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Abstract

This paper constructs a North–South quality-ladder model in which foreign direct investment (FDI) is determined by the endogenous location choice of firms, and examines analytically how strengthening patent protection in the South affects welfare in the South. Strengthening patent protection increases the South's welfare by enhancing innovation and FDI, but it also allows the firms with patents to charge higher prices for their goods, which decreases welfare. However, the model shows that the former positive welfare effect outweighs the latter negative effect. Moreover, introducing the strictest form of patent protection in the South, that is, harmonizing patent protection in the South with that in the North, may maximize welfare in the South as well as in the North. Further, a similar result can also be obtained in a nonscale effect model.

Keywords: foreign direct investment, innovation, intellectual property rights protection, welfare analysis

JEL classification: F43, O33, O34

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1 Introduction

Recently, many developing countries have been encouraged to strengthen intellectual property rights (IPR) protection. An agreement on Trade-Related Aspects of Intellectual Property Rights (the TRIPs agreement) claims that all World Trade Organization (WTO) member countries should adopt a set of minimum standards on IPR, including patents and copyrights. Strengthening IPR protection is often a requirement for developing countries to enter the WTO. However, most developing countries fear that stronger domestic protection of IPR may damage their economies.[1] Empirical studies show that strengthening IPR protection in developing countries tends to cause an income transfer from developing countries that have few or no patents to developed countries, which have many patents.[2] However, in order to judge whether strengthening IPR protection in developing countries is beneficial or harmful in practice, it is important to examine how strengthening IPR protection in developing countries affects their welfare, not their income. That is, does strengthening IPR protection in developing countries harm their welfare?

The present paper examines how strengthening patent protection in a developing country affects its welfare, considering all the effects through changes in endogenous variables. To do this, we use a North–South quality-ladder model in which both innovation and technology transfers are endogenous. In our model, the main mode of technology transfer is assumed to be foreign direct investment (FDI). There are types of technology transfer that occur from a developed country (hereafter referred to as the North) to a developing country (hereafter, the South), such as FDI, licensing, illegal imitation, and outsourcing. In particular, inward FDI is increasing greatly in developing countries. FDI data from the UNCTAD World Investment Report show that inward FDI stock in developing countries increased at an annual rate of

[1] A panel data study of the index of patent protection by Park (2008) shows that many developing countries, as well as developed countries, strengthened patent protection between 1990 and 2005. For instance, Brazil, China, and India strengthened patent protection within this period to about three times its average 1960–1990 level. However, the indexes of patent protection of some developing countries in South-east Asia and Latin America remain below those of developed countries such as the U.S. and Japan.

[2] McCalman (2001) estimated the income transfers brought about by patent harmonization as a result of the TRIPs agreement. His results imply that only a few developed countries, including the U.S., could benefit from cross-country income transfers by strengthening patent protection, whereas all other countries would experience income losses from TRIPs; for instance, the net transfer from Brazil amounts to 28% of GDP. Moreover, Yang and Maskus (2001a) and Park and Lippoldt (2005) examined how U.S. receipts of royalties and license fees depend on IPR protection in the recipient countries, and showed that strengthening IPR protection has statistically significant positive influences on licensing receipts.

about 10% from 1980 to 2007.

The present analysis obtains the following two main results. First, the model shows that strengthening patent protection in the South enhances FDI and innovation, and raises the wage rate in the South. The reason for these results is as follows. Strengthening patent protection in the South enhances FDI because it enables multinationals to charge higher prices and obtain higher profits. Moreover, the enhancement of FDI further promotes innovation in the North by reducing the labor demand of the production sectors in the North and directing more labor resources to research and development (R&D). On the other hand, strengthening patent protection in the South raises the wage rate in the South.

Second, using the results of the above positive analysis, the present model shows that strengthening patent protection in the South increases welfare in both the South and the North. That is, we show that strengthening patent protection in the South can be a Pareto-improving policy for the North and the South. Moreover, we obtain the following important result for an assessment of global patent protection: harmonizing patent protection policy in the South with that existing in the North— that is, applying the strictest patent protection—can maximize welfare in the South. This result implies that, in contrast to the developing countries' apprehension that stronger IPR protection damages their welfare, patent harmonization is beneficial to developing countries that have few patents.

In our model, strengthening patent protection in the South affects welfare through three channels, as follows. The first channel is *through enhancing innovation*: strengthening patent protection promotes innovation and consequently raises welfare. The second channel is *through the change in nominal spending*: as mentioned above, strengthening patent protection raises the wage rate in the South and thereby increases the nominal spending of Southern consumers, which raises welfare in the South. By contrast, strengthening patent protection lowers the wage rate in the North and thereby lowers the nominal spending of Northern consumers, which reduces welfare in the North. The third channel is *through changing the prices of goods*: the sign of this effect is indeterminate because strengthening patent protection affects the prices of goods positively and negatively. The third channel can be decomposed into the following three effects. First, there is a welfare effect that occurs *through promoting FDI*: strengthening patent protection lowers the prices of some goods by increasing the proportion of goods produced by multinationals in the South, which produce cheaper goods than do the firms located in the North. Therefore, a rise in the proportion of FDI firms raises welfare. The second welfare effect caused by the change in prices occurs *through raising the wage in the South*: strengthening patent protection raises the wage rate in the South and enables production firms to charge higher prices for their goods by raising the marginal costs of rival firms, which consequently reduces welfare. The third welfare effect occurs *through reducing competition*: strengthening patent protection allows multinationals to charge higher prices for their goods because it reduces competition with nonpatentees, which reduces welfare. As a result of this analysis, we show that the positive welfare effects can outweigh the negative welfare effects.

In the theoretical literature on technology transfer, a number of studies have examined how strengthening IPR protection affects innovation and FDI. Such studies include those of Helpman (1993), Lai (1998), Glass and Saggi (2002), Glass and Wu (2007), Mondal and Gupta (2008), and Dinopoulos and Segerstrom (2010).[3] However, with the exception of Helpman (1993), none of these authors conducted complete analyses of welfare.[4] Unlike these studies, our study analyzes the welfare effect of strengthening patent protection in the South by using a simple model with no transitional dynamics.[5] Further, we also conduct welfare analysis by using the extended model without scale effects; although the extended model exhibits transitional dynamics, we can conduct a complete analysis of welfare in the same way as did Helpman (1993), who evaluated the effect on welfare of marginally strengthening IPR protection.

One of the few studies dealing with the welfare effect of IPR protection in developing countries is that of Helpman (1993), who conducted two welfare analyses: first, a welfare analysis in a North–South model in which the only mode of technology transfer is illegal imitation; and second, a welfare analysis in a model in which the means of technology transfer is FDI.[6] The former analysis examined how lowering the probability of imitation of Northern products by Southern firms, which is achieved by introducing tighter IPR in the South, affects welfare levels in both the South and the North. The results showed that tighter IPR reduces welfare in the South mainly by hampering innovation. The latter analysis, which is more relevant to the present paper because it deals with FDI, showed a similar result to the first analysis

[3]As can FDI, licensing can play an important role in technology transfer in the development process, as has occurred in, for example, Japan, Korea, and Taiwan. Some studies have constructed North–South growth models in which the mode of technology transfer is not FDI but licensing; see, for example, Yang and Maskus (2001b), Tanaka et al. (2007), and Futagami et al. (2007).

[4] Although Dinopoulos and Segerstrom (2010) conducted welfare analysis, theirs was limited to the comparison between steady states.

[5]Although this feature of our model is based on some special assumptions, the policy implications of our model are useful because few existing studies in this literature deal with the welfare effect of IPR protection.

[6]Extending the model of Helpman (1993), Grinols and Lin (2006) analyzed the welfare effect of strengthening patent protection in the South. However, the equilibrium paths in their model are so complex that they rely on numerical analysis. In addition, in contrast to the present model, their model does not include FDI.

without FDI; that is, tighter IPR in the South necessarily reduces welfare in the South.

Why does the result of the present paper contrast with the pessimistic result from the FDI model of Helpman? The main reason is that the present paper assumes that innovation is determined endogenously, whereas Helpman's FDI model assumes for simplicity that innovation is exogenous. By introducing the endogenous determination of innovation, our model can capture an important positive welfare effect of strengthening IPR protection, that is, the welfare effect that occurs through enhancing innovation, which is not taken into account in Helpman's FDI model. The main result of the present paper implies that the negative conclusion regarding the welfare effect of strengthening patent protection in the South may change significantly when the welfare effect that occurs through innovation is taken into consideration.

We briefly describe how patent protection is incorporated into our model. There are two instruments of patent policy: patent length and patent breadth. Patent length refers to the duration for which a patentee can sell the patented product monopolistically. Patent breadth refers to the scope of products that patentees can prevent firms without patents from producing and selling. We focus on the effects of extending patent breadth to evaluate analytically the welfare effect of increasing patent protection in the South. More concretely, in a quality-ladder model, patent breadth represents the degree of quality that the government permits firms other than the patentee to produce and, thus, patent breadth determines the markup charged by multinationals. Hence, in the present quality-ladder model, broadening patent breadth operates as would raising the markup of multinationals; that is, broadening patent breadth allows the multinationals to charge higher prices and obtain higher profits, which raises the number of multinationals and increases both the labor demand and the wage rate in the South.[7]

In contrast to the present paper, which deals with optimal patent breadth, and unlike studies that use the imitation rate as the parameter of IPR protection, some studies analyzed the effects of patent

[7] The TRIPs agreement requires WTO member countries to strengthen patent protection not only in regard to patent length but also in regard to patent breadth. According to Maskus (2000, pp. 21–22), TRIPs mandated that, in adjudicating process patent infringement cases, the burden of proof is reversed; that is, the defendant must demonstrate that his or her process does not infringe the plaintiff's patent. In general, proving process infringement is difficult, and this reduces the scope of products that firms without a patent can produce and sell, and the consequence of this is an extension of patent breadth. On the other hand, TRIPs requires extending the coverage of products that are patentable and the strengthening of patent enforcement; WTO member countries must extend patent protection to important areas of technology such as chemical products and processes, pharmaceutical products and processes, and food products and they must make more effort to expose patent infringements. We can interpret these requirements as strengthening patent protection in regard to patent breadth. See Maskus (2000, Ch. 2) for details.

length. Dinopoulos and Kottaridi (2008) focused their analysis on the effect of changing patent length.[8] They analyzed the effects of patent harmonization, under which the strength of the South's patent protection is raised to the level of the North's patent protection, and obtained the important result that patent harmonization raises the long-run growth rate and improves the relative wage in the South. However, Dinopoulos and Kottaridi (2008) did not evaluate the welfare effects of strengthening patent protection because of the complexity of their model with a finite patent length.[9] Grossman and Lai (2004) also used patent length as the policy instrument of patent policy. They analyzed why IPR tends to be better protected in the North than in the South by using a North–South model in which R&D is conducted in both countries. However, they assumed quasilinear utility in order to conduct welfare analysis. Furthermore, their model is not a growth model, unlike the present model and that of Dinopoulos and Kottaridi (2008).

The rest of the paper is structured as follows. In Section 2, we describe the model. In Section 3, we derive the equilibrium path of the model and show that strengthening patent protection promotes both innovation and FDI. In Section 4, we consider the effect of stronger patent protection on the welfare of consumers in both the South and the North. In Section 5, we examine how R&D subsidies influence the welfare effect of strengthening patent protection, and we investigate how the welfare effect of strengthening patent protection would change in a nonscale effect version of the model. In Section 6, we provide concluding remarks.

2 The Model

We develop a dynamic general equilibrium model such that FDI is introduced into a quality-ladder model. Our model has the same basic structure as that of Grossman and Helpman (1991, Ch. 12).

[8] Most of the existing studies dealing with patent protection, such as that of Kwan and Lai (2003), use the imitation rate as the parameter of patent protection. In contrast to these models that incorporate a constant imitation rate, which depends on the degree of IPR protection determined by governments, the recent growth model of the endogenous strength of protection constructed by Eicher and García-Peñalosa (2008) assumes that private R&D firms can raise the degree of enforcement by allocating labor to developing institutions that prevent the infringement of their IPR. They showed that endogenizing IPR protection generates multiple equilibria; that is, there is a high-growth equilibrium with stronger IPR protection and a low-growth equilibrium with weaker IPR protection.

[9] In theoretical analyses of patent length, the dynamic properties of the equilibrium paths tend to be complicated. For example, Futagami and Iwaisako (2007) investigated analytically the characteristics of the equilibrium paths of an economy incorporating a finite patent length and showed that, even if the production structure is a simple AK type, the equilibrium paths exhibit oscillations.

Consider an economy comprising two countries, the North and the South, which are denoted by N and S, respectively. The population size of country $i \in \{N, S\}$ is given by L_i, and each agent supplies one unit of his or her labor inelastically at each point in time. There is a continuum of goods, indexed by $\omega \in [0, 1]$, that are produced in the North or the South. Each product is classified by a number of "generations" $j = 0, 1, 2, \cdots$ and each generation progresses one step ahead if innovation occurs in the industry. Therefore, product ω of generation j can be produced after the jth innovation in industry ω. As we explain subsequently, innovation occurs as a result of successful R&D efforts by firms. We assume that products of different generations have different "qualities". The quality of product ω of generation j is given by $q_j(\omega) = \lambda^j$, where the increment in quality between generation j and $j + 1$, $\lambda > 1$, is identical for all products. In addition, we assume that one unit of labor produces one unit of output in each country and industry, irrespective of the generation number j. We choose our units appropriately so that, at time $t = 0$, the generation number is zero and quality is unity for all goods.

2.1 Consumers

Consumers living in country $i \in \{N, S\}$ have the following lifetime utility: $U_i = \int_0^\infty e^{-\rho t} \log u_{i,t} dt$, where ρ is a common subjective discount rate and $\log u_{i,t}$ represents instantaneous utility at time t. We specify the instantaneous utility function as $\log u_{i,t} = \int_0^1 \log \left[\sum_j q_j(\omega) d^i_{j,t}(\omega) \right] d\omega$, where $d^i_{j,t}(\omega)$ denotes the individual's consumption of good ω of generation j at time t.[10] The representative consumer maximizes his or her lifetime utility subject to the following budget constraint:

$$\int_0^\infty e^{-\int_0^t r_s ds} E_{i,t} dt = A_{i,0} + \int_0^\infty e^{-\int_0^t r_s ds} w_{i,t} dt, \tag{1}$$

where r_t is the interest rate that consumers in both countries face at time t, $A_{i,0}$ is the initial asset holdings of a consumer in country i, and $w_{i,t}$ denotes the wage in country i. The term $E_{i,t}$ represents the flow of spending at time t, which is given by $E_{i,t} = \int_0^1 \left[\sum_j p_{j,t}(\omega) d^i_{j,t}(\omega) \right] d\omega$, where $p_{j,t}(\omega)$ is the price of product ω of generation j at time t.

[10] In this model, we implicitly assume that the product with a quality level that lies between those of the latest generation and the second-latest generation can be produced and consumed. However, as mentioned below in Subsection 2.2, because of the pricing behavior of the firms holding the patent on the latest-generation product, only the product of the latest generation is produced and consumed in each goods sector. Thus, for simplicity, we describe the instantaneous utility as if a product with a quality level that lies between those of the latest generation and the second-latest generation could not be consumed because, in equilibrium, consumers do not consume such a product.

This consumer's utility maximization problem can be solved in two stages. In the first stage, the consumer allocates his or her spending $E_{i,t}$ to maximize $\log u_{i,t}$, given prices at time t. To solve this static problem, the consumer allocates identical expenditure shares to all products. Then, for each product, the consumer chooses the single generation $j = J_t(\omega)$ that carries the lowest quality-adjusted price $p_{j,t}(\omega)/q_j(\omega)$. This implies the following static demand function:

$$d^i_{j,t}(\omega) = \begin{cases} E_{i,t}/p_{j,t}(\omega) & \text{for } j = J_t(\omega), \\ 0 & \text{otherwise.} \end{cases}$$

In the second stage, the consumer chooses the time pattern of spending to maximize his or her lifetime utility. Such intertemporal utility maximization requires that $\dot{E}_{i,t}/E_{i,t} = r_t - \rho$. By treating aggregate spending as the numeraire, we normalize $E_t \equiv E_{N,t}L_N + E_{S,t}L_S = 1$ for all t so that the interest rate r_t always corresponds to the subjective discount rate ρ.[11]

2.2 Production

We assume that each economy has a single primary production factor, namely labor. The amounts of total labor supplied in the North and the South are constant and are denoted by L_N and L_S, respectively. As is the case in most related studies, we assume that labor is immobile between the North and the South. Labor is devoted to the production of goods in both the North and the South. In addition, in the North, labor is devoted to innovative activities to develop a higher quality product. We assume that state-of-the-art products cannot be invented in the South.

If a Northern firm succeeds in inventing a state-of-the-art good, it can take out a patent for the good in both countries and supply the good monopolistically. In contrast to the typical setting adopted by, for example, Grossman and Helpman (1991), we assume that firms in country $i \in \{N, S\}$ other than the inventor of the latest-generation product have the technological capacity to make a product with a level of quality that lies between those of the latest generation and the second-latest generation by imitating the product without undertaking R&D if and only if the inventor is located in country i. However, the existence of the patent legally guards the inventor from imitation. Thus, as mentioned below, the highest level of product quality that other firms can produce and sell legally depends on the degree of patent protection in the country.

[11] This normalization is convenient for examining the dynamic behavior of the economy. See Grossman and Helpman (1991, Ch. 12).

