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Abstract

This paper examines the welfare-maximizing degree of patent protection in a growth model where the engines of economic growth are R&D and public services. We find that an increase in public services enhances the positive and negative effects of strengthening patent protection on R&D and the volume of production, respectively. However, if public services are relatively small, the negative welfare effect associated with the decrease in production volume tends to outweigh the positive welfare effect from the increase in the growth rate, and so the welfare-maximizing degree of patent protection tends to be lower. This result provides one possible explanation for why developing countries tend to prefer weaker patent protection.

Keywords: endogenous growth, patent protection, public services, welfare analysis

JEL classification: O34, O38, O40

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1 Introduction

Since the Agreement on TRIPs, patent protection has strengthened in both developing and developed countries. According to Park (2008), however, in many developing countries, including African countries and some Southeast Asian countries such as Thailand and Indonesia, the indices of patent protection remain relatively low.[1] The purpose of this paper is to examine why most developing countries do not prefer stronger patent protection.

It is commonly thought that developing countries are unwilling to strengthen patent protection because it impedes the domestic use of technologies created by developed countries. To consider this effect of patent protection, many early studies examine the effects of strengthening patent protection in North–South two-country models where technologies typically transfer from developed to developing countries (Gustafsson and Segerstrom 2011, Iwaisako et al. 2011, and Tanaka and Iwaisako 2012). On the other hand, infrastructure services exert a strong impact on the productivity of private inputs and the rate of return on capital, as discussed in Agénor and Moreno-Dodson (2006), and these also have a significant impact on the effects of strengthening patent protection. Thus, we anticipate that differences in infrastructure services can play an important role in explaining differences in the degree of patent protection. Hence, this paper focuses on the role of public services supplied by governments as the main reason for the differences in the strength of patent protection commonly found across countries.

By introducing productive public spending into a variety expansion-type R&D-based growth model following Rivera-Batiz and Romer (1991), we construct a model where both R&D and public services are the engines of economic growth. By examining the welfare-maximizing degree of patent protection, we show that a country that can maintain only a lower level of public services prefers weaker patent protection as long as the ratio of public services to output is not too high.[2] This result then explains why developing countries prefer the weaker protection of patents. In addition, this result is also consistent with the empirical evidence; we identify a positive correlation between the ratio of government spending to GDP and an index of patent protection as also shown in Figure 1.[3]

[1]The values of the patent protection indices in Thailand and Indonesia were 2.66 and 2.77 in 2005, while in most OECD countries these same values exceed 4.

[2] In fact, we find a small but positive correlation between per capita GDP and the ratio of government spending to GDP, as also shown in Appendix A.

[3] The data source for the ratio of government spending to GDP is the United Nations' National Accounts Main Aggregates Database, while the data source for the index of patent protection is Park (2008). The latest data for the index of patent protection are for 2005 and thus we use the 2005 data. We obtain data for both variables for 118 countries. The scatterplot in Figure 1 depicts the relation between the ratio of government spending to GDP and index of patent protection for 118 countries in 2005, for which we can easily discern a positive correlation.

