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Abstract

This paper develops an overlapping-generation model featuring four types of households: single female, single male, one-breadwinner couple and two-breadwinner couple. The paper considers majority voting over public pension in the presence of derived pension rights for one-breadwinner couples. In an economy with a low intertemporal elasticity of substitution, borrowing-constrained one-breadwinner couples may prefer a lower tax rate than do other types of households, although the former attain a higher benefit-to-cost ratio of public pension than do others. Changes in the gender wage gap, the level of derived pension rights, and the fraction of two-breadwinner couples produce an inverse U-shaped relationship between the relevant variable and the tax rate.

Keywords: Borrowing constraint; Marital status; Gender wage gap; Derived pension rights; Political economy

JEL Classification: D72, H55, J12

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1 Introduction

Most OECD (Organization for Economic Co-operation and Development) countries offer pension benefits for non-working spouses and divorcees. The benefits, called derived pension rights, include (i) survivors' benefits for widows; (ii) benefits for divorced spouses; and (iii) spousal benefits as a supplement to a worker's benefit (Choi, 2006; Leroux and Pestieau, 2011). These benefits imply that derived pension rights have an intra-generational redistribution component from working singles and two-breadwinner couples to one-breadwinner couples. Thus, recent pension reforms in many OECD countries that attempt to link contributions and benefits more closely (OECD, 2011) may provoke an intra-generational conflict over pension policy.

Despite the conflict among singles and couples caused by derived pension rights, there are few studies focusing on these rights in the political-economic literature. Exceptions are the works of Leroux and Pestieau (2012) and Leroux, Pestieau, and Racionero (2011). Leroux and Pestieau (2012) consider an economy composed of couples who maximize the joint lifetime utility of a husband and a wife. A husband always works regardless of his labor productivity, while a wife chooses whether or not to work depending on her reservation wage. Under this framework, Leroux and Pestieau (2012) demonstrate an interaction between a wife's labor supply decision and pension policy preferences, and they show that a pension system with derived pension rights is likely to emerge as a voting equilibrium outcome.

Leroux, Pestieau, and Racionero (2011) assume that the degree of derived pension rights is fixed. Instead, they allow for the presence of single males and females and examine how the degree of derived pension rights affects tax burden policy preferences for public pensions and thus a resulting pension system via voting. Their results are as follows: (1) a reduction of derived pension rights results in a smaller tax burden for public pension, and (2) an increase in the share of two-breadwinner couples have two opposing effects on the pension burden, where the net effect may be positive or negative depending on other economic factors.

The results in Leroux, Pestieau, and Racionero (2011) provide significant policy predictions for public pensions. However, their results heavily depend on the following two assumptions: quasi-linear utility and no borrowing constraints. The first assumption, which is often adopted in the political-economic analyses of social security (see, for example, Conde-Ruiz and Galasso, 2003, 2004, 2005; Borck, 2007), makes the analysis tractable, but draws attention away from the considerable effect of the intertemporal elasticity of substitution on a household's decisions concerning saving and voting (Casamatta et al., 2000; Conde-Ruiz and Profeta, 2007; Cremer et al., 2007; Arawatari and Ono, 2011). The second assumption allows for borrowing against future pension benefits, which is difficult to support from the empirical viewpoint (Diamond and Hausman, 1984; Mulligan and Sala-i-Martin, 1999). The aim of this paper is to relax these two assumptions in the framework of Leroux, Pestieau, and Racionero (2011) and to provide new insight into derived pension rights from a political-economic viewpoint.

For analytical purposes, we will extend the framework of Leroux, Pestieau, and Racionero (2011) in the following two ways. First, the preferences of each household are represented by a utility function with a constant intertemporal elasticity of substitution. Second, each household is unable to borrow against its future pension benefits. Under this extended framework, we show the following two results. First, in an economy where an intertemporal elasticity of substitution is below one, one-breadwinner couples who benefit from public pensions may prefer a lower, rather than higher, tax rate than single females who owe net burden, because of the presence of borrowing constraints. Borrowing-constrained one-breadwinner couples want to choose a low tax rate to keep their after-tax income level as high as possible. There is then a voting equilibrium, much like an ends-against-the-middle equilibrium, in which single females, along with the old, form a coalition against the others.

Secondly, when the intertemporal elasticity of substitution is below one, an inverse U-shaped relationship is created between the relevant variable and the tax rate due to the gender wage gap, the level of derived pension rights, and the ratio of two-breadwinner couples. Near the maximum of the inverse U-shaped curve, the decisive voter is borrowing-unconstrained on one side and borrowing-constrained on the other side. This two-toned effect, as well as the ends-against-the-middle equilibrium, both of which were not shown in Leroux, Pestieau, and Racionero (2011), are derived by the presence of a borrowing constraint associated with a low intertemporal elasticity of substitution.

The organization of this paper is as follows. Section 2 describes the economic environment. Section 3 demonstrates the utility maximization of singles, one-breadwinner couples, and two-breadwinner couples. Section 4 presents the political institution and pension policy preferences of the young and the old. Section 5 characterizes the political equilibrium. Section 6 performs a comparative statics analysis and shows how gender wage gap, derived pension rights, and the share of two-breadwinner couples affect the equilibrium pension policy. Section 7 provides concluding remarks. Proofs are provided in the Appendix.

2 The Economic Environment[1]

Consider a discrete time economy in which time is denoted by $t = 0, 1, 2, \cdots$. The economy is comprised of overlapping generations of individuals, each of whom lives for two defined periods: youth and old age. Each generation is composed of a continuum of agents. Specifically, in each generation, there are males and females; the size of each gender population is normalized to unity. Thus, the total population size of each generation is two.

Each generation consists of four different categories of households: single males, single females, one-breadwinner couples, and two-breadwinner couples. The total population of each generation is divided as follows: $(1-\varphi)$ single females, φ females in couples where μ of them are workers and $1-\mu$ of them are non-workers, $(1-\varphi)$ single males and φ males in couples. The allocation of households is fixed over time. For simplicity, marriage decisions are not factored into the analysis.

Each agent is endowed with one unit of labor in youth and retires in old age. Males supply labor regardless of their marital status, while only females who are single or belong to two-breadwinner couples supply labor. Females who belong to one-breadwinner couples do not supply labor; instead, they devote their time to home production and leisure, both of which are assumed not to have an effect on utility or household income.

In this economy, there are two types of heterogeneity between males and females: wage and longevity. These types are characterized by the parameter pairs (w^m, π^m) for males and (w^f, π^f) for females, such that

$$\begin{cases} (w^m, \pi^m) = (w, \pi), \ \pi \in (0,1); \\ (w^f, \pi^f) = (\alpha w, 1), \ \alpha \in (0,1), \end{cases}$$

where w^i $(i = f, m)$ represents the wage, and π^i represents the probability of surviving in old age. The term $\alpha \in (0,1)$ represents the gender wage gap; an increase in α implies a reduction of the gender wage gap. The term $\pi \in (0,1)$ represents the longevity difference between males and females. It is assumed that females have a longer life span than males but obtain a lower wage.

Individuals contribute to the pension system during youth and receive a pension benefits in old age. Following the convention in the literature, we present the loss of efficiency in taxation by assuming convex costs for collecting taxes (see, for example, Casamatta, Cremer, and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Borck, 2007; Cremer et al., 2007). The actual tax revenue from the young is therefore given by

$$\tau(1-\tau) \cdot [w + \alpha w(1-\varphi) + \alpha w \varphi \mu],$$

[1] Since the framework is based on that in Leroux, Pestieau, and Racionero (2011), a part of description in this section follows Leroux, Pestieau, and Racionero (2011).

where the terms w, $\alpha w(1-\varphi)$, and $\alpha w \varphi \mu$ in the square brackets correspond to the contributions by males, single females, and females who belong to two-breadwinner couples, respectively. The term $(1-\tau)$ is the distortionary factor. The assumption of distortionary taxation is made solely to ensure an interior solution to preferred tax rates and otherwise plays no role.

Let p denote pension benefits for contributors; let γp denote pension benefits for non-contributors, where $\gamma \in [0,1]$ represents the level of derived pension rights. The total pension payments are

$$p \cdot [\pi + (1-\varphi) + \varphi\mu + \gamma\varphi(1-\mu)].$$

The pension benefit for males is $p\pi$, rather than p, because their length of life in old age is assumed to be $\pi \in [0,1)$.

Under the assumption of a balanced budget, the government budget constraint becomes

$$p = w\chi(\cdot)\tau(1-\tau), \tag{1}$$

where

$$\chi(\cdot) \equiv \frac{1+\alpha(1-\varphi+\varphi\mu)}{\pi+1-\varphi+\varphi\mu+\gamma\varphi(1-\mu)}.$$

The tax rate τ is determined via majority voting, whereas the degree of derived pension rights γ is assumed to be fixed at the constitutional level. Voting over γ will be discussed in Section 7.[2]

3 Economic Decisions

Let $j = f, m, c1$, and $c2$ denote single females, single males, one-breadwinner couples, and two-breadwinner couples, respectively. In this section, we illustrate the economic decisions on saving by singles and couples. An old agent does not make any economic decisions because his/her saving is predetermined during youth.

3.1 Singles

Each single agent is assumed to receive utility from private consumption. The lifetime utility function of a type-j ($j = f, m$) single young agent is specified by:

$$U^j = \frac{(c^j)^{1-\sigma}-1}{1-\sigma} + \beta \cdot \frac{(d^j)^{1-\sigma}-1}{1-\sigma},$$

[2]The paper makes several assumptions: the fraction of each type of household is fixed; the pension benefits are equal for all contributors; and the derived pension rights are fixed. We employ these assumptions to keep the comparability with the result in Leroux, Pestieau, and Racionero (2011), and to shed light on the roles of borrowing constraint and an intertemporal elasticity of substitution, both of which were abstracted away in Leroux, Pestieau, and Racionero (2011).

where c^j is consumption in youth, d^j is consumption in old age, $\beta \in (0,1)$ is a discount factor, and $\sigma(>0)$ is the inverse of the intertemporal elasticity of substitution. A lower $1/\sigma$ implies a lower intertemporal elasticity of substitution.[3]

Type-j's ($j = f, m$) individual budget constraints in youth and in old age are respectively given by,

$$c^j + s^j \leq (1-\tau)w^j,$$
$$d^j \leq s^j + \pi^j p,$$

where s^j is saving, τ is the income tax rate, and p is the per capita pension benefit. If $j = f$, then $w^f = \alpha w$ and $\pi^f = 1$; if $j = m$, then $w^m = w$ and $\pi^m = \pi$. For the tractability of analysis, we assume that the gross rate of interest is equal to one. In addition, we assume borrowing constraints, that is, $s^j \geq 0$. This constraint precludes the possibility of borrowing when young against future pension benefits (Diamond and Hausman, 1984; Mulligan and Sala-i-Martin, 1999).

A type-j young agent maximizes his/her utility subject to his/her budget constraint and the borrowing constraint. When $s^j > 0$, the first-order condition for an interior solution is $d^j = (\beta)^{1/\sigma} c^j$. This condition determines an interior solution of saving by a type-j agent. By taking the borrowing constraint into account, the saving function of a type-j agent becomes

$$s^j = \max\left\{0, \frac{(\beta)^{1/\sigma}}{1+(\beta)^{1/\sigma}}\left[(1-\tau)w^j - \frac{\pi^j p}{(\beta)^{1/\sigma}}\right]\right\},\ j = f, m. \tag{2}$$

The saving function (2) and the government budget constraint (1) imply that there is a critical rate of tax such that

$$s^f > 0 \Leftrightarrow \tau < \hat{\tau}^f \equiv \frac{\alpha(\beta)^{1/\sigma}}{\chi(\cdot)};$$
$$s^m > 0 \Leftrightarrow \tau < \hat{\tau}^m \equiv \frac{(\beta)^{1/\sigma}}{\pi\chi(\cdot)}.$$

The critical rate for single males, $\hat{\tau}^m$, is higher than that for single females, $\hat{\tau}^f$, because single males obtain higher wages and live shorter than do single females.

