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Abstract

This paper analyzes the interactions between labor and housing (and land) markets in a city. We develop a monocentric city model involving land development and frictional unemployment. Unemployment, the spatial structure of a city, land development, housing demand, prices of housing and land are all endogenously determined in the model. We then characterize two different spatial configurations, spatial mismatch equilibrium in which unemployed workers are located far from jobs and integrated equilibrium in which unemployed workers live in areas close to jobs. To better understand how two equilibria are affected by labor market parameters, such as search intensity, wage, job finding rate, job destruction rate, and so on, we implement a comparative steady state analysis. We further explored the effects of policies such as a tax on land development to subsidize residents, a subsidy to reduce residents' commuting costs, and a subsidy to improve unemployed workers' benefits.

JEL Classification: R14, R21, R28

Key words: Land development, City structure, Search frictions, Spatial mismatch

1 Introduction

It is commonly observed that the intensity of land development varies a lot between different areas of modern cities: buildings close to a city center are usually taller than those in a city fringe. It is also observed that the incidence of unemployment is unevenly distributed between different areas within cities. In several U.S. large cities, the unemployment rates are higher in city centers than

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in suburbs.[1] In other countries, we also observe the variation of unemployment rate in different areas within a city, although it becomes more complex.[2] For instance, in Japan, the three largest metropolitan areas (MAs) exhibit different patterns:[3] in the second largest city, Osaka MA, the unemployment rate is higher in the city center than in suburbs (in 2005, it is 0.108 in the central city and 0.068 in the suburb cities). In the third largest city, Nagoya MA, the opposite holds true (the unemployment rate in 2005 is 0.055 in the central city and 0.044 in suburb cities). In the largest city, Tokyo MA, we observe no systematic spatial difference in the unemployment rate (in 2005, it is 0.056 in the central city and 0.056 in the suburb cities). As first shown by Wasmer and Zenou [21] and further investigated by subsequent studies (see Zenou [26]), interactions between land and labor markets can explain these spatial differences in the unemployment rate. They also proved that such interactions result in market inefficiency, indicating possible positive roles of policies in improving welfare of residents.

In this paper, we uncover effects of various policies on land use patterns and welfare by constructing a monocentric city model that involves unemployment and land use. To be more precise, we consider a city where all jobs are located in the unique central business district (CBD). Employed workers and unemployed workers coexist in the city due to search frictions in the labor market. Developers rent land from absentee landlords and supply housing service. Each worker obtains utility from the numèraire and housing consumption while she/he needs to commute to the CBD in order to work when employed and in order to search for a job when unemployed. By comparing the maximum housing price that each type of workers is willing to pay at each location, we obtain two types of equilibrium: spatial mismatch equilibrium in which unemployed workers locate far away from the CBD and integrated equilibrium where unemployed workers are close to the CBD. By using this framework, we analyze policy effects on land use/development and welfare of residents in the two types of equilibrium. Here, we focus on the following three policies: (a) a land development policy that taxes housing development in the city to subsidize residents, (b) a transportation policy that improves the transportation infrastructure in the city, and (c) an income transfer from employed workers to unemployed workers.

Our paper is related with long tradition of the literature of monocentric city models. This

[1] The discussion about this phenomenon can be found in the literature on spatial mismatch initiated by the seminal work of Kain [10].

[2] Zenou [25] made a careful comparative review of the spatial distributions of unemployment in U.S. cities and European cities.

[3] Here, we define the Japanese MAs as the Urban Employment Areas proposed by Kanemoto and Tokuoka [12].

literature dates back at least to the classic monocentric city model by Alonso [1], Mills [14], and Muth [15], which has become the standard framework to explain the observed regularities in the real world cities, such as the variation of land development intensity and housing (land) price. Brueckner [3] provides a unified treatment of these models, which is built around the key observation that difference of commuting costs within an urban area is balanced by the difference of houisng prices. It is shown that the model in Brueckner [3] does an excellent job of predicting the internal structure of cities and explaining intercity differences of spatial structure. To account for the observed pattern of higher-income groups locating more peripherally, Hartwick [6] and Wheaton [23] extends the monocentric city model with homogeneous workers to incorporate multiple income groups. When the income elasticity of land consumption exceeds the income elasticity of commuting costs, households of greater income select locations near the city fringe as a compensation to land consumption. Although these studies provided complete analysis of city structure under a perfectly competitive labor market, unemployment was not introduced in them, implying that there is no scope for welfare improving policies.

Given the prevailing spatial variations in labor market conditions, recent studies examined interactions between the labor market and the housing (land) market. Wasmer and Zenou [21] developed an urban search model by introducing a land market into the search-matching model.[4] In their model, workers' search efficiency is negatively affected by their distance to jobs. The endogeneous location of workers in the city reflects the trade-off between commuting costs, land rents, and the surplus associated with search. They show that there are two possible spatial configurations in the city. The unemployed workers may be located close to the CBD or the city fringe far away from jobs. Sato [18][19] considered the heterogeneity of workers in the background of urban labor market. In all these models, the land and housing markets are not fully modelled: they simply assumed that there is no land development in the city and each worker consumes fixed units of land. Coulson et al. [4] explained the spatial mismatch by developing a search matching model for a city with central and suburban labor markets. They assume that search is costly and the costs of setting up a firm is higher in the central markets. These two key assumptions ensure that unemployment rate is higher in the central area of a city than the suburban area. However, the city structue is still exogeneously given in their model. Our model is the most closely related to Smith and Zenou [20] and Xiao [24]. Smith and Zenou [20] extended the model described in Wasmer and Zenou [21] by endogenizing the job search intensity and housing consumption whereas they

[4] A complete introduction of the search-matching framework can be found in Pissarides [16].

treated land development exogenously, and Xiao [24] endogenized land development in the context of a monocentric city with search frictions in the labor market whereas he assumed fixed housing consumption.[5] In contrast, we endogenized both demand and supply sides of the housing market in order to examine the full relationship between housing and labor markets, which is indispensable to policy analysis.

The paper proceeds as follows. Section 2 proposes the model. Section 3 characterizes two spatial configurations, spatial mismatch equilibrium and integrated equilibrium. Section 4 explains the results of comparative steady state analysis that are usefull in understanding the policy effects. Section 5 explores the performance of policies. Section 6 concludes.

2 Model

2.1 Spatial structure

We extend the basic framework of an urban search model developed by Smith and Zenou [20]. Consider a closed city where there is a continuum of workers of size one. Workers are either employed or unemployed. An employed worker works and obtains wage income whereas an unemployed worker seaches for a job. We follow Smith and Zenou [20] in assuming the city structure and commuting behaviors of each type of workers: we consider a linear monocentric city, normalize the land endowment at each location to one, and assume that land is owned by absentee landlords. An employed worker commutes to the CBD to work and her/his commuting cost is tx, where t is a positive constant and x is the distance from the CBD. An unemployed worker commutes to the CBD to search for a position and get interviewed by firms posting vacancies. She/he bears the commuting cost stx, where $s \in (0, 1)$ represents the search intensity (such as frequency of job interviews). Because our primary purpose is to analyze the housing development in the monocentric city with job search, we simplify the framework of Smith and Zenou [20] by assuming that s is exogenous whereas we endogenize the supply of housing service.

2.2 Developers

Housing service is supplied by developers, who rent land L from absentee landlords and combine it with capital K to supply housing service. The production function of housing service is defined as $G(x) = L^{\gamma}K^{1-\gamma}$, where γ is a positive constant satisfying $0 < \gamma < 1$. The housing service

[5] For the earlier studies relating the labor market to the land (housing) market, see Zenou [26].

produced at location x can be written as $g(x) = S^{1-\gamma}$ in which $S = K/L$ is the capital-land ratio. A developer at location x, behaving as a price taker, determines the supply of housing service in order to maximize its profit:

$$\max_S \Pi(x) = R(x)S^{1-\gamma} - rS - \phi(x),$$

where $R(x)$, r, and $\phi(x)$ are the price of housing service at location x, the capital price, and the land rent at x, respectively. We assume that the capital market is global and the city is sufficiently small that the capital price is exogenously given.

The standard profit maximization yields the investment per acre of land and the supply of housing service as[6]

$$S(x) = \left[\frac{(1-\gamma) R(x)}{r}\right]^{1/\gamma}, \tag{1}$$

$$g(x) = \left[\frac{(1-\gamma) R(x)}{r}\right]^{(1-\gamma)/\gamma}. \tag{2}$$

The constant returns to scale in housing service production imply that the equilibrium profit of each developer is zero. From this zero profit condition, the bid rent, i.e., the maximum land rent that a developer can pay, $\Phi(x)$, is determined by $R(x)S(x)^{1-\gamma} - rS(x) - \Phi(x) = 0$. Inserting (1) into it, we obtain

$$\Phi(x) = \gamma \left(\frac{1-\gamma}{r}\right)^{(1-\gamma)/\gamma} R(x)^{1/\gamma}. \tag{3}$$

Because we assumed that landlords live outside of the city, land rents received by them can be seen as a leakage of welfare in the city.

2.3 Workers

There are e employed workers and u unemployed workers, implying that $e+u = 1$. When employed, each worker obtains the wage income w whereas she/he obtains b when unemployed. b represents the income of self-employment or unemployment benefits. We assume that both w and b are exogenous.

Workers obtain utility from consumption of housing service and the numéraire. We apply the instantaneous utility function of the Cobb-Douglas form:

$$u = \alpha \ln h + (1-\alpha) \ln z, \tag{4}$$

where $0 < \alpha < 1$. h and z describe the levels of housing and numéraire consumption, respectively.

[6] Notice that the land endowment at each location x is normalized to one.

2.4 Asset values and steady state conditions

Letting $R(x)$ denote the price of housing service, the budget constraint of a worker is

$$I = z + R(x)h + \tau x,$$

where I and τ are given by

$$I = \begin{cases} w & \text{if employed} \\ b & \text{if unemployed} \end{cases}, \qquad \tau = \begin{cases} t & \text{if employed} \\ st & \text{if unemployed} \end{cases}.$$

Although our framework is dynamic, consumption levels are determined by instant utility maximization. Each worker, taking prices and $g(x)$ as given, maximizes u with respect to z and h subject to the above budget constraint. Utility maximization leads to the standard demand functions:

$$z = (1-\alpha)(I - \tau x), \qquad h = \frac{\alpha(I - \tau x)}{R(x)}. \tag{5}$$

The resulting indirect utility is given by

$$v(x) = B + \ln(I - \tau x) - \alpha \ln R(x), \tag{6}$$

where $B \equiv \ln\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right]$.