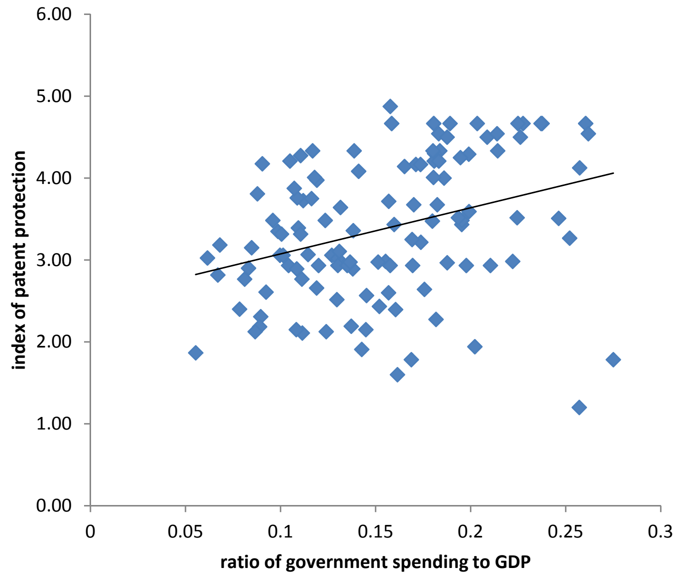

Figure 1: Ratio of government spending to GDP and patent protection

There are some important studies relating to the present analysis. Grossman and Lai (2004), for example, derived the Nash equilibrium patent length in a two-country game and showed that the country with more abundant human capital chooses a longer patent length. However, unlike the present work, this was not a growth model. Chu and Lai (2012) also examined the effects of productive public services in an R&D-based growth model. However, they focused on the effects of defense R&D as a public service on growth and welfare and derived only the welfare-maximizing level of defense R&D. Finally, Chu, Cozzi, and Gali (2011) examined optimal patent protection in developing countries and showed that the optimal degree of patent protection increases as an economy endogenously develops toward the global technology frontier. In contrast with the present study, which examines how the optimal patent protection depends on the public services a country can provide, their study examined how the optimal patent protection depends on the stage of development. Thus, their study and the present analysis are complementary.

2 Model

Following Barro (1990), we assume that governments can increase the productivity of private firms by providing public services. The final good is produced using labor and intermediate goods as follows: $Y_t = {L_{Y,t}}^{1-\alpha} \int_0^{N_t} (G_t X_{i,t})^{\alpha} \, di, \ 0 < \alpha < 1$, where $L_{Y,t}$ denotes the volume of labor devoted to final good production, G_t is productive public spending, and $X_{i,t}$ is the volume of intermediate good i devoted to final good production. The market for the final good is assumed to be perfectly competitive and we take the final good as numeraire. Letting $P_{i,t}$ denote the price of intermediate good i, we obtain the conditions for profit maximization as follows:

$$
\begin{aligned}
w_t &= (1-\alpha){L_{Y,t}}^{-\alpha} \int_0^{N_t} (G_t X_{i,t})^{\alpha} \, di, &(1)\\
P_{i,t} &= \alpha {L_{Y,t}}^{1-\alpha} {G_t}^{\alpha} {X_{i,t}}^{\alpha-1}. &(2)
\end{aligned}
$$

One unit of labor produces one unit of each intermediate good and a single firm holding the patent monopolistically supplies each intermediate good. Now we must consider how governments protect patents. Generally, governments control the degree of patent protection through patent length and breadth. However, for simplicity, we assume that the patent length is fixed and infinite and that governments control the degree of patent protection using only patent breadth.[4] Following Goh and Olivier (2002), Iwaisako and Futagami (2012), and Chu et al. (2012), we assume that the broader the government makes patent breadth, the more difficult it is to produce imitative goods. We specify the unit cost of producing imitative goods as βw_t ($\beta \in (1, 1/\alpha]$), where strengthening patent protection increases the value of β. Each firm that produces an intermediate good charges a price such that producers of imitative goods cannot earn positive profits, as follows: $P_{i,t} = \beta w_t$. Because prices are symmetric across intermediate goods, the volumes of production are also symmetric. Letting X_t denote the volume of production, from the demand for each intermediate good (2), X_t satisfies

$$
\beta w_t = \alpha {L_{Y,t}}^{1-\alpha} {G_t}^{\alpha} {X_t}^{\alpha-1}. \tag{3}
$$

The intermediate goods firms are symmetric, and thus the profits are symmetric. Letting π denote the profit, we obtain $\pi = (\beta - 1) w_t X_t$.

We turn now to R&D activities. Devoting a units of the final good in the time interval dt, R&D firms can invent dt units of intermediate goods. Letting V_t denote the patent value of one intermediate good

[4] A number of studies have specified the probability of imitation as a parameter in the enforcement of patents and have examined the effects of strengthening patent protection; these include Eicher and García–Peñalosa (2008), Furukawa (2007), Horii and Iwaisako (2007), and Gangopadhyay and Mondal (2012).

and the stock value of a firm producing one intermediate good, we obtain the R&D equilibrium condition as follows: $V_t = a$. Next, we consider the equilibrium condition of the stock value of a firm (the price of the patent). Here we assume that the profit of firms is taxed at rate τ. As discussed below, the tax revenue is spent on public services. If households possess one unit of stock in the time interval dt, they can obtain a profit of $(1-\tau)\pi_t dt$ and a capital gain or a loss of $\dot{V}_t$. Alternatively, they can invest V_t units of funds in a risk-free asset, and obtain $r_t V_t$, where r_t is the interest rate of the risk-free asset. Therefore, in equilibrium, the no-arbitrage condition must hold as follows: $r_t V_t = (1-\tau)\pi_t + \dot{V}_t$.