In the present paper, we assume that the inventor of a latest-generation product can select the location of production; that is, the firm determines whether to produce the good in the North or shift production to the South by undertaking FDI. In particular, we assume that the Northern firm, which is the inventor of a latest-generation product, can shift production from the North to the South instantaneously and at no cost if the firm chooses to undertake FDI.[12] If the firm elects to shift production to the South, it can use Southern labor, which is cheaper than Northern labor. This allows the firm to obtain higher profits at each point in time. However, a firm that chooses to undertake FDI faces more intense competition from potential rivals than does a patentee located in the North. This is because patent protection is assumed to be weaker in the South than in the North. We assume that a firm can freely export its product from one country to the other without incurring transportation costs or facing tariffs.

Before considering how patentees decide whether to undertake FDI, we must consider how governments protect patents in the North and the South. Generally, there are two policy instruments influencing the degree of patent protection. One is the patent length, which determines for how long the patentee can produce and sell the product exclusively. The other is the patent breadth, which determines the scope of products that the patentee can prevent other firms from producing and selling. In the quality-ladder model, because products of different qualities within the same product line are perfect substitutes, patent breadth represents the degree of quality that the government permits other producers to produce.[13] In practice, governments control both policy variables. However, for simplicity, we assume that the patent length is fixed and infinite and that governments control the degree of patent protection by using only the patent breadth.[14]

[12] In similar studies of FDI, Lai (1998) and Glass and Wu (2007) make a similar assumption. We can extend the model to include the cost of FDI. However, as long as the cost of FDI is small, introducing such a cost into the model would not change the result that strengthening patent protection in the South can improve the welfare of consumers in the North and the South.

[13] Strictly speaking, the concept of patent breadth includes leading breadth and lagging breadth. Leading breadth specifies the level of superiority of a product (compared with the patented product) that producers without the patent are legally permitted to produce and sell. Lagging breadth specifies how inferior a product must be compared with the patented product for the producers without the patent to legally produce and sell it. The definition of patent breadth used in this paper corresponds to lagging breadth. O'Donoghue and Zweimüller (2004) examined how leading breadth affects innovation and welfare in a closed economy. On the other hand, similarly to us, Li (2001) analyzed the effect on innovation of lagging breadth in a quality-ladder model. However, he analyzed a closed economy.

[14] Judd (1985), Iwaisako and Futagami (2003), and Futagami and Iwaisako (2007) examined how patent length affects social welfare. As shown by Futagami and Iwaisako (2007), under the assumption of finite patent length, the equilibrium paths are complicated.

In the present paper, we incorporate patent breadth as follows. When the state-of-the-art quality of product ω is given by $q_j(\omega)$, firms in country i other than the patentee of the state-of-the-art-quality product cannot legally produce product ω with a higher quality than $q_j(\omega)/\beta_i$, where $\beta_i \in [1, \lambda]$. Then, β_i can be interpreted as representing the patent breadth in country i. In this setting, a higher β_i implies a broader patent breadth: if β_i is equal to λ, then patent protection in country i is at its maximum; if β_i is equal to unity, then patent protection in country i is nonexistent.[15]

Under the rules of patent policy, the pricing strategy of a firm operating in country i depends on the patent breadth in that country. The optimal price level for the firm holding the patent for a state-of-the-art good is such that the other firms cannot earn positive profit by entering the market for that good. That is, the leader firm chooses to adopt a limit pricing strategy. More concretely, the patentee of the latest generation of product ω, the quality of which is equal to $q_j(\omega)$, adopts a pricing strategy such that the quality-adjusted price of the good is no higher than the quality-adjusted price charged by the other producers. If the patentee is operating in country i, the other producers can legally produce product ω with quality of no more than $q_j(\omega)/\beta_i$. Therefore, if the patentee charges a price p that satisfies $p/q_j(\omega) \leq p'/[q_j(\omega)/\beta_i]$, where p' denotes the price set by the other producers, then the patentee can exclude the other producers from the market. Because the lowest price that the other producers can charge is equal to their marginal cost, the limit price of the patentee is given by $p = \beta_i MC$, where MC denotes the marginal cost of the other firms. This implies that a greater patent breadth enables the patentee to charge a higher price; in particular, when β_i takes its highest value, λ, the patentee can raise the price to λMC, whereas, when β_i takes its lowest value, unity, the patentee must lower the price to the level of marginal cost.[16]

Under the patent breadth policy described above, we derive the optimal pricing strategy and the profit of the patent holders producing in the North and the South. In the rest of the paper, we refer to patent holders producing in the North as "Northern leaders" and refer to patent holders shifting production

[15]We can consider a patent breadth that is broader than λ. However, $\beta_i > \lambda$ implies that the patent for the state-of-the-art-quality product prevents the production of the good of the second-latest generation in the same product line; that is, the patent prevents production of the product invented by the previous innovator. Such a large patent breadth seems unrealistic. Thus, in our analysis, we assume that the patent breadth implied by $\beta_i = \lambda$ is the strictest form of patent protection.

[16]Gilbert and Shapiro (1990) assumed that the patent authority could raise patentee profits by increasing patent breadth. Hence, they represented the level of patent breadth by the size of the profit flow. Similarly, Goh and Olivier (2002) assumed that the patent authority could indirectly raise the legal marginal cost of producing a patented good illegally by increasing patent breadth. Thus, they represented the level of patent breadth by the scale of this legal cost.

to the South as "multinationals". First, let us consider the pricing behavior of Northern leaders. We assume that patent protection is strictest in the North; that is, patent breadth is at its highest: $\beta_N = \lambda$. Then, Northern firms other than the patent holder of the latest-generation product ω are prohibited from producing product ω at a quality level exceeding that of the second-latest generation product. Therefore, the strongest potential rival to the patent holder of the latest-generation product ω is necessarily the patent holder of the second-latest-generation product that chooses to operate in the South. This implies that the marginal cost of the strongest rival is equal to the wage in the South, $w_{S,t}$. Thus, the optimal strategy of the patentees of the latest-generation product that decide to produce their goods in the North is to set their prices to $p_{N,t} = \lambda w_{S,t}$. Hence, the instantaneous profit of Northern leaders is:

$$\pi_{N,t} = (\lambda w_{S,t} - w_{N,t}) \frac{1}{\lambda w_{S,t}} = 1 - \frac{w_{N,t}}{\lambda w_{S,t}}. \tag{2}$$

Second, let us consider the pricing behavior of multinationals. If patent protection is strong enough in the South, as well as in the North, then the optimal price for multinationals in the South, $p_{F,t}$, is the same as that of Northern firms: $p_{F,t} = \lambda w_{S,t}$. However, patent breadth may be lower in the South than in the North. Suppose that patent breadth in the South β_S takes a value of $\beta \in [1, \lambda]$. That is, a Southern firm other than the multinational of industry ω is permitted to produce product ω at a quality of $q_j(\omega)/\beta$ when the quality of the state-of-the-art good produced by the multinational is given by $q_j(\omega)$. Then, multinationals are obliged to cut their prices to $p_{F,t} = \beta w_{S,t} (\leq \lambda w_{S,t})$, which is lower than $p_{N,t}$, unless there is maximum patent protection in the South.[17] The price that multinationals can charge depends on the extent of patent breadth in the South, as argued by Goh and Olivier (2002). If the Southern government increases patent breadth, Southern firms other than multinationals can produce only lower quality products. Consequently, multinationals can charge a higher price the greater patent breadth is extended, that is, when patent protection becomes stricter in the South. From the pricing rule, the profit

[17] As mentioned above, production at a quality level beyond the patent breadth specified in each country is assumed to be prohibited effectively in each country. Moreover, we assume that products can be distributed and sold freely in both countries; this assumption is supported by evidence of parallel imports for many products. Therefore, multinationals cannot charge a higher price in the North than in the South; if they charge a higher price, the other firms can sell the products that they buy from multinationals to consumers in the North at a lower price than that charged by multinationals in the North, as a consequence of which multinationals would lose sales in the North. Thus, multinationals are obliged to cut their price in the Northern market to the price level prevailing in the South, $\beta w_{S,t}$.

flow of multinationals is given by:

$$\pi_{F,t} = (\beta w_{S,t} - w_{S,t})\frac{1}{\beta w_{S,t}} = 1 - \frac{1}{\beta} \quad (\geq \pi_{N,t}). \tag{3}$$

As this equation shows, the higher is β, the greater is the profit flow to multinationals. Consequently, the stronger is patent protection in the South, the greater is the incentive to shift production to the South.

2.3 R&D and FDI

Next, we consider the behavior of R&D firms. Following Grossman and Helpman (1991), we assume the following R&D process: if a Northern firm devotes $a_N \tilde{I}$ units of Northern labor for a time interval of length dt to research on product ω, it succeeds in developing the next generation product ω with probability $\tilde{I}dt$. In other words, $\tilde{I}$ represents the instantaneous probability of success in R&D. Although R&D firms can choose their levels of R&D, investing in R&D imposes on R&D firms labor costs that are proportional to $\tilde{I}$. If a firm succeeds in developing the new generation of a good, then it can take out a patent for that generation of product. For a finite size of R&D activities in equilibrium, the expected gain from R&D must not exceed the cost of R&D. Thus, letting $v_{N,t}$ denote the market value of the patent, we have:

$$v_{N,t} \leq w_{N,t} a_N \quad \text{with equality if} \quad I_t > 0, \tag{4}$$

where I_t denotes the innovation rate in the economy at time t, which is common to all industries.

Once a Northern firm succeeds in inventing a new-generation good, the firm can become a multinational by shifting production to the South at no cost. Therefore, as long as both Northern leader firms and multinationals exist in equilibrium, the market values of these firms are equal; that is, the following equality holds at each point in time:

$$v_{N,t} = v_{F,t}, \tag{5}$$

where $v_{F,t}$ denotes the market value of multinationals.

Next, we consider no-arbitrage conditions. The shareholders of a Northern leader firm earn dividends $\pi_{N,t}dt$ and capital gains $\dot{v}_{N,t}dt$ over a time interval of length dt. Moreover, the Northern leader firm is exposed to the risk of being leapfrogged by the development of the next-generation good by another Northern firm at the innovation rate I_t over that time interval. Thus, shareholders face making a capital loss of $v_{N,t}$ with probability $I_t dt$. Therefore, we obtain the following no-arbitrage condition between the stock of the patentee of a state-of-the-art product in the Northern market and a riskless asset:[18]

$$r_t v_{N,t} = \pi_{N,t} + \dot{v}_{N,t} - I_t v_{N,t}. \tag{6}$$

The shareholders of a multinational earn dividends $\pi_{F,t}dt$ and capital gains $\dot{v}_{F,t}dt$ over a time interval of length dt. The multinational is also exposed to the risk of being leapfrogged by a Northern firm at the innovation rate I_t. Thus, its shareholders face making a capital loss of $v_{F,t}$ with probability $I_t dt$. The no-arbitrage condition between the stock of a multinational and a riskless asset is:

$$r_t v_{F,t} = \pi_{F,t} + \dot{v}_{F,t} - I_t v_{F,t}. \tag{7}$$

2.4 The Labor Market

First, we consider the labor market in the South. Southern labor is demanded for production by multinationals that have patents for their state-of-the-art products. We let $n_{F,t}$ denote the measure of industries in which multinationals produce these state-of-the-art products. Because each multinational demands $1/(\beta w_{S,t})$ units of Southern labor, the aggregate labor demand of multinationals is $n_{F,t}/(\beta w_{S,t})$. Therefore, the labor market clearing condition in the South is:

$$\frac{n_{F,t}}{\beta w_{S,t}} = L_S. \tag{8}$$

In the North, labor is devoted not only to production but also to R&D activities. Letting $n_{N,t}$ represent the measure of industries in which Northern firms produce the state-of-the-art-quality products, the labor demand for production in the North is given by $n_{N,t}/(\lambda w_{S,t})$. In addition, because R&D firms target all goods, the labor demand for R&D activities is given by $a_N I_t(n_{F,t} + n_{N,t})$. Because $n_{F,t} + n_{N,t} = 1$, the labor market clearing condition in the North is:

$$\frac{n_{N,t}}{\lambda w_{S,t}} + a_N I_t = L_N. \tag{9}$$

3 Market Equilibrium Paths

In this section, we derive the equilibrium paths of both economies. That is, we explain the determination of the measure of firms choosing to undertake FDI, $n_{F,t}$, the wage rates in the South and the North, $w_{S,t}$

[18] If the Northern firm shifts production to the South and becomes a multinational, the gain is $(v_{F,t} - v_{N,t})$; however, this is zero from (5). Hence, even if we allow Northern firms to choose their FDI, the no-arbitrage condition remains unchanged.

and $w_{N,t}$, and the innovation rate, I_t. As shown in this section, our model has no transitional dynamics and, thus, these variables are constant over time.

First, we consider the equilibrium determination of FDI. From (5), the market values of Northern leaders and multinationals are equal at each point in time. Therefore, in what follows, we let v_t denote the market value of all firms; that is, $v_t \equiv v_{N,t} = v_{F,t}$. Because (5) holds at each point in time, we obtain $\dot{v}_{F,t}/v_{F,t} = \dot{v}_{N,t}/v_{N,t}$. Substituting the no-arbitrage conditions (6) and (7) into this equation yields:

$$\pi_{F,t} = \pi_{N,t}. \tag{10}$$

Using (2) and (3), the equilibrium condition for FDI, (10) requires the following relationship between $w_{N,t}$ and $w_{S,t}$:

$$w_{S,t} = \frac{\beta}{\lambda} w_{N,t}. \tag{11}$$

This means that the relative wage in the South, $w_{S,t}/w_{N,t}$ equals $\beta/\lambda \in [1/\lambda, 1]$ and is an increasing function of β.[19] The reason is as follows. Greater patent breadth in the South (a higher β) guarantees multinationals higher profits, as shown by (3). Moreover, because multinationals and Northern leaders must earn equal profits, greater patent breadth in the South must also raise the profits of Northern leaders. In order for the profits of Northern leaders to be higher, the total production cost of Northern leaders must be lower because their revenue is fixed. The total production cost of Northern leaders, which is equal to $w_{N,t}/(\lambda w_{S,t})$, is inversely proportional to the relative wage in the South and, hence, the relative wage in the South must be increasing with β.

Second, from the condition for labor market equilibrium in the South, (8), the relationship between

[19] The upper limit of the relative Northern wage rate implied by this model is determined by λ. However, this level is probably unrealistically low, given a plausible estimate of λ; related literature, such as the study of Şener (2008), suggests that λ is between 1.05 and 1.4. This is a common feature of quality-ladder North–South models, as explained by Gustafsson and Segerstrom (2010). In the quality-ladder North-South model, the production of goods that have been imitated in the South can move back to the North due to successful innovation by a Northern firm, and therefore, the price charged by the Northern firm becomes λw_S, which depends on the Southern wage rate. Furthermore, it must not be lower than the Northern wage rate, w_N in order to guarantee positive profit for Northern innovators, and thus this imposes the upper limit of the Northern relative wage rate: $\lambda > w_N/w_S$. Gustafsson and Segerstrom (2010) released the relative wage from the condition by utilizing a variety-expansion North-South model, where the production of goods that have been imitated in the South cannot move back to the North by Northern firms' innovation. Moreover, they showed that introducing the differences in R&D technologies between innovation in the North and imitation in the South into the models, the model can account for an arbitrarily large Northern relative wage.

the measure of multinationals and the wage rate in the South is positive as follows:

$$n_{F,t} = \beta L_S w_{S,t}. \tag{12}$$

This is because a rise in the wage in the South raises the price of the good of a multinational, $p_{F,t} = \beta w_{S,t}$, which reduces the demand for the labor used to produce this good, $1/(\beta w_{S,t})$. The decrease in labor demand in the South enables more firms to conduct FDI and increases $n_{F,t}$. In the same way, strengthening patent protection in the South increases the measure of multinationals.

Third, we consider the relationship between the innovation rate, I_t, and the wage rate in the South, $w_{S,t}$. By substituting $n_{F,t} + n_{N,t} = 1$ and (12) into the condition for the clearing of the Northern labor market (9), we obtain the innovation rate as follows:

$$I_t = \frac{L_N}{a_N} - \frac{1 - n_{F,t}}{a_N \lambda w_{S,t}} = \frac{L_N + (\beta/\lambda)L_S}{a_N} - \frac{1}{a_N \lambda w_{S,t}}. \tag{13}$$

These expressions show that the relationship between I_t and $w_{S,t}$ is positive.

Next, by using the expressions derived so far, we derive the equilibrium value of a firm, v_t. As long as the innovation rate is positive, (4) holds with equality: $w_{N,t} = v_t/a_N$. By substituting this, (3), (11), and (13) into (7), we obtain the following equilibrium dynamics of v_t: $\dot{v}_t = \left[\frac{L_N + (\beta/\lambda)L_S}{a_N} + \rho\right] v_t - 1$. This differential equation has the unique steady state, v, which is given by:

$$v = \frac{a_N}{L_N + (\beta/\lambda)L_S + \rho a_N}. \tag{14}$$

This unique steady state v is unstable and, therefore, v_t diverges to positive or negative infinity unless v_t is v. Hence, the equilibrium value of v_t must immediately jump to v at the initial point in time, because v_t is a jump variable. Because v_t is constant over time, we can show that the other variables are also constant over time; that is, our North–South model economy has no transitional dynamics. However, its tractability enables us to use the model for welfare analysis.[20]

[20] There are two straightforward ways to extend the model. One is to allow Southern firms to copy the technology of multinationals. Assuming this would make the model more realistic because, then, Southern firms other than multinationals could produce goods. However, this extension injects transitional dynamics into the model because the measure of goods imitated and produced by Southern firms is a state variable. This complicates the equilibrium path and makes welfare analysis less tractable. The other possible extension is to introduce a finite patent length as in, for example, Dinopoulos and Kottaridi (2008). The introduction of a finite patent length would affect the welfare effects of raising patent breadth and might change our welfare results. This would also inject into the model transitional dynamics, perhaps including oscillations, as Judd (1985) and Futagami and Iwaisako (2007) found. Hence, this extension would make welfare analysis far more difficult.