Time is continuous and we assume the off-the-job search. The opportunity of landing a job for an unemployed worker arrive according to a Poisson process at an exogenous rate $s\lambda$ (> 0).[7] Each job is destroyed according to a Poisson process at an exogenous rate δ (> 0). Let $W(x)$ and $U(x)$ denote the asset value of an employed worker residing at x and that of an unemployed worker residing at x, respectively. $W(x)$ and $U(x)$ are given by

$$\begin{aligned} \rho W(x) &= v(x) + \delta(U_{\max} - W(x)), \\ \rho U(x) &= v(x) + s\lambda(W_{\max} - U(x)). \end{aligned} \tag{7}$$

where ρ is the exogenous discount rate. $U_{\max}$ and $W_{\max}$ represent the results of workers' maximization regarding location choice (i.w., $U_{\max} = \max_x U(x)$ and $W_{\max} = \max_x W(x)$). Equation (7) implies that the flow capital cost of an employed or unemployed worker at location x is equal to the instantaneous utility plus the utility derived from the change of her/his economic status.

We focus on the steady state. From the assumptions on the job matching and destruction process, we have the steady state condition the job flows:

$$\delta e = s\lambda u.$$

[7] λ can be endogenized in the same way as Smith and Zenou [20] without altering our main results.

Combined with $e + u = 1$, this yields

$$e = \frac{s\lambda}{\delta + s\lambda}, \qquad u = \frac{\delta}{\delta + s\lambda}. \tag{8}$$

It can be seen that an increase in job destruction rate δ reduces the number of employed workers and raises the number of unemployed workers. For search effort s and job finding rate λ, they have an opposite effect on e and u. An increase of s and λ leads to a larger e and a smaller u.

2.5 Bid rent functions and spatial structure

Workers can relocate costlessly within the city, which is a standard assumption in urban economics.[8] This assumption implies that there is no incentive for workers to relocate in equilibrium. Therefore, in equilibrium, all employed workers enjoy the same level of value ($W(x) = W^{\max} = W$) and this also holds true for unemployed workers ($U(x) = U^{\max} = U$).

In order to determine the equilibrium location of workers, we use the concept of bid rents, defined as the maximum price of housing service at location x that each type of worker is willing to pay in order to reach her/his respective level of equilibrium utility (in this paper, asset value). Plugging (6) into (7), the bid rents Ω of employed and unemployed worker can be expressed as follows:

$$\begin{aligned} \alpha \ln \Omega_w(x) &= B + \ln(w - tx) + \delta(U - W) - \rho W, \\ \alpha \ln \Omega_u(x) &= B + \ln(b - stx) + s\lambda(W - U) - \rho U. \end{aligned} \tag{9}$$

In order to see the location pattern in equilibrium, it is sufficient to examine the slope of the two bid rent functions at the intersection of $\Omega_w(x)$ and $\Omega_u(x)$. Differentiating $\Omega_w(x)$ and $\Omega_u(x)$ and evaluating them at $\Omega_w(x) = \Omega_u(x) = \Omega(x)$, we can see that

$$\Omega'_w(x) = -\frac{\Omega_w(x)}{\alpha}\frac{t}{w - tx}, \qquad \Omega'_u(x) = -\frac{\Omega_u(x)}{\alpha}\frac{st}{b - stx},$$

which imply that $\Omega'_w(x) < 0$ and $\Omega'_u(x) < 0$. Therefore, we have two bid rent functions which are downward sloping with respect to the distance from the CBD. We can see which is steeper than the other at the intersection of the two bid rent functions:

$$\mathbf{sgn}\left[\Omega'_w(x) - \Omega'_u(x)\right]\Big|_{\Omega_w(x)=\Omega_u(x)=\Omega(x)} = \mathbf{sgn}\left[sw - b\right].$$

From this, we have the following proposition:[9]

[8] For the effects of relocation costs in urban job search models, see Zenou [26] and Kawata [13].

[9] When $sw = b$, both employed and unemployed workers reside in mixed. Although we can characterize such equilibrium, this case yields no interesting results.

Proposition 1 *Employed workers reside closer to the CBD than unemployed workers if $sw < b$. The opposite holds true if $sw > b$.*

The intuition of Proposition 1 is straightforward. Given wage and unemployment benefits, the spatial structure of the city is determined by unemployed workers' search effort s. If the unemployed workers search less frequently, the commuting costs induced by search are lower. Then, employed workers have stronger incentive to reduce commuting costs and propose higher bid rents for the apartments close to the CBD than the unemployed workers. Therefore, the area near the CBD is occupied by employed workers and the area near the edge of the city is allocated to unemployed workers. However, if unemployed workers search more frequently, we then obtain a city with a different structure in which unemployed workers reside in the area close to the jobs while employed workers live far away from the CBD. Following the tradition in the urban job search models (see Zenou [26]), we call the former spatial structure the spatial mismatch equilibrium and the latter the integrated equilibrium.[10] In both cases, the residential area is divided into two zones: in one zone, only employed workers reside, and unemployed workers live in the other zone. We call the former the employment zone (EZ) and the latter the unemployment zone (UZ).

In the following analysis, we focus on the two possibilities in Proposition 1.[11]

3 Equilibrium

As noted before, in equilibrium, all employed workers enjoy the same utility level W and all unemployed workers enjoy the same utility levels U. Plugging $W_{\max} = \max_x W(x) = W$, $U_{\max} = \max_x U(x) = U$ and (2) into (7), we obtain

$$\begin{aligned} \rho W &= B + \ln(w - tx) - \alpha \ln R(x) + \delta(U - W), \\ \rho U &= B + \ln(b - stx) - \alpha \ln R(x) + s\lambda(W - U), \end{aligned} \tag{10}$$

We have two different spatial configurations (i) spatial mismatch equilibrium and (ii) integrated city equilibrium. We will deal with these two cases sequentially.

[10] The standard spatial mismatch hypothesis argues that the spatial disconnection between residential locations in inner cities and suburban job opportunities account for the adverse labor market outcomes of low skilled minorities (see the seminal work by Kain [10] and the empirical surveys by Jencks and Mayer [9], Holzer [7], Kain [11], Ihlanfeldt and Sjoquist [8]).

[11] If we endogenize the search intensity, much richer equilibrium configurations are possible. See Smith and Zenou [20] for details.

3.1 (i) Spatial mismatch equilibrium

We start from characterizing the spatial mismatch equilibrium (the case of $sw < b$). In this equilibrium, the market housing price is determined by

$$R(x) = \begin{cases} \Omega_w(x) & \text{for } x \in (0, \widehat{x}] \\ \Omega_u(x) & \text{for } x \in (\widehat{x}, \overline{x}] \end{cases}, \tag{11}$$

where $\widehat{x}$ is the intersection of $\Omega_w(x)$ and $\Omega_u(x)$. Hence, the market housing price at the center of the city $x = 0$ is determined by the bid rent of an employed worker, and that at the edge of the city $x = \overline{x}$ is determined by the bid rent of an unemployed worker. Here, we set the opportunity cost of land use to one. Therefore, we have

$$R(0) = \Omega_w(0), \qquad R(\overline{x}) = \Omega_u(\overline{x}) = 1.$$

Substituting these equations and (11) into (10), we obtain

$$\rho W = B + \ln w - \alpha \ln \Omega_w(0) - \delta(W - U)$$

$$\rho U = B + \ln(b - st\overline{x}) + s\lambda(W - U).$$

We can rewrite the bid rent functions (9) by using these asset values as

$$\Omega_w(x) = \Omega_w(0) \left(\frac{w - tx}{w} \right)^{1/\alpha}, \qquad \Omega_u(x) = \left(\frac{b - stx}{b - st\overline{x}} \right)^{1/\alpha}. \tag{12}$$

This shows that the bid rents of the employed and the unemployed workers are decreasing with the distance to the CBD. Solving $\Omega_w(x) = \Omega_u(x)$, we can see that $\widehat{x}$ is determined as

$$\widehat{x} = \frac{w \left[(b - st\overline{x})\, \Omega_w(0)^\alpha - b \right]}{t \left[(b - st\overline{x})\, \Omega_w(0)^\alpha - sw \right]}. \tag{13}$$

The remaining endogenous variables to be fixed are $\Omega_w(0)$ and $\overline{x}$, which are determined by the population conditions. The population density $d(x)$ at location x is described by the housing supply over the housing demand $g(x)/h(x)$. From (5) and (2), we obtain $d(x)$ as

$$d(x) = \begin{cases} \frac{1}{\alpha} \left(\frac{1-\gamma}{r} \right)^{(1-\gamma)/\gamma} \Omega_w(0)^{1/\gamma} w^{-1/(\alpha\gamma)} \left(w - tx \right)^{1/(\alpha\gamma) - 1} & \text{for } x \in (0, \widehat{x}] \\ \frac{1}{\alpha} \left(\frac{1-\gamma}{r} \right)^{(1-\gamma)/\gamma} \left(b - st\overline{x} \right)^{-1/(\alpha\gamma)} \left(b - stx \right)^{1/(\alpha\gamma) - 1} & \text{for } x \in (\widehat{x}, \overline{x}] \end{cases}. \tag{14}$$

We know that all employed workers reside in the employment zone $(0, \widehat{x}]$ and all unemployed workers live in the unemployment zone $(\widehat{x}, \overline{x}]$. Therefore, the numbers of employed and unemployed workers are described as

$$e = \int_0^{\widehat{x}} d(x)dx, \qquad u = \int_{\widehat{x}}^{\overline{x}} d(x)dx \tag{15}$$

Substituting (13) and (14) into (15) and solving the two equations, we obtain $\Omega_w(0)$ and $\overline{x}$ as

$$\Omega_w(0) = [1 + \phi t\,(su + e)]^{\gamma}\,, \qquad b - st\overline{x} = \frac{sw}{[1 + \phi t\,(su + e)]^{\alpha\gamma}} + \frac{b - sw}{[1 + \phi tsu]^{\alpha\gamma}}, \tag{16}$$

where $\phi = \frac{1}{\gamma}\,[r/\,(1-\gamma)]^{(1-\gamma)/\gamma}$.

