Discussion Papers In Economics And Business

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Abstract

In this paper, we develop an endogenous growth model with two countries in which the international trade of differentiated goods requires different trade costs and equilibrium wages in the two countries. If the labor productivity in one country's agricultural sector is higher than that of the other country, the wages will also be higher.

In this model, there is a case in which the small country has a higher share of manufacturing firms than the larger country, and the innovation sector locates in the small country, since the cost for production of the manufacturing sector and innovation sector is higher in the large country than in the small country.

First, when trade costs are high, the share of manufacturing firms in a large country increases with a decline in trade costs. However, the share then decreases with a decline in trade costs when trade costs are low. Finally, all firms agglomerate in the small country with lower production costs. If trade costs are very high, the innovation sector will locate in the small country. If trade costs take an intermediate value, it will locate in the large country. If trade costs become very low, it will re-locate in the small country. The growth rate moves non-monotonously in a W-shaped curve when there is a reduction in trade costs. This happens because the growth rate is affected by the number of manufacturing firms and the location of the innovation sector.

JEL Classification: F0, O31.

Key words: market size, wage differential, growth rates, innovation sector.

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1 Introduction

In this paper, a Grossman-Helpman (1991) [5] and Romer (1990) [11]-type endogenous growth model that has two countries is developed. The activity of the innovation sector results in an increased variety of differentiated goods. In our model, there is some degree of knowledge spillover between the two countries. Given the knowledge spillover, the innovation costs in a particular country decrease as the number of firms locating there increases. We assume that labor productivity in the agricultural sector are different between two countries, which leads to different equilibrium wages in the two countries. In our model, there are three sectors, agricultural, manufacturing, and innovation, and there are two countries. In the agricultural sector, homogenous goods are produced only by labor with constant return production functions. These homogenous goods are traded internationally with no trade costs. Therefore, the price of these goods is the same in both countries. We assume that labor productivity in the agricultural sectors differs in the two countries. As a result, the equilibrium wages in the two countries differ.

If the equilibrium wage in two countries is the same, the country with a large market always absorbs more firms than the other one. Therefore, when the equilibrium wage in two countries is the same, the innovation sector always locates in the large market country, and the growth rates increase with agglomeration of manufacturing firms in that country. In this case, the share of manufacturing firms in the large market country increases with a decline in trade costs.

In the standard international trade theory, there are comparative advantages between two countries. However, in the standard new economic geography literature, such as that by Fujita, Krugman, and Venables (1999), there are not comparative advantages between two countries. Some studies have examined the endogenous growth-new economic geography models, such as those by Baldwin, Martin, and Ottaviano (2001), Martin and Ottaviano (1999), (2001), Yamamoto (2003), and Minniti and Parello (2011). However, in each of these studies, it is assumed that there are no comparative advantages between two countries and that the production costs of the agricultural and manufacturing sectors are the same. No study has investigated the effects of the production cost differential in the context of the endogenous growth - new economic geography. It is important to study the endogenous growth - new economic geography model in such a context. In this paper, we present a model of endogenous growth and new economic geography in which production cost differentials are introduced.

If wages in the large market country are higher than those in the other country, the higher wages will lower the share of manufacturing firms in the large market country. Let us assume that trade costs are initially high and gradually decrease. In this case, the share of manufacturing firms in the large country increases with a decline in trade costs. In a range of trade costs, we observe full agglomeration in the large country. However, when trade costs become lower, full agglomeration in the large country is broken, and some manufacturing firms switch their location from the large to the small market country. Finally, we observe that all manufacturing firms agglomerate in the small market country when trade costs are very low. In this process, the innovation sector changes its location twice when critical numbers of manufacturing firms are reached. When trade costs are very high, the innovation sector locates in the small market country. When the trade costs have an intermediate value, the innovation sector locates in the larger market country. When trade costs are very low, the innovation sector locates once again in the small market country. The growth rate moves non-monotonously in this process. The growth rate is raised by the number of firms of the same country because of the spillover of local knowledge. Our model shows that not only the market size but also the production costs of the manufacturing sector determine the location of the innovation sector, which derives economic growth.

Our model is closely related to that of Martin and Ottaviano (1999) [9]. When shipping of differentiated goods incurs trade costs, more firms locate in the large market country than in the small market country to economize on trade costs. In addition, if the equilibrium wages in two countries are the same, the innovation costs are always lower in the country that has the large market. This occurs because the higher the number of firms that locate in a particular country, the lower the innovation costs in that country due to the partial spillover of local knowledge. Thus, when the equilibrium wage is the same in the two countries, the growth rate increases monotonically with agglomeration in the large market country. Our paper expands on the ideas of Martin and Ottaviano (1999) [9]; in our model, wages in the two countries differ from each other. In this paper, we show that wage differences between two countries have a similar effect to market size differences on the locational share of manufacturing firms. In other words, it is more profitable for firms to locate in a country with lower production costs. If wages in a small market country are sufficiently low, the small market country will have a higher share of manufacturing firms than the large market country. Furthermore, the innovation sector will locate in the small market country, since the production costs for the manufacturing sector and innovation sector will be higher in the large market country than in the small market country. In this case, agglomeration in the small market country fosters the economic growth rate. The growth rate reaches its maximum value when all of the firms locate in the small (lower wage) country. This occurs when the trade costs are very low. Thus, our paper has richer implications and presents the mechanism of the history of economic development and agglomeration. Although it has been reported in many studies that trade liberalization results in increased growth rates, there are cases in the process of economic development in which the regulation of trade has resulted in increased growth rates. For example, Komiya et al. (1984) argued that the Japanese government regulated trade for the protection and expansion of domestic industry between 1950 and 1970. Due to this regulation, Japan achieved rapid growth. Therefore, there are cases in which trade liberalization increases the growth rate, and there are cases in which trade liberalization lowers the growth rate. By introducing a wage differential, we can explain why both situations occur in the real world.

This paper is organized as follows. In the next section, we present the basic model. In Section 3, we analyze the model and present the steady state equilibrium. In Section 4, we report the effects of trade costs, wage difference, and market size on the location of manufacturing firms, the innovation sector, and the growth rate. Section 5 is the conclusion.

2 The Model

In this section, we introduce the model. There are two countries, 1 and 2. Variables referring to 1 have the subscript 1, and those referring to 2, the subscript 2. Each country is endowed with a fixed amount of labor, L_1 and L_2 ($L_1 > L_2$). Thus, country 1 has a larger amount of labor than country 2. This also means that country 1 has a larger market than country 2. Labor can be used to produce homogenous agricultural goods, differentiated manufactured goods, and blueprints. While labor can be mobile between sectors in the same country, it cannot be mobile between different countries. For variety to be achievable, a blueprint has to be invented. The blueprint is then protected by a patent that cannot expire that initially belongs to the country in which innovation took place. Once the blueprints are invented, the patent can be sold to any firm located in either country. The innovation and the production process are, therefore, conducted by different economic agents and possibly in different countries.