Note that v depends negatively on β, as shown in (14). This is because strengthening patent protection causes capital losses to rise as leapfrogging by other firms increases because of the promotion of innovation.

Using the free entry condition, $w_{N,t} = v_t / a_N$, and (11), we can compute the following equilibrium wage rates in the North and South, which are constant over time:

$$w_N = \frac{1}{L_N + (\beta/\lambda)L_S + \rho a_N}, \tag{15}$$

$$w_S = \frac{\beta/\lambda}{L_N + (\beta/\lambda)L_S + \rho a_N}, \tag{16}$$

where w_N and w_S are the equilibrium values of $w_{N,t}$ and $w_{S,t}$, respectively. In the rest of the paper, the variables without the subscript "t" represent equilibrium values. From (15) and (16), we can prove the following proposition.

Proposition 1. *Strengthening patent protection in the South raises the wage rate in the South and lowers the wage rate in the North.*

Proposition 1 can be interpreted as follows. As shown by (14), strengthening patent protection enhances innovation and therefore reduces the value of Northern firms and multinationals. From the condition for zero profit in R&D, the reduction in the reward for innovation must bring about a decrease in the cost of R&D, which lowers the wage in the North. On the other hand, as shown by (11), strengthening patent protection in the South raises the relative wage in the South. Because the increase in the relative wage of the South is sufficiently large to exceed the decrease in the wage in the North, the wage in the South must increase.

By substituting (16) into (12) and (13), we can derive the equilibrium values of $n_{F,t}$ and I_t as follows:

$$n_F = \frac{(\beta^2/\lambda)L_S}{L_N + (\beta/\lambda)L_S + \rho a_N}, \tag{17}$$

$$I = \frac{\beta - 1}{\beta} \frac{L_N + (\beta/\lambda)L_S}{a_N} - \frac{\rho}{\beta}. \tag{18}$$

Differentiating (17) and (18) with respect to β yields:

$$\frac{dn_F}{d\beta} = \frac{(\beta/\lambda)L_S}{L_N + (\beta/\lambda)L_S + \rho a_N} \left[2 - \frac{(\beta/\lambda)L_S}{L_N + (\beta/\lambda)L_S + \rho a_N} \right] > 0,$$

$$\frac{dI}{d\beta} = \frac{1}{\beta^2}\left(\frac{L_N}{a_N} + \rho\right) + \frac{L_S}{\lambda a_N} > 0. \tag{19}$$

Therefore, we can show that both n_F and I are increasing functions of β. Consequently, strengthening patent protection in the South necessarily promotes FDI and innovation in the North.

Proposition 2. *Strengthening patent protection in the South promotes both innovation and FDI.*

How can we interpret Proposition 2? As Proposition 1 shows, strengthening patent protection raises the wage in the South. Therefore, strengthening patent protection raises the price of a good produced by a multinational, $p_F = \beta w_S$, and thereby reduces the demand for the labor used to produce this good, $1/(\beta w_S)$. Consequently, to keep the Southern labor market in equilibrium, more firms must become multinationals and shift production to the South than before the policy change. In addition, strengthening patent protection raises the wage in the South, thereby raising the price of a good produced by a Northern leader, $p_N = \lambda w_S$, and thus reducing the demand for the labor used to produce this good, $1/(\lambda w_S)$. Because of both the decrease in labor demand by Northern leaders and the increased production shift to the South, stronger patent protection in the South decreases total demand for the labor used to produce goods in the North. Consequently, stronger patent protection increases labor demand for R&D activity and raises the equilibrium innovation rate.[21]

Our finding that strengthening patent protection in the South enhances FDI is consistent with empirical results. For example, Lee and Mansfield (1996) estimated the relationship between the volume of FDI flows and the strength of IPR protection, and found that they are positively correlated. A number of theoretical studies contain results similar to ours: Vishwasrao (1994) and Žigić (1998) showed that weaker patent protection in the South may reduce technology transfers in a partial equilibrium. Lai (1998) and Dinopoulos and Segerstrom (2010) obtained this result in a dynamic general equilibrium model.[22]

[21] We implicitly assume the existence of an interior solution, in which $0 < n_F < 1$ and $I > 0$. As discussed in Appendix C, $0 < n_F < 1$ is guaranteed if the parameters satisfy $(\lambda - 1)L_S < L_N + \rho a_N$. Further, assuming that $\beta > \beta_{min}$, where $\beta_{min} \equiv 2^{-1}(1 - \lambda L_N/L_S) + \sqrt{2^{-2}(1 - \lambda L_N/L_S)^2 + \lambda(L_N + \rho a_N)/L_S}$, and $(\lambda - 1)(L_N + L_S) > \rho a_N$ guarantees $I > 0$. In the rest of the paper, we focus on the interior solution by assuming that these parameter conditions are satisfied.

[22] In contrast, Glass and Saggi (2002) showed that strengthening IPR protection impedes innovation and FDI in a dynamic general equilibrium. The dynamic general equilibrium results probably differ because of differences in assumptions about imitation and IPR protection; like us, Lai (1998) and Dinopoulos and Segerstrom (2010), assume that imitation is costless, whereas Glass and Saggi (2002) assume that imitation is costly and that strengthening IPR protection increases the cost of imitation.

4 Welfare Analysis

In the previous section, we showed that strengthening patent protection raises the relative wage in the South. However, the Southern government's main concern is the welfare of its consumers: if strengthening patent protection in the South improves the South's welfare, a Southern government would be keen to implement the policy. Otherwise, it has an incentive to relax patent protection. Thus, in this section, we examine the welfare effects of strengthening patent protection in the South.

First, we derive the spending of the representative consumer living in country $i \in \{N, S\}$. As mentioned in Subsection 2.1, because we treat total spending as the numeraire, that is, $E_t = 1$ for all t, the interest rate is equal to the subjective discount rate, $r_t = \rho$. Given the intertemporal utility maximization of each consumer, the per capita spending of a consumer living in any country is constant over time. Therefore, we let E_i denote the spending of a consumer in country i. Because spending levels and the wage rate in each country are constant over time, the intertemporal budget constraint (1) is reduced to $E_i = \rho A_{i,0} + w_i$ for $i \in \{N, S\}$. Multiplying both sides of these budget constraints by the population and adding the constraints yields $\rho(A_{N,0}L_N + A_{S,0}L_S) + w_N L_N + w_S L_S = 1$, where we use $E_t \equiv E_{N,t}L_N + E_{S,t}L_S = 1$. Letting A_0 denote the total initial asset holdings, that is, $A_0 \equiv A_{N,0}L_N + A_{S,0}L_S$, we can derive the value of A_0 as follows: $A_0 = [1 - (w_N L_N + w_S L_S)]/\rho$. Because country i's share of asset holdings must be given as an initial condition, we let $\zeta \in [0, 1]$ denote the share of assets held by Northern consumers, that is, $\zeta \equiv A_{N,0}L_N/A_0$.[23] By substituting these relationships into E_i, we obtain the following equilibrium values of E_N and E_S:

$$E_N = \zeta \frac{1 - w_S L_S}{L_N} + (1 - \zeta) w_N, \tag{20}$$

$$E_S = (1 - \zeta) \frac{1 - w_N L_N}{L_S} + \zeta w_S. \tag{21}$$

Next, we rewrite the instantaneous utility as follows:

$$\log u_{i,t} = \int_0^1 \log \lambda^{J_t(\omega)} d_t^i(\omega) d\omega = (\log \lambda) \int_0^1 J_t(\omega) d\omega + \int_0^1 \log d_t^i(\omega) d\omega. \tag{22}$$

Because the latest generation of a product sells for the lowest quality-adjusted price in the product line, $J_t(\omega)$ corresponds to the generation number of the latest generation of product ω. Thus, the first term of

[23] When the share of assets held by Southern consumers $(1 - \zeta)$ equals $(\beta - 1)(\beta/\lambda)L_S/(\rho a_N)(> 0)$, balanced trade is obtained; that is, $(1 - n_F)E_S L_S = n_F E_N L_N$.

(22) is equal to $\log\lambda$ multiplied by the total number of innovations obtained in all industries by time t. Because the rate of innovation is constant over time in our model, we can rewrite this term as:

$$(\log\lambda)\int_0^1 J_t(\omega)d\omega = (\log\lambda)It. \tag{23}$$

The second term of (22) can be rewritten as follows: $\int_0^1 \log d_t^i(\omega)d\omega = n_F \log d_{F,t}^i + (1-n_F)\log d_{N,t}^i$, where $d_{F,t}^i$ and $d_{N,t}^i$ denote the demand for goods produced by multinationals and Northern leaders, respectively. By using $d_{F,t}^i = E_i/p_F = E_i/(\beta w_S)$ and $d_{N,t}^i = E_i/p_N = E_i/(\lambda w_S)$, we obtain:

$$\int_0^1 \log d_t^i(\omega)d\omega = \log E_i - \log w_S - (\log\beta)n_F - (\log\lambda)(1-n_F). \tag{24}$$

As shown above, E_i, w_S, n_F, and I are all constant on the equilibrium path. Hence, from (22)–(24), $(\log\lambda)I$ represents the utility growth rate. Substituting (22)–(24) into the lifetime utility function, we obtain the welfare of each consumer in country $i \in \{N,S\}$ as follows:

$$U_i(\beta) = \frac{1}{\rho}\left[\frac{\log\lambda}{\rho}I + \log E_i - \log w_S - (\log\beta)n_F - (\log\lambda)(1-n_F)\right], \tag{25}$$

where $U_i(\beta)$ denotes the welfare of each consumer in country i when patent breadth in the South is β. This shows that the welfare of an individual depends on the innovation rate, nominal spending, the wage in the South, which in turn determines the prices of goods, the measure of multinationals, and patent breadth in the South. The welfare levels of a Northern individual and a Southern individual differ only because their nominal spending, E_i, differs.

To determine whether increasing patent breadth in the South raises welfare, we differentiate (25) with respect to breadth β. The derivative of $U_i(\beta)$ is given by:

$$\begin{aligned}\frac{dU_i(\beta)}{d\beta} = \frac{1}{\rho}\Bigg\{ & \underbrace{\frac{\log\lambda}{\rho}\frac{dI}{d\beta}}_{\substack{\text{innovation-enhancing effect}\\(+)}} \underbrace{+\frac{1}{E_i}\frac{dE_i}{d\beta}}_{\substack{\text{nominal spending effect}\\(+)\text{ or }(-)}} \\ & + \Bigg[\underbrace{(\log\lambda - \log\beta)\frac{dn_F}{d\beta}}_{\substack{\text{FDI-promoting effect}\\(+)}} \underbrace{-\frac{1}{w_S}\frac{dw_S}{d\beta}}_{\substack{\text{marginal cost effect}\\(-)}} \underbrace{-\frac{n_F}{\beta}}_{\substack{\text{competition-reducing effect}\\(-)}} \Bigg]\Bigg\}.\end{aligned} \tag{26}$$

As shown on the right-hand side (RHS) of (26), an increase in patent breadth in the South affects the welfare of both countries through the following three channels. The first channel is the welfare effect that occurs *through enhancing innovation*, which is indicated by the first term on the RHS of (26). As shown in Proposition 2, increasing patent breadth promotes innovation and raises welfare. We refer to this effect as the *innovation-enhancing effect*. This effect has a positive influence on the welfare of both countries. The second channel is the welfare effect that occurs *through the change in nominal spending*, which is indicated by the second term on the RHS of (26). As shown by (20), (21), and Proposition 1, increasing patent breadth raises nominal spending in the South, but reduces nominal spending in the North. Thus, it affects the welfare of both countries. We refer to this effect as the *nominal spending effect*. The third channel is the welfare effect that occurs *through changing the prices of goods*, which is indicated by the three terms in square brackets on the RHS of (26).

The sign of the sum of the three terms in the square brackets is indeterminate because increasing patent breadth has both positive and negative effects on the prices of goods in the following ways. First, extending patent breadth increases the proportion of goods that multinationals produce, as shown in Proposition 2. Because patent breadth is smaller in the South than in the North, the price of the goods produced by multinationals (βw^S) is lower than the price of the goods produced by Northern leaders (λw^S). This means that an increase in patent breadth improves the welfare of both countries by increasing the proportion of FDI firms, n_F. We refer to this positive welfare effect as the *FDI-promoting effect*, which is indicated by the first term in the square brackets. Second, from (16), increasing patent breadth raises the wage rate in the South. This causes a rise in the marginal costs of followers, which allows Northern leaders and multinationals to charge higher prices, and this reduces the welfare of both countries. We refer to this negative effect as the *marginal cost effect*, which is indicated by the second term in the square brackets. Third, increasing patent breadth in the South enables multinationals to raise the price of their goods directly, which reduces welfare. This is because increasing patent breadth permits Southern firms other than multinationals to produce only goods of lower quality. This means that multinationals can outcompete other firms even if multinationals charge a higher price for their goods. For this reason, increasing patent breadth reduces the welfare of both countries. We refer to this negative effect, which is shown by the last term in the square brackets, as the *competition-reducing effect*.

If the positive welfare effects outweigh the negative welfare effects, then strengthening patent protection in the South raises welfare. As shown in the subsequent subsections, whether this is the case depends on the values of the parameters.

4.1 The Effect on the South's Welfare

First, we explore the effect of strengthening patent protection on the South's welfare. We determine the parameter values that cause the strictest patent protection to maximize the welfare of Southern consumers. In the rest of this subsection, as a benchmark, we focus on the case in which consumers in the South have no assets at the initial point in time; that is, $\zeta = 1$. In Subsection 4.3, we analyze the general case in which $\zeta \neq 1$.

We can summarize the results of our analysis of the South's welfare in the case of $\zeta = 1$ as follows.[24]

Proposition 3. *Suppose that consumers in the South have no assets at the initial point in time. Then, the strictest patent protection in the South ($\beta_S = \lambda$) maximizes the welfare of consumers in the South if and only if the parameters satisfy* $(\log \lambda)(L_N + \lambda L_S + \rho a_N)/(\lambda^2 \rho a_N) \geq L_S/\left(L_N + L_S + \rho a_N\right)$.

Proof. If $\zeta = 1$, we obtain the following equality from (21) and (26) in the range of $\beta \in (\beta_{min}, \lambda]$: $dU_S(\beta)/d\beta = \left[f(\beta) + (\log \lambda - \log \beta)\left(dn_F/d\beta\right)\right]/\rho$, where $f(\beta) \equiv \left[(\log \lambda)/\rho\right]\left(dI/d\beta\right) - \left(n_F/\beta\right)$. It is straightforward to show that $f'(\beta) < 0$ from (17) and (19), and therefore $f(\beta) \geq 0$ for all $\beta \in (\beta_{min}, \lambda]$ if $f(\lambda) \geq 0$. Because $(\log \lambda - \log \beta)\left(dn_F/d\beta\right) \geq 0$, this implies that $dU_S(\beta)/d\beta \geq 0$ for all $\beta \in (\beta_{min}, \lambda]$ if $f(\lambda) \geq 0$. Meanwhile, if $f(\lambda) < 0$, then $dU_S(\lambda)/d\beta < 0$. This is because $(\log \lambda - \log \beta)\left(dn_F/d\beta\right) = 0$ when $\beta = \lambda$. As a result, the strictest patent protection ($\beta_S = \lambda$) maximizes the welfare of consumers in the South if and only if $f(\lambda) \geq 0$. By rewriting $f(\lambda) \geq 0$, we obtain $(\log \lambda)(L_N + \lambda L_S + \rho a_N)/(\lambda^2 \rho a_N) \geq L_S/\left(L_N + L_S + \rho a_N\right)$. □

If $\zeta = 1$, then $dU_S(\beta)/d\beta$ is simplified to the sum of the innovation-enhancing effect, the FDI-promoting effect, and the competition-reducing effect because $E_S = w_S$ from (21). The innovation-enhancing effect necessarily offsets the competition-reducing effect for all $\beta \in (\beta_{min}, \lambda]$ if and only if the parameters satisfy the condition in Proposition 3. The FDI-promoting effect is necessarily nonnegative and is zero when $\beta = \lambda$. Therefore, the strictest patent protection maximizes welfare in the South if and only if the condition in Proposition 3 holds.

[24] For Propositions 3–5, we assume that the degree of patent protection in the South is no weaker than the level of protection below which Northern R&D activities cease; that is, $\beta \in (\beta_{min}, \lambda]$. For some set of parameter values, a low value of β that prevents innovation could maximize welfare. However, if the cost of innovation is sufficiently low, there is no such set of parameter values. For instance, by assuming $a_N < (\lambda - 1)(L_N + L_S)^2/\left\{\left[(\lambda + 1)(L_N + L_S) + \lambda L_S\right]\rho\right\}$, we can show that any low value of β that prevents innovation will maximize neither the welfare of Southern consumers nor that of Northern consumers. The proof is given in Appendix D.

Clearly, the larger is Northern labor, L_N, the more likely is the condition in Proposition 3 to hold. Therefore, Proposition 3 implies that the strictest patent protection improves the welfare of consumers in the South if labor is more abundant in the North. Why might strengthening patent protection raise the welfare of Southern consumers in such a case? An increased abundance of labor in the North intensifies the promotion of the innovation effect of strengthening patent protection, $dI/d\beta$, which is represented by the left-hand side (LHS) of the condition in Proposition 3. Moreover, an increased abundance of Northern labor decreases the proportion of multinationals, n_F, and, consequently, weakens the competition-reducing effect of strengthening patent protection, $-n_F/\beta$, which is represented by the RHS of the condition in Proposition 3. Thus, the more abundant is labor in the North, the more intensive is the positive welfare effect of strengthening patent protection.