With the saving function and the private and government budget constraints, we can obtain the consumption functions of a type-j $(= f, m)$ agent in youth and in old age. We use the functions to obtain the indirect utility function of type-j singles, denoted by $V^j(j = f, m)$:

$$V^j = \begin{cases} V^j_{s>0} \equiv \frac{1}{1-\sigma}\left(\frac{1}{1+(\beta)^{1/\sigma}}\right)^{-\sigma} \cdot \left[(1-\tau)w^j + \pi^j w\chi(\cdot)\tau(1-\tau)\right]^{1-\sigma} - \frac{1+\beta}{1-\sigma} & \text{if } \tau < \hat{\tau}^j; \\ V^j_{s=0} \equiv \frac{1}{1-\sigma}\left((1-\tau)w^j\right)^{1-\sigma} + \frac{\beta}{1-\sigma}\left\{\pi^j w\chi(\cdot)\tau(1-\tau)\right\}^{1-\sigma} - \frac{1+\beta}{1-\sigma} & \text{if } \tau \geq \hat{\tau}^j. \end{cases}$$

[3] For $j = m$, the second-period utility might be more appropriately written as $\pi\beta\{(d^j)^{1-\sigma} - 1\}/(1-\sigma)$ since π is interpreted as the probability of surviving to the second period of life. Following Borck (2007), we assume away the effect of π on the second-period utility for the tractability of analysis.

The function $V_{s>0}^j$ ($j = f, m$) denotes the indirect utility of a type-j young household when it saves some portion of income, and $V_{s=0}^j$ denotes the indirect utility when it is faced with a borrowing constraint and saves nothing. The term in square brackets in the equation of $V_{s>0}^j$ represents the lifetime income; the first and the second terms on the right-hand side in the equation for $V_{s=0}^j$ represent the utilities of consumption in youth and in old age, respectively; the constant term, $(1+\beta)/(1-\sigma)$, summarizes the parameters unrelated to the political decision on taxes.

3.2 Couples

We next consider consumption decisions by couples. Following Leroux, Pestieau, and Racionero (2011), we adopt the unitary model of a household that has only one set of preferences:

$$U^j = 2 \cdot \left\{ \frac{(c^j)^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \frac{(d^j)^{1-\sigma} - 1}{1-\sigma} \right\}, j = c1, c2.$$

Under this specification, spouses play cooperatively and share their resources over their lifecycle.[4]

A couple chooses consumption and saving to maximize the household utility subject to the budget constraints in youth and old age:

$$\begin{aligned} 2c^j + s^j &\leq (1-\tau)w^j, \\ 2d^j &\leq s^j + (\pi + \gamma^j)p. \end{aligned}$$

The borrowing constraint is $s^j \geq 0$, where $w^{c1} = w$, $w^{c2} = (1+\alpha)w, \gamma^{c1} = \gamma$ and $\gamma^{c2} = 1$.

In the first period of life, a husband and/or wife work and earn the after-tax wage income, $(1-\tau)w^j$. A couple consumes a part of the after-tax wage and saves the rest for old-age consumption. In the second period of life, the couple obtains the return from savings, s^j, the pension benefit paid to the husband, πp, and that to the wife, $\gamma^j p$.

By taking the borrowing constraint into account, the saving function of the type-j couple becomes

$$s^j = \max\left\{0, \frac{(\beta)^{1/\sigma}}{1+(\beta)^{1/\sigma}} \left[(1-\tau)w^j - \frac{(\pi+\gamma^j)p}{(\beta)^{1/\sigma}}\right]\right\}, j = c1, c2 \tag{3}$$

The saving function (3) and the government budget constraint (1) imply that there is a

[4] An alternative model of a household is to assume that "members of a family cannot cooperate because they cannot communicate with one another, and so the best that they can do is to behave according to the definition of a Nash equilibrium" (Ermisch, 2003, p.22). This alternative modeling may produce a result different from that achieved in the current model. However, we adopt the unitary model to keep the comparability with the result in Leroux, Pestieau and Racionero (2011). The author would like to thank one of the referees for pointing out an alternative option.

critical tax rate for couples such that

$$s^{c1} > 0 \Leftrightarrow \tau < \hat{\tau}^{c1} \equiv \frac{(\beta)^{1/\sigma}}{(\pi+\gamma)\chi(\cdot)};$$
$$s^{c2} > 0 \Leftrightarrow \tau < \hat{\tau}^{c2} \equiv \frac{(1+\alpha)(\beta)^{1/\sigma}}{(\pi+1)\chi(\cdot)}.$$

With the saving function and the private and government budget constraints, we can obtain the consumption functions in youth and in old age. We use these functions to derive a type-j couple's indirect utility function:

$$V^j = \begin{cases} V^j_{s>0} & \text{if} \quad \tau < \hat{\tau}^j; \\ V^j_{s=0} & \text{if} \quad \tau \geq \hat{\tau}^j; \end{cases}$$

where:

$$V^j_{s>0} \equiv \frac{1}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma} \cdot \left(\frac{1}{1+(\beta)^{1/\sigma}}\right)^{-\sigma} \cdot \left[(1-\tau)w^j + (\pi+\gamma^j)w\chi(\cdot)\tau(1-\tau)\right]^{1-\sigma} - \frac{2(1+\beta)}{1-\sigma};$$
$$V^j_{s=0} \equiv \frac{1}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma} \cdot \left((1-\tau)w^j\right)^{1-\sigma} + \frac{\beta}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma} \cdot \left\{(\pi+\gamma^j)w\chi(\cdot)\tau(1-\tau)\right\}^{1-\sigma} - \frac{2(1+\beta)}{1-\sigma}$$

The function $V^j_{s>0}$ denotes the indirect utility of a type-j couple when it saves in youth, and $V^j_{s=0}$ denotes the indirect utility when it is faced with a borrowing constraint and saves nothing in youth. The interpretation of each term in these equations follows that offered for singles.

4 The Political Institution and Policy Preferences

The tax rate τ is determined by individuals through a political process of majority voting. Elections take place every period and all living individuals, both young and old, cast a ballot over τ. The tax preferences of young individuals are represented by the indirect utility functions presented in the previous section. The tax preferences of old agents are determined by the size of the pension because their saving when young is predetermined and has no critical effect on voting behavior. Every individual has zero mass, and thus no individual vote can change the outcome of the election. We thus assume individuals vote sincerely.

The majority voting game is intrinsically dynamic because it describes the interaction among successive generations. To address this feature, we assume commitment, or in other words, once-and-for-all-voting. Here, voters determine the constant sequence of the parameters: $\tau_t = \tau_{t+1} = \tau$ for all t, where τ_t denotes the tax rate in period t (see, for example, Casamatta, Cremer and Pestieau, 2000; Cond-Ruiz and Profeta, 2007). We can view the full commitment solution as the solution that includes intergenerational interaction because the full commitment solution can be supported as the subgame perfect equilibrium in repeated voting (see, for example, Conde-Ruiz and Galasso, 2003, 2005; Poutvaara, 2006).

Given the stationary environment, the current model presents a static voting game. Therefore, the median voter theorem can be applied to the voting game. To find the voting equilibrium, we need to show that tax preferences of voters are single-peaked. As for the tax preferences of old voters, their objective is to maximize their pension benefits regardless of their marriage status, labor supply, and saving. Although the benefit levels differ between old agents, the factor related to political decision is common to all old agents and is specified by the Laffer curve $\tau(1-\tau)$. Thus, the tax preferences of the old are single-peaked; their preferred tax rate, denoted by τ^{oj}, is $\tau^{oj} = 1/2\ \forall j = f, m, c1, c2$.

4.1 Policy Preferences of the Young

Next, let us consider the preferences of the young. To show that the preferences of a young agent who belongs to a type-j ($j = f, m, c1, m2$) household are single peaked, we should note that the following three properties hold. First, $\partial^2 V^j_{s>0}/\partial\tau^2 < 0$ and $\partial^2 V^j_{s=0}/\partial\tau^2 < 0$ hold; that is, $V^j_{s>0}$ and $V^j_{s=0}$ are single peaked. Second, the indirect utility V^j of a young agent in a type-j household is continuous at $\tau = \hat{\tau}^j$: $V^j_{s>0}\big|_{\tau=\hat{\tau}^j} = V^j_{s>0}\big|_{\tau=\hat{\tau}^j}$, $j = f, m, c1, c2$. Third, the slope of $V^j_{s>0}$ at $\tau = \hat{\tau}^j$ is equivalent to that of $V^j_{s=0}$ at $\tau = \hat{\tau}^j$:

$$\left.\frac{\partial V^j_{s>0}}{\partial\tau}\right|_{\tau=\hat{\tau}^j} = \left.\frac{\partial V^j_{s=0}}{\partial\tau}\right|_{\tau=\hat{\tau}^j},\ j = f, m, c1, c2.$$

The details of the calculation are given in Appendix A.1. The three properties imply that V^j has a unique maximum. In what follows, we derive the conditions that determine the tax rates preferred by four types of households.

First, consider a preferred tax rate by a young single female agent. Suppose that she prefers a positive tax rate: $\tau > 0$. She chooses τ that satisfies $\partial V^f_{s>0}/\partial\tau = 0$ when she is borrowing-unconstrained; and τ that satisfies $\partial V^f_{s=0}/\partial\tau = 0$ when she is borrowing-constrained. After some calculation, we can find that the preferred tax rate by single females satisfies:

$$LHS \equiv 1 - 2\tau = RHS^f \equiv \begin{cases} \frac{\alpha}{\chi(\cdot)} & \text{if } \tau < \hat{\tau}^f, \\ \frac{1}{\beta}\left(\frac{\alpha}{\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{if } \tau \geq \hat{\tau}^f. \end{cases} \quad (4)$$

The terms LHS and RHS^f represent the *marginal efficiency loss of taxation* and the *marginal cost-to-benefit ratio of redistribution*, respectively. Single females choose the tax rate that balances these terms.

Condition (4) determines the preferred tax rate by single females provided that they prefer a positive tax rate. However, single females may want to prefer no taxation and thus no redistribution via pension when the marginal cost of redistribution is larger than the marginal benefit of redistribution at $\tau = 0$, that is, when $\partial V^f_{s>0}/\partial\tau\Big|_{\tau=0} \leq 0$ ($\Leftrightarrow$ $(1-\alpha\pi)/\alpha\varphi(1-\mu) \leq \gamma$) holds. Therefore, the preferred tax rate by single females, denoted by τ^f, is summarized as:

$$\begin{cases} \tau^f = 0 & \text{if} \quad \frac{1-\alpha\pi}{\alpha\varphi(1-\mu)} \leq \gamma; \\ \tau^f(>0) \ \text{satisfies (4)} & \text{otherwise.} \end{cases}$$

Following the same procedure, we can immediately find that $\partial V^m_{s>0}/\partial\tau|_{\tau=0} \leq 0$ always holds. The preferred tax rate by single males, denoted by τ^m, becomes $\tau^m = 0$.[5]

The next task is to find preferred tax rates by couples. As in the case of a single female, we first suppose that couples prefer a positive tax rate and compute their preferred tax rate as follows. A preferred tax rate by one-breadwinner couples satisfies:

$$LHS \equiv 1 - 2\tau = RHS^{c1} \equiv \begin{cases} \frac{1}{(\pi+\gamma)\chi(\cdot)} & \text{if} \quad \tau < \hat{\tau}^{c1}, \\ \frac{1}{\beta}\left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{if} \quad \tau \geq \hat{\tau}^{c1}; \end{cases} \tag{5}$$

and a preferred tax rate by two-breadwinner couples satisfies:

$$LHS \equiv 1 - 2\tau = RHS^{c2} \equiv \begin{cases} \frac{1+\alpha}{(\pi+1)\chi(\cdot)} & \text{if} \quad \tau < \hat{\tau}^{c2}, \\ \frac{1}{\beta}\left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{if} \quad \tau \geq \hat{\tau}^{c2}. \end{cases} \tag{6}$$

Taking into account of the case where couples prefer no redistribution, we find that a preferred tax rate by one-breadwinner couples, denoted by τ^{c1}, satisfies:

$$\begin{cases} \tau^{c1} = 0 & \text{if} \quad \gamma \leq \frac{1-\alpha\pi}{1+\alpha}; \\ \tau^{c1}(>0) \ \text{satisfies (5)} & \text{otherwise;} \end{cases}$$

and a preferred tax rate by two-breadwinner couples, denoted by τ^{c2}, satisfies:

$$\begin{cases} \tau^{c2} = 0 & \text{if} \quad \frac{1-\alpha\pi}{1+\alpha} \leq \gamma; \\ \tau^{c2}(>0) \ \text{satisfies (6)} & \text{otherwise.} \end{cases}$$

Two remarks are in order. First, single males always prefer no taxation and thus no redistribution because they always pay more than they receive. Second, the threshold value of the derived pension rights for one-breadwinner couples coincides with that for two-breadwinner couples. However, the order of preferences of the one-breadwinner couples is the total opposite to that of the two-breadwinner couples because the one-breadwinner

[5] After some calculation, we obtain:

$$\frac{\partial V^m_{s>0}}{\partial\tau}\bigg|_{\tau=0} \leq 0 \Leftrightarrow 1 \leq \frac{1}{\pi\chi(\cdot)} \Leftrightarrow \underbrace{(\pi\alpha - 1)(1-\varphi(1-\mu))}_{LHS} \leq \underbrace{\varphi(1-\mu)\gamma}_{RHS},$$

where $LHS < 0 < RHS$.

couples obtain more benefits whereas the two-breadwinner couples obtain fewer benefits as the degree of derived pension right becomes higher.

