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Koichi Futagami* and Yasuhiro Nakamoto†

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Abstract: We construct a small open economy model with a renewable resource. Households have an endogenous time preference rate that depends on the level of the renewable resource in the domestic economy. Although households know that the degree of own patience depends on its resource, we assume that households believe that they cannot control the motion of the aggregate renewable resource. This is because they think that their impact is negligible so that there exists an externality in the form of the patience of the households. Based on this framework, we analyze the dynamic character of the steady state and show that the equilibrium path may be indeterminate. We next examine the welfare effects of tax policies. Finally, we investigate socially optimal tax policies.

Key words: Endogenous time preference rate; Indeterminacy; Renewable resources; Optimal tax policy

JEL classification: F41; H21; Q28

*Faculty of Economics, Osaka University, 1–7 Machikaneyama, Toyonaka 650–0043 Japan. E-mail address: futagami@econ.osaka-u.ac.jp; Tel: +81–6–6850–5266.

†*Corresponding author*: Faculty of Economics, Kyushu Sangyo University, 2-3-1 Matsukadai, Higashi-ku, Fukuoka, 813-8503, Japan. E-mail address: nakamoto@ip.kyusan-u.ac.jp; Tel: +81–92–673–5228.

1 Introduction

Exportable renewable resources represented by timber and fisheries are very valuable for many economies. The International Trade Statistics Yearbook (2004) provides data on these exportable resources. For example, in Finland, one of the most important exportable resources is lumber, which earned \$1,838 million in 2004. In Chile, marine products are important exportable commodities that earned \$2,000 million in 2004, while the marine products exports of Thailand earned \$2,600 million.[1]The existence of such renewable resources cannot be ignored because these resources occupy an important position in those economies. In fact, it is indispensable that small economies harvest their natural resources to acquire foreign exchange. Table 1 shows data on the exports of Finland, Portugal, and Sweden from their forestry sectors, with gradual increases in the volume of their exports of wood fuel, roundwood, and industrial roundwood.

Can the level of harvested natural resources in such economies continue to increase indefinitely? To answer this question, we have to consider how people evaluate the level of natural resources in the future. If people care about the present living more, the level of harvested natural resources increases and then these natural resources are further exported abroad. On the other hand, if they care about the future living more, they may try to preserve the level of natural resources by refraining from harvesting or by investing in those natural resources. Then, the exported quantities of these natural resources will decrease in the short term. This means that the degree of their time preference, or patience, largely influences the level of harvesting of natural resources in the economy.[2] Hence, we must pay attention to people's time

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[1]Elíasson and Turnovsky (2004) use the examples of Iceland and New Zealand as follows: *An obvious example is that of Iceland where in the year 2000 the Fish and Fish Processing sector was around 10% of GDP, having been 15% just a decade earlier...Another example is New Zealand, which has developed a substantial forestry sector, with much of the timber being harvested for export.*

[2]For example, Field (1994) shows as an example that the present value of the future valuation is largely affected by the discount rate. At an annual discount rate of 1%, the discounted present value of obtaining \$100 in a century's time is \$36.78, while at an annual discount rate of 6%, the

preference when considering the economic performance of an economy dependent on natural resources.[3]

Conversely, natural resources can affect people's time preference. In the behavioral evidence, for instance, Viscusi et al. (2008) examine the relation between time preference rate and water quality by using logit model and mixed logit model. They conclude that visitors to water bodies have low rates of discount, while those who do not visit have high rate of discount and low valuations of water quality. It concludes that people who are familiar to natural resources have low rates of discount.

Alternatively, in the current paper, we intuitively consider that when the natural resource endowment of the economy dwindles, people may begin to worry about such future decreases in their natural resources. In other words, people become less impatient when the volume of their natural resources diminishes. Based on the idea, households know that the degree of own patience depends on the aggregate natural resource in the domestic country; however, we assume that households believe that they cannot control the movement of the aggregate natural resource because they think that their impact is negligible.[4] Because of the negligibility, households take it as given when maximizing their behavior. It means that there exists an externality produced by natural resources on the patience of the households.[5] We call this character a *discount externality* in the present paper.

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present value is only $0.29

[3]Karp (2005) examines the role of hyperbolic discounting and concludes that the higher the rate of discount, the lower is the level of abatement.

[4]This argument is similar to those in many papers which incorporate consumption externalities and capital ones (e.g., Futagami and Shibta (1997) and Liu and Turnovsky (2005)). For instance, one example of this kind of problem is the consumption externality; a household's utility function depends on the average level of consumption in whole economy as well as private consumption. Because households cannot control the aggregate level of consumption, a sequence of its average level is exogenously given when the maximization of utility is carried out.

[5]Lines (2005) incorporates an environmental externality into individuals' patience where the environmental externality is a stock variable as in our setting (see page 353 in Lines (2005)). He examines the existence and local stability of an equilibrium in an overlapping generations model.

We construct a small open economy model with a renewable natural resource by incorporating an endogenous time preference rate that depends on the whole level of the natural resource in the domestic economy. The evolution of the renewable natural resource in the economy has the following characters. First, the renewable resource is harvested in the small economy. Second, it is supposed that all of the harvested natural resource is exported. Third, people can invest in the natural resource to increase its level of output.

Based on this framework, we may summarize the main results of our analysis as follows. First, we explore the dynamic characters of the equilibrium.[6] It is shown that there is a unique steady state. However, the equilibrium path converging to the steady state is not uniquely determined. There can be multiple equilibrium paths, that is, the stability of the steady state can exhibit indeterminacy.[7] Second, we analyze the effects of tax policies on the welfare level. For the policy maker, it is important to maximize welfare by imposing tax policies in an economy with externalities. We cannot calculate these welfare effects mathematically because the economy does not follow a unique stable transitional adjustment path. Hence, we

[6]Incorporating the renewable resources in endogenously growth models, many researchers pay attention to whether sustainable growth is achievable. For example, Agihon and Howitt (1998), Ayong Le Kama (2001) and Wirl (2004) present simple growth models incorporating renewable resources as an input in the production function. Elíasson and Turnovsky (2004) examine impacts of some shocks such as technological improvement on economic performance in a small open economy with a production externality. López, et al. (2007) show that there is no unique relationship between the endowment of natural resources and the rate of economic growth when the engine of the growth is human capital accumulation.

[7]So far, many researchers have examined the indeterminacy of the steady state in dynamic general equilibrium models. For example, Meng (2006) shows how the indeterminacy of the steady state arises in one sector model by incorporating an endogenous time preference rate that depends on the average levels of consumption and the capital stock. Weder (2001) and Bian and Meng (2004) examine the dynamic character of equilibrium at the steady state in a small open economy. Bian and Meng (2004) in particular make use of the endogenous time preference rate that depends on the average level of consumption. Itaya (2007) shows that the indeterminacy arises in an endogenous growth model with environmental externalities.

make a comparison of the welfare levels between the steady states with different tax rates. We show that the welfare level of the steady state with a lower rate of a consumption tax is higher than that of the steady state with a higher rate of consumption tax, whereas the welfare level of the steady state with a lower rate of a tax on investment in the natural resource is lower than that of the steady state with a higher rate of tax on the investment in the natural resource. Finally, we examine the socially optimal tax policies.