The economy consists of a unit continuum of identical households, each of which consists of L consumers. Each consumer inelastically supplies one unit of labor at each time point. The lifetime utility of the household is given by $U = \int_0^\infty e^{-\rho t} \ln c_t dt$, where c_t is per capita consumption and ρ is the subjective discount rate. We assume that both labor income and corporate income are taxed at the rate τ. The intertemporal budget constraint is given by $\int_0^\infty \exp(-\int_0^t r_s ds) c_t dt = A_0 + \int_0^\infty \exp(-\int_0^t r_s ds)(1-\tau) w_t dt$, where r_t is the interest rate, A_0 is the initial per capita asset holdings, and w_t denotes the wage. The necessary condition for the maximization of the household's utility is given by the following Euler equation: $\frac{\dot{c}_t}{c_t} = r_t - \rho$.

We assume that the government provides public services, G_t, to keep the ratio of public services to output constant at g (that is, $G_t = gY_t$) and that the budget for the government balances at each point of time. As discussed, the government collects tax revenues by imposing taxes on labor and corporate income at the constant rate of τ; that is, the tax revenue is $\tau(w_t L + \pi_t N_t) = \tau Y_t$. Thus, the tax rate equals the public services–output ratio; that is, $\tau = g$.

3 The market equilibrium path

In this section, we derive the market equilibrium path in this economy.

To start with, we derive the equilibrium allocation of labor. From (1) and (3), we obtain $\frac{N_t X_t}{L_{Y,t}} = \frac{\alpha/\beta}{1-\alpha}$. Combining this and the labor market equilibrium condition, $L = L_{Y,t} + \int_0^{N_t} X_{i,t} di$, yields $L_{Y,t} = \frac{1-\alpha}{1-\alpha+(\alpha/\beta)} L$ and $N_t X_t = \frac{\alpha/\beta}{1-\alpha+(\alpha/\beta)} L$. The production function of the final good can be rewritten as $Y_t = L_{Y,t}^{1-\alpha} N_t (G_t X_t)^\alpha$, and the efficient ratio is given by $\frac{N_t X_t}{L_{Y,t}} = \frac{\alpha}{1-\alpha}$, and thus, $\beta = 1$ obtains the efficient allocation of labor. Furthermore, we can see that the volume of production decreases as β increases. Substituting these and $G_t = gY_t$ into the production function, we obtain output as follows:

$$Y_t = g^{\frac{\alpha}{1-\alpha}} A(\beta) N_t, \quad \text{where} \quad A(\beta) \equiv \left[\frac{(\alpha/\beta)^\alpha (1-\alpha)^{1-\alpha}}{1-\alpha+(\alpha/\beta)} L \right]^{\frac{1}{1-\alpha}}. \tag{4}$$

This shows that the reduced-form production function is of an AK structure, and thus, the ratio of consumption to the number of invented goods is constant on the equilibrium path in the present model. Here, we let χ_t denote the ratio; that is, $\chi_t \equiv C_t/N_t$. Consequently, the growth rate γ and the ratio of aggregate consumption to the number of invented goods χ are given by[5]

$$\gamma = (1-g)\frac{\beta-1}{\beta}\alpha\frac{g^{\frac{\alpha}{1-\alpha}}A(\beta)}{a} - \rho, \tag{5}$$

and

$$\chi = \left(1 - \frac{\beta-1}{\beta}\alpha\right)(1-g)g^{\frac{\alpha}{1-\alpha}}A(\beta) + a\rho. \tag{6}$$

Before conducting the welfare analysis, we examine how strengthening patent protection affects economic growth and the consumption level. First, we can show that strengthening patent protection enhances economic growth, as follows:

$$\frac{\partial\gamma}{\partial\beta} = (1-g)g^{\frac{\alpha}{1-\alpha}}\frac{1}{a}\left[\frac{\alpha}{\beta^2}A(\beta) + \frac{\beta-1}{\beta}\alpha A'(\beta)\right] > 0. \tag{7}$$

Strengthening patent protection lowers average productivity $Y_t/N_t = g^{\frac{\alpha}{1-\alpha}}A(\beta)$. However, the positive effect obtained through increasing profit overwhelms this negative effect.[6] We also show that strengthening patent protection reduces the ratio of consumption to the number of invented goods, as follows:

$$\frac{\partial\chi}{\partial\beta} = (1-g)g^{\frac{\alpha}{1-\alpha}}\left[-\frac{\alpha}{\beta^2}A(\beta) + \left(1 - \frac{\beta-1}{\beta}\alpha\right)A'(\beta)\right] < 0, \tag{8}$$

