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Abstract:

Pricing of public utilities has long been discussed after Hotteling (1947), and most preceding arguments have provided a negative answer to the question to attain a Pareto-efficient allocation in an economy with non-convex production possibilities. Contrasting to these, Kamiya (1995) provided an argument that it is possible to devise a pricing mechanism of non-convex technology good(s) such that the equilibrium allocation under the pricing mechanism is always Pareto-efficient. The present note intends to examine a small question to find how Kamiya's arguemnt differs from the preceding, with an intention to clarify how the efficiency property of his pricing mechanism is secured. The reconsideration however leads to a negative result that Kamiya's pricing mechanism will fail to assure the efficiency property in a simple illustrative economy considered by himself. We first confirms that the simple example given in Kamiya contradicts his main theorem, and then review Kamiya's proving argument and examine where any slip remains.

Key words: Public utility pricing, non-convex economy, Pareto-efficient allocation

JEL classification: D51, D61, H42

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1. Introduction

Pricing of public utilities has long been discussed after Hotteling (1947), and the succeeding arguments in a general equilibrium setting, like Guesnerie (1975), Beato and Mas-Colell (1985), and Vohra (1990) etc., have confirmed that while marginal-cost pricing is necessary for attaining the Pareto-efficient allocation, the efficiency is not assured even with marginal-cost pricing when production of public utilities shows increasing returns to scale. Calsamiglia (1977) also confirmed the fact in an abstract way to examine the possibility of an abstract resource allocation process that satisfies some necessary properties supposing a simplified economy. Contrasting to these, Kamiya (1995) provided an argument that it is possible to devise a pricing mechanism of non-convex technology good(s) such that the equilibrium allocation under the pricing mechanism is always Pareto-efficient.

Reviewing the preceding arguments as above and comparing their arguments with Kamiya's, it can be considered that there may remain a contradictory situation, while Kamiya states: "In this paper, if there exists an equilibrium then it is Pareto optimal. Thus if, in an economy, there does not exist a decentralized mechanism which leads to Pareto optimal allocations, then there does not exist an equilibrium in the economy when the nonconvex firms follow our nonlinear pricing rule. (Kamiya, p.550)"

The purpose of the present note is to reexamine a small question as above, with an intention to clarify how the efficiency property of Kamiya's pricing mechanism is secured. The reconsideration however leads to a negative result such that Kamiya's pricing mechanism will fail to assure the efficiency property in a simple illustrative economy considered by himself (Kamiya, p.551, summarized in Fig.1). We thus firstly confirms that the simple example given in Kamiya's Fig.1 contradicts his main theorem (Theorem 1, reproduced in Section 2.1 below). Then we review Kamiya's proving argument and examine where any slip remains.

2. The Model Economy and a Contradicting Example

2.1 Kamiya's Model Economy and Efficient Price Mechanism

We begin by explaining the model economy considered in Kamiya, which is replicated as follows: there are $\ell_1 + \ell_2$ goods in the economy, the first ℓ_1 goods, called P goods, are produced by firms with nonconvex production sets (the natural monopolies) and the last ℓ_2 goods, called C goods, are produced by firms with convex production sets (the competitive firms). Price vector of C goods is denoted by $q \in R_+^{\ell_2}$. The firms with nonconvex production sets are called P firms and the firms with convex production sets are called C firms. The number of P firms is ℓ_1 and the jth P firm produces only jth good, $j = 1, \ldots, \ell_1$, using C goods. (See assumption A2ii below.) The number of C firms is n and each C firm produces some C goods using the other goods. (See assumption A2i below.) $Y_j \subset R^{\ell_1+\ell_2}$, $j = 1, \ldots, \ell_1$, and $Z_k \subset R^{\ell_1+\ell_2}$, $k = 1, \ldots, n$, denote the production set of the jth P firm and the production set of the kth C firm, respectively. There are m consumers and the ith consumer has a quasi-concave utility function $u_i : R_+^{\ell_1+\ell_2} \to R$, an initial endowments $\omega_i \in R_+^{\ell_1+\ell_2}$ and a share holding in the kth C firm, θ_{ik}, satisfying $\theta_{ik} \geqq 0$ and $\Sigma_{i=1}^{m} \theta_{ik} = 1$ for $i = 1, \ldots, m$ and $k = 1, \ldots, n$.