The remainder of the paper is structured as follows. Section 2 describes the model. Section 3 examines the effects of tax policies on the welfare level. Section 4 characterizes the socially optimal tax policies. Finally, Section 5 concludes.

2 The basic framework

2.1 The model

Suppose that there is a continuum of identical, infinitely lived households in a small open economy in which the total population, I is constant over time and its size is normalized to unity. This economy consists of firms and a government as well as households. The economy is endowed with a stock of a renewable resource. We assume that the level of the renewable resource held by a household shows n_t, while the level of the renewable resource held by all households in the domestic economy expresses N_t where subscript t stands for time. The renewable resource held by a household evolves according to the following equation:

$$\dot{n}_t = \Gamma(i_t) + G(n_t) - z_t, \tag{1}$$

where i_t is the investment in the natural resource, $\Gamma(\cdot)$ represents the efficiency function of investment, and z_t is the rate of harvest for an input for production. We suppose that $G(n_t)$ denotes the reproduction function of the renewable natural resource held by the household.[8] Following some existing papers, this function has an

[8]Because all households are assumed to be identical, the investment function and the reproduction function do not differ among households.

inverted U shape with $G(0) = G(\bar{n}) = 0$, where $\bar{n}$ is the *carrying capacity* of the natural resource and is the level at which its growth ceases.[9] Further, the reproduction function $G(n_t)$ is supposed to be a strictly concave function (i.e., $G'' < 0$). Thus, there is a unique value $\hat{n}$ at which $G'(\hat{n}) = 0$, where $\hat{n}$ expresses the level of the renewable resource providing the maximum sustained yield.

The level of the natural resource can be increased by investing a final good according to the efficiency function $\Gamma(i_t)$. This function $\Gamma(i_t) : \Re_+ \to \Re_+$ is twice continuously differentiable, strictly increasing, strictly concave with respect to the investment i_t, and satisfies the Inada conditions, that is, $\lim_{i_t \to 0} \Gamma'(i_t) = \infty$ and $\lim_{i_t \to \infty} \Gamma'(i_t) = 0$. As for the investment i_t, López et al. (2007) mention that investments in natural resources are supposed to involve tree planting, fish replenishment including aquaculture investments, protection or cleaning-up of ecosystems, soil protection including terracing drainage, and agricultural fallowing.

In this economy, there are two final goods. One is produced by physical capital and labor according to the neoclassical production function per capita, $f(k)$. The other is produced by using the harvested natural resource according to the production function $h(z_t)$. The production function $h(\cdot)$ is strictly increasing, twice continuously differentiable, concave with respect to the harvested natural resource, and furthermore satisfies the Inada conditions. We assume that all production of the commodity is exported. Thus, the households in this economy do not consume any of their natural resources such as lumber or fisheries.[10] Then, the preference of the

[9]See, for example, Ayong Le Kama (2001), Elíasson and Turnovsky (2004), Koskela et al. (2002), López et al. (2007), and Wirl (2004).

[10]When households do consume export commodities, their preference is written as:

$$U = \int_0^\infty [u(c_t) + v(n_t) + w(\text{export goods})]e^{-\Theta_t} dt,$$

where $w(\cdot)$ represents the utility function. In this case, lifelong utility of households is different from that in the present paper. However, even if we assume that the households consume some of the export commodities, the essence of this model is not changed. This is because, in a small open economy, the relative price of consumption commodities can be exogenously given.

households is given by:

$$U = \int_0^{+\infty} [u(c_t) + v(n_t)] \exp[-\Theta_t] dt, \tag{2}$$

where $u(\cdot)$ and $v(\cdot)$ represent the instantaneous utility functions of private consumption c_t and the natural resource n_t, respectively. The households are assumed to obtain higher utility as the stock of the natural resources held by respective households increases. In other words, they feel happy when the level of own natural resources held by the household increases. Furthermore, Θ_t stands for the integral value of the instantaneous rate of time preference from the initial time to the current time. The instantaneous utility functions $u(\cdot)$ and $v(\cdot)$ are twice continuously differentiable, strictly increasing, and strictly concave in terms of c_t and n_t, respectively. In addition, these functions satisfy the Inada conditions.

Next, we consider the endogenous discount rate, which is defined by:

$$\Theta_t \equiv \int_0^t \rho(N_v) dv, \ \frac{d\Theta_t}{dt} = \rho(N_t), \ \Theta_0 = 0, \tag{3}$$

where $\rho(N_t) : \Re_+ \to \Re_{++}$ represents the instantaneous time preference rate. We assume that this time preference rate depends on the aggregate level of the renewable resource in the domestic economy. In addition, we consider that the households become more patient as the natural resource stock decreases. This is because the households come to fear extinction of their natural resource in the future. The function $\rho(\cdot)$ has the following properties:

$$\rho(N_t) > 0, \ \frac{\partial \rho(N_t)}{\partial N_t} > 0, \ \frac{\partial^2 \rho(N_t)}{\partial N_t^2} > 0, \tag{4a}$$

$$\lim_{N_t \to \bar{N}} \rho(N_t) = \rho(\bar{N}), \ \text{and}, \ \lim_{N_t \to 0} \rho(N_t) = \epsilon, \tag{4b}$$

where ϵ and $\rho(\bar{N})$ are the lower bound and the upper bound of the discount function, respectively. Assumption (4a) shows that the discount function has the character of increasing marginal impatience with respect to the natural resource, implying that as the level of the natural resource is smaller, the rate of the time preference is small (i.e., the households become more patient). Assumption (4b) shows that there exist lower and upper bounds of the discount function so that the rate of time preference takes positive values, which implies that the optimization problem of the households satisfies the transversality conditions.

The world interest rate r is exogenously given and is constant for a small open economy. Perfect competition prevails in the economy. Because the total number of the population in this economy is unity, the aggregate level of capital stock is shown by K. Thus, the profit maximizing conditions are given by $r = f'(k)$ and $w = f(k) - kf'(k)$, where k is the level of the domestic capital stock per capita, and w is the wage rate. Therefore, the level of the domestic capital stock k becomes constant. Furthermore, we suppose that the renewable resource is harvested by households. Then, by making use of commodities produced by the domestic capital stock $f(k)$, the households in the economy have options for either consumption or investing in the renewable resource. Then, the accumulation of the foreign assets held by the domestic household $\dot{b}_t$ is given by:

$$\dot{b}_t = rb_t + (1 - \tau_y)(f(k) + ph(z_t)) - (1 + \tau_c)c_t - (1 + \tau_i)i_t + T, \tag{5}$$

where p is the relative price of the harvested natural resource measured by the price of consumption goods. For the small open economy, the relative price p is assumed to be constant.

