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Abstract

This paper investigates the decision of an automaker concerning the alternative promotion of a hybrid vehicle (HV) and a full electric vehicle (EV). We evaluate the HV project by considering the option to change promotion from the HV to the EV in the future. The results not only extend previous findings concerning American options on multiple assets, but also include several new implications. One notable observation is that the increased market demand for EVs can accelerate the promotion of the HV because of the embedded option.

JEL Classifications Code: C61, G13, G31, O32.

Keywords: real options, American options on multiple assets, exercise region, alternative projects, hybrid and electric vehicles.

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1 Introduction

The global financial crisis beginning in 2007 has increased uncertainty about future market demand in many industries throughout the world. As a result, it has becoming increasingly important for firm project management to take into account uncertainty and flexibility in the future. The real options approach, in which option pricing theory is applied to capital budgeting decisions, better enables us to find an optimal investment strategy and undertake project valuation in this environment than is possible under more classical methods.

Using a real options approach, this paper investigates an automaker's decisions concerning investment timing and project choice. Recently, increased concerns about the environmental impact of gasoline fueled cars, along with increasing fuel prices and the Green New Deal, have promoted interest in hybrid and electric vehicles.[1] This is because both types of vehicles are perceived to be more environmentally friendly and cheaper to maintain and run than comparable gasoline fueled vehicles. For example, the share of HV sales in total monthly car sales in Japan reached 12% in May, 2009. However, although interest in HVs is now overwhelming, most experts predict that they are only a temporary development that will ultimately be replaced by EVs (see [4]). Under these circumstances, an automaker faces an urgent problem: namely, what type of vehicle, a HV or an EV, should it seek to promote, and when should it undertake the large-scale market promotion of the vehicle chosen? For example, Toyota is currently undertaking the substantial sales promotion of its HV, the Prius, while Nissan has announced that its EV, the Lief, will be for sale from 2010.

This paper models the investment timing and project choice decision of an automaker as follows. The automaker has an option to promote two alternatives[2], namely the HV and the EV, with sunk costs at some arbitrary timing. If and only if the automaker chooses the HV does it have the option to change its promotion from the HV to the EV with sunk costs. That is, the HV project is evaluated by taking into consideration the value of the option to replace the HV with the EV in the future. For brevity, we assume the cash flows from the HV and EV projects follow a bidimensional geometric Brownian motion (GBM). Then, the investment problem becomes a problem combining a max-call

[1] In this paper, EV denotes a full electric vehicle, while HV denotes a hybrid vehicle that employs both an engine and a battery. In turn, EVs are classified into several types depending on the type of battery. Accordingly, the analysis considers not only fuel cells (which arguably have far to go before commercialization in vehicles) but also lithium–ion batteries, currently one of the most promising battery technologies owing to its lightweight and power. See [4] for details.

[2] It is assumed that the firm cannot promote both projects at the same time. Although some automakers invest in both HV and EV technologies in the real world, most give emphasis to either HVs or EVs. For example, Toyota emphasizes HVs while Nissan emphasizes EVs.

option (studied by [7, 3, 12]) with a spread option (studied by [3, 12, 8]).[3] Indeed, the alternative project choice corresponds to the max-call option, while the replacement of HV with EV corresponds to the spread option.

For the combined problem, we show the analytical properties of the project value and the investment region. Along with a theoretical contribution to the literature on American options on multiple assets, the results lead to several implications concerning the automaker's strategy. One notable finding is that the investment region for the HV is not necessarily monotonic with respect to the market demand for EVs. This results from an interaction between the max-call option and the spread option embedded in the problem. In terms of the max-call option, increased market demand for EVs discourages the promotion of the HV because the EV becomes more favorable. Conversely, in terms of the spread option, increased market demand for EVs encourages the promotion of the HV because the value of the spread option to replace the HV with the EV increases. This tradeoff determines the investment region for the HV. In particular, we find that the latter effect (encouragement of the HV) can dominate the former effect (discouragement of the HV) if the market demand for EVs is small and the correlation between the EV and the HV is low or negative. This is consistent with the promotion strategies used by automakers such as Toyota and Honda that emphasize the promotion of HVs despite the imminent dominance of EVs.

Although this paper is intended to better understand the decision by automakers on investment timing and project choice, the use of this model is not restricted to a particular industry. For example, the model also applies, say, to a developer's decisions concerning the renovation and rebuilding of condominiums. That is, the developer accounts for not only the value of the renovated condominium but also the value of the option to rebuild it in the near future.

The paper is organized as follows. Section 2 introduces the setup and the preliminary results in two cases; the alternative choice between the HV and the EV (Section 2.1) and the replacement of the HV with the EV (Section 2.2). Section 3.1 shows several properties of the value function and the stopping region for the combined problem. Section 3.2 presents numerical results and implications concerning the investment timing and project choice decision of an automaker. Section 4 concludes the paper.