The above-mentioned two properties are qualitatively identical to those demonstrated by Leroux, Pestieau and Racionero (2011). However, by taking account of borrowing constraint and an intertemporal elasticity of substitution, we can find non-monotone effects of gender wage gap, derived pension rights and the fraction of two-breadwinner couples on the equilibrium tax, which were not observed in Leroux, Pestieau and Racionero (2011). This point will be investigated in the following sections. Before going to the next section, we would like to consider more in detail the roles of borrowing constraint and an intertemporal elasticity of substitution in the determination of a preferred tax rates.

4.2 The Role of Marginal Cost-to-benefit Ratio of Redistribution

The result in Section 4.1 presents the preferred tax rates by single females, single males, one-breadwinner couples, and two-breadwinner couples, respectively. The intuition for the corner solution is described in Section 4.1. To understand the reasoning behind the interior solution, let us consider the single female's condition, (4), as an example. In particular, we focus on the parameters α and γ, representing the gender wage gap and the degree of derived pension rights, respectively. They are observed on the right-hand side of (4); they affect the marginal cost-to-benefit ratio of redistribution.

There are two opposing effects on the ratio via the terms α and γ. First, given a tax rate, an increase in α (i.e., an increase in females' wage) imposes a further tax burden on a single female via the term α on the numerator of the right-hand side in (4); and an increase in γ (i.e., an increase in the degree of derived pension rights) produces fewer benefits for a single female via the term $\chi(\cdot)$. Greater burden and fewer benefits give her an incentive to choose a lower tax rate and thus a smaller size of redistribution, resulting in a negative effect on the preferred tax rate. Second, an increase in α augments wage income for single females, and thus pension benefits in old age, because the total wages on which the tax is levied are increased. This augmentation gives a young single female an incentive to choose a higher tax rate, resulting in a positive effect on the preferred tax rate via the term $\chi(\cdot)$.

When a young single female is borrowing-unconstrained, she can reallocate income freely across periods. Because of this intertemporal reallocation of income, the positive effect is compensated for by the negative effect regardless of the degree of intertemporal elasticity of substitution. Therefore, increases in α and γ result in a higher marginal cost-to-benefit ratio of redistribution and thus a lower preferred tax rate when a young single female is borrowing-unconstrained. The result holds regardless of $1/\sigma$ because the objective for a borrowing-unconstrained household is to maximize lifetime income, which is independent of $1/\sigma$.

When a single female is borrowing-constrained, the positive effect is not necessarily compensated for by the negative one. The borrowing-constrained single female wants to consume more when young, but her demand is restricted by the borrowing constraint. In this situation, the borrowing-constrained individual attaches a large weight to the utility gain of an increase in her wage. This effect might lead to a situation in which the positive effect overcomes the negative one, resulting in a lower, rather than higher, marginal cost-to-benefit ratio of redistribution and thus a higher preferred tax rate in response to an increase in α or γ.

Which effect outweighs the other depends on the degree of an intertemporal elasticity of substitution. A lower elasticity implies a stronger incentive for single females to smooth consumption over periods. Because of this incentive, borrowing-constrained single females attach a smaller weight to the positive effect on youthful consumption via a decrease in her preferred tax rate as the elasticity becomes lower. In other words, the positive effect on the preferred tax rate is more likely to overcome the negative one as the elasticity becomes lower.

The net effect depends on the degree of the intertemporal elasticity of substitution. When the elasticity is high, that is, $1/\sigma \geq 1$, the net effect on the tax is negative. An increase in α or γ results in a lower preferred tax rate by borrowing-constrained single females. In contrast, when the elasticity is low, that is, $1/\sigma < 1$, the net effect becomes positive. An increase in α or γ leads to a higher preferred tax rate.

In concluding this section, we note that the tax rates preferred by the young are lower than those preferred by the old who choose $\tau = 1/2$. The result of this phenomenon is that the decisive voter with respect to τ belongs to the young generation because the population size of the young is larger than that of the old given the death of some males in early life. Given this result, we focus on young agents' preferences over τ and consider the determination of τ in majority voting in the next section.

5 Political Equilibrium

This section characterizes the political equilibrium of the majority voting. In what follows, an "agent" implies a "young agent" if not otherwise specified.

5.1 Political Environment

We impose the following assumption to proceed with the analysis.

Assumption 1: $\max\left\{\frac{1-\pi}{4(1-\mu)}, \frac{1-\pi}{4\mu}\right\} < \varphi < \frac{1+\pi}{2}$.

Assumption 1 ensures that one who prefers the highest tax rate among the young becomes a decisive voter. To understand the argument stemming from Assumption 1, recall that (i) young agents are ranked in terms of their preferred tax rates; (ii) all the old prefer $\tau = 1/2$ which is higher than any other preferred tax rates by the young. However, the old cannot be majority voters because the number of the old population, 2π, is less than that of the young population, 2.

Given the abovementioned argument, we can find the decisive voter from the young. In particular, by imposing Assumption 1, we can determine the identity of the decisive voter from an agent who prefers the highest tax rate among the young. For example, suppose that a single female prefers the highest tax rate among the young. She can be a decisive voter if the number of single females plus the old is larger than half of the population, that is, if $(1-\varphi)+(1+\pi) > (3+\pi)/2 \Leftrightarrow \varphi < (1+\pi)/2$. Following the same argument, we can say that an agent who belongs to a one-breadwinner couple becomes a decisive voter if $2\varphi(1-\mu)+(1+\pi) > (3+\pi)/2 \Leftrightarrow \varphi > (1-\pi)/4(1-\mu)$; and an agent who belongs to a two-breadwinner couple becomes a decisive voter if $2\varphi\mu+(1+\pi) > (3+\pi)/2 \Leftrightarrow \varphi > (1-\pi)/4\mu$. The three conditions are summarized as in Assumption 1.[6]

Hereafter, we will focus on the parameter σ, which represents the inverse of the intertemporal elasticity of substitution, and consider two cases separately: a high elasticity case ($1/\sigma \geq 1$) and a low elasticity case ($1/\sigma < 1$). We adopt this classification because the order of preferences for the tax rate depends critically on the degree of elasticity. Because the former case includes the case of Leroux, Pestieau, and Racionero (2011) as a special one, we leave it to Appendix A.5. In what follows here, we will focus exclusively on the latter case.

5.2 An Economy with $1/\sigma < 1$

To determine the decisive voter over τ when $1/\sigma < 1$, we recall the conditions demonstrated in Section 4.1 that determine the preferred tax rates by four types of households. Because $\gamma = (1-\alpha\pi)/(1+\alpha)$ is the threshold value of the derived pension rights common to one-breadwinner and two-breadwinner couples, we consider two cases, $\gamma \in [0, (1-\alpha\pi)/(1+\alpha)]$ and $\gamma \in ((1-\alpha\pi)/(1+\alpha), 1]$ in turn.

5.2.1 Low Level of Derived Pension Rights: $0 \leq \gamma \leq \frac{1-\alpha\pi}{1+\alpha}$

First, we consider the case of a low level of derived pension rights. In this case, single males and one-breadwinner couples prefer no taxation, whereas single females and two-breadwinner couples prefer taxation. Figure 1 illustrates the conditions (4) and (6) that

[6] A single male cannot be a decisive voter because he always prefers no taxation.

determine the preferred tax rates by single females and two-breadwinner couples, respectively. As depicted in the figure, there is a critical value of τ, $\tilde{\tau}^{f,c2} \in \left(\hat{\tau}^f, \hat{\tau}^{c2}\right)$, such that RHS^f and RHS^{c2} intersect at $\tau = \tilde{\tau}^{f,c2}$. By direct calculation, we obtain:

$$\tilde{\tau}^{f,c2} \equiv \left(\frac{(1+\alpha)\beta}{(\pi+1)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{\alpha}{\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}.$$

[Figure 1 about here.]

The tax rate preferred by a type-j ($j = f, c2$) household is determined by the crossing point of LHS and RHS^j. Given the assumption of household distribution in Assumption 1, the decisive voter over τ is the one who prefers the highest tax rate among the young households. Based on the illustration in Figure 1, the decisive voter over τ when $1/\sigma < 1$ and $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$ is determined as follows.

Lemma 1. *Suppose that $1/\sigma < 1$ and $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$ hold. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(i) *a type-f single female agent if $\chi(\cdot) \leq 2\left(\frac{(1+\alpha)\beta}{\pi+1}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{1+\alpha}{\pi+1}$;*

(ii) *a type-$c2$ agent who belongs to a two-breadwinner couple, otherwise.*

Proof. See Appendix A2.

To understand the mechanism behind the result, recall the condition that produces an equilibrium in which a type$-f$ single female agent becomes a decisive voter in Lemma 1. The condition is rewritten as:

$$\frac{1 - \frac{\alpha}{1+\alpha}\varphi(1-\mu)}{1 - \frac{\varphi(1-\mu)(1-\gamma)}{\pi+1}} \leq 2\,(\beta)^{\frac{1}{\sigma}} \cdot (\pi+1)^{1-\frac{1}{\sigma}} \cdot \left(\frac{\alpha}{1+\alpha}\right)^{\frac{\sigma-1}{\sigma}} + 1, \tag{7}$$

where the left-hand side is decreasing in α, and the right-hand side is increasing in α. Therefore, the condition (7) states that a type-f agent is more likely to become a decisive voter when α is higher (that is, when the gender wage gap is smaller).

The intuition behind the condition (7) is as follows: For a low degree of derived pension rights such that $\gamma \in [0, (1-\alpha\pi)/(1+\alpha)]$, single females expect a high level of pension benefits in old age, which gives them a disincentive to save. Thus, single females would be borrowing constrained for a low degree of derived pension rights. In this situation, they owe greater tax burden as α becomes higher (i.e., as their wage becomes higher). However, when $1/\sigma < 1$, an increase in α results in a lower marginal cost-to-benefit ratio of redistribution and thus a higher preferred tax rate by borrowing-constrained single females as demonstrated in Section 4.2. Therefore, type-f agents prefer a higher tax rate than type-$c2$ agents when α is high such that (7) holds.

5.2.2 High Level of Derived Pension Right: $\frac{1-\alpha\pi}{1+\alpha} < \gamma \leq 1$

Next, let us consider the case of a high level of derived pension rights. In this case, single males and two-breadwinner couples prefer no taxation, whereas single females and one-breadwinner couples prefer taxation. Figure 2 illustrates the conditions (4) and (5) that determine the preferred tax rates by single females and one-breadwinner couples, respectively.

[Figure 2 about here.]