3.2 (ii) Integrated equilibrium

In the integrated equilibrium (the case of $sw > b$), the market housing price is determined by

$$R(x) = \begin{cases} \Omega_u(x) & \text{for } x \in (0, \widehat{x}] \\ \Omega_w(x) & \text{for } x \in (\widehat{x}, \overline{x}] \end{cases}, \tag{17}$$

where $\widehat{x}$ is again the intersection of $\Omega_w(x)$ and $\Omega_u(x)$. Hence, the market housing price at the center of the city $x = 0$ is determined by the bid rent of an unemployed worker, and that at the edge of the city $x = \overline{x}$ is determined by the bid rent of an employed worker:

$$R(0) = \Omega_u(0), \qquad R(\overline{x}) = \Omega_w(\overline{x}) = 1.$$

Similarly to the previous section, these equations and (10) lead to

$$\begin{aligned} \rho W &= B + \ln(w - t\overline{x}) + \delta(U - W), \\ \rho U &= B + \ln b - \alpha \ln \Omega_u(0) + s\lambda(W - U). \end{aligned} \tag{18}$$

By substituting (18) into (9), we obtain

$$\Omega_w(x) = \left(\frac{w - tx}{w - t\overline{x}}\right)^{1/\alpha}, \qquad \Omega_u(x) = \Omega_u(0)\left(\frac{b - stx}{b}\right)^{1/\alpha}. \tag{19}$$

Solving $\Omega_w(x) = \Omega_u(x)$, it can be seen that $\widehat{x}$ is determined as

$$\widehat{x} = \frac{b\,[(w - t\overline{x})\Omega_u(0)^{\alpha} - w]}{t\,[s(w - t\overline{x})\Omega_u(0)^{\alpha} - b]}. \tag{20}$$

From (5) and (2), the population density $d(x)$ at location x is determined by

$$d(x) = \begin{cases} \frac{1}{\alpha}\left(\frac{1-\gamma}{r}\right)^{(1-\gamma)/\gamma} \Omega_u(0)^{1/\gamma} b^{-1/(\alpha\gamma)} (b - stx)^{1/(\alpha\gamma)-1} & \text{for } x \in (0, \widehat{x}] \\ \frac{1}{\alpha}\left(\frac{1-\gamma}{r}\right)^{(1-\gamma)/\gamma} (w - t\overline{x})^{-1/(\alpha\gamma)} (w - tx)^{1/(\alpha\gamma)-1} & \text{for } x \in (\widehat{x}, \overline{x}] \end{cases}. \tag{21}$$

The location of two types of workers in the integrated equilibrium is opposite to that in the spatial mismatch equilibrium. Using (21), the numbers of employed and unemployed workers are described as

$$e = \int_{\widehat{x}}^{\overline{x}} d(x)dx, \qquad u = \int_0^{\widehat{x}} d(x)dx.$$

From these equations, we can see that $\Omega_u(0)$ and $\overline{x}$ are determined by

$$\Omega_u(0) = [1 + \phi t(su + e)]^{\gamma}, \qquad w - t\overline{x} = \frac{1}{s}\left\{\frac{b}{[1 + \phi t(su + e)]^{\alpha\gamma}} + \frac{sw - b}{[1 + \phi te]^{\alpha\gamma}}\right\}. \tag{22}$$

The following proposition summarizes the above arguments:

Proposition 2 *The equilibrium is characterized by a tuple* $(g(x), \Phi(x), R(x), \Omega_w(x), \Omega_u(x), \widehat{x}, \overline{x}, R(0), e, u)$*: the numbers of two types of workers* (e, u) *are determined by (8). In the spatial mismatch equilibrium,* $R(0) = \Omega_w(0)$*, and (16) determines* $(\overline{x}, \Omega_w(0))$*. The bid rent functions* $(\Omega_w(x), \Omega_u(x))$ *and* $\widehat{x}$ *are given by (12) and (13), respectively, which determines the market housing price* $R(x)$ *as in (11). This , in turn, determines the development level* $g(x)$ *and the market land price* $\Phi(x)$ *as described in (2)and (3). In the integrated equilibrium,* $R(0) = \Omega_u(0)$*, and (22) determines* $(\overline{x}, \Omega_w(0))$*. Equations (19) and (20) determines* $(\Omega_w(x), \Omega_u(x))$ *and* $\widehat{x}$*, which, combined with (17), (2), and (3), gives* $R(x)$, $g(x)$, and $\Phi(x)$.

The two equilibrium city configurations are shown in Figure 1.

[Figure 1 around here]

4 Comparative Steady States

In this section, we explain the results of comparative steady states that help us understand the policy effects. The goal of our analysis is to deduce the impacts of search intensity, wage, and unemployment benefits on the spatial structure of the city ($\overline{x}$ and $\widehat{x}$), housing prices ($\Omega_0(x)$, $\Omega_w(x)$, and $\Omega_u(x)$), land prices ($\Phi_w(x)$ and $\Phi_u(x)$), land development ($g_w(x)$ and $g_u(x)$), employed and unemployed workers' lifetime utilities (W and U). These arguments are keys to understand the policy analysis that will be given in the next section. The results of comparative steady states for other parameters, t, δ, and λ, are given in Appendix.[12] Our results are comparable to those shown in Wheaton [22][23] and Brueckner [3].

4.1 An increase in the job search intensity

The results of comparative steady states regarding s are summarized in the following proposition.

[12] The proof of Proposition 3, 4 and A1 are given in a technical appendix that is available upon request.

Proposition 3 *The effects of changes in s are summarized as follows:*

[Table 1 around here]

Henceforth, we use '+' to represent a positive impact, '-' a negative impact, '0' no impact, and '?' an ambiguous impact. Now let's explain the impacts on all variables.

In the spatial mismatch equilibrium, the effects of an increase in the job search intensity s are shown in Fig. 2.

[Fig. 2 around here]

When s increases, employment in the city increases, which intensifies the competition for housings in the employment zone, EZ, and raises $\Omega_w(x)$ for $x \in (0, \widehat{x}]$. This can be confirmed in (16) and (12). Facing a higher housing price, the profit-maximizing developers then provide more housing. Therefore, land development in the EZ, $g_w(x)$, becomes larger. To produce more housing, developers increase the input of land. Hence, the land price in the EZ, $\Phi_w(x)$, also becomes higher. With an increase in land input, the threshold between EZ and UZ (the unemployment zone) moves from $\widehat{x}$ to a more distant location from the CBD, $\widehat{x}'$, and the employment zone gets larger from EZ to EZ' in Fig. 2.

The impact of s in the UZ on housing prices, $\Omega_u(x)$, land prices, $\Phi_u(x)$, and housing supplies, $g_u(x)$, is ambiguous. Using (12), it can be shown that the housing price curve in the UZ moves up and rotates in a clockwise direction with a rising s. The effect of s on $\Omega_u(x)$ depends on its impact on the size of the city, $\overline{x}$. If $\overline{x}$ decreases with the increase of s, then housing prices, land prices, and housing supplies increase in the area close to the EZ. In the area close to the edge of the city, $\Omega_u(x)$, $\Phi_u(x)$, and $g_u(x)$ decrease. However, if $\overline{x}$ becomes larger with an increase in s, then $\Omega_u(x)$, $\Phi_u(x)$, and $g_u(x)$ increase at each location in the UZ. In Fig. 2, we only show the case in which $\overline{x}$ becomes smaller.

Although the comparative static analysis is complicated in the UZ, the intuition is simple. An increase in s means that an unemployed worker increases the frequency of commuting to the CBD. To save commuting costs, she/he prefers a closer location to the EZ. As a result, housing prices in such locations are bid up, which in turn encourages developers to increase housing supplies there. Land prices in these locations also become higher. In the locations close to the city fringe, an unemployed worker would like to propose a higher bid rent if and only if the city becomes larger and commuting costs become higher.

Now consider the impacts on lifetime utilities of employed workers and unemployed workers, W and U. From (10), we can show that an increase in s reduces the instantaneous utility of employed workers by increasing housing prices in the EZ. However, its effect on the expected loss of becoming unemployed $(U - W)$ is ambiguous. The net effect on W is then ambiguous. For unemployed workers, the effects of s on their instantaneous utility and the expect gain from being employed in future are ambiguous. Hence, the net effect on U is ambiguous.

In the integrated equilibrium, with an increase in s, the housing price curve moves up and rotates in a clockwise direction, which then leads to an ambiguous impact on $\Omega_u(x)$, $\Phi_u(x)$, $g_u(x)$, and $\widehat{x}$. Intuitively, an increase in s raises the number of employed workers, which raises the demands for housing in the EZ. Then $\Omega_w(x)$, $\Phi_w(x)$, and $g_w(x)$ become larger. It can be shown that the city becomes larger when s increases.

4.2 An increase in income

The next proposition summarizes the results of comparative steady states regarding w and b.

Proposition 4 *The effects of changes in* w and b *are summarized as follows:*

[Table 2 around here]

An increase in w

The effects of an increase in the wage rate w in the spatial mismatch equilibrium are described in Fig. 3.

[Fig. 3 around here]

Using (16), it can be shown that an increase in w has no impact on the housing price at $x = 0$ and leads to a larger $\overline{x}$. Changes in the housing prices associated to a rise in w can be explained by using (12): in the EZ, an increase in w makes the housing price curve rotate in a counterclockwise direction around its intercept at the vertical axis. In the unemployment zone (UZ), it moves the housing price curve up and then rotates it in a clockwise direction. In the appendix, we show that the threshold $\widehat{x}$ becomes larger as w increases. Intuitively, a higher wage raises the housing demands of employed workers, which then increases housing prices in the EZ, $\Omega_w(x)$. Facing higher housing prices, developers would like to supply more housing to the market. To do so, they need to increase the input of land and propose higher bids for land. Hence, both $g_w(x)$ and $\Phi_w(x)$ increase. The EZ is also extended to EZ'.

From (12), we see that w has no direct impact on housing prices in the UZ. However, it still affects $\Omega_u(x)$ indirectly through its effects on the size of the city. With a rising w, $\overline{x}$ becomes larger and the city expands to $\overline{x}'$. It implies that each commuting trip to the CBD becomes more expensive for unemployed workers. To reduce expenditures on commuting, unemployed workers located in the UZ prefer apartments close to the EZ. Therefore, they propose higher bids for apartments in the area near the EZ. In the technical appendix, we show that an increase in w leads to an increase in $\Omega_u(x)$ at all locations in the UZ. On the supply side, as those in the EZ, developers increase the housing supply $g_u(x)$ and raises their bids for land $\Phi_u(x)$.

The impacts of w on workers' lifetime utilities, W and U, are ambiguous. An increase of w has two opposite effects on W (U). On the one hand, a higher w leads to an increase (decrease) of the employed (unemployed) workers' instantaneous utility, which has a positive (negative) impact on W (U). One the other hand, it increases the difference $W - U$, which is a loss (gain) for the employed (unemployed) workers in future. As a result, if the job destruction rate δ is low, an increase in w raises the employed workers' lifetime utility W. If the job acquisition rate $s\lambda$ is high, an increase in w also raises the unemployed workers' lifetime utility.

In the integrated equilibrium, a change in w doesn't affect unemployed workers' housing demand, so $\Omega_u(x)$, $\Phi_u(x)$, and $g_u(x)$ in the UZ are not affected. As a result, $\widehat{x}$ is unaltered. In the EZ, housing prices $\Omega_w(x)$ increases with a rising w. According to (2), (3) and (22), $g_w(x)$, $\Phi_w(x)$, and $\overline{x}$ also increase. It can be shown that an increase in w also raises W and U.

An increase in b

In the spatial mismatch equilibrium, an increase in the unemployment benefit b has no impacts on $\Omega_w(x)$, $g_w(x)$, $\Phi_w(x)$, and $\overline{x}$. This is because of our simple market, in which unemployment is from search frictions and unemployment benefits have no impact on employment. In the UZ, an increase in b rotates unemployed workers' bid rent curve in a counterclockwise direction around the intersection of two bid-rent curves. Hence, it raises $\Omega_u(x)$, which in turn raises $\Phi_u(x)$, and $g_u(x)$.

