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Dynamic analysis of reductions in public debt in an endogenous growth model with public capital*

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Abstract

We construct an endogenous growth model that includes productive public capital and government debt. We assume that the government debt-to-GDP ratio is gradually adjusted to a target level, reflecting the permanent commitment rules in the Stability and Growth Pact or the Maastricht Treaty in the EU (i.e., the well-known 60% rule). These rules affect government borrowing and public investment. Here, we examine the welfare implications of the permanent commitment rules. We find that fiscal consolidation based on the rules improves social welfare. Moreover, the improvement in welfare accelerates as fiscal consolidation progresses more rapidly. Lastly, we also discuss and derive the optimal long-run debt-to-GDP ratio.

JEL classification: E62, H54, H63

Keywords: Fiscal consolidation; Debt policy rule; Public capital; Welfare; Endogenous growth

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1 Introduction

Government debt has increased in the EU countries since the onset of the 2008–2009 financial crisis owing to discretionary fiscal expansion. For example, in Greece, public debt as a share of GDP equaled 166.1% in 2012, and the country has subsequently suffered severe government financial failure. The debt-to-GDP ratios in Italy, Ireland, and Portugal also exceeded 100% in 2012. Since then, many EU member countries have increased their efforts to implement fiscal consolidation.

To reduce the risk of such government financial failure, the EU has implemented the Maastricht Treaty, which set two types of fiscal rules that work as permanent constraints on fiscal policies. The Maastricht Treaty states that EU member states must keep their government *deficit-to-GDP ratio* below 3% and their *debt-to-GDP ratio* below 60%. The debt reduction benchmark (rule) was introduced by the reform of the Stability and Growth Pact (SGP), the so-called Six-Pack, in December 2011. The rule states that member states whose current debt-to-GDP ratio is above the 60% threshold must reduce their ratios to 60% by an average rate of one-twentieth per year. We study these fiscal rules from a theoretical perspective, focusing in particular on the *debt/GDP rules*.

These fiscal rules on government borrowing are related to public investment policies, because government borrowing is linked directly to public investment, which affects economic growth. The IMF (2014) suggests that investment in productive public capital is important in countries with a high degree of public investment efficiency, because the positive effects of such investments on potential growth are substantially stronger. Bom and Ligthart (2014) estimate the average output elasticity of public capital, which suggests that public capital is undersupplied in OECD economies. Indeed, according to plans set out in the Stability and Convergence Programs (SCPs) in the EU, which were submitted to the Commission and Council in Spring 2012, the member countries are planning further fiscal consolidation based on expenditure cuts, including reductions in public investment. Thus, it is important that we study how to reduce government debt and create the fiscal space for public investment.

Many recent studies consider permanent fiscal rules and public investment using endogenous growth models. For example, Greiner and Semmler (2000) investigate the long-run growth effects of public investment policies under *deficit/GDP rules*. Ghosh and Mourmouras (2004) extend the Greiner and Semmler framework into welfare analysis. In contrast, Futagami, Iwaisako, and Ohdoi (2008) and Minea and Villieu (2013) focus on *debt/GDP rules*, under which the government debt relative to the size of the economy is adjusted gradually to a target level in the long run. These authors show that permanent fiscal rules may be important determinants of long-run economic growth and welfare, as emphasized by Barro (1990).[1]

Because we focus on the *debt/GDP rule*, the works of Futagami et al. (2008) and Minea and Villieu (2013) are more relevant to our study. These authors successfully provide policy implications of the *debt/GDP rule*. However, there is still room for discussion. First, they focus on the current *flow* of public services rather than the *stock* of public capital or its

[1]There is considerable literature on how fiscal debt policies affect macroeconomic stability, growth, and welfare in an endogenous growth model. See Bruce and Turnovsky (1999), Greiner (2007, 2010, 2012a, 2012b), Yakita (2008b), Arai (2011), Kamiguchi and Tamai (2012), and Teles and Mussolini (2014).

accumulation.[2] Second, they do not derive and discuss an optimal debt-to-GDP ratio. Third, they do not investigate the welfare effects of debt reduction. Thus, the present study addresses the following: (i) the effects of the *debt/GDP rule* on the accumulation of public capital; (ii) the optimal long-term debt-to-GDP ratio; and (iii) the welfare effects of debt reduction, including transition dynamics. Further, we pay attention to the *pace* of debt reduction (the *timeline* of debt reduction) because the timeline of a debt adjustment affects both government borrowing and public investment.

For our purpose, we construct an endogenous growth model that includes public debt finance, in line with the aforementioned literature. In our model, the growth engine is productive public capital, as in Futagami et al. (1993) and Turnovsky (1997). Public capital accumulates through public investment financed by issuing bonds and taxes on capital income, labor income, and household consumption. For a government to reduce its debt to a target level, it must cut spending or increase tax rates. We focus mainly on the former case, because fiscal consolidation based only on spending cuts is discussed widely in the context of many European countries.[3] However, we do also briefly examine the latter case because fiscal consolidation in Spain, Austria, Cyprus, France, Malta, and Romania is relatively evenly balanced between these two methods.

First, we show that a steady state that is locally saddle stable exists. In contrast to the models in Futagami et al. (2008) and Minea and Villieu (2013), our model includes the possibilities that growth rates become negative and that the no-Ponzi game condition of the government breaks because we consider the *stock* of public capital. Thus, we derive the necessary and sufficient condition for strictly positive long-run growth.

Second, we derive the optimal long-term debt-to-GDP ratio. This ratio depends on the tax rates on wage income and consumption, as well as the share of public investment in total government spending. Furthermore, we find that the target debt ratio set by the SGP and Maastricht Treaty, namely 60%, might be much higher than the optimal level.

Finally, we investigate the welfare effects of debt reduction by considering the transition dynamics. For these analyses, we calibrate the model to the data of Greece as an example of a country with a very high debt-to-GDP ratio. In the benchmark case, where debt reduction is based on expenditure cuts only, reducing the debt-to-GDP ratio to 60% improves social welfare. This is because debt reduction not only releases resources to private consumption in the short run, but also creates fiscal space to increase public investment in the long run. In addition, we suggest two policy implications. First, the improvement in welfare increases as the pace of debt reduction increases. Second, lowering the target debt-to-GDP ratio from 60% to the optimal level increases the improvement in welfare.

Then, we also examine the welfare effects of debt reduction based on tax increases and expenditure cuts. In this case, welfare does not always improve. In addition, even when welfare does improve, the welfare gains are lower than those under expenditure cuts only.

[2] Fisher and Turnovsky (1998) note that "*... it is open to the criticism that insofar as productive government expenditures are intended to represent public infrastructure, such as roads and education, it is the accumulated stock, rather than the current flow, that is relevant.*".

[3] The European Commission proposes using spending cuts rather than tax increases for fiscal consolidation because past evidence indicates that expenditure-based consolidation (spending cuts) tends to have greater success. According to *Public finances in EMU – 2012*, on average, the SCPs of both the Euro area and the EU27 are based primarily on spending cuts. In addition, Greece, whose debt-to-GDP ratio is the highest in the EU, announced deep spending cuts in their 2011 budget, under IMF and EU supervision.

This is because although the increases in taxes mitigate the decrease in public investment in the short run, the negative welfare effects caused by the tax increases become stronger, mainly owing to their negative effects on savings and consumption growth.

Some fiscal consolidation strategies are discussed in exogenous growth models, for example, Coenen et al. (2008), Forni et al. (2010), Papageorgiou (2012), Bi et al. (2013), Cogan et al. (2013), and Erceg and Lindé (2013). These authors focus mainly on the effects of fiscal consolidation on transitional dynamics. Then, less normative welfare analyses of the permanent debt policy rule have been conducted. Our study differs from these prior studies in the following ways. We consider the long-run growth effects of public investment that are linked to the permanent debt policy rule. This means that fiscal consolidation is part of a long-run objective to balance government borrowing and to create the fiscal space for positive expenditure on long-run growth. More importantly, we investigate welfare analyses with the permanent debt policy rule, and derive and discuss the long-run optimal debt-to-GDP ratio. Here, we focus on the pace at which consolidation should be implemented.

The remainder of this paper is organized as follows. Section 2 presents the basic model. Section 3 derives the equilibria. Section 4 examines the long-run optimal debt-to-GDP ratio. Section 5 presents the welfare effects of expenditure-based consolidation. Section 6 introduces an increase in taxes, along with expenditure cuts. Section 7 modifies the model settings and checks the robustness of the results. Section 8 concludes the paper.

2 Model

Our model is based on those of Futagami et al. (2008) and Minea and Villieu (2013). Our economy is populated by infinitely long-lived representative households who have an infinite planning horizon and perfect foresight. Time is continuous and denoted as $t \geq 0$. We assume there is no population growth and that the population size is normalized to one, as in Futagami et al. (2008) and Minea and Villieu (2013).

2.1 Production Structure

There is a continuum of competitive firms whose size is normalized to one. Firm j produces a single final good using the production technology given by $Y_{j,t} = AK_{j,t}^{\alpha}(h_t L_{j,t})^{1-\alpha}$ ($0 < \alpha < 1$), where $Y_{j,t}$, $K_{j,t}$, and $L_{j,t}$ represent the output level, private capital, and labor input of firm j, respectively. In addition, h_t represents the labor productivity at time t. Through profit maximization, factor prices become equal to the marginal products: $R_t = \alpha A(K_{j,t}/L_{j,t})^{\alpha-1}h_t^{1-\alpha}$ and $w_t = (1-\alpha)A(K_{j,t}/L_{j,t})^{\alpha}h_t^{1-\alpha}$, where R_t and w_t denote the rental price of capital and wage rates, respectively.

Following Kalaitzidakis and Kalyvitis (2004) and Yakita (2008a), we assume that the aggregate private capital, $K_t = \sum_j K_{j,t}$, and public capital, $K_{g,t}$, have positive external effects on labor productivity, and specify $h_t = K_t^{1-\epsilon}K_{g,t}^{\epsilon}$ ($\epsilon \in (0,1)$). Since $K_{j,t}/L_{j,t} = K_t/L_t$ and $L_t = \sum_j L_{j,t}$ hold in equilibrium, the aggregate output and factor prices in period t are written, respectively, as

$$Y_t = Ak_{g,t}^{\beta}K_t, \tag{1a}$$

$$R_t = \alpha Ak_{g,t}^{\beta}, \tag{1b}$$

$$w_t = (1-\alpha)Ak_{g,t}^{\beta}K_t, \tag{1c}$$

where $\beta \equiv \epsilon(1-\alpha)$ and $k_{g,t}(\equiv K_{g,t}/K_t)$ is the ratio of public capital to private capital. Since we have $L_t = 1$ in equilibrium (as shown later), we omit L_t in the above equations.

