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**Abstract**

This study presents an overlapping-generations model with altruism towards children. We characterize a Markov-perfect political equilibrium of voting over two policy issues, public education for the young and social security for the old. The model potentially generates two types of political equilibria, one favoring public education and the other favoring social security. One equilibrium is selected by the government to maximize its objective. It is shown that (i) longevity affects equilibrium selection and relevant policy choices; and (ii) private education as an alternative to public education and a Markov-perfect political equilibrium can generate the two types of equilibria.

- Keywords: Public education; Social security; Intergenerational conflict
- JEL Classification Number: H52, H55, I22.

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# 1 Introduction

With increasing life expectancy, many developed countries have experienced a shift towards a progressively older population over the past several decades. This demographic change has induced an increase in the political power of the elderly, and social security expenditure for them is expected to increase (Casey et al., 2003). This may imply a reduction in spending on the young (e.g., public education) because of government budgetary constraints (Poterba, 1997, 1998; Fernandez and Rogerson, 2001; Harris, Evans, and Schwab, 2001).

Such predictions are not always possible when altruistic concern for one's offspring is considered (Cattaneo and Wolter, 2009). Greater longevity means that parents can enjoy the human capital of their children longer in later life, making public and private investment in education more attractive for the working population. In addition, the effects of the increased political power of the elderly on social security expenditure is not straightforward. A greater social security burden on the working population discourages them from investing in education, which may result in a smaller tax base and a lower level of social security benefits in the long run. Furthermore, all these effects interact with each other. The following question arises: how does a conflict of interest between generations affect political decision-making on social security and public education (and hence, human capital accumulation) in the long run? This study aims to answer this question from a political economy viewpoint.

For analysis, this paper presents an overlapping-generations model with uncertain lifetimes and altruism towards children. In each generation, there are identical individuals who live for at most three periods: young, middle, and old. An individual produces one offspring during the middle period and dies at the end of the middle period with some probability. A middle-age individual is endowed with a stock of human capital that also defines his/her labor capacity. He/she sets the allocation of disposable income between his/her personal consumption and investment in furthering the child's education. An individual who lives throughout old age can receive and consume social security benefits. The level of an offspring's human capital is determined by public education and the parents' human capital and private investment. Social security and public education are financed using taxation on the middle.

Within this framework, we consider a probabilistic voting (à la Lindbeck and Weibull, 1987) in which in each period, the middle and the old participate in voting. Here, the government in power maximizes a political objective function of the weighted sum of the utilities of the middle and the old (see, e.g., Grossman and Helpman (1998), Hassler et al. (2005), and Song, Storesletten and Zilibotti (2012) for applications for the overlapping generations models). In particular, this paper restricts its attention to Markov-perfect equilibria, where voters condition their strategies on payoff-relevant state variables (i.e., human capital in the present model). This concept of equilibrium captures the forward-looking behavior of the middle, who expect the benefit of social security when they reach old age.

The present model demonstrates two types of political equilibria: the presence of private investment in education and the absence of public education, and vice versa. In both equilibria, social security is provided to old individuals. It is shown that the former equilibrium is realized if the efficiency of public education is relatively low compared to that of private education; otherwise, the latter equilibrium is observed.

Voters' preferences are affected by longevity. In particular, longevity has effects on the political determination of social security through the following three factors: (i) the weight on the utility of old-age social security that benefits the old; (ii) the tax burden of the middle to finance the current old-age social security payments; and (iii) the sum of the weights on the utilities of the offspring's human capital and old-age social security that are expected to benefit the current middle in their old age. The first factor works to increase old-age social security, whereas the second factor works to reduce it. The effect created by the first factor is offset by that created by the second factor. Therefore, there remains an effect created by the third factor, which includes the forward-looking behavior of agents.

The third factor implies a negative effect of longevity on old-age social security. This somewhat counterintuitive result arises as follows. Greater longevity implies a larger weight on the utility of old-age social security that the middle receive in their old age. To maintain a certain level of future social security, the middle must sustain the human-capital level of their offspring and thus invest privately and/or publicly in education. Then, the middle have an incentive to reduce the tax burden for old-age social security payments today and instead increase private and/or public investment in education. Therefore, greater longevity results in a lower level of old-age social security.

Longevity also affects public education, but its effect is non-monotone. In a political equilibrium with public education, greater longevity results in a higher level of public education spending. However, a further increase in longevity induces voters to prefer old-age social security to public education and thus to leave educational investment to the private sector. In other words, a further increase in longevity puts the economy into a state with no public education. Overall, greater longevity produces a non-monotone effect on public education spending. This non-monotone effect is peculiar to the present model that includes two alternatives for educational investments.

This study employs a Markov-perfect equilibrium to demonstrate the forward-looking behavior of voters. In order to examine the role of this equilibrium concept, we investigate an alternative concept of equilibrium, that is, myopic voting, which is often employed in the literature (see, e.g., Holtz-Eakin, Lovely, and Tosun, 2004; Tosun, 2008; Gradstein and Kaganovich, 2004; Boldrin, 2005). In this voting scheme, voters take future policy as given. Under this alternative concept, we find that the model fails to demonstrate a political equilibrium with the presence of public education. In other words, the model shows only a political equilibrium with the absence of public education, which is empirically an implausible scenario. The result suggests that the Markov-perfect equilibrium is key to demonstrating the two types of political equilibria with empirically relevant properties.

The remainder of this paper is organized as follows. We first present a literature review. Thereafter, Section 2 presents the model. Section 3 demonstrates individual decision-making on education and then characterizes an economic equilibrium. Section 4 demonstrates a period-$t$ political equilibrium. Section 5 investigates how longevity affects education and social security policies. Section 6 shows the existence and stability of a steady-state political equilibrium. Section 7 undertakes the analysis using an alternative assumption of myopic voting. Section 8 concludes.

## 1.1 Literature Review

The present work can be seen as integrating two bodies of literature. The first is concerned with public education as a means of redistribution and its possible effects on human capital accumulation in political economy models of public education. These models, however, do not include private educational investment as a choice for individuals (Glomm and Ravikumar, 1995, 2001; Glomm, 2004; Gradstein and Kaganovich, 2004; Boldrin, 2005; Palivos and Varvarigos, 2013). Several studies overcome this issue by comparing an economy with public education to one with private education (Glomm and Ravikumar, 1992; Saint-Paul and Verdier, 1993; Gradstein and Justman, 1997; de la Croix and Doepke, 2004) or by considering parents' choice of public and/or private education to maximize their altruistic utility (Stiglitz, 1974; Epple and Romano, 1996; Gradstein and Justman, 1996; Hoyt and Lee, 1998; Glomm and Ravikumar, 1998; Cardak, 2004; Bearse, Glomm, and Patterson, 2005; de la Croix and Doepke, 2009; Kunze, 2014).

Most of these studies capture public education as public expenditure on children financed by taxation on adults. In other words, they focus on a forward intergenerational transfer from parents to their children. However, in the real world, there is another intergenerational transfer that works in the opposite direction: income transfer from the young to the elderly, such as social security. The present study focuses on this alternative public spending and investigates how an intergenerational conflict over these two policy issues affects human capital accumulation and the allocation of government spending on public education and social security through individual decision-making regarding education. In particular, the paper sheds light on the role of longevity on political decision-making.

The second body of literature focuses on two-issue voting in the presence of intergenerational conflict, such as two types of income redistribution. Examples include redistribution within a generation and between different generations (Conde-Ruiz and Galasso, 2005; Bassetto, 2008; Gonzalez-Eiras and Niepelt, 2008); public goods provision and social security (Creedy and Moslehi, 2009; Creedy, Li and Moslehi, 2011); public capital and social security (Konrad, 1995; Bellettini and Berti Ceroni, 1999); and medicare services and social security (Bethencourt and Galasso, 2008). In particular, the present paper is concerned with works on public education that benefits the young and social security that benefits the elderly (Bearse, Glomm, and Janeba, 2001; Soares, 2006; Iturbe-Ormaetxe and Valera, 2012; Kaganovich and Meier, 2012; Kaganovich and Zilcha, 2012; Naito, 2012). However, these studies assume either a vote over public education for a given social security benefit or over the allocation of tax revenue for a given tax rate. In other words, the two-dimensional voting aspect is reduced to one dimension. Therefore, these studies do not indicate how the size of the government (i.e., the tax rate) and the allocation of government spending between different generations are jointly determined through voting in the presence of an intergenerational conflict.