The city also becomes larger. As for U, an increase in b has a positive impact on an unemployed worker's instantaneous utility and a negative impact on her expected gain from being employed. However, the net effect is always positive. Finally, although b has no impact on an employed worker's instantaneous utility, an increase of b still increases W by decreasing the expected loss from the change of her economic status.

In the integrated equilibrium, the impact of b in the integrated city is similar to the impact of w in the spatial mismatch equilibrium. Applying the same argument, we show that an increase in b increases $\Omega_u(x)$, $\Omega_w(x)$, $\Phi_u(x)$, $\Phi_w(x)$, $g_u(x)$, $g_w(x)$, $\widehat{x}$ and $\overline{x}$. The impacts on W and U are ambiguous.

5 Policy issues

With the results of comparative steady states in hand, we move to the analysis of policy effects on housing development and welfare in this economy. Here, we consider three policies: land development tax, transportation policy, and income transfer. In so doing, we relate the performance of each policy to the search intensity of unemployed workers. Because the model in the previous section is already complex and becomes intractable once we introduce these policies, we resort to numerical analysis in this section.

The values of parameters we use are as follows. The wage rate of employed workers is $w = 30$ and the value of leisure is $b = 10$. Therefore, we have a spatial mismatch equilibrium when $s \leq 1/3$ and an integrated equilibrium when $s > 1/3$. The job finding rate is $\lambda = 2/5$ while the job destruction rate is $\delta = 1/50$. The discount rate is $\rho = 0.05$. Since the amount of non-land input can be measured in any unit, we normalize r to 1. Commuting cost per unit of distance is $t = 1/10$. The parameter in workers' utility function is $\alpha = 0.15$. In this paper, we set the search intensity as $s = 0.25(< 1/3)$ or $s = 0.8(> 0.8)$. We can obtain the qualitatively same results under different values of s.[13] In Table 3, we summarize the two equilibira under these parameter values.

[Table 3 around here]

Note here that the unemployment rate is higher in the spatial mismatch equilibrium (16.7%) where we set the search intensity as $s = 0.25$ than in the integrated equilibrium (5.9%) where we set the

[13] The results under different values of s are available upon request.

search intensity as $s = 0.8$. This is consistent with the conventional image that cities with spatial mismatch suffer from unemployment more than cities without it.

5.1 Land development tax

We first consider the effects of a land development tax. Consider a city government that collects taxes from land development and redistributes them to consumers in a lump-sum fashion. The main question is whether such development tax can possibly improve the welfare.

Denote by η^L the tax rate on housing development and by ζ the lump-sum transfer to a worker. Then, the profit function of a developer at x becomes as

$$\Pi(x) = \left(1 - \eta^L\right) R(x) S^{1-\gamma} - rS - \phi(x).$$

By solving the profit maximization problem and using the housing production function, we obtain the supply of housing as follows

$$g(x) = \left[\frac{(1-\gamma)}{r}\left(1 - \eta^L\right) R(x)\right]^{(1-\gamma)/\gamma}. \tag{23}$$

By using the zero-profit condition, we obtain the following bid-rent function

$$\Phi(x) = \gamma \left(\frac{1-\gamma}{r}\right)^{(1-\gamma)/\gamma} \left[\left(1 - \eta^L\right) R\left(x\right)\right]^{1/\gamma}. \tag{24}$$

We can see that the development tax reduces the supply of housing service at each location x given housing price $R(x)$. However, as shown in the following analysis, the development tax also indirectly affects housing supplies through its effects on housing prices. Assume that tax revenues are evenly redistributed among residents by the city government. The lump-sum subsidy ζ^L is determined by the city government's budget constraint

$$\eta^L \int_0^{\overline{x}} R\left(x\right) g(x) dx = \zeta^L.$$

With the lump sum transfer ζ^L, the budget constraints of the employed workers and the unemployed workers are respectively given by

$$w + \zeta^L = z + R\left(x\right) h + tx,$$
$$b + \zeta^L = z + R\left(x\right) h + stx.$$

Now we can derive the equilibrium under land development policy. As before, we have two different equilibria. In the case of the spatial mismatch equilibrium, the housing prices at $x = 0$

and $\overline{x}$ are given by

$$\Omega_w(0) = \left[1 + \phi\left(1 - \eta^L\right)^{-(1-\gamma)/\gamma} t\,(e + su)\right]^{\gamma}, \tag{25}$$

$$b + \zeta^L - st\overline{x} = \frac{s\left(w + \zeta^L\right)}{\left[1 + \phi\,(1 - \eta^L)^{-(1-\gamma)/\gamma}\, t\,(su + e)\right]^{\alpha\gamma}} + \frac{b - sw + (1 - s)\,\zeta^L}{\left[1 + \phi\,(1 - \eta^L)^{-(1-\gamma)/\gamma}\, tsu\right]^{\alpha\gamma}}.$$

Substituting (25) into (12), (11), (13) and replacing w by $w + \zeta^L$ and b by $b + \zeta^L$, we obtain the equilibrium solutions of $\Omega_w(x)$, $\Omega_u(x)$, $R(x)$, and $\widehat{x}$ when there is a land development policy. In the integrated equilibrium, the housing price at $x = 0$ and $\overline{x}$ are equal to

$$\Omega_u(0) = \left[1 + \phi\left(1 - \eta^L\right)^{-(1-\gamma)/\gamma} t\,(e + su)\right]^{\gamma}, \tag{26}$$

$$w + \zeta^L - t\overline{x} = \frac{1}{s}\left\{\frac{b + \zeta^L}{\left[1 + \phi\,(1 - \eta^L)^{-(1-\gamma)/\gamma}\, t\,(su + e)\right]^{\alpha\gamma}} + \frac{sw - b - (1 - s)\,\zeta^L}{\left[1 + \phi\,(1 - \eta^L)^{-(1-\gamma)/\gamma}\, te\right]^{\alpha\gamma}}\right\}.$$

Similarly, the equilibrium solutions of $\Omega_w(x)$, $\Omega_u(x)$, $R(x)$, and $\widehat{x}$ can be obtained by substituting (26) into (19), (17), (20) and then replacing w by $w + \zeta^L$ and b by $b + \zeta^L$.

Using the parameters given above, we examine the effect of land development tax by evaluating the derivatives of endogenous variables with respect to ζ^L at $\zeta^L = 0$. If these derivatives are positive, the land development tax affects endogenous variables positively. In Table 3, we report the effects of land development tax on asset values (W, U, and $SW \equiv eW + uU$), total land rent (TLR), land development ($g(x)$), and threshold locations ($\widehat{x}$ and $\overline{x}$) for two values of job search intensity, s.

[Table 4 around here]

The first and second columns show the effects of development tax on the spatial mismatch and integrated equilibrium, respectively. The impacts of land development policy are complicated. On the one hand, the subsidy ζ^L increases w and b. According to the results in Proposition 4, an increase in w has ambiguous impacts on (resp. increases) W and U while an increase in b raises (resp. has ambiguous impacts on) W and U in the spatial mismatch equilibrium (resp. in the integrated equilibrium). On the other hand, the tax rate η^L has a direct influence on housing prices, which affects workers' utilities through changing their housing demands. As shown in Table 4, the land development policy raises W, U, and SW for both types of equilibrium.

Table 4 also shows the impacts on TLR received by landlords, or the leakage from urban welfare. We found that the tax on land development reduces the leakage for all levels of s. In the spatial mismatch equilibrium, on the one hand, according to Proposition 4, an increase in w leads to a higher $\Omega_w(x)$ and a higher $\Omega_u(x)$ while an increase in b has no impact on $\Omega_w(x)$ and positive impacts $\Omega_u(x)$. In the integrated equilibrium, the effects of w and b get reversed. By increasing w or b, an increase in η^L raises land rents at all locations across the city. On the other hand, as shown in (24), an increase in η^L reduce land rents directly. Our numerical analysis shows that the negative effect of an increase in η^L dominates its positive effect for both types of equilibrium.

Let us now study housing supply $g(x)$ expressed by (23). Our focus is on housing supplies in the center of the EZ and the UZ. As in the analysis of the effect on TLR, an increase of η^L has two opposite effects on housing supplies. The result of our numerical analysis indicates that in the spatial mismatch equilibrium, the positive effect dominates the negative effect in the center of the UZ while the latter dominates the former in the center of the EZ. The development tax encourages land development in the suburbs whereas it discourages land development in the center of the city. Put differently, land development tax makes the city spatially dispersed in the spatial mismatch equilibrium. Unlike in the spatial mismatch equilibrium where the EZ is close to the CBD, it is the UZ that is close to the CBD in the integrated equilibrium. In such a case, the marginal impact of η^L on housing supply in the center of the UZ is negative while it is positive in the center of the EZ.

The city dispersion is confirmed by the results regarding $\widehat{x}$ and $\overline{x}$: the land development tax encourages developers to develop more housings at locations far from the CBD and raises $\widehat{x}$ and $\overline{x}$ for both types of equilibrium although the impacts on $\widehat{x}$ are much smaller than those on $\overline{x}$.

5.2 Transportation policy

The second policy to be considered is to reduce the commuting costs in the city. Suppose that the city government collects taxes η^T from each worker and use them as investment in transportation infrastructure in order to improve the commuting traffic conditions. With the transportation policy, the commuting cost per unit t is assumed to be reduced to $t/(1+\eta^T)$. Then, an employed worker's budget constraint can be written as

$$w - \eta^T = z + R(x)\,h + \frac{tx}{1+\eta^T},$$

while an unemployed worker has the following budget constraint

$$b - \eta^T = z + R(x)\,h + \frac{stx}{1+\eta^T}.$$

Replacing w by $w - \eta^T$, b by $b - \eta^T$, and t by $t/(1 + \eta^T)$ in the equilibrium conditions that we have solved in Section 3, we then obtain the spatial mismatch equilibrium and the integrated equilibrium under the transportation policy. Using the parameters given above, let's show how the investment to transportation infrastructure influences the equilibria.

The third and fourth columns of Table 4 report the results. Again, they can be explained using the results of comparative steady states. Indeed, an increase in η^T reduces w, b, and t. According to Proposition 4, a decrease in w has ambiguous impacts on (resp. decreases) W and U while a decrease in b reduces (resp. has ambiguous impacts on) W and U in the spatial mismatch equilibrium (resp. in the integrated equilibrium). As described in Appendix, a decrease in the commuting cost, t, leads to a rise in W and U. The net effects on W, U, and SW are then ambiguous. However, our numerical analysis shows that the positive effects dominate the negative effects, and the transportation policy raises W, U, and SW for both types of equilibrium.