2.2 Households

The utility function of a representative household is specified as

$$U_0 = \int_0^{\infty} \frac{C_t^{1-\sigma}}{1-\sigma} e^{-\rho t} dt, \tag{2}$$

where C_t is the household's consumption at time t, and $\rho(>0)$ and $1/\sigma$ denote the subjective discount rate and intertemporal elasticity of substitution, respectively. Since most empirical evidence suggests that the intertemporal elasticity of substitution is relatively small, we assume that $\sigma > 1$. The household inelastically supplies one unit of labor. The household's budget constraint is given by

$$\dot{W}_t = (1-\tau_{r,t})r_t W_t + (1-\tau_{w,t})w_t - (1+\tau_{c,t})C_t, \tag{3}$$

where W_t denotes assets, r_t is the interest rate, $\tau_{c,t}(\geq 0)$ is the consumption tax rate, and $\tau_{r,t}(\tau_{w,t}) \in [0,1)$ is the interest (labor) income tax rate. Taking r_t, w_t, $\tau_{r,t}$, $\tau_{w,t}$, and $\tau_{c,t}$ as given, the household maximizes (2) subject to (3), which yields the Euler equation and the transversality condition:

$$\dot{C}_t = \frac{1}{\sigma}\left[(1-\tau_{r,t})r_t - \rho - \frac{\dot{\tau}_{c,t}}{1+\tau_{c,t}}\right]C_t, \tag{4a}$$

$$\lim_{t\to\infty} C_t^{-\sigma}W_t \exp(-\rho t) = 0. \tag{4b}$$

2.3 Government

The government imposes taxes on income and consumption, and issues bonds, B_t, to finance public expenditure, G_t. We restrict our attention to the case of $G_t > 0$. The budget constraint of the government is

$$\dot{B}_t = r_t B_t + G_t - (\tau_{r,t}r_t W_t + \tau_{w,t}w_t + \tau_{c,t}C_t), \tag{5}$$

where B_t represents the government's outstanding debt and $\dot{B}_t$ denotes newly issued government bonds. The government constraint must satisfy the no-Ponzi game condition, $\lim_{t\to\infty} B_t \exp[-\int_0^t r_v dv] = 0$.[4] A constant fraction, $\theta \in (0,1)$, of G_t is used for public capital investment, $I_{g,t}$, that is, $\theta G_t = I_{g,t}$. The evolution of public capital is given by

$$\dot{K}_{g,t} = I_g - \delta_g K_{g,t} = \theta G_t - \delta_g K_{g,t}, \tag{6}$$

[4] It is well known that this no-Ponzi game condition of the government becomes equivalent to the household's transversality condition in the steady state.

where $\delta_g \in (0,1)$ is the depreciation of public capital. In Section 7, we briefly discuss the case in which households derive their utility from public services, $C_{g,t} \equiv (1-\theta)G_t$, which is not considered in the benchmark model.

Following Futagami et al. (2008) and Minea and Villieu (2013), we assume that the government adjusts its bonds gradually to the target level. Let us define $b_t \equiv B_t/Y_t$ and assume that the government adjusts b_t according to the following rule:

$$\dot{b}_t = -\phi(b_t - \bar{b}), \tag{7}$$

where $\bar{b}$ and $\phi(>0)$ denote the target level of government bonds and the adjustment coefficient of the rule, respectively. We consider the case of $\bar{b} > 0$. If b_t is larger than $\bar{b}$, the government has to reduce its debt according to the difference between the current and target levels of b: $b_t - \bar{b}$. If the adjustment coefficient, ϕ, takes a large (small) value, the government adjusts b_t to the target level at a fast (slow) pace. This policy rule indicates the long-run commitment of the SGP debt reduction benchmark, reformed in 2011, under which member states whose current debt-to-GDP ratio is above the 60% threshold must reduce their ratio 60% (see the second paragraph of the introduction).[5] Our investigation focuses on the rule that the SGP expects.

The benchmark model considers fiscal consolidation based on expenditure cuts, and the government sets constant tax rates, $\tau_{r,t} = \tau_r$, $\tau_{w,t} = \tau_w$, and $\tau_{c,t} = \tau_c$. Then, g_t varies so that it satisfies (5) and (7). This is because, according to *Public finances in EMU – 2012*, the consolidations set out in the SCPs for both the Euro area and the EU27 are based primarily on expenditure cuts. In contrast, in Spain, Austria, Cyprus, France, Malta, and Romania, fiscal consolidation is relatively evenly balanced between spending cuts and tax increases, and is primarily tax-based in Belgium and Italy. Therefore, we examine fiscal consolidation based on both expenditure cuts and tax increases in Section 6.

3 Equilibrium

3.1 Dynamic System

The labor market equilibrium condition is $L_t = 1$. The asset market clears as $W_t = K_t + B_t$. Let us assume that private capital depreciates at the rate $\delta_k \in (0,1)$, where the interest rate is $r_t = R_t - \delta_k$.[6] Substituting these into (5) and using (1a), (1b), and (1c), we obtain

$$\dot{B}_t = (1-\tau_r)(R_t - \delta_k)B_t - (\tilde{\tau}Y_t + \tau_c C - G_t) + \tau_r \delta_k K_t, \tag{8}$$

where $\tilde{\tau} \equiv \alpha\tau_r + (1-\alpha)\tau_w$. The goods market equilibrium condition is given by

$$\dot{K}_t = Y_t - C_t - G_t - \delta_k K_t. \tag{9}$$

[5] Such a policy rule would not expect uncertain fiscal policy behavior such as policy regime changes. However, there may be some discussion on uncertain fiscal consolidation, as studied by Bi et al. (2013). There may also be other discussions on the rule. For example, recent high-debt countries that are in the process of reducing their debt can have difficulty obeying the rule in the short run because of the adverse effects that contractionary fiscal policies and the increase in the interest payment have on GDP.

[6] This is because, by the no-arbitrage argument in the asset market, government bonds have the same rate of return as capital.

Let us define $c_t \equiv C_t/K_t$ and $g_t \equiv G_t/K_t$. The definition of b_t implies $\dot{b}_t = \dot{B}_t/Y_t - (\dot{Y}_t/Y_t)b_t$. Substituting (1a), (1b), (7), and (8) into this expression, we obtain

$$\dot{b}_t = -\phi(b_t - \bar{b}) = \left[(1-\tau_r)\left\{\alpha A k_{g,t}^{\beta} - \delta_k\right\} - \frac{\dot{Y}_t}{Y_t}\right] b_t - \tilde{\tau} - \tau_c \frac{c_t}{A k_{g,t}^{\beta}} + \frac{g_t}{A k_{g,t}^{\beta}} + \frac{\tau_r \delta_k}{A k_{g,t}^{\beta}}. \quad (10)$$

Using (1a), (6), and (9), we obtain

$$\dot{k}_{g,t} = (\theta + k_{g,t})g_t - A k_{g,t}^{1+\beta} + c_t k_{g,t} + (\delta_k - \delta_g)k_{g,t}. \quad (11)$$

From $Y_t = A k_{g,t}^{\beta} K_t$, (9), and (11), we derive the GDP growth rate as

$$\frac{\dot{Y}_t}{Y_t} = \beta \frac{\dot{k}_{g,t}}{k_{g,t}} + A k_{g,t}^{\beta} - c_t - g_t - \delta_k = (1-\beta)(A k_{g,t}^{\beta} - c_t - \delta_k) - [(1-\beta)k_{g,t} - \theta\beta]\frac{g_t}{k_{g,t}} - \beta\delta_g. \quad (12)$$

Substituting (12) into (10), and solving for g_t, we obtain

$$\begin{aligned} g_t &= \frac{\Gamma(b_t, c_t, k_{g,t}) - \alpha b_t (A k_{g,t}^{\beta})^2 - \phi(b_t - \bar{b}) A k_{g,t}^{\beta} + (1-\beta)(A k_{g,t}^{\beta} - c_t) A k_{g,t}^{\beta} b_t - \beta(\delta_g - \delta_k) A k_{g,t}^{\beta} b_t}{1 + [(1-\beta)k_{g,t} - \theta\beta]\, b_t A k_{g,t}^{\beta-1}} \\ &\equiv g(b_t, c_t, k_{g,t}), \end{aligned} \quad (13)$$

where $\Gamma(b_t, c_t, k_{g,t}) \equiv [\tilde{\tau} + \tau_r(\alpha A k_{g,t}^{\beta} - \delta_k)b_t] A k_{g,t}^{\beta} + \tau_c c_t - \tau_r \delta_k$. The numerator on the right-hand side (RHS) of (13) represents the components of g_t. The first term $\Gamma(b_t, c_t, k_{g,t})$ is the tax revenue of the government. The second term represents the interest payment on government debt. The third term shows that, given b_t, if the government reduces $\bar{b}$, public spending must fall in the short run. When ϕ is larger, the short-run decreases in g_t are also larger because the government must reduce its debt more rapidly, and hence its budget becomes tight. We call this effect of ϕ *the timeline effect*. The fourth term implies that a high growth of Y_t has a positive effect on public spending.[7] The last is the term related to depreciation.

Substituting $\dot{\tau}_c = 0$ and (1b) into (4a), and using (1a), (9), and (13), we obtain

$$\dot{c}_t = \left[c_t + g(b_t, c_t, k_{g,t}) - \left\{1 - \frac{1}{\sigma}(1-\tau_r)\alpha\right\} A k_{g,t}^{\beta} - \frac{\rho}{\sigma} + \left\{1 - \frac{1-\tau_r}{\sigma}\right\}\delta_k\right] c_t. \quad (14)$$

Using (1a), (6), (9), and (13), we then have

$$\dot{k}_{g,t} = (\theta + k_{g,t}) g(b_t, c_t, k_{g,t}) - A k_{g,t}^{1+\beta} + c_t k_{g,t} + (\delta_k - \delta_g)k_{g,t}. \quad (15)$$

Equations (7), (14), and (15), together with the initial values, $k_{g,0}$ and b_0, and the transversality condition, (4b), characterize the dynamics of the economy.

[7]The goods market equilibrium condition, (9), shows that, given G_t, when $A k_{g,t}^{\beta} - c_t$ becomes large, the growth rate of private capital rises. When K_t grows at a high rate, Y_t tends to grow more quickly. In this case, the government requires relatively little effort to reduce $b_t \equiv B_t/Y_t$ to $\bar{b}$. Then, a large increase in Y_t enables the government to increase g_t.

3.2 Steady-State Equilibrium

This subsection derives the steady-state equilibrium in which c_t, $k_{g,t}$, and b_t become constant over time. We omit the time index, t, from the variables that become constant over time in the steady state. Setting $\dot{c}_t = 0$, $\dot{k}_{g,t} = 0$, and $b_t = \bar{b}$ in (14) and (15), we obtain

$$c^* + g^* = \left[1 - \frac{1}{\sigma}(1-\tau_r)\alpha\right] A(k_g^*)^\beta + \frac{\rho}{\sigma} - \left[1 - \frac{1-\tau_r}{\sigma}\right]\delta_k \tag{16}$$

$$c^* = A(k_g^*)^\beta - \left(1 + \frac{\theta}{k_g^*}\right) g^* - (\delta_k - \delta_g). \tag{17}$$

We use an asterisk to represent a steady-state variable. From (16) and (17), we obtain

$$g^* = \frac{(1-\tau_r)\alpha A(k_g^*)^{1+\beta} - [\rho + (1-\tau_r)\delta_k - \sigma\delta_g]k_g^*}{\sigma\theta}. \tag{18}$$

From (1b) and (4a), we derive the long-run growth rate, $\gamma^* = \dot{C}_t/C_t = \dot{K}_t/K_t = \dot{K}_{g,t}/K_{g,t}$:

$$\gamma^* = \frac{1}{\sigma}\left[(1-\tau_r)\left\{\alpha A(k_g^*)^\beta - \delta_k\right\} - \rho\right]. \tag{19}$$

Then, we have $\gamma^* > 0$ if and only if $k_g^* > [\{\rho + (1-\tau_r)\delta_k\}/\{(1-\tau_r)\alpha A\}]^{\frac{1}{\beta}} \equiv k_{g,\gamma}$.

Using (17), (19), $\gamma^* = \dot{K}_t/K_t = \dot{K}_{g,t}/K_{g,t}$, and $b_t = \bar{b}$, we can rearrange (10) as

$$\sigma\left[1 + \tau_c\left(1 + \frac{\theta}{k_g^*}\right)\right] g^* = \omega(k_g^*)A(k_g^*)^\beta - \sigma(\delta_k - \delta_g)\tau_c - \sigma\tau_r\delta_k, \tag{20}$$

where $\omega(k_g^*) \equiv (1-\sigma)(1-\tau_r)\alpha\bar{b}A(k_g^*)^\beta + \sigma(\tilde{\tau} + \tau_c) - [(1-\sigma)(1-\tau_r)\delta_k + \rho]\bar{b}$. Substituting (18) into (20), we obtain

$$\theta\left[\Omega(k_g)Ak_g^\beta + \zeta\right] = (1+\tau_c)k_g\left[(1-\tau_r)\alpha Ak_g^\beta - \{\rho + (1-\tau_r)\delta_k\} + \sigma\delta_g\right], \tag{21}$$

where $\Omega(k_g) \equiv \omega(k_g) - \tau_c(1-\tau_r)\alpha$ and $\zeta \equiv \tau_c\{\rho + (1-\tau_r)\delta_k\} - \sigma(\tau_c + \tau_r)\delta_k$. This equation determines k_g^*. Substituting k_g^* into (17), we obtain c^*. Let us denote the RHS and the left-hand side (LHS) of (21) as $\Pi(k_g)$ and $\Lambda(k_g)$, respectively. Appendices A and B prove the next proposition.