The present study, therefore, attempts to integrate both literatures in an analytical framework. Some recent works that share these concerns are Kemnitz (2000), Rangel (2003), Levy (2005), Poutvaara (2006), Bernasconi and Profeta (2012), Gonzalez-Eiras and Niepelt (2012) and Lancia and Russo (2013). However, these differ from the present study in that (1) there is no private education as an alternative choice (Kemnitz, 2000; Gonzalez-Eiras and Niepelt, 2012; Lancia and Russo, 2013) or no human capital accumulation (Rangel, 2003; Levy, 2005), (2) there is a focus on the intragenerational conflict rather than the intergenerational one (Bernasconi and Profeta, 2012), and (3) there is no analysis on the effect of increasing longevity on the allocation of government spending (Poutvaara, 2006). By contrast, this paper demonstrates how an intergenerational conflict over two policy issues (i.e., public education and social security) affects human capital accumulation and the allocation of government spending in the presence of private education as an alternative to public education. The paper then shows that private education as an alternative to public education and forward-looking behavior of voters are key to demonstrating the two types of political equilibria.

# 2 The Model

We consider a discrete-time overlapping-generations model that starts at time 0. Individuals live for at most three periods: young, middle, and old. An individual dies at the end of his/her middle age with probability $1-p$ and lives throughout old age with probability

$p \in (0, 1)$. A higher $p$ means greater longevity, which is interpreted as population aging.[1] Young and middle individuals are identical within each generation, and each middle individual produces one offspring.[2] There is no population growth, and the number of young individuals is assumed to be constant. Young individuals are economically inactive except that they consume education.

Consider a middle individual at time $t$ (i.e., generation $t$). He/she is endowed with a stock of human capital $h_t$, which also defines his/her labor capacity. Given the labor income tax $\tau_t$, a middle individual sets his/her allocation of disposable income ($(1-\tau_t)h_t$) between consumption in the middle period ($c_t^m$) and private investment in his/her child's further education ($z_t$) subject to the budget constraint:

$$c_t^m + z_t \leq (1 - \tau_t)h_t.$$

In old age, an individual receives social security $b_{t+1}$ and consumes it. The budget constraint in old age is

$$c_{t+1}^o \leq b_{t+1},$$

where $c_{t+1}^o$ denotes consumption in old age. We assume that there is no means of storing private goods for old-age consumption.[3] Figure 1 depicts the structure of the model.

[Figure 1 here.]

The level of the offspring's human capital, $h_{t+1}$, is determined by the parent's human capital, $h_t$, material private input, $z_t$, and public expenditure on education, $e_t$. The human capital production function is assumed to have the following form:

$$h_{t+1} = A \cdot (h_t)^\eta \cdot (z_t + (1 - \gamma)e_t)^\delta ,$$

where $A$, $\eta$ and $\delta$ are constant and satisfy $A > 0$, $\eta \in (0, 1), \gamma \in (0, 1), \delta \in (0, 1)$, and $\eta + \delta \in (0, 1)$. The function implies that private input $z_t$ is a perfect substitute for the publicly provided input $e_t$.[4]

[1]There are two aspects of population aging: an increase in longevity, which is basically out of the control of individuals, and a decline in fertility rates, which is the outcome of individual decision-making. The current study focuses on the former aspect to examine the effect of an exogenous change in the demographic structure on education and social security policies via voting.

[2]Given the assumption of identical individuals within a generation, we ignore intragenerational political conflict in the present study. Instead, we focus on the conflict between generations.

[3]This assumption is unusual in the literature, but it enables us to demonstrate the intergenerational conflict over the two policy issues (i.e., public education for the young and social security for the old) in a tractable manner.

[4]In general, private education serves as a substitute and a complement to public education (Glomm and Kaganovich, 2003; Bearse, Glomm, and Patterson, 2005). The latter role is not included in the present analysis.

The assumption $\gamma \in (0, 1)$ implies that public education is less efficient than private education is. In other words, the rate of return to investment in public education is lower than that in private education (Gradstein and Justman, 1996). This assumption reflects the fact that public education provides standardized (rather than individualized) education to each child. Because of this uniformity of public school education, each child is unable to receive the type of education suitable for his or her needs. The lack of individualized education programs in public education results in a lower return to investment compared to private education.

An individual in generation $t$ derives utility from consumption in middle and old ages, $c_t^m$ and $c_{t+1}^o$, respectively, and from his/her child's anticipated future income, $h_{t+1}$. We assume that parents do not care about the welfare of their children and only about their human capital. Generation $t$'s preferences are specified by the following expected lifetime utility function:

$$\ln c_t^m + p \cdot \left\{ \theta \ln h_{t+1} + (1-\theta) \ln c_{t+1}^o \right\},$$

where $\theta \in (0, 1)$ and $1-\theta$ denote the relative weights on the utility of the child's anticipated future income and of old-age consumption, respectively. A middle individual in generation $t$ chooses $c_t^m$ and $z_t$ to maximize his/her expected lifetime utility subject to the budget constraints in the middle and old ages given $(1-\tau_t)h_t$, $b_{t+1}$, and $e_t$.

In each period, the government raises tax revenues to finance the provision of uniform public schooling for all children, $e_t$, and social security, $b_t$. The government budget constraint is given by

$$\tau_t h_t = e_t + p b_t,$$

where $\tau_t h_t$ is tax revenue from the middle, $e_t$ is public expenditure on education, and $pb_t$ is expenditure on social security (here, the expenditure on social security, $b_t$, is multiplied by $p$ because the number of old individuals is $p$ in each period).

The timing of events in period $t$ is as follows. First, middle and old individuals vote on the tax rate ($\tau_t$) and on expenditures on public education and social security ($e_t$ and $b_t$, respectively). Second, each middle individual sets an allocation of disposable income between consumption and private education subject to his/her budget constraints. We solve the model using backward induction in the following two sections.

# 3 Economic Equilibrium

This section demonstrates a middle individual's decision on consumption and private investment in education and its consequence for utility and human capital accumulation. Thus, we first define the economic equilibrium as the outcome of a middle individual's utility-maximizing behavior.

**Definition 1.** Given a sequence of tax rates and the sizes of redistribution and public education, $\{\tau_t, e_t, b_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations, $\{c_t^m, z_t, c_t^o, h_{t+1}\}_{t=0}^{\infty}$, with an initial condition $h_0(>0)$ such that (i) in each period, a middle individual maximizes his/her lifetime utility subject to the budget constraints, non-negativity constraint of investment in private education, and the human capital production function, and (ii) the government budget is balanced in each period.

The problem of a middle individual in period $t$ is as follows:

$$
\begin{aligned}
&\max_{c_t^m, z_t \in [0,(1-\tau_t)h_t]} \ln c_t^m + p\left\{\theta \ln h_{t+1} + (1-\theta)\ln c_{t+1}^o\right\} \\
&\text{subject to} \\
&\qquad c_t^m + z_t \leq (1-\tau_t)h_t, \\
&\qquad c_{t+1}^o \leq b_{t+1}, \\
&\qquad h_{t+1} = A \cdot (h_t)^{\eta} \cdot (z_t + (1-\gamma)e_t)^{\delta}, \\
&\text{given } h_t, \tau_t, b_{t+1} \text{ and } e_t,
\end{aligned}
$$

where the first and second constraints are the budget constraints in middle and old ages, respectively, and the third constraint is the human capital production function.

