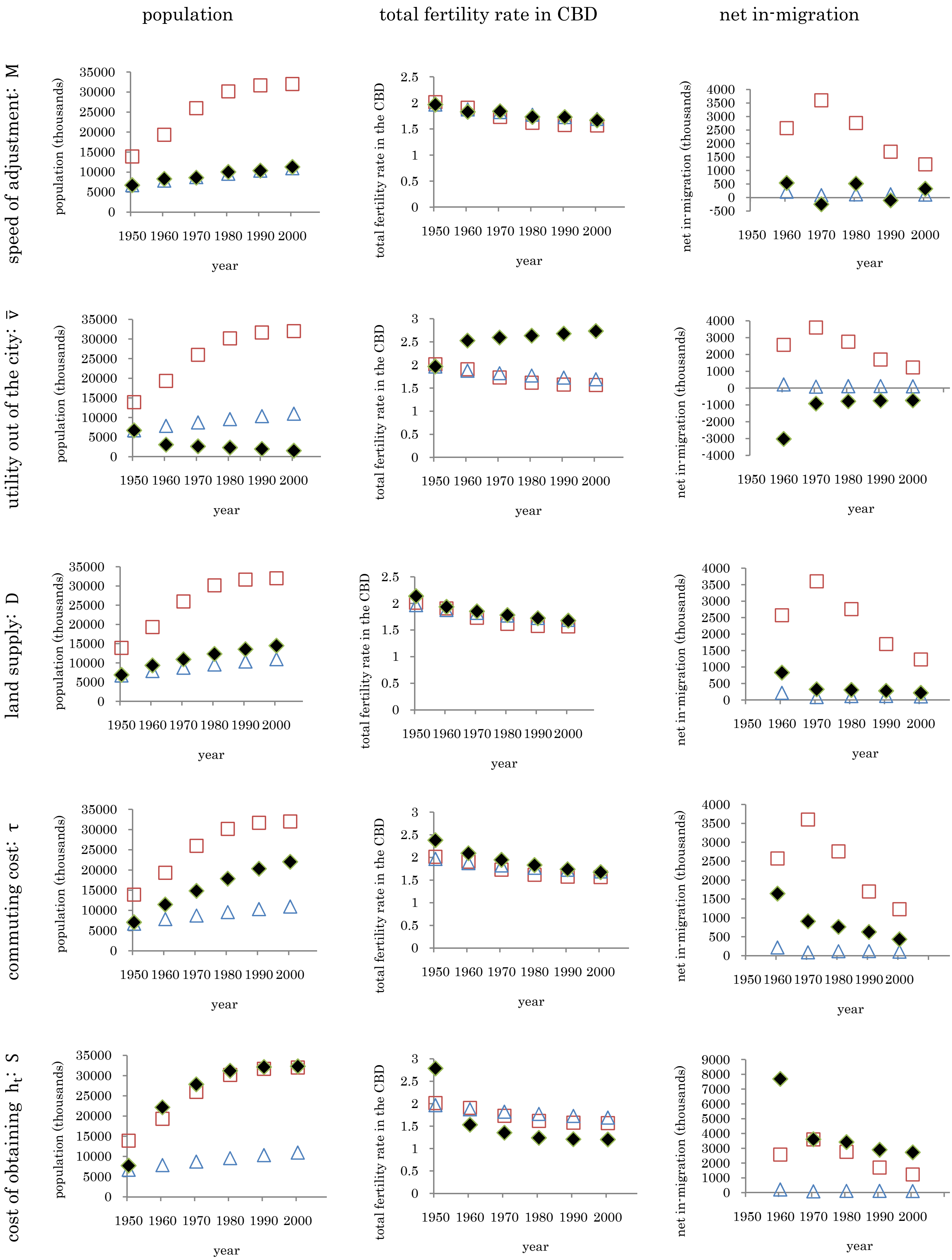


Figure 9. Counterfactuals on Nagoya MA (continued).
□: Tokyo, △: Nagoya, ◆: Counterfactual Nagoya