As to consumers' preferences and producers' production technologies the following is assumed:

A1. For $i \in M \equiv \{1, \ldots, m\}$

(i) for $x_i \in R_+^{\ell_1+\ell_2}$ and $\varepsilon \in R_{++}$, there exists $x'_i \in R_+^{\ell_1+\ell_2}$ such that $x_{ih} = x'_{ih}$ for $h = 1, \ldots, \ell_1$, $\| x_i - x'_i \| < \varepsilon$, and $u_i(x'_i) > u_i(x_i)$, where $\| \cdot \|$ denotes the Euclidian norm,

(ii) $\omega_{ih} = 0$ for $h \in L_1 \equiv \{1, \ldots, \ell_1\}$ and $\omega_{ih} > 0$ for $h \in L \backslash L_1$, $L \equiv \{1, \ldots, \ell_1 + \ell_2\}$.

A2. (i) For $k \in N \equiv \{1, \ldots, n\}$, $Z_k \subset R^{\ell_1+\ell_2}$ is a closed convex set such that $R_-^{\ell_1+\ell_2} \subset Z_k$, and $z_k \in Z_k$ implies that $z_{kj} \leq 0$ for $j \in L_1$.

(ii) For $j \in L_1$, there exists a function $g_j : V \to R_+$ such that $Y_j = K_j - R_+^{\ell_1+\ell_2}$, where V is an open set such that $R_+^{\ell_2} \subset V$ and $K_j = \{y_j \in R^{\ell_1+\ell_2} \mid y_{jh} = 0$ for $h \in L_1 \backslash \{j\}$, $y_j^C \in R_-^{\ell_2}$, and $y_{jj} = g_j(-y_j^C)\}$, with y_j^C designating the vectors of the last ℓ_2 coordinates of y_j. Moreover, g_j is C^2 for $y_j^C \in R_-^{\ell_2}$ such that $g_j(-y_j^C) > 0$.

(iii) For $j \in L_1$, $g_j(0) = 0$, g_j is strictly quasi-concave for $y_j^C \in R_-^{\ell_2}$ such that $g_j(-y_j^C) > 0$, is increasing in $-y_j^C$, i.e., for $-y_j^C \in R_+^{\ell_2}$,

$$\frac{\partial g_j(-y_j^C)}{\partial(-y_{jh})} \geqslant 0,\ h \in L \backslash L_1,$$

and, for $-y_j^C \in R_+^{\ell_2}$ such that $g_j(-y_j^C) > 0$,

$$\frac{\partial g_j(-y_j^C)}{\partial(-y_{jh})} > 0,\ h \in L \backslash L_1.$$

(iv) For $a \in R_+$, there exists $y_j^C \in R_-^{\ell_2}$ such that $a = g_j(-y_j^C)$.

(v) For $y_j^C \in R_-^{\ell_2}$ such that $g_j(-y_j^C) > 0$, let $H(y_j^C) \equiv \{h \in \{1, \ldots, \ell_2\} \mid y_{jh} = 0\}$. Then $Dg_j(-y_j^C)$ and e_h, $h \in H(y_j^C)$, are linearly independent, where $Dg_j(-y_j^C)$ is the gradient of g_j at $-y_j^C$ and e_h is the hth unit vector in R^{ℓ_2}.

The prices of P goods are determined by the following nonlinear pricing rule: first, the consumers and C firms are asked to report their demands for the jth P goods, $\bar{x}_{ij}$ and $\bar{z}_{kj}$, for $i \in M$, $j \in L_1$, and $k \in N$. (Note that P goods are inputs for C firms so that $\bar{z}_{kj}$ is nonpositive.) Defining $\bar{v}_j \equiv \Sigma_{i=1}^m \bar{x}_{ij} - \Sigma_{k=1}^n \bar{z}_{kj}$, the following function $\beta_j : R_+ \times R_{++}^{\ell_2} \times R_{++}$ $(\ni (y_{jj}, q, \bar{v}_j)) \to R$ is supposed, for given $q \in R_{++}^{\ell_2}$ and $\bar{v}_j \in R_{++}$, which satisfies;