[3] Numerous studies propose new methods of computing the prices of American options on multiple assets, including max-call and spread options. However, other than [7, 8], there are few studies concerning their application to real options.

2 Preliminaries

Consider an automaker that has an option to invest in a project i. Consider two kinds of projects $i = 1$ (HV) and $i = 2$ (EV). When the firm conducts project i at time t with sunk cost $I_i(> 0)$, it receives a cash flow $X_i(t)$ for the ongoing project i. For analytical purposes, this analysis builds on the continuous time model with exogenous cash flows $X(t)$, which follows a bidimensional GBM

$$\mathrm{d}X_i(t) = \mu_i X_i(t)\mathrm{d}t + \sigma_i X_i(t)\mathrm{d}B_i(t), \tag{1}$$

where $(B_1(t), B_2(t))$ is a two-dimensional Brownian Motion (BM) with correlation coefficient ρ satisfying $|\rho| < 1$. The drift μ_i and the volatility $\sigma_i(> 0)$ represent the mean growth rate and the volatility of the cash flow from project i. As usual, we assume $\max(\mu_1, \mu_2) < r$ for convergence, where $r(> 0)$ denotes the constant discount rate. Mathematically, the model is built on the filtered probability space $(\Omega, \mathcal{F}, P; \mathcal{F}_t)$ generated by $(B_1(t), B_2(t))$. The set $\mathcal{F}_t$ is the available information set in time t, and a firm optimizes its investment strategy under this information. Let $T(> 0)$ denote the maturity of the option to choose HV or EV. Although we can theoretically take $T = \infty$ (a perpetual option), we assume recent circumstances oblige the automaker to make a short-term decision. Therefore, we take a finite T, set to 1 year in the numerical example. In preparation for the main results in Section 3, we present two earlier results in this section.

2.1 Choice among HV and EV

As a benchmark, we consider the case of no replacement of the HV with the EV. The automaker has an option to invest between the HV and the EV at an arbitrary time before maturity T. However, the firm cannot execute both projects. The option value at time $t(< T)$ with $X(t) = x \in \mathbb{R}^2_{++}$ is equal to the value function of the optimal stopping problem as follows:

$$V_A(x, t) = \sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}^x_t[\mathrm{e}^{-r(\tau - t)} \max\left(\frac{X_1(\tau)}{r - \mu_1} - I_1, \frac{X_2(\tau)}{r - \mu_2} - I_2, 0\right)], \tag{2}$$

where $\mathcal{T}_{t,T}$ denotes the set of all stopping times τ satisfying $\tau \in [t, T]$ and $E^x_t[\cdot]$ is the expectation conditional on $X(t) = x$. Throughout the paper, the subscript $_A$ denotes "Alternative choice." Problem (2) has been essentially investigated in [7, 3] (also refer to Section 6 in [6]).[4] The optimal stopping time $\tau_A(t)$ in problem (2) becomes:

$$\tau_A(t) = \inf\{s \geq t \mid X(s) \in \mathcal{S}_A(s) = \mathcal{S}_{A,1}(s) \cup \mathcal{S}_{A,2}(s)\}, \tag{3}$$

[4]In relation to problem (2), [5] investigated investment of different scales under a one-dimensional state variable. As an alternative, [11, 10] examined preemptive competition where two firms strategically preempt a project between two alternative projects using one-dimensional and bidimensional models, respectively.

where the stopping region $\mathcal{S}_{A,i}(s)$ is defined by:

$$\mathcal{S}_{A,i}(s) = \{x \in \mathbb{R}^2_{++} \mid V_A(x,s) = \frac{x_i}{r-\mu_i} - I_i\}. \tag{4}$$

If $X(t) \in \mathcal{S}_{A,i}(t)$, the firm makes immediate investment in project i. Both the value function $V_A(t)$ and the stopping region $S_A(t)$ cannot be derived in a closed form for this bidimensional problem. However, for $t < T$, the following properties concerning $V_A(x,t)$ and $S_A(t)$ hold (see Section 6 in [6]).

(Convexity of the value function) $V_A(x,t)$ is convex with respect to $x \in \mathbb{R}^2_{++}$.

(Convexity of each stopping region) $S_{A,i}(t)$ is a convex set.

(Monotonicity of each stopping region) $x \in S_{A,1}(t) \Rightarrow x' \in S_{A,1}(t)$ $(\forall x'_1 \geq x_1, \forall x'_2 \in (0, x_2])$. $x \in S_{A,2}(t) \Rightarrow x' \in S_{A,2}(t)$ $(\forall x'_1 \in (0, x_1], \forall x'_2 \geq x_2)$.

(Behavior on the indifference line) $x_1/(r-\mu_1) - I_1 = x_2/(r-\mu_2) - I_2 \Rightarrow x \notin S_A(t)$.