The current case could be divided at most into the following three sub-cases: (2-a) a case of $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \min\left\{\frac{1-\alpha\pi}{\alpha}, 1\right\}\right]$ (see Panel (a)); (2-b) a case of $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, \min\left\{\frac{1-\alpha\pi}{\alpha\varphi(1-\mu)}, 1\right\}\right)$ provided that $1 < \alpha(1+\pi)$ holds (see Panel (b)); and (2-c) a case of $\gamma \in \left[\frac{1-\alpha\pi}{\alpha\varphi(1-\mu)}, 1\right]$ (see Panel (c)) provided that $\frac{1-\alpha\pi}{\alpha(1-\mu)} < \varphi$ holds. As depicted in the figure, there are critical values of the tax, $\tilde{\tau}^{f,c1}$ for the case (2-a), and $\tilde{\tau}^{c1,f}$ for the case (2-b), defined by:

$$\tilde{\tau}^{f,c1} \equiv \left(\frac{\beta}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{\alpha}{\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}, \tilde{\tau}^{c1,f} \equiv \left(\frac{\alpha\beta}{\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}$$

respectively. As in the previous subsection, we can characterize the political equilibrium for the current case as follows.

Lemma 2. *Suppose that $1/\sigma < 1$ and $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, 1\right]$ hold. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(i) *a type-f single female agent if:*

$$\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \min\left\{\frac{1-\alpha\pi}{\alpha}, 1\right\}\right] \text{ and } \chi(\cdot) \leq 2\left(\frac{\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\pi+\gamma},$$

or if:

$$\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, 1\right], 1 < \alpha(1+\pi) \text{ and } \chi(\cdot) > 2(\alpha\beta)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + \alpha;$$

(ii) *a type-$c1$ agent who belongs to a one-breadwinner couple otherwise.*

Proof. See Appendix A3.

The main difference from the previous case is that a type-$c2$ agent cannot be a decisive voter; instead, a type-$c1$ agent can be a decisive voter under a certain condition. The change of the decisive voter is due to that, in the current case, a type-$c1$ agent attains a higher pension benefit because of a higher level of derived pension rights relative to the previous case. This relationship results in a lower marginal cost-to-benefit ratio of redistribution for a type-$c1$ agent compared to a type-$c2$ agent. Therefore, a type-$c1$ agent prefers a higher tax rate than does a type-$c2$ agent.

The two conditions in Lemma 2(i) implies that for a lower α, a single female agent prefers a higher tax rate; and she is more likely to be a decisive voter (see Appendix A3). This result is qualitatively different from that in the previous, low-derived pension case: for a higher α, a single female agent prefers higher tax rate; and she is more likely to be a decisive voter. The difference between the two cases comes from the saving behavior affected by the degree of derived pension rights.

To understand the role of derived pension rights, suppose that the degree of derived pension rights is higher than the critical value $(1-\alpha\pi)/(1+\alpha)$. Under this situation, single females are less likely to be borrowing constrained because they expect a lower level of pension benefits, which gives them an incentive to save more for old-age consumption. In this situation, a decrease in α results in a decrease of the cost-to-benefit ratio of redistribution for unconstrained single females; and this gives them an incentive to choose a higher tax rate. Therefore, when γ is higher than the critical value $(1-\alpha\pi)/(1+\alpha)$, a single female agent is more likely to be a decisive voter for a lower α.

Alternatively, suppose that the degree of derived pension rights is lower than the critical value $(1-\alpha\pi)/(1+\alpha)$. Under this assumption, single females are more likely to be borrowing constrained because they expect a higher level of pension benefits and thus find less need to save for old-age consumption. In this situation, an increase in α results in an increase of the cost-to-benefit ratio of redistribution for constrained single females; but this gives them an incentive to choose a higher, rather than a lower, tax rate because single females are borrowing constrained and are faced with a low intertemporal elasticity of substitution. Therefore, when γ is lower than the critical value $(1-\alpha\pi)/(1+\alpha)$, a single female agent is more likely to be a decisive voter for a higher α.

5.2.3 A Decisive Voter When $1/\sigma < 1$

The results established so far are summarized as follows.

Proposition 1. *Suppose that $1/\sigma < 1$ holds. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(i) *a type-f single female agent if:*

$$\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right] \text{ and } \chi(\cdot) \leq 2\left(\frac{(1+\alpha)\beta}{\pi+1}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{1+\alpha}{\pi+1}; \text{ or}$$

$$\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \min\left\{\frac{1-\alpha\pi}{\alpha}, 1\right\}\right] \text{ and } \chi(\cdot) \leq 2\left(\frac{\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\pi+\gamma}; \text{ or}$$

$$\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, 1\right], 1 < \alpha(1+\pi) \text{ and } \chi(\cdot) > 2(\alpha\beta)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + \alpha;$$

(ii) *a type-$c2$ agent who belongs to a two-breadwinner couple if:*

$$\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right] \text{ and } \chi(\cdot) > 2\left(\frac{(1+\alpha)\beta}{\pi+1}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{1+\alpha}{\pi+1};$$

(iii) *a type-$c1$ agent who belongs to a one-breadwinner couple otherwise.*

Proof. The proof is immediate from Lemmas 1 and 2.

Proposition 1 presents the determination of the identity of the decisive voter in majority voting. A type-f single female agent becomes a decisive voter if the degree of derived pension rights is low and the gender wage gap is small as demonstrated in the first condition of the statement (i); or if the degree of derived pension right is high and the gender wage gap is large as demonstrated in the second and third conditions of the statement (i). Interpretations for these results are discussed in the previous subsections.

A type-$c2$ agent who belongs to a two-breadwinner couple becomes a decisive voter if the degree of derived pension right is low such that the first condition in the statement (ii) holds; and the benefit-to-burden ratio of public pension, denoted by $\chi(\cdot)$, is high such that the second condition in the statement (ii) holds. Because $\chi(\cdot)$ is increasing in μ, the latter condition implies that a type-$c2$ agent is more likely to be a decisive voter when the fraction of type-$c2$ agents is larger in the economy.

Finally, a type-$c1$ agent who belongs to a one-breadwinner couple becomes a decisive voter if the conditions in statements (i) and (ii) of Proposition 1 fail to hold. In particular, all types except type-c1 agents prefer no taxation and thus no redistribution via pension when $\gamma > \frac{1-\alpha\pi}{\alpha\varphi(1-\mu)}$ holds (see Panel (c) of Figure 2). This condition implies that the gender wage gap is small, the fraction of one-breadwinner couples is large, and the longevity of men is high. All these factors imply greater benefits via pension compared to the cost of taxation for type-$c1$ agents. Therefore, type-$c1$ agents have an incentive to choose taxation on working agents although all the other types of agents find it optimal to choose no taxation.

The effect of the derived pension rights on the identity of the decisive voter is qualitatively equivalent to that demonstrated in Leroux, Pestieau and Racionero (2011). However, the effect of gender wage gap on the identity of the decisive voter is different from that in Leroux, Pestieau and Racionero (2011). As the gender wage gap becomes smaller, the marginal cost-to-benefit ratio of redistribution for single females becomes larger. However, they prefer a larger size of redistribution as long as they are borrowing constrained with a low intertemporal elasticity of substitution (Lemma 1). This counterintuitive result was not demonstrated in Leroux, Pestieau and Racionero (2011) who assume a quasi-linear utility and no borrowing constraint. This different effect is further investigated in the next section.

6 Gender Wage Gap, Derived Pension Rights and the Fraction of Two-breadwinner Couples

Given the characterization of the political equilibrium in Section 5, we investigate how the tax rate changes in response to recent trends in developed economies: a reduction of the gender wage gap, a reduction of derived pension rights, and an increase in the fraction of two-breadwinner couples. The aim of the analysis is to explore the roles of the borrowing constraint and the intertemporal elasticity of substitution on the determination of the tax rate, which were not demonstrated in Leroux, Pestieau and Racionero (2011). The analysis also aims to discuss policy implications of the result.

6.1 A Reduction of the Gender Wage Gap

First, we investigate the effect of a reduction of the gender wage gap on the determination of the tax rate.

Proposition 2: *In an economy with $1/\sigma < 1$ where the decisive voter is a type-j ($j = f, c1,$ or $c2$) agent, a reduction of the gender wage gap (i.e., an increase in α) locally produces an inverse U-shaped relationship between α and τ around $\hat{\tau}^j$.*

Proof. See Appendix A.4.

To understand the mechanism behind the result in Proposition 2, we first note that there is a critical value of α for a type-j agent, denoted by α^j ($j = f, c1, c2$): α^j is the α that makes a type-j agent choose $\tau = \hat{\tau}^j$ (see Figure 3). Around this critical value, there is a change in the pattern of saving as discussed in the previous sections. In particular, for $j = f$ and $c2$, an agent is borrowing constrained for $\alpha \leq \alpha^j$; and he/she is borrowing unconstrained for $\alpha > \alpha^j$ (see Panel (a) of Figure 3). For $j = c1$, an agent is borrowing unconstrained for $\alpha \leq \alpha^j$; and he/she is borrowing constrained for $\alpha > \alpha^j$ (see Panel (b) of Figure 3). The detail of the mechanism behind the change of saving pattern around α^j is shown in Appendix A.4.1.

[Figure 3 about here.]

Proposition 2 says that starting from a low value of α, borrowing-constrained type-f or type-$c2$ agent prefers a higher tax rate as α becomes larger (that is, as the gender wage gap becomes narrower). However, he/she becomes borrowing-unconstrained once α approaches α^j; and he/she prefers a lower tax rate as α is further increased. Type-j's ($j = f, c2$) preferences over the tax qualitatively changes at $\alpha = \alpha^j$. As for a type-$c1$ agent, he/she is borrowing unconstrained for α below his/her critical value α^{c1}. For this range of α, he/she prefers a higher tax rate as α becomes larger. However, he/she becomes borrowing constrained when α is beyond the critical value α^{c1}. He/she turns to choose a lower tax rate in response to a further increase in α

The result described so far could be understood in the following way. We first consider the effect of an increase in α on the marginal cost-to-benefit ratio of redistribution, denoted by $RHS^j (j = f, c1, c2)$, when an agent is borrowing-unconstrained. After some calculation, we obtain

$$\frac{\partial RHS^f}{\partial \alpha} > 0, \frac{\partial RHS^{c1}}{\partial \alpha} < 0, \frac{\partial RHS^{c2}}{\partial \alpha} > 0 \text{ if } s^j > 0 \ (j = f, c1, c2).$$

When an agent is borrowing-unconstrained, a reduction of the gender wage gap (i.e., an increase in α) increases the marginal cost-to-benefit ratio of redistribution for type-f and type-$c2$ agents, whereas it decreases the ratio for type-$c1$ agents. The difference is due to the fact that single females and two-breadwinner couples owe an additional tax burden when there is an increase in females' wages, whereas one-breadwinner couples owe no additional burden. Because increased tax revenue is returned to all types of agents as lump-sum pension benefits, single females and two-breadwinner couples pay more than they receive, whereas one-breadwinner couples pay nothing but receive additional benefits. Therefore, a reduction of the gender wage gap results in a higher marginal cost-to-benefit ratio of redistribution and thus a lower preferred tax rate for type-f and type-$c2$ agents, whereas it results in a lower marginal cost-to-benefit ratio of redistribution and thus a higher preferred tax rate for type-$c1$ agents.

The opposite result holds when an agent is borrowing-constrained because the effect of α on RHS^j is reversed as demonstrated in Section 4.2. A reduction of the gender wage gap results in lower RHS^f and RHS^{c2} and thus in higher preferred tax rates by type-f and type-$c2$ agents, whereas it results in a higher RHS^{c1} and thus in a lower preferred tax rate by type-$c1$ agents. Therefore, there is an inverse U-shaped relationship between α and τ^j around the critical value of the tax, $\hat{\tau}^j$, that divides the status of saving (see Panel (c) of Figure 3).

6.2 A Reduction of Derived Pension Rights

Next, we consider the effect of a reduction of derived pension rights on the determination of the tax rate.

Proposition 3: *In an economy with* $1/\sigma < 1$ *where the decisive voter is a type-*j *(*$j = f, c1,$ *or* $c2$*) agent, a reduction of derived pension rights (i.e., a decrease in* γ*) produces an inverse U-shaped relationship between* γ *and* τ *around* $\hat{\tau}^j$*.*

Proof. See Appendix A.4.

To understand the result in Proposition 3, we first note that there is a critical value of γ, γ^j, for a type-j agent: γ^j is the γ that makes a type-j agent choose $\tau = \hat{\tau}^j$. Around this critical value, there is a change in the patterns of saving. In particular, for $j = f, c2$, an agent is borrowing constrained for $\gamma \leq \gamma^j$; he/she is borrowing unconstrained for $\gamma > \gamma^j$. For $j = c1$, an agent is borrowing unconstrained for $\gamma < \gamma^j$; he/she is borrowing constrained for $\gamma \geq \gamma^j$.