1. $\beta_j(y_{jj}; q, \bar{v}_j)$, a function of y_{jj}, is expressed by a finite number of parameters,
2. β_j is concave in $y_{jj} \in R_+$,
3. $\beta_j(\bar{v}_j; q, \bar{v}_j) = C_j(\bar{v}_j, q)$,

4. if $y_{jj} > y'_{jj}$ then $\beta_j(y_{jj}; q, \bar{v}_j) > \beta_j(y'_{jj}; q, \bar{v}_j)$,

5. $\beta_j(y_{jj}; q, \bar{v}_j) \leqslant C_j(y_{jj}, q)$ for $y_{jj} \in R_+$,

6. β_j is differentiable with respect to y_{jj} at $y_{jj} = \bar{v}_j$, and $\partial\beta_j(\bar{v}_j; q, \bar{v}_j)/\partial y_{jj}$ $= \partial C_j(\bar{v}_j, q)/\partial y_{jj}$.

In the above the following notations are used; R_{++} and $R^{\ell_2}_{++}$ are the interiors of R_+ and $R^{\ell_2}_+$, respectively. For $y_{jj} \in R_+$, $q \in R^{\ell_2}_{++}$ and $j \in L_1$,

$$C_j(y_{jj}, q) \equiv \min -q \cdot y^C_j \quad \text{s.t.} \quad y_{jj} = g_j(-y^C_j) \text{ and } -y^C_j \in R^{\ell_2}_+.$$

Assuming the setting as above, Kamiya proves the following theorem;

Theorem 1 (Kamiya). Under A1 and A2, if an $m(\ell_1 + \ell_2) + \ell_1(\ell_1 + \ell_2)$ $+n(\ell_1 + \ell_2) + \ell_2$ tuple $((x^*_i), (y^*_j), (z^*_k), q^*) \in R^{m(\ell_1+\ell_2)}_+ \times \Pi^{\ell_1}_{j=1} Y_j \times \Pi^n_{k=1} Z_k \times R^{\ell_2}_{++}$ is an equilibrium, then $((x^*_i), (y^*_j), (z^*_k))$ is a Pareto optimal allocation. Moreover, (x^*_i) is individually rational, i.e., $u_i(x^*_i) \geqslant u_i(\omega_i)$ for all $i \in M$.

For a later reference let us replicate the definition of an equilibrium by Kamiya;

Definition 1 (Kamiya). An $m(\ell_1 + \ell_2) + \ell_1(\ell_1 + \ell_2) + n(\ell_1 + \ell_2) + \ell_2$ tuple $((x^*_i), (y^*_j), (z^*_k), q^*) \in R^{m(\ell_1+\ell_2)}_+ \times \Pi^{\ell_1}_{j=1} Y_j \times \Pi^n_{k=1} Z_k \times R^{\ell_2}_{++}$ is said to be an equilibrium if

(i) $x^*_i \in \arg\max\{u_i(x_i) \mid \Sigma^{\ell_1}_{j=1} \zeta_j(x^*_{ij}, \bar{v}^*_j, x_{ij}, q^*) + q^* \cdot x^C_i \leqq q^* \cdot \omega^C_i$ $+\Sigma^n_{k=1} \theta_{ik} \pi_k((z^*_{kj}), (\bar{v}^*_j), q^*), x_i \in R^{\ell_1+\ell_2}_+\}$ for $i \in M$, where $\bar{v}^*_j = \Sigma^m_{i=1} x^*_{ij} - \Sigma^n_{k=1} z^*_{kj}$ for $j \in L_1$,

(ii) $y^{*C}_j \in \arg\min\{-q^* \cdot y^C_j \mid y^*_{jj} = g_j(-y^C_j) \text{ and } y^C_j \in R^{\ell_2}_-\}$ for $j \in L_1$,

