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Ryo Arawatari† and Tetsuo Ono‡

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Abstract

This paper introduces an overlapping-generations model with earnings heterogeneity and borrowing constraints. The labor income tax and the allocation of tax revenue across social security and forward intergenerational public goods are determined in a bidimensional majoritarian voting game played by successive generations. The political equilibrium is characterized by an ends-against-the-middle equilibrium where low- and high-income individuals form a coalition in favor of a low tax rate and less social security while middle-income individuals favor a high tax rate and greater social security. Government spending then shifts from social security to public goods provision if higher wage inequality is associated with the borrowing constraint and a low interest-rate elasticity of consumption.

Keywords: Borrowing constraint; Social security; Public goods provision; Ends-against-the-middle equilibrium; Wage inequality

JEL Classification: H41; H55; D72

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1 Introduction

Almost all OECD countries have experienced some increase in wage inequality over the past few decades. Standard political economy theory suggests that higher wage inequality results in a larger volume of redistribution as the decisive voter becomes less well-off as wage inequality increases (Romer, 1975; Roberts, 1977; Meltzer and Richard, 1981; Alesina and Rodrik, 1994; Benabou, 1996; Persson and Tabellini, 1994). This prediction still holds when the focus is on intergenerational redistribution, such as pay-as-you-go (PAYG) social security. That is, the intragenerational redistribution component of the PAYG social security system makes this program palatable to low-income young individuals (Conde-Ruiz and Galasso, 2003, 2005; Bethencourt and Galasso, 2008).

The empirical evidence, however, does not necessarily support the abovementioned theoretical predictions. OECD cross-country data show that the volume of redistribution is negatively correlated with wage inequality (Gottschalk and Smeeding, 1997; Chen and Song, 2009). For instance, the United Kingdom and the United States feature higher wage inequality and smaller redistribution whereas the Nordic countries display lower wage inequality and larger redistribution. In fact, Pineda and Rodriguez (2006) found a strong negative correlation between redistribution and the share of capital in GDP where the share is considered an indicator of income inequality.

Several theories have been provided to explain the negative correlation between inequality and redistribution. Examples include political bias toward the rich (Benabou, 2000); the prospect of upward mobility by low-income agents (Benabou and Ok, 2001; Arawatari and Ono, 2009); and lobbying by rich capitalists (Rodriguez, 2004). These studies, however, abstract away forward intergenerational public goods provision (for example, public education, environmental maintenance, pure science and other public services for future generations) as alternative public spending and thus say nothing about how the *intergenerational* conflict over the composition of public spending is affected by the increase in *intragenerational* inequality.

In sum, most contributions addressing the negative correlation focus on a single policy issue. A notable exception is Levy (2005), who develops a two-dimensional political economy model with endogenous formation of parities. In essence, Levy (2005) considers political conflicts over income redistribution that benefits the poor and public education that benefits the young. One of the striking results is that higher income inequality may decrease, rather than increase, the tax rate in the young-majority equilibrium. However, her analysis ignores predictions about the share of redistribution and public education in government because no provision of public education arises in equilibrium.[1]

[1] In Levy (2005), public provision of education arises only in the old-majority equilibrium. However, inequality and redistribution have a positive correlation in that equilibrium, and this does not readily fit

The present paper contributes to this literature by predicting how the shares of social security and forward intergenerational public goods in government expenditure are affected by inequality within a generation. For this purpose, we introduce an overlapping-generations model with heterogeneous agents. In this economy, young workers are of three types of income: low, middle and high. Because they are not permitted to borrow in youth as a result of imperfect financial markets, lower-income individuals are more likely to be borrowing constrained. Young workers then pay a fixed proportion of their labor income to the government, and the tax revenue is divided into PAYG social security payments from which the old can benefit and public goods provision that the old cannot capture.

The tax rate and the allocation of tax revenue between social security and public goods provision are determined in a two-dimensional majoritarian voting game played by the young and the old. Voters cast a ballot over the labor income tax, which finances social security and public goods provision, and over the allocation of tax revenue between social security and public goods provision. Under this type of voting game, the existence of a Condorcet winner of the majority voting game is not necessarily guaranteed because of the multidimensionality of the issue space. To deal with this problem, we utilize the concept of a structure-induced equilibrium (Shepsle, 1979) with the notion of once-and-for-all voting, which is applied to an overlapping-generations framework by Conde-Ruiz and Galasso (2003, 2005).

Based on the abovementioned concept of equilibrium, we consider the voting behavior of each type of individual. The preferences of the old are identical across all types of individuals because they owe no tax burden, receive the same level of social security benefit, and cannot capture the benefit of forward intergenerational public goods. Instead, they prefer the tax rate that attains the top of the Laffer curve and full use of the revenue for social security. In contrast, the preferences of the young depend on their income type because the tax burden differs across the types of income. In particular, the key factors to their preferences are the borrowing constraint and the interest-rate elasticity of consumption.

To understand the role of these two factors, consider the case where only low-income individuals are faced with the borrowing constraint. They wish to consume more in their youth, but cannot because of the borrowing constraint. In this situation, a higher tax rate produces two opposing effects: a negative effect that results in lower after-tax income, and thus the utility loss of taxation in youth, and a positive effect that produces higher social security benefit, and thus the utility gain of taxation in old age.

When the interest-rate elasticity is high, substitution across periods is easy: the utility loss of taxation in youth is compensated for by the utility gain of taxation in old age.

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with the available empirical evidence.

Therefore, low-income individuals choose a higher tax rate than middle- and high-income individuals. However, when the interest-rate elasticity is low, compensation is not necessarily available for low-income individuals because substitution across periods is difficult. Low-income individuals then prefer a lower tax rate than middle-income individuals. This results in an ends-against-the-middle equilibrium where low- and high-income individuals form a coalition in favor of a low tax rate while middle-income individuals favor a high tax rate.

Given the characterization of political equilibrium, we investigate how the shares of social security and forward intergenerational public goods in government expenditure are altered in response to changes in wage inequality. We show that the mean-preserving reduction of the decisive voter's wage creates an inverse U-shaped relationship between the decisive voter's wage and the share of social security when the interest-rate elasticity is low. The positive correlation arises when the decisive voter's wage is high and thus he/she is borrowing unconstrained, while the negative correlation arises when his/her wage is low and thus he/she is borrowing constrained. The same relation also holds between wage inequality and the size of social security. Therefore, the interest-rate elasticity and the borrowing constraint are the key factors needed to demonstrate the negative correlation between wage inequality and the share (or size) of social security.

The organization of this paper is as follows. Section 2 introduces the model and characterizes the economic equilibrium. Section 3 develops the political system, introduces the equilibrium concept of the voting game and demonstrates the voting behavior of each individual. Section 4 characterizes the political equilibrium. Section 5 examines how wage inequality affects the tax rate, the allocation of tax revenue and the levels of social security and public goods provision via the political process. Section 6 briefly undertakes the analysis under a generalized framework. Section 7 provides some concluding remarks.

2 The Economic Environment

Consider a discrete time economy where time is denoted by $t = 0, 1, 2 \cdots$. The economy is made up of overlapping generations of individuals, each of whom lives two periods: youth and old age. The size of a generation born in period t, called generation t, is denoted by N_t. Population grows at a constant rate $n > 0 : N_{t+1} = (1 + n)N_t$ for all $t \geq 0$. Within each generation, there are three types of agents according to ability, low, middle and high ($j = L, M, H$), whose proportions are respectively ρ^L, ρ^M and ρ^H, where $\sum_j \rho^j = 1$ and ρ^j satisfies the following assumption.

Assumption 1. $\rho^j > n/\{2(1 + n)\}$, $j = L, M, H$.

Assumption 1 ensures that a young individual who prefers the highest tax rate among young individuals becomes the decisive voter. To understand the argument stemming from Assumption 1, suppose that a type-k ($k = L, M$ or H) prefers the highest tax rate. As explained below, all the old have the same preferences over policies and choose a higher tax rate than any young agent. When the young and the old participate in voting, the sum of the type-k young and the old is given by $N_t\rho^k + N_{t-1}$, which is greater than half of the population in period t, $(N_t + N_{t-1})/2$, under the assumption of $\rho^k > n/2(1+n)$. This implies that the decisive voter becomes the old or the type-k young. However, the old cannot become the decisive voter because the population size of the old is smaller than that of the young under the assumption of $n > 0$. Therefore, the type-k young individual becomes the decisive voter. Figure 1 provides an example of preferences over the tax rate.

[Figure 1 about here.]

2.1 Individuals

Each individual is assumed to receive utility from private consumption and publicly provided goods. The utility function of a type-j young individual in period t is given by:

$$U_t^j = \frac{(c_t^{yj})^{1-\sigma} - 1}{1-\sigma} + \eta\frac{(g_t)^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \left[c_{t+1}^{oj} + \eta\frac{(g_{t+1})^{1-\sigma} - 1}{1-\sigma}\right],$$

where c_t^{yj} is consumption in youth, c_{t+1}^{oj} is consumption in old age, g_t is per capita public goods in period t, $\eta(> 0)$ is the parameter representing the preference for public goods, $\beta \in (0, 1]$ is the discount factor and $\sigma(> 0)$ is the inverse of the elasticity of young-age consumption with respect to the interest rate. Following the literature (Conde-Ruiz and Galasso, 2005; Borck, 2007; Bethencourt and Galasso, 2008), we assume a quasi-linear utility function for analytical tractability.[2]

Each individual works in youth and retires in old age. The wage income is related to working ability. The wage of a type-j individual is given by $w^j (j = H, M, L)$, where w^j is constant over time and $w^L < w^M < w^H$. The average of the wage is denoted by $\bar{w} \equiv \rho^L w^L + \rho^M w^M + \rho^H w^H$.

Type-j's individual budget constraints in youth and old age are given by, respectively:

$$c_t^{yj} + s_t^j \leq (1 - \tau_t)w^j,$$
$$c_{t+1}^{oj} \leq Rs_t^j + b_{t+1},$$

[2] In Section 6, we briefly analyze the case where the utility of old-age consumption is given by $\beta(c_{t+1}^{oj})^{1-\sigma}/(1-\sigma)$. We will show that the main results are qualitatively unchanged under this alternative assumption.

where s_t^j is saving, τ_t is the income tax rate in period t, R is the gross interest rate and b_{t+1} is the per capita social security benefit in old age. We impose the restriction of nonnegative savings, that is:

$$s_t^j \geq 0.$$

This rules out the possibility of borrowing in youth against future social security benefits (Diamond and Hausman, 1984; Conde-Ruiz and Profeta, 2007).

We also assume that (i) the interest rate is exogenous and (ii) each individual receives the same amount of old age social security benefits regardless of contributions in their youth. The first assumption abstracts away the general equilibrium effect via the interest rate investigated by, for example, Cooley and Soares (1999) and Boldrin and Rustichini (2000). However, this simplification enables us to more simply demonstrate the analytical solution of the model. The second assumption abstracts away from the choice of social security systems (for example, Bismarckian vs. Beveridgean) analyzed by, for example, Borck (2007), Conde-Ruiz and Profeta (2007) and Cremer et al. (2007). We adopt the second assumption to concentrate on the allocation of tax revenue over social security and public goods provision as the main focus of the paper.

The representative type-j young individual maximizes his/her utility subject to the budget constraints and the restriction of nonnegative saving. When $s_t^j > 0$, the first-order condition for an interior solution is $(c_t^{yj})^{-\sigma} = \beta R$ and thus defines the optimal saving decision of a type-j individual given by $s_t^j = (1-\tau_t)w^j - (\beta R)^{-1/\sigma}$. By taking the borrowing constraint into account, the saving function of a type-j individual is:

$$s_t^j = \max\left\{0, (1-\tau_t)w^j - (\beta R)^{-1/\sigma}\right\}. \tag{1}$$

Eq. (1) indicates that the saving decision depends on the current tax rate τ_t, but is independent of the future tax rate τ_{t+1} and the proportion of tax revenues devoted to social security in old age, denoted by λ_{t+1}. This property comes from the assumption of a linear utility function of old-age consumption. Because of this property, we can easily demonstrate the joint political determination of the tax rate τ and the proportion λ.

We should note that Conde-Ruiz and Galasso (2003, 2005) and Bethencourt and Galasso (2008) cut the link between the current saving decision and future policy variables by assuming no consumption in youth; that is, households save all of their income in their youth and consume it in their old age. In contrast, the current paper introduces the saving decisions of households because our focus is on the borrowing constraint.

The saving function (1) implies that there is a critical rate of tax such that:

$$s_t^j \gtreqless 0 \Leftrightarrow \tau_t \lesseqgtr \hat{\tau}(w^j) \equiv 1 - \frac{1}{(\beta R)^{1/\sigma} w^j}. \tag{2}$$

Figure 2 illustrates the relation between savings and the tax rate for each type of an individual. A type-j individual chooses positive saving when the tax is below the critical rate. However, when the tax is above the critical rate, a type-j individual faces a borrowing constraint and can save nothing in youth. The critical rate of tax is higher when the wage income is larger because, given a tax rate common to all types of individuals, a higher ability individual receives a higher level of disposable income.

[Figure 2 about here.]

2.2 The Government

In each period, the government collects tax revenue from the young by imposing an income tax. Following the convention in the literature, we present the efficiency loss of taxation by assuming convex costs of collecting taxes (see, for example, Casamatta, Cremer and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Cremer et al., 2007). The actual tax revenue is therefore given by $\tau_t(1-\tau_t)(\rho^L w^L + \rho^M w^M + \rho^H w^H) = \tau_t(1-\tau_t)\bar{w}$, where the term $(1-\tau_t)$ is the distortionary factor. The assumption of distortionary taxation is solely to ensure an interior solution to preferred tax rates and otherwise plays no role.