The government has three kinds of tax policies. The government imposes a consumption tax, an investment tax, and an income tax on the households. Let τ_c be the tax rate on the consumption, τ_y the tax rate on the income, and τ_i the tax rate on the investment in the natural resource. We assume that these tax rates are constant over time. The government is assumed to keep to the following balanced budget constraint:

$$I \times T = I \times \{\tau_y(f(k) + ph(z_t)) + \tau_i i_t + \tau_c c_t\}. \tag{6}$$

This equation means that the government collects tax revenue from domestic households and returns it to the households by lump-sum transfers where I shows the total number of domestic population.

We now consider the maximizing problem of the representative household in a small open economy. In the present paper, we assume that households know that their time preference depends on the aggregate level of the renewable resource; however, the households believe that they cannot control the motion of the aggregate renewable resource, as they think that their impact is negligible.[11] Hence, when the households optimize, they take it as given due to their negligibility. We call this externality a *discount externality* in this paper.[12] Maximizing the discounted sum of the lifelong utility (2) subject to (1) and (5) leads to the following first-order conditions:

$$u'(c_t) = (1+\tau_c)q_t, \tag{7a}$$

$$(1-\tau_y)q_t p h'(z_t) = \eta_t, \tag{7b}$$

$$\eta_t \Gamma'(i_t) = (1+\tau_i)q_t, \tag{7c}$$

$$r = -\frac{\dot{q}_t}{q_t} + \dot{\Theta}_t, \tag{7d}$$

$$\frac{v'(n_t)}{\eta_t} + G'(n_t) = -\frac{\dot{\eta}_t}{\eta_t} + \dot{\Theta}_t, \tag{7e}$$

where η_t and q_t represent the costate variables associated with equations (1) and (5), respectively. In particular, note that the effects of the natural resource on the time preference rate are not included in the first-order conditions due to the discount externality.

Equation (7a) equates the marginal utility of consumption to the shadow value of the foreign assets. Equation (7b) equates the marginal value of the harvested natural resource z_t to the shadow value of the natural resource η_t. Equation (7c) equates the marginal efficiency of investment in the natural resource to the cost of the investment. Equations (7d) and (7e) show the arbitrage conditions in this economy. Equation (7d) is the arbitrage condition, which equates the rate of return on the

[11]This is a similar argument in the existing papers with consumption externalities and capital ones; a household's utility is affected by the average levels of consumption or capital as well as private consumption (e.g., Futagami and Shibata (1998) and Liu and Turnovsky (2005)). These papers assume that households believe that they cannot influence the aggregate consumption or the aggregate capital stock so that their average levels are exogenously given when they optimize.

[12]Lines (2005) incorporates the environmental externality in the discount function where the environmental externality is a stock variable like that in our setting.

foreign assets r to the rate of return on consumption. Equation (7e), which is the key equation in the present paper, represents the arbitrage condition of the investment in the natural resource. This equation (7e) equates the marginal benefit of the natural resource to the rate of return on investment in the natural resource. Please note that the households are assumed to make their decision without considering the degree to which their patience depends on the whole level of the natural resource stock.

Finally, the next two equations are the transversality conditions for the foreign assets and the natural resource, which are satisfied by assumption (4b):

$$\lim_{t\to\infty} q_t b_t e^{-\Theta_t} = 0 \text{ and } \lim_{t\to\infty} \eta_t n_t e^{-\Theta_t} = 0. \tag{7f}$$

2.2 Equilibrium

This subsection shows that there exists a unique steady state and characterizes the equilibrium paths. In particular, we show that the equilibrium path of this economy can be indeterminate, although the steady state is uniquely determined. Firstly, because the total number of households is unity (i.e., $I = 1$), the renewable natural resource is shown by $n_t = N_t$. Hence, we can rewrite that the rate of the endogenous time preference depends on n_t. In this time, noting the discount externality, we define a competitive equilibrium as follows.

Definition. *A competitive equilibrium is a sequence of allocations,* $\{c_t, b_t, n_t, z_t, i_t, k_t\}_{t=0}^{\infty}$, *such that, given initial conditions* b_0, k_0 *and* n_0, *a given set of exogenous prices* $\{w,\ r,\ p\}$ *and the perfect foresight of the households, then the representative household's utility is maximized, the firm's profits are maximized, the government budget constraint is balanced at each time, and all markets are cleared.*

As described in the above definition, due to the perfect foresight of the households, the rate of time preference moves along time as indicated by (3) where $n_t = N_t$. Making use of equations (6) and (7), we can summarize the macroeconomic equilibrium in terms of three dynamic equations for c_t, n_t, and i_t. First, substituting equation (7a) into (7d), we obtain the well-known dynamic equation of consumption as follows:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma_c}(r - \rho(n_t)), \tag{8}$$

where $\sigma_c \equiv -u''(c_t)c_t/u'(c_t)$ represents the elasticity of the intertemporal substitution of consumption. Please note that the time preference rate depends on the level of the renewable resource.

Next, by using (7c) and differentiating η_t with respect to time, we obtain the following equations:

$$\eta_t = \frac{(1+\tau_i)q_t}{\Gamma'(i_t)}, \text{ and, } \dot{\eta}_t = \frac{(1+\tau_i)\dot{q}_t}{\Gamma'(i_t)} - \frac{(1+\tau_i)q_t\Gamma''(i_t)\dot{i}_t}{(\Gamma'(i_t))^2}.$$

Note that the tax rate τ_i is constant over time. Substituting these equations and the dynamic equation of consumption into equation (7e), we obtain the dynamic equation of the investment i_t:

$$\frac{\dot{i}_t}{i_t} = \frac{1}{\sigma_i}\left(r - G'(n_t) - \frac{1+\tau_c}{1+\tau_i}\frac{v'(n_t)\Gamma'(i_t)}{u'(c_t)}\right), \tag{9}$$

where $\sigma_i \equiv -\Gamma''(i_t)i_t/\Gamma'(i_t)$.