Solving the utility maximization problem leads to the following private education decision:

$$
z_t = \max\left\{0, \frac{1}{1+p\theta\delta}\left[p\theta\delta(1-\tau_t)h_t - (1-\gamma)e_t\right]\right\}. \tag{1}
$$

Equation (1) indicates that the investment decision depends on an individual's human capital $h_t$ and on government policy variables $\tau_t$ and $e_t$. In particular, a middle individual chooses to invest privately in education if his/her human capital is high, the tax rate is low, and/or the level of public education is low. Otherwise, he/she chooses no private investment in education and spends disposable income for his/her own consumption. Therefore, the consumption function in middle age is given by

$$
c_t^m = \min\left\{(1-\tau_t)h_t, \frac{1}{1+p\theta\delta}\left[(1-\tau_t)h_t + (1-\gamma)e_t\right]\right\},
$$

where the first and second arguments in the brackets correspond to consumption when $z = 0$ and $z > 0$, respectively.

Equation (1) indicates that a middle individual invests in education if and only if $p\theta\delta(1-\tau_t)h_t - (1-\gamma)e_t > 0$. Using the government budget constraint, the condition is rewritten as follows:

$$
z_t > 0 \Leftrightarrow h_t > \left(\frac{1-\gamma}{p\theta\delta} + 1\right) \cdot e_t + pb_t.
$$

This condition states that lower levels of public education and social security benefits produce a larger income effect, thereby giving a middle individual incentive to invest more in education.

Considering the condition $z_t > 0$, we can write the indirect utility function of a middle individual as follows:

$$V_t^m = \begin{cases} V_{t,z>0}^m \text{ if } h_t > \left(\frac{1-\gamma}{p\theta\delta}+1\right)e_t + pb_t, \\ V_{t,z=0}^m \text{ if } h_t \leq \left(\frac{1-\gamma}{p\theta\delta}+1\right)e_t + pb_t, \end{cases}$$

where $V_{z>0}^m$ and $V_{z=0}^m$ are the indirect utility functions of a period-$t$ middle-aged individual born in the previous period (i.e., period $t-1$) when $z_t > 0$ and $z_t = 0$, respectively. These are given by

$$\begin{aligned} V_{t,z>0}^m &= (1+p\theta\delta)\ln(h_t - \gamma e_t - pb_t) + p(1-\theta)\ln b_{t+1} \\ &+ \left[\ln\frac{1}{1+p\theta\delta} + p\theta\delta\ln\frac{p\theta\delta}{1+p\theta\delta} + p\theta\ln A + p\theta\eta\ln h_t\right]; \quad (2) \\ V_{t,z=0}^m &= \ln(h_t - e_t - pb_t) + p\theta\delta\ln e_t + p(1-\theta)\ln b_{t+1} \\ &+ \left[p\theta\ln A + p\theta\delta\ln(1-\gamma) + p\theta\eta\ln h_t\right]. \quad (3) \end{aligned}$$

The old do not make an economic decision. They receive social security benefits and consume them. In addition, they receive the utility of their offspring's human-capital level. The indirect utility of an old individual alive at time $t$ is as follows:

$$V_t^o = (1-\theta)\ln b_t + \theta\ln h_t, \quad (4)$$

where the first and second terms on the right-hand side represent the utilities of social security and the offspring's human capital, respectively.

Using the government budget constraint and the condition $z_t > 0$, the human capital production equation is given by the following:

$$h_{t+1} = H(h_t, e_t, b_t) = \begin{cases} H_{z>0}(h_t, e_t, b_t) \equiv A(h_t)^\eta\left[\frac{p\theta\delta}{1+p\theta\delta}(h_t - \gamma e_t - pb_t)\right]^\delta \\ \qquad \text{if } h_t > \left(\frac{1-\gamma}{p\theta\delta}+1\right)e_t + pb_t; \\ H_{z=0}(h_t, e_t) \equiv A(h_t)^\eta\left[(1-\gamma)e_t\right]^\delta \\ \qquad \text{if } h_t \leq \left(\frac{1-\gamma}{p\theta\delta}+1\right)e_t + pb_t. \end{cases} \quad (5)$$

Equation (5) implies that there exists a critical level of human capital, $((1-\gamma)/p\theta\delta + 1)e_t + pb_t$, that determines educational investment behavior. For a given set of policy variables, $e$ and $b$, human capital accumulates according to the first equation in (5) when the stock of human capital is above the critical level. In addition, it evolves according to the second equation in (5) when the stock of human capital is below the critical level. The critical level depends on public education and social security, both of which are determined through voting (as demonstrated in the next section).

# 4 Political Equilibrium

Public education $e$ and old-age social security $b$ are determined by individuals through a political process. Elections occur every period, and all middle and old individuals cast a ballot on $e$ and $b$. Individuals' preferences for the two policy issues are represented by the indirect utility functions in Eqs. (2) and (3) for the middle and by those in Eq. (4) for the old.

The issue space is bi-dimensional, and thus, a majoritarian voting game equilibrium may not exist. To resolve this problem, the present paper assumes probabilistic voting à la Lindbeck and Weibull (1987) in the demonstration of the political mechanisms. In each period, the middle and the old participate in voting, and the government in power maximizes a political objective function that reflects the preferences of the middle and the old. Formally, the political objective function in period $t$ is given by the following:

$$\Omega_t = pV_t^o + V_t^m,$$

where $p$ (attached to the utility of the old, $V_t^o$) is the relative weight of the old measured as a percentage of the population in the economy. The government's problem is maximizing $\Omega_t$ subject to the human capital production function, Eq. (5), given $h_t$.[5]

This study restricts its attention to a Markov-perfect equilibrium as described in Krusell, Quadrini, and Rios-Rull (1997) and applied to a political economy analysis (e.g., Grossman and Helpman, 1998; Hassler et al., 2003, 2005; Hassler, Storesletten, and Zilibotti, 2007; Gonzalez-Eiras and Niepelt, 2008, 2012; Song, 2011, 2012; Song, Storesletten, and Zilibotti, 2012). Voters condition their strategies only on payoff-relevant state variables. In the current framework, human capital $h$ is the payoff-relevant state variable. Therefore, the expected level of social security for the next period included in the utility function of the middle, $V_t^m$, is given by $b_{t+1} = \tilde{B}(h_{t+1}) : \Re_{++} \to \Re_+$. We can now define a period-$t$ political equilibrium as follows:

**Definition 2.** A *period-t political equilibrium* is a pair of functions, $\{B, E\}$, where $B$ and $E$ are two policy rules, $b_t = B(h_t) : \Re_{++} \to \Re_+$ and $e_t = E(h_t) : \Re_{++} \to \Re_+$, respectively, such that (i) $B$ and $E$ are solutions to the government's problem for a given expectation of $b_{t+1} = \tilde{B}(h_{t+1})$ and (ii) $\tilde{B} = B$ holds.

Next, we characterize a period-$t$ political equilibrium, that is, a voting outcome in some period $t$. For this purpose, we seek the voters' preferred policies when $z_t > 0$ (Section 4.1) and $z_t = 0$ (Section 4.2). In Section 4.3, we summarize the two cases to characterize a

[5] An explicit microfoundation for this model is explained in Persson and Tabellini (2000, Chapter 3) and Acemoglu and Robinson (2005, Appendix). Song, Storesletten, and Zilibotti (2012, Appendix B) show the process of deriving the political objective function in an overlapping generations framework.

period-$t$ political equilibrium. Section 4.4 examines how the period-$t$ political equilibrium outcome is affected by the increased longevity of agents. In Section 4.5, we review the long-run consequences and show the existence and stability of a steady-state political equilibrium.

## 4.1 Voters' Preferred Policy When $z_t > 0$

Suppose that a middle individual in period $t$ invests a portion of his/her disposable income in education: $z_t > 0$. Because the preferences are specified by the logarithmic utility function, we conjecture a linear policy function of social security in period $t+1 : b_{t+1} = B_0 \cdot h_{t+1}$, where $B_0 \in (0, \infty)$ is a constant parameter. Under this conjecture, the social security benefit is given by the following:

$$\begin{aligned} b_{t+1} &= B_0 A H_z(h_t, e_t, b_t) \\ &= B_0 A (h_t)^{\eta} \left( \frac{p\theta\delta}{1+p\theta\delta} \right)^{\delta} (h_t - \gamma e_t - p b_t)^{\delta}, \end{aligned} \tag{6}$$

where the second line is derived using the equation of human capital production (5).