Given the saving pattern of each agent, we first consider the effect of the derived pension rights on the marginal cost-to-benefit ratio of redistribution when an agent is borrowing unconstrained:

$$(-1)\frac{\partial RHS^f}{\partial \gamma} < 0, (-1)\frac{\partial RHS^{c1}}{\partial \gamma} > 0, (-1)\frac{\partial RHS^{c2}}{\partial \gamma} < 0 \text{ if } s^j > 0 \ (j = f, c1, c2)$$

We multiply the derivatives by (-1) to demonstrate the qualitative effect of a decrease in γ.

A reduction of derived pension rights (i.e., a decrease in γ) increases the pension benefits for type-f and type-$c2$ agents, lowers the marginal cost-to-benefit ratio of redistribution, and thus raises their preferred tax rate. Therefore, type-f and type-$c2$ agents prefer a higher tax rates in response to a reduction of the degree of derived pension rights as long as γ is above the critical value γ^j ($j = f, c2$). In contrast, such a reduction decreases the pension benefits for type-$c1$ agents, raises their cost-to-benefit ratio of redistribution and thus lowers their preferred tax rate. Type-$c1$ agent prefers a lower tax rate in response to a reduction of γ as long as γ is below a critical value γ^{c1}.

The above result is reversed when an agent is borrowing-constrained as demonstrated in the case of a change in α. Therefore, the two opposing effects result in an inverse U-shaped relationship between γ and τ^j around the critical value of γ, γ^j. The preferred tax rate is maximized and is given by $\tau = \hat{\tau}^j$ at $\gamma = \gamma^j$.

The result in Proposition 3 differs from that demonstrated in Leroux, Pestieau and Racionero (2011). They numerically showed that a reduction of derived pension rights (i.e., a decrease in γ) results in decreases in the tax rate and pension size for plausible set of parameters. Their monotone result comes from the assumption of quasi-linear utility and no borrowing constraint. The current paper takes away these assumptions and shows a non-monotone effect of the derived pension rights on the equilibrium tax.

6.3 An Increase in the Fraction of Two-breadwinner Couples

Finally, we examine the effect of the share of two-breadwinner couples on the equilibrium tax rate.

Proposition 4: *In an economy with* $1/\sigma < 1$ *where the decisive voter is a type-*j *(*$j = f, c1,$ *or* $c2$*) agent, an increase in the fraction of two-breadwinner couples (i.e., an increase in* μ*)* (a) *locally produces an inverse U-shaped relationship between* μ *and* τ *around* $\hat{\tau}^j$ *if* $\gamma \neq (1-\alpha\pi)/(1+\alpha)$*;* (b) *has no effect on the equilibrium tax if* $\gamma = (1-\alpha\pi)/(1+\alpha)$.

Proof. See Appendix A.4.

To understand the result in Proposition 4, let us first consider how the benefit-to-burden ratio of public pension, denoted by $\chi(\cdot)$, is affected by the parameter μ:

$$\chi(\cdot) \equiv \frac{1+\alpha\,(1-\varphi+\varphi\mu)}{\pi+1-\varphi+\varphi\mu+\gamma\varphi(1-\mu)}.$$

On one hand, an increase in μ results in an increase of tax revenue from the working females and thus in an increase per capita pension benefit. This effect is observed in the numerator of $\chi(\cdot)$. On the other hand, an increase in μ implies a larger size of two-breadwinner couples and a smaller size of one-breadwinner couples. Because the level of pension rights is larger for working females than for non-working females, an increase in μ results in a decrease of per capita pension benefits. This effect is observed in the denominator of $\chi(\cdot)$.

The former positive effect on $\chi(\cdot)$ is independent of γ, whereas the latter negative effect on $\chi(\cdot)$ is dependent of γ. Specifically, the negative effect becomes larger as γ becomes smaller. Therefore, the negative effect overcomes the positive one when the level of derived pension rights is low such that $\gamma < (1-\alpha\pi)/(1+\alpha)$: that is, $\partial\chi(\cdot)/\partial\mu < 0$ holds if $\gamma < (1-\alpha\pi)/(1+\alpha)$; the opposite holds if $\gamma > (1-\alpha\pi)/(1+\alpha)$: that is, $\partial\chi(\cdot)/\partial\mu > 0$ holds if $\gamma > (1-\alpha\pi)/(1+\alpha)$. The two opposing effects are offset each other if $\gamma = (1-\alpha\pi)/(1+\alpha)$.

In what follows, we consider the case of $\gamma < (1-\alpha\pi)/(1+\alpha)$ and the case of $\gamma > (1-\alpha\pi)/(1+\alpha)$ in turn. First, for a low degree of derived pension rights such that $\gamma < (1-\alpha\pi)/(1+\alpha)$, type $j = f$ and $j = c2$ agents prefer a positive tax rate. A low degree of derived pension rights for non-working females produces a large size of per capita pension benefits for working females. In addition, a low μ (i.e., a low share of two-breadwinner couples) implies a high level of $\chi(\cdot)$ (i.e., a high benefit-to-burden ratio of public pension) and thus a high level of per capita public pension. These two effects give types $j = f$ and $j = c2$ agents a disincentive to save for their old-age consumption. Thus, they are borrowing constrained for $\gamma < (1 - \alpha\pi)/(1 + \alpha)$ and $\mu < \mu^j$. However, for $\mu > \mu^j$, a level of per capita pension benefits becomes low; and this negative effect on the size of pension overcomes the positive effect produced by a low degree of derived pension rights. Therefore, types $j = f$ and $j = c2$ agents are borrowing unconstrained for $\gamma < (1 - \alpha\pi)/(1 + \alpha)$ and $\mu > \mu^j$.

Based on the abovementioned argument, we can now show the effect of μ on the preferred tax rate by a type-j ($j = f, c2$) agent when $\gamma < (1-\alpha\pi)/(1+\alpha)$. First, suppose that $\mu > \mu^j$: a type-j ($j = f, c2$) agent is borrowing unconstrained. In this situation, a higher μ results in a lower size of $\chi(\cdot)$, and thus a larger marginal cost-to-benefit ratio of redistribution, RHS^j ($j = f, c2$) in Eqs. (4) and (6). A type-j ($j = f, c2$) agents prefers a lower tax rate as μ becomes larger for the range of $\mu > \mu^j$. However, the opposite result holds for the range of $\mu < \mu^j$ because he/she is borrowing constrained. Therefore, around μ^j, there arises an inverse U-shaped relationship between μ and the preferred tax rate by the decisive voter.

Next, consider the case of a high degree of derived pension rights such that $\gamma > (1 - \alpha\pi)/(1 + \alpha)$: type-$f$ and type-$c1$ agents prefer a positive tax rate. A high degree of derived pension rights gives a high level of per capita pension benefits for type-$c1$ agents. However, while it gives a low level of per capita pension benefits for type-f agents, this negative effect on pension size is compensated for by the effect of μ via $\chi(\cdot)$ because a higher μ results in a larger size of $\chi(\cdot)$ and thus a larger size of per capita pension, $\partial\chi(\cdot)/\partial\mu > 0$. Therefore, for a high μ such that $\mu > \mu^j$, a type-j ($j = f, c1$) agent obtains a large sized pension benefit; and this gives him/her a disincentive to save for old-age consumption, thereby resulting in being borrowing constrained for $\mu > \mu^j$. Because $\partial\chi(\cdot)/\partial\mu > 0$ holds, a larger μ results in a lower marginal cost-to-benefit ratio of redistribution for a constrained type-j agent and thus leads to a lower preferred tax rate by him/her. The opposite result holds for $\mu < \mu^j$ because a type-j ($j = f, c1$) agent is borrowing unconstrained due to a low size of pension benefits. A larger μ results in a lower marginal cost-to-benefit ratio of redistribution for a type-j unconstrained agent and thus results in a higher preferred tax rate.

6.4 Discussion

We have analyzed the effects of changes in α (gender wage gap), γ (the degree of derived pension rights) and μ (the fraction of two-breadwinner couples) on the equilibrium tax rate, and shown the inverse U-shaped relationship between the tax rate and parameters.

Based on the result demonstrated so far, in this subsection we briefly discuss the following three issues: (i) the effect of longevity gap between males and females on the equilibrium tax rate, (ii) the effect of a decisive voter's switch on the equilibrium tax rate, and (iii) policy implications of the result.

The current framework assumes that females live two periods, but males die at the end of youth with probability $1-\pi$. The parameter π represents the longevity difference: a smaller π implies a larger longevity difference between males and females. From equations (4), (5) and (6), we can immediately find that a smaller π (i.e., a larger longevity gap) results in a lower marginal cost-to-benefit ratio of redistribution for type-f and type-$c2$ agents, while it results in a higher marginal cost-to-benefit ratio of redistribution for type-$c1$ agents. Given this result, we can apply the analysis and result established so far and conclude that type-f and type-$c2$ agent prefer a higher tax rate in response to an increase in longevity gap when they are borrowing unconstrained, whereas they prefer a lower tax rate when they are borrowing constrained. The opposite result holds for type-$c1$ agents. Therefore, a change in longevity difference also produces an inverse U-shaped effect on the equilibrium tax.

The result in this section, combined with the result in Section 5, suggests that the equilibrium tax would show two peaks in response to changes in parameters. For illustrative purpose, assume $\gamma < (1-\pi)/2$: the result in Lemma 1 holds for any $\alpha \in [0,1]$, and the decisive voter is a type-f or type-$c2$ agent depending on the value of α. First, we take a sufficiently low value of α such that the decisive voter is a borrowing-constrained type-$c2$ agent. His/her preferred tax rate is increased in response to an increase in α (i.e., a reduction of gender wage gap), and peaks at α^{c2}. A further increase in α makes him/her to be borrowing-unconstrained; and to prefer a lower tax rate (Proposition 2). When the value of α approaches beyond the critical level, the decisive voter changes from a borrowing-unconstrained type-$c2$ agent to a borrowing-constrained type-f agent. The same argument holds for a type-f agent: her preferred tax rate attains a peak at α^{f} (Proposition 2). Therefore, there are two peaks of the equilibrium tax rates in response to changes in α.

Finally, we discuss the policy implications of the results. Many developed countries have been faced with increasing burden of public pension for the past decades. It has been argued that population aging is one of the most causes for this increasing burden. The local comparative statics analysis and result in this section provide an alternative view: decreases in the gender wage gap, the reduction of derived pension rights, and an increase in the fraction of two-breadwinner couples, observed in developed countries for the past decades, also provide a political incentive to increase the tax burden of public pension. However, our analysis and result suggest that further increases in these parameters may result in a decrease or a non-monotone change in tax burden. This possibility could be further investigated in future research.

7 Concluding Remarks

This paper developed an overlapping-generation model based on that of Leroux, Pestieau and Racionero (2011). The model includes four types of households: single female, single male, one-breadwinner couple and two-breadwinner couple. The paper introduced a borrowing constraint into their model and generalized the model by assuming a utility function with a constant intertemporal elasticity of substitution. Under this generalized framework, we consider majority voting over public pension policy in the presence of derived pension rights, and investigate how the borrowing constraint and intertemporal elasticity of substitution affect the preferences of each household over pension and the resulting equilibrium pension policy.

The paper showed the following two results. First, in an economy where an intertemporal elasticity of substitution is below one, one-breadwinner couples may prefer a lower, rather than higher, tax rate than do single females because of the presence of borrowing constraints. There is an equilibrium, much like an ends-against-the-middle equilibrium, where the old and single females form a coalition against the others.

Second, the gender wage gap, the level of derived pension rights, and the fraction of two-breadwinner couples create an inverse U-shaped relationship between the relevant variable and the tax rate when the intertemporal elasticity of substitution is below one. This two-toned effect was derived via a borrowing constraint associated with a low intertemporal elasticity of substitution.