The government uses the tax revenue for pay-as-you-go (PAYG) social security payments along with forward intergenerational public goods such as environmental preservation and pure science. The proportion $\lambda_t \in [0,1]$ of tax revenue is devoted to PAYG social security benefits and the remainder $(1-\lambda_t)$ is devoted to public goods provision. The PAYG social security is then an intergenerational transfer from the young to the old within a period. The budget constraint is $\lambda_t N_t \tau_t(1-\tau_t)\bar{w} = N_{t-1} b_t$. The per capita social security benefit in period t, b_t, is given by:

$$b_t = (1+n)\lambda_t \tau_t (1-\tau_t)\bar{w}.$$

The formation of public goods requires investment one period ahead of time. This assumption reflects the idea that education, pure science, and investment in the environment do not obtain immediate results. Importantly, the current young can enjoy the outcomes of any investment in the future, while the current old cannot enjoy it while they are still alive. The budget constraint is $(1-\lambda_t)N_t\tau_t(1-\tau_t)\bar{w} = (N_t + N_{t+1})g_{t+1}$. The per capita pubic goods provision in period $t+1$, g_{t+1}, is given by:

$$g_{t+1} = \frac{1+n}{2+n}(1-\lambda_t)\tau_t(1-\tau_t)\bar{w}.$$

2.3 The Economic Equilibrium

We define the economic equilibrium as follows:

Definition 1. For a given sequence of tax rates and social security shares in government expenditure, $\{\tau_t, \lambda_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations, $\{c_t^{yj}, c_t^{oj}, s_t^j\}_{j=L,M,H}^{t=0,\cdots,\infty}$ with the initial condition $s_0^j (j = L, M, H)$, such that: (i) in every period a type-j individual maximizes his/her utility subject to the budget constraints and the nonnegativity constraint of saving, (ii) the social security budget and the public goods budget are balanced every period and (iii) the goods market clears every period.

From (1) and the private and government budget constraints, the consumption functions of a type-j individual in youth and old age are given by, respectively:

$$c_t^{yj} = \begin{cases} (\beta R)^{-1/\sigma} & \text{if } \tau_t < \hat{\tau}(w^j) \\ (1-\tau_t)w^j & \text{if } \tau_t \geq \hat{\tau}(w^j) \end{cases}$$

$$c_{t+1}^{oj} = \begin{cases} R\{(1-\tau_t)w^j - (\beta R)^{-1/\sigma}\} + (1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w} & \text{if } \tau_t < \hat{\tau}(w^j) \\ (1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w} & \text{if } \tau_t \geq \hat{\tau}(w^j). \end{cases}$$

Because of the assumption of a quasi-linear utility function, the consumption in youth is type-independent and constant over time when the tax is below the critical rate.

The utility level obtained by individuals in economic equilibrium can be represented by their indirect utility functions. We can use the above-mentioned consumption functions to obtain an indirect utility function of a type-j young individual:

$$V_t^{yj} = \begin{cases} V_{t,s>0}^{yj} & \text{if } \tau_t < \hat{\tau}(w^j) \\ V_{t,s=0}^{yj} & \text{if } \tau_t \geq \hat{\tau}(w^j), \end{cases} \tag{3}$$

where:

$$\begin{aligned} V_{t,s>0}^{y,j} \equiv & \frac{\left((\beta R)^{-1/\sigma}\right)^{1-\sigma} - 1}{1-\sigma} + \beta \left[R\left\{(1-\tau_t)w^j - (\beta R)^{-1/\sigma}\right\} + (1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w}\right] \\ & + \beta\eta \frac{\left(\frac{1+n}{2+n}(1-\lambda_t)(1-\tau_t)\tau_t\bar{w}\right)^{1-\sigma} - 1}{1-\sigma}, \\ V_{t,s=0}^{y,j} \equiv & \frac{\left((1-\tau_t)w^j\right)^{1-\sigma} - 1}{1-\sigma} + \beta(1+n)\lambda_{t+1}(1-\tau_{t+1})\tau_{t+1}\bar{w} \\ & + \beta\eta \frac{\left(\frac{1+n}{2+n}(1-\lambda_t)(1-\tau_t)\tau_t\bar{w}\right)^{1-\sigma} - 1}{1-\sigma}. \end{aligned}$$

$V_{t,s>0}^{y,j}$ denotes the indirect utility of a type-j young individual when he/she saves some portion of his/her income, and $V_{t,s=0}^{y,j}$ denotes the indirect utility when he/she is faced with a borrowing constraint and saves nothing. For each indirect utility function, the first term on the right-hand side shows the utility of consumption in youth, the second term shows the utility of consumption in old age and the third term shows the utility of public goods in old age. Variables unrelated to political decisions are removed from the indirect utility functions.

For a type-j old individual in period t, the indirect utility function is:

$$V_t^{o,j} \equiv (1+n)\lambda_t(1-\tau_t)\tau_t\bar{w}, \tag{4}$$

where the right-hand side shows the PAYG social security benefits. Old individuals have the same indirect utility function regardless of their type because their saving in youth is predetermined and the level of public goods they enjoy is predetermined one period in advance. Therefore, old individuals have the same preferences for the tax rate, τ, and the share of PAYG social security, λ.

3 The Political Institution and Voting

The tax rate τ and the proportion λ are determined by individuals through a political process of majoritarian voting. Elections take place every period and all individuals alive, both young and old, cast a ballot over τ, the income tax, and λ, the share of social security in government expenditure. Individual preferences over the two issues are represented by the indirect utility functions at Eqs. (3) and (4) for the young and the old, respectively. Every individual has zero mass, and thus no individual vote can change the outcome of the election. We thus assume individuals vote sincerely.

This majoritarian voting game has two significant characteristics. First, the issue space is bidimensional (τ and λ), and thus the Nash equilibria of a majoritarian voting game may fail to exist. To deal with this feature, we use the concept of issue-by-issue voting, or *structure-induced equilibrium*, as formalized by Shepsle (1979) and applied by Conde-Ruiz and Galasso (2003, 2005) for the framework of overlapping generations.

Second, the game is intrinsically dynamic because it describes the interaction among successive generations. To deal with the second feature, we assume full commitment, i.e., once-and-for-all voting. That is, voters determine the constant sequence of the parameters: $\tau_t = \tau_{t+1} = \tau$ and $\lambda_t = \lambda_{t+1} = \lambda$ for all t as in Casamatta, Cremer and Pestieau (2000) and Conde-Ruiz and Profeta (2007). We can view the full commitment solution as the solution including intergenerational interaction because the full commitment solution can be supported as the subgame perfect equilibrium (see, for example, Conde-Ruiz and Galasso, 2003, 2005).

Given the stationary environment, the current model presents a static voting game. Therefore, the result in Shepsle (1979) can be applied to obtain the sufficient conditions for the existence of a structure-induced equilibrium. In particular, if preferences are single peaked along every dimension of the issue space, a sufficient condition for (τ^*, λ^*) to be an equilibrium of the voting game with full commitment is that τ^* represents the outcome of majority voting over the jurisdiction τ when the other dimension is fixed at its level λ^*, and vice versa. Taking second derivatives of V_t^{yj} and $V_t^{o,j}$ with respect to τ and λ, we immediately find that $\partial^2 V^{o,j}/\partial\tau^2 < 0, \partial^2 V^{o,j}/\partial\lambda^2 < 0, \partial^2 V^{y,j}/\partial\tau^2 < 0$, and $\partial^2 V^{y,j}/\partial\lambda^2 < 0$, showing that preferences are single peaked.

The old choose τ to maximize $V^{o,j}$ in (4) given λ, and λ to maximize $V^{o,j}$ in (4) given τ. Their preferred rate of tax and the share of PAYG social security are respectively given by:

$$\tau^{oj} = \frac{1}{2} \text{ and } \lambda^{oj} = 1 \text{ for all } j.$$

Maximization is realized when the tax rate is set to attain the top of the Laffer curve, $\tau(1-\tau)$, and the maximized tax revenue is used exclusively for PAYG social security.

In what follows, we sequentially investigate the preferred tax and share of government expenditure by the borrowing-unconstrained and borrowing-constrained young.

3.1 Voting by the Borrowing-unconstrained Young

The borrowing-unconstrained young choose τ to maximize $V_{s>0}^{y,j}$ in (3) given λ, and λ to maximize $V_{s>0}^{y,j}$ in (3) given τ. First, we consider the choice of τ. The first derivative of $V_{s>0}^{y,j}$ with respect to τ is given by:

$$\begin{aligned}\frac{\partial V_{s>0}^{y,j}}{\partial\tau} &= -\beta R w^j + \beta(1+n)\lambda(1-2\tau)\bar{w} \\ &\quad + \beta\eta\left\{\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma}\frac{1+n}{2+n}(1-\lambda)(1-2\tau)\bar{w},\end{aligned}$$

where the first term on the right-hand side shows a marginal decrease in old-age consumption, the second term shows a marginal increase in social security benefits and the third term shows a marginal utility benefit of an increase in public goods provision produced by an increase in tax revenue.

The borrowing-unconstrained young choose the τ that balances marginal costs and benefits in terms of utility, that is, that attains $\partial V_{s>0}^{y,j}/\partial\tau = 0$, which is equivalent to:

$$\begin{aligned}\beta R w^j &= \beta(1+n)\lambda(1-2\tau)\bar{w} \\ &\quad + \beta\eta\left\{\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma}\frac{1+n}{2+n}(1-\lambda)(1-2\tau)\bar{w}. \qquad (5)\end{aligned}$$

The left-hand side of (5) is constant whereas the right-hand side of (5), denoted by RHS, is featured by $\lim_{\tau\to 0} RHS = +\infty$ and $RHS|_{\tau=1/2} = 0$. Therefore, the preferred tax rate by the borrowing-unconstrained young is set within the range $(0, 1/2)$ as illustrated in Panel (a) of Figure 3.

[Figure 3 about here.]

Next, we consider the choice of λ. The first derivative of $V_{s>0}^{y,j}$ with respect to λ is given by:

$$\frac{\partial V_{s>0}^{y,j}}{\partial \lambda} = \beta(1+n)(1-\tau)\tau\bar{w} - \beta\eta\left\{\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma} \cdot \frac{1+n}{2+n}(1-\tau)\tau\bar{w},$$

where the first term on the right-hand side shows the marginal increase in the benefit of social security given by an increase in the share of social security, and the second term is the marginal loss of utility of public goods given by a decrease in the share of public goods provision. The share of PAYG social security, λ, is chosen to balance the marginal benefit and loss in terms of utility. However, under a certain condition, the share λ could be zero because the marginal loss of public goods in terms of utility may overcome the marginal benefit of PAYG social security at $\lambda = 0$.

Based on the above argument, the preferred share of the borrowing-unconstrained young satisfies $\partial V_{s>0}^{y,j}/\partial\lambda = 0$ when $\lambda > 0$ and $\partial V_{s>0}^{y,j}/\partial\lambda \leq 0$ when $\lambda = 0$; that is:

$$\lambda = \begin{cases} 0 & \text{if } \tau \in [0, \underline{\tau}] \\ 1 - \frac{2+n}{(1+n)(1-\tau)\tau\bar{w}}\left(\frac{\eta}{2+n}\right)^{1/\sigma} & \text{if } \tau \in \left(\underline{\tau}, \frac{1}{2}\right] \end{cases}, \tag{6}$$

where:

$$\underline{\tau} \equiv \frac{1 - \sqrt{1 - \frac{4(2+n)}{(1+n)\bar{w}}\left(\frac{\eta}{2+n}\right)^{1/\sigma}}}{2}.$$

Panel (c) of Figure 3 illustrates the graph of (6). The range of τ is limited to $(0, 1/2)$ because the preferred tax rate by the old is equal to $1/2$ and that by the young is less than $1/2$. When the tax rate is below the critical rate $\underline{\tau}$, the tax revenue is too small, so that the marginal benefit of raising the share of PAYG social security is lower than the marginal cost of reducing the share of public goods provision for any $\lambda \in [0, 1]$. Therefore, the borrowing-unconstrained young choose no expenditure to social security, $\lambda = 0$, if $\tau \leq \underline{\tau}$.

When the tax rate is above the critical rate $\underline{\tau}$, the tax revenue is sufficient, so that there is a share $\lambda \in (0, 1)$ that balances the marginal benefits and costs. The optimal share for the borrowing-unconstrained young increases as the tax rate increases. The optimal share attains its highest value at $\tau = 1/2$, where the tax revenue is maximized.

3.2 Voting by the Borrowing-constrained Young

The borrowing-constrained young choose τ to maximize $V_{s=0}^{y,j}$ in (3) given λ, and λ to maximize $V_{s=0}^{y,j}$ in (3) given τ. First, we consider the choice of τ. The first derivative of $V_{s=0}^{y,j}$ with respect to τ is:

$$\frac{\partial V_{s=0}^{y,j}}{\partial \tau} = -((1-\tau)w^j)^{-\sigma}w^j + \beta(1+n)\lambda(1-2\tau)\bar{w} + \beta\eta\left\{\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma}\frac{1+n}{2+n}(1-\lambda)(1-2\tau)\bar{w},$$

where the first term on the right-hand side shows the marginal cost of a decrease in disposable income in youth, the second term shows the marginal increase in social security benefits, and the third term shows the marginal utility benefit of an increase in public goods provision produced by an increase in tax revenue.

The borrowing-constrained young choose τ that balances the marginal costs and benefits in terms of utility. That is, they choose τ that attains $\partial V_{s=0}^{y,j}/\partial\tau = 0$, which is equivalent to:

$$\frac{(w^j)^{1-\sigma}}{(1-\tau)^{\sigma}} = \beta(1+n)\lambda(1-2\tau)\bar{w} + \beta\eta\left\{\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma}\cdot\frac{1+n}{2+n}(1-\lambda)(1-2\tau)\bar{w}. \tag{7}$$

Panel (b) of Figure 3 illustrates the determination of τ that maximizes the utility of the borrowing-constrained young.