Proposition 1

Suppose that $\Lambda(k_g^\dagger) > \Pi(k_g^\dagger)$ and (A.5) or (A.6) are satisfied, where $k_g^\dagger$, (A.5), and (A.6) are defined in Appendix A. There exists a locally saddle-stable steady state where we have $\gamma^* > 0$ and the no-Ponzi game condition of the government is satisfied.

In contrast to the models in Futagami et al. (2008) and Minea and Villieu (2013), our model includes the possibilities that the growth rates become negative and the no-Ponzi game condition of the government breaks in the long run. This is because we consider the *stock* of public capital rather than the flow of public services. A sufficient public capital accumulation is needed to sustain a large government debt. Conditions (A.5) or (A.6) ensure such a public capital accumulation.[8]

Next, we examine the long-run effects of reductions in $\bar{b}$. Appendix C proves the following proposition.

Proposition 2

A decrease in the target debt ratio, $\bar{b}$, promotes economic growth and increases public expenditure in the long run. That is, $dk_g^*/d\bar{b} < 0$, $d\gamma^*/d\bar{b} < 0$, and $dg^*/d\bar{b} < 0$.

The intuition behind these policy effects is simple. As $\bar{b}$ decreases, outstanding public debt reduces in the long run. The government's interest payments also reduce, which loosens its budget constraint and creates fiscal space to increase public investment. In the long run, g increases and, hence, the long-run growth rate increases. Note that in the short run, decreases in $\bar{b}$ reduce g (see (13)). However, as b_t decreases to $\bar{b}$, the government's interest payment steadily decreases, which means it can gradually increase public expenditure. The next subsection examines the transitional effects of reductions in $\bar{b}$.

3.3 Transitional Dynamics

Here, we examine the effects of $\bar{b}$ on the transition path and show that these effects depend heavily on the timeline effect of ϕ, which has no effect in the long run.

We consider the following scenario: the economy is initially in the steady state with $\bar{b} = \bar{b}_{init}$, where $\bar{b}_{init}$ denotes the initial level of $\bar{b}$. Then, we have $b_0 = \bar{b}_{init}$. At time 0, the government reduces $\bar{b}$ from $\bar{b}_{init}$ to $\bar{b}_{new}$ unexpectedly, where $\bar{b}_{new}$ is the level of $\bar{b}$ after the policy change. Then, the economy begins to move toward the new steady state.

Parameter Values

The complexity of the model does not allow us to obtain an analytical solution. Thus, we conduct a numerical analysis. The debt-to-GDP ratio in the new steady state, $\bar{b}_{new}$, is set to 0.6 following the target debt ratio in the SGP and the Maastricht Treaty. We use the following five values of ϕ: $\phi = 0.01, 0.025, 0.05, 0.075, 0.1$. The other parameter values are chosen based on the data of Greece (see Appendix D) as an example of a country with a very high debt-to-GDP ratio. Table 1 summarizes the parameter values. Under these parameter values, we calculate the values of the consumption-to-GDP ratio (C_0/Y_0), the total capital-to-GDP ratio ($(K_0 + K_{g,0})/Y_0$), and the ratio of tax revenue to output in the initial steady state (see Appendix D). Table 2 compares these calculations with the data averages in Greece for the period 2000–2008.[9] Since the initial steady state of the model is

[8]Dioikitopoulos and Kalyvitis (2008) also assess the possibility of a negative long-run growth rate in an endogenous growth model of productive public capital in Proposition 1 on pages 3766–3767. With the depreciations of public and private capital, they show that the government should be cautious in using scarce public expenditure to maintain the public capital that sustains the economic growth. However, rather than government borrowing, they consider a balanced budget. In our model, we would rather put a caveat on huge government debt that hinges on sustainable economic growth.

[9]The data average for the capital-to-GDP ratio is based on the AMECO database. The data average for tax revenue is based on the OECD Revenue Statistics.

in line with the data, it is a reasonable starting point for our policy experiments. Finally, note that the long-run growth rate in the initial steady state takes a positive value under the (parameter) values in Table 1, namely $\gamma^*_{init} = 0.035$ (see Appendix D), and then (A.5) or (A.6) in Proposition 1 is satisfied. Reductions in $\bar{b}$ also satisfy either of them in the long run because of Proposition 2.

[Tables 1 and 2]

Effects of Reductions in $\bar{b}$ on Transitional Dynamics

Here, we analyze the transition paths numerically using the relaxation algorithm.[10] Panel (a) in Figure 1 shows that the debt-to-GDP ratio decreases monotonically toward its new steady-state value. As ϕ increases, the debt-to-GDP ratio decreases at a higher rate.

[Figure 1]

Just after the policy change, the ratio of public investment to output, $I_{g,t}/Y_t$, drops sharply (see Panel (b)). To reduce b_t, the government must initially reduce its expenditure, as shown by the term $\phi(b_t - \bar{b})$ in (13). However, as b_t steadily declines, the interest payments on government debt gradually decrease, which loosens the government's budget constraint and creates fiscal space to increase public investment. Then, $I_{g,t}/Y_t$ gradually increases and eventually exceeds the initial level (see the last paragraph in subsection 3.2). For larger values of ϕ, the initial drops in $I_{g,t}/Y_t$ are also larger because the government must cut a larger amount of its expenditure to reduce its debt at a higher rate. Higher values of ϕ also mean that it takes less time for $I_{g,t}/Y_t$ to recover its initial level.

Panel (c) shows that the growth rate of private capital, K_t, jumps just after the policy change because the decline in public expenditure (investment) releases resources to the private sector. Then, it gradually decreases to its new steady-state value.

The dynamics of public investment and private capital drive those of $k_{g,t}$. During the early stage of the transition, $k_{g,t}$ gradually decreases because of the reductions in $I_{g,t}/Y_t$ and the increases in K_t. However, as $I_{g,t}/Y_t$ gradually increases, $k_{g,t}$ begins to increase, eventually exceeding its initial level in the long run. Since the growth rate of consumption is a function of $k_{g,t}$ (see (1b) and (4a)), it decreases during the early stage of the transition, but then begins to increase in the latter stages, eventually exceeding its initial level (see Panel (d)). As ϕ increases, the decline in the growth rate of consumption in the early stage of the transition is large, whereas it also takes less time until the growth rate recovers its initial level. The Euler equation (4a) shows that the interest rate exhibits the transition path similar to that of the growth rate of consumption.[11]

Panel (e) provides the transitional paths of c_t. From this panel, we know the effects of the policy change on the initial consumption level, C_0, because $C_0 \equiv c_0 K_0$. For all values

[10] Trimborn et al. (2008) detail the relaxation algorithm. They also provide MATLAB programs for the relaxation algorithm, freely downloadable at http://www.wiwi.uni-siegen.de/vwli/forschung/relaxation/the_relaxation_method.html?lang=de.

[11] The real interest rate becomes somewhat larger than the standard level. This comes from the simple setting of the model where capital inflow is absent. If one carefully examines this situation in the real world, huge models that incorporate the capital market in the open economy and the exchange rates and other fiscal and monetary policies in each of the countries might be necessary. However, this is beyond the scope of this study. Here, we investigate those effects that are essential to social welfare and an optimal target debt ratio, which have received less attention. Thus, our discussion restricts the movement of the real interest rate.

of ϕ, C_0 jumps just after the policy change because the decline in public expenditure (investment) releases resources to the private sector. As ϕ increases, these crowding-in effects become stronger and the initial increase in C_0 becomes large.

4 Optimal Debt-to-GDP Ratio

In the SGP or the Maastricht Treaty, the target debt-to-GDP ratio is set at 60%. However, this level might be different from the optimal level. This section examines the long-run optimal debt-to-GDP ratio that can attain the first-best welfare maximizing allocation. The long-run optimal debt-to-GDP ratio can be derived by considering the optimal fiscal policy in the steady state. To this end, we solve a social planner's problem.

Distortions from the following external effects exist in this economy: the positive external effect of K_t and that of $K_{g,t}$ (see $Y_{j,t} = AK_{j,t}^{\alpha}(h_t L_{j,t})^{1-\alpha}$ and $h_t = K_t^{1-\epsilon}K_{g,t}^{\epsilon}$ in Subsection 2.1). Therefore, the decentralized economy cannot attain a first-best allocation in the long run. To attain the optimum allocation, the following fiscal policies are required. First, the capital income tax rate τ_r is needed to remove the distortion from the external effect of K_t. Second, the other policy instruments, τ_c, τ_w, θ, and $\bar{b}$, are needed to remove the external effects of $K_{g,t}$.

A social planner maximizes the household's discounted lifetime utility, subject to the economy's aggregate resource constraints, (6) and (9). Because $\theta G_t = I_{g,t}$ (see (6)) and (1a), the resource constraint (9) can be rewritten as

$$\dot{K}_t = AK_{g,t}^{\beta}K_t^{1-\beta} - \theta^{-1}I_{g,t} - C_t - \delta_k K_t. \tag{22}$$

The social planner maximizes (2), subject to (6) and (22) with respect to C_t, $I_{g,t}$, K_t, and $K_{g,t}$. From the first-order conditions, we obtain

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\sigma}\left[(1-\beta)Ak_{g,t}^{\beta} - \delta_k - \rho\right], \tag{23}$$

$$(1-\beta)Ak_g^{\beta} - \delta_k = \theta\beta Ak_g^{\beta-1} - \delta_g. \tag{24}$$

The condition (24) indicates that the marginal product of private capital is equal to that of public capital. The LHS of (24) is monotonically increasing in k_g, whereas the RHS is monotonically decreasing in k_g. Then, k_g^{opt} is uniquely determined. Substituting (24) into (23), we obtain the following optimal long-run growth rate of consumption:

$$\gamma^{opt} = \frac{1}{\sigma}\left[(1-\beta)A(k_g^{opt})^{\beta} - \delta_k - \rho\right]. \tag{25}$$

The government in the decentralized economy chooses the target debt ratio, $\bar{b}$, to replicate the first-best outcome in the long run.

Let us compare the corresponding relationships (21) and (19) with (24) and (25). The steady-state equilibrium will replicate the first-best optimum: $k_g^* = k_g^{opt}$ and $\gamma^* = \gamma^{opt}$ if and only if

$$1 - \tau_r^{opt} = \frac{(1-\beta)A(k_g^{opt})^{\beta} - \delta_k}{\alpha A(k_g^{opt})^{\beta} - \delta_k}, \tag{26}$$

and

$$\theta \left[\Omega(k_g^{opt})A(k_g^{opt})^{\beta} + \zeta\right] = (1+\tau_c)k_g^{opt}\left[(1-\tau_r^{opt})\alpha A(k_g^{opt})^{\beta} - \{\rho + (1-\tau_r^{opt})\delta_k\} + \sigma\delta_g\right]. \tag{27}$$

Recall that $\beta = \epsilon(1-\alpha)$. Condition (26) shows that $\tau_r^{opt} < 0$ when $0 \leq \epsilon < 1$, whereas $\tau_r^{opt} = 0$ when $\epsilon = 1$. The former result corresponds to the usual policy implication that subsidies on capital are required to remove the external effects of K_t (see Barro and Sala-i-Martin (2004), pp 216-217). On the other hand, the latter case corresponds to a manifestation of the well-known result of Chamley (1986) that capital should be untaxed when there are no externalities from K_t.