In addition, the condition in Proposition 3 necessarily holds irrespective of the amount of labor in the North, as long as L_S/a_N is sufficiently large that $L_S/a_N \geq \lambda\rho/\left[(\log\lambda)(2\lambda-1)\right]$.[25] Therefore, Proposition 3 shows that the strictest patent protection improves the welfare of consumers in the South if labor is more abundant in the South and if the productivity of R&D is higher in the North. The reason is as follows. The more abundant is labor in the South and the higher is the productivity of R&D in the North, the larger is the innovation-enhancing effect of strengthening patent protection, $dI/d\beta$. However, higher R&D productivity and Southern labor abundance increase the proportion of multinationals, n_F, the consequence of which is an intensification of the competition-reducing effect of strengthening patent protection. Thus, increased labor abundance in the South and increased productivity of R&D in the North have two opposing effects on welfare. However, if L_S/a_N is sufficiently large to satisfy $L_S/a_N \geq \lambda\rho/\left[(\log\lambda)(2\lambda-1)\right]$, the former positive effect outweighs the latter negative effect.

Proposition 3 is important in the following two respects. First, it seems widely supposed that strengthening patent protection, particularly by equating levels of protection in the North and South, lowers welfare in the South. However, our result contradicts this intuitive supposition, and shows that, far from being harmful, strengthening patent protection in the South, even to the extent that its level of protection matches that of the North, may benefit the South.[26] In other words, this paper provides a ratio-

[25]The proof is given in Appendix E.

[26] A possible explanation for the intuitive supposition is that the benefits conferred by the innovation-enhancing effect are underestimated because this effect is difficult to discern in practice. In addition, as mentioned in the conclusion, incorporating enforcement costs to implement patent protection, which we ignore in this paper, into the analysis may reduce the probability that the strictest protection of patent maximizes the welfare in the South.

nale for strengthening patent protection, which is a policy implemented in many developing countries.[27]

Second, our result contrasts with that of Helpman (1993), the seminal paper that examines the effect of strengthening IPR protection on welfare. Helpman examined the welfare effect of strengthening IPR protection both in an endogenous innovation model, in which the only mode of technology transfer is illegal imitation, and in an exogenous innovation model, in which FDI is the mode of technology transfer. Helpman concluded from both models that stronger IPR protection in the South necessarily damages welfare in the South. We obtain important implications from our results that contrast with the results of Helpman's two models. First, comparing our model with Helpman's model in which imitation is the only mode of technology transfer suggests that the effect of strengthening IPR protection on the South's welfare depends on the main mode of technology transfer: when the main mode of technology transfer is FDI, strengthening IPR protection is likely to raise the South's welfare. Second, comparing our model with Helpman's model incorporating FDI suggests that taking the innovation-enhancing effect into account may reverse the sign of the total welfare effect of strengthening IPR protection: if the positive effect on innovation of strengthening IPR protection is taken into account, increasing patent protection might raise the South's welfare.

4.2 The Effect on the North's Welfare

Next, we examine how strengthening patent protection in the South affects the North's welfare. To do this, we first show that the effect on the North's welfare of increasing patent breadth is independent of the initial distribution of assets, ζ. From (15), (16), and (20), E_N/w_S becomes $E_N/w_S = [1 + (\zeta\rho a_N/L_N)]\,(\lambda/\beta)$. Then, the effect of increasing patent breadth on $\log(E_N/w_S)$ is given by:

$$\frac{1}{E_N}\frac{dE_N}{d\beta} - \frac{1}{w_S}\frac{dw_S}{d\beta} = -\frac{1}{\beta} < 0. \tag{27}$$

This shows that the effect of increasing patent breadth on $\log(E_N/w_S)$, that is, the sum of the nominal spending effect and the marginal cost effect, is independent of the initial distribution of assets, ζ. Because the innovation rate, I, and the measure of multinationals, n_F, are independent of ζ, the magnitudes of the innovation-enhancing effect, the FDI-promoting effect, and the competition-reducing effect do not depend on ζ. Thus, the effect on the North's welfare, $dU_N(\beta)/d\beta$, is independent of ζ.

[27] Park's (2008) panel data on the index of patent protection show that many developing countries strengthened patent protection between 1990 and 2005.

As shown in the following proposition, $dU_N(\beta)/d\beta$ is positive for all $\beta \in (\beta_{min}, \lambda]$ if the parameters satisfy an inequality.

Proposition 4. *The strictest patent protection in the South ($\beta_S = \lambda$) maximizes the welfare of consumers in the North if the parameters satisfy $L_N \geq [\lambda/(\log\lambda) - 1]\,\rho a_N$.*

Proof. By substituting (27) into (26), we can rewrite $dU_N(\beta)/d\beta$ as follows:

$$\frac{dU_N(\beta)}{d\beta} = \frac{1}{\rho}\left[\frac{\log\lambda}{\rho}\frac{dI}{d\beta} - \frac{n_F}{\beta} - \frac{1}{\beta} + (\log\lambda - \log\beta)\frac{dn_F}{d\beta}\right]. \tag{28}$$

If $[(\log\lambda)/\rho]\,(dI/d\beta) - (n_F/\beta) - (1/\beta) > 0$, then $dU_N(\beta)/d\beta > 0$ because $(\log\lambda - \log\beta)\,(dn_F/d\beta) \geq 0$. From (17) and (19), we find that the following relationship holds:

$$\begin{aligned}
\frac{\log\lambda}{\rho}\frac{dI}{d\beta} - \frac{n_F}{\beta} - \frac{1}{\beta} &= \frac{\log\lambda}{\rho}\left[\left(\frac{L_N}{a_N} + \rho\right)\frac{1}{\beta^2} + \frac{L_S}{\lambda a_N}\right] - \frac{(\beta/\lambda)L_S}{L_N + \rho a_N + (\beta/\lambda)L_S} - \frac{1}{\beta} \\
&> \frac{\log\lambda}{\rho}\left[\left(\frac{L_N}{a_N} + \rho\right)\frac{1}{\beta^2} + \frac{L_S}{\lambda a_N}\right] - \frac{(\beta/\lambda)L_S}{L_N + \rho a_N} - \frac{1}{\beta} \\
&= \frac{1}{\beta}\left[L_N + \rho a_N + \frac{\beta^2}{\lambda}L_S\right]\left[\frac{\log\lambda}{\rho a_N \beta} - \frac{1}{L_N + \rho a_N}\right].
\end{aligned} \tag{29}$$

If $L_N \geq [\lambda/(\log\lambda) - 1]\,\rho a_N$, then the parameters satisfy $(\log\lambda)/(\rho a_N \beta) \geq 1/(L_N + \rho a_N)$ for all $\beta \in [1, \lambda]$. Therefore, from (28) and (29), we can conclude that $dU_N(\beta)/d\beta > 0$ for any value of $\beta \in (\beta_{min}, \lambda]$ if the parameters satisfy $L_N \geq [\lambda/(\log\lambda) - 1]\,\rho a_N$. □

The strictest patent protection maximizes Northern welfare when the quantity of Northern labor is relatively large for the same reasons as given in the interpretation of Proposition 3. That is, an increased abundance of Northern labor enhances the innovation-enhancing effect of strengthening patent protection and, thus, the positive welfare effects outweigh the negative welfare effects.

The condition in Proposition 4 is stricter than the condition in Proposition 3.[28] Hence, we can show that when $\zeta = 1$, strengthening patent protection makes both the South and the North better off as long as the parameters satisfy the condition in Proposition 4. In other words, as long as labor resources in the North are sufficiently abundant or as long as the productivity of R&D is sufficiently high to satisfy

[28]The LHS of the inequality in Proposition 3 is an increasing function of L_N and the RHS is a decreasing function of L_N. Substituting $L_N = [\lambda/(\log\lambda) - 1]\rho a_N$ into the inequality in Proposition 3 reveals that the inequality holds if $L_N = [\lambda/(\log\lambda) - 1]\rho a_N$. Thus, the inequality in Proposition 3 holds whenever $L_N \geq [\lambda/(\log\lambda) - 1]\rho a_N$.

the condition, then raising patent protection in the South to the level enjoyed in the North is a Pareto-improving policy.[29]

With respect to the effect on the welfare of consumers in the North, the results of the present paper are consistent with those of Helpman's FDI model. In his exogenous innovation model, Helpman concluded that tightening IPR protection when the main mode of technology transfer is FDI benefits the North if the imitation rate in the South is sufficiently low. Our endogenous innovation model implies that maximum patent protection in developing countries is globally optimal for the consumers in the North if labor resources in the North are sufficiently abundant or if the productivity of R&D is sufficiently high. This result implies that Helpman's conclusion about Northern welfare is robust to incorporating the innovation-enhancing effect of strengthening IPR protection. In that sense, our Proposition 4 complements Helpman's conclusion about Northern welfare.

4.3 Generalizing the Initial Distribution of Assets

In order to generalize the result described in Proposition 3, we show that the strictest patent protection can maximize the welfare of consumers in the South even when $\zeta \neq 1$. When $\zeta \neq 1$, the nominal spending effect and the marginal cost effect do not cancel each other out. Thus, one must consider these welfare effects of strengthening patent protection. From (16) and (21), E_S/w_S becomes $E_S/w_S = 1 + [(1-\zeta)\lambda\rho a_N/(\beta L_S)]$. Hence, the effect of strengthening patent protection on $\log(E_S/w_S)$ is:

$$\frac{1}{E_S}\frac{dE_S}{d\beta} - \frac{1}{w_S}\frac{dw_S}{d\beta} = -\frac{1-\zeta}{\beta L_S/(\lambda\rho a_N) + 1 - \zeta}\frac{1}{\beta} < 0. \tag{30}$$

This means that the higher is the initial level of assets held by Southern consumers (the larger is $(1-\zeta)$), the greater is the negative effect of strengthening patent protection on $\log(E_S/w_S)$. Therefore, the total effect of strengthening patent protection on the welfare of Southern consumers is more likely to be negative the higher is the initial level of assets held by Southern consumers. However, we can show

[29] The RHS of the inequality in Proposition 4 is a U-shaped function of λ, and the restriction for the other parameters becomes stringent when λ takes extremely large or small values. However, unless λ is an extreme value, the ranges of values for L_N, a_N, and ρ that satisfy the inequality are sufficiently broad. We assess the stringency of the inequality by using a numerical example. Following the numerical analysis of Şener (2008), we use the range $[1.05, 1.4]$ for λ so that the value of $\lambda - 1$ representing the markup in this model can be consistent with empirical data. Assuming that ρ is 0.05, if $L_N/a_N > 1.02604$, the inequality in Proposition 4 holds. Further, L_N/a_N must satisfy the conditions that $(\lambda-1)L_S < L_N + \rho a_N$ and $(\lambda-1)(L_N+L_S) > \rho a_N$, which are assumed in Section 3. However, as long as L_S/a_N is less than 21.5207, values of L_N/a_N that exceed 1.02604 satisfy the two conditions. Thus, the parametric restrictions are not particularly stringent.

that the strictest patent protection maximizes the welfare of Southern consumers for any $\zeta \in [0, 1]$ if the parameters satisfy the condition given in Proposition 4. Combining this result with that stated in Proposition 4 enables us to state the following proposition.

Proposition 5. *The strictest patent protection in the South ($\beta_S = \lambda$) maximizes the welfare of consumers in both the South and the North for any initial asset distribution between the North and the South if the parameters satisfy $L_N \geq [\lambda/(\log \lambda) - 1]\, \rho a_N$.*

Proof. The proof relating to the welfare of Northern consumers is given in Proposition 4. Therefore, it is sufficient for this proof to show that $dU_S(\beta)/d\beta > 0$ for any values of $\beta \in (\beta_{min}, \lambda]$ and $\zeta \in [0, 1]$ if the parameters satisfy $L_N \geq [\lambda/(\log \lambda) - 1]\, \rho a_N$. We can rewrite $dU_S(\beta)/d\beta$ for any $\zeta \in [0, 1]$ as follows:

$$\begin{aligned} \frac{dU_S(\beta)}{d\beta} &= \frac{1}{\rho}\left[\frac{\log\lambda}{\rho}\frac{dI}{d\beta} - \frac{n_F}{\beta} + \left(\frac{1}{E_S}\frac{dE_S}{d\beta} - \frac{1}{w_S}\frac{dw_S}{d\beta}\right) + (\log\lambda - \log\beta)\frac{dn_F}{d\beta}\right] \\ &> \frac{1}{\rho}\left[\frac{\log\lambda}{\rho}\frac{dI}{d\beta} - \frac{n_F}{\beta} - \frac{1}{\beta} + (\log\lambda - \log\beta)\frac{dn_F}{d\beta}\right], \end{aligned} \tag{31}$$

where the inequality holds because $(1/E_S)\,(dE_S/d\beta) - (1/w_S)\,(dw_S/d\beta) > -1/\beta$ from (30). Note that the RHS of (31) is equal to $dU_N(\beta)/d\beta$. Therefore, using the same proof as in Proposition 4, we can show that $dU_S(\beta)/d\beta > dU_N(\beta)/d\beta > 0$ for any values of $\beta \in (\beta_{min}, \lambda]$ and $\zeta \in [0, 1]$ if the parameters satisfy $L_N \geq [\lambda/(\log \lambda) - 1]\, \rho a_N$. □

Proposition 5 shows that as long as labor resources in the North are sufficiently abundant or as long as the productivity of innovation is sufficiently high to satisfy the condition, then raising patent protection in the South to the level enjoyed in the North is a Pareto-improving policy irrespective of the distribution of assets between the North and the South. To establish Proposition 3, we assumed that Southern consumers initially held no assets. As shown by (21), holding no initial assets implies that no assets are ever held, which appears to imply that the proposition is dependent on restricting the distribution of assets. However, if the inequality in Proposition 5 holds, the strictest patent protection in the South is optimal, regardless of the distribution of assets between countries. That is, even if Southern consumers hold assets, the strictest patent protection in the South can maximize the welfare of Southern consumers.

The result given in Proposition 5 differs from that obtained by Grossman and Lai (2004). They showed that maximum patent protection tends to be suboptimal for the South. The difference between their results and ours is largely attributable to the difference in the size of the dynamic benefit of strengthening patent protection in the South. By using a quality-ladder type model, we assume that innovation occurs through the improvement of existing goods. In our model, the invention of a latest-generation good generates permanent social benefit by providing fundamental knowledge that is freely available to the R&D of that industry even after obsolescence. Having assumed that the invention of a good has no influence on other R&D, Grossman and Lai (2004) ignore this "shoulders-of-giants" effect in the R&D process. Taking this effect into account implies that strengthening patent protection in the South can generate dynamic benefits supplementary to those identified by Grossman and Lai because patent protection increases the fundamental knowledge available for future R&D by promoting innovation. Thus, in our model, stronger patent protection in the South can improve the welfare of the South if R&D resources in the North are sufficient and if R&D is sufficiently productive. Moreover, even patent harmonization, which requires maximum patent protection in the South, can improve welfare of the South.[30]

5 Discussion

5.1 R&D Subsidies and Patent Protection in the South

In Proposition 5, we concluded that, provided a certain parameter condition is satisfied, the strictest patent protection in the South can maximize welfare in both the North and the South. In this subsection, we briefly consider the possibility that the strictest patent protection is suboptimal for the South. Suppose that the Southern government implements patent protection that is weaker than the maximum, and does not cooperate with the Northern government. Can the Northern government implement a policy that induces the Southern government to maximize its patent protection in this case?

One policy option is for the North to raise R&D productivity, perhaps by improving higher education. Higher education is important for training researchers who develop new products. Improvements in higher education that successfully raise researchers' abilities to do R&D will increase the produc-

[30] If one introduces into our model imitation by Southern firms of multinationals' technology, our welfare results might change for the following three reasons. First, in such a model, strengthening patent protection in the South would affect welfare through channels other than (26) because it would be bound to affect the progress of imitation and the measure of imitated products. Second, imitation by Southern firms would change the magnitudes of the five welfare effects derived in our model. Third, introducing imitation into our model would generate transitional dynamics, as explained in footnote 20. Thus, if imitation activities are sufficiently pervasive and convergence to the steady state is sufficiently slow, our welfare results might change.

tivity of R&D in the economy. According to our model, an improvement in the productivity of R&D, which is represented by a decrease in a_N, reinforces the innovation-enhancing effect. Hence, such an improvement can reduce the stringency of the parameter constraint that must be satisfied if the strongest patent protection is to maximize the welfare of Southern consumers, as shown in Proposition 5. Therefore, a policy that increases R&D productivity, including an improvement in higher education, gives the Southern government an incentive to maximize its patent protection.

Another policy option to induce the South to enact maximum patent protection is to subsidize R&D. We can analyze the effect of an R&D subsidy by somewhat modifying our model.[31] Suppose that the Northern government subsidizes entrepreneurs by $100 \times s_R$ percent of R&D costs, where $s_R \in [0, 1)$. We assume that the subsidies are financed by a lump-sum tax levied on Northern consumers, and assume that the Northern government runs a balanced budget at each point in time. Under these assumptions, strengthening patent protection in the South still affects the welfare of the South through the five channels shown in (26). Among those channels, the innovation-enhancing effect tends to be represented by an inverted U-shaped function of s_R: the effect strengthens as s_R increases up to a certain point, after which it weakens. In addition, the R&D subsidy exacerbates the negative competition-reducing effect. However, subsidizing R&D mitigates the negative effect of the combined nominal spending and marginal cost effects, and also increases the positive FDI-promoting effect. Thus, if the positive effects of introducing an R&D subsidy on $dU_S(\beta)/d\beta$ outweigh the negative effects, then introducing an R&D subsidy may make the strongest patent protection optimal for the South.