Under the assumption of $z_t > 0$ and the expectation of $b_{t+1}$ in (6), the objective function of the period-$t$ government becomes

$$\Omega_{t,z>0} = p(1-\theta) \ln b_t + (1+p\delta) \ln (h_t - \gamma e_t - p b_t),$$

where unrelated terms are omitted from the expression. The first term on the right-hand side, $(1-\theta)\ln b_t$, denotes the utility of social security benefits for the period-$t$ old weighted by the number of old, $p$. The second term denotes the expected utility of the period-$t$ middle. In particular, it is the sum of the utilities of their consumption $((1+p\theta\delta)\ln(h_t - \gamma e_t - p b_t))$ and social security benefits they receive in their old age, $p(1-\theta)\delta \ln(h_t - \gamma e_t - p b_t)$. The latter benefit is specific to the model considering the Markov-perfect political equilibrium.

The problem of the government in period $t$ is choosing a pair of $(b_t, e_t)$ that maximize $\Omega_{t,z>0}$. The first-order conditions with respect to $e_t$ and $b_t$ are

$$\begin{aligned} e_t &: (1+p\delta) \frac{-\gamma}{h_t - \gamma e_t - p b_t} \leq 0, \\ b_t &: \frac{p(1-\theta)}{b_t} = (1+p\delta) \frac{p}{h_t - \gamma e_t - p b_t}. \end{aligned}$$

These conditions lead to

$$e_t = 0 \text{ and } b_t = \frac{1-\theta}{1+p((1-\theta)+\delta)} h_t. \tag{7}$$

Therefore, $B = \tilde{B}$ holds if $B_0 = (1-\theta) \cdot (1+p((1-\theta)+\delta))^{-1}$. That is, function (7) constitutes a period-$t$ political equilibrium as long as $B_0 = (1-\theta) \cdot (1+p((1-\theta)+\delta))^{-1}$.

We substitute the solution in (7) into the condition $z_t > 0$ in (5) and find that the condition holds for any $h_t > 0$.[6] The result established thus far is summarized as follows.

**Lemma 1.** *For any $h_t > 0$, there is a solution to the period-t government problem distinguished by $z_t > 0, e_t = 0$ and $b_t = (1-\theta) \cdot (1 + p((1-\theta)+\delta))^{-1} \cdot h_t$.*

The result in Lemma 1 implies that given the expectation that the middle invest privately in education, the government finds it optimal to invest nothing in public education and to use all tax revenue for old-age social security for any level of $h_t$. Faced with this government policy, the middle choose to invest privately in education to maximize utility. Zero public investment in education, which seems to be an extreme result, is partly because of the specifications of the utility and human capital production functions. However, it may be viewed as demonstrating an economy where government spending is in favor of old-age social security.

## 4.2 Voters' Preferred Policy When $z_t = 0$

Alternatively, suppose that a middle individual in period $t$ privately invests nothing in education: $z_t = 0$. Following the same procedure as in the case of $z_t > 0$, we conjecture a linear function of social security in period $t+1$ : $b_{t+1} = B_1 \cdot h_{t+1}$ where $B_1 \in (0, \infty)$ is a constant parameter. Under this conjecture, the social security benefit is given by

$$b_{t+1} = B_1 A (h_t)^{\eta} (1-\gamma)^{\delta} (e_t)^{\delta}.$$

The objective function of the period$-t$ government becomes

$$\Omega_{t,z=0} = p(1-\theta) \ln b_t + \ln (h_t - e_t - pb_t) + p\delta \ln e_t,$$

where unrelated terms are omitted from the expression. The first term on the right-hand side, $(1-\theta)\ln b_t$, denotes the utility of social security benefits for period-$t$ old weighted by the number of old, $p$. The second term denotes the utility of consumption for the period-$t$ middle, and the third term is part of the expected utility of old-age social security for the period-$t$ middle.

The problem for the government in period $t$ is choosing a pair of $(b_t, e_t)$ that maximize $\Omega_{t,z=0}$. The first-order conditions with respect to $b_t$ and $e_t$ are

$$e_t : \frac{1}{h_t - e_t - pb_t} = \frac{p\delta}{e_t},$$
$$b_t : \frac{p(1-\theta)}{b_t} = \frac{p}{h_t - e_t - pb_t}.$$

[6] The condition $z_t > 0$ becomes

$$z_t > 0 \Leftrightarrow h_t > \left( \frac{1-\gamma}{p\theta\delta} + 1 \right) e_t + pb_t \Leftrightarrow 1 > \frac{p(1-\theta)}{1 + p((1-\theta)+\delta)}.$$

The last inequality holds for any set of parameters.

These conditions lead to

$$e_t = \frac{p\delta}{1 + p\left((1-\theta)+\delta\right)} h_t \text{ and } b_t = \frac{1-\theta}{1 + p\left((1-\theta)+\delta\right)} h_t. \tag{8}$$

Therefore, $B = \tilde{B}$ if $B_1 = (1-\theta) \cdot (1 + p\left((1-\theta)+\delta\right))^{-1}$.

Plugging the solution in (8) into the condition $z_t = 0$, we find that $z_t = 0$ holds if and only if $\theta \leq 1 - \gamma \Leftrightarrow \gamma \leq 1 - \theta$ holds. Therefore, we obtain the following result.

**Lemma 2.** *Suppose that $\gamma \leq 1-\theta$ holds. Then, for any $h_t > 0$, there is a solution to the period-t government problem distinguished by $z_t = 0, e_t = p\delta \cdot (1+p\left((1-\theta)+\delta\right))^{-1} \cdot h_t$ and $b_t = (1-\theta) \cdot (1 + p\left((1-\theta)+\delta\right))^{-1} \cdot h_t$.*

The result in Lemma 2 states that given the expectation that the middle privately invest nothing into education, the government finds it optimal to use a part of the tax revenue for public education as long as the efficiency of public education is sufficiently high that $\gamma \leq 1-\theta$. Here, a lower $\gamma$ implies a higher efficiency of public education. Faced with this policy, the middle chooses to privately invest nothing into education to maximize utility. Compared to the result in Lemma 1, the result in Lemma 2 demonstrates an economy with government spending in favor of public education.

## 4.3 Period-$t$ Political Equilibrium

The results in Lemmas 1 and 2 suggest that there is a unique solution distinguished by $z > 0$ if $\gamma > 1-\theta$; furthermore, there may exist two solutions distinguished by $z > 0$ and $z = 0$ if $\gamma \leq 1-\theta$. The case $\gamma \leq 1-\theta$ implies that there are two local maxima for the political objective function. The government can choose the better of the two local maxima by controlling policies.

To select the equilibrium for the case $\gamma \leq 1-\theta$, we substitute the policy functions in Eq. (7) when $z > 0$ and in Eq. (8) when $z = 0$ into the political objective functions $\Omega_{z>0}$ and $\Omega_{z=0}$, respectively. For a given $h_t$, we have the following relation:

$$\Omega_{t,z>0} \gtreqless \Omega_{t,z=0} \Leftrightarrow V^m_{t,z>0} \gtreqless V^m_{t,z=>0},$$

where $V^m_{t,z>0}$ and $V^m_{t,z=>0}$ are defined in (2) and (3), respectively. This relation holds because the old-age social security functions are equivalent in both cases ($z > 0$ and $z = 0$) for a given $h_t$. Thus, $V^o_t$ when $z > 0$ is equal to $V^o_t$ when $z = 0$.

Using a direct calculation, we obtain[7]

$$V^m_{t,z>0} \gtreqless V^m_{t,z=>0} \Leftrightarrow \gamma \gtreqless 1-\theta\cdot\phi(p), \tag{9}$$

where $\phi(p)$ is defined as

$$\phi(p) \equiv \left(\frac{1+p\delta}{1+p\theta\delta}\right)^{(1+p\delta)/p\delta} > 1,$$

and $1-\theta > 1-\theta\cdot\phi(p)$ holds. Therefore, we can conclude that the government chooses the policy in Lemma 1 if $\gamma \in (1-\theta\cdot\phi(p),1)$, that in Lemma 2 if $\gamma \in (0,1-\theta\cdot\phi(p))$, and is indifferent between the two if $\gamma = 1-\theta\cdot\phi(p)$. The result is summarized in the following proposition.