From (26) and (27), we obtain the optimal target debt ratio as follows:

$$\begin{aligned}\bar{b}^{opt} =& \frac{\theta^{-1}(1+\tau_c)k_g^{opt}\left[(1-\tau_r^{opt})\alpha A(k_g^{opt})^{\beta} - \{\rho + (1-\tau_r^{opt})\delta_k\} + \sigma\delta_g\right] - \zeta}{\left\{(1-\sigma)(1-\tau_r^{opt})\alpha A(k_g^{opt})^{\beta} - [(1-\sigma)(1-\tau_r^{opt})\delta_k + \rho]\right\} A(k_g^{opt})^{\beta}} \\ &+ \frac{\tau_c(1-\tau_r^{opt})\alpha - \sigma(\tilde{\tau}+\tau_c)}{(1-\sigma)(1-\tau_r^{opt})\alpha A(k_g^{opt})^{\beta} - [(1-\sigma)(1-\tau_r^{opt})\delta_k + \rho]}.\end{aligned} \tag{28}$$

In the case of the parameter set in Table 1: $(1-\alpha, A, \beta, \sigma, \rho, \theta, \delta_k, \delta_g)$=(0.5645, 0.4650, 0.0345, 2.5, 0.0412, 0.1590, 0.028, 0.031), we obtain $(k_g^{opt}, \tau_r^{opt}) = (0.0056, -1.4567)$. Figure 2 shows the relationship between $\bar{b}^{opt}$ and the set of tax rates (τ_c, τ_w). When (τ_c, τ_w)=(0.1293, 0.2818), as in Table 1, $\bar{b}^{opt}$ is in the region of a negative value. Only when both τ_c and τ_w are sufficiently high do we have $\bar{b}^{opt} > 0$.

[Figure 2]

To simplify the discussion, we consider the case of no depreciation in both private and public capital ($\delta_k = \delta_g = 0$). In this case, we obtain $k_g^{opt} = \theta\beta/(1-\beta)$ and $\tau_r^{opt} = (\alpha+\beta-1)/\alpha$ from (24) and (26). Therefore, (28) reduces to

$$\bar{b}^{opt} = \frac{[\sigma\tilde{\tau} + (\sigma-1)\tau_c - \beta]A\left(\frac{\theta\beta}{1-\beta}\right)^{\beta} + \frac{\rho(\tau_c+\beta)}{1-\beta}}{\left[(\sigma-1)(1-\beta)A\left(\frac{\theta\beta}{1-\beta}\right)^{\beta} + \rho\right] A\left(\frac{\theta\beta}{1-\beta}\right)^{\beta}}, \tag{29}$$

where $\tilde{\tau} \equiv (1-\alpha)\tau_w + \alpha + \beta - 1$. Because $\sigma > 1$ is assumed, $\bar{b}^{opt}$ is obviously increasing in both τ_w and τ_c. After some calculation, we can easily show $\partial\bar{b}^{opt}/\partial\theta < 0$ when $\bar{b}^{opt} > 0$. Thus, we obtain the following proposition.

Proposition 3

If the depreciation of both private and public capital are zero, we have
(i) $\frac{\partial \bar{b}^{opt}}{\partial \tau_w} > 0$ and $\frac{\partial \bar{b}^{opt}}{\partial \tau_w} > 0$, and (ii)$\frac{\partial \bar{b}^{opt}}{\partial \theta} < 0$, when $\bar{b}^{opt} > 0$.

The intuitive explanation of this policy implication is simple. When tax revenue is sufficiently large, the government's debt finance can become positive as the optimal decision. On the other hand, an increase in θ promotes public investment and leads to a rise in k_g^*, which increases the interest rate of government debt. In order to reduce the cost of this interest payment, the planner decreases $\bar{b}^{opt}$ in the long run.

5 Welfare Analysis: Expenditure Cuts Only

Focusing only on the steady state, we obtain the long-run optimal debt-to-GDP ratio in the previous section. To conduct a complete welfare analysis of debt reduction, we have to fully consider the transitional dynamics. We consider the same scenario as that in Subsection 3.3.

Our welfare measure is (2). Using $C_t = C_0 \exp[\int_0^t \gamma_{C,v} dv]$, where $\gamma_{C,t} (\equiv [(1-\tau_r)\{\alpha A k_{g,t}^{\beta} - \delta_k\} - \rho]/\sigma)$ is the growth rate of consumption, we rewrite (2) as

$$U_0 = \frac{C_0^{1-\sigma}}{1-\sigma} \int_0^{\infty} \exp\left[(1-\sigma)\int_0^t \gamma_{C,v} dv - \rho t\right] dt.$$

This equation shows that decreases in $\bar{b}$ affect the welfare level through their effects on $C_0 = c_0 K_0$ and the paths of $\gamma_{C,t}$. Because the effects of the policy change on C_0 and the transitional paths of $\gamma_{C,t}$ depend heavily on the timeline effect of ϕ, as we observed in Subsection 3.3, the welfare effects of reductions in $\bar{b}$ are also influenced by ϕ.

We examine the welfare effects numerically using the relaxation algorithm. The initial steady state is the same as that considered in Subsection 3.3. Again, we consider five values of ϕ: 0.01, 0.025, 0.05, 0.075, and 0.1. The case in which $\phi = 0.05$ matches the debt reduction benchmark introduced by the SGP (see the introduction).

Let us denote the welfare level without the policy change as $U_{0,N}^* \equiv (c_N^* K_0)^{1-\sigma}/\{(1-\sigma)[\rho - \gamma_{C,N}^*(1-\sigma)]\}$, where c_N^* and $\gamma_{C,N}^*$ are the initial steady-state values of c and γ_C, respectively. To calculate $U_{0,N}^*$, we set $K_0 = 1$. The welfare level immediately after the policy change is denoted as U_0^{**}. Appendix E presents the calculation procedure. The welfare gains (losses) of the policy change are measured by $\Delta U_0 \equiv (U_0^{**} - U_{0,N}^*)/|U_{0,N}^*|$.

Results

The second column in Table 3 provides the long-run growth rates of the new steady states. Because the growth rate in the initial steady state is 0.035, debt reduction increases the long-run growth rate, as shown in Proposition 2. However, the growth effect is rather modest. This reflects the small elasticity of output with respect to public capital, $\beta = \epsilon(1-\alpha) = 0.0345$.[12]

[Table 3]

The row labeled as ΔC_0 represents percentage changes in the initial consumption levels. As discussed in Subsection 3.3, as ϕ increases, the initial increases in C_0 become large.

With regard to the welfare effects, Table 3 reveals the following results: (i) for all values of ϕ, a reduction of $\bar{b}$ to 60% improves welfare; and (ii) as ϕ increases, welfare improvements become large. This is because when ϕ is larger, more resources are released to the private sector and, hence, the initial increases in C_0 become large.

If the consumption growth rate remains constant at the initial rate, how much of an increase in initial consumption is required to attain the same level of welfare improvements as that of ΔU_0? The row labeled as ΔC_{equiv} shows the required increases in initial

[12] Bom and Ligthart (2014) estimate that the average output elasticity of public capital amounts to 0.106 if we consider OECD countries. Such a large output elasticity of public capital can bring about the larger long-run growth effect of debt reductions.

consumption. In all cases, ΔC_{equiv} is smaller than ΔC_0. This indicates that the decline in the consumption growth rate during the early stage of the transition has a strong negative welfare effect. However, ΔC_0 is large enough to counteract this negative effect.

Furthermore, Table 3 shows that by setting the target level, $\bar{b}_{new}$, to zero, welfare improves even further. This is because the long-run optimal target debt ratio is smaller than 0.6 under the parameter sets of Table 1 (see the statements below Eq. (28) in Section 4).

6 Expenditure Cuts with Tax Increases

Thus far, we have discussed debt reductions based only on expenditure cuts because the consolidations set out in the SCPs for both the Euro area and the EU27 are primarily based on this method, according to the *Public finances in EMU – 2012* (See footnote 3 in the introduction). The goal of this section is to compare the welfare effects between debt reductions based only on expenditure cuts and those based on both expenditure cuts and tax increases. This is motivated by the following points. As noted earlier, in countries such as Spain, Austria, Cyprus, France, Malta, and Romania, consolidations are relatively evenly spread between expenditure cuts and tax increases. If the government could increase tax rates when attempting to decrease the debt-to-GDP ratio, the reductions in public investment just after the policy change might be mitigated. Therefore, with tax increases, the effects of debt reductions might be different from those examined thus far.

We assume that the government unexpectedly increases tax rates at the same time as it decreases $\bar{b}$. See Appendix F for more details. From then on, the tax rates remain unchanged because, in practice, frequent changes in tax rates might be difficult to implement politically. We denote increases in the rates of interest income, labor income, and consumption taxes as $\Delta\tau_r$, $\Delta\tau_w$, and $\Delta\tau_c$, respectively. We assume that only one tax rate is changed, while the other two rates remain unchanged. We choose values of $\Delta\tau_r$, $\Delta\tau_w$, and $\Delta\tau_c$ so that the ratio of tax revenue to output in the new steady state becomes larger than that in the initial steady state by $x(=0,\ 1,\ 2,\ 3)\%$. Naturally, $x = 0$ implies $\Delta\tau_r = \Delta\tau_w = \Delta\tau_c = 0$.

We use the same parameter values as in the previous sections. Figure 3 shows the transitional dynamics under $\bar{b}_{new} = 0.6$ and $\phi = 0.05$. The solid lines represent debt reductions without any tax increases ($x = 0$). The panels in the first column present the results obtained when only τ_r is increased. The effects of increases in τ_w and τ_c are shown in the second and last columns, respectively.

As x becomes larger, the initial decline in the ratio of public investment to output is mitigated (see Panels (a)-1, (b)-1, and (c)-1). Panels (a)-2, 3, and 4 show that because increases in τ_r have a negative distortionary effect on households' saving, the growth rates of private capital and consumption decrease significantly and the initial increases in private consumption become larger. Panels (b)-2 and (c)-2 show that an increase in either τ_w or τ_c reduces the early increase in the growth rate of K_t. Panels (b)-4 and (c)-4 further show that the initial increases in C_0 are lower. This is because more resources are devoted to public investment as a result of the tax increases. Panels (b)-3 and (c)-3 show that when either τ_w or τ_c increase, the decreases in the growth rate of consumption in the early stage are mitigated because increased public investments have a positive effect on the interest rate.

Table 4 presents the welfare effects. The results without a tax increase are presented in the row labeled "Benchmark." With a tax increase, the welfare gains from the debt reduction are depressed and, in many cases, welfare deteriorates. In all cases, the debt reduction based only on expenditure cuts generates the largest welfare gains. As shown in Panel (a)-3 in Figure 3, an increase in τ_r has a large and negative distortionary effect on the consumption growth rate, which has a significant negative welfare effect. When either τ_w or τ_c increase, there is less of an initial increase in C_0, which decreases the negative welfare effects.[13]

[Figure 3 and Table 4]

7 Extension of the Model: Government Consumption into Utility

Thus far, we have assumed that government consumption expenditure, $C_{g,t} \equiv (1-\theta)G_t$, which accounts for a large part of government spending, is not valued by households. Reductions in $\bar{b}$ initially have a negative effect on $C_{g,t}$. Hence, if households derive their utility from $C_{g,t}$, the initial welfare loss becomes larger and a reduction in $\bar{b}$ might have a negative welfare effect. Incorporating government consumption into utility may be an important modification.

However, if we assume non-separability between utilities from private and government consumption, based on the empirical findings of Ni (1995), equilibrium indeterminacy may arise.[14] The multiplicity of equilibrium paths makes it difficult to investigate the welfare effects. If the utility function takes a separable form with respect to C_t and $C_{g,t}$, equilibrium indeterminacy never arises. Then, we consider the following utility function:

$$U_0 = \int_0^\infty \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \psi \ln(C_{g,t}) \right) e^{-\rho t} dt, \tag{30}$$

where ψ is a positive constant and $C_{g,t} \equiv (1-\theta)G_t$ denotes government consumption. This modification has no influence on the dynamic system of the economy and the steady state. The results thus far, except those regarding welfare effects, hold perfectly, even under this modification.