5.2 Reexamination in a Nonscale Effect Model

In this subsection, we introduce population growth into the model and examine how our basic results would change. In practice, population grows in many countries. However, so far, we have assumed that the quantity of labor supplied is constant over time. In addition, the model developed in the preceding sections exhibits a scale effect; that is, an increase in the size of the labor force raises the innovation rate and the growth rate of utility. Therefore, in line with the settings used by Segerstrom (1998), who developed a closed economy quality-ladder model without a scale effect, we incorporate positive population growth into our model and examine the robustness of our main results.

We first describe the settings of the nonscale effect model and derive the effects of strengthening

[31] In Appendix F, we analyze the effect of an R&D subsidy in our model.

patent protection on innovation and FDI. We retain the notation used and the assumptions made in the original model if possible. Following earlier studies, which use two-country nonscale effect models, such as Dinopoulos and Segerstrom (2010), we assume that the labor population grows at the same constant rate, n, in both the North and South, and that $0 < n < \rho$. We let $\bar{L}_N$ and $\bar{L}_S$ denote the initial levels of population in the North and South, respectively. Letting L_t denote the total world population at time t, we can represent L_t as $L_t = (\bar{L}_N + \bar{L}_S)e^{nt}$. Further, we define $\tilde{L}_i$ as the ratio of the population of country $i \in \{N, S\}$ to the world population; that is, $\tilde{L}_i = \bar{L}_i/(\bar{L}_N + \bar{L}_S)$. Suppose that the initial measure of members of each household in country $i \in \{N, S\}$ is unity; in other words, the measure of households in country $i \in \{N, S\}$ equals $\bar{L}_i$. We assume that the number of members of each household grows at the constant rate n in both countries. Then, the lifetime utility of each representative household in country $i \in \{N, S\}$ is given by $U_i = \int_0^\infty e^{-(\rho-n)t} \log u_{i,t} dt$. The modified budget constraint is $\int_0^\infty e^{-\int_0^t (r_s - n)ds} E_{i,t} dt = A_{i,0} + \int_0^\infty e^{-\int_0^t (r_s - n)ds} w_{i,t} dt$. Thus, the demand for each product and the Euler equation for each household are the same as those in Section 2.1. We let c_t denote per capita global spending; that is, $c_t \equiv E_t/L_t$. In Section 5.2, unlike in the original model, we use the per capita global spending as the numeraire and normalize $c_t = E_t/L_t = E_{N,t}\tilde{L}_N + E_{S,t}\tilde{L}_S = 1$ for all t so that r_t always equals ρ. Hence, the per capita spending of households in country i, $E_{i,t}$, is constant for all t.

In order to remove the scale effect from the original model, we need to change an assumption on the cost of R&D. Following Segerstrom (1998), we assume that R&D costs increase with the aggregate volume of R&D over time. Concretely, we replace the cost of R&D, a_N, with $a_N X_t(\omega)$, where $X_t(\omega)$ represents the difficulty of doing R&D in industry ω, which is assumed to evolve as follows:

$$\frac{\dot{X}_t(\omega)}{X_t(\omega)} = \mu I_t(\omega), \tag{32}$$

where μ is a parameter that affects the growth rate of the difficulty of doing R&D: an increase in μ implies an increase in the growth rate of the difficulty of doing R&D. Under this assumption, the innovation rate and the growth rate of the utility level do not depend on the population level in the long run as shown below.[32]

Under the assumption of increasing difficulty of R&D, the free entry condition for R&D, (4), changes to $v_{N,t}(\omega) = w_{N,t} a_N X_t(\omega)$, where $v_{N,t}(\omega)$ means that the stock values are not necessarily symmetrical among industries ex ante. However, assuming that the R&D difficulty index, $X_t(\omega)$, is symmetrical across industries at the initial point in time makes the innovation rate $I_t(\omega)$ symmetrical across industries

[32] The earlier nonscale effect models including Segerstrom (1998) exhibit this property.

and, thus, $v_{N,t}(\omega)$ is also symmetrical because of the no-arbitrage condition. As a result, no variable depends on ω in equilibrium. Therefore, we henceforth omit ω from the equilibrium conditions, including the free entry condition for R&D:

$$v_{N,t} = w_{N,t} a_N X_t. \tag{33}$$

The other equilibrium conditions are as follows. Because of the change in the normalization, the profits of Northern firms and multinationals, respectively, change to $\pi_{N,t} = [1 - w_{N,t}/(\lambda w_{S,t})] L_t$ and $\pi_{F,t} = (1 - 1/\beta) L_t$. The no-arbitrage conditions for the stocks of Northern leaders and multinationals, (6) and (7), respectively, remain unchanged. Thus, the equilibrium condition for FDI, (10), holds, and we thereby obtain the equilibrium wage rate in the South as (11). The labor market equilibrium conditions in the South and the North are $n_{F,t} L_t/(\beta w_{S,t}) = L_{S,t}$ and $n_{N,t} L_t/(\lambda w_{S,t}) + a_N X_t I_t = L_{N,t}$.

Although incorporating increasing R&D costs purges the model of the scale effect, it also introduces transitional dynamics into the model. This complicates analysis of the effects of policy changes because one must consider their effects on the transition. However, by linearizing the market equilibrium path around the balanced growth path (BGP), we can show that strengthening patent protection does raise innovation and the growth rate on the transitional path, while it does not affect them in the long run.

Proposition 6. *We consider the nonscale effect model. Suppose that the economy is initially on the BGP. Strengthening patent protection marginally in the South promotes innovation in the short run, although the positive effect approaches zero in the long run. Moreover, strengthening patent protection marginally in the South promotes FDI in both the short run and the long run.*

Proof. See Appendix A. □

Using the results of the positive analysis, we now conduct welfare analysis in the nonscale effect model. To examine analytically how strengthening patent protection affects welfare, we adopt an approach similar to the one used by Helpman (1993). We assume that, when patent breadth is changed, the economy is on the BGP. We then evaluate the effect on welfare of Northern and Southern households of a marginal increase in β. The instantaneous utility of any household in each country remains unchanged as in the original model: $\log u_{i,t} = (\log \lambda) \int_0^t I_\tau d\tau + \log E_i - \log w_{S,t} + (\log \lambda - \log \beta) n_{F,t} - \log \lambda$. Because instantaneous utility comprises five parts, we can decompose the total welfare effect into the following five parts as we did in the original model:

$$\frac{dU_i(\beta)}{d\beta} = \int_0^\infty e^{-(\rho-n)t}\Big\{ \underbrace{(\log\lambda)\frac{d}{d\beta}\left(\int_0^t I_\tau d\tau\right)}_{\substack{\text{innovation-enhancing effect}\\(+)}} \underbrace{+\frac{d\log E_i}{d\beta}}_{\substack{\text{nominal spending effect}\\(+)\text{ or }(-)}}$$
$$+\Big[\underbrace{(\log\lambda-\log\beta)\frac{dn_{F,t}}{d\beta}}_{\substack{\text{FDI-promoting effect}\\(+)}} \underbrace{-\frac{d\log w_{S,t}}{d\beta}}_{\substack{\text{marginal cost effect}\\(+)\text{ or }(-)}} \underbrace{-\frac{n_{F,t}}{\beta}}_{\substack{\text{competition-reducing effect}\\(-)}} \Big]\Big\}dt. \tag{34}$$

Each of these effects corresponds to a welfare effect in the original model which exhibits scale effects, (26). As proved in Appendix B, the signs of some of the welfare effects are determinate as shown in (34).

As was the case for the original model, by showing that the innovation-enhancing effect outweighs the competition-reducing effect and the combined negative effect of the nominal spending and marginal cost effects, we can show that strengthening patent protection improves the welfare of Northern and Southern households. The following proposition summarizes the results of our welfare analysis of the nonscale effect model.[33]

Proposition 7. *We consider the nonscale effect model. Suppose that the economy is initially on the BGP. Strengthening patent protection marginally in the South raises the welfare of households in both North and South for all* $\beta \in (1,\lambda]$*, if the parameters satisfy either (i):* $\frac{n}{\rho}\frac{\log\lambda}{\mu} > \Omega$*, or (ii):* $\tilde{L}_S < \left[\frac{\lambda}{\lambda-1}\frac{n}{\rho}\frac{\log\lambda}{\mu} - \Omega\right]\left[\Omega - \frac{n}{\rho}\frac{\log\lambda}{\mu}\right]^{-1}\Omega^{-1}$*, where* $\Omega \equiv \frac{\lambda+\mu[(\rho/n)-1]}{1+\mu[(\rho/n)-1]}$.

Proposition 7 implies that when the South's population share is lower (and the North's is higher), strengthening patent protection in the South tends to improve the welfare of the Northern and Southern households. The reason is similar to that in the original model; the higher is the North's share of the labor population, the more likely is the positive innovation-enhancing welfare effect to outweigh the negative competition-reducing welfare effect. This implies that the welfare effects of patent protection in the South are robust to the incorporation of a nonscale effect.[34]

[33] See Appendix H for the proof.

[34] As Şener (2008) pointed out, the policy results may depend on how the scale effect is purged from the model. According to Şener (2008), there are three main approaches to removing scale effects: incorporating diminishing technological opportunities, as undertaken by Segerstrom (1998); incorporating rent protection activities, as proposed by Dinopoulos and Syropoulos (2007); and incorporating variety expansion, as undertaken by, for example, Dinopoulos and Thompson (1998). We adopt the first of

6 Conclusion

We developed a North–South quality-ladder model, in which foreign direct investment (FDI) is the main channel of technology transfer. We then conducted not only a positive analysis but also a welfare analysis. Despite the fact that welfare analysis is crucial for assessing policies, few previous theoretical studies on intellectual property rights (IPR) protection in developing countries have conducted such an analysis. However, by focusing our analysis on patent breadth, we examined analytically how strengthening patent protection in the South affects welfare in the South. We showed that strengthening patent protection can raise welfare not only in the North but also in the South.

This result contrasts with that of Helpman (1993), whose pioneering study examined the effect of stronger IPR protection on welfare. Helpman concluded that stronger IPR protection in the South necessarily damages welfare in the South, regardless of whether the mode of technology transfer is illegal imitation or FDI. However, our results differed markedly from Helpman's in this respect. Thus, our result provides a theoretical basis for strengthening patent protection in the South.

To simplify the analysis and obtain clearer results, we abstracted from two factors whose incorporation would make the model more realistic. The first is the costs of enforcing patent protection. In practice, resources are required to enforce patent protection. For instance, the government must allocate labor to institutions applying laws dealing with patent infringements. Hence, increasing patent breadth requires more labor resources. Incorporating the costs associated with patent protection might result in our finding that strengthening patent protection improves Southern welfare being subject to more stringent restrictions. The second factor is trade barriers between North and South. Trade barriers, in the form of tariffs, for example, are expected to affect both the benefits and costs of stronger patent protection in the South. For example, the existence of tariffs might induce tariff-jumping FDI from the North and might intensify the positive FDI-promoting welfare effect of strengthening patent protection. Moreover, increasing the number of multinationals by strengthening patent protection might reduce the South's tariff revenue, thereby reducing the welfare of Southern consumers. Incorporating tariffs into the model might also have other effects that are not easily predictable or obvious.[35] Although these extensions are

these approaches. Adopting either of the other two approaches is beyond the scope of the present paper. However, examining the effect of these other approaches on the results is worthwhile.

[35]Related studies examined the effect of trade liberalization using a North–South quality-ladder model. By extending the symmetric North–North model of Dinopoulos and Segerstrom (1999) to a North–South model, Grieben (2005) examined how Northern and Southern tariffs affect wage inequality in the North. Grieben and Şener (2009) examined how tariff reductions

beyond the scope of the present paper, they are worth examining.

Appendix A: Proof of Proposition 6

In this appendix, we prove Proposition 6.

To describe the equilibrium path, we define the following two variables: $z_t \equiv L_t/(a_N X_t)$ and $y_t \equiv L_t/v_t$, each of which is constant on the BGP. Note that z_t is a state variable, whereas y_t is a jump variable. From (11) and (33), the wage rates in the North and the South are represented by using z_t and y_t:

$$w_{N,t} = \frac{z_t}{y_t} \quad \text{and} \quad w_{S,t} = \frac{\beta}{\lambda}\frac{z_t}{y_t}. \tag{35}$$

In addition, from (35) and the labor market equilibrium conditions, the innovation rate and the measure of multinationals in the market equilibrium are expressed by using z_t and y_t as follows:

$$I_t = \tilde{L}(\beta)z_t - \frac{1}{\beta}y_t, \tag{36}$$

$$n_{F,t} = \frac{\beta^2}{\lambda}\tilde{L}_S\frac{z_t}{y_t}, \tag{37}$$

where $\tilde{L}(\beta) \equiv \tilde{L}_N + (\beta/\lambda)\tilde{L}_S$. Given (7), (32), (36), and (37), the market equilibrium path in this nonscale effect model can be characterized by the dynamic system for z_t and y_t as follows:

$$\frac{\dot{z}_t}{z_t} = n - \mu\left[\tilde{L}(\beta)z_t - \frac{1}{\beta}y_t\right], \tag{38}$$

$$\frac{\dot{y}_t}{y_t} = -\tilde{L}(\beta)z_t + y_t - (\rho - n). \tag{39}$$

All of the other endogenous variables are determined by the values of z_t and y_t. From (38) and (39), z_t and y_t are constant on the BGP and satisfy:

$$z = \frac{1}{(\beta-1)\tilde{L}(\beta)}\left(\frac{\beta n}{\mu} + \rho - n\right) \quad \text{and} \quad y = \frac{\beta}{\beta-1}\left(\frac{n}{\mu} + \rho - n\right), \tag{40}$$

where z and y are the values on the BGP. As in the original model, terms without the subscript "t" represent values on the BGP.

By conducting comparative statics, we can derive the long-run effects of an increase in β on I_t and $n_{F,t}$. Suppose that the economy is on the BGP until patent protection is changed at time 0. From (36) and

affect Northern innovation and Southern imitation in a North–South model in which Northern monopolists engage in rent-protection activities, which were originally proposed by Dinopoulos and Syropoulos (2007).

(40), on the BGP, we obtain $I = n/\mu$, which is independent of β. Thus, strengthening patent protection does not affect the long-run innovation rate. Meanwhile, differentiating (40) with respect to β yields:

$$z_\beta \equiv \frac{dz}{d\beta} = -\frac{y}{\beta(\beta-1)\tilde{L}(\beta)} - \frac{\tilde{L}_S}{\lambda\tilde{L}(\beta)}z < 0, \tag{41}$$

$$y_\beta \equiv \frac{dy}{d\beta} = -\frac{y}{\beta(\beta-1)} < 0. \tag{42}$$

By using (37), (41), and (42), we obtain the effect of strengthening patent protection on n_F as follows: $dn_F/d\beta = n_F\left\{\tilde{L}_N/[\beta\tilde{L}(\beta)] + n/[\mu(\beta-1)\tilde{L}(\beta)z]\right\} > 0$, where we use the relationship that $y = \beta\tilde{L}(\beta)z - (\beta n/\mu)$ on the BGP from (38). Hence, strengthening patent protection in the South increases the measure of multinationals in the long run.

Unlike the original model, the nonscale effect model has transitional dynamics because z_t is a state variable. Therefore, we must take the effects on the transition into consideration to evaluate the overall effects of a policy. To do so, we derive the linearized system of z_t and y_t in the neighborhood of the BGP and compute the transition paths of I_t and $n_{F,t}$. The linearized system of (38) and (39) is given by:

$$\begin{pmatrix} \dot{z}_t \\ \dot{y}_t \end{pmatrix} = \begin{pmatrix} -\mu\tilde{L}(\beta)z & \frac{\mu}{\beta}z \\ -\tilde{L}(\beta)y & y \end{pmatrix} \begin{pmatrix} z_t - z \\ y_t - y \end{pmatrix}. \tag{43}$$

Let J denote the Jacobian matrix of the dynamic system on the RHS of (43). The determinant of J is negative as follows: $\det J = -(\beta-1)\mu\tilde{L}(\beta)zy/\beta < 0$. Therefore, one characteristic root is negative and the other is positive and, thus, the BGP is a saddle point. Because y_t is a jump variable, whereas z_t is a state variable, the market equilibrium path is uniquely determined in this nonscale effect model. Moreover, we can show that the negative root of the characteristic equation is smaller than $-n$. We let ν denote the negative characteristic root and we let $h = [1, \Lambda]^T$ denote the characteristic vector corresponding to ν. Solving $Jh = \nu h$ for Λ yields $\Lambda = \beta[\nu/(\mu z) + \tilde{L}(\beta)]$. Using the characteristic root and vector, we obtain the market equilibrium path, including the transition, as follows: $z_t = z + (z_0 - z)\,e^{\nu t}$ and $y_t = y + (z_0 - z)\,\Lambda e^{\nu t}$. By differentiating these expressions with respect to β, we can describe the responses of z_t and y_t to a marginal increase in β as the following functions of time t:

$$\frac{dz_t}{d\beta} = z_\beta\left(1 - e^{\nu t}\right), \tag{44}$$

$$\frac{dy_t}{d\beta} = y_\beta - z_\beta\Lambda e^{\nu t}, \tag{45}$$

where $z_0 = z$ is used because we assume that the economy is on the BGP at the initial point in time and that z_t is not jumpable.