The monotonicity of each stopping region ensures the intuition that a higher market demand for HVs (resp. EVs) and a lower demand for EVs (resp. HVs) encourages the promotion of the HV (resp. EV). The last property means that, in the situation where the value of the HV project is the same as that of the EV, the automaker waits and sees which project is more promising. For other detailed properties, refer to [12, 6].

2.2 The replacement of the HV with the EV

Now, we consider that the automaker is promoting the HV and has an option to replace the HV with the EV. Assume that the replacement requires sunk costs I_R. For consistency, we also assume that $I_R > \max(I_2 - I_1, 0)$. Throughout the subscript $_R$ denotes "Replacement of the HV with the EV." As mentioned earlier, maturity T for the option to initiate the HV or the EV project is reasonably considered as a short-term decision, while the replacement of the HV with the EV may take place in the longer term. We then consider the infinite maturity for the replacement option.

The option value at time t with $X(t) = x \in \mathbb{R}^2_{++}$ is equal to the value function of the time-homogeneous optimal stopping problem[5] as follows:

$$V_R(x) = \sup_{\tau \in \mathcal{T}_{t,\infty}} \mathbb{E}^x_t[\mathrm{e}^{-r(\tau-t)} \left(\frac{X_2(\tau)}{r-\mu_2} - \frac{X_1(\tau)}{r-\mu_1} - I_R \right)]. \tag{5}$$

The project value including the ongoing HV project at time t becomes:

$$\underbrace{\frac{X_1(t)}{r-\mu_1}}_{\text{perpetual HV}} + \underbrace{V_R(X(t))}_{\text{option value}}. \tag{6}$$

Problem (5) is essentially the same as the value function of an American spread option (refer to [9, 6]). In terms of real options, [8] used a similar problem to find the optimal

[5]Assume that in (5) the payoff is zero in the case of $\tau = \infty$.

timing in changing the method of nuclear waste disposal. According to [8], the optimal stopping time $\tau_R(t)$ in problem (5) allows a closed form:

$$\tau_R(t) = \inf\{s \geq t \mid \mathcal{S}_R = \{X_2(s) \geq c_1 X_1(s) + c_2\}\}, \tag{7}$$

where $c_1(> 0), c_2(> I_R)$ are constants derived in a closed form (for details, see [8]). Although the value function $V_R(x)$ cannot be derived in a closed form, the following properties are known. For other detailed properties, refer to [12, 6].

(Convexity of the value function) $V_R(x)$ is convex with respect to $x \in \mathbb{R}^2_{++}$.

Furthermore, we provide the following lemmas concerning $V_R(x)$ for the next section.

Lemma 1

$$V_R(x') - V_R(x) \leq \frac{x_1 - x'_1}{r - \mu_1} + \frac{x'_2 - x_2}{r - \mu_2} \quad (\forall x'_1 \in (0, x_1], \forall x'_2 \geq x_2).$$

Proof First, note that for any $x \in \mathbb{R}^2_{++}$

$$\frac{-1}{r - \mu_1} \leq \frac{\partial V_R}{\partial x_1}(x) < 0 \tag{8}$$

and

$$0 < \frac{\partial V_R}{\partial x_2}(x) \leq \frac{1}{r - \mu_2}, \tag{9}$$

where the equalities in (8) and (9) hold if and only if $x \in \mathcal{S}_R(t)$. These are readily proved by the differentiability (refer to [3]) and convexity of $V_R(x)$ and $V_R(x) = x_2/(r - \mu_2) - x_1/(r - \mu_1) - I_1$ $(x \in \mathcal{S}_R(t))$. By (8), (9), and the convexity of $V_R(x, t)$, we have for any $x'_1 \leq x_1$ and $x'_2 \geq x_2$:

$$\begin{aligned} V_R(x) &\geq V_R(x') + \frac{\partial V_R}{\partial x_1}(x')(x_1 - x'_1) + \frac{\partial V_R}{\partial x_2}(x')(x_2 - x'_2) \\ &\geq V_R(x') - \frac{x_1 - x'_1}{r - \mu_1} - \frac{x'_2 - x_2}{r - \mu_2}, \end{aligned}$$

where the last inequality completes the proof. □

Lemma 2 Fix any $x \notin \mathcal{S}_R(t)$. There exist constants $a_1 \in (0, 1/(r - \mu_1))$, $a_2 \in (0, 1/(r - \mu_2))$, and $a_3 \geq I_1$ (which may depend on x) such that

$$\frac{x'_1}{r - \mu_1} + V_R(x') - I_1 \geq a_1 x'_1 + a_2 x'_2 - a_3 \quad (\forall x' \in \mathbb{R}_{++}),$$

where the equality holds when $x' = x$.