Finally, substituting equation (7b) into (7c) yields:

$$ph'(z_t)\Gamma'(i_t) = \frac{1+\tau_i}{(1-\tau_y)}. \tag{10}$$

Solving this equation for the harvested resources z_t, we obtain z_t as a function of i_t, τ_y, and τ_i. We derive this function as follows:

$$z_t = z(i_t, \tau_y, \tau_i). \tag{11a}$$

As for this function, we can obtain the following derivatives:

$$\frac{\partial z_t}{\partial i_t} = -\frac{\Gamma''(i_t)h'(z_t)}{\Gamma'(i_t)h''(z_t)} < 0, \tag{11b}$$

$$\frac{\partial z_t}{\partial \tau_y} = \frac{h'(z_t)}{(1-\tau_y)h''(z_t)} < 0, \tag{11c}$$

$$\frac{\partial z_t}{\partial \tau_i} = \frac{h'(z_t)}{(1+\tau_i)h''(z_t)} < 0. \tag{11d}$$

Substituting function (11a) into the evolution of natural resource (1) yields:

$$\dot{n}_t = G(n_t) + \Gamma(i_t) - z(i_t, \tau_y, \tau_i). \tag{12}$$

Let c^*, n^*, and i^* denote the steady-state levels of consumption, the natural resource, and the investment in the natural resource, respectively. Then, a stationary solution (c^*, n^*, i^*) of the dynamic equations of (8), (9), and (12) is characterized by $\dot{c}_t = \dot{n}_t = \dot{i}_t = 0$ and is defined as follows:

$$r = \rho(n^*), \tag{13a}$$

$$G(n^*) = z(i^*, \tau_y, \tau_i) - \Gamma(i^*), \tag{13b}$$

$$r - G'(n^*) = \frac{1+\tau_c}{1+\tau_i} \frac{v'(n^*)\Gamma'(i^*)}{u'(c^*)}. \tag{13c}$$

¿From (13c), the sign of $r - G'(n^*)$ must be positive at the steady state. We assume this in the following.

Then we obtain the following proposition with respect to the steady state in this economy.

Proposition 1. *Suppose that the world interest rate, r, is larger than the lower bound of the discount rate, ϵ, and that the following inequality holds: $r < \rho(\bar{n})$. Then there exists a unique steady state in this economy.*

Proof. See Appendix A.■

Next, we examine the stability of the steady state. Let J denote the Jacobian matrix of the dynamic equations linearized around the steady state. Then we obtain:

$$\begin{bmatrix} \dot{c}_t \\ \dot{n}_t \\ \dot{i}_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & u'(c^*)\rho'(n^*)/u''(c^*) & 0 \\ 0 & G'(n^*) & \Gamma'(i^*) - \partial z(i^*, \tau_y, \tau_i)/\partial i \\ \Omega_1 & \Omega_2 & \Omega_3 \end{bmatrix}}_{J} \begin{bmatrix} c_t - c^* \\ n_t - n^* \\ i_t - i^* \end{bmatrix}, \tag{14}$$

where Ω_1, Ω_2, and Ω_3 are given by:

$$\Omega_1 = -\frac{1+\tau_c}{1+\tau_i} \frac{\Gamma'(i^*)^2 v'(n^*) u''(c^*)}{\Gamma''(i^*) u'(c^*)^2} < 0, \tag{15a}$$

$$\Omega_2 = \frac{\Gamma'(i^*)}{\Gamma''(i^*)} \left(G''(n^*) + \frac{1+\tau_c}{1+\tau_i} \frac{v''(n^*)\Gamma'(i^*)}{u'(c^*)} \right) > 0, \tag{15b}$$

$$\Omega_3 = \frac{1+\tau_c}{1+\tau_i} \frac{\Gamma'(i^*) v'(n^*)}{u'(c^*)} = r - G'(n^*) > 0. \tag{15c}$$

Then the characteristic equation of the matrix J is given by:

$$-\lambda^3 + \mathrm{Tr}J\lambda^2 - \mathrm{B}J\lambda^1 + \mathrm{Det}J = 0, \tag{16}$$

where the coefficients are given by:

$$\mathrm{Tr}J = r > 0, \tag{17a}$$

$$\mathrm{B}J = G'(n^*)(r - G'(n^*)) + \underbrace{\left(\frac{\partial z(\cdot)}{\partial i} - \Gamma'(i^*)\right)\frac{\Gamma'(i^*)}{\Gamma''(i^*)}\left(G''(n^*) + \frac{1+\tau_c}{1+\tau_i}\frac{v''(n^*)\Gamma'(i^*)}{u'(c^*)}\right)}_{(-)}, \tag{17b}$$

$$\mathrm{Det}J = \Omega_1 \frac{u'(c^*)\rho'(n^*)}{u''(c^*)}(\Gamma'(i^*) - \partial z(\cdot)/\partial i) > 0, \tag{17c}$$

where $\mathrm{Tr}J$ and $\mathrm{Det}J$ indicate the trace and the determinant of this matrix, respectively. To examine stability, we make use of the application of 'Ruth's theorem' to a third-order polynomial as shown in Benhabib and Perli (1994).

Theorem (Benhabib and Perli (1994)) *The number of roots of the polynomial in (16) with positive real parts is equal to the number of variations of sign in the scheme:*

$$-1, \quad \mathrm{Tr}J, \quad -\mathrm{B}J + \frac{\mathrm{Det}J}{\mathrm{Tr}J}, \quad \mathrm{Det}J. \tag{18}$$

First, both of the signs of determinant $\mathrm{Det}J$ and trace $\mathrm{Tr}J$ are positive. Thus, when the sign of $\mathrm{B}J$ is negative, there exist two roots with positive real parts and one negative root. Because the economy has one stock variable n_t and two unpredetermined variables, c_t and i_t, there exist multiple equilibrium paths converging to the steady state. In other words, the dynamic character of the steady state exhibits indeterminacy. $\mathrm{B}J$ takes a negative value if the following inequality holds:

$$G'(n^*)(r - G'(n^*)) < \left(\Gamma'(i^*) - \frac{\partial z(\cdot)}{\partial i}\right)\frac{\Gamma'(i^*)}{\Gamma''(i^*)}\left(G''(n^*) + \frac{1+\tau_c}{1+\tau_i}\frac{v''(n^*)\Gamma'(i^*)}{u'(c^*)}\right), \tag{19}$$

where the sign of the right-hand side is positive. Note that the sign of $r - G'(n^*)$ is positive at the steady state.

We now summarize the above analysis in the next proposition.[13]

Proposition 2. *In the small open country, the equilibrium path of this economy is indeterminate if inequality (19) is satisfied.*

Considering the sufficient condition of indeterminacy, the character of the equilibrium at the steady state tends to be indeterminate as $G'(n^*)$ becomes smaller, which means that the speed of reproduction at the steady state is close to its maximum speed, $G'(\hat{n})$.

3 Welfare comparison associated with tax policies

In this section, we examine the effects of tax policies on the level of welfare. Because the stability of the steady state of this economy exhibits indeterminacy, we focus on a comparison of welfare between different steady states associated with different tax rates.

We first investigate the long-run effects of tax policies on consumption and the

[13]We can present a simple example of the character of the equilibrium by using the specified functions supposing that the rates of income tax, consumption tax, and investment tax are zero.

The discount function is given by $\rho(n_t) = \epsilon + dn_t + gn_t^2$, where ϵ represents the lower bound of this function as the level of natural resources approaches to zero, and d and g are parameters of curvature. In particular, we restrict the range of the parameters d and g to satisfy the assumption of the discount factor given by (4a) and (4b). Then, it is enough that the parameters of d and g take positive values.