(iii) $z^*_k \in \arg\max\{q^* \cdot z^C_k - \Sigma^{\ell_1}_{j=1} \zeta_j(-z^*_{kj}, \bar{v}^*_j, -z_{kj}, q^*) \mid z_k \in Z_k\}$ for $k \in N$,

(iv) $\Sigma^{\ell_1}_{j=1} y^{*C}_j + \Sigma^n_{k=1} z^{*C}_k + \Sigma^m_{i=1} \omega^C_i \geqq \Sigma^m_{i=1} x^{*C}_i$,

(v) $y^*_{jj} = \Sigma^m_{i=1} x^*_{ij} - \Sigma^n_{k=1} z^*_{kj}$, $j \in L_1$,

(vi) $x^*_{ij} > 0$ and $-z^*_{kj} > 0$ for $i \in M$, $j \in L_1$ and $k \in N$.

In Definition 1 above the following notations and assumptions are used and/or supposed: first, the ith consumer maximizes the utility subject to the budget constraint as follows,

$$\max_{x_i \in R_+^{\ell_1+\ell_2}} u_i(x_i) \text{ s.t. } \Sigma_{j=1}^{\ell_1} \zeta_j(\bar{x}_{ij}, \bar{v}_j, x_{ij}, q) + q \cdot x_i^C$$

$$\leqq q \cdot \omega_i^C + \Sigma_{k=1}^{n} \theta_{ik} \pi_k((\bar{z}_{kj}), (\bar{v}_j), q),$$

where x_i^C and ω_i^C designate the vectors of the last ℓ_2 coordinates of x_i and ω_i respectively, and $\zeta_j(\bar{x}_{ij}, \bar{v}_j, x_{ij}, q)$ shows ith consumer's outlay for x_{ij} by the pricing rule for P goods that is given by utilizing above-explained β_j function as follows,

$$\zeta_j(\bar{x}_{ij}, \bar{v}_j, x_{ij}, q) = \beta_j(x_{ij} \tfrac{\bar{v}_j}{\bar{x}_{ij}}, q, \bar{v}_j) \tfrac{\bar{x}_{ij}}{\bar{v}_j}.$$

Second, the prices of C goods are determined competitively. Third, the kth C firm maximizes the profit on the production set as follows,

$$\max_{z_k \in Z_k} q \cdot z_k^C - \Sigma_{j=1}^{\ell_1} \zeta_j(-\bar{z}_{kj}, \bar{v}_j, -z_{kj}, q)$$

where z_k^C denotes the vectors of the last ℓ_2 coordinates of z_k and $\zeta_j(-\bar{z}_{kj}, \bar{v}_j, -z_{kj}, q)$ is the firm's outlay for z_{kj} given by utilizing the function β_j for P goods as follows,

$$\zeta_j(-\bar{z}_{kj}, \bar{v}_j, -z_{kj}, q) = \beta_j(-z_{kj} \tfrac{\bar{v}_j}{-\bar{z}_{kj}}, q, \bar{v}_j) \tfrac{-\bar{z}_{kj}}{\bar{v}_j}.$$

The profit for the kth C firm is denoted by $\pi_k((\bar{z}_{kj}), (\bar{v}_j), q)$. Last, the jth P firm minimizes the cost of producing y_{jj} for given y_{jj} and q as follows,

$$\min_{y_j^C \in R_-^{\ell_2}} -q \cdot y_j^C \text{ s.t. } y_{jj} = g_j(-y_j^C)$$

2.2 A Contradicting Example

As noted in Introduction a simple economy illustrated by Kamiya himself satisfies our present purpose to examine whether his Theorem 1 holds in general. For this, let us reproduce here also the illustrative economy: an economy with two goods, one P firm and one consumer is supposed and C firm is assumed away. There are a single input, good 2, and a single output, good 1. The price of good 2 is normalized to be one. The production set of the P firm, $Y_1 \subset R^2$, the initial endowment, $\omega_1 \in R_+^2$, and the utility function of the consumer, $u_1 : R_+^2 \to R$, are illustrated as in Kamiya's Fig.1, which is also reproduced here as Fig.A[1)].