Proof By the convexity of $V_R(x)$, we have for any $x' \in \mathbb{R}^2_{++}$:

$$\begin{aligned} & \frac{x'_1}{r - \mu_1} + V_R(x') - I_1 \\ \geq\ & \frac{x'_1}{r - \mu_1} + V_R(x) + \frac{\partial V_R}{\partial x_1}(x)(x'_1 - x_1) + \frac{\partial V_R}{\partial x_2}(x)(x'_2 - x_2) - I_1 \\ =\ & \underbrace{\left(\frac{1}{r - \mu_1} + \frac{\partial V_R}{\partial x_1}(x)\right)}_{a_1} x'_1 + \underbrace{\frac{\partial V_R}{\partial x_2}(x)}_{a_2} x'_2 + \underbrace{V_R(x) - \frac{\partial V_R}{\partial x_1}(x) x_1 - \frac{\partial V_R}{\partial x_2}(x) x_2 - I_1}_{-a_3}. \end{aligned}$$

By (8), (9), and $x \notin \mathcal{S}_R(t)$, we have $a_1 \in (0, 1/(r-\mu_1))$ and $a_2 \in (0, 1/(r-\mu_1))$. Considering the limit $x_1' \downarrow 0, x_2' \downarrow 0$, we have $a_3 \geq I_1$. $\square$

Note that in Lemma 2 the right-hand side is the first order Taylor approximation to the left-hand function for x' near a fixed point x.

3 Main Results

This section combines Sections 2.1 and 2.2. The automaker initiates the promotion of either the HV or the EV before maturity T. If and only if the automaker chooses the HV does it have an option to change its promotions from the HV to the EV in future. Before the initiation of any promotion, the option value at time $t(< T)$ with $X(t) = x \in \mathbb{R}^2_{++}$ is equal to the value function of the optimal stopping problem:

$$V_{AR}(x,t) = \sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}_t^x[\mathrm{e}^{-r(\tau-t)} \max\left(\underbrace{\frac{X_1(\tau)}{r-\mu_1} + V_R(X(\tau))}_{\text{equation (6)}} - I_1, \frac{X_2(\tau)}{r-\mu_2} - I_2, 0\right)]. \tag{10}$$

Throughout the paper, the subscript $_{AR}$ denotes "Alternative choice including the replacement option." The optimal stopping time $\tau_{AR}(t)$ in problem (10) then becomes:

$$\tau_{AR}(t) = \inf\{s \geq t \mid X(s) \in \mathcal{S}_{AR}(s) = \mathcal{S}_{AR,1}(s) \cup \mathcal{S}_{AR,2}(s)\}, \tag{11}$$

where each stopping region $\mathcal{S}_{AR,i}(s)$ is defined by:

$$\mathcal{S}_{AR,1}(s) = \{x \in \mathbb{R}^2_{++} \mid V_{AR}(x,s) = \frac{x_1}{r-\mu_1} + V_R(x) - I_1\}. \tag{12}$$

and

$$\mathcal{S}_{AR,2}(s) = \{x \in \mathbb{R}^2_{++} \mid V_{AR}(x,s) = \frac{x_2}{r-\mu_2} - I_2\}. \tag{13}$$

Problem (10) differs from problem (2) in that the value of the HV includes the spread option value $V_R(X(\tau))$. This paper focuses on the effects of the spread option to replace the HV with the EV.

3.1 Theoretical results

The following proposition provides the properties of the value function $V_{AR}(x,t)$ and the stopping region $\mathcal{S}_{AR}(t)$ in problem (10) for $t < T$.

Proposition 1
(Convexity of the value function) $V_{AR}(x,t)$ is convex with respect to $x \in \mathbb{R}^2_{++}$.
(Convexity of the stopping region for EV) $\mathcal{S}_{AR,2}(t)$ is a convex set.
(Monotonicity of the stopping region for EV) $x \in \mathcal{S}_{AR,2}(t) \Rightarrow x' \in \mathcal{S}_{AR,2}(t)$ $(\forall x_1' \in$

$(0, x_1], \forall x_2' \geq x_2)$.

(Behavior on the indifference curve) $x_1/(r-\mu_1) + V_R(x) - I_1 = x_2/(r-\mu_2) - I_2 \Rightarrow x \notin \mathcal{S}_{AR}(t)$.

(Comparison with the case of no replacement) $V_{AR}(x,t) \geq V_A(x,t)$, $\mathcal{S}_{AR,1}(t) \supset \mathcal{S}_{A,1}(t)$, and $\mathcal{S}_{AR,2}(t) \subset \mathcal{S}_{A,2}(t)$.

Proof For simplicity, we denote the payoff function of the combined option by

$$f(x) := \max\left(\frac{x_1}{r-\mu_1} + V_R(x) - I_1, \frac{x_2}{r-\mu_2} - I_2, 0\right).$$

(Convexity of the value function) By the convexity of $V_R(x)$, the payoff function $f(x)$ is also convex. Because the payoff function is convex, the value function $V_{AR}(x,t)$ is convex with respect to $x \in \mathbb{R}^2_{++}$ (by Proposition A.6 in [3], or equivalently, Proposition 88 in [6]).