Using the same parameter values as in Subsections 3.3 and Section 5 and considering the same scenario as in Section 5, we conduct a welfare analysis of reductions in $\bar{b}$ for several values of ψ.[15] Table 5 presents $\Delta U_0 \equiv (U_0^{***} - U_{0,N}^*)/|U_{0,N}^*|$, where U_0^{***} denotes the utility just after the policy change, and $U_{0,N}^*$ denotes the utility in the initial steady state (as earlier).[16] Table 5 also shows the results obtained under $\psi = 0$, which are

[13] Note that τ_w and τ_c do not distort households' behaviors. However, they affect public investments through the government budget constraint and the debt rule (see (13).).

[14] Using a model with utility-generating government consumption, Guo and Harrison (2008) show that equilibrium indeterminacy arises under a balanced budget.

[15] Since this modification has no effect on the dynamic system and the steady-state equilibrium, the initial and new steady states are the same as in Subsection 3.3 and Section 5 under the same parameter values.

[16] See Appendix E for the calculation of U_0^{***}.

naturally the same as those in Table 3. Even under this modification, the reductions in $\bar{b}$ improve welfare for all values of ϕ and $\bar{b}_{new}$, and these improvements in welfare increase as ϕ increases and $\bar{b}_{new}$ decreases.

If the government increases tax rates, as described in Section 6, the reductions in $\bar{b}$ might have different welfare effects if $\psi > 0$, because tax increases mitigate the negative effects on $C_{g,t}$. Assuming $\psi = 0.5$, we conduct the same policy experiment as in Section 6 and present the results in Table 6. The table shows that, even when households place a high value on government consumption ($\psi = 0.5$), debt reductions based only on expenditure cuts generate the largest positive welfare gains in all cases.

[Tables 5 and 6]

8 Conclusion

Following Futagami et al. (2008) and Minea and Villieu (2013), we considered a simple debt policy rule in an endogenous growth model. Under the policy rule, the government gradually reduces its debt to the target level. Departing from these two studies, we investigate (i) the effects of the debt policy rule on the accumulation of public capital, (ii) the optimal target debt ratio in the long run, and (iii) the welfare analysis of the debt reduction that fully considers the transition dynamics. We obtained the following results.

1. A steady state that is locally saddle stable exists. However, in contrast to the models in Futagami et al. (2008) and Minea and Villieu (2013), our model includes the possibilities that the growth rates become negative and that the no-Ponzi game condition of the government breaks in the long run. This result occurs because we consider the *stock* of public capital rather than the flow of public services.

2. The optimal target debt ratio is uniquely determined. The ratio depends on the tax rates on wage income and consumption, as well as the share of public investment in total government spending. The target debt ratio set by the SGP and Maastricht Treaty, namely 60%, might be higher than the optimal level.

3. Debt reduction based only on expenditure cuts improves welfare. As the pace of debt reduction increases, welfare improves further. Second, lowering the target debt ratio of 60% to the optimal level increases the improvement in welfare.

4. Under fiscal consolidation based on expenditure cuts with a tax increase, welfare does not always improve. Even when welfare does improve, the welfare gains are lower than those under expenditure cuts only.

5. Even when households derive their utility from public services, the above results hold.

There remain areas for further study. First, we do not consider the heterogeneity of agents. Bastagli et al. (2012) discuss how some governments may increase the top marginal income tax rates and avoid the decrease in redistributive transfers during fiscal consolidation periods. Incorporating the heterogeneity of agents may be an interesting extension. Second, we do not consider intergenerational conflicts. Expenditure cuts or tax increases may worsen the welfare of the current generation. However, if the consolidation is not conducted appropriately, the burden on future generations may increase. Therefore, incorporating intergenerational conflicts remains for future research. Third, some policy shifts after a debt reduction would remain as an important policy issue. For example, the government could use the policy space created by the debt reduction to decrease tax rates.

Appendix

A Proof of Propositions 1-(a): Existence of the Steady State

The first and second derivatives of $\Pi(k_g)$ (the RHS of (21)) are

$$\Pi'(k_g) = (1+\tau_c)\left[(1-\tau_r)(1+\beta)\alpha A k_g^{\beta} - \{\rho + (1-\tau_r)\delta_k\} + \sigma\delta_g\right], \tag{A.1}$$

$$\Pi''(k_g) = \alpha\beta(1+\tau_c)(1+\beta)(1-\tau_r)A k_g^{\beta-1} > 0. \tag{A.2}$$

Therefore, $\Pi(k_g)$ is a convex and strictly increasing (decreasing) function of k_g for $k_g \geq (<)[1/(1+\beta)]^{\frac{1}{\beta}}\underline{k}_g$, where $\underline{k}_g \equiv [1-\sigma\delta_g/\{\rho+(1-\tau_r)\delta_k\}]^{\frac{1}{\beta}}\, k_{g,\gamma}$. Note that $\underline{k}_g \equiv \left[1 - \frac{\sigma\delta_g}{\rho+(1-\tau_r)\delta_k}\right]^{\frac{1}{\beta}} k_{g,\gamma} = \left[\frac{\rho+(1-\tau_r)\delta_k-\sigma\delta_g}{(1-\tau_r)\alpha A}\right]^{\frac{1}{\beta}}$ and $\underline{k}_g < k_{g,\gamma}$. If $1 > \sigma\delta_g/\{\rho+(1-\tau_r)\delta_k\}$, $\Pi(k_g)$ is equal to zero both when $k_g = 0$ and $k_g = \underline{k}_g \equiv [1-\sigma\delta_g/\{\rho+(1-\tau_r)\delta_k\}]^{\frac{1}{\beta}}\, k_{g,\gamma}$. On the other hand, if $1 \leq \sigma\delta_g/\{\rho+(1-\tau_r)\delta_k\}$, $\Pi(k_g)$ is equal to zero only when $k_g = 0$. Figures 4 and 5 show that $\Pi(k_g)$ is a convex function of k_g.

[Figures 4 and 5]

Next, we consider the properties of $\Lambda(k_g)$ (the LHS of (21)). Seemingly, we have $\Lambda(0) = \theta\zeta$. The first derivative of $\Lambda(k_g)$ is

$$\Lambda'(k_g) = \theta\beta A k_g^{\beta-1}\left[-2(\sigma-1)(1-\tau_r)\alpha\bar{b}A k_g^{\beta} + \eta\right], \tag{A.3}$$

where $\eta \equiv \sigma(\tilde{\tau}+\tau_c) - [(1-\sigma)(1-\tau_r)\delta_k + \rho]\bar{b} - \tau_c(1-\tau_r)\alpha$. If $\eta > 0$, $\Lambda'(k_g) < (=) > 0$ if and only if $k_g > (=) < \tilde{k}_g$. If $\eta < 0$, $\Lambda'(k_g) < 0$ for $k_g \geq 0$. The second derivative of $\Lambda(k_g)$ is

$$\Lambda''(k_g) = \theta\beta A k_g^{\beta-2}\left[-2(2\beta-1)(\sigma-1)(1-\tau_r)\alpha\bar{b}A k_g^{\beta} + (\beta-1)\eta\right]. \tag{A.4}$$

If $\eta > 0$, $\Lambda''(k_g)$ has the following properties. When $\frac{1}{2} \leq \beta < 1$, $\Lambda''(k_g) < 0$ holds for all $k_g(\geq 0)$. When $0 < \beta < \frac{1}{2}$, $\Lambda''(k_g) \leq 0$ holds for $0 < k_g < \tilde{k}_g \leq \breve{k}_g \equiv [(1-\beta)/(1-2\beta)]^{1/\beta}\tilde{k}_g$, and $\Lambda''(k_g) > 0$ for $k_g > \breve{k}_g$. If $\eta < 0$, the following properties holds for $\Lambda''(k_g)$. When $\frac{1}{2} \leq \beta < 1$, $\Lambda''(k_g) > 0$ holds for $0 < k_g \leq \breve{k}_g$, and $\Lambda''(k_g) \leq 0$ for $k_g > \breve{k}_g$.

In summary, $\Lambda(k_g)$ has the following properties. When $\eta > 0$, $\Lambda(k_g)$ is an increasing and concave function for $k_g < \tilde{k}_g \equiv [\eta/\{2(\sigma-1)(1-\tau_r)\alpha\bar{b}A\}]^{\frac{1}{\beta}}$, while it decreases with k_g for $k_g > \tilde{k}_g$ (see Figure 4). On the other hand, when $\eta \leq 0$, $\Lambda(k_g)$ becomes monotonically decreasing for $k_g > 0$ (see Figure 5).

A steady state E^* always exists if and only if $\Lambda(k_g^{\dagger}) > \Pi(k_g^{\dagger})$, where $k_g^{\dagger}$ is the value of $k_g(< k_g^*)$ that satisfies $\Pi'(k_g) = \Lambda'(k_g)$. Appendix B shows that this steady state is saddle stable.[17] Then, as shown in Figures 4 and 5, this steady state value k_g^* satisfies $\gamma^* > 0$ if and only if $\Lambda(k_{g,\gamma}) > \Pi(k_{g,\gamma})$ or $\Lambda'(k_{g,\gamma}) > \Pi'(k_{g,\gamma})$ when $\Lambda(k_{g,\gamma}) < \Pi(k_{g,\gamma})$. That is,

$$\frac{\rho + (1-\tau_r)\delta_k}{(1-\tau_r)\alpha A} < \Theta, \tag{A.5}$$

[17] It is possible that another steady state in which $k_g < k_g^*$ exists if $\zeta < 0$ (see Figures 4 and 5). Let us denote the value of k_g in this steady state as $k_{g,low}$. We find that this steady state is unstable in Appendix B. Therefore, we find that the economy converges to the saddle stable economy k_g^* if $k_{g,0} > k_{g,low}$. Nevertheless, if $k_{g,0} < k_{g,low}$, the economy can fail to develop, in which case the government in the economy with low public capital should avoid the fiscal policy of $\zeta < 0$.

or

$$\Theta < \frac{\rho + (1-\tau_r)\delta_k}{(1-\tau_r)\alpha A} < \Upsilon \quad \text{when} \quad \frac{\rho + (1-\tau_r)\delta_k}{(1-\tau_r)\alpha A} > \Theta, \tag{A.6}$$

where

$$\Theta \equiv \left\{ \frac{\theta\left[(\tilde{\tau} + \tau_c - \rho\bar{b})\{\rho + (1-\tau_r)\delta_k\} - (1-\tau_r)(\tau_c + \tau_r)\alpha\delta_k\right]}{(1-\tau_r)(1+\tau_c)\alpha\delta_g} \right\}^{\beta},$$

$$\Upsilon \equiv \left\{ \frac{\theta\beta A[\eta - 2(\sigma-1)\{\rho + (1-\tau_r)\delta_k\}\bar{b}]}{(1+\tau_c)[\beta\{\rho + (1-\tau_r)\delta_k\} + \sigma\delta_g]} \right\}^{\frac{\beta}{1-\beta}}.$$

Under (A.5) or (A.6), the conventional transversality condition (no-Ponzi game condition of the government): $(1-\sigma)\gamma^* < \rho$ for $\sigma > 1$ is satisfied in the long run because of $\gamma^* > 0$. Both Θ and Υ are decreasing in $\bar{b}$. Then, the government can break the fiscal balance in the long run if $\bar{b}$ is large enough to break (A.5) or (A.6).