The intertemporal utility function of the consumer in country s ($s = 1, 2$) is as follows:

$$U_s = \int_0^{\infty} e^{-\rho t} \left(Y_{st} + \mu \log M_{st}\right) dt, \tag{1}$$

where

$$M_{st} = \left[\int_0^{n_{1t}} m_{1st}\left(i\right)^{\frac{\sigma-1}{\sigma}} di + \int_0^{n_2} m_{2st}\left(i\right)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}, \ \sigma > 1. \tag{2}$$

Here, Y_{st} is the consumption of agriculture goods at time t, M_{st} is the consumption of the composite of manufactured goods at time t, ρ is the subjective discount rate, and μ is the constant parameter. $m_{rst}\left(i\right)$ denotes the consumption of the variety i th manufactured goods produced by a firm in country r ($r = 1, 2$). n_{rt} is the number of varieties produced by a firm in country r at time t. n_{rt} is also the number of operating firms in country r at time t. $N_t \equiv n_{1t} + n_{2t}$ denotes the total number of varieties at time t. σ is the constant parameter that represents the elasticity of substitution among differentiated goods. Following Grossman and Helpman (1991), the market has been characterized by free financial movements between two countries. Thus, the interest rate of both countries is the same at all times ($r_{1t} = r_{2t} = r_t$). The intertemporal optimization behavior of the consumer brings about the next equation

$$r_t = \rho. \tag{3}$$

We can derive the following instantaneous demand functions (we take homogeneous goods as the numeraire, such that $p_A = 1$),

$$M_{st} = \frac{\mu}{P_{st}}, \tag{4}$$

$$P_{st} = \left(\int_0^{n_{1t}} p_{1st}(i)^{1-\sigma} di + \int_0^{n_{2t}} p_{2st}(i)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}, \tag{5}$$

$$Y_{st} = E_{st} - \mu, \tag{6}$$

$$m_{rst}\left(i\right) = \frac{\mu {P_{st}}^{\sigma-1}}{p_{rst}\left(i\right)^{\sigma}}, \tag{7}$$

where P_{st} is called the 'price index' in country s at time t. $p_{rst}\left(i\right)$ is the consumer price of variety i, which is produced in r and consumed in s, and E_{st} represents the instantaneous expenditure of a consumer in country s at time t.

Here, we describe the production structure of the agricultural sector. A homogeneous agriculture goods market is perfectly competitive. We assume that the productivities of labor in the agricultural sector differ between the two countries. In country 1, a_1 units of agriculture goods are produced with 1 unit of labor. In country 2, 1 unit of labor produces a_2 units of agricultural goods. We assume that the international trade of homogenous goods incurs no trade costs. Therefore, since we assume that agricultural goods are produced in both countries at the equilibrium[1], the equilibrium wages in the two countries become $w_1 = a_1$, $w_2 = a_2$.

In the manufacturing goods sector, manufacturing firms operate under Dixit-Stiglitz [2]-type monopolistic competition. Each firm produces differentiated goods, and each variety is produced by one firm. To start a production activity, a firm in country r is required to buy one unit of a patent produced by the innovation sector at market price v_{rt}, which plays the role of fixed costs for the firms. Moreover, a firm locating in a country uses c units of labor in its country as the marginal input to produce one unit of manufactured goods. Potential firms can freely enter into a production activity as long as the operating profits are positive and can choose to locate in a country where profits are higher. Under this production structure, each manufacturing firm sets the following constant markup price:

$$p_r = \frac{\sigma}{\sigma-1} \cdot cw_r = \frac{\sigma}{\sigma-1} \cdot ca_r, \ r = 1, 2. \tag{8}$$

$cw_r = ca_r$ represents the marginal costs for the manufacturing firms. The international trade of manufactured goods incurs 'iceberg'-type trade costs. If a firm sends x units of goods to a foreign country, it must dispatch τx units of

[1] We assume that μ is sufficiently small so that the total demand for agricultural goods is sufficiently large, that is, $(E_{1t} - \mu) L_1 + (E_{2t} - \mu) L_2 \geq a_1 L_1$ and $(E_{1t} - \mu) L_1 + (E_{2t} - \mu) L_2 \geq a_2 L_2$. In this model, $E_{1t} \geq a_1$ and $E_{2t} \geq a_2$. Therefore, we assume that $(a_1 - \mu) L_1 + (a_2 - \mu) L_2 \geq a_1 L_1$ and $(a_1 - \mu) L_1 + (a_2 - \mu) L_2 \geq a_2 L_2$.

goods. $\tau > 1$ represents the trade costs. Thus, consumer prices are $p_{rs} = p_r$ if $r = s$, and $p_{rs} = \tau p_r$ if $r \neq s$. With constant markup pricing, the operating profit of each firm and the price index are written as,

$$\pi_{rt} = \frac{cw_r}{\sigma - 1} q_{rt} = \frac{ca_r}{\sigma - 1} q_{rt}, \tag{9}$$

$$P_{it} = \frac{\sigma c}{\sigma - 1} \cdot \left(n_{it} a_i^{1-\sigma} + n_{jt} \phi a_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \ i = 1, 2.\ j = 1, 2.\ i \neq j, \tag{10}$$

where $\phi \equiv \tau^{1-\sigma} \in (0, 1)$ represents the freeness of trade.

We assume that the capital market is perfectly competitive. We assume that there are risk-free assets and their interest rate is r_t. The value of the firm (which is the market price of the patent) is equalized to the present value of the sum of discounted profit over time. From (9), it represents

$$v_{rt} = \int_t^\infty e^{-r(\iota - t)} \cdot \frac{ca_r}{\sigma - 1} q_{r\iota} d\iota. \tag{11}$$

Differentiating (11) with respect to t, we obtain the no- arbitrage condition for capital investment v_{rt}.

$$\frac{ca_r}{\sigma - 1} \cdot q_{rt} + \dot{v}_{rt} = r v_{rt}. \tag{12}$$

In the innovation sector, we assume that innovation firms produce 1 unit of a patent by using I_s units of labor. For innovators in country s, the innovation costs for a patent are written as $w_s I_{st}$ $(s = 1, 2)$. Innovators choose their own location with no relocation cost. Then, innovators choose s, where they can minimize innovation costs $w_s I_{st}$. If $w_1 I_{1t} < (>) w_2 I_{2t}$, then the innovator locates in country 1(2). We assume that I_s depends on the number of home and foreign varieties of manufacturing firms, as follows:

$$I_{it} = \frac{\eta}{n_{it} + \delta n_{jt}}, \ i = 1, 2.\ j = 1, 2.\ i \neq j, \tag{13}$$

where $\delta \in (0, 1)$ represents the degree of international knowledge spillover[2]. In this innovation technology, the agglomeration of manufacturing firms in a country lowers the innovation costs in this country, since we assume that $\delta \in (0, 1)$. Let us define $s \equiv \frac{n_1}{n_1 + n_2}$ as the share of manufacturing firms in country 1. We can check $\frac{\partial w_1 I_{1t}}{\partial s} < 0$ and $\frac{\partial w_2 I_{2t}}{\partial s} > 0$ as $N \equiv n_1 + n_2$ fixed. A cost-minimizing innovator chooses its location according to the share of manufacturing firms in country 1, $s \equiv n_1/N$, and the relative wage in country 1 to country 2, $\frac{a_1}{a_2}$. Therefore, we can describe the location behavior of innovators as follows:

$$\text{The location of the innovator is} \begin{cases} \text{country 2} & \text{if} \quad \left(s, \frac{a_1}{a_2}\right) \in \left\{ s, \frac{a_1}{a_2} | 0 \leq s \leq \widehat{s}, 0 < \frac{a_1}{a_2} \right\} \\ \text{country 1} & \text{if} \quad \left(s, \frac{a_1}{a_2}\right) \in \left\{ s, \frac{a_1}{a_2} | \widehat{s} \leq s \leq 1, 0 < \frac{a_1}{a_2} \right\} \end{cases}, \tag{14}$$