where we use the fact that $A'(\beta) < 0$. Strengthening patent protection lowers average productivity by bringing about a distortion in labor allocation. This enhances R&D investment and therefore necessarily reduces consumption. Next, we examine how increasing public services affects the growth-enhancing effect of patent protection, $\frac{\partial\gamma}{\partial\beta}$, and the consumption-reducing effect, $\frac{\partial\chi}{\partial\beta}$. An increase in the public services–output ratio, g, has two opposing effects, as shown in Barro (1990). That is, while it increases productivity and output, it also raises the tax rate, weakens the incentive for R&D, and thereby reduces after-tax income. Therefore, both the growth rate and the ratio of consumption to the number of invented goods display an inverted-U shape with respect to g, as we can see from (5) and (6). Put strictly, if the public services–output ratio is so low that $g < \alpha$, an increase in the public services–output ratio increases both the growth rate and the ratio of consumption to the number of invented goods; otherwise, an increase in g reduces both.

Furthermore, inspecting (7) and (8), we can easily see that if the public services–output ratio is lower (higher) than α, increasing the public services–output ratio enhances (weakens) both the growth-enhancing effect and the consumption-reducing effect *proportionally*.

[5]See Appendix B for a detailed derivation.

[6]See Appendix C for the proof and property of $A(\beta)$.

4 Welfare-maximizing patent policy

We can easily calculate the lifetime utility as a function of patent breadth, β. From the fact that $c_t = \chi N_t/L = (N_0/L)\chi e^{\gamma t}$, we can obtain the lifetime utility: $U = \frac{1}{\rho}\left[\ln\chi + \frac{\gamma}{\rho}\right] + \frac{\ln(N_0/L)}{\rho}$, and thus the welfare-maximizing condition of β satisfies[7] [8]

$$\frac{\rho}{\chi} = -\frac{\frac{\partial\gamma}{\partial\beta}}{\frac{\partial\chi}{\partial\beta}}. \tag{9}$$

As shown in Appendix D, we obtain the following proposition:

Proposition 1 *If the public services–output ratio is low (high) such that $g < \alpha$ ($g > \alpha$), an increase in the public services–output ratio raises (reduces) the welfare-maximizing degree of patent protection.*

We can understand the reason for this result by using the welfare-maximizing condition (9), as follows. We focus on the case where $g < \alpha$: that is, an increase in g raises both the growth rate and the ratio of consumption to the number of invented goods.[9] The increase in g strengthens both the growth-enhancing effect and the consumption-reducing effect of strengthening patent protection. However, these positive and negative effects are proportional, as discussed in the previous section, and therefore, the ratio of the two effects $\frac{\partial\gamma}{\partial\beta}/\frac{\partial\chi}{\partial\beta}$ does not depend on g. Consequently, the increase in g has no impact on the RHS of (9). On the other hand, as shown in $\frac{\partial U}{\partial\beta} = \frac{1}{\rho}\left[\frac{1}{\chi}\frac{\partial\chi}{\partial\beta} + \frac{1}{\rho}\frac{\partial\gamma}{\partial\beta}\right]$, the marginal utility of the level of consumption is diminishing while the marginal utility of the growth rate is constant. The increase in g thus reduces the marginal utility of χ, $\frac{1}{\chi}$ and weakens the consumption-reducing effect. As a result, an increase in g raises the welfare-maximizing degree of patent protection.

From the proposition, a country with a high level of public services prefers stronger patent protection unless the public services–output ratio is so high that $g > \alpha$. Conversely, a poor country that cannot maintain a high level of public services prefers weaker patent protection. This result may then explain why patent protection in many developing countries is relatively weak. On the other hand, and as shown in Park (2008), some rapidly growing countries, such as China and Vietnam, have strengthened patent protection since 1990. Using the results from the present model, we can explain this tendency as follows. Because of economic development, these countries have become able to maintain a high level of public

[7] The economy jumps to the balanced growth path at time 0 so that N_0 is exogenously given at time 0.

[8] We can prove that the welfare function is concave in patent breadth as shown in Appendix E.

[9] Using 2010 data (the most recent available in the United Nations' National Accounts Main Aggregates Database), we find that the ratio of government spending is less than 0.3 in most countries. We also confirm this tendency in 2005, as shown in Figure 1. In the present model, we consider α as the capital share, and thus $g < \alpha$ is the empirically relevant case.

services, and thus their welfare-maximizing patent protection has increased in strength.[10] Moreover, the TRIPs agreement provides developing countries with an extension of time for strengthening patent protection, for which the result in this paper provides something of a rationale.