Utilizing this example Kamiya states that "the marginal cost pricing equilibria are a, b and c in the figure and only a is Pareto optimal...a cannot be achieved as a competitive equilibrium (Kamiya, p.550)". He says also that if the consumer's budget line is like the dotted line in the figure under a nonlinear pricing rule, a is an equilibrium of the model.

Now let us confirm point c in the above example drawn in Fig.1 also satisfies the conditions for equilibrium reproduced in Section 2.1 above if the nonlinear pricing rule is appropriately defined while fulfilling the conditions for β_j function set by Kamiya. First, the nonlinear pricing rule for P goods set by Kamiya says that β_j functions and corresponding pricing rules are defined for different levels of production of jth P good, i.e. depending on various values of $\bar{v}_j$s, as shown in the definition of β_j above[2)]. Second, suppose then that β_j function corresponding to point c is given by the *bold* dotted line, named β^c, in Fig.A here[3)] and that it satisfies all the conditions for β_j function specified in Section 2.1 above. Also, denote the coordinates of point c by $(y^{Cc} + \omega^C, y^{Pc})$ where (y^{Cc}, y^{Pc}) is the input-output vector of the P firm at point c and $(\omega^C, 0) \equiv \omega_1$. Third, note that the set of the allocation and price $((x^{Cc}, x^{Pc}), (y^{Cc}, y^{Pc}), 1)$ satisfies the following, where $(x^{Cc}, x^{Pc}) = (y^{Cc} + \omega^C, y^{Pc})$ and the price for the competitive good is normalized to 1;

i) First, the budget constraint for the consumer is given by the following inequality,

$$\zeta_1(x^{Pc}, \bar{v}_1^c, x, 1) + x^{Cc} \leqq \omega^C \tag{1}$$

where $\bar{v}_1^c = x^{Pc}$, x denotes consumer's demand for P good, and

$$\zeta_1(x^{Pc}, \bar{v}_1^c, x, 1) = \beta^c(x\tfrac{\bar{v}_1^c}{x^{Pc}}; 1, \bar{v}_1^c) \tag{2}$$

$\beta^c(x^{Pc}\frac{\bar{v}_1^c}{x^{Pc}}; 1, \bar{v}_1^c) = C(y^{Pc}, 1)$ by construction with cost function C for the P firm. (1) is due to that there exists neither competitive firm thus nor production profit, and (2) holds since the consumer only demands the P good. Then, an indifference curve tangent to the production frontier $Y^P + \omega$ at point c is tangent also to the budget constraint above at the point by the construction of β^c above, or equivalently by conditions 3 and 6 for β_j. Thus (x^{Cc}, x^{Pc}) maximizes the utility under the budget constraint (1).

ii) It is clear that all the other conditions for the equilibrium (ii), (iv),(v) and the former half of (vi) are satisfied, while the conditions (iii) and the latter half of (vi) can be dismissed since there is no competitive firm (n is zero and N empty).

Thus point c with β_j defined as above, i.e. β^c, for P good satisfies the conditions for the equilibrium, and a contradiction remains.

2.3 How Inefficient Equilibrium Remains

It is thus of an importance to reconsider why the efficiency result of the proposed nonlinear pricing rule was asserted. One will be that β_j functions and the nonlinear pricing rule defined utilizing β_js for P goods are considered corresponding to various production points $\bar{v}_j = y_{jj}$. That is, while confining our attention to the above-reviewed example, if the only nonlinear pricing rule or β_j function is given as Kamiya's dotted line in Fig.1 or Fig.A, point a will represent the unique equilibrium under the pricing rule and the efficiency obtains[4)]. However, they are considered also referring to point c as their definitions above (pp.4-5) say and the argument in Section 2.2 shows.