(Convexity of the stopping region for EV) Take any $\lambda \in (0,1), x \in \mathcal{S}_{AR,2}(t)$, and $y \in \mathcal{S}_{AR,2}(t)$. By the convexity of $V_{AR}(x,t)$ with respect to $x \in \mathbb{R}^2_{++}$, we have

$$\begin{aligned} V_{AR}(\lambda x + (1-\lambda)y, t) &\leq \lambda V_{AR}(x,t) + (1-\lambda)V_{AR}(y,t) \\ &= \lambda\left(\frac{x_2}{r-\mu_2} - I_2\right) + (1-\lambda)\left(\frac{y_2}{r-\mu_2} - I_2\right) \\ &= \frac{\lambda x_2 + (1-\lambda)y_2}{r-\mu_2} - I_2, \end{aligned}$$

where the last inequality implies $\lambda x + (1-\lambda)y \in S_{AR,2}(t)$, i.e., the convexity of the stopping region $S_{AR,2}(t)$.

(Monotonicity of the stopping region for EV) Take any $x \in \mathbb{R}^2_{++}$, $x_1' \in (0, x_1]$, and $x_2' \geq x_2$. By Lemma 1, we have:

$$\frac{x_1'}{r-\mu_1} + V_R(x') - I_1 \leq \frac{x_1}{r-\mu_1} + V_R(x) - I_1 + \frac{x_2' - x_2}{r-\mu_2}. \tag{14}$$

By (14) we have:

$$\begin{aligned} f(x') &= \max\left(\frac{x_1'}{r-\mu_1} + V_R(x') - I_1, \frac{x_2}{r-\mu_2} - I_2 + \frac{x_2'-x_2}{r-\mu_2}, 0\right) \\ &\leq \underbrace{\max\left(\frac{x_1}{r-\mu_1} + V_R(x) - I_1, \frac{x_2}{r-\mu_2} - I_2, 0\right)}_{=f(x)} + \frac{x_2'-x_2}{r-\mu_2} \\ &= f(x) + \frac{x_2'-x_2}{r-\mu_2}. \end{aligned} \tag{15}$$

Then, for $x \in \mathcal{S}_{AR,2}(t)$, we have:

$$\begin{aligned}
V_{AR}(x',t) &= \sup_{\tau\in\mathcal{T}_{t,T}} \mathbb{E}_t^{(1,1)}[\mathrm{e}^{-r(\tau-t)} f(x_1'X_1(\tau), x_2'X_2(\tau))] \\
&\le \sup_{\tau\in\mathcal{T}_{t,T}} \mathbb{E}_t^{(1,1)}[\mathrm{e}^{-r(\tau-t)} \left(f(x_1X_1(\tau), x_2X_2(\tau)) + \frac{(x_2'-x_2)X_2(\tau)}{r-\mu_2} \right)] \qquad (16) \\
&\le \underbrace{\sup_{\tau\in\mathcal{T}_{t,T}} \mathbb{E}_t^{(1,1)}[\mathrm{e}^{-r(\tau-t)} f(x_1X_1(\tau), x_2X_2(\tau))]}_{=V_{AR}(x,t)} + \sup_{\tau\in\mathcal{T}_{t,T}} \mathbb{E}_t^{(1,1)}[\mathrm{e}^{-r(\tau-t)} \frac{(x_2'-x_2)X_2(\tau)}{r-\mu_2}] \\
&= V_{AR}(x,t) + \frac{(x_2'-x_2)}{r-\mu_2} \\
&= \frac{x_2}{r-\mu_2} + \frac{(x_2'-x_2)}{r-\mu_2} \qquad (17) \\
&= \frac{x_2'}{r-\mu_2}, \qquad (18)
\end{aligned}$$

where (16) and (17) follow from (15) and $x \in \mathcal{S}_{AR,2}(t)$, respectively. The last inequality (18) implies $x' \in \mathcal{S}_{AR,2}(t)$, and hence we have $x \in \mathcal{S}_{AR,2}(t) \Rightarrow x' \in \mathcal{S}_{AR,2}(t)$ $(\forall x_1' \in (0, x_1], \forall x_2' \ge x_2)$.[6]