Next, we consider the choice of λ. The first derivative of $V_{s=0}^{y,j}$ with respect to λ is given by:

$$\frac{\partial V_{s=0}^{y,j}}{\partial \lambda} = \beta(1+n)(1-\tau)\tau\bar{w} - \beta\eta\left\{\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right\}^{-\sigma}\cdot\frac{1+n}{2+n}(1-\tau)\tau\bar{w},$$

which is equivalent to that of the young without borrowing constraints: $\partial V_{s=0}^{y,j}/\partial\lambda = \partial V_{s>0}^{y,j}/\partial\lambda$. This equality stems from the quasi-linear utility function. An increase in the share λ creates a marginal utility benefit irrespective of whether the borrowing constraint is binding. Therefore, the preferred share of the borrowing-constrained young is given by:

$$\lambda = \begin{cases} 0 & \text{if } \tau\in[0,\underline{\tau}] \\ 1-\frac{2+n}{(1+n)(1-\tau)\tau\bar{w}}\left(\frac{\eta}{2+n}\right)^{1/\sigma} & \text{if } \tau\in\left(\underline{\tau},\frac{1}{2}\right] \end{cases}, \tag{8}$$

which is equivalent to that of the borrowing-unconstrained young.[3]

4 The Political Equilibrium

The previous section analyzed the voting behavior of each type of individual along the two dimensions of the issue space, τ and λ. Given that preferences are single peaked for each issue, we can now apply Shepsle's (1979) result and characterize the structure-induced equilibrium of the game. To proceed with the analysis, we impose the following assumption.

Assumption 2: $1 > \frac{4(2+n)}{(1+n)\bar{w}} \left(\frac{\eta}{2+n}\right)^{1/\sigma}$.

This assumption implies that the preferred share in (6) and (8), which attains the highest value at $\tau = 1/2$, takes a positive value at $\tau = 1/2$. Therefore, the assumption ensures that a political equilibrium exists with $\lambda > 0$ for a certain range of τ; otherwise, $\lambda = 0$ holds for any $\tau \in [0, 1]$, implying a trivial outcome of no provision of PAYG social security.

The structure-induced equilibrium outcome is found as follows. First, we determine the decisive voter over λ and calculate his/her most preferred share, denoted by $\lambda^{dec}(\tau)$, as a function of the tax rate τ, where the superscript *dec* indicates the decisive voter. Second, we determine the decisive voter over τ and calculate his/her most preferred tax rate, denoted by $\tau^{dec}(\lambda)$, as a function of the share parameter λ. Finally, we find the point where these reaction functions $\lambda^{dec}(\tau)$ and $\tau^{dec}(\lambda)$ cross. This point corresponds to the structure-induced equilibrium outcome of the voting game.

First, consider the political determination of λ. The decisive voter over λ is a young individual because (i) the population size of the young is larger than that of the old and (ii) all young individuals have the same preferences for λ regardless of their type, as shown in (6) and (8). Therefore, from (6) and (8), the decisive voter's reaction function $\lambda^{dec}(\tau)$ is given by:

$$\lambda^{dec}(\tau) = \begin{cases} 0 & \text{if } \tau \in [0, \underline{\tau}] \\ 1 - \frac{2+n}{(1+n)(1-\tau)\tau\bar{w}} \left(\frac{\eta}{2+n}\right)^{1/\sigma} & \text{if } \tau \in \left(\underline{\tau}, \frac{1}{2}\right]. \end{cases} \tag{9}$$

Next, consider the political determination of τ. The decisive voter over τ belongs to the young generation because (i) young individuals choose lower tax rates than the old and (ii) the population size of the young is larger than that of the old. In particular, to determine the type of the decisive voter, we focus on the parameter σ representing the inverse of the interest-rate elasticity of young-age consumption and consider two cases

[3]The equivalence stems from the assumption of a quasi-linear utility function of old-age consumption. In Section 6, we show how the preferences over λ change when this assumption is removed.

separately: a high elasticity ($1/\sigma \geq 1$ in Subsection 4.1) and a low elasticity ($1/\sigma < 1$ in Subsection 4.2).

We adopt the above classification because the order of preferences for the tax rate critically depends on the degree of interest-rate elasticity. For the case of $1/\sigma \geq 1$, a lower-income young individual prefers a higher tax rate. However, for the case of $1/\sigma < 1$, a low-income young individual may prefer a lower tax rate than middle-income (or middle- and high-income) individuals. For each case, we show the existence and uniqueness of a structure-induced equilibrium of the voting game and explain the mechanism underlying the result.

4.1 The Case of a High Interest-rate Elasticity of Consumption ($1/\sigma \geq 1$)

To determine the type of a decisive voter over τ in the case of $1/\sigma \geq 1$, we consider the preferred tax rate of a type-j young in the following way. When $\tau < \hat{\tau}(w^j)$, he/she saves part of his/her income, and his/her preference for τ follows (5). When $\tau \geq \hat{\tau}(w^j)$, he/she saves nothing and his/her preference over τ follows (7). From (5) and (7), given λ, the preferred tax rate by the type-j young satisfies the following condition:

$$\underbrace{\beta(1+n)\lambda(1-2\tau)\bar{w} + \beta\eta\left\{\frac{1+n}{2+n}(1-\lambda)\bar{w}\right\}^{1-\sigma}\frac{1-2\tau}{((1-\tau)\tau)^{\sigma}}}_{LHS}$$
$$= \underbrace{\begin{cases} \beta R w^j & \text{if } \tau < \hat{\tau}(w^j) \\ \frac{(w^j)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \tau \geq \hat{\tau}(w^j), \end{cases}}_{RHS^j} \tag{10}$$

where LHS and RHS^j show marginal benefit and loss of taxation in terms of utility, respectively.

Figure 4 illustrates the condition (10) that determines the preferred tax rate by a type-j young ($j = L, M, H$) individual. The left-hand side of (10), denoted by LHS, is decreasing in τ and is independent of the type of young individual. In contrast, the right-hand side of (10), denoted by RHS^j, is nondecreasing in τ, and dependent on the type of young individual and featured by $RHS^H \geq RHS^M \geq RHS^L$, where an equality holds if and only if $\sigma = 1$. The kink point of $\tau = \hat{\tau}(w^j)$ implies that a type-j young can save part of his/her income if $\tau < \hat{\tau}(w^j)$ and nothing if $\tau \geq \hat{\tau}(w^j)$. It is immediately obvious from Figure 4 that given λ, a lower-income young individual prefers a higher tax rate: $\tau^{yH} < \tau^{yM} < \tau^{yL}$ for all $\lambda \in [0, 1]$, where $\tau^{yj} (j = L, M, H)$ denotes the preferred tax rate of a type-j young individual.

[Figure 4 about here.]

Given the assumption of demographic structure (Assumption 1) and the fact $\tau^{yH} < \tau^{yM} < \tau^{yL} < \tau^{oj}$, the decisive voter over τ is the one who prefers the highest tax rate among young individuals, that is, a type-L young individual. Therefore, the reaction function of $\tau, \tau^{dec}(\lambda)$, is implicitly given by (10) with $j = L$. To find the crossing point of the two reaction functions, $\lambda^{dec}(\tau)$ and $\tau^{dec}(\lambda)$, we substitute (9) into (10) with $j = L$ to obtain:

$$y(\tau; \bar{w}, n) = z(\tau; w^L),$$

where:

$$y(\tau; \bar{w}, n) = \begin{cases} (1+\beta)\eta \left\{ \frac{1+n}{2+n} \bar{w} \right\}^{1-\sigma} \frac{1-2\tau}{((1-\tau)\tau)^{\sigma}} & \text{if } \tau \leq \underline{\tau} \\ \beta(1+n)(1-2\tau)\bar{w} & \text{if } \tau > \underline{\tau} \end{cases}$$

$$z(\tau; w^L) = \begin{cases} \beta R w^L & \text{if } \tau < \hat{\tau}(w^L) \\ \frac{(w^L)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \tau \geq \hat{\tau}(w^L). \end{cases}$$

Solving $y(\tau; \bar{w}, n) = z(\tau; w^L)$ for τ leads to the tax rate in a structure-induced equilibrium of the voting game. The corresponding λ is obtained by substituting the equilibrium τ into the reaction function λ^{dec} in (9).

Proposition 1. *Suppose that $1/\sigma \geq 1$ holds. There exists a unique structure-induced equilibrium of the voting game such that the decisive voter over τ is a type-L young individual.*

Proof. See Appendix 8.1.

Figure 5 illustrates two examples. Panel (a) depicts the case where the wage of type-L individuals is high such that they can save part of their income in youth for future consumption. In this case, the equilibrium tax rate represented by the crossing point of $y(\tau; \bar{w}, n)$ and $z(\tau; w^L)$ is below the critical rate of $\hat{\tau}(w^L)$. In contrast, panel (b) illustrates the case where the wage of type-L individuals is low such that they save nothing in their youth. The equilibrium tax rate is given above the critical rate of $\hat{\tau}(w^L)$.

[Figure 5 about here.]

4.2 The Case of a Low Interest-rate Elasticity of Consumption ($1/\sigma < 1$)

Next, consider the case of a low interest-rate elasticity such that $1/\sigma < 1$. The decisive voter over λ is equivalent to that in the previous case: that is, the reaction function

$\lambda^{dec}(\tau)$ is given by (9). However, the decisive voter over τ may differ from the previous case: that is, the order of preferred tax rates may change depending on the value of λ.

To determine the decisive voter over τ, let us recall the condition (10) that determines the tax rate preferred by a type-j young individual for a given λ. The graphs of (10) for $j = L, M, H$ are illustrated in Figure 6. The main difference from the previous case is that RHS^i and $RHS^j (i \neq j)$ cross at some tax rate $\tau \in (0, 1/2)$. This is because when a type-j individual is borrowing constrained, the slope of RHS^j becomes steeper as the elasticity $1/\sigma$ becomes lower. Thus, there are two critical values of τ, $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$, such that RHS^L and RHS^H cross at $\tau = \tilde{\tau}^{LM}$ and RHS^M and RHS^H cross at $\tau = \tilde{\tau}^{MH}$. By direct calculation, we obtain:

$$\tilde{\tau}^{LM} \equiv 1 - \left(\frac{(w^L)^{1-\sigma}}{\beta R w^M} \right)^{1/\sigma} \text{ and } \tilde{\tau}^{MH} \equiv 1 - \left(\frac{(w^M)^{1-\sigma}}{\beta R w^H} \right)^{1/\sigma},$$

where $\hat{\tau}(w^L) < \tilde{\tau}^{LM} < \hat{\tau}(w^M) < \tilde{\tau}^{LM} < \hat{\tau}(w^H)$ (see Figure 6). The derivation of $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$ is given in Appendix 8.2.

[Figure 6 about here.]

The tax rate preferred by a type-j young is determined by the crossing point of LHS and RHS of (10). RHS is independent of λ while LHS is strictly increasing in λ. Thus, the tax rate preferred by a type-j young depends on the size of λ. Overall, he/she prefers a higher tax rate when λ is higher.

The order of tax rates preferred by the three types of agents is changed by the size of λ as illustrated in Figure 6. First, when λ is low such that LHS of (10) crosses RHS of (10) with $j = L$ within the range $(0, \tilde{\tau}^{LM}]$, the tax rates preferred by the young are ordered by $\tau^{yH} < \tau^{yM} < \tau^{yL}$, where $\tau^{yj} (j = L, M, H)$ denotes the preferred tax rate by type-j young: the type-L young individual becomes the decisive voter. Second, when λ attains a middle value such that LHS of (10) crosses RHS of (10) with $j = M$ within the range $(\tilde{\tau}^{LM}, \tilde{\tau}^{MH}]$, the tax rates preferred by the young are ordered by $\tau^{yH} < \tau^{yL} < \tau^{yM}$ or $\tau^{yL} \leq \tau^{yH} < \tau^{yM}$: the decisive voter in this case is the type-M young individual. Finally, when λ is high such that LHS of (10) crosses RHS of (10) with $j = H$ within the range $[\tilde{\tau}^{MH}, 1/2]$, the tax rates preferred by the young are ordered by $\tau^{yL} < \tau^{yM} < \tau^{yH}$: the decisive voter becomes the type-H young individual.

Given the abovementioned feature, the reaction function of τ, $\tau = \tau^{dec}(\lambda)$, is now implicitly given by:

$$\beta(1+n)\lambda(1-2\tau)\bar{w} + (1+\beta)\eta \left\{ \frac{1+n}{2+n}(1-\lambda)\bar{w} \right\}^{1-\sigma} \cdot \frac{1-2\tau}{((1-\tau)\tau)^{\sigma}} = \tilde{z}(\tau; w^L, w^M, w^H), \quad (11)$$

where:

$$\tilde{z}(\tau; w^L, w^M, w^H) \equiv \begin{cases} \beta R w^L & \text{if } \tau < \hat{\tau}(w^L) \\ \frac{(w^L)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \hat{\tau}(w^L) \leq \tau \leq \tilde{\tau}^{LM} \\ \beta R w^M & \text{if } \tilde{\tau}^{LM} < \tau < \hat{\tau}(w^M) \\ \frac{(w^M)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \hat{\tau}(w^M) \leq \tau \leq \tilde{\tau}^{MH} \\ \beta R w^H & \text{if } \tilde{\tau}^{MH} < \tau < \hat{\tau}(w^H) \\ \frac{(w^H)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{if } \hat{\tau}(w^H) \leq \tau < \frac{1}{2}. \end{cases}$$

The graph of the function $\tilde{z}$ is illustrated by the bold curve in Figure 6.

We substitute the reaction function of $\lambda^{dec}(\tau)$ given by (9) into the left-hand side of (11) to obtain the condition that determines the equilibrium tax rate:

$$y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H).$$

Figure 7 illustrates the graphs of $y(\tau; \bar{w}, n)$ and $\tilde{z}(\tau; w^L, w^M, w^H)$. Solving $y(\tau; \bar{w}, n) = \tilde{z}(\tau; w^L, w^M, w^H)$ for τ leads to the tax rate in a structure-induced equilibrium of the voting game. The corresponding λ is obtained by substituting the equilibrium τ into the reaction function λ^{dec} in (9).