Next, the reproduction function of natural resources following Koskela et al. (2002) is given by $G(n_t) = A \times n_t - (1/2)b \times n_t^2$, where A and b are positive parameters. In particular, the growth rate of reproduction $G'(n_t)$ is given by $(A - b \times n_t)$, which implies that, when the stock of the renewable resources provides the maximum sustained yield $G'(\hat{n}) = 0$, the level of the natural resource is $\hat{n} = A/b$.

Assume that the steady-state rate of time preference rate is 0.04. That is, $\rho(n^*) = 0.04$. When the free parameters are given by $\epsilon = 10^{-6}$, $d = 0.05$, $g = 0.2$, $b = 1.5$, and $A = 0.5$, we obtain $n^* = 0.34$ and $G'(n^*) = -0.0090$, which shows that the sign of BJ is negative. In this case, the equilibrium path is indeterminate.

natural resource. Making use of (13a)–(13c), we obtain the following lemma.

Lemma 1. *(i) The consumption tax and the income tax have negative impacts on consumption in the long run, whereas the investment tax has positive impacts on consumption. That is,* $dc^*/d\tau_c < 0$, $dc^*/d\tau_y < 0$, *and* $dc^*/d\tau_i > 0$. *(ii) The consumption tax does not affect investment in the natural resource, whereas the income tax and the investment tax have negative impacts on investment. That is,* $di^*/d\tau_c = 0$, $di^*/d\tau_y < 0$, *and* $di^*/d\tau_i < 0$. *(iii) The consumption tax does not influence the harvesting of the natural resource, whereas the income tax and the investment tax have negative impacts. That is,* $dz^*/d\tau_c = 0$, $dz^*/d\tau_y < 0$, *and* $dz^*/d\tau_i < 0$. *(iv) The tax policy does not affect the renewable natural resources in the long run. That is,* $dn^*/d\tau_y = dn^*/d\tau_c = dn^*/d\tau_i = 0$.

Proof. See Appendix B. ■

We give explanations for the effects of tax policies on consumption. When the rate of the consumption tax rises, the steady-state level of consumption decreases because the price of consumption goods increases. When the rate of income tax increases, the steady-state level of consumption decreases because overall output decreases. On the other hand, the increase in the investment tax rate has two opposite effects. One is the substitution effect of this tax. An increase in the investment tax decreases the level of investment in the natural resource, and thus raises the level of consumption. The other is the indirect negative effect of this tax on consumption. The increase in the investment tax decreases the harvest level and thus lowers the level of income. Although there are these positive and negative impacts of the investment tax on consumption, we can calculate that the positive effect of the investment tax on consumption dominates the negative.[14]

Let us consider the effects of tax policies on investment in the natural resource. Equations (13a) and (13b) determine the steady-state level of investment in the natural resource. Because these equations do not include the consumption tax, the consumption tax does not have any impact on the long-run level of the investment as

[14] The proof is given in Appendix B.

stated in Lemma 1 (ii). On the other hand, the income tax and the investment tax have negative impacts on the long-run level of the investment in the natural resource. When the rate of income tax rises, the decrease in total income reduces the level of investment in the natural resource. Because an increase in the rate of investment tax produces an increase in the cost of investing in the natural resource, it decreases the level of investment in the natural resource in the long run.

Taking account of (11a), we can next examine the effects of taxes on the harvesting of the natural resource. The consumption tax does not have any effect on harvesting because this tax does not affect the level of investment in the natural resource. On the other hand, the income tax and investment tax have two opposite impacts on the harvesting of the natural resource. One is the direct negative effect of the tax policies, and the other is the indirect positive effect through the negative impact on investment in the natural resource. We can show that the direct negative effect of the income tax or the investment tax on the harvesting of the natural resource dominates the indirect positive effect.[15]

Finally, let us consider the effects of the income tax, the consumption tax, and the investment tax on the natural resource. These tax policies do not have any effect on the natural resource. This is because the equality of the constant world interest rate and the time preference rate uniquely determines the steady-state level of the natural resource as shown by (13a).

Making use of these results, we can conduct a welfare comparison with respect to the income tax, the consumption tax, and the investment tax.

Proposition 3. *The welfare level at the steady state with a higher rate of the consumption tax or the income tax is lower than the welfare level at the steady state with lower rates of these taxes. On the other hand, the welfare level at the steady state with a higher rate of the investment tax is higher than the welfare level at the steady state with a lower rate of the investment tax.*

[15]The proof is given in Appendix B.

Proof. By differentiating the welfare level of the household, we obtain the following:

$$\frac{dU}{d\tau_j} = \frac{1}{\rho(n^*)} u'(c^*) \frac{dc^*}{d\tau_j}, \ \ j = c, y, i. \tag{20}$$

Thus we can obtain Proposition 3.■

4 The centrally planned economy

Because households neglect that the discount factor depends on the level of the natural resource stock, the macroeconomic equilibrium may not coincide with the social optimum. In this section, we first consider the first best outcome, namely, when the social planner maximizes the intertemporal utility of the representative household taking account of this externality. First, we calculate the first-order conditions derived by the maximizing problem of the social planner and investigate the distortions caused by the discount externality. Next, we examine the socially optimal tax policies that enable the decentralized equilibrium to replicate the dynamic equations of the centrally planned economy.

4.1 Comparison of steady-state equilibria

Let the variables with the upper bar denote those in the centrally planned economy. Maximizing the intertemporal utility shown by equation (2) subject to (1), (3), (5), and (6) yields the first-order conditions of the central planner as follows:

$$u'(\bar{c}_t) = \bar{q}_t, \tag{21a}$$

$$\bar{q}_t p h'(\bar{z}_t) = \bar{\eta}_t, \tag{21b}$$

$$\bar{\eta}_t \Gamma'(\bar{i}_t) = \bar{q}_t, \tag{21c}$$

$$r = -\frac{\dot{\bar{q}}_t}{\bar{q}_t} + \rho(\bar{n}_t), \tag{21d}$$

$$\frac{u(\bar{c}_t) + v(\bar{n}_t)}{\bar{\lambda}_t} = -\frac{\dot{\bar{\lambda}}_t}{\bar{\lambda}_t} + \rho(\bar{n}_t), \tag{21e}$$

$$\frac{v'(\bar{n}_t)}{\bar{\eta}_t} + G'(\bar{n}_t) - \frac{\bar{\lambda}_t \rho'(\bar{n}_t)}{\bar{\eta}_t} = -\frac{\dot{\bar{\eta}}_t}{\bar{\eta}_t} + \rho(\bar{n}_t), \tag{21f}$$

where $\bar{\eta}_t$, $\bar{\lambda}_t$, and $\bar{q}_t$, represent the shadow prices associated with (1), (3), and (5), respectively.