[2] In Hirose and Yamamoto (2007) [6], knowledge spillover from a foreign country is asymmetric in two countries. We can extend our model with the assumption of asymmetric knowledge spillover.

where $\widehat{s} \equiv \frac{a_1 - a_2\delta}{(1-\delta)(a_1+a_2)} = \frac{\frac{a_1}{a_2}-\delta}{(1-\delta)\left(\frac{a_1}{a_2}+1\right)}$. $\frac{\partial \widehat{s}}{\partial\left(\frac{a_1}{a_2}\right)} = \frac{1+\delta}{\left(\frac{a_1}{a_2}+1\right)\left(\frac{a_1}{a_2}-\delta\right)} > 0$. As the productivity for agricultural goods of country 1 relative to that of country 2 becomes larger, wages in country 1 become higher relative to those in country 2. Thus, if a_1/a_2 is sufficiently large, innovation firms locate in country 2. On the other hand, if the share of manufacturing firms in country 1 is large, positive technological externality lowers the innovation costs in country 1. Then, if country 1 absorbs many manufacturing firms relative to country 2 (s is large), the innovation sector locates in country 1.

3 Equilibrium conditions and steady states

3.1 Market clearing conditions

In this section, we characterize and solve the equilibrium conditions of the model. The total production quantities of a firm of each variety in countries 1 and 2 must satisfy $q_1 = L_1 m_{11} + \tau L_2 m_{12}$ and $q_2 = L_2 m_{22} + \tau L_1 m_{21}$, respectively. Introducing (7), (8), and (5), the market clearing conditions for each variety of goods are

$$q_{1t}(s) = a_1^{-\sigma} \cdot \frac{(\sigma-1)}{\sigma c N_t} \cdot \mu \left(\frac{L_1}{s_t a_1^{1-\sigma} + (1-s_t)\phi a_2^{1-\sigma}} + \frac{\phi L_2}{(1-s_t) a_2^{1-\sigma} + s_t \phi a_1^{1-\sigma}} \right) \quad (15)$$

$$q_{2t}(s) = a_2^{-\sigma} \cdot \frac{(\sigma-1)}{\sigma c N_t} \cdot \mu \left(\frac{L_2}{(1-s_t) a_2^{1-\sigma} + s_t \phi a_1^{1-\sigma}} + \frac{\phi L_1}{s_t a_1^{1-\sigma} + (1-s_t)\phi a_2^{1-\sigma}} \right) \quad (16)$$

For the following analysis, we define $A \equiv \left(\frac{a_1}{a_2}\right)^{1-\sigma}$ and $\lambda = L_1/(L_1+L_2)$. With this definition, we can see that $\frac{dA}{d\left(\frac{a_1}{a_2}\right)} < 0$, $\lim_{\frac{a_1}{a_2}\to 0} A = \infty$, $\lim_{\frac{a_1}{a_2}\to\infty} A = 0$ and $1/2 < \lambda < 1$.

We must clarify the parameter conditions in which the firm's location equilibrium becomes an interior solution or a corner solution. The difference of operating profits across two countries is

$$\Phi(s) \equiv \pi_1 - \pi_2 = \frac{\mu(L_1+L_2)}{\sigma N} \left[\frac{(A-\phi)\lambda}{sA + (1-s)\phi} + \frac{(\phi A - 1)(1-\lambda)}{(1-s) + s\phi A} \right]. \quad (17)$$

$\Phi(s)$ is a decreasing function of s. A manufacturing firm determines its location taking s as given. When $\Phi(s) > (<) 0$, a firm locates in country 1 (2). When $\Phi(s) = 0$, it is indifferent for a firm wheter it locates in country 1 or 2. Since enterance and exit of manufacturing firms are free, firms enter into one country as long as its profits are positive and higher than the other country. We define s^* as an equilibrium value of s. First, if $\Phi(0) \le 0$, then $s^* = 0$. Second, if $\Phi(1) \ge 0$, then $s^* = 1$. Third, if $\Phi(0) > 0$ and $\Phi(1) < 0$, then s^* is an interior solution ($0 < s^* < 1$) and $\Phi(s^*) = 0$. In order to determine which equilibrium is realized, we analyze the model by identifying three cases: (a)$0 < A \le \phi$, (b)$\frac{1}{\phi} \le A < \infty$ and (c)$\phi < A < \frac{1}{\phi}$.

(a) The case of $0 < A \leq \phi$ leads to both $A - \phi \leq 0$ and $\phi A - 1 < 0$ in the numerators of (17). Thus, $\Phi(s) < 0$for all $s \in [0,1]$ and for all $\lambda \in \left(\frac{1}{2}, 1\right)$. Then, $\Phi(0) < 0$ for all $\lambda \in \left(\frac{1}{2}, 1\right)$. This means that $s^* = 0$. Then, when $0 < A \leq \phi$, all manufacturing firms agglomerate in country 2.

(b) The case of $\frac{1}{\phi} \leq A < \infty$ leads to both $A - \phi > 0$ and $\phi A - 1 \geq 0$ in the numerator of (17). Thus, $\Phi(s) > 0$ for all $s \in [0,1]$ and for all $\lambda \in \left(\frac{1}{2}, 1\right)$.Then, $\Phi(1) > 0$ for all $\lambda \in \left(\frac{1}{2}, 1\right)$. This means $s^* = 1$. Therefore, in the case of $\frac{1}{\phi} \leq A < \infty$, all manufacturing firms agglomerate in country 1.

(c) The case of $\phi < A < \frac{1}{\phi}$ leads to both $A - \phi > 0$ and $\phi A - 1 < 0$. The sign of (17) is not clear. However, since (17) is a decreasing function of s, it has interior solutions in some parameter conditions: $\Phi(0) > 0$ and $\Phi(1) < 0$. The conditions of $\Phi(0) > 0$ and $\Phi(1) < 0$ are equivalent to $(0 <) \frac{\phi(1-\phi A)}{A(1+\phi)(1-\phi)} < \lambda < \frac{1-\phi A}{(1+\phi)(1-\phi)} (< 1)$. Under this condition, s^*satisfies $0 < s^* < 1$ and $\Phi(s^*) = 0$. Otherwise, it has corner solutions. In other words, if $\lambda \leq \frac{\phi(1-\phi A)}{A(1+\phi)(1-\phi)}$, then, $s^* = 0$, and, if $\lambda \geq \frac{1-\phi A}{(1+\phi)(1-\phi)}$, then, $s^* = 1$.