The transportation policy affects TLR through reducing workers' income and commuting costs. We have shown that a decrease in w (resp. b) reduces housing prices across the city in the spatial mismatch equilibrium (resp. in the integrated equilibrium) while a decrease in b (resp. w) only reduces housing prices in the UZ (resp. EZ) in the spatial mismatch equilibrium (resp. in the integrated equilibrium). According to (3), land rents are affected in the same way. We have also shown in Appendix that a decrease in t ambiguously affects housing prices and hence, TLR. As a result, the net effects of η^T are ambiguous. Still, in our numerical analysis, η^T decreases TLR. Similarly, η^T has ambiguous effects on housing supply $g(x)$. However, our numerical exercise indicates that an investment in transportation infrastructure induces developers to supply less housing.

Theoretically, the impacts of η^T on $\widehat{x}$ and $\overline{x}$ are also ambiguous. On the one hand, a decrease in w (resp. b) decreases $\widehat{x}$ and $\overline{x}$ in the spatial mismatch equilibrium (resp. in the integrated equilibrium) while a decrease in b (resp. w) only decreases $\overline{x}$ in the spatial mismatch equilibrium (resp. in the integrated equilibrium). On the other hand, a decrease of commuting cost has ambiguous impacts on $\widehat{x}$ and $\overline{x}$. Hence, the net impact is not clear. However, the transportation policy increases both $\widehat{x}$ and $\overline{x}$ in a similar way in the spatial mismatch equilibrium whereas it increases $\overline{x}$ more than $\widehat{x}$ in the integrated equilibrium. This implies that the transportation policy enlarges EZ.

5.3 Transfer to unemployed workers

We finally consider a policy that aims at subsidizing unemployed workers. Suppose that the subsidy to unemployed workers is financed by taxes on employed workers. Denote by η^w the tax rate and by ζ^w the subsidy rate. Then an employed worker's budget constraint can be written as

$$w - \eta^w = z + R(x)\,h + tx,$$

whereas an unemployed worker faces the following budget constraint

$$b + \zeta^w = z + R(x)\,h + stx.$$

In equilibrium, the revenue of the city government is equal to its expenditure

$$\zeta^w = \frac{e\eta^w}{u}.$$

The model is exactly as before but we now replace w by $w - \eta^w$ and b by $b + e\eta^w/u$. If $\eta^w = 0$, we then goes back to the model introduced in Section 2. Now let's analyze the impact of an increase in s on the equilibrium outcomes when there is a transfer policy.

The effects on the spatial equilibrium outcomes are shown in the fifth and sixth columns. The transfer from employed workers to unemployed workers reduces w and raises b. The comparative steady states show that the net impacts on W and U are ambiguous. Here, we find that an increase in η^w raises W, U, and SW. Similarly, the comparative steady states show that the net effect of transfer on TLR is analytically ambiguous. However, our numerical analysis finds negligible impacts of η^w on TLR in the spatial mismatch equilibrium and negative effects in the integrated equilibrium.

Finally, let us analyze the impacts on the city structure. The transfer from employed workers to unemployed workers reduces housing demands of employed workers and increases that of unemployed workers. However, the housing supply is mostly unaffected. Hence, the EZ shrinks and the UZ expands. This is represented by decreases in $\widehat{x}$ in the spatial mismatch equilibrium and by increases in it in the integrated equilibrium.

6 Concluding remarks

This paper developed a monocentric city model with search frictions in a labor market and development in a land market. Our model captures some important features in the real world cities:

in equilibrium, we obtain two different spatial configurations, a segregated city and an integrated city, which are roughly correspond to "old cities" and "new or edge cities" in the US (Wasmer and Zenou [21]). We then explore the interactions between the labor market and the land (housing) market. We also investigated how policies that are likely to improve welfare are associated to land development and city structure. Analysis showed that the effects of policies on development patterns and on land rents (welfare of landlords) can be quite different between the two spatial configurations.

To better understand cities in the real world, some extensions of our model could be made. For example, a detailed labor market structure with endogenous search frictions could be introduced in the monocentric model. Another possible extension is to relax the assumption of monocentric city and consider a city with a subcenter.

References

[1] Alonso, W., 1964, *Location and Land Use*, Cambridge: Harvard University Press.

[2] Anderson, J.E., 1993, Land Development, Externalities, and Pigouvian Taxes, *Journal of Urban Economics* 33(1),1-9.

[3] Brueckner, J.K., 1987, The Structure of Urban Equilibria: A Unified Treatment of the Muth-Mills Model, ch. 20, 821-845 in: E.S. Mills, eds., *Handbook of Regional and Urban Economics*, vol. 2, Elsevier.

[4] Coulson, E., D. Laing, and P. Wang, 2001, Spatial Mismatch in Search Equilibrium, *Journal of Labor Economics* 19, 949-972.

[5] Fujita, M., 1989, *Urban Economic Theory*, Cambridge: Cambridge University Press.

[6] Hartwick, J., U. Schweizer, and P. Varaiya, 1976, Comparative Static Analysis of Residential Economy with Several Classes, *Journal of Economic Theory* 13, 396-413.

[7] Holzer, H., 1991, The Spatial Mismatch Hypothesis: What Has the Evidence Shown?, *Urban Studies* 28, 105-122.

[8] Ihlanfeldt, K. and D. Sjoquist, 1998, The Spatial Mismatch Hypothesis: A Review of Recent Studies and Their Implications for Welfare Reform, *Housing Policy Debate* 9, 849-892.

[9] Jencks, C. and S. Mayer, 1990, Residential Segregation, Job Proximity, and Black Job Opportunities, in: L.E. Lynn and M.G.H. Mcgeary, eds., *Inner-city Poverty in the United States*, 187–222. Washington, DC: National Academy Press.

[10] Kain, J.F., 1968, Housing Segregation, Negro Employment, and Metropolitan Decentralization, *Quarterly Journal of Economics* 82, 175-797.

[11] Kain, J.F., 1992, The Spatial Mismatch Hypothesis: Three Decades Later, *Housing Policy Debate* 3, 371-460.

[12] Kanemoto, Y., and K. Tokuoka, 2002, The Proposal for the Standard Definition of the Metropolitan Area in Japan [In Japanese.], *Journal of Applied Regional Science* 7, 1–15.

[13] Kawata, K., 2012, Explicit Spatial Friction and Worker Heterogeneity in Urban Search-matching Model, mimeo.

[14] Mills, E.S., 1967, An Aggregate Model of Resource Allocation in a Metropolitan Area, *American Economic Review, Papers and Proceedings* 57, 197-210.

[15] Muth, R.F., 1969, *Cities and Housing*, Chicago: University of Chicago Press.

[16] Pissarides, C.A., 2000, *Equilibrium Unemployment Theory, Second Edition*, M.I.T Press, Cambridge.

[17] Rossi-Hansberg, E., P.-D. Sarte, and R. Owens, 2010, Housing Externalities, *Journal of Political Economy* 118, 485-535.

[18] Sato, Y., 2001, Labor Heterogeneity in an Urban Labor Market, *Journal of Urban Economics* 50, 313-337

[19] Sato, Y., 2004, City Structure, Search, and Workers' Job Acceptance Behavior, *Journal of Urban Economics* 55, 350-370.

[20] Smith, T.E. and Y. Zenou, 2003, Spatial Mismatch, Search Effort, and Urban Spatial Structure, *Journal of Urban Economics* 54, 129-156.

[21] Wasmer, E. and Y. Zenou, 2002, Does City Structure Affect Job Search and Welfare?, Journal of Urban Economics 51,515-541.

[22] Wheaton, W.C., 1974, A Comparative Static Analysis of Urban Spatial Structure, *Journal of Economic Theory* 9, 223-237.

[23] Wheaton, W.C., 1976, On the Optimal Distribution of Income among Cities, *Journal of Urban Economics* 3, 31-44.

[24] Xiao, W., 2012, Search Frictions, Unemployment, and Housing in Cities: Theory and Policies," mimeo.

[25] Zenou, Y., 2000, Unemployment in Cities. in: J.-M. Huriot and J.-F. Thisse, eds., *Economics of Cities. Theoretical Perspectives*, Cambridge: Cambridge University Press, ch. 10, 343-389.

[26] Zenou, Y., 2009, *Urban Labor Economics*, Cambridge: Cambridge University Press.

Appendix: Full Results of Comparative Steady States

In this appendix, we explain the results of basic comparative steady states analysis regarding t, δ, and λ. The goal of our analysis is to deduce the impacts of these parameters on the spatial structure of the city ($\overline{x}$ and $\widehat{x}$), housing prices ($\Omega_0(x)$, $\Omega_w(x)$, and $\Omega_u(x)$), land prices ($\Phi_w(x)$ and $\Phi_u(x)$), land development ($g_w(x)$ and $g_u(x)$), employed and unemployed workers' lifetime utilities (W and U).

The next proposition summarizes the results of comparative steady states in the spatial mismatch equilibrium and integrated equilibrium.

Proposition A1 *The effects of changes in* t, δ, and λ *are summarized as follows:*

	W	U	$\Omega_w(x)$	$\Omega_u(x)$	$R(0)$	$\Phi_w(x)$	$\Phi_u(x)$	$\overline{x}$	$\widehat{x}$	$g_w(x)$	$g_u(x)$
spatial mismatch equilibrium											
t	-	-	?	?	+	?	?	?	?	?	?
δ	?	?	-	?	-	-	?	?	-	-	?
λ	?	?	+	?	+	+	?	?	+	+	?
integrated equilibrium											
t	-	-	?	?	+	?	?	?	?	?	?
δ	?	?	-	-	-	-	-	-	+	-	-
λ	?	?	+	+	+	+	+	+	-	+	+

Table A1. Comparative steady states regarding t, δ, and λ

An increase in t

We start from the spatial mismatch equilibrium. In the EZ, an increase in the commuting cost t affects housing prices $\Omega_w(x)$ in two different ways. On the one hand, it intensifies the competition for housing at $x = 0$, which then increases $\Omega_w(0)$, the housing price in the CBD. According to (12), an higher $\Omega_w(0)$ moves the housing price curve in the EZ up. On the other hand, the housing price curve rotates in a clockwise direction with a rising t. Therefore, the impact on $\Omega_w(x)$ and $\widehat{x}$ is ambiguous. However, we still can conclude that an increase in t at least raises the prices of housing at locations near the CBD. In turn, housing supplies and land prices in this area also increase. Intuitively, when t increases, the commuting trip becomes more expensive for employed workers. To reduce their commuting costs, employed workers prefer to live close to the CBD. Therefore, they propose higher bids for apartments in these locations. Housing supplies and land prices in these locations also increase with a rising t. In the UZ, the bid rent curve of unemployed workers also moves up and rotates in a clockwise direction with a rising t. Thus, the impacts on $\Omega_u(x)$, $g_u(x)$, and $\overline{x}$ are ambiguous. Concerning the utility level of workers, we show that an increase of commuting costs always reduces W and U.