We compare these first-order conditions with those in the decentralized economy, (7a)–(7e). First, because the social planner takes the discount externality into account, the dynamic equation of the shadow value of patience is given by equation (21e). Next, the third term of the left-hand side of equation (21f) shows the additional term derived by taking account of the discount externality. Noting that the shadow value of patience is included in the additional term, we need to consider the dynamic equation of the shadow value of the patience. Then there exist four dynamic equations of the centrally planned economy, which are given by:

$$\frac{\dot{\bar{c}}_t}{\bar{c}_t} = \frac{1}{\bar{\sigma}_c}(r - \rho(\bar{n}_t)), \tag{22a}$$

$$\dot{\bar{n}}_t = G(\bar{n}_t) + \Gamma(\bar{i}_t) - z(\bar{i}_t), \tag{22b}$$

$$\frac{\dot{\bar{i}}_t}{\bar{i}_t} = \frac{1}{\bar{\sigma}_i}\left(r - \frac{v'(\bar{n}_t)\Gamma'(\bar{i}_t)}{u'(\bar{c}_t)} - G'(\bar{n}_t) + \frac{\bar{\lambda}_t\rho'(\bar{n}_t)\Gamma'(\bar{i}_t)}{u'(\bar{c}_t)}\right), \tag{22c}$$

$$\frac{\dot{\bar{\lambda}}_t}{\bar{\lambda}_t} = \frac{u(\bar{c}_t) + v(\bar{n}_t)}{\bar{\lambda}_t} - \rho(\bar{n}_t), \tag{22d}$$

where $\bar{\sigma}_c \equiv -\bar{c}_t u''(\bar{c}_t)/u'(\bar{c}_t)(> 0)$ and $\bar{\sigma}_i \equiv -\Gamma''(\bar{i}_t)\bar{i}_t/\Gamma'(\bar{i}_t)(> 0)$ represent the elasticity of the marginal utility and the elasticity of the marginal efficiency of the investment. In particular, the last term of the right-hand side of equation (22c) includes the shadow value of patience $\bar{\lambda}_t$. Thus, the dynamic equation (22d) for this shadow value must be taken into account.

Setting the rates of the income, consumption, and investment taxes in equations (9) and (12) to zero, we compare the dynamic equations of the decentralized economy with those of the social optimum. Then, we obtain the following proposition.

Proposition 4. *Suppose that both of the economies are at the steady state. The steady-state levels of the natural resource and the investment in the natural resource are the same in both economies. The steady-state level of consumption in the decentralized economy is smaller than that in the centrally planned economy.*

Proof. When the tax rates are zero, we can obtain the same equation as (10) from equations (21b) and (21c). From the dynamic equations of consumption and the natural resource at the steady state with zero tax rates (i.e., equations (8), (12), (22a), and (22b)), the steady-state levels of the natural resource and investment in the natural resource are the same in both economies, that is, $n^* = \bar{n}^*$ and $i^* = \bar{i}^*$.[16]

We next compare the levels of consumption. From the equations $\dot{i}_t = 0$ and $\dot{\bar{i}}_t = 0$, we obtain the following.

$$\text{The decentralized economy: } r = \frac{v'(n^*)\Gamma(i^*)}{u'(c^*)} + G'(n^*). \tag{23a}$$

$$\text{The centrally planned economy: } r = \frac{v'(\bar{n}^*)\Gamma(\bar{i}^*)}{u'(\bar{c}^*)} + G'(\bar{n}^*) - \frac{\bar{\lambda}^*\rho'(\bar{n}^*)\Gamma'(\bar{i}^*)}{u'(\bar{c}^*)}. \tag{23b}$$

Because the steady-state levels of the natural resource and investment in the decentralized economy are identical to those in the centrally planned economy, we can subtract (23a) from (23b) and can obtain:

$$v'(\bar{n}^*)\Gamma(\bar{i}^*)\left(\frac{1}{u'(c^*)} - \frac{1}{u'(\bar{c}^*)}\right) = -\frac{\bar{\lambda}^*\rho'(\bar{n}^*)\Gamma'(\bar{i}^*)}{u'(\bar{c}^*)}. \tag{24}$$

Because the right-hand side of (24) takes a negative value, the level of consumption in the decentralized economy is smaller than that of the social optimum, that is, $c^* < \bar{c}^*$. ■

This proposition states that the steady-state levels of the natural resource and the investment in the natural resource are the same in both economies. This is because the endogenous time preference rate determines the same level of the natural resource due to (8) and (22a). In contrast with this result, the steady-state level of consumption in the decentralized economy is smaller than that in the centrally planned economy. This is because the discount function has the character of increasing marginal impatience with respect to the level of the natural resource (i.e., $\rho'(n_t) > 0$). When the level of the natural resource is large enough, the social planner largely discounts the future stream of utility compared with the households of the decentralized economy because they do not take account of this effect. As a result, the steady-state level of consumption in the centrally planned economy is larger than that of the decentralized economy. This indicates why the steady state

[16]The asterisks represent the steady-state level of the social optimum.

with the higher rate of investment tax has the larger welfare level (see Proposition 3). A higher rate of investment tax reduces the level of investment and thus raises the level of consumption.

4.2 The socially optimal tax policies

In the last subsection, we have confirmed that the discount externality creates a distortion in resource allocation, which means that the government should develop a tax policy to achieve the social optimum.

Let us consider a tax structure such that the decentralized economy replicates the optimal path of the centrally planned economy *over time*. The equivalence between the path in the decentralized economy and that in the centrally planned economy means $\bar{c}_t = c_t$, $\bar{n}_t = n_t$, $\bar{i}_t = i_t$, and $\bar{\lambda}_t = \lambda_t$ at every point in time. Thus, to achieve this, we allow the rates of the three kinds of tax, $\tau_y(t)$, $\tau_c(t)$, and $\tau_i(t)$ to be time varying. Then we obtain the following lemma with respect to the consumption tax rate.

Lemma 2. *The rate of the consumption tax is constant over time to mimic the socially optimal path in the centrally planned economy.*

Proof. The replication of the social optimum requires $\dot{q}_t/q_t = \dot{\bar{q}}_t/\bar{q}_t$, which leads to $\bar{q}_t = \alpha q_t$ where α is an arbitrary constant. Thus, considering equations (7a) and (21a), the optimal rate of consumption tax is an arbitrary constant, that is, $\alpha = \tau_c = constant$. ■

Taking account of this lemma, the rate of consumption tax must be constant over time, that is, $\tau_c(t) = \tau_c =$ arbitrary constant.[17] Hence, we set the tax rate of consumption at zero through time, that is, $\tau_c(t) = 0$.