B Proof of Proposition 1-(b): Stability of the Steady State

Approximating (7), (14), and (15) linearly around the steady states, we obtain

$$\begin{pmatrix} \dot{b} \\ \dot{c} \\ \dot{k}_g \end{pmatrix} = \begin{pmatrix} -\phi & 0 & 0 \\ J_{cb} & J_{cc} & J_{ck_g} \\ J_{k_g b} & J_{k_g c} & J_{k_g k_g} \end{pmatrix} \begin{pmatrix} b_t - \bar{b} \\ c_t - c^* \\ k_{g t} - k_g^* \end{pmatrix}. \tag{B.1}$$

Then, $J = (J_{ij})$ denotes the coefficient matrix of the former system:

$$\begin{aligned}
J_{cb} =& g_1(\bar{b}, c^*, k_g^*)c^*, \quad J_{cc} = (1 + g_2(\bar{b}, c^*, k_g^*))c^*, \\
J_{ck_g} =& \left[g_3(\bar{b}, c^*, k_g^*) - \frac{[\sigma - (1-\tau_r)\alpha]\beta A(k_g^*)^{\beta-1}}{\sigma} \right] c^*, \\
J_{k_g b} =& (\theta + k_g^*)g_1(\bar{b}, c^*, k_g^*), \quad J_{k_g c} = (\theta + k_g^*)g_2(\bar{b}, c^*, k_g^*) + k_g^*, \\
J_{k_g k_g} =& g^* + (\theta + k_g^*)g_3(\bar{b}, c^*, k_g^*) - (1+\beta)A(k_g^*)^{\beta} + c^* + \delta_k - \delta_g.
\end{aligned} \tag{B.2}$$

The structure of the first column of J means that one of the eigenvalues of J is $-\phi < 0$. The remaining two eigenvalues of J are those of the matrix $\bar{J}$, derived by deleting the first row and column from J. To check for stability, we examine the sign of $det\bar{J} = J_{cc}J_{k_g k_g} - J_{k_g c}J_{ck_g}$. Using (B.2) with (17), (18), and (21), we obtain

$$det\bar{J} = \frac{c^*\Psi(k_g^*)}{\left[1 + \left\{(1-\beta)k_g^* - \theta\beta\right\}\bar{b}A(k_g^*)^{\beta-1}\right]\sigma},$$

where

$$\begin{aligned}
\Psi(k_g^*) \equiv& \theta\beta A(k_g^*)^{\beta-1}\left[2(1-\sigma)(1-\tau_r)\alpha\bar{b}A(k_g^*)^{\beta} + \eta\right] \\
&- (1+\tau_c)\left[(1-\tau_r)\alpha(1+\beta)A(k_g^*)^{\beta} - \{\rho + (1-\tau_r)\delta_k\} + \sigma\delta_g\right].
\end{aligned}$$

If we use $\Lambda'(k_g^*)$ and $\Pi'(k_g^*)$, the above equation can be rewritten as

$$det\bar{J} = \frac{c^*}{\left[1 + \left\{(1-\beta)k_g^* - \theta\beta\right\}\bar{b}A(k_g^*)^{\beta-1}\right]\sigma}\left[\Lambda'(k_g^*) - \Pi'(k_g^*)\right].$$

Figure 4 shows that in the steady state E^*, $\Lambda'(k_g^*) < \Pi'(k_g^*)$ holds and, hence, $det\bar{J} < 0$. One of the eigenvalues of $\bar{J}$ is positive and the other is negative. Then, the steady state is locally saddle-point stable. When $\zeta < 0$, another steady state in which $k_g = k_{g,low}$ can exist (see footnote 17). However, this steady state is unstable because $\Lambda'(k_{g,low}) > \Pi'(k_{g,low})$ holds and, hence, $det\bar{J} > 0$.

C Proof of Proposition 2

From the LHS of (21), the magnitude of the shift in $\Lambda(k_g)$ when $\bar{b}$ increases is $\sigma\theta A(k_g^*)^\beta[(1-\sigma)\gamma^* - \rho]$. Because $\sigma > 1$, it takes a negative value for $\gamma^* > 0$. Thus, when $\bar{b}$ falls, $\Lambda(k_g)$ shifts upward, as shown in Figures 4 and 5. As a result, k_g^* increases. Then, the long-run growth rate, γ^*, increases (see (19)). Using (6) and the definitions of g, k_g, and γ, we have that $\theta g^* = (\gamma^* + \delta_g)k_g^*$. Reductions in $\bar{b}$ unambiguously increase g^*. Thus, Proposition 2 holds.

D Parameter Values

We choose parameter values using the data of Greece. The labor income share is computed from the EU KLEMS database using data of labor compensation and gross value added, which yield an average labor income share of 56.43% in the period 2000–2007.[18] Thus, we set $1-\alpha = 0.5643$. According to the AMECO database, the average ratio of government expenditure (including public investment) to GDP is 0.2170 in the period 2000–2008.[19] During the same period, the average ratio of public investment to GDP is 0.0345. Since we have $\theta =$ public investment/government expenditure, we set $\theta = 0.1590 (= 0.0345/0.2170)$, which means that investment by the government amounts to 15.9% of its total expenditure. Following Baxter and King (1993), the elasticity of output with respect to public capital is set to the ratio of public investment to GDP, namely $\beta \equiv \epsilon(1-\alpha) = 0.0345$, which yields $\epsilon = 0.0611$. The intertemporal elasticity of substitution, $1/\sigma$, is set to 0.4. Following Kollintzas amd Vassilatos (2000), we set $\delta_k = 0.028$ and $\delta_g = 0.031$. These depreciation rates are consistent with those of Papageorgiou (2012).

We calculate the tax rates based on a modified version of the methodology proposed by Mendoza et al. (1994), as described in Carey and Tchilinguirian (2000) in paragraphs 16–28, which yields average tax rates for 2000–2008 in Greece as $\tau_r = 0.2178$, $\tau_w = 0.2818$, and $\tau_c = 0.1293$. The results largely agree with those of Papageorgiou (2012). The only difference is that he assumed self-employed earnings to be an imputed wage.

According to the AMECO database, the debt-to-GDP ratio of Greece in 2008 was 1.13, and the average annual compound growth rate of GDP for the period 2000–2008 was about 3.5%. Then, we set $b_{init} = 1.13$. The long-run growth rate in the initial steady state is set as $\gamma_{init}^* = 0.035$. Using (19), (21), and $G/Y = g/(Ak_g^\beta)$, we choose the values of A and ρ such that $G_0/Y_0 = 0.2170$, $B_0/Y_0 (= b_0) = 1.13$, and $\gamma_{init}^* = 0.035$ hold in the initial steady state, which yields $A = 0.465$ and $\rho = 0.0412$.

[18]The EU KLEMS database is freely accessible at http://www.euklems.net/.

[19]The AMECO database is freely accessible at http://ec.europa.eu/economy_finance/db_indicators/ameco/index_en.htm.

E Welfare Effects of $\bar{b}$

To calculate the value of U_0^{**}, we calculate the dynamic path and initial value of $U_t \equiv \int_t^\infty e^{-\rho(v-t)}C_v^{1-\sigma}/(1-\sigma)dv$ using the relaxation algorithm. However, we cannot calculate the dynamic path and initial value of U_t directly because U_t does not remain constant in the steady state. Let us define $X_t \equiv U_t/K_t^{1-\sigma}$. Since $C_t \equiv c_t K_t$, we have

$$\dot{X}_t = \rho X_t - \frac{c_t^{1-\sigma}}{1-\sigma} - (1-\sigma)(Ak_{g,t}^\beta - c_t - g_t - \delta_k)X_t.$$

In this case, X_t becomes constant over time in the high-growth steady state. Then, we calculate the dynamic path and the initial value of X_t using the relaxation algorithm. Since K_0 is normalized to one, we have that $U_0^{**} = X_0$.

If the utility function is given by (30), as in Section 7, we further define $X_{c_g,t} \equiv \int_0^\infty e^{-\rho t}\psi \ln(C_{g,t})dt - (\psi/\rho)\ln K_t$. Since $C_{g,t} \equiv (1-\theta)g_t K_t$, we have

$$\dot{X}_{c_g,t} = \rho X_{c_g,t} - \psi \ln(1-\theta)g_t - \frac{\psi}{\rho}(Ak_{g,t}^\beta - c_t - g_t - \delta_k).$$

In this case, $X_{c_g,t}$ becomes constant over time in the steady state, and we can calculate the dynamic path and initial value of $X_{c_g,t}$. Since $K_0 = 1$, we have that $U_0^{***} = X_0 + X_{c_g,0}$ under (30).

F Debt Reduction based on Tax Increases and Expenditure Cuts

After the increase in the tax rates, the budget constraint of the government becomes

$$\begin{aligned}\dot{B}_t &= r_t B_t + G_t - [(\tau_r + \Delta\tau_r)r_t W_t + (\tau_w + \Delta\tau_w)w_t + (\tau_c + \Delta\tau_c)C_t] + T_t \\ &= (1-\tau_r)r_t B_t - (\tilde{\tau}Y_t + \tau_c C_t - G_t) - \{\Delta\tau_r r_t(K_t + B_t) + \Delta\tau_w w_t + \Delta\tau_c C_t - T_t\},\end{aligned}$$

where the lump-sum transfer to the households is denoted as T_t, which we will specify later. In addition, we have $I_{g,t} = \theta G_t$.

In this setting, (13) is modified as

$$g_t = g(b_t, c_t, k_{g,t}) + \frac{\Delta_t Ak_{g,t}^\beta}{1 + [(1-\beta)k_{g,t} - \theta\beta]\, b_t Ak_{g,t}^{\beta-1}} \equiv \tilde{g}(b_t, c_t, k_{g,t}, \Delta_t), \tag{F.1}$$

where $g(b_t, c_t, k_{g,t})$ is defined as in (13) and $\Delta_t \equiv \{\Delta\tau_r r_t(K_t + B_t) + \Delta\tau_w w_t + \Delta\tau_c C_t - T_t\}/Y_t$ represents the direct effect of tax increases on public expenditure. If $\Delta_t > 0$, the tax increases mitigate the negative effect of the debt reduction on public expenditure. We formulate T_t in such a way that $\Delta_t > 0$ gradually diminishes and eventually disappears as the fiscal consolidation progresses (i.e., b_t approaches $\bar{b}_{new}$). Keeping in mind that the interest and wage rates can be rewritten as $r_t = \alpha Ak_{g,t}^{\beta-1}k_{g,t} - \delta_k$ and $w_t = (1-\alpha)Ak_{g,t}^{\beta-1}K_{g,t}$, respectively, we define $r_t^{**} \equiv \alpha A{k_g^{**}}^{\beta-1}k_{g,t} - \delta_k$ and $w_t^{**} = (1-\alpha)A{k_g^{**}}^{\beta-1}K_{g,t}$, where ${k_g}^{**}$ denotes the new steady-state value of $k_{g,t}$. Using these definitions, we specify T_t as

$$T_t = \Delta\tau_r r^{**}(K_t + B_t) + \Delta\tau_w w_t^{**} + \Delta\tau_c c^{**}K_t, \tag{F.2}$$

where c^{**} is the new steady-state value of c_t. Under (F.2), Δ_t is expressed as

$$\Delta_t \equiv \{\alpha\Delta\tau_r + (1-\alpha)\Delta\tau_w + \Delta\tau_r \alpha A k_{g,t}^{\beta} b_t\} \left[1 - \left(\frac{k_{g,t}}{k_g^{**}}\right)^{1-\beta}\right] + \Delta\tau_c \left[\frac{c_t - c^{**}}{A(k_{g,t})^{\beta}}\right].$$

When $\Delta\tau_r = \Delta\tau_w = \Delta\tau_c = 0$, we have $\Delta_t = 0$. From the discussion in subsection 3.2, we plausibly guess that, even with tax increases, debt reductions raise the steady-state value of k_g and, hence, $k_g^{**} > k_{g,0}$ holds. The discussion in Subsection 3.3 reasonably suggests that c_t increases initially in response to debt reductions. Then, under (F.2), we expect that Δ_t takes a positive value just after the policy change. As the fiscal consolidation progresses, $k_{g,t}$ and c_t converge to k_g^{**} and c^{**}, respectively. Then, Δ_t gradually diminishes and eventually disappears.