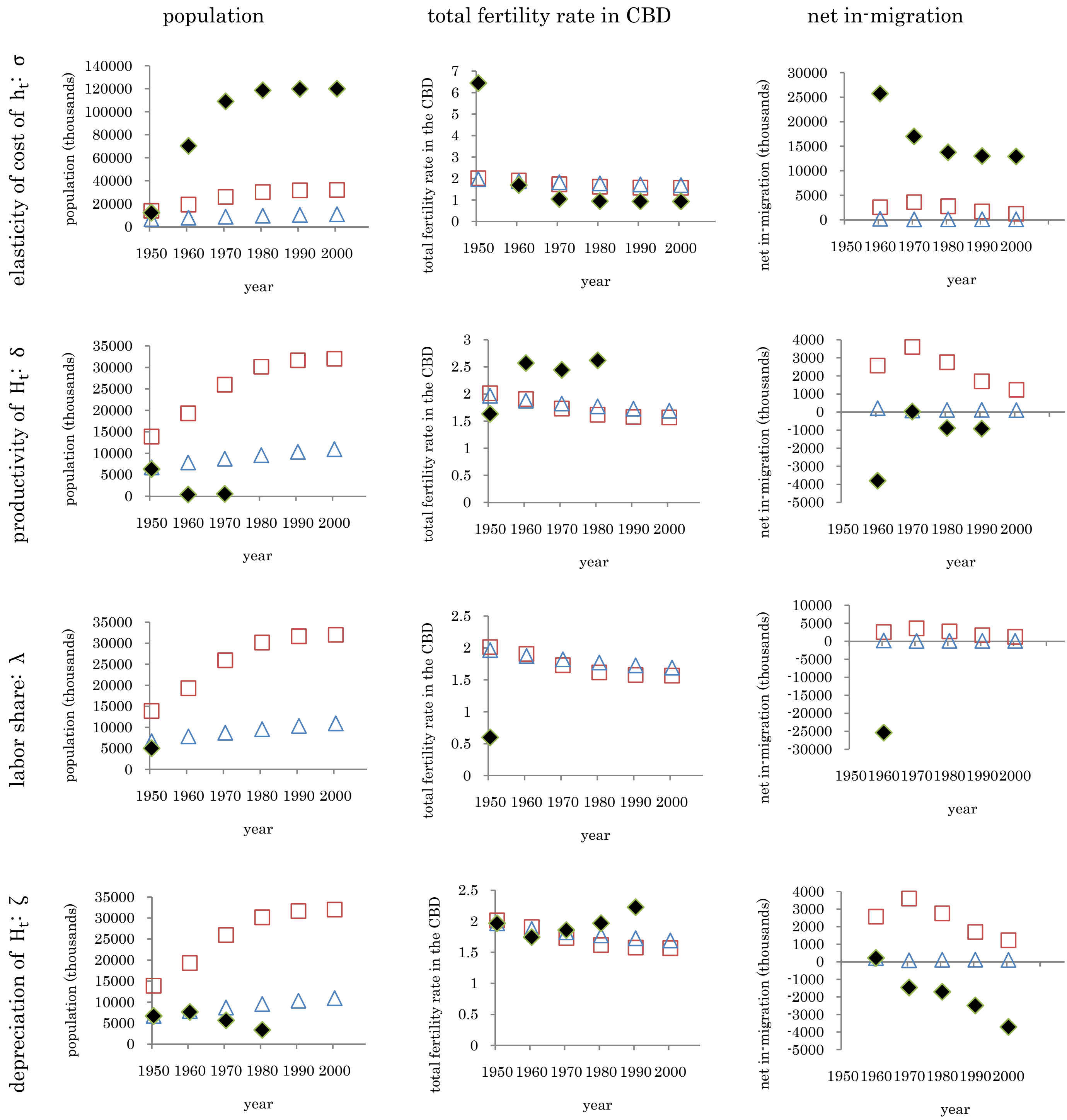


Figure 9. Counterfactuals on Nagoya MA (continued).

□: Tokyo, △: Nagoya, ◆: Counterfactual Nagoya