(Behavior on the indifference curve) Take any $x \in \mathbb{R}^2_{++}$ satisfying $x_1/(r-\mu_1) + V_R(x) - I_1 = x_2/(r-\mu_2) - I_2$. Note that $x \notin \mathcal{S}_R(t)$ because of the assumption $I_R > \max(I_2 - I_1, 0)$. Using the constants a_1, a_2, and a_3 in Lemma 2, we have for any $s \in (t, T]$:

$$\begin{aligned}
& V_{AR}(x,t) \\
\ge\ & \sup_{\tau\in\mathcal{T}_{t,T}} \mathbb{E}_t^x[\mathrm{e}^{-r(\tau-t)} \max\left(a_1X_1(\tau) + a_2X_2(\tau) + a_3, \frac{X_2(\tau)}{r-\mu_2} - I_2, 0 \right)] \\
\ge\ & \mathrm{e}^{-r(s-t)}\mathbb{E}_t^x[\max\left(a_1X_1(s) + a_2X_2(s) + a_3, \frac{X_2(s)}{r-\mu_2} - I_2 \right)] \\
\ge\ & \underbrace{\mathrm{e}^{-r(s-t)}\mathbb{E}_t^x[\frac{X_2(s)}{r-\mu_2} - I_2]}_{\uparrow x_2/(r-\mu_2)-I_2\ (s\downarrow t)} \\
& + \underbrace{\mathrm{e}^{-r(s-t)}\mathbb{E}_t^x[\max\left(a_1X_1(s) - \left(\frac{1}{r-\mu_2} - a_2 \right) X_2(s) + a_3 - I_2, 0 \right)]}_{\downarrow 0\ (s\downarrow t)}. \qquad (19)
\end{aligned}$$

In the right-hand side of (19), the first term $\uparrow x_2/(r-\mu_2) - I_2$ $(s \downarrow t)$ at a finite rate while the second term[7] $\downarrow 0$ $(s \downarrow t)$ at a rate that increases to infinity in the limit (refer to Lemma B.1 in [3], or equivalently, Lemma 91 in [6]). Therefore, there exists some $s \in (t, T]$ such that the right-hand side of (19) is strictly larger than $x_2/(r-\mu_2) - I_2$. This implies that $V_{AR}(x,t) > x_2/(r-\mu_2) - I_2 = x_1/(r-\mu_1) + V_R(x) - I_1$, i.e., $x \notin \mathcal{S}_{AR}(x,t)$.

(Comparison with the case of no replacement) The inequality $V_{AR}(x,t) \ge V_A(x,t)$

[6] We used a similar method of the proof of Proposition A.3 in [3], or equivalently, Proposition 85 in [6].

[7] This is equal to the value of the European spread option with maturity s.

is clear. We have for any $x \in \mathcal{S}_{A,1}(t)$:

$$\begin{aligned}
& V_{AR}(x,t) \\
\leq \;& \sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}_t^x[\mathrm{e}^{-r(\tau-t)}\left(V_R(X(\tau)) + \max\left(\frac{X_1(\tau)}{r-\mu_1} - I_1, \frac{X_2(\tau)}{r-\mu_2} - I_2, 0\right)\right)] \\
\leq \;& \sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}_t^x[\mathrm{e}^{-r(\tau-t)} V_R(X(\tau))] + \underbrace{\sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}_t^x[\mathrm{e}^{-r(\tau-t)} \max\left(\frac{X_1(\tau)}{r-\mu_1} - I_1, \frac{X_2(\tau)}{r-\mu_2} - I_2, 0\right)]}_{=V_A(x,t)} \\
= \;& V_R(x) + V_A(x,t) \\
= \;& V_R(x) + \frac{x_1}{r-\mu_1} - I_1,
\end{aligned}$$

where the last inequality implies $x \in \mathcal{S}_{AR,1}(t)$. We have for any $x \in \mathcal{S}_{AR,2}(t)$:

$$\begin{aligned}
V_A(x,t) & \leq V_{AR}(x,t) \\
& = \frac{x_2}{r-\mu_2} - I_2,
\end{aligned}$$

where the last inequality implies $x \in \mathcal{S}_{A,2}(t)$. □

Proposition 1 extends previous findings by [7, 3, 1] allowing only a linear function to a case allowing a nonlinear function $V_R(x)$. The difference from problem (2) in Section 2.1 is that the investment region for the HV, $\mathcal{S}_{AR,1}(t)$, does not necessarily satisfy either convexity or monotonicity. The monotonicity of $\mathcal{S}_{AR,2}(t)$ brings about the straightforward fact that an increased demand for EVs and a decreased demand for HVs accelerates the automaker's investment in the EV. However, this monotonicity does not necessarily hold for the HV because of the spread option to replace the HV with the EV. Indeed, as will be shown in the numerical example, increased demand for the EV may encourage the promotion of the HV because the value of the spread option increases. Note that $\mathcal{S}_{AR,1}(t)$ includes $\mathcal{S}_{A,1}(t)$, which has both monotonicity and convexity (see the final property in Proposition 1). On the other hand, the behavior on the indifference curve is inherited from the option without replacement. The automaker delays the decision on project choice when the market demand $X(t)$ lies on the curve where both project values are equal. The property of $\mathcal{S}_{AR,1}(t) \supset \mathcal{S}_{A,1}(t)$ and $\mathcal{S}_{AR,2}(t) \subset \mathcal{S}_{A,2}(t)$ supports the natural intuition that the potential replacement of the HV with the EV encourages (resp. discourages) the HV (resp. EV).