An increase in t in the integrated equilibrium has the same impacts as in the spatial mismatch equilibrium.

An increase in δ and λ

Finally, take another two labor market parameters δ and λ. In the spatial mismatch equilibrium, an increase in job destruction rate δ leads to a decrease of employment, which weakens the competition for housing in the EZ and reduces housing prices $\Omega_w(x)$. In turn, housing supplies, $g_w(x)$, land prices, $\Phi_w(x)$, and the size of EZ, $\widehat{x}$, also become smaller with a rising δ. From (16), we see that the effect of δ on $\overline{x}$ is ambiguous. According to (19), (2), and (3), the effects of δ on housing prices, housing supplies, and land prices in the UZ are also ambiguous due to the indeterminate change of $\overline{x}$. The impact of λ is just the opposite of the impact of δ. Both δ and λ have ambiguous impacts on W and U.

In the integrated equilibrium, an increase in δ leads to a decrease in the total number of employed workers and searchers, $e + su$, which then reduces the housing price at $x = 0$. Using (19), (2) and (3), we show that $\Omega_u(x)$, $\Phi_u(x)$, and $g_u(x)$ decrease. An increase in the number of unemployed workers increases the size of UZ, $\widehat{x}$. According to (22), however, its impact on $\overline{x}$ is negative, which in turn decreases $\Omega_w(x)$, $\Phi_w(x)$, and $g_w(x)$. As in the spatial mismatch equilibrium, the changes following an increase of λ are just the reverse of the impacts of an increase in δ.

	W	U	$\Omega_w(x)$	$\Omega_u(x)$	$R(0)$	$\Phi_w(x)$	$\Phi_u(x)$	$\overline{x}$	$\widehat{x}$	$g_w(x)$	$g_u(x)$
					spatial mismatch equilibrium						
s	?	?	+	?	+	+	?	?	+	+	?
					integrated equilibrium						
s	?	?	+	?	+	+	?	+	?	+	?

Table 1. Comparative steady states regarding s

	W	U	$\Omega_w(x)$	$\Omega_u(x)$	$R(0)$	$\Phi_w(x)$	$\Phi_u(x)$	$\overline{x}$	$\widehat{x}$	$g_w(x)$	$g_u(x)$
					spatial mismatch equilibrium						
w	?	?	+	+	0	+	+	+	+	+	+
b	+	+	0	+	0	0	+	+	0	0	+
					integrated equilibrium						
w	+	+	+	0	0	+	0	+	0	+	0
b	?	?	+	+	0	+	+	+	+	+	+

Table 2. Comparative steady states regarding w and b

	spatial mismatch ($s = 0.25$)	integrated ($s = 0.8$)
Asset value of employed (W)	56.55	57.95
Asset value of unemployed (U)	50.12	55.14
Social welfare (SW)	55.48	57.79
Unemployment rate (u)	0.167	0.059
Housing supply of EZ ($g_w(x)$)	0.54	0.54
Housing supply of UZ ($g_u(x)$)	0.50	0.59

Table 3. Comparison between the spatial mismatch equilibrium and integrated equilibrium

Notes: Housing supplies, $g_w(x)$ and $g_u(x)$ are evaluated at the centers of EZ and UZ, respectively.

	Land development policy		Transportation policy		Transpfer policy	
	spatial mismatch	integrated	spatial mismatch	integrated	spatial mismatch	integrated
	$(s = 0.25)$	$(s = 0.8)$	$(s = 0.25)$	$(s = 0.8)$	$(s = 0.25)$	$(s = 0.8)$
Asset value of employed ($\partial W/\partial \tau$)	3.31	2.72	-0.44	-0.32	0.61	1.00
Asset value of unemployed ($\partial U/\partial \tau$)	4.87	3.45	-0.87	-0.49	3.80	5.19
Social welfare ($\partial SW/\partial \tau$)	3.57	2.77	-0.51	-0.33	1.14	1.24
Total land rent ($\partial TLR/\partial \tau$)	-1.70	-1.61	-0.06	-0.65	0.00	-2.06
Housing supply of EZ ($\partial g_w(x)/\partial \tau$)	-9.95	1.65	-0.03	-0.03	0.00	0.00
Housing supply of UZ ($\partial g_u(x)/\partial \tau$)	30.05	-0.51	0.00	-0.08	0.00	0.00
Border between EZ and UZ ($\partial \hat{x}/\partial \tau$)	0.42	0.10	0.75	0.02	-0.21	0.20
Edge of the city ($\partial \bar{x}/\partial \tau$)	1.97	37.00	0.71	0.88	0.04	-0.04

Table 4. Effects of policies

Notes: Housing supplies, $g_w(x)$ and $g_u(x)$ are evaluated at the centers of EZ and UZ, respectively.

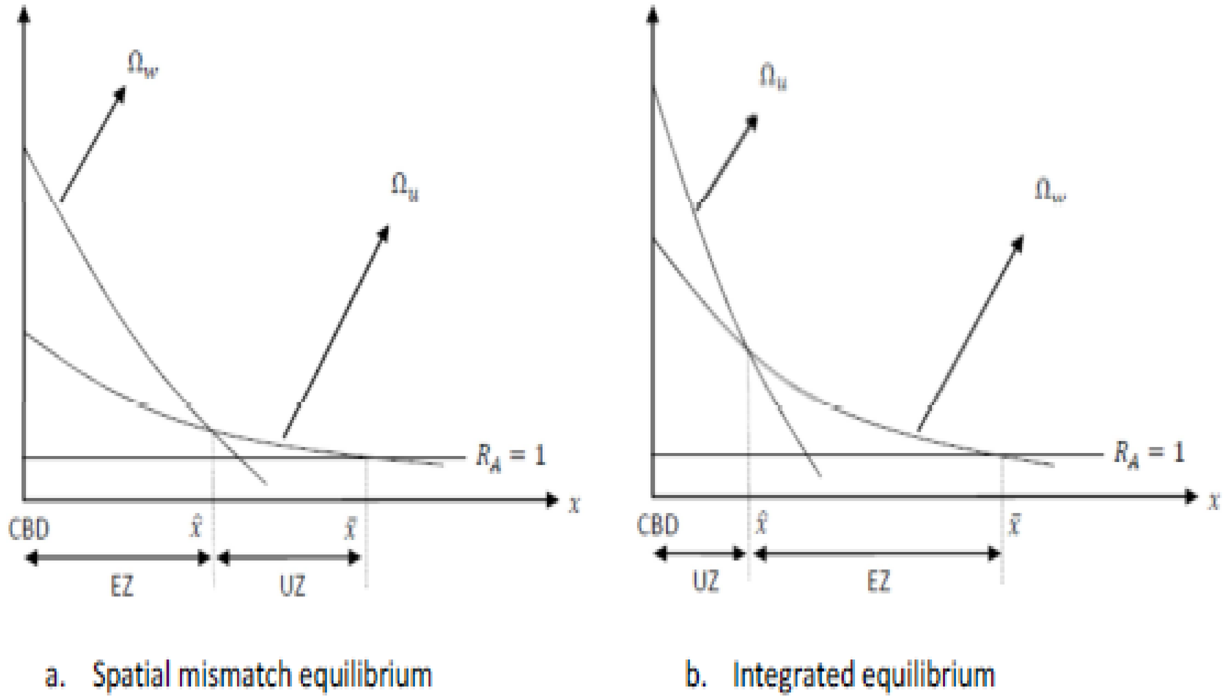

Fig. 1. Urban structure equilibria.

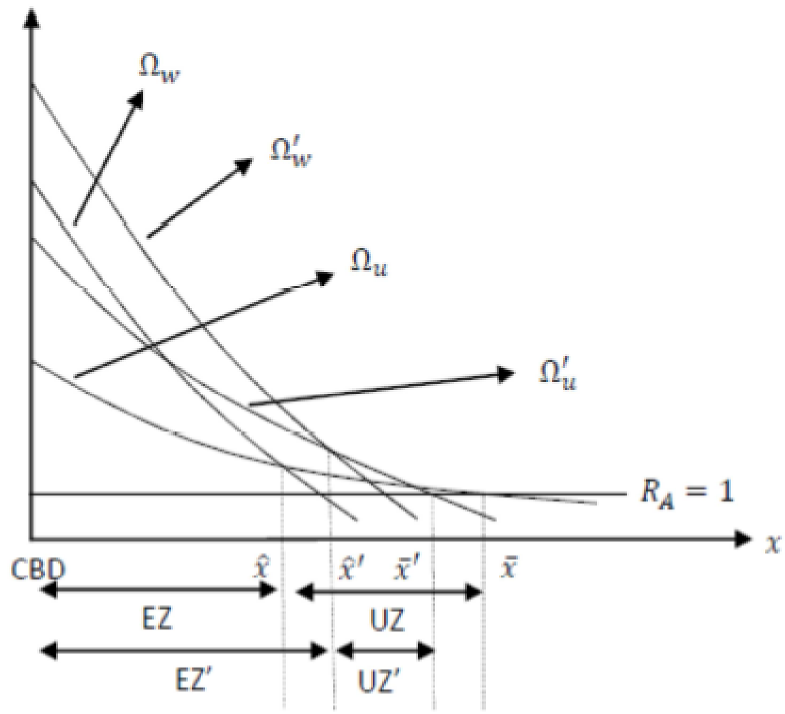

Fig. 2. Search intensity and the city structure in the spatial mismatch equilibrium.

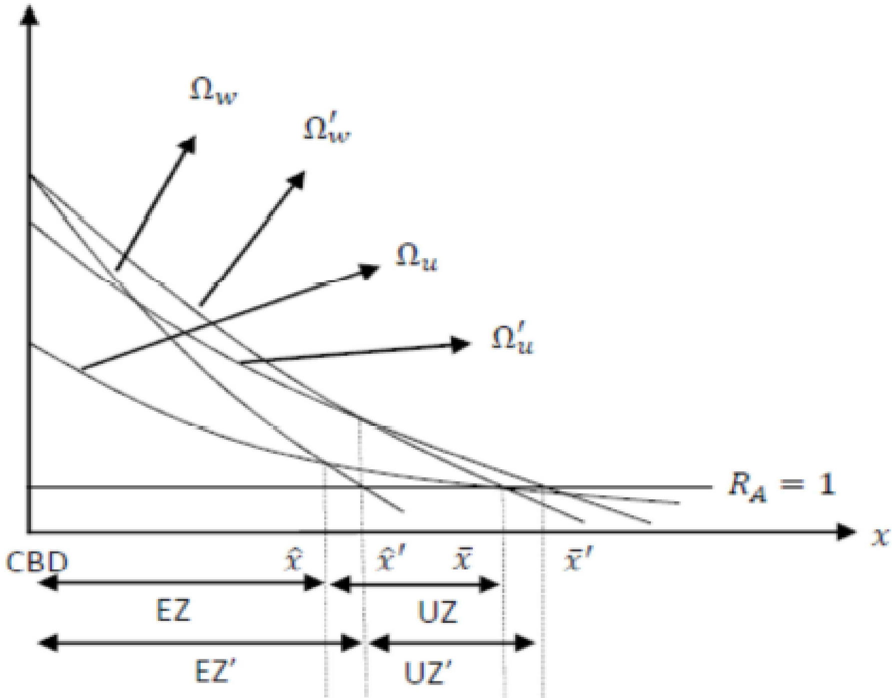

Fig. 3. Wage and the city structure in the spatial mismatch equilibrium.