Finally, we derive the complete paths of the effects of a change in β on I_t and $n_{F,t}$ from the paths of z_t and y_t. By using (36), (44), and (45), we obtain: $dI_t/d\beta = (\tilde{L}_S z/\lambda) + \tilde{L}(\beta) z_\beta + (y/\beta^2) - (y_\beta/\beta) + \left[-\tilde{L}(\beta) + (\Lambda/\beta)\right] z_\beta e^{\nu t}$. By substituting (41), (42), and $\Lambda = \beta[\nu/(\mu z) + \tilde{L}(\beta)]$ into this equation, we obtain:

$$\frac{dI_t}{d\beta} = (-\nu)\frac{-z_\beta}{\mu z} e^{\nu t} > 0. \tag{46}$$

This shows that a rise in β enhances innovation in the short run, although the positive effect vanishes in the long run because $\nu < 0$. Likewise, by using (37), (38), (41), (42), (44), and (45), we obtain:

$$\frac{dn_{F,t}}{d\beta} = n_F \left[\frac{\tilde{L}_N}{\beta \tilde{L}(\beta)} + \frac{n}{\mu(\beta - 1)\tilde{L}(\beta) z} + (y - z\Lambda)\frac{-z_\beta}{zy} e^{\nu t}\right]. \tag{47}$$

The first two terms on the RHS of (47) represent the long-run effect on $n_{F,t}$, which is positive. Moreover, from $\Lambda = \beta[\nu/(\mu z) + \tilde{L}(\beta)]$ and (40), we obtain $y - z\Lambda = -\beta(\nu + n)/\mu$. Because $\nu < -n$ as mentioned above, $y - z\Lambda > 0$ holds. This implies that the last term on the RHS of (47) is positive, and thus, a rise in β promotes FDI in the short run as well as in the long run.

Appendix B: The Welfare Effects in the Nonscale Effect Model

In this appendix, we evaluate each of the constituent welfare effects in (34).

First, we can derive the magnitude of the welfare effect through enhancing innovation from (46) as follows: $\int_0^\infty e^{-(\rho-n)t}(\log\lambda)\left[d\left(\int_0^t I_\tau d\tau\right)/d\beta\right] dt = \{[1/(\rho-n)] - 1/(\rho-n-\nu)\}(\log\lambda)(-z_\beta)/(\mu z) > 0$. An increase in β enhances innovation in the short run. Thus, the innovation-enhancing effect is positive in the nonscale effect model, as in the original model.

Second, we evaluate the welfare effect through nominal spending. Because per capita spending is constant over time in each country, the intertemporal budget constraint can be reduced to $E_i = (\rho - n)(A_{i,0} + W_i)$, where $W_i = \int_0^\infty e^{-(\rho-n)t} w_{i,t} dt$ for $i \in \{N, S\}$. Global total initial asset holdings, A_0, equal the total volume of stocks at the initial point in time: $A_0 = v_0 \times (n_{N,0} + n_{F,0}) = L_0/y_0$. By using $\zeta \equiv A_{N,0}\bar{L}_N/A_0$, which denotes the share of assets held by Northern households at the initial time period, the initial asset holdings of a consumer in the North and South are given by $A_{N,0} = \zeta/(\tilde{L}_N y_0)$ and $A_{S,0} = (1-\zeta)/(\tilde{L}_S y_0)$, respectively. By substituting these expressions into E_i and then differentiating with respect to β, we obtain the following expression for the change in $\log E_i$ caused by an increase in β:

$$\frac{1}{E_i}\frac{dE_i}{d\beta} = (1-\phi_i)\left(-\frac{1}{y}\frac{dy_0}{d\beta}\right) + \phi_i \frac{1}{W_i}\frac{dW_i}{d\beta}, \tag{48}$$

where $\phi_i \in [0,1]$ denotes the ratio of labor income to total household wealth in country i; that is, $\phi_i = W_i/[E_i/(\rho - n)]$. In (48), we use $y_0 \simeq y$ because the change in the level of a variable following a marginal increase in β is infinitesimal. Because the change in $\log E_i$ is constant over time, the magnitude of the nominal spending effect is given by the RHS of (48) multiplied by $1/(\rho - n)$, the sign of which is ambiguous.

Third, we evaluate the welfare effect through promoting FDI. Given (47), a rise in β increases $n_{F,t}$ at each point in time from period 0. Therefore, the FDI-promoting effect is clearly nonnegative.

Fourth, we evaluate the welfare effect through raising marginal cost. By using (35), (41), (42), (44), and (45), we obtain: $(1/w_S)(dw_{S,t}/d\beta) = (1/\beta) + (z_\beta/z) - (y_\beta/y) + [(y - z\Lambda)(-z_\beta)/(zy)]e^{\nu t} = \{\lambda\tilde{L}_N - \mu\,[(\rho/n) - 1]\,\tilde{L}_S\}\{\beta + \mu\,[(\rho/n) - 1]\}^{-1}(\lambda\tilde{L}_N + \beta\tilde{L}_S)^{-1} + [(y - z\Lambda)\,(-z_\beta)/(zy)]e^{\nu t}$. The first term on the RHS of this equation represents the long-run effect on $w_{S,t}$, the sign of which is ambiguous. The sign of the second term is positive. Therefore, the sign of $dw_{S,t}/d\beta$ is ambiguous. The welfare effect through raising marginal cost is given by $\int_0^\infty e^{-(\rho-n)t}(-d\log w_{S,t}/d\beta)dt = -(1/w_S)\int_0^\infty e^{-(\rho-n)t}(dw_{S,t}/d\beta)dt = -1/[(\rho - n)W_S](dW_S/d\beta)$. The sign of this effect is ambiguous.

Finally, we can derive the welfare effect through reducing competition simply as follows: $\int_0^\infty e^{-(\rho-n)t}(-n_{F,t}/\beta)dt = -n_F/[\beta(\rho - n)] < 0$. We have used $n_{F,t} = n_F$ because the change in $n_{F,t}$ following a marginal increase in β is infinitesimal. Thus, the competition-reducing effect is negative.

Appendix C: The Conditions for $0 < n_F < 1$ and $I > 0$

In this appendix, we show that we can guarantee $0 < n_F < 1$ when $(\lambda - 1)L_S < L_N + \rho a_N$ holds, and that $I > 0$ when $\beta > \beta_{min}$ holds. Moreover, we show that $\lambda > \beta_{min}$ if $(\lambda - 1)(L_N + L_S) > \rho a_N$.

First, we consider a parameter condition to exclude the equilibrium in which all firms possessing patents move to the South. If patent protection is sufficiently strong in the South, that is, if β is sufficiently high (or close to λ), all firms that succeed in innovation may choose to become multinationals and shift their production to the South. In this case, no firm engages in production in the North, so that the proportion of multinationals, $n_{F,t}$, is equal to unity. In order to exclude such an extreme case, we assume that $(\lambda - 1)L_S < L_N + \rho a_N$. Once we assume that the values of the parameters satisfy this inequality, n_F is less than unity even under the strictest patent protection in the South ($\beta = \lambda$) from (17).

By contrast, when patent protection is sufficiently weak in the South, that is, when β is sufficiently low, no firm can conduct R&D ($I_t = 0$). Specifically, from (18), innovation rate I takes a value of zero if and only if:

$$\beta \leq \beta_{min} \equiv 2^{-1}(1 - \lambda L_N/L_S) + \sqrt{2^{-2}(1 - \lambda L_N/L_S)^2 + \lambda(L_N + \rho a_N)/L_S}.$$

Furthermore, if $\beta_{min} > \lambda$, no firm conducts R&D even under the strictest patent protection in the South. In order to exclude such an extreme case, we assume that the values of the parameters satisfy $(\lambda - 1)(L_N + L_S) > \rho a_N$. Once we assume that the values of the parameters satisfy this inequality, I is positive, at least under $\beta = \lambda$ from (18).

Appendix D: Proof of Footnote 24

In this appendix, we prove the assertion in footnote 24. It is sufficient for the proof to show that $U_i(\lambda)$ is higher than $U_i(\bar{\beta})$ for any $\bar{\beta} \in [1, \beta_{min}]$ if $a_N < (\lambda - 1)(L_N + L_S)^2 / \{[(\lambda + 1)(L_N + L_S) + \lambda L_S]\,\rho\}$.

To verify this, we first compute the welfare of each consumer when there is no innovation; that is, $U_i(\bar{\beta})$ for $\bar{\beta} \in [1, \beta_{min}]$. If $I_t = 0$, then $n_{F,t} = 1 - \lambda L_N w_{S,t}$ holds from (9). Substituting this equation into (8), we obtain $w_S = 1/(\lambda L_N + \bar{\beta} L_S)$ and $n_F = \bar{\beta} L_S / (\lambda L_N + \bar{\beta} L_S)$ for $\bar{\beta} \in [1, \beta_{min}]$. This implies that $w_N = \lambda / [\bar{\beta}(\lambda L_N + \bar{\beta} L_S)]$ for $\bar{\beta} \in [1, \beta_{min}]$ because (11) holds even if $I_t = 0$. By substituting w_S and w_N into (20) and (21), we have:

$$E_N = \frac{\zeta(\bar{\beta} - 1)(\lambda L_N + \bar{\beta} L_S) + \lambda L_N}{\bar{\beta} L_N (\lambda L_N + \bar{\beta} L_S)},$$

$$E_S = \frac{(1 - \zeta)(\bar{\beta} - 1)(\lambda L_N + \bar{\beta} L_S) + \bar{\beta} L_S}{\bar{\beta} L_S (\lambda L_N + \bar{\beta} L_S)},$$

for $\bar{\beta} \in [1, \beta_{min}]$. Thus, from these equations and (25), we can compute $U_i(\bar{\beta})$ for $\bar{\beta} \in [1, \beta_{min}]$ as follows:

$$\begin{aligned} U_N(\bar{\beta}) &= \frac{1}{\rho} \left\{ \log \frac{\zeta(\bar{\beta} - 1)(\lambda L_N + \bar{\beta} L_S) + \lambda L_N}{\bar{\beta} L_N} \right. \\ &\qquad \left. + \frac{\bar{\beta} L_S}{\lambda L_N + \bar{\beta} L_S} \left(\log \lambda - \log \bar{\beta} \right) - \log \lambda \right\}, \end{aligned}$$

$$\begin{aligned} U_S(\bar{\beta}) &= \frac{1}{\rho} \left\{ \log \frac{(1 - \zeta)(\bar{\beta} - 1)(\lambda L_N + \bar{\beta} L_S) + \bar{\beta} L_S}{\bar{\beta} L_S} \right. \\ &\qquad \left. + \frac{\bar{\beta} L_S}{\lambda L_N + \bar{\beta} L_S} \left(\log \lambda - \log \bar{\beta} \right) - \log \lambda \right\}. \end{aligned}$$

Next, we derive the welfare of each consumer when $\beta = \lambda$; that is, $U_i(\lambda)$. Substituting (15) and (16) into (20) and (21) yields the equilibrium values of each consumer's spending when $\beta = \lambda$:

$$E_N = \frac{1}{L_N + L_S + \rho a_N} \left(\frac{\zeta \rho a_N}{L_N} + 1 \right), \tag{49}$$

$$E_S = \frac{1}{L_N + L_S + \rho a_N} \left[\frac{(1 - \zeta) \rho a_N}{L_S} + 1 \right]. \tag{50}$$

Therefore, substituting (16)–(18) and (49) into (25) yields:

$$U_N(\lambda) = \frac{1}{\rho} \left\{ \frac{\log \lambda}{\rho} \left[\frac{\lambda - 1}{\lambda} \frac{L_N + L_S}{a_N} - \frac{\rho}{\lambda} \right] + \log \left(\frac{\zeta \rho a_N}{L_N} + 1 \right) - \log \lambda \right\}.$$

Likewise, from (16)–(18), (25), and (50), we obtain:

$$U_S(\lambda) = \frac{1}{\rho}\left\{\frac{\log\lambda}{\rho}\left[\frac{\lambda-1}{\lambda}\frac{L_N+L_S}{a_N}-\frac{\rho}{\lambda}\right]+\log\left[\frac{(1-\zeta)\rho a_N}{L_S}+1\right]-\log\lambda\right\}.$$

Next, we compare $U_i(\bar{\beta})$ with $U_i(\lambda)$. By subtracting $U_N(\bar{\beta})$ from $U_N(\lambda)$, we have:

$$\begin{aligned} U_N(\lambda)-U_N(\bar{\beta}) &= \frac{1}{\rho}\left\{\frac{\log\lambda}{\rho}\left[\frac{\lambda-1}{\lambda}\frac{L_N+L_S}{a_N}-\frac{\rho}{\lambda}\right]+\log\left(\frac{\zeta\rho a_N}{L_N}+1\right)\right. \\ &\quad \left. -\log\frac{\zeta(\bar{\beta}-1)(\lambda L_N+\bar{\beta}L_S)+\lambda L_N}{\bar{\beta}L_N}-\frac{\bar{\beta}L_S}{\lambda L_N+\bar{\beta}L_S}\left(\log\lambda-\log\bar{\beta}\right)\right\}. \quad (51)\end{aligned}$$

Note that the sum of the second and third terms in (51) satisfies the following inequality:

$$\begin{aligned} &\log\left(\frac{\zeta\rho a_N}{L_N}+1\right)-\log\frac{\zeta(\bar{\beta}-1)(\lambda L_N+\bar{\beta}L_S)+\lambda L_N}{\bar{\beta}L_N} \\ &= \log\frac{\zeta\rho a_N\bar{\beta}+\bar{\beta}L_N}{\zeta(\bar{\beta}-1)(\lambda L_N+\bar{\beta}L_S)+\lambda L_N} \\ &\geq \log\frac{\left[\zeta(\bar{\beta}-1)(\lambda L_N+\bar{\beta}L_S)\bar{\beta}/\lambda\right]+\bar{\beta}L_N}{\zeta(\bar{\beta}-1)(\lambda L_N+\bar{\beta}L_S)+\lambda L_N} \\ &= \log\bar{\beta}-\log\lambda, \end{aligned}$$

where the inequality on the third line uses the fact that $\rho a_N \geq (\bar{\beta}-1)(\lambda L_N+\bar{\beta}L_S)/\lambda$ must hold because $(\beta-1)[L_N+(\beta/\lambda)L_S]/(\beta a_N)-(\rho/\beta)\leq 0$ for $\beta\leq\beta_{min}$ from (18). Thus, $U_N(\lambda)-U_N(\bar{\beta})$ must satisfy the following inequality for any $\bar{\beta}\in[1,\beta_{min}]$:

$$\begin{aligned} U_N(\lambda)-U_N(\bar{\beta}) &\geq \frac{1}{\rho}\left\{\frac{\log\lambda}{\rho}\left[\frac{\lambda-1}{\lambda}\frac{L_N+L_S}{a_N}-\frac{\rho}{\lambda}\right]+\log\bar{\beta}-\log\lambda\right. \\ &\quad \left. -\frac{\bar{\beta}L_S}{\lambda L_N+\bar{\beta}L_S}\left(\log\lambda-\log\bar{\beta}\right)\right\} \\ &= \frac{1}{\rho}\left\{(\log\lambda)\left[\frac{\lambda-1}{\lambda}\frac{L_N+L_S}{\rho a_N}-\frac{1}{\lambda}-1-\frac{\bar{\beta}L_S}{\lambda L_N+\bar{\beta}L_S}\right]\right. \\ &\quad \left. +(\log\bar{\beta})\left(1+\frac{\bar{\beta}L_S}{\lambda L_N+\bar{\beta}L_S}\right)\right\} \\ &> 0, \end{aligned}$$

where the last inequality uses the condition that $a_N<(\lambda-1)(L_N+L_S)^2/\left\{[(\lambda+1)(L_N+L_S)+\lambda L_S]\,\rho\right\}$. Therefore, the welfare of each Northern consumer is higher when $\beta=\lambda$ than when there is no innovation if $a_N<(\lambda-1)(L_N+L_S)^2/\left\{[(\lambda+1)(L_N+L_S)+\lambda L_S]\,\rho\right\}$.

We can also show that $U_S(\lambda)-U_S(\bar{\beta})$ is positive if the condition is satisfied. Subtracting $U_S(\bar{\beta})$ from $U_S(\lambda)$ yields:

$$\begin{aligned} U_S(\lambda)-U_S(\bar{\beta}) &= \frac{1}{\rho}\left\{\frac{\log\lambda}{\rho}\left[\frac{\lambda-1}{\lambda}\frac{L_N+L_S}{a_N}-\frac{\rho}{\lambda}\right]+\log\left[\frac{(1-\zeta)\rho a_N}{L_S}+1\right]\right. \\ &\quad \left. -\log\frac{(1-\zeta)(\bar{\beta}-1)(\lambda L_N+\bar{\beta}L_S)+\bar{\beta}L_S}{\bar{\beta}L_S}-\frac{\bar{\beta}L_S}{\lambda L_N+\bar{\beta}L_S}\left(\log\lambda-\log\bar{\beta}\right)\right\}. \quad (52)\end{aligned}$$

The sum of the second and third terms in (52) satisfies the following inequality:

$$
\begin{aligned}
&\log\left[\frac{(1-\zeta)\rho a_N}{L_S}+1\right]-\log\frac{(1-\zeta)(\bar\beta-1)(\lambda L_N+\bar\beta L_S)+\bar\beta L_S}{\bar\beta L_S}\\
&\quad=\log\frac{(1-\zeta)\rho a_N\bar\beta+\bar\beta L_S}{(1-\zeta)(\bar\beta-1)(\lambda L_N+\bar\beta L_S)+\bar\beta L_S}\\
&\quad\geq\log\frac{\left[(1-\zeta)(\bar\beta-1)(\lambda L_N+\bar\beta L_S)\bar\beta/\lambda\right]+\bar\beta L_S}{(1-\zeta)(\bar\beta-1)(\lambda L_N+\bar\beta L_S)+\bar\beta L_S}\\
&\quad>\log\bar\beta-\log\lambda,
\end{aligned}
$$

where the first inequality uses the fact that $\rho a_N \geq (\bar\beta-1)(\lambda L_N+\bar\beta L_S)/\lambda$ must hold because $(\beta-1)[L_N+(\beta/\lambda)L_S]/(\beta a_N)-(\rho/\beta)\leq 0$ for $\beta\leq\beta_{min}$ from (18). Thus, $U_S(\lambda)-U_S(\bar\beta)$ must satisfy the following inequality for any $\bar\beta\in[1,\beta_{min}]$:

$$
\begin{aligned}
U_S(\lambda)-U_S(\bar\beta) \;&>\; \frac{1}{\rho}\left\{\frac{\log\lambda}{\rho}\left[\frac{\lambda-1}{\lambda}\frac{L_N+L_S}{a_N}-\frac{\rho}{\lambda}\right]+\log\bar\beta-\log\lambda\right.\\
&\qquad\left.-\frac{\bar\beta L_S}{\lambda L_N+\bar\beta L_S}\left(\log\lambda-\log\bar\beta\right)\right\}\\
&=\; \frac{1}{\rho}\left\{(\log\lambda)\left[\frac{\lambda-1}{\lambda}\frac{L_N+L_S}{\rho a_N}-\frac{1}{\lambda}-1-\frac{\bar\beta L_S}{\lambda L_N+\bar\beta L_S}\right]\right.\\
&\qquad\left.+(\log\bar\beta)\left(1+\frac{\bar\beta L_S}{\lambda L_N+\bar\beta L_S}\right)\right\}\\
&>\; 0,
\end{aligned}
$$

where the last inequality uses the condition that $a_N<(\lambda-1)(L_N+L_S)^2/\{[(\lambda+1)(L_N+L_S)+\lambda L_S]\,\rho\}$. Thus, the welfare of each Southern consumer is higher when $\beta=\lambda$ than when there is no innovation, if $a_N<(\lambda-1)(L_N+L_S)^2/\{[(\lambda+1)(L_N+L_S)+\lambda L_S]\,\rho\}$.