3.2 Numerical results and implications

This subsection provides a numerical example with some implications. As reported in [4], it is difficult to forecast the future market share of HVs and EVs. What appears to be certain is that EVs have more potential and more volatile than HVs in the future, but at present the market demand for EVs is much lower. Considering this, we set the base parameter values as: $\mu_1 = 1\%, \sigma_1 = 20\%$ for the HV, $\mu_2 = 5\%, \sigma_2 = 40\%$ for the EV, and the discount rate $r = 8\%$. The sunk costs I_i ($i = 1, 2$) are not essential because they are adjustable by $X(t) = x$. We set $I_1 = 100$ for the HV, $I_2 = 100$ for the EV, and $I_R = 100$ for the replacement. The maturity of the option is reasonably set as $T - t = 1$ year. In the numerical procedure, we make a discretization with 200 time steps per 1 year, and use a bivariate version of the lattice binomial method (see [2]). We compute the perpetual spread option $V_R(x)$ using a value iteration algorithm in the first step, and using the computed $V_R(x)$ compute the max-call option $V_{AR}(x, t)$ backward from maturity T using a dynamic programming algorithm.

Figure 1[8] illustrates the investment region $\mathcal{S}_{AR}(t)$ along with $\mathcal{S}_A(t)$ and $\mathcal{S}_R(t)$. In Figure 1, the investment region for the HV, $\mathcal{S}_{AR,1}(t)$, does not satisfy monotonicity when the market demand for EVs, x_2, is small (see $X(t) = x \approx (9, 2)$). This finding leads to the following implication. In the current circumstance, where the market demand for EVs is much lower than that of HVs, an increase in the market demand for EVs (or equivalently, a technical innovation in the EV) can also accelerate the promotion of HVs because of the effect of the spread option to replace the HV with the EV. This supports the promotion strategies of automakers such as Toyota and Honda that emphasize the promotion of HVs as a temporary measure before the introduction of EVs.[9] In general, whether an increase in the demand for EVs encourages HVs or not is determined by the tradeoff between the max-call option (discouragement of HVs) and the spread option (encouragement of HVs). According to computations using a wide range of parameter values, a larger gap $\mu_2 - \mu_1$ and lower replacement costs $I_R(> I_2 - I_1)$ make the effect of the spread option (encouragement of HVs) dominant for a small x_2. In Figure 1, $\mathcal{S}_{AR,1}(t)$ does not satisfy convexity. This results from the nonlinearity of the indifference curve.

Another important feature of $\mathcal{S}_{AR,1}(t)$ is the sensitivity with respect to the correlation coefficient ρ. Figure 2 depicts the investment regions $\mathcal{S}_{AR}(t)$ with $\rho = -0.5, 0$, and 0.5. It has been numerically verified in [7, 6] that $S_{A,i}(t)$ grows monotonically with ρ. This is because a higher ρ decreases the value of the option to postpone the project choice and therefore hastens the investment in each project. In contrast, in Figure 2, the investment region $\mathcal{S}_{AR,1}(t)$ does not present monotonicity. In the combined problem, a higher ρ decreases not only the value of the option to postpone the decision, but also the value of the spread option embedded in the HV project. The latter effect of a higher ρ decreases the project value of the HV and then delays its promotion. Indeed, in the current circumstance where $X(t) = x \approx (9, 2)$ in Figure 2, a lower ρ encourages the promotion of the HV. Given the prospect that HVs will be replaced with EVs (ρ is negative), an

[8]Technically, we compute the lattice model with 400 time steps for maturity $T = 2$ year, and Figure 1 shows the investment regions $\mathcal{S}_{AR}(t)$, $\mathcal{S}_A(t)$, and $\mathcal{S}_R(t)$ for $t = 1$ year.

[9]Of course, Nissan's emphasis on EVs is not necessarily criticized if one considers strategic competition against Toyota and Honda, carmakers that have taken a lead in HV technologies. This strategic interaction is one of several important issues to be addressed in future work.

automaker then accelerates the investment in the HV. This also supports the rapid spread of the promotion of HVs in present circumstances. Lastly, we remark that $V_{AR}(x,t)$ is about 1.5 times higher than $V_A(x,t)$ for $X(t) = x \approx (9,2)$ due to the spread option, though the paper focuses on the investment strategy rather than the value.