Throughout the analysis, we assumed that the degree of derived pension rights is fixed. This assumption can be relaxed by assuming a structure-induced Nash equilibrium of voting (for example, Conde-Ruiz and Galasso, 2003; 2005; Casamatta, Cremer and Pestieau, 2005; Conde-Ruiz and Profeta, 2007; Bethencourt and Galasso, 2008). In this voting equilibrium, one-breadwinner couples prefer a full derived pension right, whereas others prefer no right. Thus, the full derived pension right is realized if the number of one-breadwinner couples is larger than a half of the population; no derived pension right is realized otherwise. However, in the real world, the degree of the derived pension right is set between these two extreme solutions. There is a need to add an institutional feature to demonstrate a more realistic situation: this task is left as future work.

A Appendix

A.1 Single-peaked Preferences of the Young

In this appendix, we prove that preferences of a type-f young agent are single peaked. The proof applies to other types of young agents.

The proof proceeds as follows. First, we show that both $V^f_{s>0}$ and $V^f_{s=0}$ are single peaked over τ. Then we demonstrate that $\partial V^f_{s>0}/\partial\tau = \partial V^f_{s=0}/\partial\tau$ and $V^f_{s>0} = V^f_{s=0}$ hold at $\tau = \hat{\tau}^f$, implying that V^f has a unique maximum over the whole range of τ and thus that V^f is single peaked over τ.

The first and the second derivatives of $V^{y,j}_{s>0}$ and $V^{y,j}_{s=0}$ with respect to τ are

$$
\begin{aligned}
\frac{\partial V^f_{s>0}}{\partial\tau} &= \left(\frac{1}{1+(\beta)^{1/\sigma}}\right)^{-\sigma} \cdot [(1-\tau)\alpha w + w\chi(\cdot)\tau(1-\tau)]^{-\sigma} \cdot \{-\alpha w + w\chi(\cdot)(1-2\tau)\};\\
\frac{\partial^2 V^f_{s>0}}{\partial\tau^2} &= \left(\frac{1}{1+(\beta)^{1/\sigma}}\right)^{-\sigma} \cdot (-\sigma) \cdot [(1-\tau)\alpha w + w\chi(\cdot)\tau(1-\tau)]^{-\sigma-1} \cdot \{-\alpha w + w\chi(\cdot)(1-2\tau)\}^2\\
&+ \left(\frac{1}{1+(\beta)^{1/\sigma}}\right)^{-\sigma} \cdot [(1-\tau)\alpha w + w\chi(\cdot)\tau(1-\tau)]^{-\sigma} \cdot (-2)w\chi(\cdot)\\
&< 0;\\
\frac{\partial V^f_{s=0}}{\partial\tau} &= [(1-\tau)\alpha w]^{-\sigma} \cdot (-\alpha w) + \beta \cdot [w\chi(\cdot)\tau(1-\tau)]^{-\sigma} \cdot w\chi(\cdot) \cdot (1-2\tau);\\
\frac{\partial^2 V^f_{s=0}}{\partial\tau^2} &= (-\sigma) \cdot [(1-\tau)\alpha w]^{-\sigma-1} \cdot (\alpha w)^2\\
&- \sigma\beta \cdot [w\chi(\cdot)\tau(1-\tau)]^{-\sigma-1} \cdot (w\chi(\cdot))^2 \cdot (1-2\tau)^2\\
&- 2\beta \cdot [w\chi(\cdot)\tau(1-\tau)]^{-\sigma} \cdot w\chi(\cdot)\\
&< 0.
\end{aligned}
$$

The functions $V^f_{s>0}$ and $V^f_{s=0}$ are single peaked over τ because the second derivatives are negative.

Next, we show that $\partial V^f_{s>0}/\partial\tau = \partial V^f_{s=0}/\partial\tau$ and $V^f_{s>0} = V^f_{s=0}$ hold at $\tau = \hat{\tau}^f$. By direct calculation, we have:

$$
\begin{aligned}
V^{y,j}_{s>0}\big|_{\tau=\hat{\tau}^f} &= V^{y,j}_{s=0}\big|_{\tau=\hat{\tau}^f}\\
&= \frac{1}{1-\sigma} \cdot \frac{1+(\beta)^{1/\sigma}}{1} \cdot \left(1 - \frac{\alpha(\beta)^{1/\sigma}}{\chi(\cdot)}\right)^{1-\sigma} \cdot (\alpha w)^{1-\sigma} - \frac{1+\beta}{1-\sigma},\\
\left.\frac{\partial V^{y,j}_{s>0}}{\partial\tau}\right|_{\tau=\hat{\tau}^f} &= \left.\frac{\partial V^{y,j}_{s=0}}{\partial\tau}\right|_{\tau=\hat{\tau}^f}\\
&= \left(1 - \frac{\alpha(\beta)^{1/\sigma}}{\chi(\cdot)}\right)^{-\sigma} \cdot (\alpha w)^{1-\sigma} \cdot \left[-1 + \frac{\chi(\cdot)}{\alpha} + 2(\beta)^{1/\sigma}\right].
\end{aligned}
$$

With this result and the single-peakedness of $V_{s>0}^{y,j}$ and $V_{s=0}^{y,j}$ over τ, we can conclude that V^f has a unique maximum with respect to τ over the whole range of τ. Specifically, V^f is maximized at $\tau = \arg\max V_{s>0}^f$ if $\arg\max V_{s>0}^f < \hat{\tau}^f$; it is maximized at $\tau = \arg\max V_{s=0}^f$ otherwise.

■

A.2 Proof of Lemma 1

From Panel (a) of Figure 1, a type-f agent becomes a decisive voter if and only if the following condition holds:

$$1 - 2\tilde{\tau}^{f,c2} \leq \frac{1+\alpha}{(\pi+1)\chi(\cdot)}.$$

This condition is rewritten as the one in Lemma 1(i). Otherwise, a type-$c2$ agent becomes a decisive voter (see Panel (b) of Figure 1).

■

A.3 Proof of Lemma 2

Suppose that $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \min\left\{1, \frac{1-\alpha\pi}{\alpha}\right\}\right]$ holds (see Panel (a) of Figure 2). A type-f agent becomes a decisive voter if $1 - 2\tilde{\tau}^{f,c1} \leq 1/(\pi+\gamma)\chi(\cdot)$ holds, that is, if $\chi(\cdot) \leq 2\left(\frac{\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\pi+\gamma}$ holds.

Next, assume $\frac{1-\alpha\pi}{\alpha} < 1$ (i.e., $1 < (1+\alpha)\pi$) and $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, \min\left\{1, \frac{1-\alpha\pi}{\alpha\varphi(1-\mu)}\right\}\right)$ hold (see Panel (b)). A type-f agent becomes a decisive voter if $1 - 2\tilde{\tau}^{c1,f} > \alpha/\chi(\cdot)$ holds, that is, if $\chi(\cdot) > 2(\alpha\beta)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + \alpha$ holds. When these conditions fail to hold, the decisive voter becomes a type-$c1$ agent.

The conditions demonstrated above imply that a single female agent is more likely to be a decisive voter for a lower α. To confirm this statement, let us rewrite the conditions in Lemma 2(i) as follows:

$$\chi(\cdot) \leq 2\left(\frac{\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{1}{\pi+\gamma}$$

$$\Leftrightarrow \frac{\{1-\varphi(1-\mu)\}\left[(\pi+\gamma)-(1-\gamma)/\alpha\right]}{\{(\pi+1)-(1-\gamma)\varphi(1-\mu)\}(\pi+\gamma)} \leq 2\left(\frac{\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{-1}{\sigma}}; \quad (8)$$

$$\chi(\cdot) > 2(\alpha\beta)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + \alpha$$

$$\Leftrightarrow \frac{1-\alpha\{\pi+\gamma\varphi(1-\mu))\}}{(\pi+1)-(1-\gamma)\varphi(1-\mu)} > 2(\alpha\beta)^{1/\sigma}\left(\frac{1}{\pi+\gamma}\right)^{(\sigma-1)/\sigma} \quad (9)$$

The left-hand side of (8) is increasing in α whereas the right-hand side of (8) is decreasing in α; (8) is more likely to hold for a lower α. The left-hand side of (9) is decreasing in α whereas the right-hand side of (9) is increasing in α; (9) is more likely to hold for a lower α.

■

A.4 Proof of Propositions 2-4

First, we focus on the terms $\alpha/\chi(\cdot), 1/(\pi+\gamma)\chi(\cdot)$ and $(1+\alpha)/(\pi+1)\chi(\cdot)$ that affect the marginal cost-to-benefit ratios of redistribution for type-$f, c1$ and $c2$ agents, respectively: see equations (4), (5) and (6). We denote these terms as:

$$\widetilde{RHS}^{f} \equiv \frac{\alpha}{\chi(\cdot)}, \widetilde{RHS}^{c1} \equiv \frac{1}{(\pi+\gamma)\chi(\cdot)}, \widetilde{RHS}^{c2} \equiv \frac{1+\alpha}{(\pi+1)\chi(\cdot)}.$$

After some calculation, we obtain the following properties of $\widetilde{RHS}^{j}$ ($j = f, c1, c2$):

$$\frac{\partial\widetilde{RHS}^{f}}{\partial\alpha} > 0, \frac{\partial\widetilde{RHS}^{c1}}{\partial\alpha} < 0, \frac{\partial\widetilde{RHS}^{c2}}{\partial\alpha} > 0; \tag{10}$$
$$\frac{\partial\widetilde{RHS}^{f}}{\partial\gamma} > 0, \frac{\partial\widetilde{RHS}^{c1}}{\partial\gamma} < 0, \frac{\partial\widetilde{RHS}^{c2}}{\partial\gamma} > 0; \tag{11}$$
$$\frac{\partial\widetilde{RHS}^{f}}{\partial\mu} \gtreqless 0, \frac{\partial\widetilde{RHS}^{c1}}{\partial\mu} \gtreqless 0, \frac{\partial\widetilde{RHS}^{c2}}{\partial\mu} \gtreqless 0 \text{ if and only if } \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}. \tag{12}$$

Consider the political equilibrium in an economy with $1/\sigma < 1$. The decisive voter in the current case is a type-f, type-$c1$ or type-$c2$ agent (Proposition 1). Because we here consider the effects of α, γ and μ around the critical value $\hat{\tau}^{j}$ defined in Section 3, we calculate the effects of these parameters on $\hat{\tau}^{j}$ as follows:

$$\frac{\partial\hat{\tau}^{f}}{\partial\alpha} > 0, \frac{\partial\hat{\tau}^{c1}}{\partial\alpha} < 0, \frac{\partial\hat{\tau}^{c2}}{\partial\alpha} > 0; \tag{13}$$
$$\frac{\partial\hat{\tau}^{f}}{\partial\gamma} > 0, \frac{\partial\hat{\tau}^{c1}}{\partial\gamma} < 0, \frac{\partial\hat{\tau}^{c2}}{\partial\gamma} > 0; \tag{14}$$
$$\frac{\partial\hat{\tau}^{f}}{\partial\mu} \gtreqless 0, \frac{\partial\hat{\tau}^{c1}}{\partial\mu} \gtreqless 0, \frac{\partial\hat{\tau}^{c2}}{\partial\mu} \gtreqless 0 \text{ if and only if } \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}. \tag{15}$$

A.4.1 The effect of α on the equilibrium tax rate: Proof of Proposition 2

Consider first the equilibrium where the decisive voter is a type-f agent. Suppose that α is initially given such that type-f's preferred tax rate is $\tau = \hat{\tau}^{f}$. We denote α^{f} as the α that makes a type-f young agent choose $\tau = \hat{\tau}^{f}$.