Next, let us consider the optimal investment tax rate and the optimal income tax to replicate the stable path in the centrally planned economy. Differentiating

[17] Liu and Turnovsky (2005) show that, in a simple Ramsey model with consumption externalities, the optimal tax rate of consumption needed to mimic the levels of a social optimum is arbitrary if the labor supply is inelastic.

equation (7c) with respect to time, we obtain the dynamic equation of the investment in the natural resource as follows:

$$\frac{\dot{i}_t}{i_t} = \frac{1}{\sigma_i}\left(r - G'(n_t) - \frac{\dot{\tau}_i(t)}{1+\tau_i(t)} - \frac{1}{1+\tau_i(t)}\frac{v'(n_t)\Gamma'(i_t)}{u'(c_t)}\right). \tag{25}$$

Thus, by comparing (25) with (22c), we obtain the optimal rate of the investment tax $\tau_i(\infty)$ at the steady state as follows:

$$\frac{\tau_i(\infty)}{1+\tau_i(\infty)} = \frac{\bar{\lambda}^*\rho'(\bar{n}^*)}{v'(\bar{n}^*)} = \left(\frac{u(c^*)}{v(n^*)}+1\right)\frac{\left(\frac{n^*\rho'(n^*)}{\rho(n^*)}\right)}{\left(\frac{n^*v'(n^*)}{v(n^*)}\right)}. \tag{26}$$

This equation implies that the government can drive the decentralized economy to replicate the centralized economy if the following inequality holds at the steady state:

$$\left(\frac{u(c^*)}{v(n^*)}+1\right)\frac{\left(\frac{n^*\rho'(n^*)}{\rho(n^*)}\right)}{\left(\frac{n^*v'(n^*)}{v(n^*)}\right)} < 1. \tag{27}$$

If the elasticity of the endogenous time preference rate is sufficiently small, this inequality holds. Otherwise, the government cannot achieve a social optimal allocation at the steady state by any tax policies.

Furthermore, the socially optimal rate of the investment tax must evolve according to the following dynamic equation:

$$\frac{\dot{\tau}_i(t)}{1+\tau_i(t)} = \frac{v'(\bar{n}_t)\Gamma'(\bar{i}_t)}{u'(\bar{c}_t)}\left(\frac{\tau_i(t)}{1+\tau_i(t)} - \frac{\bar{\lambda}_t\rho'(\bar{n}_t)}{v'(\bar{n}_t)}\right). \tag{28}$$

Taking account of the rate of the investment tax at the steady-state level and on the socially optimal transitional path, which are given by (26) and (28), we can determine the initial rate of the investment tax.

Finally, we examine the optimal rate of the income tax through time to replicate the harvested level of the natural resource of the centrally planned economy. Substituting (7b) into (7c), we obtain equation (10) where the income tax rate and the investment tax rate change through time. Similarly, substituting (21b) into (21c) yields:

$$ph'(\bar{z}_t)\Gamma'(\bar{i}_t) = 1. \tag{29}$$

Therefore, the comparison of (10) and (29) derives the optimal relationship between the rate of the income tax and the rate of the investment tax as follows:

$$\tau_y(t) = -\tau_i(t). \tag{30}$$

When the rate of the income tax is set according to (30), the harvested level of the natural resource in the decentralized economy is the same as that in the centrally planned economy. This results in $\dot{n}_t = \dot{\tilde{n}}_t$. Then the tax policies defined above enable the decentralized economy to achieve the socially optimal path.

We can summarize these results as follows.

Proposition 5. *Suppose that the inequality (27) holds. Then, the decentralized economy can replicate the resource allocation in the centrally planned economy by setting the rates of the income tax and the investment tax according to (26), (28), and (30), and by setting the rate of the consumption tax at an arbitrary constant level.*

5 Conclusion

We have examined the effects of tax policies in a small open economy that depends on a renewable resource. In the present paper, it is supposed that the patience of households depends on the whole level of the natural resource in the domestic economy. However, households believe that they cannot control its whole level because they think that their influence is negligible. Hence, they do not consider this dependency when they optimize their consumption and investment decisions. In the present paper, we call this externality a discount externality. From this setting we obtain the following results.

First, we have shown that there exists a unique steady state and that the equilibrium path can be indeterminate.

Second, we examined the welfare levels of two different steady states corresponding to two different tax rates. The welfare level at the steady state with a higher rate of consumption or income taxes is lower than that at the steady state with lower rates of these tax rates. Furthermore, the welfare level at the steady state with a higher rate of the investment tax is higher than that at the steady state with a lower rate of the investment tax.

Finally, we examined the optimal tax policy through time to replicate the transitional path in the centrally planned economy. It is possible for the decentralized economy to replicate the optimal path when tax policies are mixed.

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Appendix

Appendix A

In this appendix, we show that the steady-state levels of consumption, the natural resource, and the investment in the natural resource are uniquely determined.

Because $\epsilon < r < \rho(\bar{n})$ in (4b) and the discount rate is an increasing function of the natural resource (4a), the steady-state level of the natural resource is uniquely determined by (13a).

We next consider the relationship between the harvested natural resource z_t and the level of investment in the natural resource i_t given the rates of the income and investment taxes. By differentiating the right-hand side of equation (13b) with respect to i_t, we can show that the right-hand side of equation (13b) is a decreasing function of the investment in the natural resource, that is, $\partial z(i^*, \tau_y, \tau_i)/\partial i - \Gamma'(i^*) < 0$. When the level of the investment in the natural resource is approaching to zero, $\Gamma'(i)$ becomes infinity, that is, $\lim_{i \to 0} \Gamma'(i) = \infty$. Therefore, due to the Inada conditions with respect to $h(z)$, we obtain $\lim_{i\to 0} z(i, \tau_c, \tau_i) = \infty$ (see equation (10)). Because $\Gamma(0) = 0$, we can show:

$$\lim_{i\to 0}[z(i, \tau_c, \tau_i) - \Gamma(i)] = \infty. \tag{A.1}$$

On the contrary, when the level of the investment in the natural resource goes to infinity, $\Gamma'(i)$ approaches to zero, that is, $\lim_{i\to\infty} \Gamma'(i) = 0$. Therefore, we obtain $\lim_{i\to 0} z(i, \tau_c, \tau_i) = 0$. Consequently, we can show the following:

$$\lim_{i\to\infty}[z(i, \tau_c, \tau_i) - \Gamma(i)] < 0. \tag{A.2}$$

Thus, equation (13b) uniquely determines a positive level of investment in the natural resource in the steady state.

Finally, let us consider $\dot{i}_t = 0$ expressed by (13c) to determine the steady-state level of consumption given the tax rates. Because the marginal utility of consumption $u'(\cdot)$ is monotonically decreasing, the steady-state level of consumption is uniquely determined under the assumption of $r > G'(n^*)$.