4 Conclusion

From a real options perspective, this paper investigated an automaker's decisions concerning investment timing and project choice between the HV and EV. We modelled the problem as the max-call option including the spread option to replace the HV with the EV. We showed the analytical properties of the project value and the investment region. A notable difference from the case of no replacement option is that the increased market demand for EVs may accelerate the promotion of not only the EV but also the HV because of the replacement option. Especially, the encouragement of the HV is predicted in the current circumstance where the market demand for EVs is much smaller than that of HVs and the correlation between the EV and the HV is low or negative. This supports the promotion strategies used by automakers such as Toyota and Honda that emphasize the promotion of HVs despite the imminent dominance of EVs.

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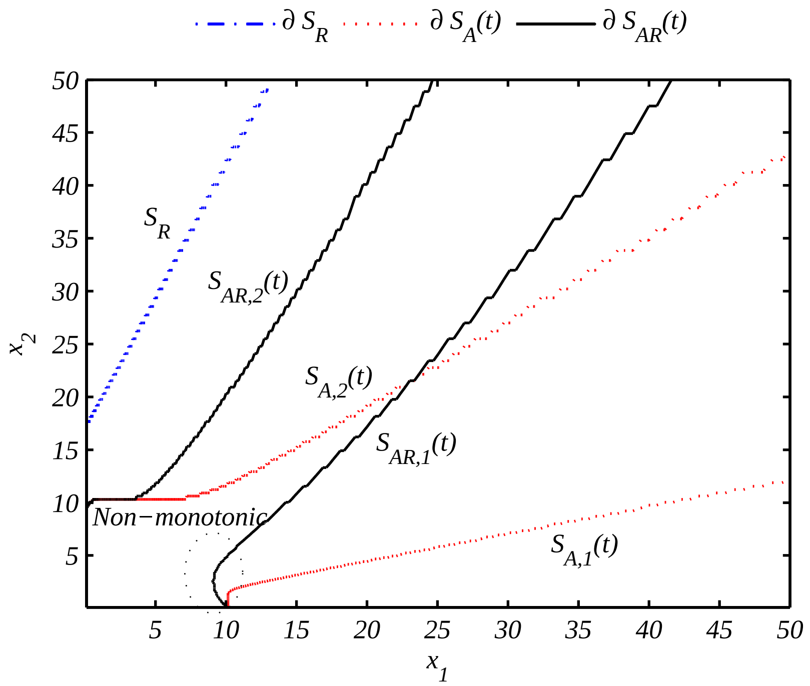

Figure 1: The boundaries of the investment regions $\mathcal{S}_{AR}(t), \mathcal{S}_A(t)$, and $\mathcal{S}_R$. The parameter values are set as $\mu_1 = 1\%, \sigma_1 = 20\%, \mu_2 = 5\%, \sigma_2 = 40\%, r = 8\%, I_1 = I_2 = I_R = 100, \rho = 0\%$.

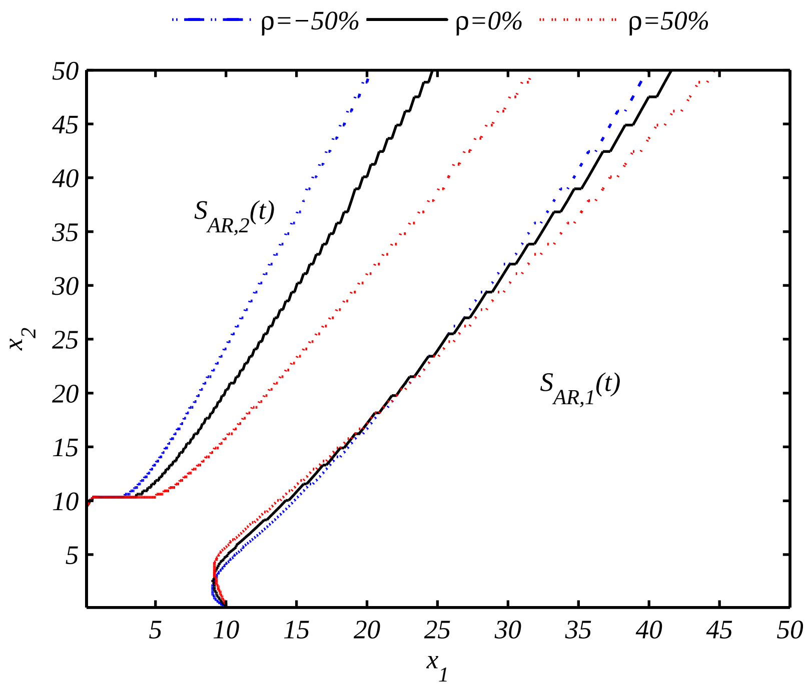

Figure 2: The boundaries of the investment regions of $\mathcal{S}_{AR}(t)$ and $\mathcal{S}_A(t)$ for correlation coefficient $\rho = -50\%, 0\%$, and 50%. The other parameter values are the same as those in Figure 1.

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