Technical Appendix (NOT FOR PUBLICATION)

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This appendix presents the proof of the results summarized in Proposition 3, 4 and A1. We start from the spatial mismatch equilibrium.

1 The spatial mismatch equilibrium

1.1 Search intensity

Effect on $\widehat{x}$ and $\overline{x}$

Using (16), we get

$$(b - st\overline{x})\,\Omega_w(0)^\alpha = sw + (b - sw)\left(\frac{1+\phi t\,(su+e)}{1+\phi tsu}\right)^{\alpha\gamma}.$$

Plugging the above equation into (13) yields

$$\widehat{x} = \frac{w}{t}\left\{1 - \left[\frac{1+\phi tsu}{1+\phi t\,(su+e)}\right]^{\alpha\gamma}\right\}. \tag{A1}$$

Let

$$\Gamma \equiv \frac{1+\phi tsu}{1+\phi t\,(su+e)}.$$

We then have

$$\widehat{x} = \frac{w}{t}\left(1 - \Gamma^{\alpha\gamma}\right).$$

Plugging $e = \frac{s\lambda}{\delta+s\lambda}$ and $u = \frac{\delta}{\delta+s\lambda}$ into the expression of Γ and differentiating it with respect to s yields

$$\frac{\partial\Gamma}{\partial s} = -\frac{\phi t\frac{\delta\lambda}{(\delta+s\lambda)^2}}{\left[1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}\right]^2} < 0,$$

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which implies $\frac{\partial \widehat{x}}{\partial s} > 0$.

Let

$$\Lambda \equiv \frac{sw}{\left[1 + \phi t\left(su + e\right)\right]^{\alpha\gamma}} + \frac{b - sw}{\left[1 + \phi tsu\right]^{\alpha\gamma}}. \tag{A2}$$

Differentiating (16) with respect to s yields

$$\frac{\partial \overline{x}}{\partial s} = -\frac{1}{st}\left[\frac{\partial \Lambda}{\partial s} + t\overline{x}\right],$$

where

$$\begin{aligned}
\frac{\partial \Lambda}{\partial s} &= w\left[1 + \phi t\frac{s\left(\delta + \lambda\right)}{\delta + s\lambda}\right]^{-\alpha\gamma} - w\left[1 + \phi t\frac{s\delta}{\delta + s\lambda}\right]^{-\alpha\gamma} \\
&\quad - \frac{\alpha\gamma\phi t\delta}{\left(\delta + s\lambda\right)^2}\left\{\frac{sw\left(\delta + \lambda\right)}{\left[1 + \phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}\right]^{\alpha\gamma+1}} + \frac{\left(b - sw\right)\delta}{\left[1 + \phi t\frac{s\delta}{\delta+s\lambda}\right]^{\alpha\gamma+1}}\right\} \\
&< w\left\{\frac{1}{\left[1 + \phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}\right]^{\alpha\gamma}} - \frac{1}{\left[1 + \phi t\frac{s\delta}{\delta+s\lambda}\right]^{\alpha\gamma}}\right\} \\
&< 0.
\end{aligned}$$

Therefore, the sign of $\frac{\partial \overline{x}}{\partial s}$ is indeterminate.

Effect on $\Omega_w(x)$, $\Phi_w(x)$ and $g_w(x)$

Using (16), it is easy to show that $\frac{\partial \Omega_w(0)}{\partial s} > 0$. Then differentiating (12) with respect to s and using $\frac{\partial \Omega_w(0)}{\partial s} > 0$ in the result, we have $\frac{\partial \Omega_w(x)}{\partial s} > 0$, which together with (2) and (3) implies $\frac{\partial g_w(x)}{\partial s} > 0$ and $\frac{\partial \Phi_w(x)}{\partial s} > 0$ for $x \in (0, \widehat{x}]$.

Effect on $\Omega_u(x)$, $\Phi_u(x)$ and $g_u(x)$

Differentiating (12) yields

$$\frac{\partial \Omega_u(x)}{\partial s} = -\frac{tx}{\alpha}\left(b - stx\right)^{\frac{1}{\alpha}-1}\left(b - st\overline{x}\right)^{-\frac{1}{\alpha}} - \frac{1}{\alpha}\left(b - stx\right)^{\frac{1}{\alpha}}\left(b - st\overline{x}\right)^{-\frac{1}{\alpha}-1}\frac{\partial\left(b - st\overline{x}\right)}{\partial s},$$

where $\frac{\partial(b-st\overline{x})}{\partial s} = \frac{\partial \Lambda}{\partial s} < 0$. Therefore, the sign of $\frac{\partial \Omega_u(x)}{\partial s}$ is indeterminate. In turn, the impact of s on $\Phi_u(x)$ and $g_u(x)$ is also ambiguous.

Effect on W and U

Plugging (12) into (10) yields

$$\rho W = B + \frac{\rho + \delta}{\rho + \delta + s\lambda}\ln w - \frac{\alpha\left(\rho + s\lambda\right)}{\rho + \delta + s\lambda}\ln \Omega_w(0) + \frac{\delta}{\rho + \delta + s\lambda}\ln(b - st\overline{x}), \tag{A3}$$

$$\rho U = B + \frac{s\lambda}{\rho + \delta + s\lambda}\ln w - \frac{\alpha s\lambda}{\rho + \delta + s\lambda}\ln \Omega_w(0) + \frac{\rho + \delta}{\rho + \delta + s\lambda}\ln(b - st\overline{x}). \tag{A4}$$

The sign of $\frac{\partial W}{\partial s}$ and $\frac{\partial U}{\partial s}$ is then indeterminate.

1.2 Wage

Effect on $\widehat{x}$ and $\overline{x}$

Differentiating (A1) with respect to w, we obtain

$$\frac{\partial \widehat{x}}{\partial w}=\frac{1-\Gamma^{\alpha\gamma}}{t}>0.$$

Differentiating (16) with respect to w yields

$$\frac{\partial \overline{x}}{\partial w}=-\frac{1}{t}\left\{\frac{1}{\left[1+\phi t\left(su+e\right)\right]^{\alpha\gamma}}-\frac{1}{\left[1+\phi tsu\right]^{\alpha\gamma}}\right\}>0.$$

Effect on $\Omega_w(x)$, $\Phi_w\left(x\right)$ and $g_w\left(x\right)$

Differentiating (12) with respect to w yields

$$\frac{\partial \Omega_w(x)}{\partial w}=\frac{\Omega_w(0)}{\alpha}\frac{tx}{w^2}\left(\frac{w-tx}{w}\right)^{1/\alpha-1}>0.$$

We then have $\frac{\partial \Phi_w(x)}{\partial w}>0$ and $\frac{\partial g_w(x)}{\partial w}>0$.

Effect $\Omega_u(x)$, $\Phi_u\left(x\right)$ and $g_u(x)$

Differentiating (12) with respect to w yields

$$\frac{\partial \Omega_u(x)}{\partial w}=\frac{st}{\alpha}\left(b-stx\right)^{1/\alpha}\left(b-st\overline{x}\right)^{-\frac{1}{\alpha}-1}\frac{\partial \overline{x}}{\partial w},$$

where $\frac{\partial \overline{x}}{\partial w}>0$. We then have $\frac{\partial \Omega_u(x)}{\partial w}>0$. In turn, we have $\frac{\partial \Phi_u(x)}{\partial w}>0$ and $\frac{\partial g_u(x)}{\partial w}>0$.

Effect on W and U

Differentiating (A3) and (A4) yields

$$\rho\frac{\partial W}{\partial w}=\frac{\rho+\delta}{\rho+\delta+s\lambda}\frac{1}{w}+\frac{\delta}{\rho+\delta+s\lambda}\frac{\partial\left(b-st\overline{x}\right)}{\partial w},$$

$$\rho\frac{\partial U}{\partial w}=\frac{s\lambda}{\rho+\delta+s\lambda}\frac{1}{w}+\frac{\rho+\delta}{\rho+\delta+s\lambda}\frac{\partial\left(b-st\overline{x}\right)}{\partial w},$$

where $\frac{\partial(b-st\overline{x})}{\partial w}<0$. Therefore, the sign of $\frac{\partial W}{\partial w}$ and $\frac{\partial U}{\partial w}$ is then indeterminate.

1.3 Unemployment benefits

Effect on $\widehat{x}$ and $\overline{x}$

From (A1), it can be seen that b has no impact on $\widehat{x}$.

Differentiating (16) yields

$$\frac{\partial \overline{x}}{\partial b}=\frac{1}{st}\left(1-\frac{\partial \Lambda}{\partial b}\right),$$

where

$$\frac{\partial \Lambda}{\partial b} = \frac{1}{(1+\phi tsu)^{\alpha\gamma}} < 1.$$

Therefore, we have $\frac{\partial \overline{x}}{\partial b} > 0$.

Effect on $\Omega_w(x)$, $\Phi_w(x)$ and $g_w(x)$

From (12), we see that b has no impact on $\Omega_w(x)$. Then it has no impact on $\Phi_w(x)$ and $g_w(x)$ either.

Effect on $\Omega_u(x)$, $\Phi_u(x)$ and $g_u(x)$

Differentiating (12) yeilds

$$\begin{aligned} \frac{\partial \Omega_u(x)}{\partial b} &= \frac{1}{\alpha}\left(\frac{b-stx}{b-st\overline{x}}\right)^{1/\alpha-1}\frac{1}{b-st\overline{x}}\left[1-\frac{1}{(1+\phi tsu)^{\alpha\gamma}}\frac{b-stx}{b-st\overline{x}}\right] \\ &> \frac{1}{\alpha}\left(\frac{b-stx}{b-st\overline{x}}\right)^{1/\alpha-1}\frac{1}{b-st\overline{x}}\left[1-\frac{1}{(1+\phi tsu)^{\alpha\gamma}}\frac{b-st\widehat{x}}{b-st\overline{x}}\right] \end{aligned}$$

for $x > \widehat{x}$. Plugging (A1) into the above inequality, we obtain $\frac{\partial \Omega_u(x)}{\partial b} > 0$. In turn, we have $\frac{\partial \Phi_w(x)}{\partial b} > 0$ and $\frac{\partial g_u(x)}{\partial b} > 0$.