Second, it will be conjectured that Kamiya's proof of the Theorem itself leaves a key to the slip. So, review Kamiya's formal arguments and consider why the above inefficiency continues to exist irrespective of the Theorem. Since his proof relies on the absurdity (contradiction), consider the question by comparing his arguments with a usual proof of the first theorem of the welfare economics utilizing the absurdity: i) Reproduce a rough story of a proof of the first welfare-economics theorem; it starts to assume the existence of a feasible allocation that assures no worse welfare to all consumers and (strictly) better welfare at least to a consumer $((x_i'), (z_k'))$ compared to an equilibrium $((x_i^*), (z_k^*), q^*)$ where (y_j') and (y_j^*) are suppressed since there exist only C producers. Then, Pareto-improving consumption means

$$q^*\Sigma_i x_i' > q^*(\Sigma_k z_k^* + \Sigma_i \omega_i)$$

Taking account of the feasibility and the equilibrium, this is rewritten

$$q^*(\Sigma_k z_k' + \Sigma_i \omega_i) > q^*(\Sigma_k z_k^* + \Sigma_i \omega_i) \quad \text{or} \quad q^*\Sigma_k z_k' > q^*\Sigma_k z_k^* \tag{3}$$

This contradicts to profit maximization at (z_k^*) by C producers, which is reduced to the starting assumption of a feasible and Pareto-improving allocation $((x_i'), (z_k'))$.

ii) Consider now Kamiya's proving argument: Kamiya, like above, begins to assume the existence of a feasible allocation $((x_i'), (y_j'), (z_k'))$ that is Pareto-improving compared to the equilibrium one $((x_i^*), (y_j^*), (z_k^*))$. This supposition implies the first strict inequality in his proof (Kamiya, p.557). Then, taking account of the feasibility of the allocation, C-firms' profit maximization and supposition concerning β_j function leads to the second and the third inequalities in his proof (Kamiya, p.557). Note here that the strict inequality in these relations due to the supposition that the allocation $((x_i'), (y_j'), (z_k'))$ is Pareto-improving compared to the equilibrium one, since the additionally considered inequalities by referring the feasibility of the allocation, C-firms' profit maximization and property of β_j functions are all weak. By consideration of the feasibility of $((x_i'), (y_j'), (z_k'))$ and increasingness of β_js, the last inequality is rewritten utilizing only β_j functions (Kamiya, p.558, inequality (1)). Kamiya's inequality (1) is expressed only by β_js, and since the variables there are in the relation of convex combination, Kamiya concludes that the inequality contradicts the assumed concavity of β_js. Note here also that weak concavity of β_js is sufficient to derive the contradiction.

iii) To make clearer the role of respective assumptions in Kamiya's proof, repeat his proving argument for the case of his example used also in Section 2.2 above supposing that the present equilibrium is point c with the pricing rule specified there and a feasible and Pareto-improving allocation is given by point a in Fig.A. For reference, denote the coordinate of point a by (x^{Ca}, x^{Pa}) and $(y^{Ca} + \omega^C, y^{Pa})$ where (x^{Ca}, x^{Pa}) denotes consumption allocation and (y^{Ca}, y^{Pa}) is the input-output vector of the P firm at point a. Of course $(x^{Ca}, x^{Pa}) = (y^{Ca} + \omega^C, y^{Pa})$. Then, since point a is better than point c, the following inequality holds,

$$\zeta_1(x^{Pc}, \bar{v}_1^c, x^{Pa}, 1) + x^{Ca} > \omega^C \tag{4}$$

since the price for C good is 1 and there is no C firm and its profit. Then, (2) and the feasibility of the allocation $((x^{Ca}, x^{Pa}), (y^{Ca}, y^{Pa}))$ give the following,

$$\beta^c(x^{Pa}\frac{\bar{v}_1^c}{x^{Pc}}; 1, \bar{v}_1^c) + y^{Ca} > 0 \tag{5}$$

Further, since $\beta^c(y^{Pa}; 1, \bar{v}_1^c) \leqq C(y^{Pa}; 1) = -1 \cdot y^{Ca}$ as is supposed and seen in Fig.A,

$$\beta^c(x^{Pa}\frac{\bar{v}_1^c}{x^{Pc}}; 1, \bar{v}_1^c) - \beta^c(y^{Pa}; 1, \bar{v}_1^c) > 0 \tag{6}$$

Since $x^{Pa} = y^{Pa}$, (6) is rewritten as,

$$\beta^c(x^{Pa}\frac{\bar{v}_1^c}{x^{Pc}}; 1, \bar{v}_1^c) - \beta^c(x^{Pa}; 1, \bar{v}_1^c) > 0 \tag{6$'$}$$