**Proposition 1** (Period-$t$ political equilibrium)

(i) *If* $\gamma > 1-\theta\cdot\phi(p)$, *there exists a unique period-t political equilibrium with* $z_t > 0$ *and* $e_t = 0$.

(ii) *If* $\gamma = 1-\theta\cdot\phi(p)$, *there are two period-t political equilibria: one is distinguished by* $z_t > 0$ *and* $e_t = 0$, *and the other is distinguished by* $z_t = 0$ *and* $e_t > 0$.

(iii) *If* $\gamma < 1-\theta\cdot\phi(p)$, *there exists a unique period-t political equilibrium with* $z_t = 0$ *and* $e_t > 0$.

Figure 2 illustrates the conditions $\gamma \gtreqless 1-\theta$ and $\gamma \gtreqless 1-\theta\cdot\phi(p)$ in a $\theta-\gamma$ space. The condition $\gamma > 1-\theta\cdot\phi(p)$ indicates that given $\gamma$, obtaining the equilibrium with $z_t > 0$ is positively correlated with the size of $\theta$. Greater parental interest in the child's education gives the parents a stronger motivation for providing education. Given the property that private education is a perfect substitute for public education, parents find it optimal to privately provide education and to use tax revenue for old-age social security instead of for public education. Therefore, the economy realizes an equilibrium where education is provided in private-sector institutions when $\gamma > 1-\theta\cdot\phi(p)$ holds.

[7]Using this direct calculation, we have

$$\begin{aligned} V^m_{t,z>0} \gtreqless V^m_{t,z=>0} &\Leftrightarrow (1+p\delta)\ln\frac{1+p\delta}{1+p\theta\delta} + p\delta\ln\theta \gtreqless p\delta\ln(1-\gamma) \\ &\Leftrightarrow \ln\left(\frac{1+p\delta}{1+p\theta\delta}\right)^{(1+p\delta)} + \ln(\theta)^{p\delta} \gtreqless \ln(1-\gamma)^{p\delta} \\ &\Leftrightarrow \left(\frac{1+p\delta}{1+p\theta\delta}\right)^{(1+p\delta)} (\theta)^{p\delta} \gtreqless (1-\gamma)^{p\delta} \\ &\Leftrightarrow \left(\frac{1+p\delta}{1+p\theta\delta}\right)^{(1+p\delta)/p\delta} \theta \gtreqless 1-\gamma. \end{aligned}$$

Using the definition of $\phi(p)$ in the text, we obtain (9).

The condition $\gamma > 1-\theta \cdot \phi(p)$ also indicates that given $\theta$, the equilibrium with $z_t > 0$ is positively correlated with $\gamma$, that is, it is negatively correlated with the relative efficiency of public education. As previously mentioned, public education is a perfect substitute for private education, but the former is less efficient and thus more costly than the latter. However, for the middle-aged voters, public education acts as a collective mechanism to increase the average human capital of subsequent generations and their old-age pension benefits. Middle-aged voters examine the costs and benefits of public education and find that costs are larger (smaller) than benefits when the efficiency of public education is below (above) the critical value, denoted by $\theta \cdot \phi(p)$. Eventually, middle-aged voters choose private education rather than public education when $\gamma$ is sufficiently high that $1-\gamma < \theta \cdot \phi(p)$ (i.e., $\gamma > 1-\theta \cdot \phi(p)$). Furthermore, they choose public education when $\gamma$ is sufficiently low that $1-\gamma > \theta \cdot \phi(p)$ (i.e., $\gamma < 1-\theta \cdot \phi(p)$).

[Figure 2 here.]

The situation described thus far is somewhat extreme, because it lacks either public or private education. However, this situation demonstrates two types of states in the real world: states with a high and low share of private education in the total education expenditure, respectively. The equilibrium with $z > 0$ ($z = 0$) can be viewed as demonstrating the former (latter) state. To examine the plausibility of these situations, we review data on OECD countries (OECD, 2013, Education at a glance 2013). Table 1 shows the relative proportions of private and public expenditure on educational institutions for all levels of education in 2010. Panel (a) presents the list of countries with an over-25 % share of private education; Panel (b) presents the list of countries with an under-10% share.

[Table 1 here.]

The evidence suggests that the group of countries with high shares of private education includes Anglo-Saxon countries (Australia, the United Kingdom, and the United States) and East-Asian countries (Japan and Korea)[8]. The group of countries with low shares includes European countries (Austria, Belgium, Denmark, Estonia, Finland, Iceland, Ireland, Italy, Portugal, and Sweden). One interpretation of this evidence is that countries in the former group features low efficiency of public education and/or larger concern of parents regarding the education of their children. Alternatively, countries in the latter group feature high efficiency of public education. In other words, the degree of efficiency of public education and parental motivation for education tend to affect decision making on education and the choice of redistribution policies. Furthermore, they are keys to

[8]An exception is Chile with a 42.1% share. The share is 24.2% in Canada, which can plausibly be included in the group of Anglo-Saxon countries.

explaining cross-country difference in the composition of education expenditures. This is a testable implication of the theory, which should be confirmed by empirical evaluation in future work.

# 5 Effects of Longevity

The analysis and results thus far suggest that longevity, represented by the parameter $p$, affects the equilibrium policies and characterizations demonstrated in Section 4. To investigate the longevity effects in more detail, we first consider the effect of an increase in $p$ on the equilibrium policies $e$ and $b$ for the cases $z > 0$ and $z = 0$ in Proposition 2. Then, we provide an interpretation of the results, examine the effects of an increase in $p$ on the threshold condition in (9), and show the overall effect of longevity on public education spending around the threshold condition. Finally, we compare our results to previous results and findings.

The following proposition summarizes the effect of greater longevity on the equilibrium policies.

**Proposition 2.** *Consider a period-t political equilibrium.*

(i) *Greater longevity results in a lower level of old-age social security.*

(ii) *Suppose that $z = 0$ holds. Greater longevity results in a higher level of public education spending.*

To understand the mechanism behind the result, we recall the political objective function when $z > 0$:

$$\Omega_{t,z>0} = \underbrace{p}_{(a.i)}(1-\theta)\ln b_t + (1+\underbrace{p\delta}_{(a.iii)})\ln\left(h_t - \gamma e_t - \underbrace{pb_t}_{(a.ii)}\right).$$

Longevity affects the determinants of old-age social security using the following three factors: (a.i) the weight on the utility of old-age social security that benefits the current old; (a.ii) the tax burden of the middle required to finance the current old-age social security payments; and (a.iii) the sum of the weights on the utilities of consumption, the offspring's human capital, and old-age social security expected to benefit the current middle in their old age. In the present specification of the model, the effect created by the first factor is offset by that created by the second one. Therefore, the effect produced by the third factor remains.

The third factor includes the discipline effect exercised by the middle voters. Greater longevity implies a larger weight on the utility of old-age social security that they will receive in their old age. To maintain a certain level of social security benefits, they need to sustain the human capital level of their offspring and thus need to invest privately and/or publicly in education. Therefore, the middle have an incentive to reduce the tax burden for old-age social security and to increase private and/or public investment in education in response to an increase in longevity. Therefore, greater longevity results in a lower level of old-age social security.

To confirm that a similar result is obtained for the case $z = 0$, recall the political objective function when $z = 0$:

$$\Omega_{t,z=0} = \underbrace{p(1-\theta)}_{(b.i)} \ln b_t + \ln\left(h_t - e_t - \underbrace{pb_t}_{(b.ii)}\right) + \underbrace{p\delta}_{(b.iii)} \ln e_t.$$

Similar to the case $z > 0$, longevity has effects on the determination of old-age social security and public education through the following three factors: (b.i) the weight on the utility of old-age social security that benefits the old; (b.ii) the tax burden on the middle to finance the current old-age social security payments; and (b.iii) the sum of the weights on the utilities of the offspring's human capital and old-age social security that is expected to benefit the current middle in their old age. As in the case $z > 0$, we find that the effect created by the first factor is offset by that created by the second factor. The remaining factor, that is, the third factor, shows that greater longevity means a larger weight on public education. Therefore, we can conclude that greater longevity results in a larger level of public education spending and thus a lower level of old-age social security for the case $z = 0$.