[Figure 7 about here.]

Proposition 2. *Suppose that $1/\sigma < 1$ holds. There exists a unique structure-induced equilibrium of the voting game such that the decisive voter over τ is:*

(i) *a type-L individual if $y(\tilde{\tau}^{LM}; \bar{w}) \leq \beta R w^M$;*

(ii) *a type-M individual if $\beta R w^M < y(\tilde{\tau}^{LM}; \bar{w})$ and $y(\tilde{\tau}^{MH}; \bar{w}) \leq \beta R w^H$; and*

(iii) *a type-H individual if $\beta R w^H < y(\tilde{\tau}^{MH}; \bar{w})$.*

Proof. See Appendix 8.3.

A noteworthy feature of the result in Proposition 2 is that under certain conditions, the middle-income (or middle- and high-income) individuals prefer a higher tax rate than the low-income individuals. In particular, if the condition in statement (ii) in Proposition 2 holds, there exists an ends-against-the-middle equilibrium where the low- and high-income young individuals form a coalition in favor of a low tax rate and the middle-income individual favoring a high tax rate becomes the decisive voter (see Figure 7). In contrast, if the condition in statement (iii) holds, the low- and middle-income individuals form a coalition and the high-income young individual becomes the decisive voter.

The key factors in the result in Proposition 2 are the borrowing constraints and the interest-rate elasticity of consumption. To understand the roles of these two factors, consider where the low-income individuals are faced with a borrowing constraint. Here, they wish to consume more in their youth, but cannot because of the borrowing constraint. In this situation, a higher tax rate produces two opposing effects: a negative effect that results in lower after-tax income and thus the utility loss of taxation in youth, and a positive effect that produces higher social security benefit and thus the utility gain in old age.

The net impact of taxation depends on the interest-rate elasticity. When the elasticity is high such that $1/\sigma > 1$, the positive effect overcomes the negative effect. The low-income individual then chooses the highest tax rate among the young and thus becomes the decisive voter. When $1/\sigma = 1$, both effects perfectly cancel out. The three types of young then choose the same tax rate. Finally, when the elasticity is low such that $1/\sigma < 1$, the negative effect may dominate the positive effect. In particular, if $\beta R w^M < y(\tilde{\tau}^{LM}; \bar{w})$ and $y(\tilde{\tau}^{MH}; \bar{w}) \leq \beta R w^H$ hold as in Proposition 2(ii), the negative effect dominates the positive effect for low-income individuals. They choose a lower tax rate than the middle-income individuals, and this results in an equilibrium where the middle-income individual is the decisive voter.

5 Effects of Income Inequality on Policy

Given the characterization of the political equilibrium in Section 4, we wish to investigate how the tax rate (τ), the share of social security (λ), the level of social security benefit (b) and public goods provision (g) change in response to changes in income inequality. In particular, we consider a mean-preserving reduction of the decisive voters' wage in order to compare two groups of countries with similar per capita income levels but different levels of income inequality. We focus on the nontrivial equilibrium with $\lambda > 0$ to observe the marginal effect on the share of social security in government expenditure.

Proposition 3. *Consider a political equilibrium with $\lambda > 0$.*

(i) *In an economy with $1/\sigma \geq 1$ where the decisive voter is a type-L young individual, a mean-preserving reduction of w^L increases the tax rate (τ), the share of social security (λ) and the level of social security benefit (b), and keeps the level of public goods (g) constant.*

(ii) *In an economy with $1/\sigma < 1$ where the decisive voter is a type-j ($j = L, M, H$) young individual, a mean-preserving change in the decisive voter's wage (w^j) locally produces inverse U-shaped relationships between w^j and the tax rate (τ), between w^j and the share of social security (λ), and between w^j and the level of social security* *benefit (b) if the equilibrium tax rate is initially given by* $\tau^{equil} = \hat{\tau}(w^j)$*; and keeps the level of public goods* (g) *constant.*

Proof. See Appendix 8.4.

Proposition 3 states that if the interest-rate elasticity is high such that $1/\sigma \geq 1$, there is a monotone relationship between the decisive voter's wage and his/her preferred tax rate. The decisive voter then prefers a higher tax rate as he/she becomes poorer. However, when the elasticity is low such that $1/\sigma < 1$, such a monotone relationship no longer holds. Once the decisive voter's wage falls below the critical level that changes his/her status from unconstrained to constrained, he/she prefers a lower tax rate as he/she becomes poorer. There is then an inverse U-shaped relationship between the decisive voter's wage and the preferred tax rate around the critical wage.

To understand the result intuitively, let us assume that the decisive voter is a type-L individual regardless of the value of σ, and consider cases where the decisive voter is borrowing unconstrained and constrained, respectively. The following argument also holds for the case of $1/\sigma < 1$ where the decisive voter is a type-M or type-H individual. Figure 8 illustrates the effects of a mean-preserving change in the decisive voter's wage on the equilibrium tax rate. Panel (a) is for the case of $1/\sigma \geq 1$; Panel (b) is for the case of $1/\sigma < 1$.

[Figure 8 about here.]

To proceed with the analysis, we first assume that the decisive voter is borrowing unconstrained: $s^L > 0$. Given a tax rate τ, a reduction of w^L decreases the after-tax income of the decisive voter in youth, but has no effect on the consumption in youth because $c^{yj} = (\beta R)^{-1/\sigma}$ if $s^L > 0$. The negative income effect is then absorbed by saving because of the assumption of a quasi-linear utility function. In other words, a reduction of w^L only has an effect on consumption in old age. The decisive voter then wishes to offset the loss of saving by increasing the PAYG social security benefits. Therefore, a mean-preserving reduction of the decisive voter's wage gives him/her an incentive to choose a higher tax rate when he/she is borrowing unconstrained.

Next, suppose that the decisive voter is borrowing constrained: $s^L = 0$: the equilibrium tax rate satisfies $y(\tau; \bar{w}, n) = (w^L)^{1-\sigma}/(1-\tau)^{\sigma}$, where the right-hand side is the marginal utility loss of taxation when the decisive voter is borrowing constrained. A marginal reduction of w^L affects the marginal utility loss of taxation such that $\partial (w^L)^{1-\sigma}/(1-\tau)^{\sigma}/\partial w^L = (1-\sigma)(w^L)^{-\sigma}/(1-\tau)^{\sigma} \gtreqless 0 \Leftrightarrow 1/\sigma \gtreqless 1$. When $1/\sigma \geq 1$, a reduction of w^L leads to a decrease in the marginal utility loss, thereby giving the decisive voter an incentive to increase the tax rate. However, when $1/\sigma < 1$ holds, a reduction of w^L leads to the opposite effect. Therefore, if $1/\sigma < 1$, a mean-preserving reduction of the decisive voter's wage gives him/her an incentive to choose a lower tax rate when he/she is borrowing constrained.

The share of social security (λ) changes in the same direction as τ because the decisive voter's reaction function $\lambda^{dec}(\tau)$, given by (9), is increasing in τ as long as $\tau \in (0, 1/2)$. The social security benefit (b) also changes in the same direction as τ because b is given by $b = (1+n)\lambda\tau(1-\tau)$ from the government budget constraint. However, the level of public goods (g), given by $g = (1+n)(1-\lambda)\tau(1-\tau)\bar{w}/(2+n)$, remains unchanged because an increase (decrease) in the tax revenue, $\tau(1-\tau)\bar{w}$, is offset by a decrease (increase) in the share of public goods provision, $(1-\lambda)$.

The result established in Proposition 3 implies that an economy with a low interest-rate elasticity (i.e., the case of $1/\sigma < 1$) exhibits an inverse U-shaped relationship between the decisive voter's wage and the level of social security, as illustrated in Panel (c) of Figure 8. When the decisive voter (i.e., a type-L individual in the current assumption) is borrowing unconstrained, a lower w^L results in a higher tax rate, a higher share of social security and thus a higher level of social security benefit. However, when the decisive voter is borrowing constrained, a lower w^L results in a lower tax rate, a lower share of social security and thus a lower level of social security benefit.

Empirical Implications

As mentioned in Section 1, standard political economy theory suggests a positive correlation between inequality and social security: higher inequality results in greater redistribution. However, cross-country data do not necessarily support this prediction. For example, Gottschalk and Smeeding (1997) and Chen and Song (2009) find a negative correlation between wage inequality and social security: put differently, countries with smaller earnings inequality have, on average, greater social security as a percentage of GDP. For example, the United Kingdom and the United States feature high income inequality, a low tax rate and a low level of social security benefits. In contrast, many continental European and Nordic countries feature low income inequality, a high tax rate and a high level of social security benefits.

In the current framework, the negative correlation arises only in the equilibrium where the following two conditions hold: (i) the interest-rate elasticity of consumption is low and (ii) the decisive voter is borrowing constrained. When one of the conditions fails to hold, the economy displays a positive correlation between income inequality and the tax rate. This is inconsistent with the empirical evidence. Therefore, our analysis suggests that these factors are the key to explaining cross-country differences in income inequality, tax rates and the share of social security.

6 A Generalized Utility Function

To this point, we have conducted the analysis by assuming a quasi-linear utility function where the utility of old-age consumption is given by βc_{t+1}^{oj}. This specification enables us to illustratively show the existence and uniqueness of the political equilibrium. However, the specification also results in (i) a saving decision unaffected by social security; and (ii) type-independent preferences over the share of social security. We introduce a nonlinear utility function of old-age consumption to resolve these problems. We show that most of the previous results still hold true under this alternative utility function. That is, under a certain condition, there exists the same ends-against-the-middle equilibrium when the interest-rate elasticity is low and the decisive voter is borrowing constrained. In this equilibrium, a mean-preserving spread of income inequality results in a lower equilibrium tax rate and a lower share of social security in government expenditure.

For the purpose of analysis, we assume the following utility function:

$$U_t^j = \frac{(c_t^{yj})^{1-\sigma} - 1}{1-\sigma} + \eta \frac{(g_t)^{1-\sigma} - 1}{1-\sigma} + \beta \left[\frac{(c_{t+1}^{oj})^{1-\sigma} - 1}{1-\sigma} + \eta \frac{(g_{t+1})^{1-\sigma} - 1}{1-\sigma} \right].$$

The main difference from the previous model is that the utility of old-age consumption is given by $\beta\{(c_{t+1}^{oj})^{1-\sigma} - 1\}/(1-\sigma)$. We maximize their lifetime utility under the budget constraints and obtain the following saving function:

$$s_t^j = \max\left\{0, \frac{(\beta R)^{1/\sigma}}{(\beta R)^{1/\sigma} + R}\left[(1-\tau_t)w^j - \frac{b_{t+1}}{(\beta R)^{1/\sigma}}\right]\right\}.$$

Saving now depends on the social security benefit b_{t+1} that gives agents a disincentive to save. We hereafter drop the time subscript because our focus is on the time-invariant policy.

We substitute the government budget constraint for social security $b = (1+n)\lambda\tau(1-\tau)\bar{w}$ into the above saving function to obtain the following condition that determines the saving behavior of a type-j individual:

$$s^j > 0 \Leftrightarrow \frac{w^j}{\bar{w}} \cdot \frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau.$$

This inequality condition states that a type-j individual is borrowing unconstrained if his/her wage is high, the tax burden is low, and/or the share of social security in government expenditure is also low.

With the saving function and the government budget constraints, we give the consumption functions of a type-j individual in youth and old age as follows:

$$c_t^{yj} = \begin{cases} \frac{R}{(\beta R)^{1/\sigma}+R}\left[(1-\tau)w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right] & \text{if } \frac{\bar{w}}{w^j}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ (1-\tau)w^j & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} \le \lambda\tau \end{cases}$$

$$c_{t+1}^{oj} = \begin{cases} \frac{(\beta R)^{1/\sigma}R}{(\beta R)^{1/\sigma}+R}\left[(1-\tau)w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right] & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ (1+n)\lambda(1-\tau)\tau\bar{w} & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} \le \lambda\tau. \end{cases}$$

Unlike the previous case, the consumption in youth is now type-dependent and is linearly related to lifetime income when agents are borrowing unconstrained.

After some calculation, we can obtain indirect utility functions of type-j young and old individuals as follows:

$$V^{yj} = \begin{cases} V_{s>0}^{y,j} \equiv \frac{\phi}{1-\sigma}\left[(1-\tau)w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right]^{1-\sigma} + \frac{\beta\eta}{1-\sigma}\left[\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{1-\sigma} \\ \qquad\qquad\qquad \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ V_{s=0}^{y,j} \equiv \frac{\left((1-\tau)w^j\right)^{1-\sigma}}{1-\sigma} + \frac{\beta}{1-\sigma}\left[(1+n)\lambda(1-\tau)\tau\bar{w}\right]^{1-\sigma} + \frac{\beta\eta}{1-\sigma}\left[\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{1-\sigma} \\ \qquad\qquad\qquad \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} \le \lambda\tau \end{cases}$$

$$V^{oj} = \begin{cases} V_{s>0}^{y,j} \equiv \frac{1}{1-\sigma}\left[Rs_{-1}^j + (1+n)\lambda(1-\tau)\tau\bar{w}\right]^{1-\sigma} & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} > \lambda\tau \\ V_{s=0}^{y,j} \equiv \frac{\beta\eta}{1-\sigma}\left[\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{1-\sigma} & \text{if } \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n} \le \lambda\tau, \end{cases}$$

where variables unrelated to political decisions are dropped from V^{yj} and V^{oj}.

The policy preferences of the old are the same as for quasi-linear utility. That is, regardless of type and saving behavior, the old wish to maximize the tax revenue from the young and use it exclusively for social security: $\tau^{oj} = 1/2$ and $\lambda^{oj} = 1$ hold for all j. Accordingly, generalization of the utility function does not affect the policy preferences of the old.