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Appendix A: Data on ratio of government spending to GDP and per capita GDP

In this appendix, in order to observe the actual relation between per capita GDP and the ratio of government spending to GDP, we provide data for these variables in Figure 2. We use the 2010 data because they are the most recent data available in the United Nations' National Accounts Main Aggregates Database. We identify a small but positive correlation between these variables.

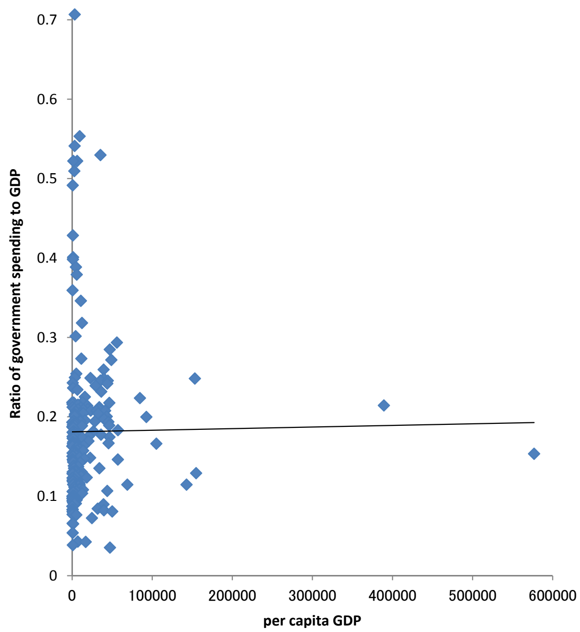

Figure 2: Per capita GDP and ratio of government spending to GDP

Appendix B: Growth rate and ratio of consumption to the number of invented goods

In this appendix, we derive the equilibrium growth rate and the equilibrium ratio of consumption to the number of invented goods.

Substituting the R&D equilibrium condition, $V_t = a$, into the no-arbitrage condition, $r_t V_t = (1 - \tau)\pi_t + \dot{V}_t$, we obtain the interest rate, as follows: $r_t = (1-g)\frac{\pi_t}{a}$. From (3), $w_t N_t X_t = \frac{\alpha}{\beta} Y_t$. Substituting this into π_t, we get $\pi_t N_t = \frac{\beta - 1}{\beta}\alpha Y_t$. Thus, the interest rate is $r_t = (1-g)\dfrac{\beta - 1}{\beta}\alpha\dfrac{g^{\frac{\alpha}{1-\alpha}} A(\beta)}{a}$. From the

Euler equation, $\dot{c}_t/c_t = r_t - \rho$, the growth rate of consumption is given by

$$\gamma = (1-g)\frac{\beta-1}{\beta}\alpha\frac{g^{\frac{\alpha}{1-\alpha}}A(\beta)}{a} - \rho.$$

The growth rate of $\chi_t (\equiv C_t/N_t)$ is given by $\frac{\dot{\chi}_t}{\chi_t} = \gamma - \frac{\dot{N}_t}{N_t}$. From the goods market equilibrium condition $Y_t = C_t + a\dot{N}_t + G_t$, $\frac{\dot{N}_t}{N_t} = (1-\tau)\frac{Y_t}{aN_t} - \frac{C_t}{aN_t} = (1-g)\frac{g^{\frac{\alpha}{1-\alpha}}A(\beta)}{a} - \frac{\chi_t}{a}$. Therefore, the differential equation of χ_t is given by $\frac{\dot{\chi}_t}{\chi_t} = \frac{\chi_t - \chi}{a}$, where

$$\chi = \left[1 - \frac{\beta-1}{\beta}\alpha\right](1-g)g^{\frac{\alpha}{1-\alpha}}A(\beta) + a\rho.$$

This unique steady state is unstable, and we can show that any path diverging to infinity violates the Euler equation and any path converging to 0 violates the transversality condition. Therefore, the unique solution of χ_t is given by $\chi_t = \chi$.

Appendix C: Proof of $A'(\beta) < 0$ and $\frac{\partial\gamma}{\partial\beta} > 0$

In this appendix, we show that $A'(\beta) < 0$ and that $\frac{\partial\gamma}{\partial\beta} > 0$.

First, $A'(\beta)$ can be calculated as follows:

$$A'(\beta) = -\frac{\beta-1}{\beta}\alpha\frac{1}{(1-\alpha)\beta+\alpha}A(\beta) < 0. \tag{10}$$

Substituting this into (7) and rewriting this yields

$$\frac{\partial\gamma}{\partial\beta} = (1-g)g^{\frac{\alpha}{1-\alpha}}\frac{1}{a}A(\beta)\frac{\alpha(1+\alpha-\alpha\beta)}{\beta\left[(1-\alpha)\beta+\alpha\right]} > 0, \tag{11}$$

where we use $\beta \leq 1/\alpha$.