Equations (14) and (15) are modified as

$$\dot{c}_t = \left[c_t + \tilde{g}(b_t, c_t, k_{g,t}, \Delta_t) - \left\{1 - \frac{1}{\sigma}(1 - \tau_r - \Delta\tau_r)\alpha\right\} A k_{g,t}^{\beta} - \frac{\rho}{\sigma} + \left\{1 - \frac{1 - \tau_r - \Delta\tau_r}{\sigma}\right\}\delta_k\right] c_t, \tag{F.3}$$

$$\dot{k}_{g,t} = (\theta + k_{g,t})\tilde{g}(b_t, c_t, k_{g,t}, \Delta_t) - A k_{g,t}^{1+\beta} + c_t k_{g,t} + (\delta_k - \delta_g) k_{g,t}. \tag{F.4}$$

The dynamic system of the economy is given by (7), (F.3), and (F.4). As in the benchmark model, there exists a unique steady state. Note that in the steady state, $\Delta\tau_w$ and $\Delta\tau_c$ disappear from the dynamic system. As long as $\Delta\tau_r = 0$, the steady state in this modified model corresponds exactly to that of the benchmark model. When $\Delta\tau_r > 0$, this is not the case, because $\Delta\tau_r$ directly influences household behavior, and so is included in (F.3).

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Table 1: Benchmark Parameters

Parameter or variable	Description	Value	Source
$1-\alpha$	Labor income share	0.5643	Data average
A	Total factor productivity	0.4650	Calibrated
ϵ	Public capital elasticity in labor productivity	0.0611	Calibrated
σ	Curvature parameter in the utility function	2.5	Set
ρ	Time discount rate	0.0412	Calibrated
$\dot{K}_{g,0}/Y_0$	Initial government investment-to-output ratio	0.0345	Data average
G_0/Y_0	Initial government expenditure-to-output ratio	0.2170	Data average
θ	Government investment-to-government expenditure ratio	0.1590	Data average
τ_r	Tax rate on capital income	0.2178	Data average
τ_w	Tax rate on labor income	0.2818	Data average
τ_c	Tax rate on consumption	0.1293	Data average
b_0	Initial debt-to-GDP ratio	1.13	Data average
δ_k	Depreciation rate for private capital	0.028	Set
δ_g	Depreciation rate for public capital	0.031	Set
ϕ	Adjustment parameter of the speed of debt reduction	0.01, 0.025, 0.05, 0.075, 0.1	Set

Table 2: Data Averages and Solutions in the Initial Steady State

Variable	Description	Data averages	Solution
C_0/Y_0	Consumption-to-output ratio	0.6985	0.6405
$(K_0 + K_{g,0})/Y_0$	Total capital-to-output ratio	3.558	2.7850
TR_0/Y_0	Tax revenue-to-output ratio	0.3262	0.3634

Table 3: Welfare Effects ($\theta = 0.1590$)

$\bar{b}_{new}$	$\gamma^*_{H,new}$		ϕ =0.01	ϕ =0.025	ϕ =0.05	ϕ =0.075	ϕ =0.1
0.6	0.03538	ΔC_0	0.382%	0.9%	1.65%	2.28%	2.82%
		ΔU_0	0.559%	1.31%	2.38%	3.26%	4%
		ΔC_{equiv}	0.374%	0.883%	1.62%	2.23%	2.76%
			(20.9)	(8.4)	(4.2)	(2.8)	(2.1)
0	0.03575	ΔC_0	0.819%	1.95%	3.62%	5.07%	6.34%
		ΔU_0	1.19%	2.8%	5.1%	7.01%	8.63%
		ΔC_{equiv}	0.803%	1.91%	3.55%	4.96%	6.21%
			(9.3)	(3.7)	(1.9)	(1.2)	(0.9)

The figures in parentheses are the years taken for the debt-to-GDP ratio to decrease by 10% from 1.13 to 1.03.

Table 4: Welfare Effects with Tax Increases

$\bar{b}_{new}$		Revenue	ϕ =0.01	ϕ =0.025	ϕ =0.05	ϕ =0.075	ϕ =0.1
0.6	Benchmark		0.559%	1.31%	2.38%	3.26%	4%
	τ_r	1% ↑	-7.35%	-6.59%	-5.52%	-4.63%	-3.88%
		2% ↑	-10.3%	-9.56%	-8.47%	-7.57%	-6.81%
		3% ↑	-13.6%	-12.8%	-11.7%	-10.8%	-10%
	τ_w	1% ↑	-0.549%	0.174%	1.2%	2.04%	2.75%
		2% ↑	-0.863%	-0.148%	0.861%	1.69%	2.39%
		3% ↑	-1.17%	-0.467%	0.529%	1.35%	2.04%
	τ_c	1% ↑	0.122%	0.874%	1.95%	2.84%	3.6%
		2% ↑	0.00149%	0.753%	1.83%	2.72%	3.49%
		3% ↑	-0.116%	0.635%	1.71%	2.61%	3.38%

Table 5: Welfare Effects: Public Services ($\theta = 0.1590$)

ψ	ϕ =0.01	ϕ =0.025	ϕ =0.05	ϕ =0.075	ϕ =0.1
0	0.559%	1.31%	2.38%	3.26%	4%
0.1	0.692%	1.55%	2.7%	3.63%	4.39%
0.3	0.904%	1.94%	3.23%	4.21%	4.99%
0.5	1.06%	2.24%	3.63%	4.65%	5.46%

Table 6: Welfare Effects: Public Services and Tax Increases

$\bar{b}_{new}$		Revenue	ϕ =0.01	ϕ =0.025	ϕ =0.05	ϕ =0.075	ϕ =0.1
0.6	Benchmark		1.06%	2.24%	3.63%	4.65%	5.46%
	τ_r	1% ↑	-6.1%	-4.95%	-3.57%	-2.56%	-1.75%
		2% ↑	-8.59%	-7.43%	-6.05%	-5.03%	-4.23%
		3% ↑	-11.3%	-10.1%	-8.73%	-7.7%	-6.89%
	τ_w	1% ↑	0.637%	1.76%	3.09%	4.07%	4.85%
		2% ↑	0.505%	1.61%	2.93%	3.9%	4.66%
		3% ↑	0.371%	1.47%	2.77%	3.72%	4.48%
	τ_c	1% ↑	0.885%	2.02%	3.38%	4.4%	5.2%
		2% ↑	0.835%	1.96%	3.32%	4.32%	5.13%
		3% ↑	0.786%	1.9%	3.25%	4.25%	5.06%

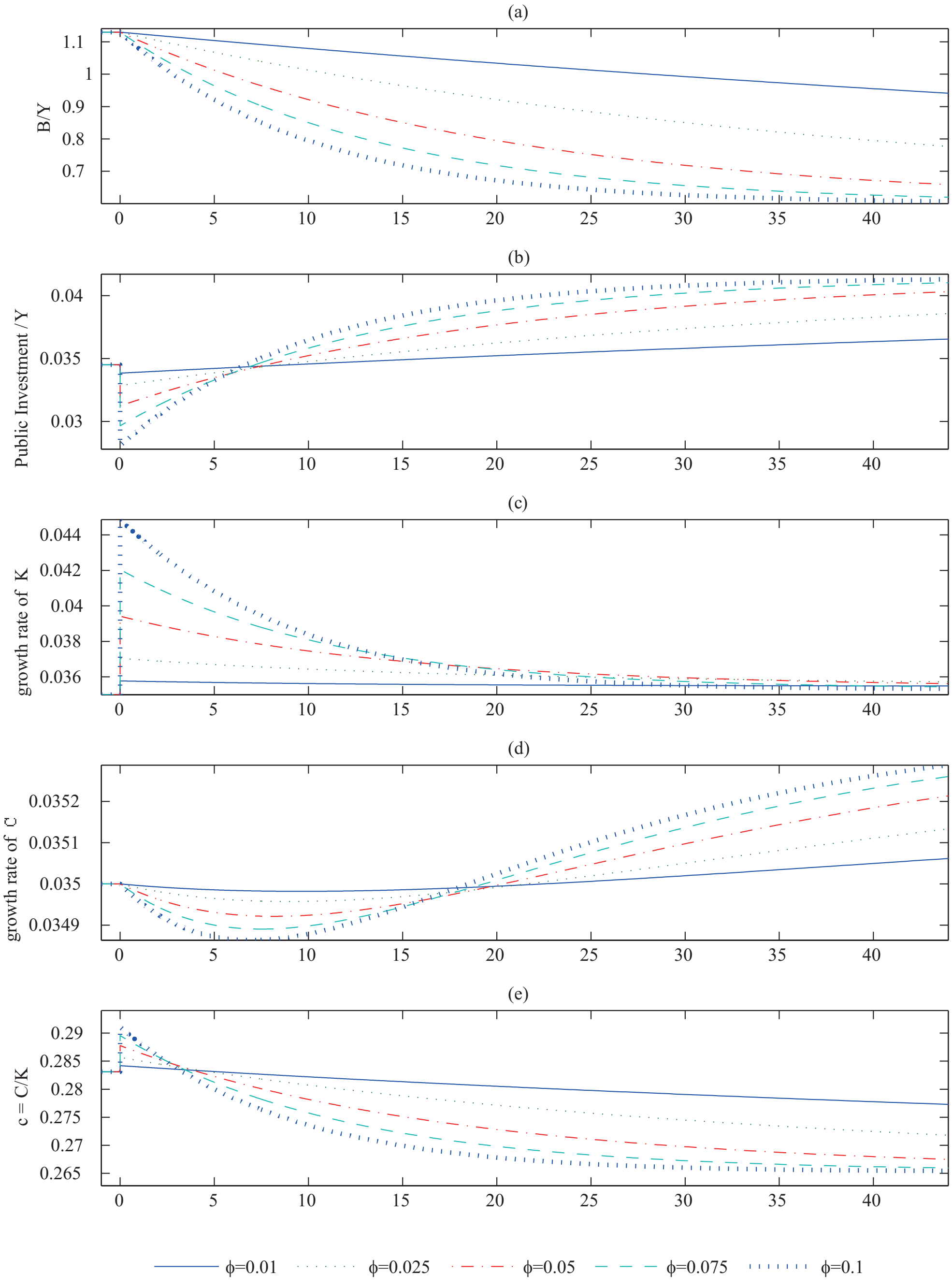


Figure 1: Transition Dynamics

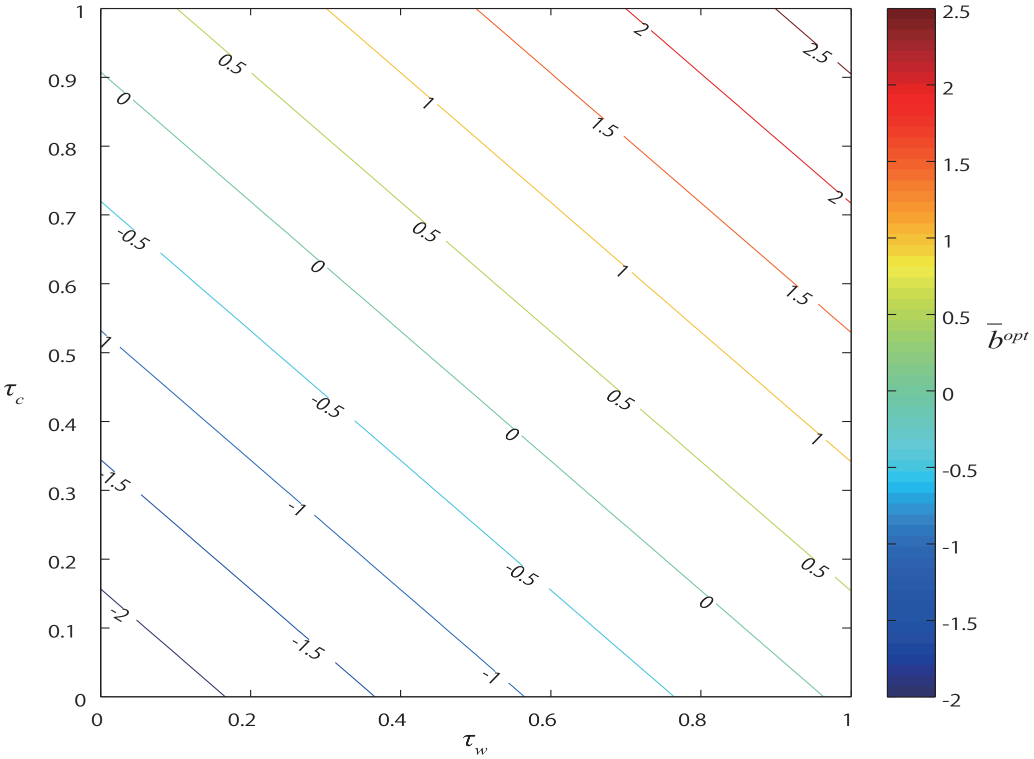


Figure 2: The Relationship between $\bar{b}^{opt}$ and (τ_c, τ_w)