We next consider the policy preferences of the young. The preferred tax rate of a type-j young individual satisfies the following first-order condition with respect to τ:

$$LHS^y = RHS^{yj} \equiv \begin{cases} RHS_{s>0}^{yj} & \text{if } \tau < \tau^*(w^j,\lambda) \equiv \frac{w^j}{\bar{w}}\cdot\frac{(\beta R)^{1/\sigma}}{1+n}\cdot\frac{1}{\lambda} \\ RHS_{s=0}^{yj} & \text{if } \tau \ge \tau^*(w^j,\lambda) \end{cases}, \tag{12}$$

where:

$$LHS^y \equiv \beta\eta\left[\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{-\sigma}\cdot\frac{1+n}{2+n}(1-\lambda)(1-2\tau)\bar{w},$$

$$RHS_{s>0}^{yj} \equiv \phi\left[(1-\tau)w^j + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right]^{-\sigma}\cdot\left[w^j - \frac{(1+n)\lambda(1-2\tau)\bar{w}}{R}\right], \text{and}$$

$$RHS_{s=0}^{yj} \equiv (1-\tau)^{-\sigma}\left(w^j\right)^{1-\sigma} - \beta\left[(1+n)\lambda(1-\tau)\tau\bar{w}\right]^{-\sigma}(1+n)\lambda(1-2\tau)\bar{w}.$$

LHS^y represents the marginal benefit of taxation in terms of the utility of public goods. This benefit is common to the three types of young agents because of the nature of public goods. RHS^{yj} represents the marginal loss of taxation plus the marginal benefit of social security in terms of the utility of consumption. However, the sum of these losses and benefits differs across agents. In particular, the following properties hold (see Appendix 8.5 for the proof):

$$\begin{cases} RHS^{yL} < RHS^{yM} < RHS^{yH} & \text{if } 1/\sigma \geq 1 \\ RHS^{yL}_{s=0} > RHS^{yM}_{s=0} > RHS^{yH}_{s=0} & \text{if } 1/\sigma < 1. \end{cases} \tag{13}$$

Similar to the previous model, the order of $RHS^{yj}_{s=0}(j = L, M, H)$ critically depends on the degree of interest-rate elasticity $1/\sigma$.[4]

Panel (a) of Figure 9 illustrates the graph of (12) when $1/\sigma \geq 1$ holds. The crossing point of LHS^y and RHS^{yj} determines the tax rate preferred by a type-j young agent. The figure shows that a lower-income young individual prefers a higher tax rate. Under the demographic structure assumption given in Assumption 1, a type-L young agent becomes the decisive voter over τ. That is, the-ends-against-the-middle equilibrium never arises when the interest-rate elasticity is high such that $1/\sigma \geq 1$.

[Figure 9 about here.]

Panel (b) of Figure 9 illustrates the graph of (12) when $1/\sigma < 1$ holds. A noteworthy feature is that lower-income young prefer a lower tax rate when agents are borrowing constrained. In particular, there may arise an equilibrium where the low- and the high-income young agents form a coalition against the middle, as illustrated in Panel (b) of Figure 9. Therefore, the low interest-rate elasticity and the borrowing constraint remain the key to the existence of the ends-against-the-middle equilibrium.

The political determination of the share of social security λ is slightly different from that in the previous quasi-linear utility case. The preferred share of a type-j young is given by:

$$\lambda^{yj} = \begin{cases} 0 & \text{if} \quad \tau \leq \tilde{\tau}(w^j) \\ \frac{1/(2+n) - (\beta\eta R/(2+n))^{1/\sigma}\left(w^j/\tau\bar{w}(1+n)\right)}{1/(2+n) + (\beta\eta R/(2+n))^{1/\sigma}/R} & \text{if } \tilde{\tau}(w^j) < \tau < \hat{\tau}(w^j) \\ \frac{1/(2+n)}{1/(2+n) + (\eta/(2+n))^{1/\sigma}} & \text{if} \quad \hat{\tau}(w^j) \leq \tau \end{cases} \tag{14}$$

[4] If $1/\sigma < 1$, the order of $RHS^{yj}_{s>0}(j = L, M, H)$ is ambiguous.

where

$$\tilde{\tau}(w^j) \equiv \left(\frac{\beta\eta R}{(2+n)}\right)^{1/\sigma} \cdot \frac{w^j(2+n)}{\bar{w}(1+n)};$$
$$\hat{\tau}(w^j) \equiv \frac{w^j(2+n)}{\bar{w}(1+n)}(\beta R)^{1/\sigma}\left\{\frac{1}{2+n}+\left(\frac{\eta}{2+n}\right)^{1/\sigma}\right\}.$$

The first two lines of the right-hand side in (14) represent the choice of λ when a type-j young is borrowing unconstrained; the third line represents the choice of λ when he/she is borrowing constrained. The derivation of (14) is given in Appendix 8.5.

When the tax burden is low, such that $\tau \leq \tilde{\tau}(w^j)$, a type-$j$ young individual can save much for his/her old-age consumption and thus finds it unnecessary to use tax revenue for social security: $\lambda = 0$. However, when the tax is above $\tilde{\tau}(w^j)$, a type-j young individual finds it optimal to offset part of their tax-induced consumption loss with a social security benefit. In particular, a lower income agent prefers a higher share of social security.

A type-j young individual is borrowing constrained when the tax rate is high such that $\tau \geq \hat{\tau}(w^j)$. Borrowing-constrained agents choose the same share of social security regardless of their type. This is because they have the same level of old-age consumption that is equal to the lump-sum pension benefit. They then choose that share to equate the marginal utilities of old-age consumption and public goods, both of which are type-independent.

Panel (c) of Figure 9 illustrates the reaction function of λ for each type of an agent. The figure shows that $\lambda^{yH}(\tau) \leq \lambda^{yM}(\tau) \leq \lambda^{yL}(\tau) < \lambda^o$ holds for any τ. Thus, under the demographic structure in Assumption 1, a type-L young agent becomes the decisive voter. We can derive the political equilibrium tax rate by substituting $\lambda = \lambda^{yL}$ into the decisive voter's first-order condition with respect to τ.

Given a brief characterization of the political equilibrium, we now compare the income inequality effects between the current and former models. In particular, we focus on the situation where the decisive voters over τ and λ are borrowing constrained. Thus, the decisive voter's choice of λ in the current framework is given by:

$$\lambda^{dec} = \frac{1}{1+(2+n)^{(\sigma-1)/\sigma}(\eta)^{1/\sigma}},$$

which is independent of τ. We substitute this into (12) for the case of $\tau \geq \tau^*(w^j, \lambda)$ and obtain the following condition that determines the equilibrium tax rate when the decisive voter is borrowing constrained:

$$\beta\eta\left[\frac{1+n}{2+n}(1-\lambda)(1-\tau)\tau\bar{w}\right]^{-\sigma}\frac{1+n}{2+n}(1-\lambda)(1-2\tau)\bar{w}$$
$$=(1-\tau)^{-\sigma}\left(w^j\right)^{1-\sigma}-\beta\left[(1+n)\lambda(1-\tau)\tau\bar{w}\right]^{-\sigma}(1+n)\lambda(1-2\tau)\bar{w},$$

where λ is now constant. Given τ, the left-hand side is independent of w^j whereas the right-hand side is decreasing (increasing) in w^j if $1/\sigma < (>)1$. Thus, a mean-preserving reduction of the decisive voter's wage decreases (increases) the equilibrium tax rate if the interest-rate elasticity is low (high) such that $1/\sigma < (>)1$. This result is qualitatively equivalent to that in the quasi-linear utility function model.

The corresponding levels of social security and public goods are given by $b = (1+n)\lambda\tau(1-\tau)\bar{w}$ and $g = ((1+n)/(2+n))(1-\lambda)\tau(1-\tau)\bar{w}$. Because λ is unchanged while τ is decreased (increased) by the mean-preserving reduction of w^j if $1/\sigma < (>)1$, b and g are decreased (increased) if the interest-rate elasticity is low (high) such that $1/\sigma < (>)1$. The main departure from the result in the previous model is that public goods provision is now nonneutral with respect to the mean-preserving expansion of income inequality.

7 Conclusion

Why is wage inequality negatively correlated with the level of social security? How does wage inequality affect the allocation of tax revenue between social security and forward intergenerational public goods provision? This paper develops a political economy model that responds to both these important questions.

Two features are crucial to our analysis and results: the interest-rate elasticity of consumption and the borrowing constraint. These features derive an ends-against-the-middle equilibrium where low- and high-income individuals form a coalition in favor of a low tax rate and middle-income individuals favor a high tax rate. In addition, higher wage inequality results in a lower level of social security and a lower share of social security in government expenditure when the decisive voter is borrowing constrained and the interest-rate elasticity is low.

The negative correlation between inequality and social security we obtain appears to fit the available empirical evidence. This correlation arises only in the equilibrium where the abovementioned conditions are effective for the decisive voter. When one of the conditions fails to hold, the economy displays a positive, rather than negative, correlation between inequality and social security. Thus, our analysis suggests that the two conditions play a key role in explaining cross-country differences in inequality and social security.

To obtain these results, we simplify the analysis by adopting a quasi-linear utility function. Because of this simplification, we can remove the link between saving and the allocation of tax revenue between social security and public goods provision. However, and as shown in our analysis, the effect of the interest-rate elasticity of consumption on the political determination of the tax rate and the allocation of tax revenue remains. In addition, Section 6 demonstrates that the main results are qualitatively unchanged under a generalized utility function except for the level of public goods. Thus, our analysis is almost robust to the assumption of a quasi-linear utility function.

8 Appendix

8.1 Proof of Proposition 1

As shown in the text, when $1/\sigma \geq 1$, the decisive voter is a type-L individual and his/her preferred tax rate satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$. The functions $y(\tau;\bar{w},n)$ and $z(\tau;w^L)$ have the following properties: $\partial y(\tau;\bar{w},n)/\partial\tau < 0, \lim_{\tau\to\infty} y(\tau;\bar{w},n) = \infty, y(1/2;\bar{w},n) = 0, \partial z(\tau;w^L)/\partial\tau \geq 0, z(0;w^L) = \max\{\beta R w^L, (w^L)^{1-\sigma}\} < \infty$, and $z(1/2;w^L) \in (0,\infty)$. These properties indicate that there exists a unique $\tau \in (0,1/2)$ that satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$. ■

8.2 The derivation of $\tilde{\tau}^{LM}$ and $\tilde{\tau}^{MH}$

The derivation of $\tilde{\tau}^{LM}$ is as follows. For the range of $(\hat{\tau}(w^L), \hat{\tau}(w^M))$, the right-hand side of (10), denoted by RHS^j, is given by:

$$RHS^j = \begin{cases} RHS^L = (w^L)^{1-\sigma}/(1-\tau)^\sigma & \text{for } j = L \\ RHS^M = \beta R w^M & \text{for } j = M. \end{cases}$$

$RHS^L < RHS^M$ holds at $\tau = \hat{\tau}(w^L)$; $RHS^L > RHS^M$ holds at $\tau = \hat{\tau}(w^M)$. Thus, there exists a unique τ, denoted by $\tilde{\tau}^{LM} \in (\hat{\tau}(w^L), \hat{\tau}(w^M))$, that satisfies $RHS^L = RHS^M$ because RHS^L is continuous and strictly increasing in τ whereas RHS^M is independent of τ. We can derive $\tilde{\tau}^{LM}$ by solving $(w^L)^{1-\sigma}/(1-\tau)^\sigma = \beta R w^M$ for τ.

By the same token, the tax rate that satisfies $RHS^M = RHS^H$ for the range of $(\hat{\tau}(w^M), \hat{\tau}(w^H))$ is derived by solving $(w^M)^{1-\sigma}/(1-\tau)^\sigma = \beta R w^H$ for τ. The solution is denoted by $\tilde{\tau}^{MH}$.

8.3 Proof of Proposition 2

As shown in the text, when $1/\sigma < 1$, the decisive voter's preferred tax rate satisfies $y(\tau;\bar{w},n) = \tilde{z}(\tau;w^L,w^M,w^H)$. The functions $y(\tau;\bar{w},n)$ and $\tilde{z}(\tau;w^L,w^M,w^H)$ have the following properties:

$$
\begin{aligned}
\partial y(\tau;\bar{w},n)/\partial\tau &< 0, \\
\lim_{\tau\to\infty} y(\tau;\bar{w},n) &= \infty, \\
y(1/2;\bar{w},n) &= 0, \\
\partial\tilde{z}(\tau;w^L,w^M,w^H)/\partial\tau &\geq 0, \\
\tilde{z}(0;w^L,w^M,w^H) &= \max\{\beta R w^L,(w^L)^{1-\sigma}\} < \infty, \\
\tilde{z}(1/2;w^L,w^M,w^H) &\in (0,\infty).
\end{aligned}
$$

These properties indicate that there exists a unique $\tau \in (0,1/2)$ satisfying $y(\tau;\bar{w},n) = \tilde{z}(\tau;w^L,w^M,w^H)$. The condition for the determination of the decisive voter over τ can be found from Figure 7. ■

8.4 Proof of Proposition 3

(i) For the case of $1/\sigma \geq 1$, the decisive voter is a type-L young individual and the equilibrium tax rate satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$ as shown in Subsection 4.1. When the mean-preserving change in w^L is considered, $y(\tau;\bar{w},n)$ is unchanged while $z(\tau;w^L)$ is decreased by a reduction of w^L. Therefore, a mean-preserving reduction of w^L results in an increase in the equilibrium tax rate that satisfies $y(\tau;\bar{w},n) = z(\tau;w^L)$. Given $\lambda^{dec}(\tau)$ is increasing in τ for $\tau \in (0,1/2)$, a mean-preserving decrease in w^L results in an increase in λ.