Appendix B

In this appendix, we examine the effects of increases in the rates of the income, consumption and investment taxes in the natural resource sector on consumption, and on the natural resource and investment in the natural resource. Totally differentiating the equations of $\dot{c}_t = \dot{n}_t = \dot{i}_t = 0$ yields:

$$\underbrace{\begin{bmatrix} 0 & -\rho'(n^*) & 0 \\ 0 & G'(n^*) & \Gamma'(i^*) - \partial z(\cdot)/\partial i \\ \Omega_1 & \Omega_2 & \Omega_3 \end{bmatrix}}_{\#A} \begin{bmatrix} dc \\ dn \\ di \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \partial z(\cdot)/\partial \tau_y & 0 & \partial z(\cdot)/\partial \tau_i \\ 0 & \Omega_4 & \Omega_5 \end{bmatrix} \begin{bmatrix} d\tau_y \\ d\tau_c \\ d\tau_i \end{bmatrix} \tag{B.1}$$

where the definition of Ω_j $(j = 1-5)$ is respectively given by:

$$\Omega_1 = \frac{1+\tau_c}{1+\tau_i}\frac{v'(n^*)\Gamma'(i^*)u''(c^*)}{(u'(c^*))^2} < 0, \tag{B.2}$$

$$\Omega_2 = -G''(n^*) - \frac{1+\tau_c}{1+\tau_i}\frac{v''(n^*)\Gamma'(i^*)}{u'(c^*)} > 0, \tag{B.3}$$

$$\Omega_3 = -\frac{1+\tau_c}{1+\tau_i}\frac{v'(n^*)\Gamma''(i^*)}{u'(c^*)} > 0, \tag{B.4}$$

$$\Omega_4 = \frac{1}{1+\tau_i}\frac{v'(n^*)\Gamma'(i^*)}{u'(c^*)} > 0, \tag{B.5}$$

$$\Omega_5 = -\frac{1+\tau_c}{(1+\tau_i)^2}\frac{v'(n^*)\Gamma'(i^*)}{u'(c^*)} < 0. \tag{B.6}$$

Thus, we obtain the following:

$$\begin{bmatrix} dc \\ dn \\ di \end{bmatrix} = \frac{1}{det}\begin{bmatrix} G'(n^*)\Omega_3 - \Omega_2(\Gamma'(i^*) - \partial z(\cdot)/\partial i) & \rho'(n^*)\Omega_3 & -\rho'(n^*)(\Gamma'(i^*) - \partial z(\cdot)/\partial i) \\ \Omega_1(\Gamma'(i^*) - \partial z(\cdot)/\partial i) & 0 & 0 \\ -G'(n^*)\Omega_1 & -\rho'(n^*)\Omega_1 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 0 & 0 \\ \partial z(\cdot)/\partial \tau_y & 0 & \partial z(\cdot)/\partial \tau_i \\ 0 & \Omega_4 & \Omega_5 \end{bmatrix}\begin{bmatrix} d\tau_y \\ d\tau_c \\ d\tau_i \end{bmatrix} \tag{B.7}$$

where *det* represents the *positive* determinant of the matrix $\#A$, which is given by $det = -\Omega_1\rho'(n^*)(\Gamma'(i^*) - \partial z^*/\partial i^*) > 0$.

Noting that the signs of $\partial z(\cdot)/\partial i$, $\partial z(\cdot)/\partial \tau_y$, and $\partial z(\cdot)/\partial \tau_i$ are negative, we can obtain the effects of tax policies on consumption, the natural resource stock, and investment in the natural resource as follows:

$$\frac{dc}{d\tau_y} = \frac{1}{det}\rho'(n^*)\Omega_3\frac{\partial z(\cdot)}{\partial \tau_y} < 0, \tag{B.8a}$$

$$\frac{dn}{d\tau_y} = 0, \tag{B.8b}$$

$$\frac{di}{d\tau_y} = -\frac{1}{det}\rho'(n^*)\Omega_1\frac{\partial z(\cdot)}{\partial \tau_y} < 0, \tag{B.8c}$$

$$\frac{dc}{d\tau_c} = -\frac{1}{det}\rho'(n^*)\left(\Gamma'(i^*) - \frac{\partial z(\cdot)}{\partial i}\right) < 0, \tag{B.8d}$$

$$\frac{dn}{d\tau_c} = 0, \tag{B.8e}$$

$$\frac{di}{d\tau_c} = 0, \tag{B.8f}$$

$$\begin{aligned}\frac{dc}{d\tau_i} &= \frac{1}{det}\left[\rho'(n^*)\Omega_3\frac{\partial z(\cdot)}{\partial \tau_i} - \rho'(n^*)\left(\Gamma'(i^*) - \frac{\partial z(\cdot)}{\partial i}\right)\Omega_5\right] \\ &= -\frac{1}{det}\Gamma'(i^*)\rho'(n^*)\Omega_5 > 0,\end{aligned} \tag{B.8g}$$

$$\frac{dn}{d\tau_i} = 0, \tag{B.8h}$$

$$\frac{di}{d\tau_i} = -\frac{1}{det}\rho'(n^*)\Omega_1\frac{\partial z(\cdot)}{\partial \tau_i} < 0. \tag{B.8i}$$

Next, let us consider the effect of these tax policies on the harvested natural resource. Differentiating (11a) with respect to the tax rates, the effects of the income, consumption, and investment taxes on the harvested natural resource are given by:

$$\begin{aligned}\frac{\partial z^*}{\partial \tau_y} &= \frac{\partial z^*}{\partial i^*}\frac{\partial i^*}{\partial \tau_y} + \frac{\partial z^*}{\partial \tau_y}, \\ &= \frac{\partial z^*}{\partial \tau_y}\left(-\frac{\Gamma''(i^*)h'(z^*)}{\Gamma'(i^*)h''(z^*)}\frac{1}{\Gamma'(i^*) - \partial z^*/\partial i^*} + 1\right), \\ &= \frac{\partial z^*}{\partial \tau_y}\frac{\Gamma'(i^*)}{\Gamma'(i^*) - \partial z^*/\partial i^*} < 0.\end{aligned} \tag{B.9a}$$

$$\frac{\partial z^*}{\partial \tau_c} = \frac{\partial z^*}{\partial i^*}\frac{\partial i^*}{\partial \tau_c} < 0, \tag{B.9b}$$

$$\begin{aligned}\frac{\partial z^*}{\partial \tau_i} &= \frac{\partial z^*}{\partial i^*}\frac{\partial i^*}{\partial \tau_i} + \frac{\partial z^*}{\partial \tau_i}, \\ &= \frac{\partial z^*}{\partial \tau_i}\left(-\frac{\Gamma''(n^*)h'(z^*)}{\Gamma'(i^*)h''(z^*)}\frac{1}{\Gamma'(i^*) - \partial z^*/\partial i^*} + 1\right), \\ &= \frac{\partial z^*}{\partial \tau_i}\frac{\Gamma'(i^*)}{\Gamma'(i^*) - \partial z^*/\partial i^*} < 0.\end{aligned} \tag{B.9c}$$

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Table 1: Export Quantity of Forestry Sector (million cubic meters).

	2000	2001	2002	2003	2004	2005
Finland	1.06	0.81	0.82	0.88	1.06	1.51
Portugal	1.14	1.62	1.64	2.04	2.02	2.56
Sweden	2.92	2.68	3.57	3.01	3.12	6.25

Data source: Food and Agriculture Organization of the United Nations.[18]

[18]We use data from the following homepage: http://www.fao.org/