With the property of $\widetilde{RHS}^{f}$ in (10) and the property of $\hat{\tau}^{f}$ in (13), we find a positive real number $\varepsilon(> 0)$ around α^{f} such that the type-f agent is borrowing-constrained for $\alpha \in$

$(\alpha^f - \varepsilon, \alpha^f)$ and borrowing-unconstrained for $\alpha \in [\alpha^f, \alpha^f + \varepsilon)$. This statement is confirmed by looking at Panel (c) of Figure 3. For a low α, a preferred tax rate by a type-f agent is determined by the crossing point of $LHS \equiv 1 - 2\tau$ and $RHS^f = (\alpha/\chi(\cdot))^{1-\sigma} \cdot (\tau)^\sigma / \beta$, showing that she is borrowing constrained. For a high α, it is determined by the crossing point of $LHS \equiv 1 - 2\tau$ and $RHS^f = \alpha/\chi(\cdot)$, showing that she is borrowing unconstrained. Therefore, the equilibrium tax rate satisfies the following condition:

$$LHS \equiv 1 - 2\tau = RHS^f \equiv \begin{cases} \frac{1}{\beta}\left(\frac{\alpha}{\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{for} \quad \alpha \in (\alpha^f - \varepsilon, \alpha^f], \\ \frac{\alpha}{\chi(\cdot)} & \text{for} \quad \alpha \in \left(\alpha^f, \alpha^f + \varepsilon\right). \end{cases}$$

Given the properties in (10) and (13), we can illustrate the effects of an increase in α on RHS^f and $\hat{\tau}^f$ as in Panel (a) of Figure 3. The illustration leads to the following result:

$$\begin{array}{l} \frac{\partial \tau^f}{\partial \alpha} > 0 \quad \text{for} \quad \alpha \in (\alpha^f - \varepsilon, \alpha^f), \\ \frac{\partial \tau^f}{\partial \alpha} < 0 \quad \text{for} \quad \alpha \in \left(\alpha^f, \alpha^f + \varepsilon\right). \end{array}$$

The result shows that an increase in α locally produces an inverse U-shaped relationship between α and τ^f around $\tau = \hat{\tau}^f$.

The analysis and result apply to the equilibrium in which the decisive voter is a type-$c2$ agent because the effects of α on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ are qualitatively similar between the two types of agents, as demonstrated in (10) and (13).

Next, consider the equilibrium where the decisive voter is a type-$c1$ agent. Suppose that α is initially given such that type-$c1$'s preferred tax rate is $\tau = \hat{\tau}^{c1}$. We denote α^{c1} as the α that makes a type-$c1$ young agent choose $\tau = \hat{\tau}^{c1}$. Because the properties of $\widetilde{RHS}^{c1}$ in (10) and $\hat{\tau}^{c1}$ in (13) are opposite to those of $\widetilde{RHS}^f$ and $\hat{\tau}^f$, the saving pattern of type-$c1$ agent around α^{c1} is also opposite to that of type-f agents around α^f. That is, we find a positive real number $\varepsilon(>0)$ around α^{c1} such that the type-$c1$ agent is borrowing-unconstrained for $\alpha \in (\alpha^{c1} - \varepsilon, \alpha^{c1})$ and borrowing-constrained for $\alpha \in [\alpha^{c1}, \alpha^{c1} + \varepsilon)$. The equilibrium tax rate satisfies the following condition:

$$LHS \equiv 1 - 2\tau = RHS^{c1} \equiv \begin{cases} \frac{1}{(\pi+\gamma)\chi(\cdot)} & \text{for} \quad \alpha \in (\alpha^{c1} - \varepsilon, \alpha^{c1}); \\ \frac{1}{\beta}\left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{for} \quad \alpha \in [\alpha^{c1}, \alpha^{c1} + \varepsilon). \end{cases}$$

Given the properties in (11) and (14), we obtain the following result:

$$\begin{array}{l} \frac{\partial \tau^{c1}}{\partial \alpha} > 0 \quad \text{for} \quad \alpha \in (\alpha^{c1} - \varepsilon, \alpha^{c1}), \\ \frac{\partial \tau^{c1}}{\partial \alpha} < 0 \quad \text{for} \quad \alpha \in (\alpha^{c1}, \alpha^{c1} + \varepsilon). \end{array}$$

The result shows that an increase in α locally produces an inverse U-shaped relationship between α and τ^{c1} around $\tau = \hat{\tau}^{c1}$.

A.4.2 The effect of γ on the equilibrium tax rate: Proof of Proposition 3

Suppose that the decisive voter is a type-j ($j = f, c1, c2$) agent. Suppose that γ is initially given such that type-j's preferred tax rate is $\tau = \hat{\tau}^j$. We denote γ^j as the γ that makes a type-j agent choose $\tau = \hat{\tau}^j$.

Under the abovementioned situation, suppose that an *increase* in γ around γ^j locally produces an inverse U-shaped relationship between γ and type-j's preferred tax rate. This assumption implies that a *decrease* in γ around γ^j also locally produces an inverse U-shaped relationship between γ and type-j's preferred tax rate. Therefore, it is sufficient to show the effect of an increase in γ on the preferred tax rate by the decisive voter.

The analysis of the effect of α applies to the current analysis because the effects of γ on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ are qualitatively similar to those of α on $\widetilde{RHS}^j$ and $\hat{\tau}^j$. Therefore, we obtain the result described in Proposition 3.

A.4.3 The effect of μ on the equilibrium tax rate: Proof of Proposition 4

Suppose that $\gamma < (1-\alpha\pi)/(1+\alpha)$ holds. The decisive voter is a type-f or type-$c2$ agent (Lemma 1). The effects of μ on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ ($j = f, c2$) are qualitatively similar to those of α on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ ($j = f, c2$). We can apply the analysis and result in Proposition 2 to the current case.

Next, suppose that $\gamma > (1-\alpha\pi)/(1+\alpha)$ holds. The decisive voter is a type-f or a type-$c1$ agent (Lemmas 2 and 3). The effects of μ on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ ($j = f, c1$) are qualitatively similar to those of α on $\widetilde{RHS}^{c1}$ and $\hat{\tau}^{c1}$. We can apply the analysis and result in Proposition 2 to the current case.

Finally, suppose that $\gamma = (1-\alpha\pi)/(1+\alpha)$ holds. The parameter μ has no effect on $\widetilde{RHS}^j$ and $\hat{\tau}^j$. A change in μ has no effect on the equilibrium tax rate.

■

A.5 The Case of a High Intertemporal Elasticity of Substitution: $1/\sigma \geq 1$

Figure 4 illustrates the conditions that determine the preferred tax rates by agents who prefer taxation for the case of $1/\sigma \geq 1$. From the observation in Figure 4, we can conclude that the decisive voter is a single female ($j = f$) agent if $\gamma \leq (1-\alpha\pi)/\alpha$ (see panels (a) and (b)); this voter is an agent who belongs to a one-breadwinner couple ($j = c1$) if $\gamma > (1-\alpha\pi)/\alpha$ (see panels (c) and (d)).

[Figure 4 about here.]

Proposition A1. *Suppose that $1/\sigma \geq 1$ holds. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(i) *a type-f, single female agent if $\gamma \leq \frac{1-\alpha\pi}{\alpha}$;*

(ii) *a type-$c1$ agent who belongs to a one-breadwinner couple otherwise.*

The result established in Proposition A1 has the following two features. First, an agent who belongs to a two-breadwinner couple cannot become a decisive voter. Such agent's marginal cost-to-benefit ratio of redistribution in terms of utility is always higher than the ratios of the other two types of households. This result implies that two-breadwinner couples prefer a lower tax rate than do other two types of young agents.

Second, which household becomes a decisive voter depends on α, π and γ that represent the gender wage gap, life expectancy of men, and the fraction of derived pension rights, respectively. Suppose that the gender wage gap is high (i.e., α is low), the life expectancy of men (π) is low, and the level of derived pension rights (γ) is low such that $\gamma \leq (1-\alpha\pi)/\alpha$. Then, the marginal cost-to-benefit ratio of redistribution in terms of utility for single females is lower than that for one-breadwinner couples because the former owe less tax burden whereas the latter receive lower pension benefits. Therefore, single females prefer a higher tax rate than do one-breadwinner couples and thus become decisive voters if $\gamma \leq (1-\alpha\pi)/\alpha$.

A.5.1 Comparative Statics Analysis

Consider the political equilibrium in an economy with $1/\sigma \geq 1$. Suppose that $\gamma \leq (1-\alpha\pi)/\alpha$ holds: the decisive voter is a type-f agent (Proposition A1). The optimality condition for a type-f agent, given by (4), indicates that a higher $\widetilde{RHS}^f$ results in a lower preferred tax rate except for the case of $1/\sigma = 1$ and $s^f = 0$:

$$\begin{cases} \frac{\partial \tau^f}{\partial \widetilde{RHS}^f} = 0 \text{ if } 1/\sigma = 1 \text{ and } s^f = 0, \\ \frac{\partial \tau^f}{\partial \widetilde{RHS}^f} < 0 \text{ otherwise.} \end{cases}$$

With the property of $\widetilde{RHS}^f$ in (10) - (12), we obtain the following result:

$$\frac{\partial \tau^f}{\partial \alpha} \leq 0, \frac{\partial \tau^f}{\partial \gamma} \leq 0,$$

and

$$\frac{\partial \tau^f}{\partial \mu} \lesseqgtr 0 \Leftrightarrow \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}.$$

Next, suppose that $\gamma > (1-\alpha\pi)/\alpha$ holds: the decisive voter is a type-$c1$ agent (Proposition A1). The optimality condition for a type-$c1$ agent, given by (5), indicates that a higher $\widetilde{RHS}^{c1}$ results in a lower preferred tax rate except the case of $1/\sigma = 1$ and $s^{c1} = 0$:

$$\begin{cases} \frac{\partial \tau^{c1}}{\partial \widetilde{RHS}^{c1}} = 0 \text{ if } 1/\sigma = 1 \text{ and } s^{c1} = 0, \\ \frac{\partial \tau^{c1}}{\partial \widetilde{RHS}^{c1}} < 0 \text{ otherwise.} \end{cases}$$

With the property of $\widetilde{RHS}^{c1}$ in (10) - (12) and the assumption of $\gamma > (1 - \alpha\pi)/\alpha$, we obtain the following result:

$$\frac{\partial \tau^{c1}}{\partial \alpha} \geq 0, \frac{\partial \tau^{c1}}{\partial \gamma} \geq 0, \frac{\partial \tau^{c1}}{\partial \mu} \geq 0.$$

The result established in this appendix indicates monotone effects of the parameters α, γ and μ on the equilibrium tax rate.

■

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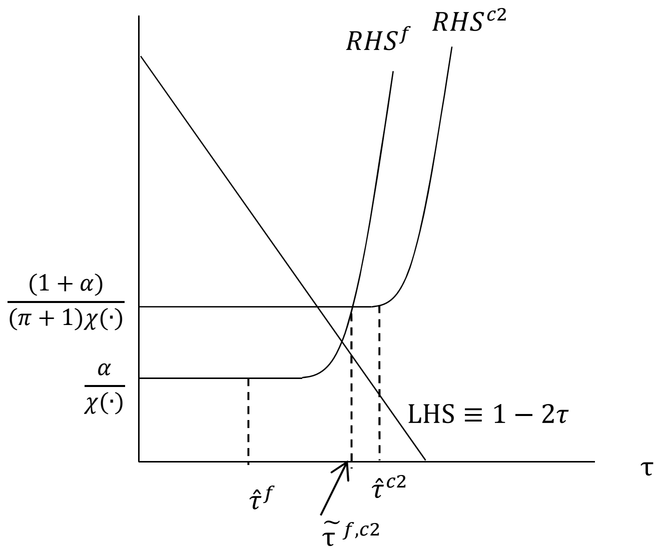
Panel (a)



Panel (b)