This analysis and its results suggest a monotone effect of longevity on old-age social security. However, the effect on public education is not straightforward, because public education spending increases with longevity when $z = 0$; however, there is no public education spending when $z > 0$. To understand the overall effect, we must investigate how an increase in $p$ affects the threshold condition $\gamma = 1 - \theta \cdot \phi(p)$ in (9).

To see the effect of $p$ on the threshold condition, we show the conditions for the three values of $p$ in Figure 2 using a graph. The corresponding values of $p$ are $p = 0.2, 0.6$, and 0.99. The figure suggests that the equilibrium with $z > 0$ is more likely to be obtained with high longevity. A higher $p$ implies a larger weight on old-age social security in the political objective function. Therefore, voters tend to prefer old-age social security to public education and to leave educational investment to the private sector as longevity increases.

Combining the results in Proposition 2 with the numerical results demonstrated in Figure 2, we obtain the following implication of longevity for public education. For a low $p$ such that the middle privately invest nothing in education, an increase in $p$ results in a higher level of public education spending. However, a further increase in $p$ results in no spending on public education, because the equilibrium allocation is changed from the one distinguished by $z = 0$ and $e > 0$ (as in Lemma 2) to that distinguished by $z > 0$ and $e = 0$ (as in Lemma 1) in response to an increase in longevity. That is, greater longevity produces a non-monotone effect on public education. This non-monotone effect is peculiar to the present model that includes two alternatives for educational investment.

The positive effect of aging on public education shown in Proposition 2(ii) is in line with the theoretical predictions in the literature. Gradstein and Kaganovich (2004) show an overall positive impact of increasing longevity on public education funding. Levy (2005) shows a relatively high level of per-capita public provision of education when the young are a minority in the population. In addition, the negative effect of aging on social security shown in Proposition 2(i) is in line with the prediction in the model by Razin, Sadka, and Swagel (2002).

These theoretical predictions are somewhat counterintuitive, and one might expect that aging voters show less support for public education that does not directly benefit the old and place more emphasis on social security. However, empirical investigations suggest that the findings are mixed regarding the effect of aging on public education (see Cattaneo and Wolter, 2009, for a literature survey). In addition, Razin, Sadka, and Swagel (2002) show that aging has a negative impact on social security in the United States and in some European countries. The predictions of the present model can therefore provide a possible explanation for these counterintuitive findings.

# 6 Steady-state Equilibrium

Having established the period-t political equilibrium, we now investigate the law of motion of human capital in the political equilibrium. In particular, we examine the existence and stability of a *steady-state political equilibrium*, where $h_t = h_{t+1}$ holds. Thus, we compute the human capital production equation by substituting the policy when $z_t > 0$ (7) into $h_{t+1} = H_{z>0}(\cdot)$ in (5) and that when $z_t = 0$ (8) into $h_{t+1} = H_{z=0}(\cdot)$ in (5). Thus, we obtain

$$h_{t+1} = \begin{cases} H_{z>0}(h_t) \equiv A\left(\frac{p\theta\delta}{1+p\theta\delta}\right)^{\delta}\left(\frac{1+p\delta}{(1-\theta)+\delta}\right)^{\delta}(h_t)^{\eta+\delta} & \text{if } z_t > 0; \\ H_{z=0}(h_t) \equiv A\left(\frac{(1-\gamma)p\delta}{(1-\theta)+\delta}\right)^{\delta}(h_t)^{\eta+\delta} & \text{if } z_t = 0, \end{cases} \tag{10}$$

where $H_{z>0}$ and $H_{z=0}$ are strictly increasing and strictly concave in $h$ with $H_{z>0}(0) = 0, H_{z=0}(0) = 0, \lim_{h\to\infty} H'_{z>0}(\cdot) = 0$, and $\lim_{h\to\infty} H'_{z=0}(\cdot) = 0$.

Given the result in Proposition 1, we can write the law of motion of human capital as follows:

$$h_{t+1} = \begin{cases} H_{z>0}(h_t) & \text{if} \quad \gamma > 1-\theta\cdot\phi(p) \\ \{H_{z>0}(h_t), H_{z=0}(h_t)\} & \text{if} \quad \gamma = 1-\theta\cdot\phi(p) \\ H_{z=0}(h_t) & \text{if} \quad \gamma < 1-\theta\cdot\phi(p). \end{cases}$$

Using this motion, we obtain the following result regarding the existence and stability of the steady-state political equilibrium.

**Proposition 3.**

(i) *If $\gamma > 1-\theta\cdot\phi(p)$, there exists a unique and stable steady-state political equilibrium with $z > 0$ and $e = 0$.*

(ii) *If $\gamma = 1-\theta\cdot\phi(p)$, there are multiple steady-state political equilibria: the steady state with $z > 0$ and $e = 0$ and that with $z = 0$ and $e > 0$.*

(iii) *If $\gamma < 1-\theta\cdot\phi(p)$, there exists a unique and stable steady-state political equilibrium with $z = 0$ and $e > 0$.*

The result in Proposition 3 implies that there is a unique political equilibrium path that stably converges to the steady state with $z > 0$ and $e = 0$ when $\gamma > 1-\theta\cdot\phi(p)$. In addition, there is a unique political equilibrium path that stably converges to the steady state with $z = 0$ and $e > 0$ when $\gamma < 1-\theta\cdot\phi(p)$. Multiple steady-state equilibria arise only when $\gamma = 1-\theta\cdot\phi(p)$. In this case, the economy either experiences a monotone convergence to either state or oscillates between the two equilibria, depending on the choice of the government in each period.

Figure 3 illustrates two numerical examples of the human capital production equation when $\gamma \leq 1-\theta$. In this situation, there are two solutions to the government problem. The government selects one of solution to maximize the objective. In each panel, a solid curve (dashed curve) presents an equation of human capital production realized (not realized) in equilibrium in accordance with the government's selected (non-selected) policy.

[Figure 3 here.]

In the present model, no switch occurs between the state with $z > 0$ and $e = 0$ and that with $z = 0$ and $e > 0$ along the equilibrium path for most sets of parameters. We observe a switch between the two states only when $\gamma = 1-\theta\cdot\phi(p)$ holds. Therefore, a question arises as to whether it is possible to show a switch between the two states along an equilibrium path for a larger set of parameters. To answer this question, we can assume that $\gamma$ depends on the human capital level, $h_t$. For example, assume that $\gamma$ is decreasing in $h_t$: the return on investment from public education increases as the human capital level increases. This assumption implies that productivity (or efficiency) of public education increases as the teachers' level in public schools (represented by the human capital level in the economy) increases.

Under this assumption, we can find a critical value of $h_t$, denoted by $\hat{h}$. For $h_t < \hat{h}$, the condition $\gamma > 1 - \theta \cdot \phi(p)$ is satisfied: there exists a unique equilibrium path with $z > 0$. For $h_t > \hat{h}$, the condition $\gamma < 1-\theta\cdot\phi(p)$ is satisfied: there exists a unique equilibrium path with $z = 0$. Therefore, we can predict that the economy attains a political equilibrium distinguished by $z > 0$ and $e = 0$ when $h_t$ is below the critical value and one distinguished by $z = 0$ and $e > 0$ when $h_t$ is above the critical value. A switch from the state with $z > 0$ and $e = 0$ to that with $z = 0$ and $e > 0$ may occur along the equilibrium path. However, the plausibility of this prediction should be tested in an empirical manner, which is beyond the scope of the present study.