From the above discussion ((a), (b), and (c)), the equilibrium share of firms in country 1 is

$$s^*(\lambda, A, \phi) = \begin{cases} 0 & \text{if} \quad (\lambda, A, \phi) \in \{\lambda, A, \phi | 0 \leq \lambda \leq \lambda_0, 0 < A < 1\} \\ \frac{A(1+\phi)(1-\phi)\lambda - \phi(1-\phi A)}{(1-\phi A)(A-\phi)} & \text{if} \quad (\lambda, A, \phi) \in \left\{\lambda, A, \phi | \lambda_0 < \lambda \leq \lambda_1, \phi < A < \frac{1}{\phi}\right\} \\ 1 & \text{if} \quad (\lambda, A, \phi) \in \{\lambda, A, \phi | \lambda_1 < \lambda \leq 1, 0 < A < 1\} \end{cases} \tag{18}$$

where $\lambda_0 \equiv \frac{\phi(1-A\phi)}{A(1+\phi)(1-\phi)}$, $\lambda_1 \equiv \frac{1-A\phi}{(1+\phi)(1-\phi)}$.

By differentiating an interior solution with respect to λ and A, we obtained the following lemma.

Lemma 1 *(1) The interior solution of $s^*(\lambda, A, \phi)$ is an increasing function of λ. That is, $\frac{\partial s^*}{\partial \lambda} > 0$. (2)The interior solution of $s^*(\lambda, A, \phi)$ is an increasing function of A. That is, $\frac{\partial s^*}{\partial A} > 0$.*

Proof. *(1) $\frac{\partial s^*}{\partial \lambda} = \frac{A(1+\phi)(1-\phi)}{(1-\phi A)(A-\phi)} \geq 0$ (Note $1 - \phi A > 0$ and $A - \phi > 0$). (2) See the Appendix.* ■

Lemma 1 says that the share of firms in country 1 rises if (1) the relative market size of country 1 becomes larger or if (2) the relative wage in country 1 becomes lower. We can depict the equilibrium share of manufacturing firms as in Figure 1.

3.2 Equilibrium locations of the innovation sector

The innovation firm determines its location according to s and $\frac{a_1}{a_2}$ as (14). Some calculations lead us to the next lemma.

Lemma 2 *(1) Let us assume that* $\frac{a_1}{a_2} < 1$*. More than half of the manufacturing firms locate in a large market country, and the innovation sector locates in this country. That is,* $s^* > \frac{1}{2}$*, and* $\widehat{s} < \frac{1}{2}$*. (2) Let us assume that* $\frac{a_1}{a_2} > 1$*. There exists* $\hat{s} \geq \frac{1}{2}$*, and* $\hat{s}$ *is raised as* $\frac{a_1}{a_2}$ *becomes larger (A becomes smaller). In addition, when A becomes smaller than* $\left(\frac{1}{\delta}\right)^{1-\sigma}$*, the innovation sector always locates in country 2, even if* $s^* = 1$*.*

Proof. *(1) (18) leads to* $s^* - \frac{1}{2} > 0$*. (14) leads to* $\hat{s} - \frac{1}{2} < 0$*. Thus, we see that* $\hat{s} < s^*$*. This means that the innovation sector locates in country 1. (2) From* $\widehat{s} = \frac{\frac{a_1}{a_2} - \delta}{(1-\delta)\left(\frac{a_1}{a_2}+1\right)}$*,* $\hat{s} - \frac{1}{2} \geq 0$*. Moreover,* $\frac{\partial \widehat{s}}{\partial\left(\frac{a_1}{a_2}\right)} = \frac{1+\delta}{\left(\frac{a_1}{a_2}+1\right)\left(\frac{a_1}{a_2}-\delta\right)} > 0$*. In addition, if* $A \leq \left(\frac{1}{\delta}\right)^{1-\sigma}$*, then* $\widehat{s} = \frac{\frac{a_1}{a_2} - \delta}{(1-\delta)\left(\frac{a_1}{a_2}+1\right)} > 1$*.* ■

For $\widehat{s}$ to be smaller than 1 for $A = \phi$, we assume that $\phi \geq \left(\frac{1}{\delta}\right)^{1-\sigma}$; that is, the parameter of the extent of knowledge spillover δ is small. The location of the innovator at equilibrium is derived by combining (14) with (18). In Figure 1, we identify the class of switch points of the innovator's location choice as the $\widehat{s}$ curve. The $\widehat{s}$ curve is the points $\left(\lambda, \frac{a_1}{a_2}, \phi\right)$, which satisfies the next equation:

$$\frac{\frac{a_1}{a_2} - \delta}{(1-\delta)\left(\frac{a_1}{a_2}+1\right)} = \frac{A(1+\phi)(1-\phi)\lambda - \phi(1-\phi A)}{(1-\phi A)(A-\phi)}. \tag{19}$$

This relation is satisfied at points $\left(\lambda = \frac{1}{2},\ A = 1\right)$, and $(\lambda = 1,\ A = \phi)$ for any ϕ. From Lemma 2 $\widehat{s}$ curve locates above $s^* = \frac{1}{2}$ line. In the case of $\left(\frac{1}{\delta}\right)^{1-\sigma} < \phi$, for any λ, the $\widehat{s}$ line lies between the $s^* = 1$ line and the $s^* = \frac{1}{2}$ line. By Lemma 1, the iso-s^* curves are downward-sloping in Figure 1. In addition, $\widehat{s}$ is increasing as A is decreasing. Therefore, we can see that the $\widehat{s}$ curve is downward-sloping. In Figure 1, the innovation sector locates in country 1 at the right and upper side from the $\widehat{s}$ curve (when country 1 has a larger market and lower wages). Conversely, the innovation sector locates in country 2 at the left and lower side of the $\widehat{s}$ curve (when country 2 has a larger market and lower wages).

In the following analysis, we restrict the values of parameters $a_1 \geq a_2$, and $\left(\frac{1}{\delta}\right)^{1-\sigma} < \phi$. This means that the wage in country 1 is higher than that in country 2 and the threshold $\hat{s}$ lies between $\frac{1}{2}$ and 1.