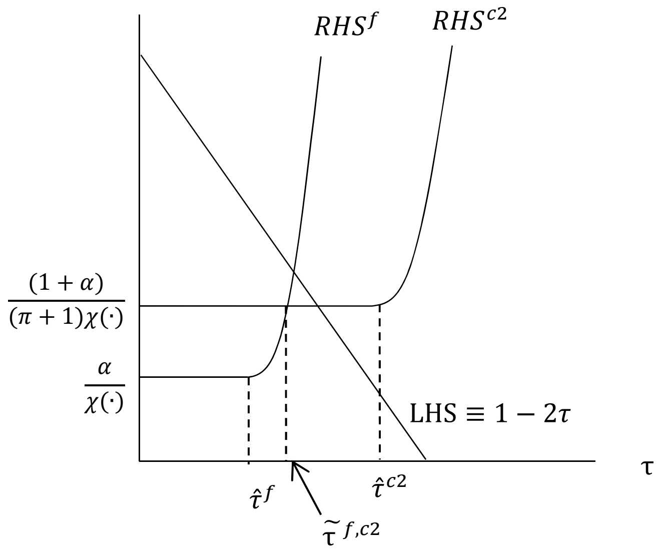
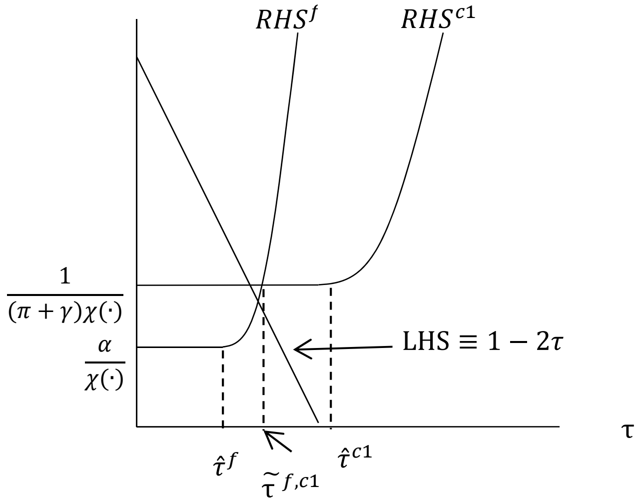
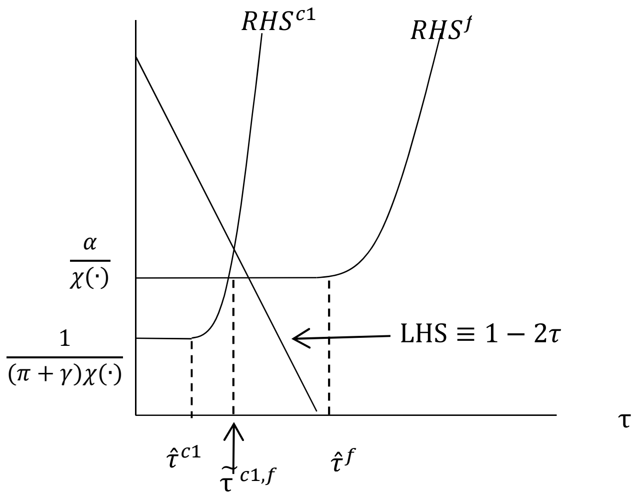


Figure 1. The figure illustrates the case of $\gamma \in [(0, (1-\alpha\pi)/(1+\alpha)]$. In Panel (a), a type-f agent is a decisive voter; in Panel (b), a type-c2 agent is a decisive voter.

Panel (a)



Panel (b)



Panel (c)

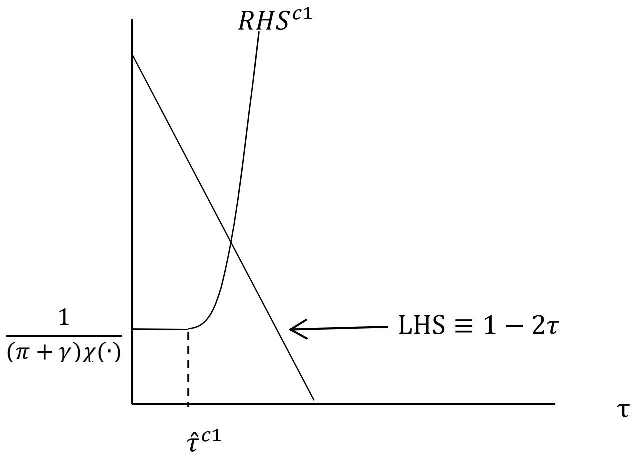


Figure 2. Panels (a), (b) and (c) illustrate cases of $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, min\left\{1, \frac{1-\alpha\pi}{\alpha}\right\}\right]$, $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, min\left\{\frac{1-\alpha\pi}{\alpha\varphi(1-\mu)}, 1\right\}\right)$ and $\gamma \in \left[\frac{1-\alpha\pi}{\alpha\varphi(1-\mu)}, 1\right]$, respectively.

Panel (a)

Panel (b)

Panel (c)

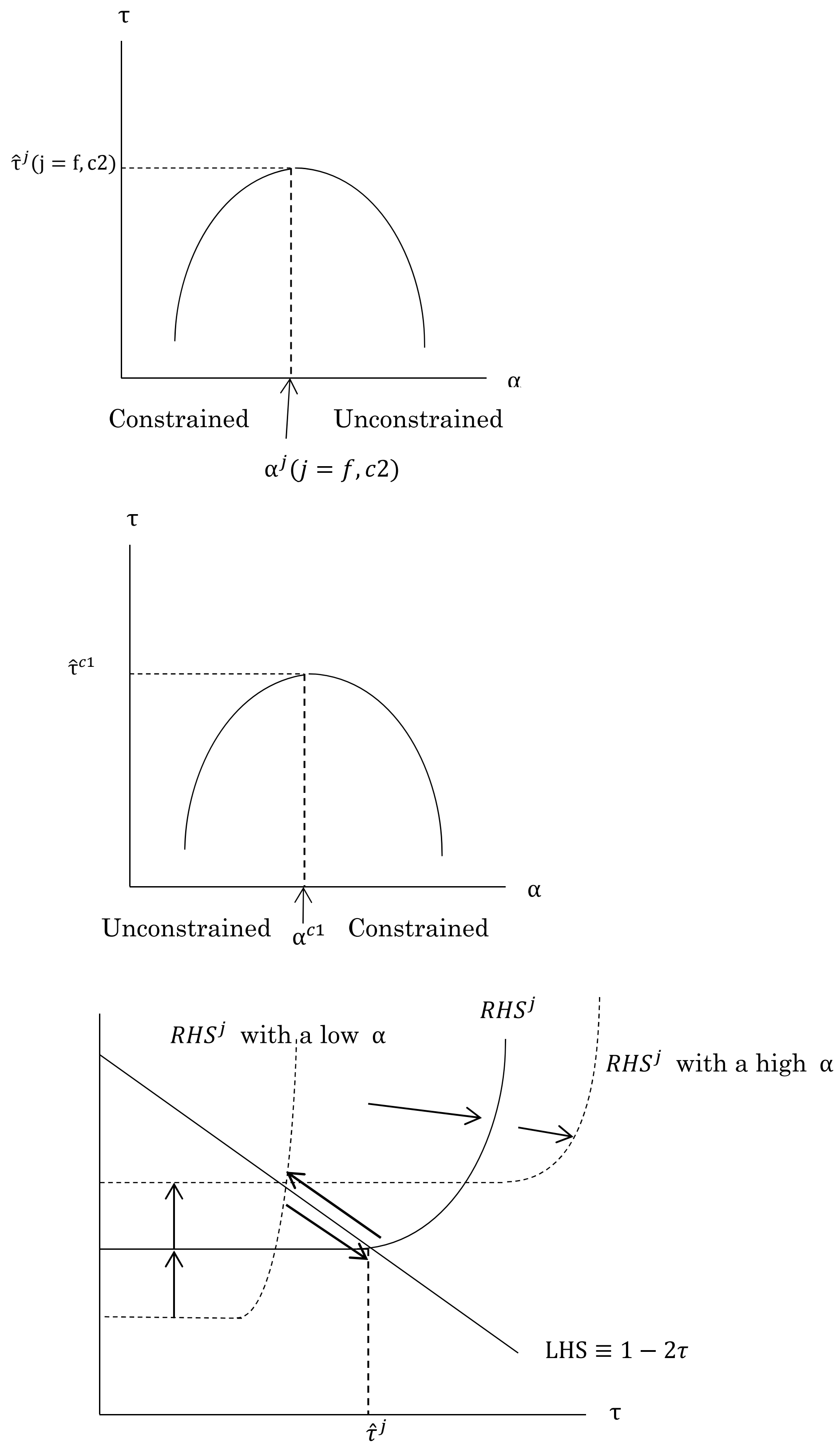


Figure 3. Panel (a) illustrates the saving pattern and the corresponding preferred tax rate by a type-j = f, c2 agent; Panel (b) illustrates the saving pattern and the corresponding preferred tax rate by a type-c1 agent; Panel (c) illustrates changes in the preferred tax rate by a type-j agent in response to an increase in α.

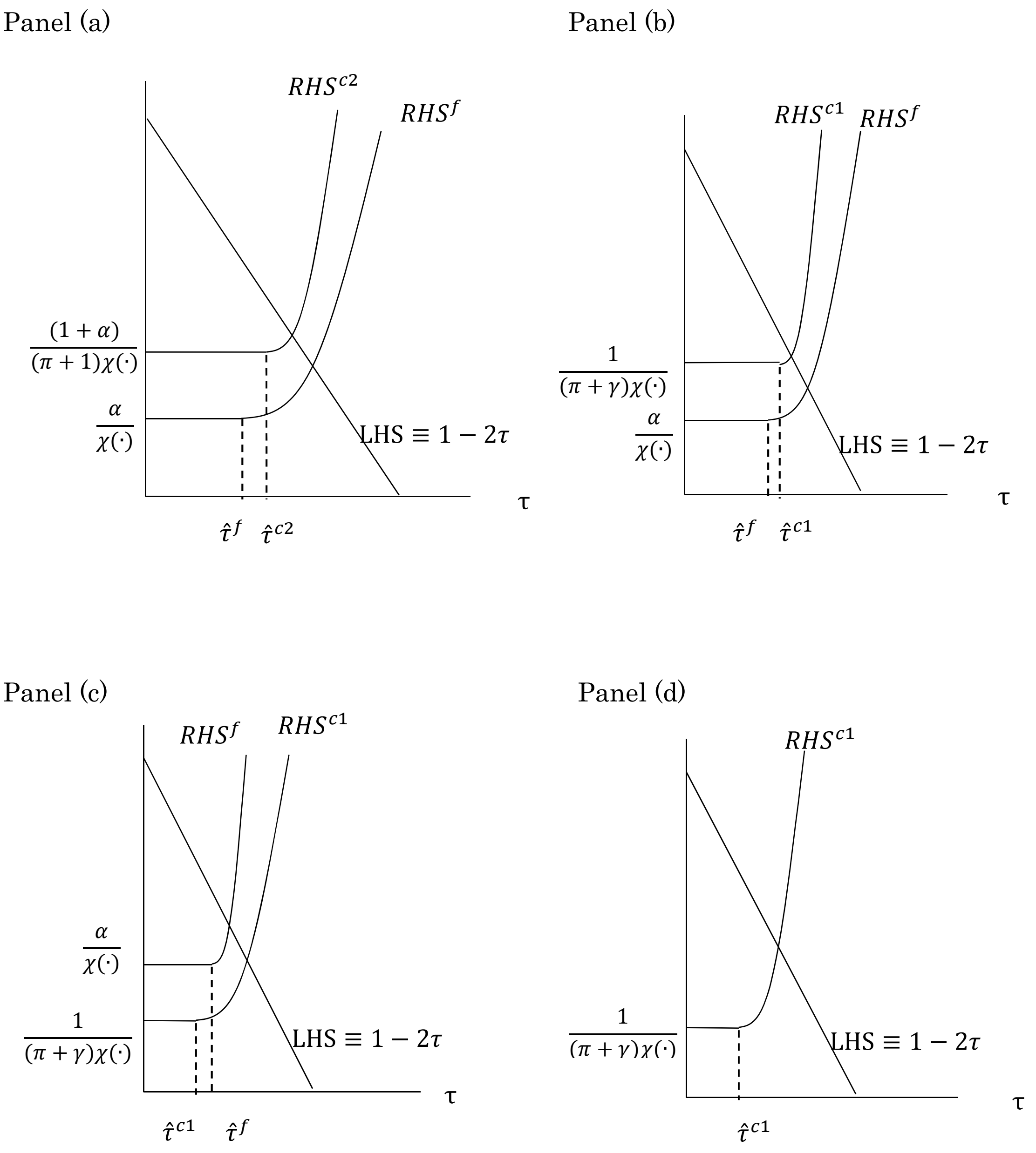


Figure 4. Panel (a) is the case of $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$; Panel (b) is the case of $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}\right]$; Panel (c) is the case of $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, \frac{1-\alpha\pi}{\alpha\varphi(1-\mu)}\right)$; and Panel (d) is the case of $\gamma \in \left[\frac{1-\alpha\pi}{\alpha\varphi(1-\mu)}, 1\right]$.