Appendix D: Proof of Proposition 1

In this appendix, we prove Proposition 1.

In the remainder of the analysis, and to focus on the case where the growth rate is positive, we limit the analysis to the range β, $(\beta_{min}, 1/\alpha]$, where β_{min} satisfies $(1-g)\frac{\beta_{min}-1}{\beta_{min}}\alpha\frac{g^{\frac{\alpha}{1-\alpha}}A(\beta_{min})}{a} = \rho$.

First, we calculate the right-hand side (RHS) of (9), $-\frac{\frac{\partial\gamma}{\partial\beta}}{\frac{\partial\chi}{\partial\beta}}$. Substituting (7) and (8) into this, we obtain

$$\begin{aligned}
-\frac{\frac{\partial\gamma}{\partial\beta}}{\frac{\partial\chi}{\partial\beta}} &= -\frac{\frac{1}{a}\left[\frac{\alpha}{\beta^2}A(\beta) + \frac{\beta-1}{\beta}\alpha A'(\beta)\right]}{\left[-\frac{\alpha}{\beta^2}A(\beta) + \left(1-\frac{\beta-1}{\beta}\alpha\right)A'(\beta)\right]} \\
&= -\frac{1}{a}\frac{\frac{\alpha}{\beta^2} + \frac{\beta-1}{\beta}\alpha\frac{A'(\beta)}{A(\beta)}}{-\frac{\alpha}{\beta^2} + \left(1-\frac{\beta-1}{\beta}\alpha\right)\frac{A'(\beta)}{A(\beta)}}.
\end{aligned}$$

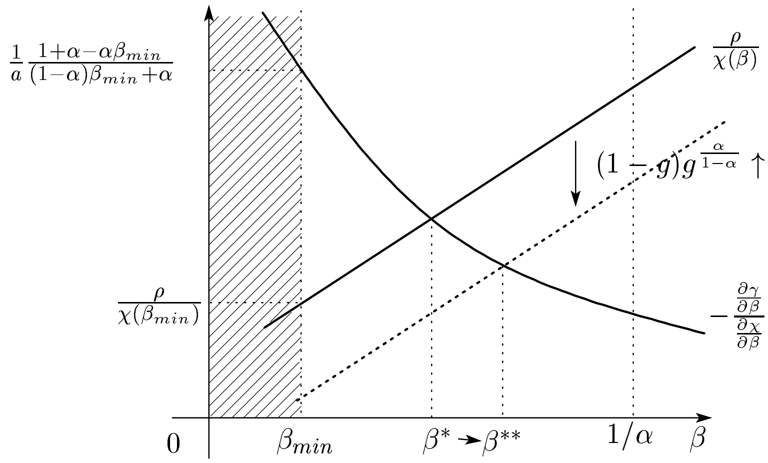

Figure 3: Determination of the welfare-maximizing patent breadth

Furthermore, substituting $A'(\beta)$ into this and rewriting it yields

$$
\begin{aligned}
-\frac{\frac{\partial\gamma}{\partial\beta}}{\frac{\partial\chi}{\partial\beta}} &= -\frac{1}{a}\frac{\frac{\alpha}{\beta^2}-\frac{\beta-1}{\beta}\alpha\frac{\beta-1}{\beta}\alpha\frac{1}{(1-\alpha)\beta+\alpha}}{-\frac{\alpha}{\beta^2}-\left(1-\frac{\beta-1}{\beta}\alpha\right)\frac{\beta-1}{\beta}\alpha\frac{1}{(1-\alpha)\beta+\alpha}} \\
&= \frac{1}{a}\frac{\frac{\alpha}{\beta^2}-\left(\frac{\beta-1}{\beta}\right)^2\alpha^2\frac{1}{(1-\alpha)\beta+\alpha}}{\frac{\alpha}{\beta^2}+\left(1-\frac{\beta-1}{\beta}\alpha\right)\frac{\beta-1}{\beta}\alpha\frac{1}{(1-\alpha)\beta+\alpha}} \\
&= \frac{1}{a}\frac{\frac{\alpha}{\beta^2}\left[1-(\beta-1)^2\alpha\frac{1}{(1-\alpha)\beta+\alpha}\right]}{\frac{\alpha}{\beta^2}\left[1+(\beta-(\beta-1)\alpha)(\beta-1)\frac{1}{(1-\alpha)\beta+\alpha}\right]} \\
&= \frac{1}{a}\frac{\left[1-(\beta-1)^2\alpha\frac{1}{(1-\alpha)\beta+\alpha}\right]}{\left[1+((1-\alpha)\beta+\alpha)(\beta-1)\frac{1}{(1-\alpha)\beta+\alpha}\right]} \\
&= \frac{1}{a\beta}\left[1-\frac{\alpha(\beta-1)^2}{(1-\alpha)\beta+\alpha}\right] \\
&= \frac{1}{a}\frac{1+\alpha-\alpha\beta}{(1-\alpha)\beta+\alpha}.
\end{aligned}
$$

Therefore, the RHS of (9) is a decreasing function of β.