Inequality $(6^{'})$ corresponds to Kamiya's inequality (1), and noting $\bar{v}_1^c = x^{Pc}$, $(6^{'})$ clearly provides a straightforward contradiction. It may be noted here that the contradiction is led without relying on concavity of β^c (i.e. β_js) (since the consumer is only one demanding P good), and that this inequality or inequality (6) comes from the fact that on the one hand (assumed) higher utility requires the consumption allocation in the better set and on the other the assumption of smallerness of β^c (or β_js) implies (total) price of P good not larger than its cost of production, i.e., $\beta^c(y^{Pa}; 1, \bar{v}_1^c) \leqq C(y^{Pa}; 1) = -1 \cdot y^{Ca}$.

Now summarize the above review. First, the story or logic leading to a contradiction for the case of Fig.1 (Fig.A here) as above will say that irrespective of the introduction of nonlinear pricing rule for P goods it seems to play little role to derive the efficiency result, and only the conditions well-known in the first welfare-eonomics theorem are utilized for the proof of Theorem 1. Though appearance differs between (3) and $(5^{'})$, this is really superfluous since (3) is arranged as follows if the maximization of profit by C firms is taken again into account,

$$q^*\Sigma_k z_k' > q^*\Sigma_k z_k^* \geqq q^*\Sigma_k z_k' \tag{3$'$}$$

which will be an expression of (3) corresponding to $(5^{'})$. It may thus be said that such a contradiction as $(5^{'})$ or $(3^{'})$ is implied in the assumption of existence of a better allocation. Second, it is noted that $(3^{'})$ is derived depending also on the convexity of production sets of producers, and as to $(6^{'})$ concavity of β^c or β_js plays the same role. Though we noted that concavity of β^c plays little roll to derive $(6^{'})$, it says simply that the relation like the equality of Kamiya just after inequality (1) is not used to derive ($6'$). However, the expression $\beta^c(x^{Pa}\frac{\bar{v}_1^c}{x^{Pc}}; 1, \bar{v}_1^c)$ means that β^c (or β_js) function(s) is applicable to point *a* and the point is representable by the function assumed to be concave. That is, the supposition of a feasible and Pareto-improving allocation together with concavity of β^c, when applied to (y^{Ca}, y^{Pa}) or point *a*, will imply that point *a* is in a convex set that will be defined by β^c. This is the same in Kamiya's proof in the sense that both the first inequality derived from the assumption of a better allocation (Kamiya, p.557) and the applicability of β_js to the allocation (Kamiya, p.558) lead to his inequality (1). If we think as this, it seems natural that the contradiction (6) or ($6'$), as well as Kamiya's Theorem 1, is led almost via the same way as the proof of the first welfare-economics theorem. Third, though the contradiction implied by inequality (6) or ($6'$) should be false since consumption allocation at point *a* is in reality superior to one at point *c*, it is implied, as noted above, by the assumption that (x^{Ca}, x^{Pa}) is better and feasible and production at (y^{Ca}, y^{Pa}) is explained by β^c. However, the last statement itself is obviously contradictory to what is given in the figure. Fourth, as is well-known, for the efficiency of an equilibrium in an economy with nonconvex production to hold, it has to be assured that there exists no feasible allocation better than the equilibrium one. However it may equal to assume the result.

Notes

1) A difference in Fig.A here from Kamiya's Fig.1 is that a proposed β^c function defined referring to point *c*, shown by bold dotted line in the figure, is added in the present one.
2) A clear illustration of this is in his Fig.2 (Kamiya, p.552, not reproduced here).
3) In this illustrative economy with one consumer, β_j function equals consumer's outlay for *P* good (see eq.(2) below), and with just two goods, they can be drawn in a figure.
4) It may be noted also that, when recalling (2) the suggested dotted line by Kamiya in

Fig.1 may imply a nonconcave β_j, while β_j's concavity is clearer in his Fig.2. Further, it will be noted that the fact that the nonlinear pricing rule is restricted to one shown by the dotted line through point *a* might be seen as if the efficient equilibrium is known in advance and the nonlinear pricing rule be defined so that the equilibrium is attained via the pricing rule.