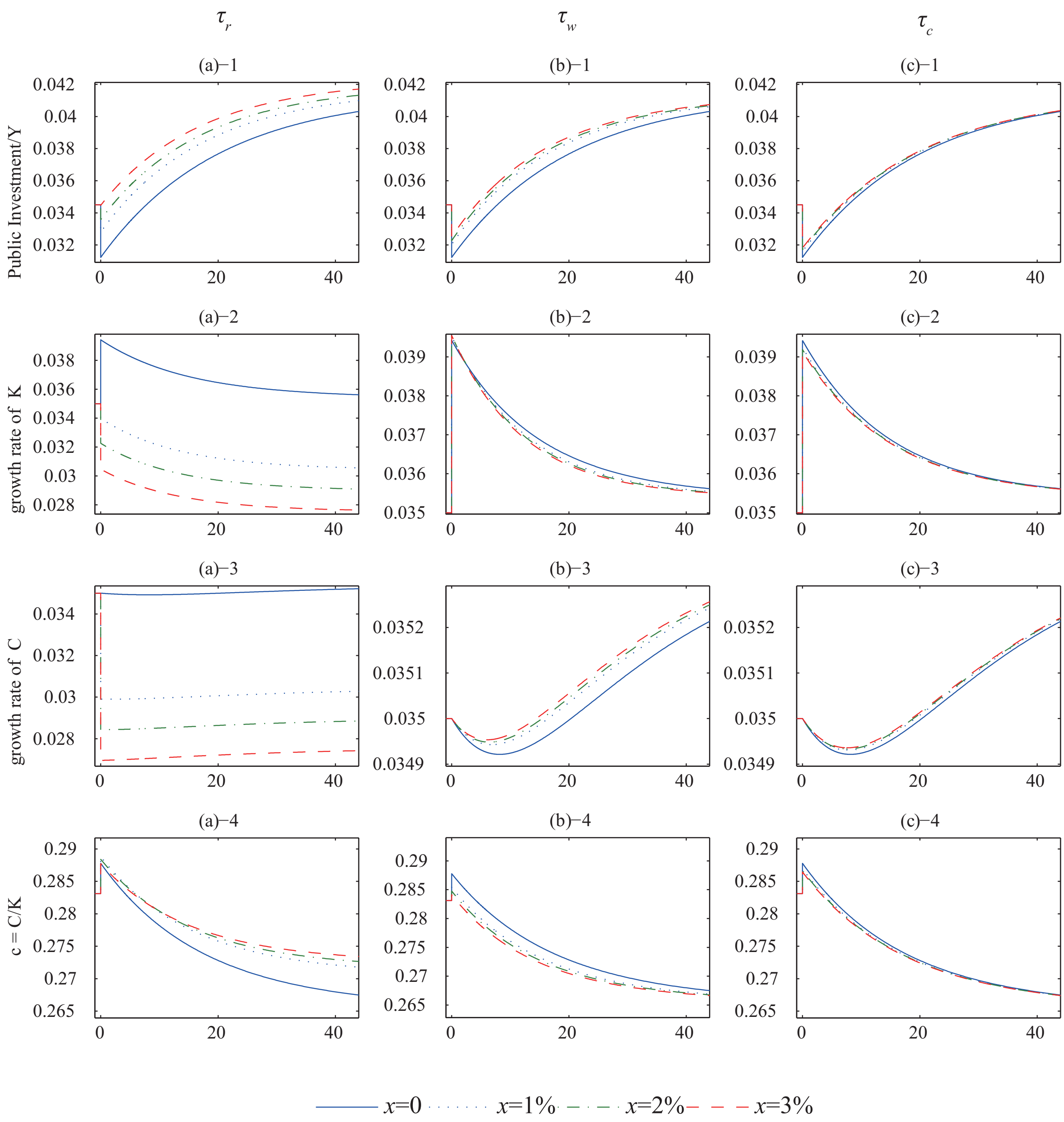
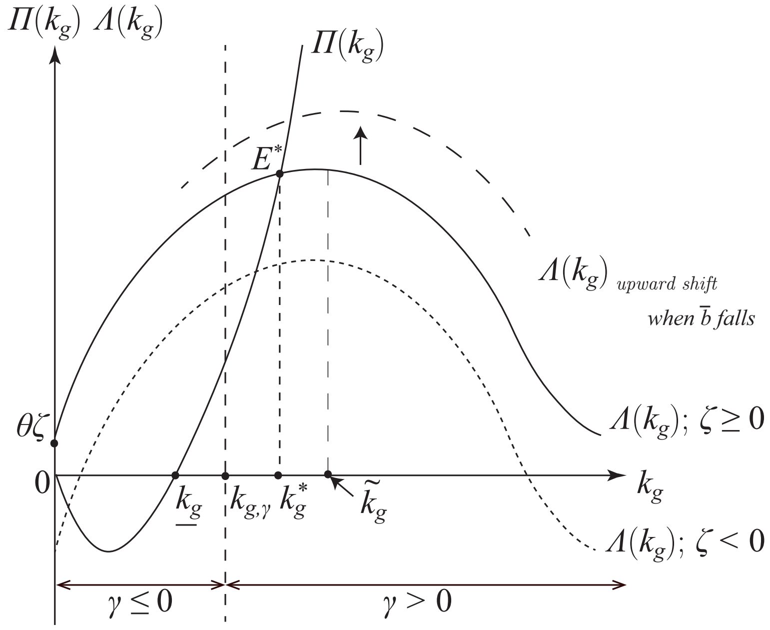


Figure 3: Transition Dynamics with Tax Increases

(a) $1 > \frac{\sigma\delta_g}{\rho+(1-\tau_r)\delta_k}$



(b) $1 \leq \frac{\sigma\delta_g}{\rho+(1-\tau_r)\delta_k}$

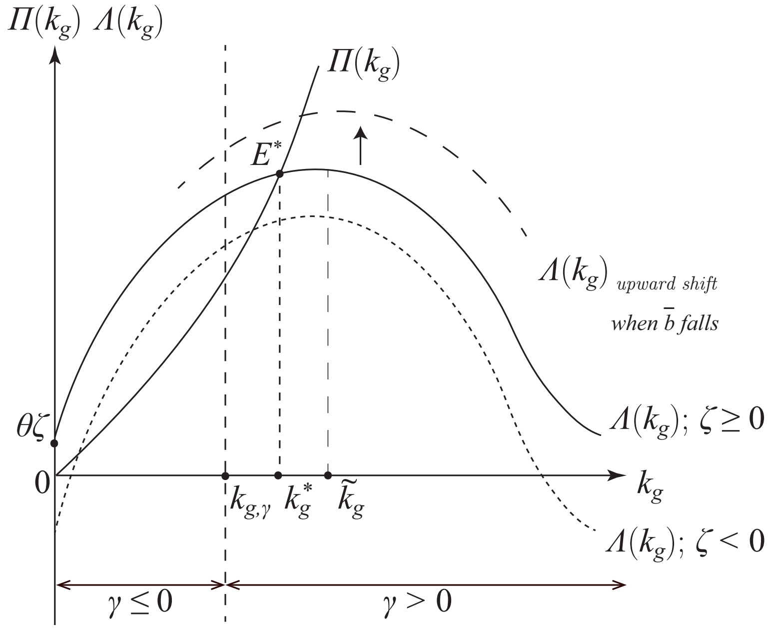
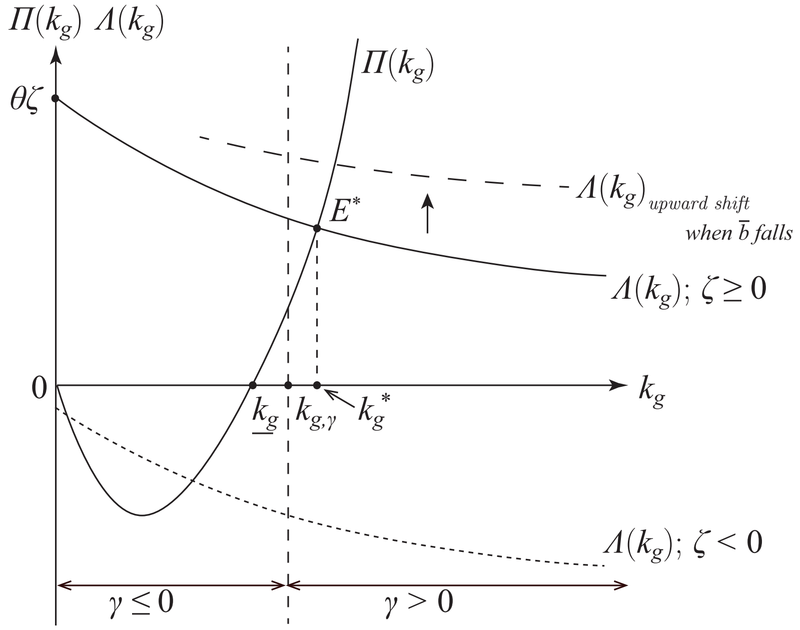


Figure 4: Steady State When $\eta > 0$

(a) $1 > \frac{\sigma\delta_g}{\rho+(1-\tau_r)\delta_k}$



(b) $1 \leq \frac{\sigma\delta_g}{\rho+(1-\tau_r)\delta_k}$

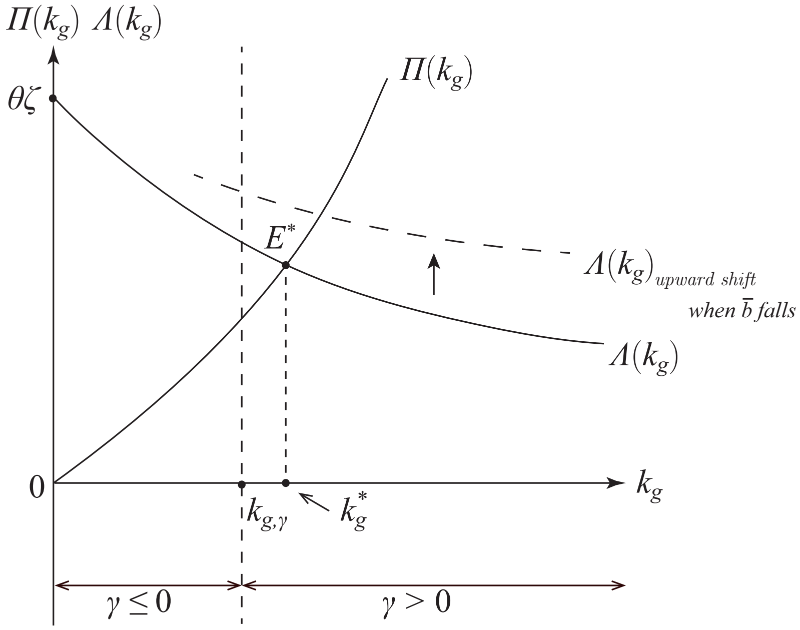


Figure 5: Steady State When $\eta \leq 0$