From the government budget constraints for social security (b) and public goods provision (g) in Subsection 2.2, b and g are given by:

$$b = (1+n)\lambda\tau(1-\tau)\bar{w}, \tag{15}$$

$$g = \frac{1+n}{2+n}(1-\lambda)\tau(1-\tau)\bar{w} = \left(\frac{(1+\beta)\eta}{\beta(2+n)}\right)^{1/\sigma}, \tag{16}$$

where the second equality in the equation (16) is obtained by substituting the decisive voter's reaction function $\lambda^{dec}(\tau)$ in (9) into the equation. (15) and (16) imply that b is increased by an increase in τ and λ, thereby implying a negative effect of w^L on b; and g is independent of τ and λ, thereby implying no effect of w^L.

(ii) For the case of $1/\sigma < 1$, the decisive voter in this case is a type-j ($j = L, M$ or H) individual depending on parameter values as shown in Subsection 4.2. To simplify the presentation, suppose that a type-L individual is a decisive voter. Note that the following argument applies for the case where a type-j ($j = M$ or H) is a decisive voter.

Assume that the equilibrium tax rate is given by $\tau^{equil} = \hat{\tau}(w^L)$: a type-$L$ individual is indifferent between saving and not saving. Under this situation, the decisive voter's wage w^L satisfies $\beta(1+n)(1-2\hat{\tau}(w^L))\bar{w} = \beta R w^L$, or:

$$R(w^L)^2 + (1+n)\bar{w}w^L - 2(1+n)\frac{\bar{w}}{(\beta R)^{1/\sigma}} = 0.$$

Solving this equation for w^L, we obtain:

$$w^L = \hat{w}^L \equiv \frac{-(1+n)\bar{w} + \sqrt{\{(1+n)\bar{w}\}^2 + 8R(1+n)\bar{w}/(\beta R)^{1/\sigma}}}{2R}.$$

Therefore, the equilibrium tax rate is given by $\tau^{equil} = \hat{\tau}(w^L)$ when a type-L individual with $w^L = \hat{w}^L$ is a decisive voter.

Let us consider a mean-preserving change of w^L around $\hat{w}^L$. As shown in Subsection 4.2, the equilibrium tax rate satisfies $y(\tau;\bar{w},n) = \tilde{z}(\tau;w^L,w^M,w^H)$ if $1/\sigma < 1$. In particular, around $w^L = \hat{w}^L$, there exists a positive real number ε such that the equilibrium tax rate satisfies the following condition:

$$\tilde{z}(\tau;w^L,w^M,w^H) = \begin{cases} \beta R w^L & \text{for } w^L \in (\hat{w}^L - \varepsilon, \hat{w}^L], \\ \frac{(w^L)^{1-\sigma}}{(1-\tau)^{\sigma}} & \text{for } w^L \in [\hat{w}^L, \hat{w}^L + \varepsilon). \end{cases}$$

We focus on the range $(\hat{w}^L - \varepsilon, \hat{w}^L + \varepsilon)$ and consider a mean-preserving change of w^L around $\hat{w}^L$. The right-hand side of the above equation is increasing in w^L within the range $(\hat{w}^L - \varepsilon, \hat{w}^L)$ and decreasing in w^L within the range $(\hat{w}^L, \hat{w}^L + \varepsilon)$. This property implies that the equilibrium tax rate attains the highest value at $w^L = \hat{w}^L$ within the range $(\hat{w}^L - \varepsilon, \hat{w}^L + \varepsilon)$. Therefore, there is an inverse U-shaped relationship between the decisive voters' wage and the equilibrium tax rate around $w^L = \hat{w}^L$. Given $\lambda^{dec}(\tau)$ is increasing in τ for $\tau \in (0, 1/2)$, there is also an inverse U-shaped relationship between the decisive voters' wage and the equilibrium share of social security around $w^L = \hat{w}^L$. Given the result for τ and λ, the result for b and g arises from (15) and (16). ■

8.5 Supplementary Explanation for Section 6

8.5.1 Derivation of (13)

In order to establish that (13) holds, we focus on the relationship between RHS^{yL} and RHS^{yM}. The result immediately applies to the relationship between RHS^{yM} and RHS^{yH}.

Let us first compare $RHS^{yL}_{s>0}$ and $RHS^{yM}_{s>0}$. We obtain:

$$\begin{aligned}
&RHS^{yL}_{s>0} \lesseqgtr RHS^{yM}_{s>0} \\
&\qquad \Leftrightarrow \left[(1-\tau)w^L + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right]^{-\sigma} \cdot \left[w^L - \frac{(1+n)\lambda(1-2\tau)\bar{w}}{R}\right] \\
&\qquad \lesseqgtr \left[(1-\tau)w^M + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}\right]^{-\sigma} \cdot \left[w^M - \frac{(1+n)\lambda(1-2\tau)\bar{w}}{R}\right] \\
&\qquad \Leftrightarrow \frac{w^L - (1+n)\lambda(1-2\tau)\bar{w}/R}{w^M - (1+n)\lambda(1-2\tau)\bar{w}/R} \lesseqgtr \left[\frac{(1-\tau)w^L + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}}{(1-\tau)w^M + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}}\right]^{\sigma}. \qquad (17)
\end{aligned}$$

The left-hand side of (17) is smaller than w^L/w^M whereas the the term in the large parentheses on the right-hand side is larger than w^L/w^M; that is:

$$\begin{aligned}
&\frac{w^L - (1+n)\lambda(1-2\tau)\bar{w}/R}{w^M - (1+n)\lambda(1-2\tau)\bar{w}/R} < \frac{w^L}{w^M}; \\
&\frac{w^L}{w^M} = \frac{(1-\tau)w^L}{(1-\tau)w^M} < \frac{(1-\tau)w^L + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}}{(1-\tau)w^M + \frac{(1+n)\lambda(1-\tau)\tau\bar{w}}{R}}.
\end{aligned}$$

Therefore, we have:

$$RHS^{yL}_{s>0} < RHS^{yM}_{s>0} \text{ if } \frac{w^L}{w^M} \leq \left(\frac{w^L}{w^M}\right)^{\sigma} \Leftrightarrow \left(\frac{w^L}{w^M}\right)^{1-\sigma} \leq 1. \qquad (18)$$

(18) indicates that $RHS^{yL}_{s>0} < RHS^{yM}_{s>0}$ holds if $\sigma \leq 1$, i.e., if $1/\sigma \geq 1$.

We next compare $RHS^{yL}_{s=0}$ and $RHS^{yM}_{s=0}$. Direct comparison leads to:

$$RHS^{yL}_{s=0} \lesseqgtr RHS^{yM}_{s=0} \Leftrightarrow \left(w^L\right)^{1-\sigma} \lesseqgtr \left(w^M\right)^{1-\sigma}.$$

Therefore, we can conclude:

$$\begin{aligned}
&RHS^{yL}_{s=0} \leq RHS^{yM}_{s=0} \text{ if } \sigma \leq 1 \Leftrightarrow 1/\sigma \geq 1, \\
&RHS^{yL}_{s=0} > RHS^{yM}_{s=0} \text{ if } \sigma > 1 \Leftrightarrow 1/\sigma < 1.
\end{aligned}$$

8.5.2 Derivation of (14)

Suppose first that the type-j young agent is borrowing unconstrained. The first-order condition for the maximization of $V^{y,j}_{s>0}$ with respect to λ is given by:

$$\frac{\partial V^{y,j}_{s>0}}{\partial \lambda} = 0 \Leftrightarrow \lambda = \lambda^{yj}_{s>0} \equiv \frac{1/(2+n) - (\beta\eta R/(2+n))^{1/\sigma}\left(w^j/\tau\bar{w}(1+n)\right)}{1/(2+n) + (\beta\eta R/(2+n))^{1/\sigma}/R} (< 1).$$

The preferred share λ is increasing in τ and is positive if and only if $\tau > \tilde{\tau}(w^j)$.

Next, suppose that the type-j young agent is borrowing constrained. The first-order condition for the maximization of $V_{s=0}^{y,j}$ with respect to λ leads to:

$$\frac{\partial V_{s>0}^{y,j}}{\partial \lambda} = 0 \Leftrightarrow \lambda = \lambda_{s=0}^{yj} \equiv \frac{1/(2+n)}{1/(2+n) + (\eta/(2+n))^{1/\sigma}}(< 1).$$

We should note that $\lambda_{s=0}^{yj}$ is constant and independent of τ. The equality holds between $\lambda_{s>0}^{yj}$ and $\lambda_{s=0}^{yj}$ at $\tau = \hat{\tau}(w^j)$. Therefore, the preferred share λ by a type-j young agent is given as (14).

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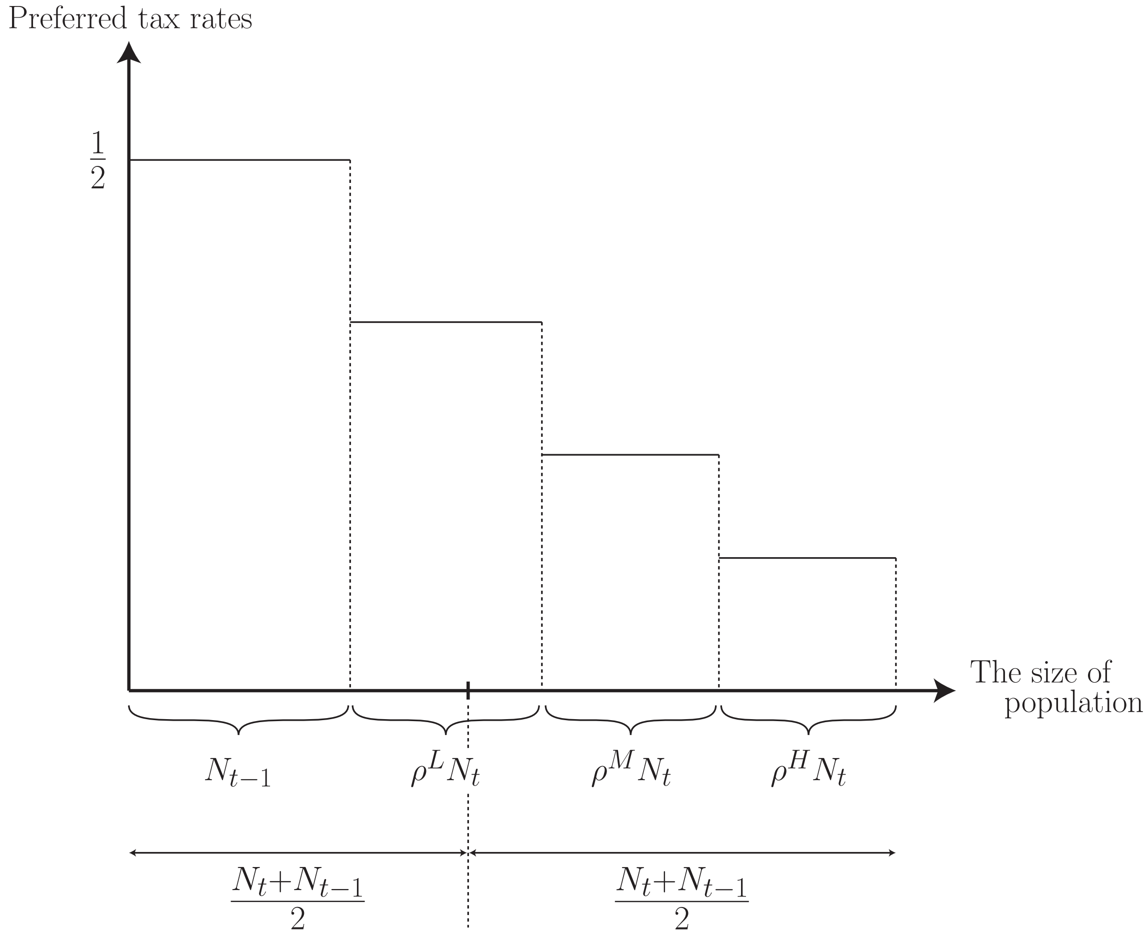

Figure 1: This figure illustrates an example of the tax rates preferred by the old and the young. In this example, a type-L young individual becomes a decisive voter.

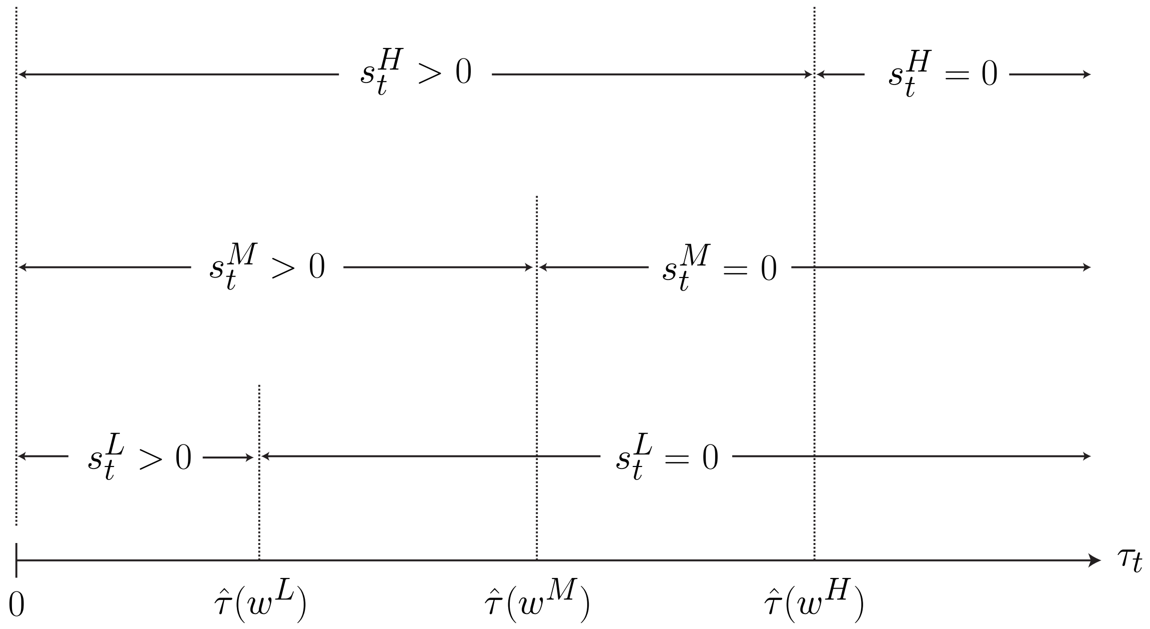

Figure 2: The relation between savings and the tax rate for each type of individual.