Another assumption may demonstrate the switch between the two states. For example, it is plausible to assume that longevity, represented by parameter $p$, is increasing in the human capital level, $h_t$ (see, e.g., Castello-Climent and Domenech, 2008). However, this assumption does not satisfy the Markov property for a given conjecture of a linear social security function, $b_{t+1} = \tilde{B}(h_{t+1}) = B_0 \cdot h_{t+1}$, where $B_0$ is a constant parameter. That is, the period-$t$ social security function, $b_t = B(h_t)$, becomes $b_t = (1 - \theta) \cdot (1 + p(h_t) \cdot ((1 - \theta) + \delta))^{-1} \cdot h_t$., which is a nonlinear function of $h_t$: $\tilde{B}(\cdot)$ is not equal to $B(\cdot)$. Therefore, the assumption that $p$ is dependent on $h_t$ is not available in the present framework.

# 7 Myopic Voting

Thus far, we have focused on the Markov-perfect political equilibrium of a voting game over two policy issues, public education for the young and social security for the old. In order to examine the role of this equilibrium concept, this section introduces an alternative political equilibrium concept adopted by many studies: *myopic voting*, where voters today take future policy as given (e.g., Gradstein and Kaganovich, 2004; Holz-Eakin, Lovely, and Tosun, 2004; Boldrin, 2005; Tosun, 2008; Kaganovich and Meier, 2012). Under this alternative voting that does not include the forward-looking behavior of agents, we find that the model demonstrates only one type of equilibrium with $z > 0$ and $e = 0$.

Under the myopic voting assumption, the objective of the period$-t$ government is to choose ($e_t$,$b_t$) to maximize $\Omega_t$, which is defined by

$$\Omega_t = \begin{cases} \Omega_{z>0} \equiv p \ln b_t + (1 + p\theta\delta) \ln(h_t - \gamma e_t - pb_t) \text{ if } h_t > \left(\frac{1-\gamma}{p\theta\delta} + 1\right) e_t + pb_t; \\ \Omega_{z=0} \equiv p \ln b_t + \ln(h_t - e_t - pb_t) + p\theta\delta \ln e_t \text{ if } h_t > \left(\frac{1-\gamma}{p\theta\delta} + 1\right) e_t + pb_t. \end{cases}$$

The function $\Omega_t$ does not include the term $p\beta \ln b_{t+1}$, representing the utility of social security benefits in the next period as a politically relevant variable, because myopic voters today (i.e., in period $t$) take the next-period policy as given. In other words, we do not include the intertemporal effect of current policy on next-period social security through the human capital accumulation equation from the analysis. Therefore, the equation of human capital accumulation that appeared as a constraint in the previous sections is not considered in the current problem formulation.

Following the same procedure as in Section 4, we consider the cases $z > 0$ and $z = 0$. First, suppose that $z > 0$ holds. The problem of the period-$t$ government is as follows:

$$\max_{b_t, e_t} p \ln b_t + (1 + p\theta\delta) \ln(h_t - \gamma e_t - p b_t).$$

The solution to this problem is

$$(e_t, b_t) = \left(0, \frac{1}{1 + p(1 + \theta\delta)} h_t\right).$$

Plugging in the solution to the condition $z_t > 0$, we have

$$h_t > \left(\frac{1-\gamma}{p\theta\delta} + 1\right) \cdot 0 + p \frac{1}{1 + p(1 + \theta\delta)} h_t.$$

This condition holds for any $h_t$ : there always exists a political equilibrium with $z > 0$.

Next, suppose that $z = 0$ holds. The problem of the period-$t$ government is

$$\max_{b_t, e_t} p \ln b_t + \ln(h_t - e_t - p b_t) + p\theta\delta \ln e_t.$$

The solution to the problem is

$$(e_t, b_t) = \left(\frac{p\theta\delta}{1 + p(1 + \theta\delta)} h_t, \frac{1}{1 + p(1 + \theta\delta)} h_t\right).$$

We substitute the solution into the condition $z = 0$ and obtain

$$z_t = 0 \Leftrightarrow 0 \leq -\gamma,$$

which never holds. Therefore, we can conclude that

**Proposition 4.** *Suppose that voters are myopic in the sense that they take future policy as given. Then, there exists a unique political equilibrium distinguished by $z > 0$ and $e = 0$.*

Why is it impossible to have an equilibrium with no private investment, $z = 0$? In order to answer this question, we recall the condition $z = 0$ in (5):

$$h_t \leq \left(\frac{1-\gamma}{p\theta\delta} + 1\right) e_t + p b_t.$$

For a given $h$, this condition requires high levels of public education and social security, both of which give individuals a disincentive to privately invest in education. However, myopic voting results in a high level of social security but a low level of public education. Moreover, the former effect is outweighed by the latter, and thus, the condition $z = 0$ fails to hold for any $h$. The model fails to show an economy distinguished by sufficient public support for education, which is an implausible scenario from an empirical point of view. The result in this subsection therefore suggests that there is a need to capture the forward-looking behavior of agents, which is peculiar to the Markov-perfect political equilibrium when we consider the two-dimensional voting on public education and old-age social security.

# 8 Conclusion

This paper developed an overlapping-generations model with altruism toward children and considers voting over two policy issues, public education for the young and social security for the old. In the model, there is a conflict of interest between generations over these policy issues. The analysis shows that the two types of political equilibria exist in the model: one with private (but not public) education and the other with public (but not private) education. In both equilibria, social security is provided to all old individuals.

One of the two equilibria is selected by the government to maximize its objective. This study first shows that longevity critically affects this selection. In addition, greater longevity results in a lower level of old-age social security in both equilibria and a higher level of spending on public education in the equilibrium with public education. These results imply that longevity produces a non-monotone effect on the provision of public education.

This study also shows the role of the forward-looking behavior assumption of agents, which is peculiar to the Markov-perfect political equilibrium. When we remove this assumption and alternatively assume myopic behavior, the model fails to show an economy distinguished by the presence of public education. In other words, the economy attains only an equilibrium distinguished by the absence of public education, which is empirically an implausible scenario. Therefore, the present analysis sheds light on the role of the Markov-perfect equilibrium in demonstrating two types of equilibria reflecting cross-country differences in public education and social security.

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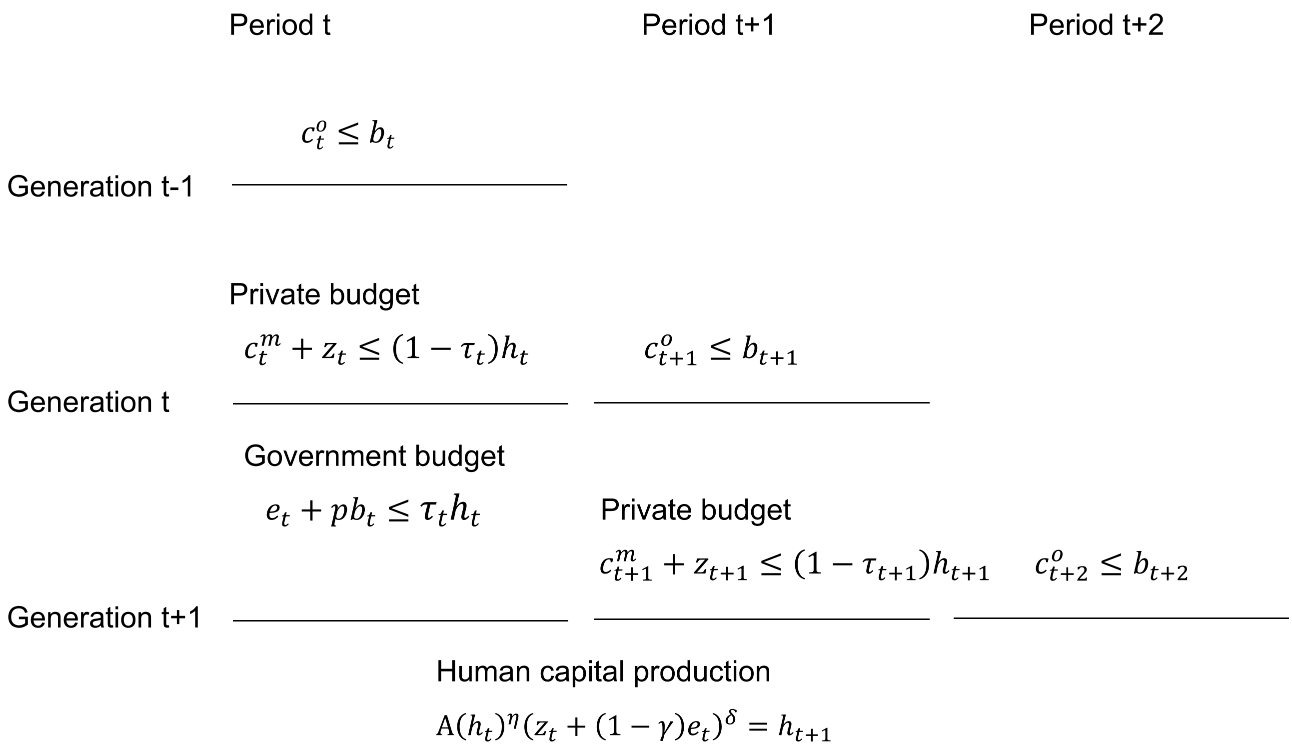


Figure 1: The structure of the model.