3.3 Growth rates at steady states

As in the study by Grossman and Helpman (1991) [5], this model has a unique steady state in which the mass of variety is expanding at a constant rate over time. Since the innovation sector is assumed to be under free entry, at equilibrium, the innovation cost must be equalized to the value of the patent. Then, $v_t = \frac{\eta a_1}{N_t(s^* + \delta(1-s^*))}$ when the innovation sector locates in country 1. s^* is constant over time from (18), and v must decrease at the same rate as N_t increases; thus, $\frac{\dot{v}_t}{v_t} = -\frac{\dot{N}_t}{N_t}$. At equilibrium, the total sales of a manufacturing firm are

$a_1 q_1 = a_1 q_2 = \frac{\mu(\sigma-1)(L_1+L_2)}{c\sigma N_t}$. Introducing these equalities into (12), we derive the growth rate at steady states (we define g as the rate of variety expanding at steady states, that is, $g \equiv \frac{\dot{N}_t}{N_t}$).

$$g\left(s, \gamma\left(\frac{a_1}{a_2}\right), a_1 + a_2\right) = \begin{cases} \frac{\mu(L_1+L_2)}{\sigma} \cdot \frac{(1-s)+\delta s}{\eta(1-\gamma)(a_1+a_2)} - \rho & \text{if} \quad \text{the innovator locates in country 2.} \\ \frac{\mu(L_1+L_2)}{\sigma} \cdot \frac{s+\delta(1-s)}{\eta\gamma(a_1+a_2)} - \rho & \text{if} \quad \text{the innovator locates in country 1.} \end{cases} \tag{20}$$

where $\gamma \equiv \frac{\frac{a_1}{a_2}}{\frac{a_1}{a_2}+1} \left(= \frac{a_1}{a_1+a_2}\right)$.

In the next section, we analyze the effect of parameters on the share of manufacturing firms and steady state growth rates.

4 Effects of parameters on the location of firms and the growth rates

4.1 Effects of the change of $\frac{a_1}{a_2}$ and $a_1 + a_2$ on s and g

In this subsection, we analyze the effect of change $\frac{a_1}{a_2}$ and $a_1 + a_2$ on the geographical configuration of firms and growth rates.

First, differentiating (20) with respect to s

$$\frac{\partial g}{\partial s} = \begin{cases} \frac{\mu(L_1+L_2)}{\sigma} \cdot \frac{-(1-\delta)}{\eta(1-\gamma)(a_1+a_2)} < 0 & \text{if} \quad \text{the innovator locates in country 2.} \\ \frac{\mu(L_1+L_2)}{\sigma} \cdot \frac{(1-\delta)}{\eta\gamma(a_1+a_2)} > 0 & \text{if} \quad \text{the innovator locates in country 1.} \end{cases} \tag{21}$$

The sign of $\frac{\partial g}{\partial s}$ depends on the location of the innovation sector. Next, the lemma represents this property.

Lemma 3 *The agglomeration of manufacturing firms in the country where the innovation sector locates enhances the economic growth rate.*

Let us assume that the innovation sector locates in country 2. When the agglomeration of a manufacturing firm in country 1 progresses, the unit requirement for producing a patent in country 2 increases, since international knowledge spillover is imperfect. The growth rates then become lower. Conversely, when the innovation sector locates in country 1, agglomeration in country 1 makes the innovation sector more efficient. Then, the growth rates become higher.

Second, differentiating (20) with respect to $\frac{a_1}{a_2}$, we obtain:[3]

$$\begin{aligned}
\frac{\partial g}{\partial\left(\frac{a_1}{a_2}\right)} &= \frac{\partial g}{\partial \gamma}\frac{\partial \gamma}{\partial\left(\frac{a_1}{a_2}\right)} \\
&= \begin{cases} \frac{\mu(L_1+L_2)}{\sigma}\cdot\frac{(1-s)+\delta s}{\eta(1-\gamma)^2(a_1+a_2)}\cdot\frac{\partial\gamma}{\partial\left(\frac{a_1}{a_2}\right)} > 0 & \text{if} \quad \text{the innovator locates in country 2.} \\ \frac{\mu(L_1+L_2)}{\sigma}\cdot\frac{-(s+\delta(1-s))}{\eta\gamma^2(a_1+a_2)}\cdot\frac{\partial\gamma}{\partial\left(\frac{a_1}{a_2}\right)} < 0 & \text{if} \quad \text{the innovator locates in country 1.} \end{cases}
\end{aligned}$$

Raising $\frac{a_1}{a_2}$ means that the relative wages in country 1 become higher. Thus, the innovation cost in country 1 becomes larger relative to that in country 2. Therefore, $\frac{\partial g}{\partial\left(\frac{a_1}{a_2}\right)}$ is negative when the innovation sector locates in country 1. Conversely, $\frac{\partial g}{\partial\left(\frac{a_1}{a_2}\right)}$ is positive when the innovation sector locates in country 2.

Third, differentiating (20) with respect to $a_1 + a_2$,

$$\frac{\partial g}{\partial(a_1+a_2)} = \begin{cases} \frac{\mu(L_1+L_2)}{\sigma}\cdot\frac{-[(1-s)+\delta s]}{\eta(1-\gamma)(a_1+a_2)^2} < 0 & \text{if} \quad \text{the innovator locates in country 2.} \\ \frac{\mu(L_1+L_2)}{\sigma}\cdot\frac{-(s+\delta(1-s))}{\eta\gamma(a_1+a_2)^2} < 0 & \text{if} \quad \text{the innovator locates in country 1.} \end{cases}$$

4.2 Effects of change of λ, $\frac{a_1}{a_2}$, and $a_1 + a_2$ on g^*

We substitute $s = s^*\left(\lambda, A\left(\frac{a_1}{a_2}\right), \phi\right)$ characterized by (18) into (20),

$$g^*\left(\lambda, \frac{a_1}{a_2}, a_1+a_2, \phi\right) \equiv g\left[s^*\left(\lambda, A\left(\frac{a_1}{a_2}\right), \phi\right), \frac{a_1}{a_2}, a_1+a_2\right]. \tag{22}$$

We differentiate (22) with respect to parameters λ, $\frac{a_1}{a_2}$, and $a_1 + a_2$.

From $\frac{\partial g^*}{\partial\lambda} = \frac{\partial g}{\partial s}\frac{\partial s^*}{\partial\lambda}$ and $\frac{\partial s^*}{\partial\lambda} > 0$, we obtain $\frac{\partial g^*}{\partial\lambda} < (>)0$ when the innovation sector locates in country 2(1). This means that, if the market size of the country in which the innovation sector locates becomes larger, manufacturing firms become more agglomerated in that country. Lemma 3 means that the innovation activity becomes more efficient, and growth rates become higher, depending on the market size of the country where the innovation sector locates.

Differentiating (22) with respect to $\frac{a_1}{a_2}$, we derive the following equations:

$$\frac{\partial g^*}{\partial(a_1/a_2)} = \begin{cases} \underbrace{\frac{\partial g}{\partial s}}_{-}\underbrace{\frac{\partial s^*}{\partial A}}_{+}\underbrace{\frac{\partial A}{\partial(a_1/a_2)}}_{-} + \underbrace{\frac{\partial g}{\partial(a_1/a_2)}}_{+} > 0 & \text{if} \quad \text{the innovator locates in country 2.} \\ \underbrace{\frac{\partial g}{\partial s}}_{+}\underbrace{\frac{\partial s^*}{\partial A}}_{+}\underbrace{\frac{\partial A}{\partial(a_1/a_2)}}_{-} + \underbrace{\frac{\partial g}{\partial(a_1/a_2)}}_{-} < 0 & \text{if} \quad \text{the innovator locates in country 1.} \end{cases}.$$

[3] Notice that $\frac{\partial\gamma}{\partial\left(\frac{a_1}{a_2}\right)} > 0$.