χ is a decreasing function of β and the left-hand side (LHS) of (9) is an increasing function of β. Therefore, (9) determines the welfare-maximizing patent breadth uniquely, as depicted in Figure 3.

Using Figure 3, we now examine how an increase in the public services–output ratio affects the welfare-maximizing patent breadth. First, an increase in g has no impact on the ratio of the growth-enhancing effect to the consumption-reducing effect, $-\frac{\partial\gamma}{\partial\beta}/\frac{\partial\chi}{\partial\beta}$, and thus the curve remains unchanged. Second, we consider the impact of g on ρ/χ. As mentioned, an increase in the public services–output ratio affects the ratio of consumption to the number of invented goods in opposite ways. If $g < \alpha$

($g > \alpha$), the increase in g increases (decreases) χ and thus shifts the curve ρ/χ downward (upward). This raises (lowers) the welfare-maximizing patent breadth, as depicted in Figure 3. Thus, we can prove Proposition 1.

Appendix E: The proof of concavity of the welfare function

In this appendix, to ensure that the second-order condition is satisfied, we prove that the welfare function $U(\beta)$ is concave in patent breadth.

The second derivative of $U(\beta)$ is given by

$$\frac{\partial^2 U}{\partial \beta^2} = \frac{1}{\rho}\left[-\frac{1}{\chi^2}\left(\frac{\partial \chi}{\partial \beta}\right)^2 + \frac{1}{\chi}\frac{\partial^2 \chi}{\partial \beta^2} + \frac{1}{\rho}\frac{\partial^2 \gamma}{\partial \beta^2}\right].$$

First, from (7) and (8), we obtain the second derivatives of γ and χ as follows:

$$\frac{\partial^2 \gamma}{\partial \beta^2} = (1-g)g^{\frac{\alpha}{1-\alpha}}\frac{1}{a}\left[-2\frac{\alpha}{\beta^2}A(\beta) + 2\frac{\alpha}{\beta^2}A'(\beta) + \frac{\beta-1}{\beta}\alpha A''(\beta)\right], \tag{12}$$

and

$$\frac{\partial^2 \chi}{\partial \beta^2} = (1-g)g^{\frac{\alpha}{1-\alpha}}\left[2\frac{\alpha}{\beta^3}A(\beta) - 2\frac{\alpha}{\beta^2}A'(\beta) + \left(1-\frac{\beta-1}{\beta}\alpha\right)A''(\beta)\right]. \tag{13}$$

Using (6), (12) and (13), we get

$$\begin{aligned}
\frac{\partial^2 \chi}{\partial \beta^2} + \frac{\chi}{\rho}\frac{\partial^2 \gamma}{\partial \beta^2} \quad = \quad & (1-g)g^{\frac{\alpha}{1-\alpha}}\Bigg(\left[2\frac{\alpha}{\beta^3}A(\beta) - 2\frac{\alpha}{\beta^2}A'(\beta) + \left(1-\frac{\beta-1}{\beta}\alpha\right)A''(\beta)\right] \\
& + \left[\left(1-\frac{\beta-1}{\beta}\alpha\right)(1-g)g^{\frac{\alpha}{1-\alpha}}A(\beta)\frac{1}{\rho a} + 1\right]\left[-2\frac{\alpha}{\beta^3}A(\beta) + 2\frac{\alpha}{\beta^2}A'(\beta) + \frac{\beta-1}{\beta}\alpha A''(\beta)\right]\Bigg) \\
= \quad & (1-g)g^{\frac{\alpha}{1-\alpha}}\Bigg(A''(\beta) \\
& + \left(1-\frac{\beta-1}{\beta}\alpha\right)(1-g)g^{\frac{\alpha}{1-\alpha}}A(\beta)\frac{1}{\rho a}\left[-2\frac{\alpha}{\beta^3}A(\beta) + 2\frac{\alpha}{\beta^2}A'(\beta) + \frac{\beta-1}{\beta}\alpha A''(\beta)\right]\Bigg)
\end{aligned}$$

Therefore, if

$$A''(\beta) + \left(1-\frac{\beta-1}{\beta}\alpha\right)(1-g)g^{\frac{\alpha}{1-\alpha}}A(\beta)\frac{1}{\rho a}\left[-2\frac{\alpha}{\beta^3}A(\beta) + 2\frac{\alpha}{\beta^2}A'(\beta) + \frac{\beta-1}{\beta}\alpha A''(\beta)\right] > 0,$$

we can show that the second derivative of $U(\beta)$ is negative.