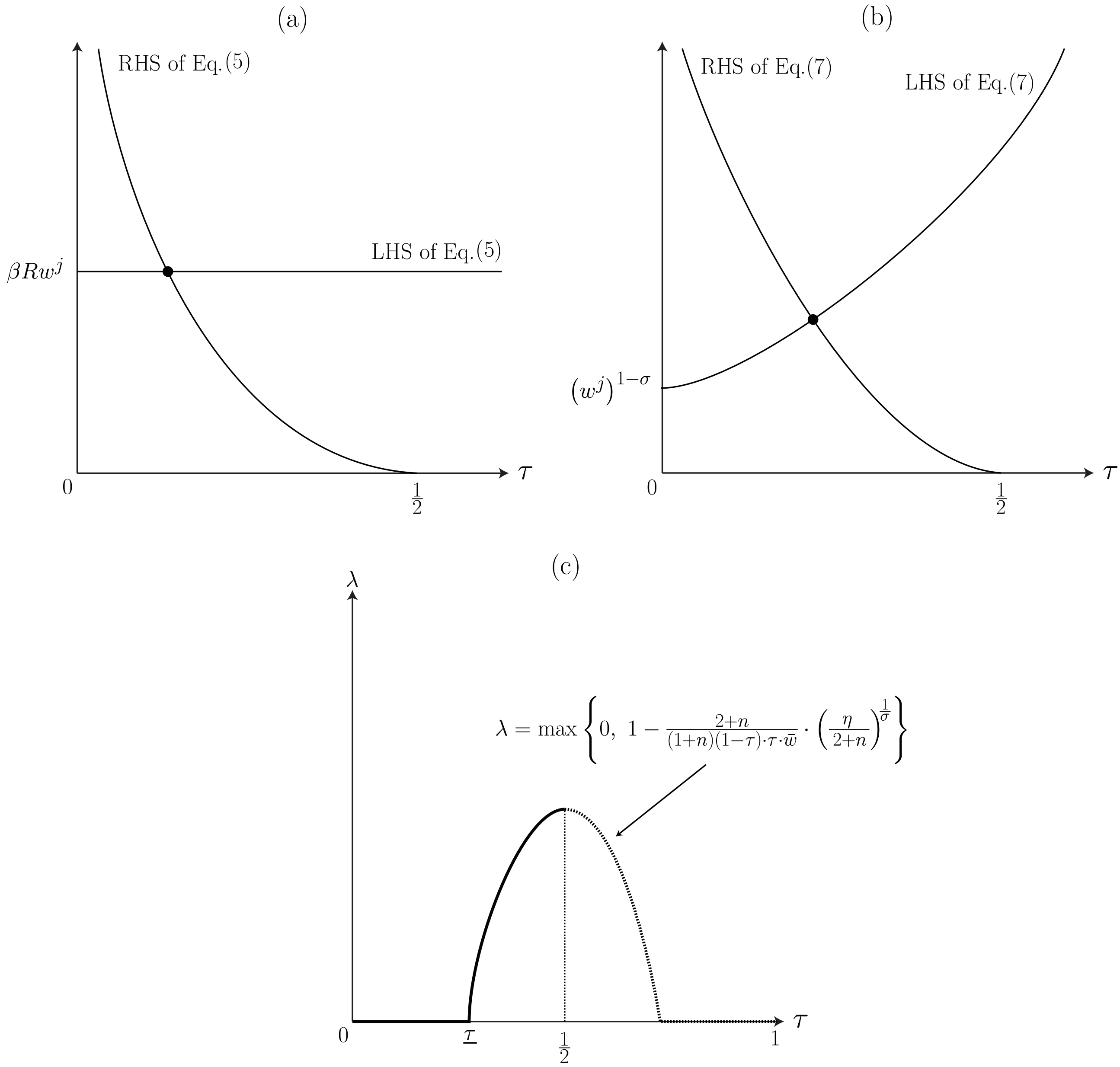

Figure 3: Panel (a) illustrates the determination of τ that maximizes the utility of the borrowing-unconstrained young. Panel (b) illustrates the determination of τ that maximizes the utility of the borrowing-constrained young. Panel (c) illustrates the share of social security preferred by the young.

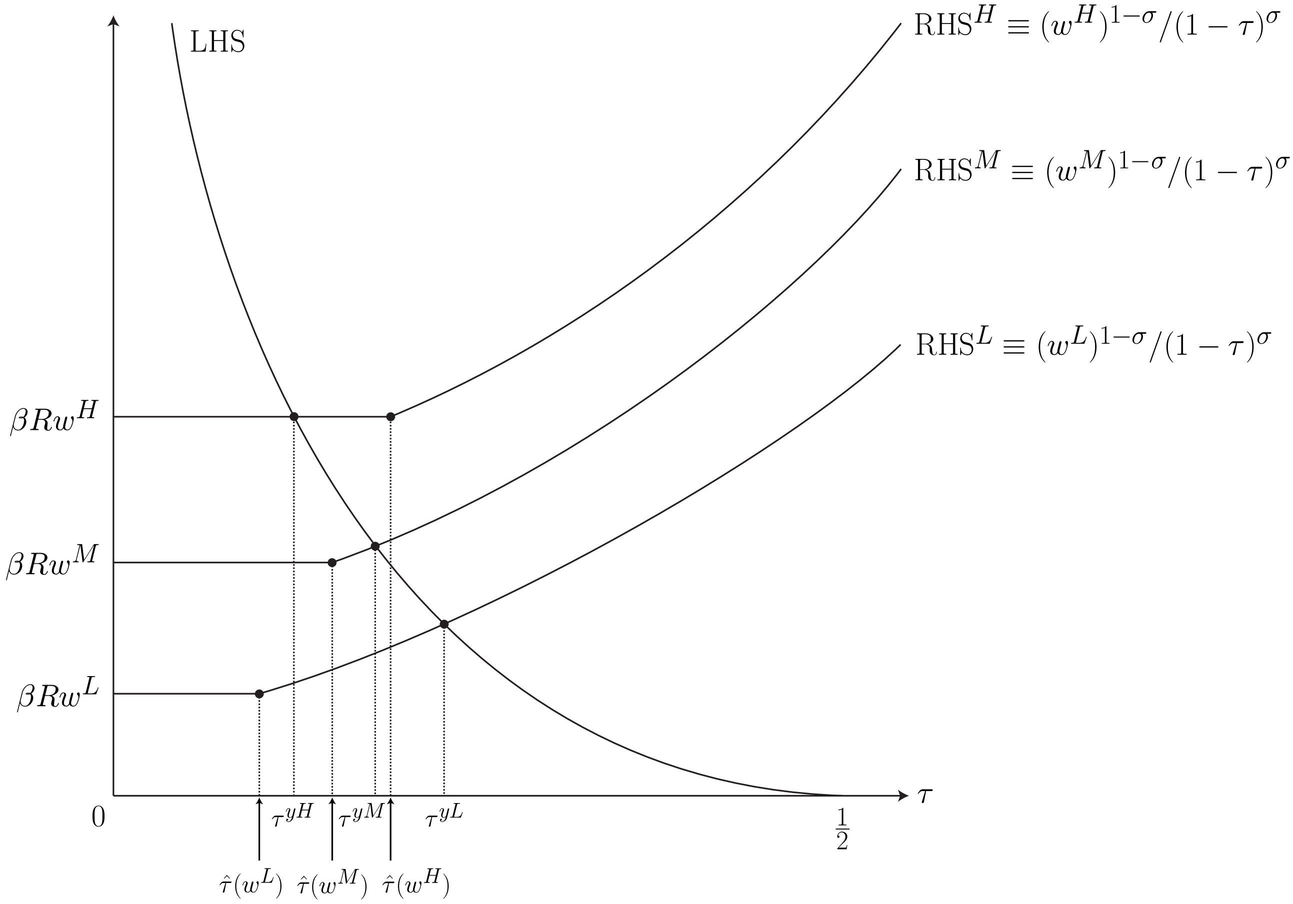

Figure 4: The tax rates preferred by the three types of young individuals in the case of $1/\sigma \geq 1$.

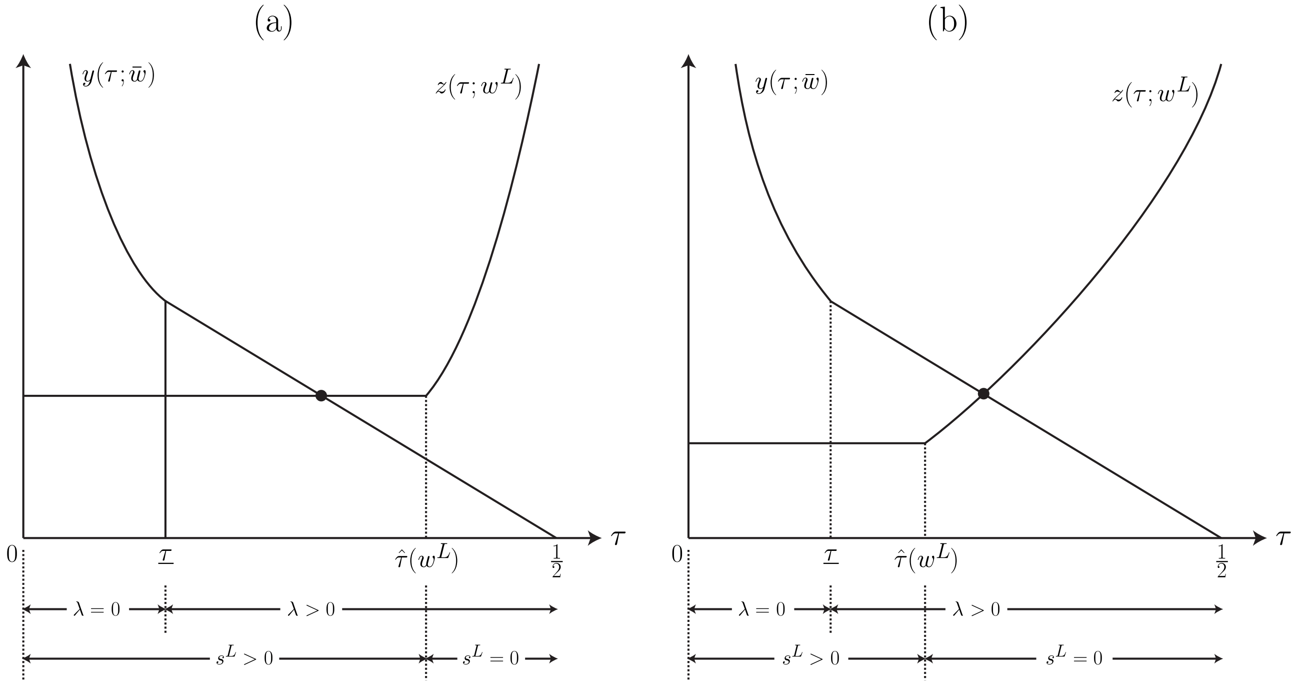

Figure 5: The figure illustrates the determination of the tax rate in the case of $1/\sigma \geq 1$. Panel (a) is for the case of $s^L > 0$ and $\lambda > 0$. Panel (b) is for the case of $s^L = 0$ and $\lambda > 0$.

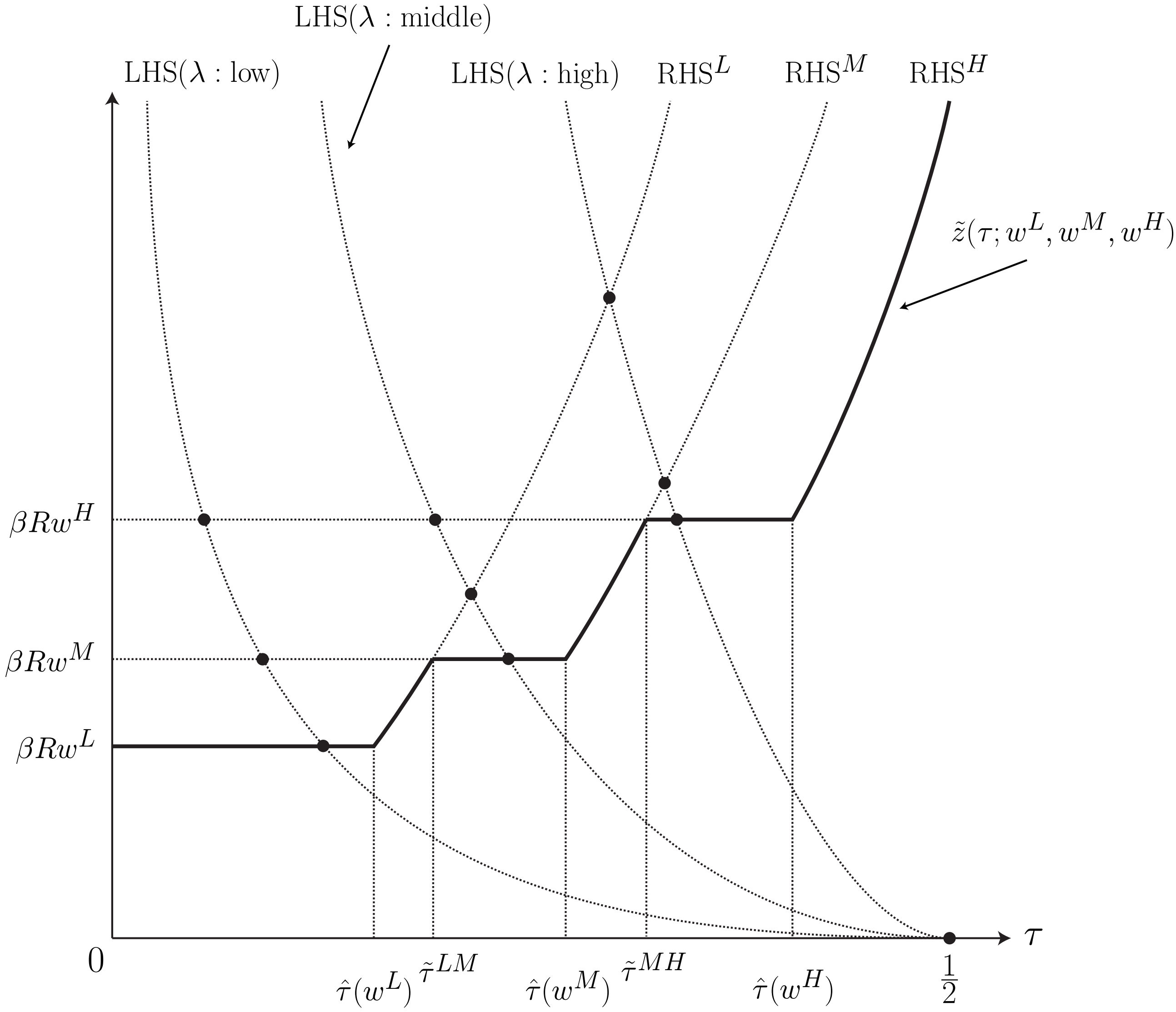

Figure 6: The tax rate preferred by a type-j young individual in the case of $1/\sigma < 1$. The bold curve illustrates the graph of $\widetilde{z}$ in (11).