We have confirmed that when the value of β is so low that no R&D is undertaken, consumer welfare is not maximized if $a_N<(\lambda-1)(L_N+L_S)^2/\{[(\lambda+1)(L_N+L_S)+\lambda L_S]\,\rho\}$.

Appendix E: The Sufficient Condition of L_S for Proposition 3

In this appendix, we show that the condition given in Proposition 3 holds irrespective of L_N, if L_S satisfies $L_S \geq \lambda\rho a_N/[(\log\lambda)(2\lambda-1)]$. From (17) and (19), we can show that $f(\lambda) = \frac{\log\lambda}{\rho}\frac{dI}{d\beta}\Big|_{\beta=\lambda} - \frac{n_F}{\lambda} = \frac{\log\lambda}{\rho}\left[\frac{1}{\lambda^2}\left(\frac{L_N}{a_N}+\rho\right)+\frac{L_S}{\lambda a_N}\right] - \frac{L_S}{L_N+L_S+\rho a_N}$ is an increasing function of $(L_N/a_N)+\rho$. Because we focus on the equilibrium in which $n_F < 1$, it follows that $(L_N/a_N)+\rho > (\lambda-1)L_S/a_N$ necessarily holds. Therefore, if $f(\lambda) \geq 0$ when $(L_N/a_N)+\rho = (\lambda-1)L_S/a_N$, then $f(\lambda)$ is necessarily positive for any value of $(L_N/a_N)+\rho$. Substituting $(L_N/a_N)+\rho = (\lambda-1)L_S/a_N$ into $f(\lambda)$, we find that $f(\lambda) \geq 0$ holds if $L_S \geq \lambda\rho a_N/\left[(\log\lambda)(2\lambda-1)\right]$.

Appendix F: Analysis of the Effects of an R&D Subsidy

In this appendix, we analyze how an R&D subsidy influences the welfare effect of strengthening patent protection in the South. To do this, we modify our model by incorporating the following three assumptions, as stated in the text. First, the Northern government subsidizes entrepreneurs by $100 \times s_R$ percent of R&D costs, where $s_R \in [0, 1)$. Second, the subsidies are financed by a lump-sum tax levied on Northern consumers. Third, the Northern government runs a balanced budget at each point in time. Taxation by the government changes the intertemporal budget constraint of the consumers in country i as follows:

$$\int_0^\infty e^{-\int_0^t r_s ds} E_{i,t} dt = A_{i,0} + \int_0^\infty e^{-\int_0^t r_s ds} w_{i,t} dt - \int_0^\infty e^{-\int_0^t r_s ds} T_{i,t} dt, \tag{53}$$

where $T_{i,t}$ is the amount of tax levied in country i at time t. Note that $T_{S,t}$ is equal to zero because Southern consumers are not taxed. To achieve the balanced budget, the Northern government must equalize its revenue and expenditure:

$$L_N T_{N,t} = s_R w_{N,t} a_N I_t, \tag{54}$$

where the LHS represents tax revenues and the RHS represents the subsidy payment, which is equal to s_R times the wage paid to workers engaging in R&D.

The subsidy influences the profitability of R&D because it decreases the costs of R&D for entrepreneurs. As a result, the following condition, which is a generalized version of (4), must be satisfied in equilibrium to ensure nonpositive profit in the R&D process:

$$v_{N,t} \leq (1 - s_R) w_{N,t} a_N \quad \text{with equality if} \;\; I_t > 0, \tag{55}$$

where the LHS represents the expected gain from R&D and the RHS represents the cost of R&D.

Because the zero-profit condition in the R&D sector is influenced by the subsidy, the values of the endogenous variables also depend on the subsidy rate. Hereafter, we focus on the equilibrium, in which the innovation rate, I_t, and production in the North, $n_{N,t} = 1 - n_{F,t}$, are strictly positive. From the zero-profit condition (55), the Northern wage is equal to $w_{N,t} = v_t / [(1 - s_R) a_N]$. Substituting this expression, (11), (12), and $n_{N,t} = 1 - n_{F,t}$ into (9) yields:

$$I_t = \frac{L_N + (\beta/\lambda) L_S}{a_N} - \frac{1 - s_R}{\beta v_t}. \tag{56}$$

From (3), (7), and (56), the dynamic equation for v_t is:

$$\dot{v}_t = \left[\frac{L_N + (\beta/\lambda) L_S}{a_N} + \rho \right] v_t + \frac{s_R}{\beta} - 1. \tag{57}$$

Because v_t is a jump variable, equation (57) shows that v_t must jump at the initial time period to the following steady-state value:

$$v = \frac{(\beta - s_R)a_N}{\beta[L_N + (\beta/\lambda)L_S + \rho a_N]}. \tag{58}$$

Therefore, from (11), (12), (55) and (56), the other endogenous variables immediately jump to their steady-state values, as is the case when $s_R = 0$. The steady-state values of $w_{N,t}$, and $w_{S,t}$ are:

$$w_N = \frac{\beta - s_R}{(1 - s_R)\beta[L_N + (\beta/\lambda)L_S + \rho a_N]}, \tag{59}$$

$$w_S = \frac{(\beta - s_R)/\lambda}{(1 - s_R)[L_N + (\beta/\lambda)L_S + \rho a_N]}, \tag{60}$$

which reduce respectively to (15) and (16) if $s_R = 0$. Furthermore, by substituting (58) and (60) into (12) and (56), we have the steady-state values of $n_{F,t}$ and I_t:

$$n_F = \frac{(\beta - s_R)(\beta/\lambda)L_S}{(1 - s_R)[L_N + (\beta/\lambda)L_S + \rho a_N]}, \tag{61}$$

$$I = \frac{\beta - 1}{\beta - s_R}\frac{L_N + (\beta/\lambda)L_S}{a_N} - \frac{(1 - s_R)\rho}{\beta - s_R}, \tag{62}$$

which are equal to (17) and (18) if $s_R = 0$.

Next, we compute the equilibrium value of spending, E_i. From (53) and $r_t = \rho$, E_i satisfies the following relationship:

$$E_i = \rho A_{i,0} + w_i - T_i, \tag{63}$$

where $T_S = 0$ and $T_N = s_R w_N a_N I/L_N$, both of which are constant over time. Summing the spending of all consumers yields:

$$E \equiv E_N L_N + E_S L_S = \rho A_0 + (L_N w_N + L_S w_S) - L_N T_N = 1, \tag{64}$$

where $A_0 \equiv A_{N,0}L_N + A_{S,0}L_S$, and the last equality holds because of the normalization that $E_t = 1$ for all t. By substituting (64) into (63), we can represent E_i as follows:

$$E_N = \zeta\frac{1 - w_S L_S}{L_N} + (1 - \zeta)(w_N - T_N),$$

$$E_S = (1 - \zeta)\frac{1 - (w_N - T_N)L_N}{L_S} + \zeta w_S, \tag{65}$$

where $\zeta \equiv A_{N,0}L_N/A_0$.

In this extended model, as in the original model, strengthening patent protection in the South affects the welfare of the South through the five channels shown in (26). First, increasing patent breadth increases welfare through enhancing innovation. The magnitude of this innovation-enhancing effect is given by:

$$\frac{\log\lambda}{\rho^2}\frac{dI}{d\beta} = \frac{\log\lambda}{\rho^2}\left[\frac{1-s_R}{(\beta-s_R)^2}\left(\frac{L_N}{a_N}+\rho\right)+\frac{L_S}{\lambda a_N}\frac{(\beta-s_R)(\beta-1)+\beta(1-s_R)}{(\beta-s_R)^2}\right] > 0. \tag{66}$$

Note that the magnitude of the innovation-enhancing effect tends to be an inverted U-shaped function of s_R. This can be shown by partially differentiating $dI/d\beta$ with respect to s_R as follows:

$$\frac{\partial^2 I}{\partial s_R \partial\beta} = \frac{-s_R\left[(2\beta-1)L_S+\lambda(L_N+\rho a_N)\right]+\beta L_S+(2-\beta)\lambda(L_N+\rho a_N)}{\lambda a_N(\beta-s_R)^3}. \tag{67}$$

Because $\beta < 2\beta - 1$ and $2-\beta < 1$, $\frac{\partial^2 I}{\partial s_R\partial\beta}$ is positive if and only if s_R is less than a critical value: $\hat{s}_R \equiv [\beta L_S + (2-\beta)\lambda(L_N+\rho a_N)]/\left[(2\beta-1)L_S+\lambda(L_N+\rho a_N)\right]$. Therefore, $dI/d\beta$ is increasing with s_R if $s_R < \hat{s}_R$, and is decreasing with s_R if $s_R > \hat{s}_R$.

Second, increasing patent breadth raises the welfare of the South through raising nominal spending. Third, increasing patent breadth decreases welfare through raising the marginal cost of production. To examine the magnitude of the sum of these two effects, it is useful to compute E_S/w_S. By substituting (54), (59), (60), and (62) into (65), we obtain:

$$\frac{E_S}{w_S} = 1 + (1-s_R)\frac{(1-\zeta)\lambda\rho a_N}{\beta L_S}. \tag{68}$$

Therefore, differentiating the logarithm of (68) with respect to β yields the sum of the nominal spending effect and the marginal cost effect as follows:

$$\frac{1}{\rho}\left(\frac{1}{E_S}\frac{dE_S}{d\beta}-\frac{1}{w_S}\frac{dw_S}{d\beta}\right) = -\frac{(1-s_R)(1-\zeta)}{[\beta L_S/(\lambda\rho a_N)+(1-s_R)(1-\zeta)]\beta\rho} < 0. \tag{69}$$

Equation (69) shows that $\frac{1}{\rho}\left(\frac{1}{E_S}\frac{dE_S}{d\beta}-\frac{1}{w_S}\frac{dw_S}{d\beta}\right)$ is increasing with s_R:

$$\frac{\partial}{\partial s_R}\left[\frac{1}{\rho}\left(\frac{1}{E_S}\frac{\partial E_S}{\partial\beta}-\frac{1}{w_S}\frac{\partial w_S}{\partial\beta}\right)\right] = \frac{(1-\zeta)L_S/(\lambda\rho a_N)}{[\beta L_S/(\lambda\rho a_N)+(1-s_R)(1-\zeta)]^2\rho} > 0. \tag{70}$$

Fourth, increasing patent breadth raises welfare through promoting FDI. By differentiating (61) with respect to β, we can derive the magnitude of the FDI-promoting effect:

$$\begin{aligned}\frac{(\log\lambda-\log\beta)}{\rho}\frac{d\bar{n}_F}{d\beta} &= \frac{(\log\lambda-\log\beta)(\beta/\lambda)L_S}{(1-s_R)[L_N+(\beta/\lambda)L_S+\rho a_N]\rho}\left[2-\frac{(\beta/\lambda)L_S+(s_R/\beta)(L_N+\rho a_N)}{L_N+(\beta/\lambda)L_S+\rho a_N}\right] \\ &\geq 0.\end{aligned} \tag{71}$$

By partially differentiating $d\bar{n}_F/d\beta$ with respect to s_R, we obtain:

$$\frac{\partial^2 \bar{n}_F}{\partial s_R \partial \beta} = \frac{(\beta/\lambda)L_S}{(1-s_R)^2[L_N + (\beta/\lambda)L_S + \rho a_N]}\left[2 - \frac{(\beta/\lambda)L_S + (1/\beta)(L_N + \rho a_N)}{L_N + (\beta/\lambda)L_S + \rho a_N}\right] > 0. \tag{72}$$

Therefore, the FDI-promoting effect is increasing with s_R.

Fifth, increasing patent breadth lowers welfare through the competition-reducing effect. From (61), the magnitude of the competition-reducing effect is given by:

$$-\frac{n_F}{\beta\rho} = -\frac{(\beta - s_R)(1/\lambda)L_S}{(1-s_R)[L_N + (\beta/\lambda)L_S + \rho a_N]\rho} < 0. \tag{73}$$

This equation shows that the competition-reducing effect is decreasing with s_R:

$$\frac{\partial}{\partial s_R}\left(-\frac{n_F}{\beta\rho}\right) = -\frac{(\beta - 1)(1/\lambda)L_S}{(1-s_R)^2[L_N + (\beta/\lambda)L_S + \rho a_N]\rho} < 0. \tag{74}$$

In summary, the effect of strengthening patent protection on welfare in the South is represented by substituting (66), (69), (71), and (73) into (26).

The marginal effect of the R&D subsidy on the welfare effect of stronger patent protection is rather complex and not necessarily monotonic. As shown in (67), the R&D subsidy increases the positive innovation-enhancing effect for $s_R < \hat{s}_R$, but decreases it for $s_R > \hat{s}_R$. In addition, as shown in (70) and (72), the R&D subsidy lessens the combined nominal spending and marginal cost effects, which are negative, but also increases the positive FDI-promoting effect. However, the subsidy exacerbates the negative competition-reducing effect, as shown in (74). The total marginal effect of the R&D subsidy is given by the net effect of these changes.

According to our numerical analysis, whether the introduction of an R&D subsidy gives the South an incentive to raise its patent protection to the maximum is ambiguous. For some parameter values, the introduction of an R&D subsidy might increase the marginal welfare effect of strengthening patent protection. For example, suppose that $a_N = 103$, $\lambda = 1.4$, $L_N = 1$, $L_S = 12$, $\rho = 0.05$, and $\zeta = 0.6$. In this case, $\frac{dU_S(\lambda)}{d\beta} < 0$ and the South's optimal patent breadth is less than λ if $s_R = 0$ (Figure 1). However, because $\left.\frac{\partial^2 U_S(\beta)}{\partial s_R \partial \beta}\right|_{s_R=0} > 0$ in this case, the Northern government can induce the South to strengthen its patent protection indirectly by introducing an R&D subsidy. In fact, if the Northern government subsidizes R&D by 10 percent of costs ($s_R = 0.1$), the sign of $\frac{dU_S(\lambda)}{d\beta}$ becomes positive and maximum patent protection becomes optimal for the South (Figure 2). By contrast, if $a_N = 85$, $\lambda = 2$, $L_N = 4$, $L_S = 0.5$, $\rho = 0.05$, and $\zeta = 0.7$, $\frac{dU_S(\lambda)}{d\beta} < 0$ for any $s_R \in [0, 1)$ (see Figure 3). Therefore, in this case, the strongest patent protection is suboptimal for the South no matter what the subsidy rate.

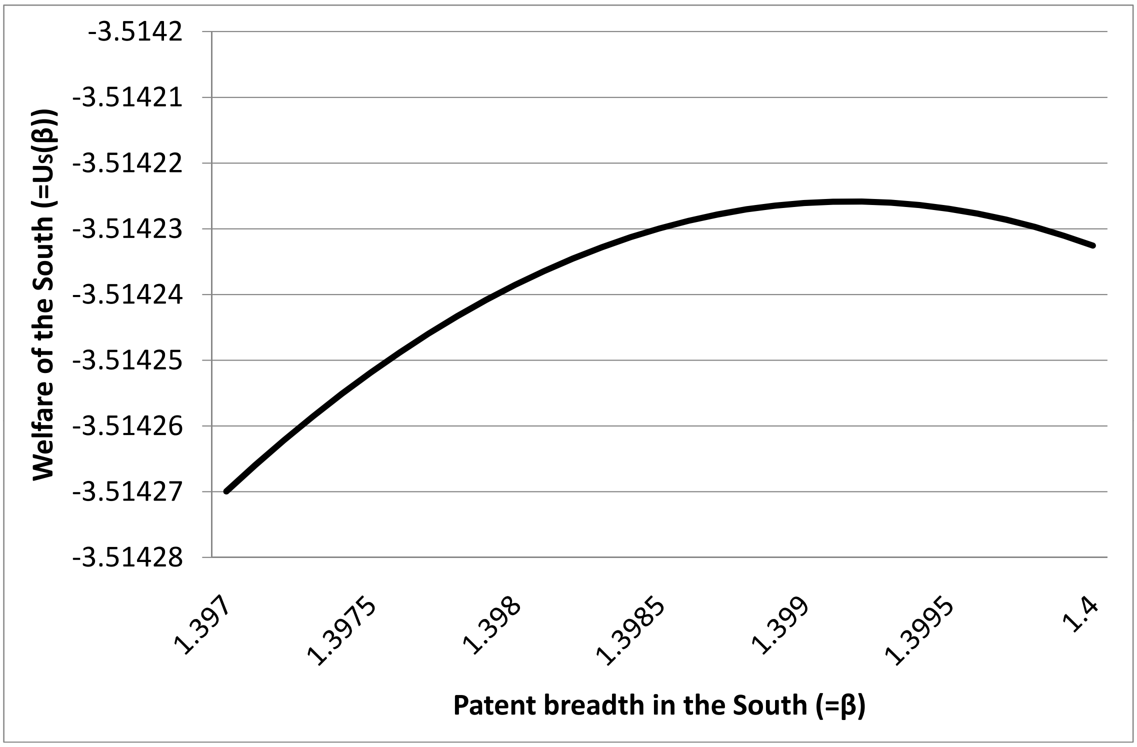

Figure 1: Patent breadth and the welfare of the South: the case of $a_N = 103$, $\lambda = 1.4$, $L_N = 1$, $L_S = 12$, $\rho = 0.05$, $\zeta = 0.6$, $s_R = 0$. Note that $\beta_{min} \simeq 1.397$ in this case.