The change of relative wages has two effects on the growth rate. One is the direct effect, in which innovation costs change. The other is the indirect effect, in which the share of manufacturing firms changes. Because these two effects have the same direction, we can see the sign of $\frac{\partial g^*}{\partial(a_1/a_2)}$. Let us assume that the innovator locates in country 1 and the relative wages in country 1 increase. The innovation costs become higher by the direct effect, and innovation technology becomes inefficient through the indirect effect of the share of firms in country 1. Therefore, the growth rates become lower when the innovation sector locates in country 1 and the relative wages in country 1 increase.

Here, $\frac{\partial g^*}{\partial(a_1+a_2)} = \frac{\partial g}{\partial(a_1+a_2)}$. In the following table, we summarize the effects of each variable on the growth rate g^*.

	country 2	country 1
λ	$-$	$+$
A	$-$	$+$
$a_1 + a_2$	$-$	$-$

In this table, country 2 and country 1 represent the country in which the innovation sector locates. It is noteworthy that A is a decreasing function of $\frac{a_1}{a_2}$, and then the sign is reversed. At corner solutions, $\frac{\partial g^*}{\partial \lambda} = 0$, from $\frac{\partial s^*}{\partial \lambda} = 0$ and $\frac{\partial g^*}{\partial(a_1/a_2)} = \frac{\partial g}{\partial(a_1/a_2)}$, from $\frac{\partial s^*}{\partial A} = 0$. In this case, the effects on the growth rate g^* are as follows:

	country 2	country 1
λ	0	0
A	$-$	$+$
$a_1 + a_2$	$-$	$-$

We depict Figure 1, in which the effects of λ and $A\left(\frac{a_1}{a_2}\right)$ on g^* are shown. The arrow in Figure 1 indicates the direction in which the growth rates are higher.

4.3 Effects of the change of ϕ on s^* and g^*

To study the effects of trade costs on the location of a manufacturing firm, with λ and A fixed, we check the sign of $\frac{\partial s^*}{\partial \phi}$. It is difficult to derive the differential of (18) directly. However, we obtain the following lemma by calculations shown in the Appendix.

Proposition 1 *(1) Let us consider that the parameters satisfy $\lambda \geq \frac{1}{1+A^2}$. When trade costs are high, s^* becomes larger by the decline in trade costs. When trade costs are sufficiently low, s^* becomes smaller than the decline in trade costs. In other words, $\frac{\partial s^*}{\partial \phi} \geq 0$ for ϕ is small and $\frac{\partial s^*}{\partial \phi} \leq 0$ for ϕ is large. Finally, when ϕ is very large, all manufacturing firms agglomerate in country 2. (2) Moreover, in the case in which parameters satisfy the relation $\lambda \geq \frac{1+2\left(1-A^2\right)^{\frac{1}{2}}}{2} \left(\geq \frac{1}{1+A^2}\right)$, when trade costs take an intermediate value, full agglomeration in country 1 is realized.*

Proof. *See Appendix.* ■

Proposition 1 says that, when trade costs are high, declining the trade costs facilitates agglomeration to the large market (and higher wage) country. On the other hand, when trade costs are small, declining the trade costs fosters agglomeration in the small market (and lower wage) country. Reduction of trade costs has two effects. One is that locating in the larger market country becomes more profitable because firms can transport manufactured goods with lower trade costs. Firms, then, need not locate in the small market country. This effect is reported in many studies of new economic geography, such as those by Martin and Ottaviano (1999) [9] and Martin and Ottaviano (2001) [10]. Another effect is that, when trade costs become lower, the market size become less important, and the wage difference becomes more important for a firm's location choice [4]. Indeed, in the $\phi = 1$ case, i.e., perfect free trade, all manufacturing firms agglomerate in country 2, where wages are lower, i.e., $s^* = 0$ for every $\lambda \in \left(\frac{1}{2}, 1\right)$ and $A \in (0, 1)$. When trade costs are high, the former effect dominates over the latter effect, and then s^* increases. With a decline in trade costs, the difference in the two effects becomes small; then, the latter effect dominates over the former effect. As a result, s^* decreases. Finally, at very low trade costs, all manufacturing firms agglomerate in country 2 (Figure 2).

Next, we study the effects of trade costs on the location of the innovation sector. From (14), with $\frac{a_1}{a_2} (\geq 1)$ fixed, the innovator determines its location according to s^* only. Then, we can draw $\hat{s}$ line horizontally between $\frac{1}{2}$ and 1 in Figure 2. By taking proper δ, for any λ and A such that $\lambda \geq \frac{1}{1+A^2}$, the relation of $\lambda \leq \hat{s}$ is satisfied. This relation means that, when trade costs are very high because the share of firms in country 1 is smaller than the threshold, the innovation sector locates in country 2. With a decline in trade costs, because s^* rises and reaches $s^* = 1$ under the parameter condition of $\lambda \geq \frac{1+2\left(1-A^2\right)^{\frac{1}{2}}}{2}$, the innovation sector changes its location from country 2 to country 1 at some ϕ (denote $\hat{\phi}_1$). Moreover, with a decline in trade costs, because full agglomeration in country 1 is broken and manufacturing firms change their location from country 1 to country 2, the innovation sector moves again from country 1 to country 2 at some ϕ (denote $\hat{\phi}_2$).

We study the economic growth rate at a steady state; Lemma 3 means that agglomeration in the country where the innovation sector locates fosters the growth rate. Thus, when innovation sector locates in country 1, the growth rate moves in the same direction as s^* moves. When the innovation sector locates in country 2, the growth rate moves in the opposite direction against s^*. In

[4] In Gao (2007) [4], because globalization makes the manufacturing/agriculture wage ratio in country 2 higher and manufacturing labor supply becomes larger, the manufacturing firms in country 2 expand, with a decline in trade costs.

addition, at the full agglomeration case, the growth rate is as follows:

$$
\begin{aligned}
g\left(1, \gamma\left(\frac{a_1}{a_2}\right), a_1+a_2\right) &= \frac{\mu\left(L_1+L_2\right)}{\sigma} \cdot \frac{1}{\eta \gamma\left(a_1+a_2\right)}-\rho, \\
g\left(0, \gamma\left(\frac{a_1}{a_2}\right), a_1+a_2\right) &= \frac{\mu\left(L_1+L_2\right)}{\sigma} \cdot \frac{1}{\eta(1-\gamma)\left(a_1+a_2\right)}-\rho .
\end{aligned}
$$

From $a_1 \geq a_2$, we can see that $\frac{1}{2} \leq \gamma < 1$, and then $1-\gamma \leq \gamma$. This implies $g(1, \bullet) \leq g(0, \bullet)$. Thus, we can draw the relation between trade costs and the growth rate as shown in Figure 3. After summarizing this discussion, we obtain the next proposition:

Proposition 2 *Consider that the parameter conditions of* $\lambda \geq \frac{1+2\left(1-A^2\right)^{\frac{1}{2}}}{2}$ *and* $\lambda \leq \hat{s}$ *are satisfied. (1) As trade costs decrease, the growth rates at the steady state follow a* W*-shaped curve; that is, they have two low growth-rate valleys. (2) When trade costs are very low, the growth rates reach their maximum value.*

5 Conclusion

In this paper, we have constructed a model in which equilibrium wages in two countries are different from each other. Differences in wage rates and market sizes generate particular patterns of growth and agglomeration in the economy. If the equilibrium wages in two countries are the same, the country which has a large market always absorbs more firms than the other country. Therefore, when the equilibrium wages in two countries are the same, the innovation sector always locates in the large market country, and growth rates are raised with agglomeration of manufacturing firms in that country. In this case, the share of manufacturing firms in the large market country increases with a decline in trade costs.