As mentioned in Appendix D, we assume that β satisfies $(1-g)\frac{\beta-1}{\beta}\alpha\frac{g^{\frac{\alpha}{1-\alpha}}A(\beta)}{a\rho} \geq 1$ in order to focus on the case where the growth rate is positive. Thus,

$$\begin{aligned}
& A''(\beta) + \left(1-\frac{\beta-1}{\beta}\alpha\right)(1-g)g^{\frac{\alpha}{1-\alpha}}A(\beta)\frac{1}{\rho a}\left[-2\frac{\alpha}{\beta^3}A(\beta) + 2\frac{\alpha}{\beta^2}A'(\beta) + \frac{\beta-1}{\beta}\alpha A''(\beta)\right] \\
& \quad > A''(\beta) + \left(1-\frac{\beta-1}{\beta}\alpha\right)\left(\frac{\beta-1}{\beta}\alpha\right)^{-1}\left[-2\frac{\alpha}{\beta^3}A(\beta) + 2\frac{\alpha}{\beta^2}A'(\beta) + \frac{\beta-1}{\beta}\alpha A''(\beta)\right].
\end{aligned} \tag{14}$$

Here, we obtain the second derivative of $A(\beta)$, as follows:

$$A''(\beta) = -\frac{\beta-2}{\beta-1}\frac{1}{(1-\alpha)\beta+\alpha}A'(\beta). \tag{15}$$

Substituting this into the LHS of the inequality, (14) can be rewritten as follows:

$$\begin{aligned}
&\left(\frac{\beta-1}{\beta}\alpha\right)\left(1-\frac{\beta-1}{\beta}\alpha\right)^{-1} \\
&\times\left[\left(\frac{\beta-1}{\beta}\alpha\right)\left(1-\frac{\beta-1}{\beta}\alpha\right)^{-1}A''(\beta)+\left[-2\frac{\alpha}{\beta^3}A(\beta)+2\frac{\alpha}{\beta^2}A'(\beta)+\frac{\beta-1}{\beta}\alpha A''(\beta)\right]\right] \\
&=\left(\frac{\beta-1}{\beta}\alpha\right)\left(1-\frac{\beta-1}{\beta}\alpha\right)^{-1}\left[-2\frac{\alpha}{\beta^3}A(\beta)+2\frac{\alpha}{\beta^2}A'(\beta)+\frac{2-\frac{\beta-1}{\beta}\alpha}{1-\frac{\beta-1}{\beta}\alpha}\frac{\beta-1}{\beta}\alpha A''(\beta)\right]
\end{aligned}$$

Therefore, if

$$-2\frac{\alpha}{\beta^3}A(\beta)+2\frac{\alpha}{\beta^2}A'(\beta)+\frac{2-\frac{\beta-1}{\beta}\alpha}{1-\frac{\beta-1}{\beta}\alpha}\frac{\beta-1}{\beta}\alpha A''(\beta), \tag{16}$$

is negative, we can show that $U''(\beta) < 0$.

Using (10) and (15), we can rewrite (16) as follows:

$$\frac{\alpha}{\beta}\left[(1-\alpha)\beta+\alpha\right]^{-3}A(\beta)F(\beta)$$

where

$$F(\beta) \equiv (2-\alpha)\alpha\beta^2 - 2\left[1+\alpha(1-\alpha)\right]\beta - \alpha^2. \tag{17}$$

The range of β is given by $(\beta_{min}, 1/\alpha]$. $F(1) = -2 < 0$, $F(1/\alpha) = -(1-\alpha)^2 - 2 < 0$, and $F(\beta)$ is a convex quadratic function. $\beta_{min} > 1$ and thus $F(\beta) < 0$ for $\beta \in (\beta_{min}, 1/\alpha]$.

Hence, $U''(\beta) < 0$ for $\beta \in (\beta_{min}, 1/\alpha]$ and thus we can show that the welfare function is concave in patent breadth.