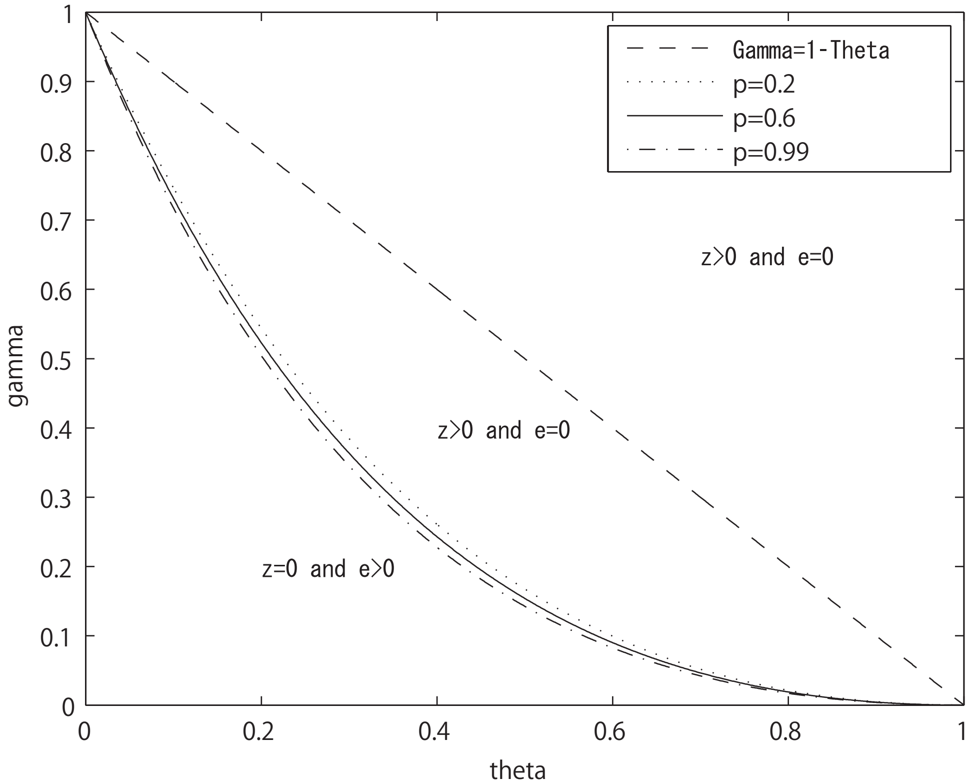


Figure 2: The dashed line presents $\gamma = 1-\theta$. The curves present $\gamma = 1-\theta\left(\frac{1+p\delta}{1+p\theta\delta}\right)^{(1+p\delta)/(p\delta)}$ for three values of $p$ : $p = 0.2, 0.6$ and $0.99$.

Panel (a)

| Country | Private | Public |
|---|---|---|
| Chile | 42.1 | 57.9 |
| Korea | 38.4 | 61.6 |
| United Kingdom | 31.4 | 68.6 |
| United States | 30.6 | 69.4 |
| Japan | 29.8 | 70.2 |
| Australia | 25.9 | 74.1 |

Panel (b)

| Country | Private | Public |
|---|---|---|
| Finland | 2.4 | 97.6 |
| Sweden | 2.5 | 97.5 |
| Belgium | 5.2 | 94.8 |
| Denmark | 5.5 | 94.5 |
| Estonia | 7.0 | 93.0 |
| Portugal | 7.4 | 92.6 |
| Ireland | 7.5 | 92.5 |
| Austria | 9.0 | 91.0 |
| Iceland | 9.6 | 90.4 |
| Italy | 9.9 | 90.1 |

Table 1: Relative proportions of public and private expenditure on educational institutions for all levels of education (2010 year). Panel (a) is the list of countries where the share of private education is more than 25% ; Panel (b) is the list of countries where the share of private education is below 10%.

Source: OECD (2013) Education at a glance 2013: OECD indicators, OECD Publishing.

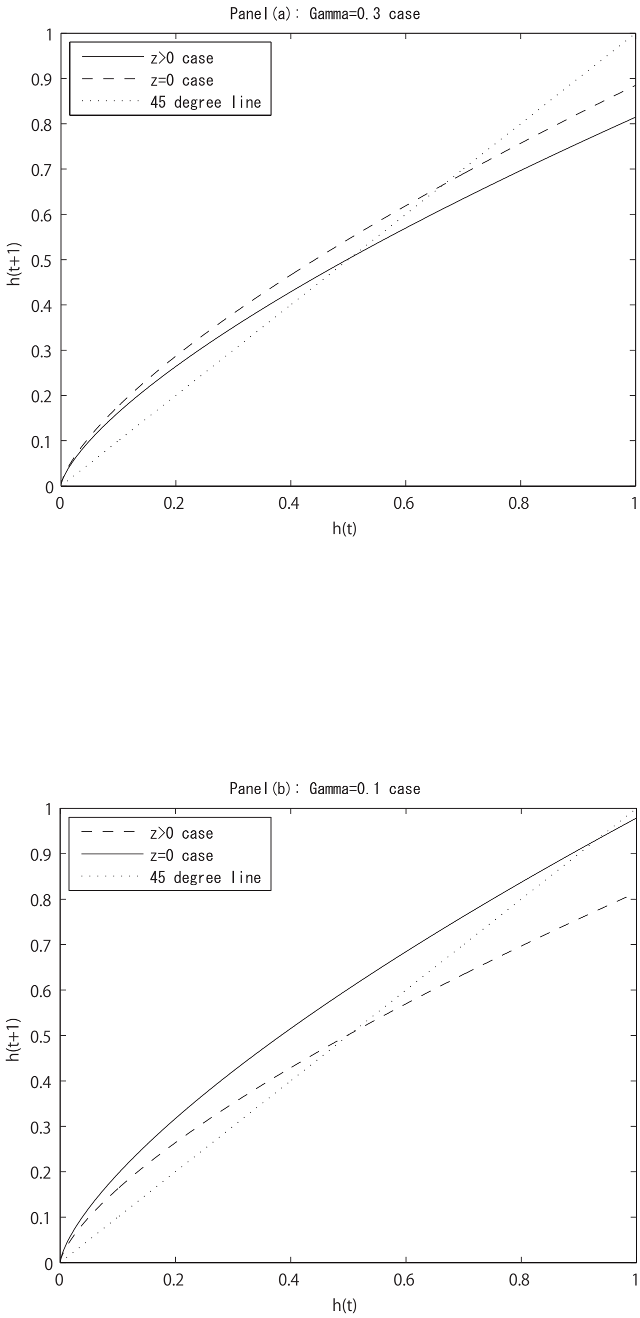


Figure 3: In each panel, a solid curve presents an equation of human capital production which is realized in equilibrium in accordance with a selected policy by the government; and a dashed curve presents one which is not realized in equilibrium because the corresponding policy is not selected by the government. The parameter values are set at $A = 2.0, p = 0.8, \theta = 0.5, \delta = 0.4$ and $\eta = 0.3$. Panel (a) with $\gamma = 0.3$ illustrates the result in Proposition 3(i); and Panel (b) with $\gamma = 0.1$ illustrates the result in Proposition 3(iii).