However, if wages in the large market country are higher than those in the other country, higher wages lower the share of manufacturing firms in the large market country. We show the relationship between the proportion of manufacturing firms and the reduction in trade costs. When trade costs are high, the share of manufacturing firms in the large market country increases with a decline in trade costs, and we observe full agglomeration in the large market country. However, when trade costs become low, full agglomeration in the large market country is broken, and some manufacturing firms locate in the small market country. We show that, finally, all manufacturing firms agglomerate in the small market country when trade costs are very low. In this process, growth rates first decrease with a decline in trade costs. After these movements, the growth rates take W-shaped movements. As we saw above, we have studied the effects of reduction in trade costs on manufacturing firms' location and economic growth rates. By introducing the difference of wages in two countries, the results become richer, and the equilibrium in which manufacturing firms agglomerate in a small market country and the innovation sector locates in that country can be observed.

Appendix

The proof that the firm share $s^* = s^*(\lambda, A, \phi)$ is an increasing function of A

We show that $s^*(\lambda, A)$ is an increasing function of A. Differentiating (18) with respect to A, we obtain

$$\begin{aligned}\frac{\partial s^*}{\partial A} &= s^*\left[\frac{(1+\phi)(1-\phi)\lambda+\phi^2}{A(1+\phi)(1-\phi)\lambda-\phi(1-\phi A)}+\frac{-\left(1+\phi^2-2\phi A\right)}{(1+\phi A)(A-\phi)}\right] \qquad (23)\\ &= \left[\frac{(1+\phi)(1-\phi)\lambda+\phi^2-\left(1+\phi^2-2\phi A\right)s^*}{(1+\phi A)(A-\phi)}\right].\end{aligned}$$

From an interior solution condition, the denominator of (23) satisfies $(1+\phi A)(A-\phi) > 0$. Therefore, we check the sign of the numerator of this equation. (18) can be transformed as follows:

$$(1+\phi)(1-\phi)\lambda+\phi^2 = \left(1-\frac{\phi}{A}-\phi(A-\phi)\right)s^*+\frac{\phi}{A}$$

Then, the numerator of (23) is

$$\begin{aligned}&(1+\phi)(1-\phi)\lambda+\phi^2-\left(1+\phi^2-2\phi A\right)s^*\\ =\ &\left(1-\frac{\phi}{A}-\phi(A-\phi)\right)s^*+\frac{\phi}{A}-\left(1+\phi^2-2\phi A\right)s^*\\ =\ &\left(\phi A-\frac{\phi}{A}\right)s^*+\frac{\phi}{A}\\ =\ &\phi\left[\frac{1}{A}(1-s^*)+s^*A\right] > 0.\end{aligned}$$

In other words, $\frac{\partial s^*}{\partial A} > 0$ has been showed.

The proof that the firm share $s^* = s^*(\lambda, A, \phi)$ is an increasing function of ϕ for small ϕ and a decreasing function of ϕ for large ϕ.

We analyze the sign of $\frac{\partial s^*}{\partial \phi}$. Totally differentiating (18), we can derive

$$ds^* = \frac{\partial s^*}{\partial \lambda}d\lambda+\frac{\partial s^*}{\partial A}dA+\frac{\partial s^*}{\partial \phi}d\phi.$$

Setting $ds^* = 0, dA = 0,$

$$0 = \frac{\partial s^*}{\partial \lambda} d\lambda + \frac{\partial s^*}{\partial \phi} d\phi$$

$$\frac{\partial s^*}{\partial \phi} = -\frac{\partial s^*}{\partial \lambda}\frac{d\lambda}{d\phi}$$

$\frac{d\lambda}{d\phi}$ is the slope of the iso-s^* curve in the $\phi-\lambda$ plane. Since $\frac{\partial s^*}{\partial \lambda} \geq 0$ from Lemma 1, if $\frac{d\lambda}{d\phi} \geq 0$, then $\frac{\partial s^*}{\partial \phi} \leq 0$; on the other hand, if $\frac{d\lambda}{d\phi} \leq 0$, then $\frac{\partial s^*}{\partial \phi} \geq 0$. We consider the case of $0 \leq A \leq 1$. We will draw the iso-s^* curves on the $\phi - \lambda$ plane (Figure 4).

(1) $s^* = 0$ curve

$$\begin{aligned} \frac{A(1+\phi)(1-\phi)\lambda - \phi(1-\phi A)}{(1-\phi A)(A-\phi)} &= 0 \\ A(1+\phi)(1-\phi)\lambda &= \phi(1-\phi A) \end{aligned}$$

$$\begin{aligned} \frac{1}{\lambda}\frac{d\lambda}{d\phi} &= \frac{-A}{1-\phi A} + \frac{1}{\phi} - \frac{1}{1+\phi} - \frac{-1}{1-\phi} \\ &= \frac{(1-\phi)^2 + 2\phi(1-A)}{\left(1-\phi^2\right)\phi(1-\phi A)} \geq 0 \end{aligned}$$

(2) $s^* = 1$curve

$$\begin{aligned} \frac{A(1+\phi)(1-\phi)\lambda - \phi(1-\phi A)}{(1-\phi A)(A-\phi)} &= 1 \\ (1+\phi)(1-\phi)\lambda &= (1-\phi A) \end{aligned}$$

$$\begin{aligned} \frac{1}{\lambda}\frac{d\lambda}{d\phi} &= \frac{-A}{1-\phi A} - \frac{1}{1+\phi} - \frac{-1}{1-\phi} \\ &= \frac{2\phi - A - A\phi^2}{(1+\phi)(1-\phi)(1-\phi A)} \end{aligned}$$

When $\phi = 0$, $\frac{1}{\lambda}\frac{d\lambda}{d\phi} = -A < 0$. When $\phi = A$, $\frac{1}{\lambda}\frac{d\lambda}{d\phi} = \frac{A-A^3}{(1-A^2)^2} > 0$. The numerator of $\frac{1}{\lambda}\frac{d\lambda}{d\phi}$ is negative for small ϕ and positive for large ϕ, while the denominator of $\frac{1}{\lambda}\frac{d\lambda}{d\phi}$ is always positive. Then, the $s^* = 1$ line is U-shaped.