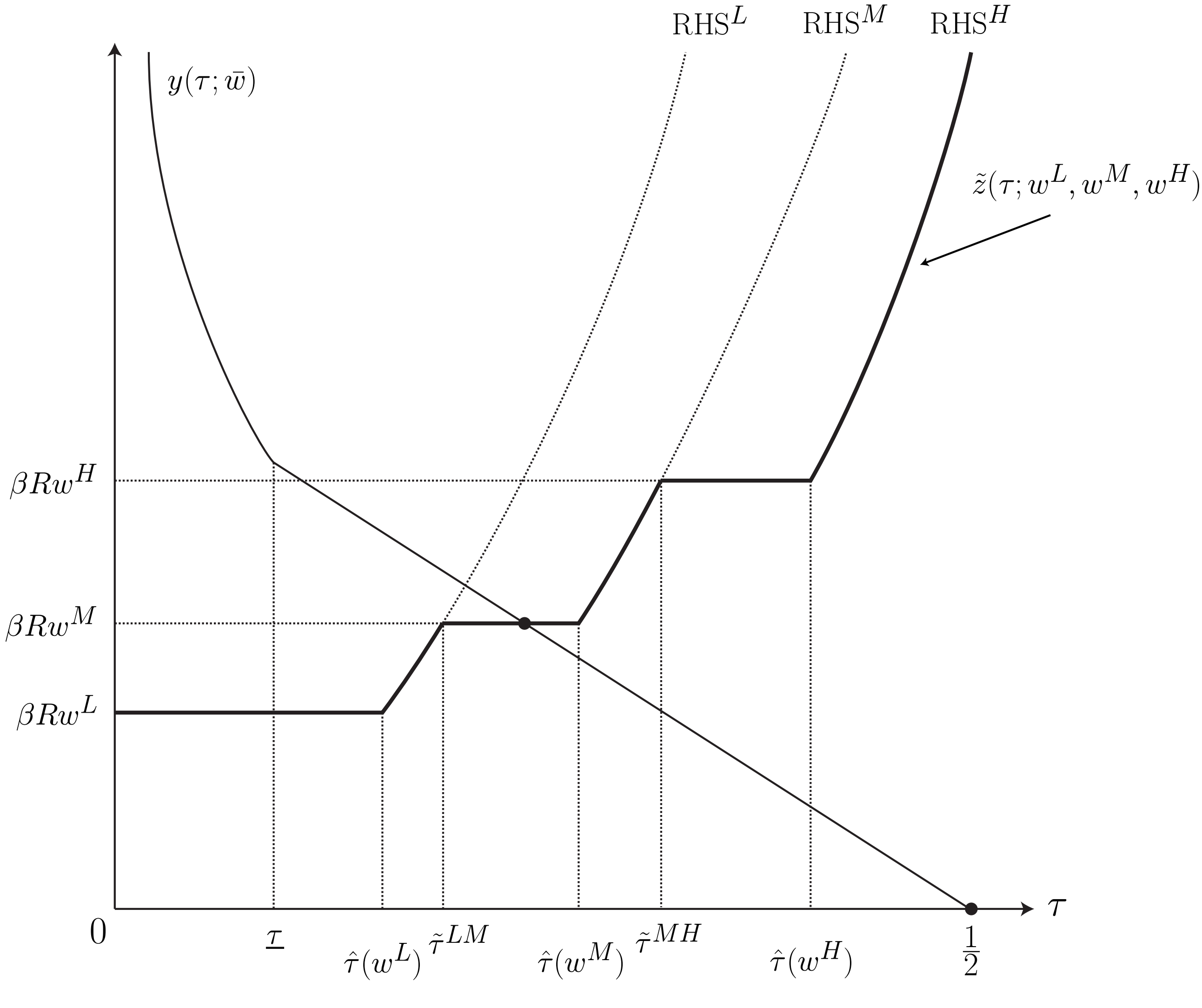

Figure 7: The determination of the tax rate in the case of $1/\sigma < 1$. The figure illustrates the case where the decisive voter is a type-M young individual.

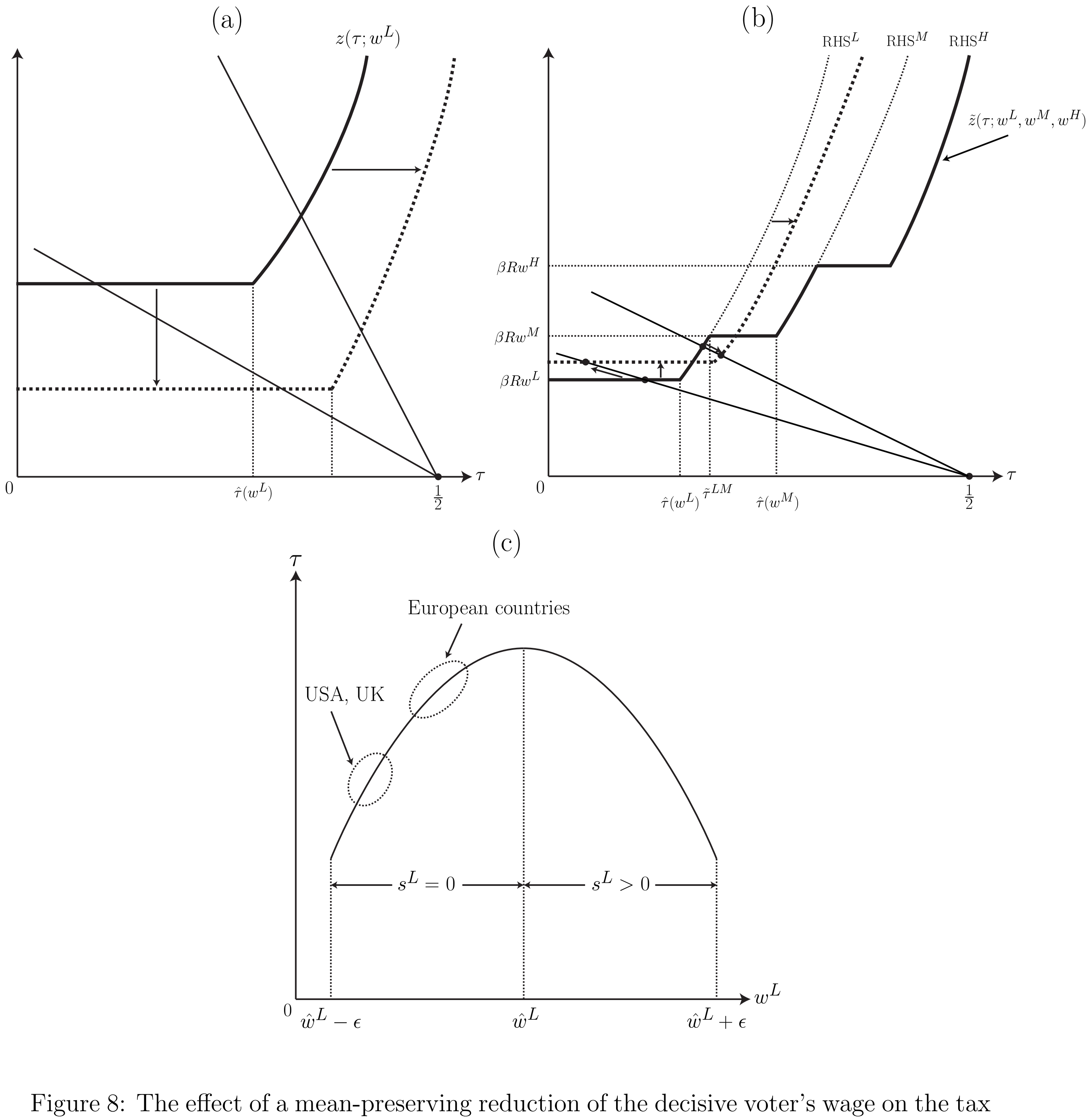

Figure 8: The effect of a mean-preserving reduction of the decisive voter's wage on the tax rate. Panel (a) illustrates the case of $1/\sigma \geq 1$. Panel (b) illustrates the case of $1/\sigma < 1$. Panel (c) illustrates the relation between the decisive voter's wage and the equilibrium tax rate in the case of $1/\sigma < 1$.

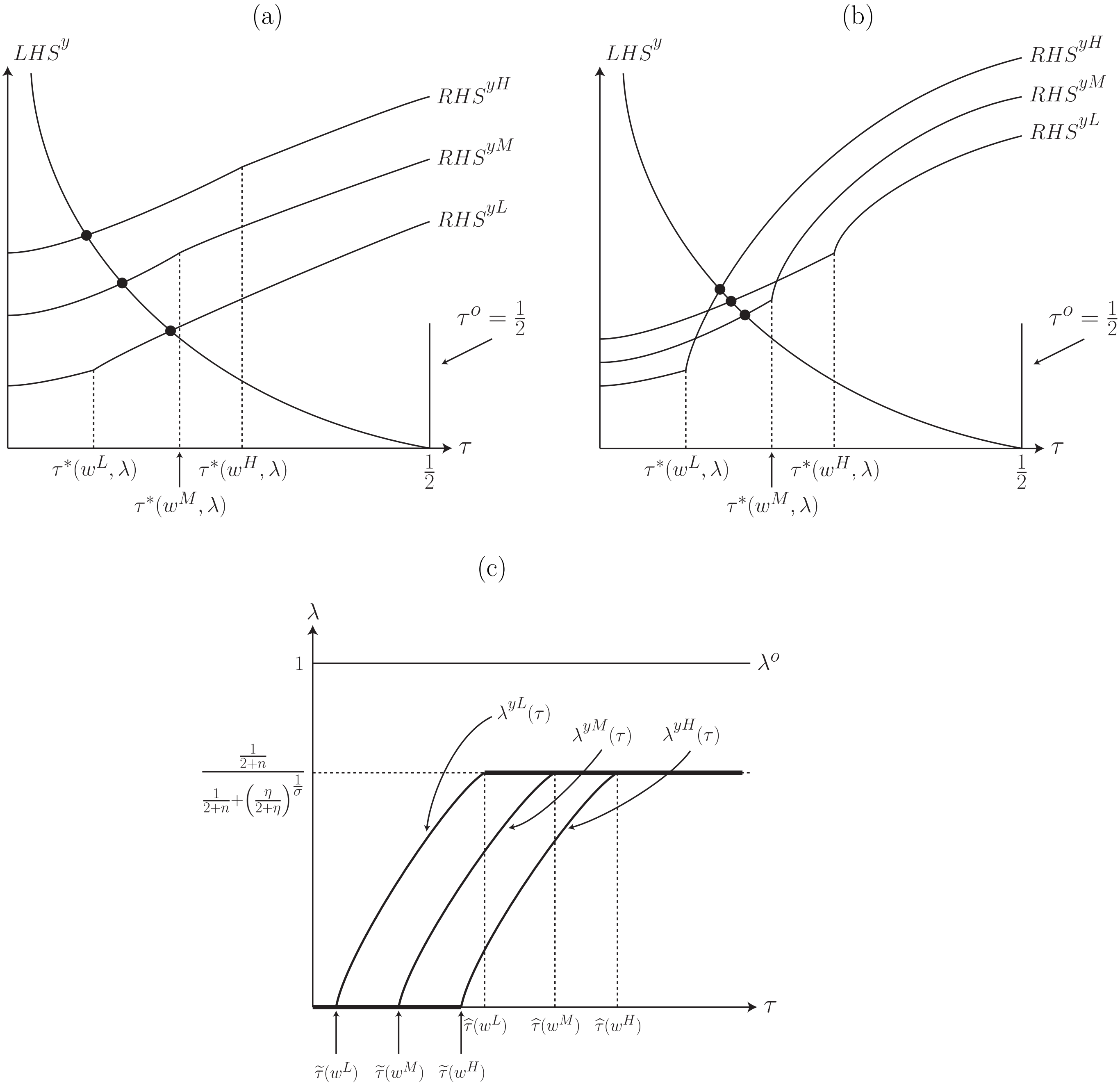

Figure 9: Panels (a) and (b) illustrate the preferred tax rates in cases where $1/\sigma \geq 1$ and $1/\sigma < 1$, respectively. Panel (c) illustrates the preferred shares of social security.