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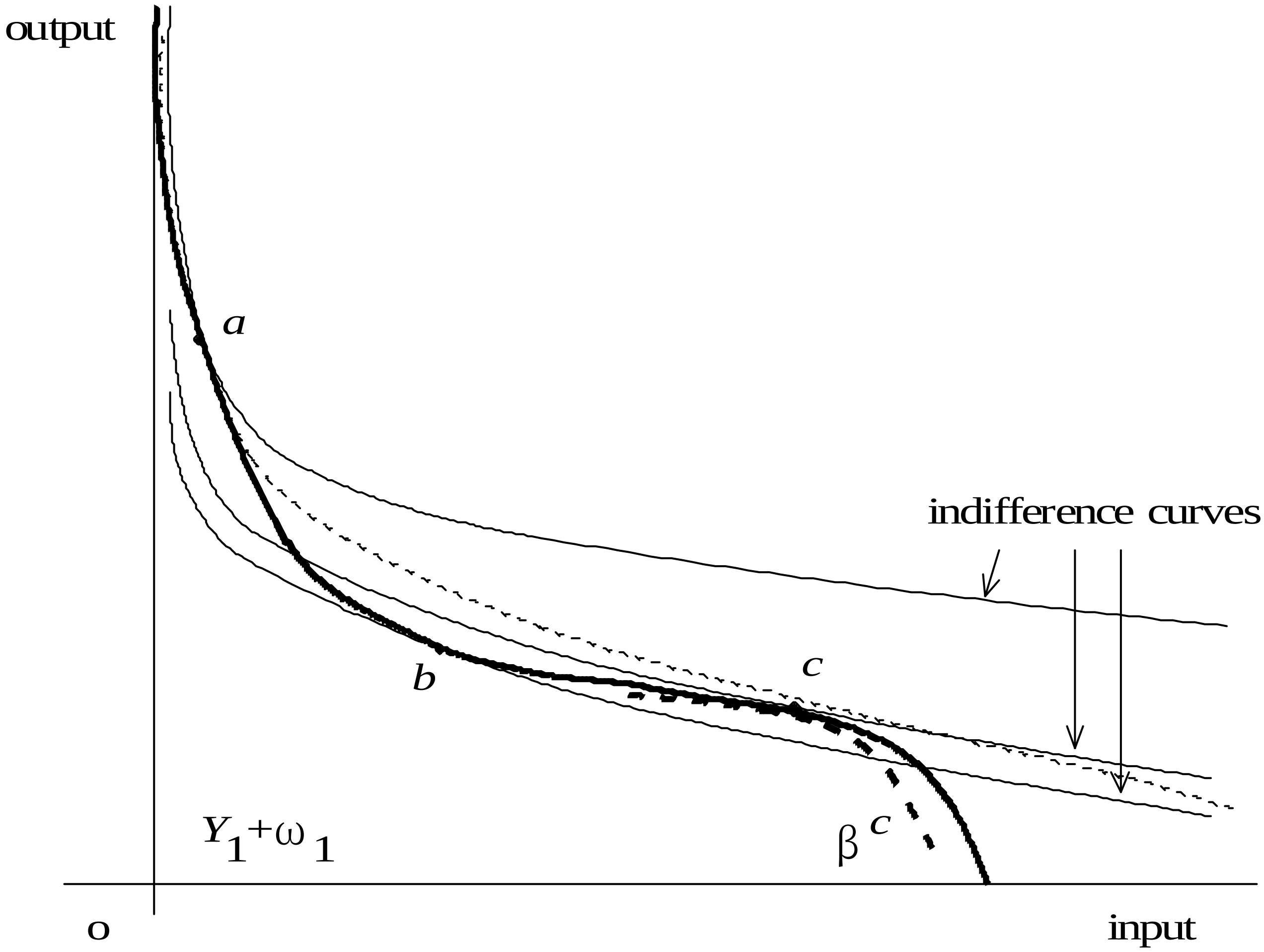

Fig.A The illustrative economy in Kamiya (1995)