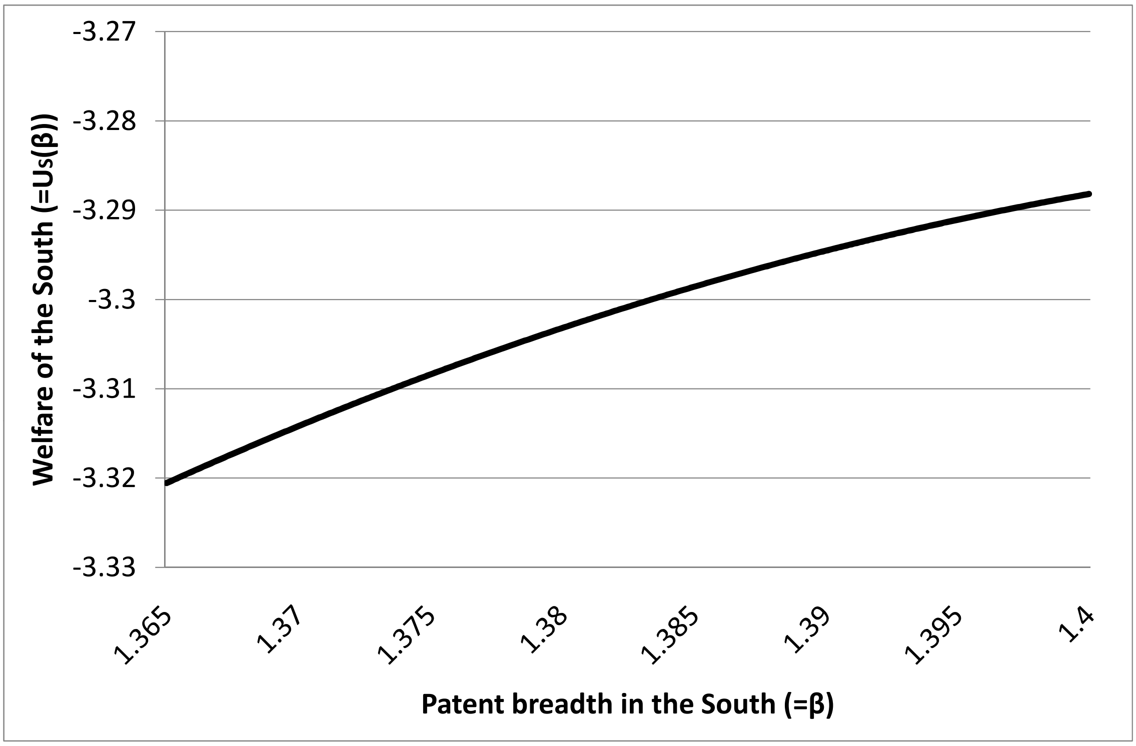

Figure 2: Patent breadth and the welfare of the South: the case of $a_N = 103$, $\lambda = 1.4$, $L_N = 1$, $L_S = 12$, $\rho = 0.05$, $\zeta = 0.6$, $s_R = 0.1$. Note that $\beta_{min} \simeq 1.365$ in this case.

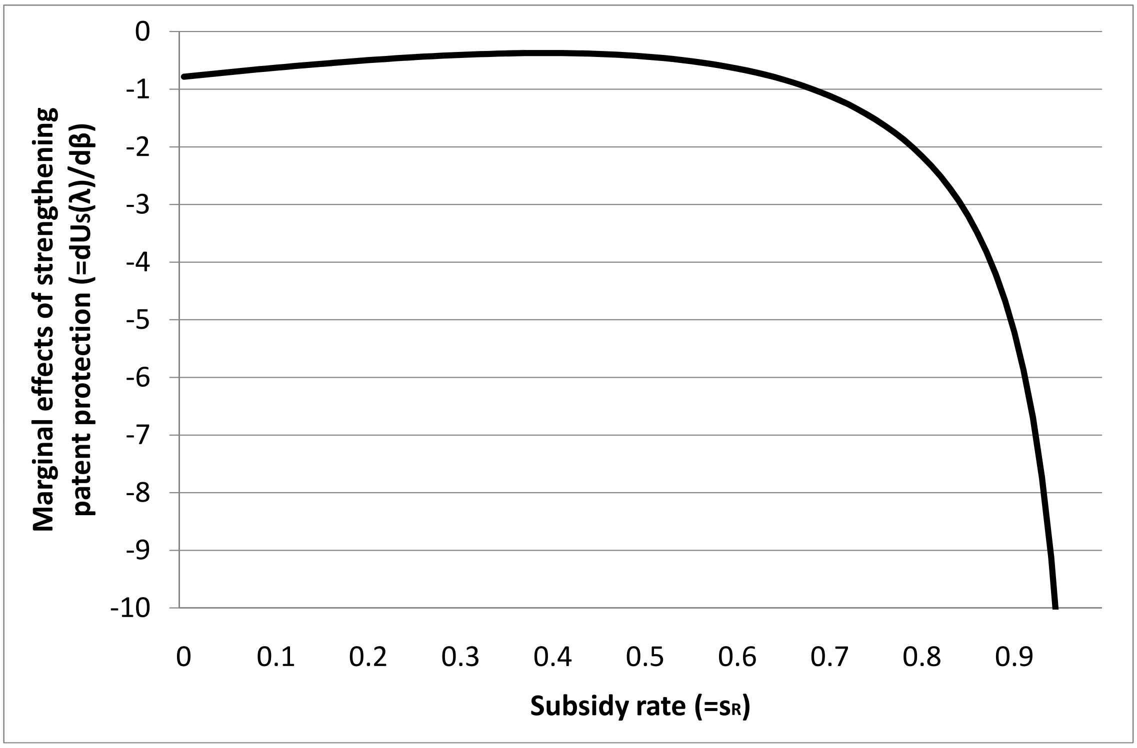

Figure 3: Subsidy rates and the marginal effects of changing patent breadth at the maximum patent protection $\beta = \lambda$: the case of $a_N = 85$, $\lambda = 2$, $L_N = 4$, $L_S = 0.5$, $\rho = 0.05$, $\zeta = 0.7$. It is required that $s_R < 0.939$ for $n_F < 1$.

Appendix G: Proof of $\nu < -n$

In this appendix, we show that $\nu < -n$. The characteristic equation of the Jacobian matrix, J, of the dynamic system (43) is given by the following quadratic equation:

$$F(x) \equiv x^2 - \left[-\mu\tilde{L}(\beta)z + y\right]x - \left(1 - \frac{1}{\beta}\right)\mu\tilde{L}(\beta)zy = 0.$$

By the definition of F, we have:

$$F(-n) = n^2 + \left[-\mu\tilde{L}(\beta)z + y\right]n - \left(1 - \frac{1}{\beta}\right)\mu\tilde{L}(\beta)zy. \tag{75}$$

From (38), $\mu\tilde{L}(\beta)z = n + (\mu y/\beta)$ holds on the BGP. Substituting this and (40) into (75) yields:

$$\begin{aligned} F(-n) &= n^2 + \left[-n - \frac{1}{\beta}\mu y + y\right]n - \left(1 - \frac{1}{\beta}\right)\left(n + \frac{1}{\beta}\mu y\right)y \\ &= -\left[(\mu - \beta)n + (\beta - 1)\left(n + \frac{1}{\beta}\mu y\right)\right]\frac{1}{\beta}y \\ &= -\left[(\mu - 1)n + \frac{\beta - 1}{\beta}\mu y\right]\frac{1}{\beta}y \\ &= -\left[n - \frac{n}{\mu} + \left(\frac{n}{\mu} + \rho - n\right)\right]\frac{1}{\beta}\mu y \\ &= -\rho\frac{1}{\beta}\mu y \\ &< 0. \end{aligned}$$

This result proves that $\nu < -n$ because ν is the negative root of the quadratic equation $F(x) = 0$.

Appendix H: Proof of Proposition 7

In this appendix, we prove Proposition 7 by deriving the parameter values at which the total welfare effect is positive.

For that purpose, we first decompose the sum of the nominal spending effect and the marginal cost effect into parts that can each be signed. Because $W_S = \beta W_N/\lambda$ holds from (35), we obtain $(1/W_S)(dW_S/d\beta) = (1/W_N)(dW_N/d\beta) + (1/\beta)$. Using this and (48), we can rewrite the sum of the nominal spending effect and the marginal cost effect for Northern households as follows:

$$\begin{aligned}&\int_0^\infty e^{-(\rho-n)t}\Big(\frac{d\log E_N}{d\beta} - \frac{d\log w_{S,t}}{d\beta}\Big)dt \\ &\qquad = \frac{1}{\rho-n}(1-\phi_N)\left(-\frac{1}{y}\frac{dy_0}{d\beta} - \frac{1}{W_N}\frac{dW_N}{d\beta}\right) - \frac{1}{\beta(\rho-n)}, \end{aligned} \tag{76}$$

where $0 \leq 1-\phi_N \leq 1$ by the definition of ϕ_i. Likewise, the corresponding expression for Southern households is:

$$\begin{aligned}&\int_0^\infty e^{-(\rho-n)t}\Big(\frac{d\log E_S}{d\beta} - \frac{d\log w_{S,t}}{d\beta}\Big)dt \\ &\qquad = \frac{1}{\rho-n}(1-\phi_S)\left(-\frac{1}{y}\frac{dy_0}{d\beta} - \frac{1}{W_N}\frac{dW_N}{d\beta}\right) - (1-\phi_S)\frac{1}{\beta(\rho-n)}, \end{aligned} \tag{77}$$

where $0 \leq 1-\phi_S \leq 1$. The second terms on the RHS of (76) and (77) are negative. Meanwhile, we can compute $(1/W_N)(dW_N/d\beta)$ in the first terms on the RHS of (76) and (77) as follows:

$$\begin{aligned}\frac{1}{W_N}\frac{dW_N}{d\beta} &= \frac{\rho-n}{w_N}\int_0^\infty e^{-(\rho-n)t}\frac{dw_{N,t}}{d\beta}dt \\ &= (\rho-n)\frac{y}{z}\int_0^\infty e^{-(\rho-n)t}\left[\frac{z_\beta}{y}\left(1-e^{\nu t}\right) - \frac{z}{y^2}\left(y_\beta - z_\beta\Lambda e^{\nu t}\right)\right]dt \\ &= \frac{z_\beta}{z}\left(1-\frac{\rho-n}{\rho-n-\nu}\right) - \frac{y_\beta}{y} + \frac{z_\beta}{y}\Lambda\frac{\rho-n}{\rho-n-\nu},\end{aligned}$$

where the second equality uses (35), (44) and (45). From this and (45), we have:

$$\begin{aligned}-\frac{1}{y}\frac{dy_0}{d\beta} - \frac{1}{W_N}\frac{dW_N}{d\beta} &= -\frac{1}{y}\left(y_\beta - z_\beta\Lambda\right) - \frac{z_\beta}{z}\left(1-\frac{\rho-n}{\rho-n-\nu}\right) + \frac{y_\beta}{y} - \frac{z_\beta}{y}\Lambda\frac{\rho-n}{\rho-n-\nu} \\ &= \left(y - z\Lambda\right)\frac{-z_\beta}{zy}\left(1-\frac{\rho-n}{\rho-n-\nu}\right) \\ &> 0,\end{aligned}$$

where the inequality holds because $y - z\Lambda = -\beta(\nu+n)/\mu > 0$ and $\nu < 0$. Consequently, the first terms on the RHS of (76) and (77) are positive. This implies that the sum of the nominal spending effect and the marginal cost effect is necessarily greater than or equal to $-1/[\beta(\rho-n)]$.

If the innovation-enhancing effect outweighs the sum of the competition-reducing effect and $1/[\beta(\rho-n)]$, the total welfare effect for Northern households is positive. This is because the FDI-promoting effect is nonnegative. Further, in this case, the total welfare effect for Southern households is also positive from (34) and (77). Hence, a rise in β improves welfare in both the North and the South if the following inequality holds:

$$\underbrace{\left(\frac{1}{\rho-n}-\frac{1}{\rho-n-\nu}\right)\left(\frac{\log\lambda}{\mu}\right)\left(\frac{-z_{\beta}}{z}\right)}_{\text{innovation-enhancing effect}} > \underbrace{\frac{n_F}{\beta(\rho-n)}}_{\text{competition-reducing effect}} + \underbrace{\frac{1}{\beta(\rho-n)}}_{\substack{\text{the negative part in the sum}\\ \text{of nominal spending effect}\\ \text{and marginal cost effect}}}. \quad (78)$$

Because $-n-\nu>0$ as shown in Appendix G, (78) holds if $\left(\frac{1}{\rho-n}-\frac{1}{\rho}\right)\left(\frac{\log\lambda}{\mu}\right)\left(\frac{-z_{\beta}}{z}\right) > \frac{n_F}{\beta(\rho-n)} + \frac{1}{\beta(\rho-n)}$. This inequality can be rewritten as:

$$\frac{n}{\rho}\frac{\log\lambda}{\mu} > \frac{n_F+1}{\beta}\left(\frac{z}{-z_{\beta}}\right). \quad (79)$$

Substituting (37), (40), and (41) into the RHS of (79) yields:

$$\frac{n_F+1}{\beta}\left(\frac{z}{-z_{\beta}}\right) = \left\{1-\left[\beta+(\beta-1)\,\frac{(\beta/\lambda)\tilde{L}_S}{\tilde{L}(\beta)}\Omega(\beta)\right]^{-1}\right\}\Omega(\beta),$$

where $\Omega(\beta)=\frac{\beta+\mu[(\rho/n)-1]}{1+\mu[(\rho/n)-1]}$. Because $\Omega(\beta)$ is an increasing function of β, the RHS of (79) is an increasing function of β. Therefore, if inequality (79) holds at $\beta=\lambda$, inequality (79) holds for all $\beta\in(1,\lambda]$. Consequently, we can show that a rise in β improves welfare in both the North and the South if (79) holds at $\beta=\lambda$.

Substituting $\beta=\lambda$ into (79) and then rewriting the resultant expression yields:

$$\frac{\Omega(\lambda)}{\lambda+(\lambda-1)\,\tilde{L}_S\Omega(\lambda)} > \Omega(\lambda)-\frac{n}{\rho}\frac{\log\lambda}{\mu}, \quad (80)$$

where we use $\tilde{L}(\lambda)=\tilde{L}_N+\tilde{L}_S=1$. This implies that inequality (79) holds irrespective of $\tilde{L}_S$ if

$$\frac{n}{\rho}\frac{\log\lambda}{\mu} > \Omega(\lambda). \quad (81)$$

If the parameters do not satisfy (81), we can rewrite (80) as follows:

$$\tilde{L}_S < \left[\frac{\lambda}{\lambda-1}\frac{n}{\rho}\frac{\log\lambda}{\mu}-\Omega(\lambda)\right]\left[\Omega(\lambda)-\frac{n}{\rho}\frac{\log\lambda}{\mu}\right]^{-1}[\Omega(\lambda)]^{-1}. \quad (82)$$

The set of values of $\tilde{L}_S$ that satisfies (82) becomes empty if λ, n/ρ, and μ do not satisfy $\lambda n(\log\lambda)/\left[(\lambda-1)\,\rho\right] > \mu\Omega(\lambda)$. However, assuming that μ is sufficiently low ensures that $\lambda n(\log\lambda)/\left[(\lambda-1)\,\rho\right] > \mu\Omega(\lambda)$ because $\mu\Omega(\lambda)=\mu+\frac{\lambda-1}{(1/\mu)+(\rho/n)-1}$ is an increasing function of μ.

Thus, if λ, n/ρ, and μ satisfy (81), or if $\tilde{L}_S$ satisfies (82), strengthening patent protection in the South improves the welfare of Northern and Southern households.

The result shows that the inequality is more likely to hold the lower is $\tilde{L}_S$. Why is this? From (46), the smaller is $\tilde{L}_S$, the weaker is the positive effect on innovation in the short run. This is because a decrease in $\tilde{L}_S$ lowers $-z_\beta/z$ from (40) and (41). Thus, the lower is $\tilde{L}_S$, the weaker is the innovation-enhancing effect. On the other hand, from (37) and (40), the smaller is $\tilde{L}_S$, the lower is the measure of multinationals, n_F and, thus, the weaker is the competition-reducing effect. Despite the fact that the smaller is $\tilde{L}_S$, the weaker are the positive and negative welfare effects, the weakening of the latter outweighs the weakening of the former. This can be verified by the fact that the RHS of (79) is an increasing function of $\tilde{L}_S$. Hence, the lower is $\tilde{L}_S$, the more the positive welfare effect outweighs the negative welfare effect.

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