(3) general $s^* \in (0,1)$ curve

$$\frac{1}{\lambda}\frac{d\lambda}{d\phi} = \frac{1-s^*}{As^* + \phi(1-s^*)} - \frac{1}{1+\phi} + \frac{1}{1-\phi} - \frac{A}{1-\phi A} \tag{24}$$

$$\lim_{\phi\to 0}\frac{1}{\lambda}\frac{d\lambda}{d\phi}=\frac{1-s^*\left(1+A^2\right)}{As^*}$$

Since $s^* = \lambda$ at $\phi = 0$, if $\lambda \geq \frac{1}{1+A^2}$, then $\frac{d\lambda}{d\phi}|_{\phi=0} \leq 0$. If $\lambda < \frac{1}{1+A^2}$, then $\frac{d\lambda}{d\phi}|_{\phi=0} > 0$. This means that, if $\lambda \geq \frac{1}{1+A^2}$, then $\frac{\partial s^*}{\partial \phi}|_{\phi=0} \geq 0$.

$$\lim_{\phi\to A}\frac{1}{\lambda}\frac{d\lambda}{d\phi}=\frac{1-s^*}{A}+\frac{A}{(1+A)(1-A)}>0$$

(24) can be transformed as follows;

$$\frac{1}{\lambda}\frac{d\lambda}{d\phi}=\frac{1-s^*(1+A)}{[As^*+\phi(1-s^*)](1+\phi)}+\frac{1-A}{(1-\phi)(1-\phi A)} \tag{25}$$

The first term of the right-hand side of (25) is positive if $s^* \leq \frac{1}{1+A}$ but negative if $s^* \geq \frac{1}{1+A}$. The second term of the right-hand side of (25) is positive. First, if $s^* \leq \frac{1}{1+A}$, then $\frac{1}{\lambda}\frac{d\lambda}{d\phi}$ is sufficiently positive for all ϕ. Thus, the iso-s^* curve is upward-sloping. Second, if $s^* > \frac{1}{1+A}$, then $\frac{1}{\lambda}\frac{d\lambda}{d\phi}$ is an increasing function of ϕ.

(Since the denominator of the first term of (25) is an increasing function, then the first term of (25) is an increasing function of ϕ (Note that $1-s^*(1+A)<0$). The second term of (25) is an increasing function of ϕ.) Thus, the iso-s^* curve is convex.

From this discussion, we can draw the iso-s^* curves on $\phi-\lambda$ plane (Figure 4). If $s^* \geq \frac{1}{1+A^2}$, then the iso-s^* curve is U-shaped. If $\frac{1}{1+A} \leq s^* < \frac{1}{1+A^2}$, then the iso-$s^*$ curve is convex and an increasing function of ϕ. If $s^* < \frac{1}{1+A}$, then the iso-s^* curve is an increasing function of ϕ.

Therefore, if $\lambda \geq \frac{1}{1+A^2}$, then $\frac{\partial s^*}{\partial\phi} \geq 0$ for small ϕ and $\frac{\partial s^*}{\partial\phi} \leq 0$ for large ϕ.

Parameter conditions for full agglomeration in country 1

When all firms agglomerate in country 1 at equilibrium, equation $s^*(\lambda, A, \phi) = 1$ for ϕ has two real number solutions. (18) can be transformed as

$$\begin{aligned}(1+\phi)(1-\phi)\lambda &= (1-\phi A),\\ \lambda\phi^2-\phi A+1-\lambda &= 0.\end{aligned}$$

For this equation to have two real number solutions, the following must be satisfied:

$$D \equiv A^2 - 4\lambda(1-\lambda) \geq 0.$$

That is,

$$\lambda \geq \frac{1+2\left(1-A^2\right)^{\frac{1}{2}}}{2}.$$

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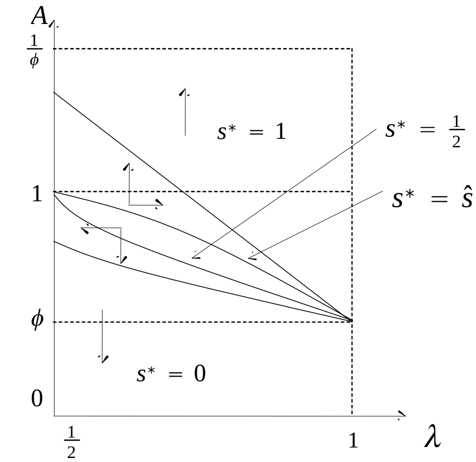
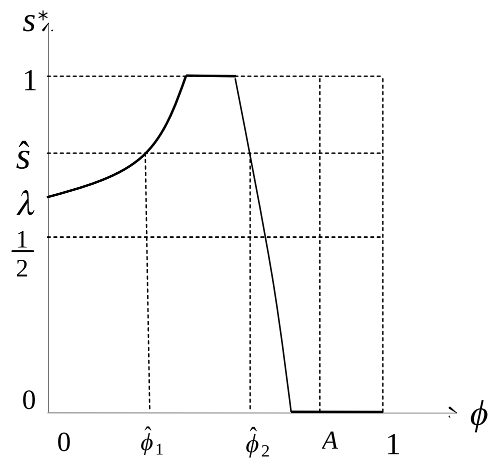
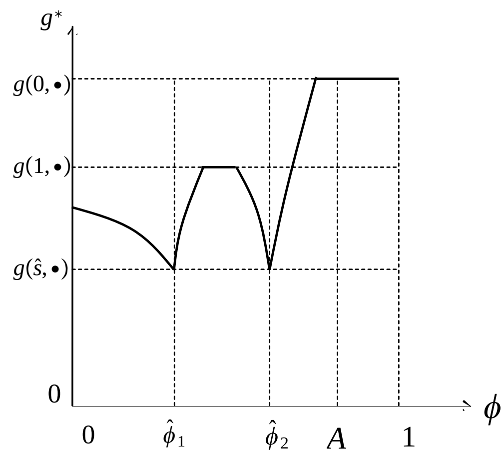


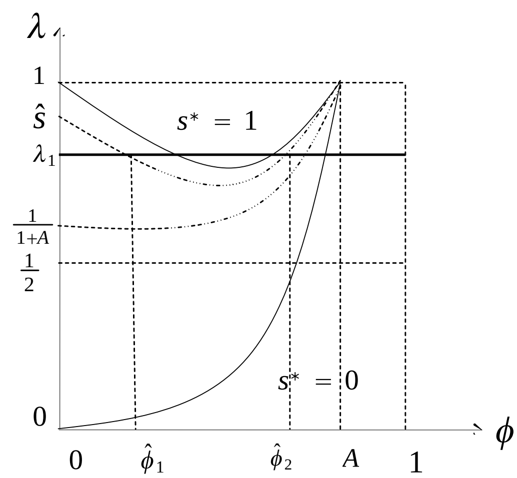
Equilibrium share of manufacturing firms



The relation of trade costs and share of manufacturing firms



The relation of trade costs and steady state growth rate



The relation of trade costs and market size and share of firms