Effect on W and U

Differentiating (A3) and (A4) yields

$$\rho\frac{\partial W}{\partial b} = \frac{\delta}{\rho+\delta+s\lambda}\frac{1}{b-st\overline{x}}\frac{\partial\left(b-st\overline{x}\right)}{\partial b},$$

$$\rho\frac{\partial U}{\partial b} = \frac{\rho+\delta}{\rho+\delta+s\lambda}\frac{1}{b-st\overline{x}}\frac{\partial\left(b-st\overline{x}\right)}{\partial b}.$$

Since $\frac{\partial(b-st\overline{x})}{\partial b} > 0$, we have $\frac{\partial W}{\partial b} > 0$ and $\frac{\partial U}{\partial b} > 0$.

1.4 Commuting cost

Effect on $\widehat{x}$ and $\overline{x}$

Differentiating (A1) with respect to t, we obtain

$$\frac{\partial \widehat{x}}{\partial t} = -\frac{w\left(1-\Gamma^{\alpha\gamma}\right)}{t^2} - \frac{w\alpha\gamma\Gamma^{\alpha\gamma-1}}{t}\frac{\partial \Gamma}{\partial t},$$

where

$$\frac{\partial \Gamma}{\partial t} = -\frac{\phi\frac{s\lambda}{\delta+s\lambda}}{\left[1+\phi t\left(su+e\right)\right]^2} < 0.$$

Therefore, the sign of $\frac{\partial \widehat{x}}{\partial t}$ is indeterminate.

Differentiating (16) with respect to t yields

$$\frac{\partial \overline{x}}{\partial t} = -\frac{1}{st}\left[\frac{\partial \Lambda}{\partial t} + s\overline{x}\right],$$

where

$$\begin{aligned}\frac{\partial \Lambda}{\partial t} &= -\alpha\gamma \left\{ sw \left[1+\phi t\left(su+e\right)\right]^{-\alpha\gamma-1} \phi\left(su+e\right) + \left(b-sw\right)\left[1+\phi t su\right]^{-\alpha\gamma-1} \phi su \right\} \\ &< 0.\end{aligned}$$

Therefore, the sign of $\frac{\partial \overline{x}}{\partial t}$ is indeterminate.

Effect on $\Omega_w(x)$, $\Phi_w(x)$ and $g_w(x)$

Differentiating (12) with respect to t, we obtain

$$\frac{\partial \Omega_w(x)}{\partial t} = \frac{\partial \Omega_w(0)}{\partial t}\left(\frac{w-tx}{w}\right)^{1/\alpha} - \frac{t}{\alpha w}\Omega_w(0)\left(\frac{w-tx}{w}\right)^{1/\alpha-1},$$

where $\frac{\partial \Omega_w(0)}{\partial t} > 0$. Therefore, the sign of $\frac{\partial \Omega_w(x)}{\partial t}$ is indeterminate. The impact on $\Phi_w(x)$ and $g_w(x)$ is then ambiguous.

Effect on $\Omega_u(x)$, $\Phi_u(x)$ and $g_u(x)$

Differentiating (12) with respect to t yields

$$\frac{\partial \Omega_u(x)}{\partial t} = -\frac{1}{\alpha}\left[\left(b-st\overline{x}\right)^{-\frac{1}{\alpha}}\left(b-stx\right)^{\frac{1}{\alpha}-1} sx + \frac{\left(b-stx\right)^{\frac{1}{\alpha}}}{\left(b-st\overline{x}\right)^{\frac{1}{\alpha}+1}}\frac{\partial\left(b-st\overline{x}\right)}{\partial t}\right],$$

where $\frac{\partial(b-st\overline{x})}{\partial t} < 0$. Therefore, the sign of $\frac{\partial \Omega_u(x)}{\partial t}$ is indeterminate. The sign of $\Phi_u(x)$ and $g_u(x)$ is indeterminate either.

Effect on W and U

Differentiating (A3) and (A4) with respect to t yields

$$\rho\frac{\partial W}{\partial t} = -\frac{\alpha\left(\rho+s\lambda\right)}{\rho+\delta+s\lambda}\frac{1}{\Omega_w(0)}\frac{\partial \Omega_w(0)}{\partial t} + \frac{\delta}{\rho+\delta+s\lambda}\frac{1}{b-st\overline{x}}\frac{\partial(b-st\overline{x})}{\partial t}$$

$$\rho\frac{\partial U}{\partial t} = -\frac{\alpha s\lambda}{\rho+\delta+s\lambda}\frac{1}{\Omega_w(0)}\frac{\partial \Omega_w(0)}{\partial t} + \frac{\rho+\delta}{\rho+\delta+s\lambda}\frac{1}{b-st\overline{x}}\frac{\partial(b-st\overline{x})}{\partial t}$$

Note that $\frac{\partial \Omega_w(0)}{\partial t} > 0$ and $\frac{\partial(b-st\overline{x})}{\partial t} < 0$. Therefore, we have $\frac{\partial W}{\partial t} < 0$ and $\frac{\partial U}{\partial t} < 0$.

1.5 Job destruction

Effect on $\widehat{x}$ and $\overline{x}$

Differentiating (A1) yields

$$\frac{\partial \widehat{x}}{\partial \delta} = -\frac{w\alpha\gamma}{t}\Gamma^{\alpha\gamma-1}\frac{\partial \Gamma}{\partial \delta},$$

where

$$\frac{\partial \Gamma}{\partial \delta} = \phi t\frac{s\lambda}{\left(\delta+s\lambda\right)^2}\frac{1+s\phi t\frac{s\lambda}{\delta+s\lambda}+\phi t\frac{s\delta}{\delta+s\lambda}}{\left(1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}\right)^2} > 0.$$

Therefore, we have $\frac{\partial \widehat{x}}{\partial \delta} < 0$.

Differentiating (16) yields

$$\frac{\partial \overline{x}}{\partial \delta} = -\frac{1}{st}\frac{\partial \Lambda}{\partial \delta},$$

where the sign of $\frac{\partial \Lambda}{\partial \delta}$ is indeterminate. Therefore, the sign of $\frac{\partial \overline{x}}{\partial \delta}$ is indetetermainate either.

Effect on $\Omega_w(x)$, $\Phi_w(x)$ and $g_w(x)$

Differentiating (12) yields

$$\begin{aligned}\frac{\partial \Omega_w(x)}{\partial \delta} &= \left(\frac{w-tx}{w}\right)^{1/\alpha}\frac{\partial \Omega_0(x)}{\partial \delta} \\ &= -\gamma\left(\frac{w-tx}{w}\right)^{1/\alpha}\left[1+\phi t\left(su+e\right)\right]^{\gamma-1}\phi t\frac{s(1-s)\lambda}{\left(\delta+s\lambda\right)^2} \\ &< 0\end{aligned}$$

for $s \in (0,1)$. We then have $\frac{\partial \Phi_w(x)}{\partial \delta} < 0$ and $\frac{\partial g_w(x)}{\partial \delta} < 0$ for $s \in (0,1)$.

Effect on $\Omega_u(x)$, $\Phi_u(x)$ and $g_u(x)$

Differentiating (12) yields

$$\frac{\partial \Omega_u(x)}{\partial \delta} = \frac{1}{\alpha}\left(\frac{b-stx}{b-st\overline{x}}\right)^{1/\alpha-1}\left[-\frac{b-stx}{\left(b-st\overline{x}\right)^2}\right]\frac{\partial\left(b-st\overline{x}\right)}{\partial \delta}.$$

Since the sign of $\frac{\partial(b-st\overline{x})}{\partial \delta}$ is ambiguous, the sign of $\frac{\partial \Omega_u(x)}{\partial \delta}$ is indeterminate. The sign of $\frac{\partial \Phi_u(x)}{\partial \delta}$ and $\frac{\partial g_u(x)}{\partial \delta}$ is indeterminate either.

Effect on W and U

The impact of δ on W **and** U is ambiguous.

1.6 Job acquisition

Effect on $\widehat{x}$

Differentiating (A1), we get

$$\frac{\partial \widehat{x}}{\partial \lambda} = -\frac{w}{t}\alpha\gamma\Gamma^{\alpha\gamma-1}\frac{\partial \Gamma}{\partial \lambda},$$

where

$$\frac{\partial \Gamma}{\partial \lambda} = -\frac{\phi ts\delta}{\left(\delta+s\lambda\right)^2}\frac{1+s\phi t}{\left(1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}\right)^2} < 0.$$

We then have $\frac{\partial \widehat{x}}{\partial \lambda} > 0$.

Effect on $\Omega_w(x)$, $\Phi_w(x)$ and $g_w(x)$

Differentiating $\Omega_w(x)$ with respect to λ yields

$$\begin{aligned}\frac{\partial \Omega_w(x)}{\partial \lambda} &= \left(\frac{w-tx}{w}\right)^{1/\alpha} \frac{\partial \Omega_0(x)}{\partial \lambda} \\ &= \gamma \left(\frac{w-tx}{w}\right)^{1/\alpha} \left[1+\phi t\left(su+e\right)\right]^{\gamma-1} \phi t \frac{s\left(1-s\right)\delta}{\left(\delta+s\lambda\right)^2}.\end{aligned}$$

Therefore, we have $\frac{\partial \Omega_w(x)}{\partial \lambda} > 0$ for $s \in (0,1)$. Then, $\frac{\partial \Phi_w(x)}{\partial \lambda} > 0$ and $\frac{\partial g_w(x)}{\partial \lambda} > 0$.

Effect on $\overline{x}$, $\Omega_u(x)$, $\Phi_u(x)$, $g_u(x)$, W and U.

We find ambiguous impact of λ on $\overline{x}$, $\Omega_u(x)$**,** $g_u(x)$, W **and** U.

2 The integrated equilibrium

2.1 Search intensity

Effect on $\widehat{x}$ and $\overline{x}$

Using (22), we get

$$(w-t\overline{x})\,\Omega_u(0)^\alpha = \frac{1}{s}\left\{b+(sw-b)\left(\frac{1+\phi t(su+e)}{1+\phi te}\right)^{\alpha\gamma}\right\}.$$

Inserting this equation into (19), we have

$$\widehat{x} = \frac{b}{st}\left[1-\left(\frac{1+\phi te}{1+\phi t(su+e)}\right)^{\alpha\gamma}\right]. \tag{A5}$$

Plugging $e=\frac{s\lambda}{\delta+s\lambda}$ and $u=\frac{\delta}{\delta+s\lambda}$ into (A5) and differentiating it with respect to s leads to

$$\frac{\partial \widehat{x}}{\partial s} = -\frac{b}{s^2 t}\left[1-\left(\frac{1+\phi t\frac{s\lambda}{\delta+s\lambda}}{1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}}\right)^{\alpha\gamma}\right] + \left(\frac{1+\phi t\frac{s\lambda}{\delta+s\lambda}}{1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}}\right)^{\alpha\gamma-1} \frac{\frac{\alpha\gamma\phi b\delta^2}{s(\delta+s\lambda)^2}}{\left[1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}\right]^2},$$

which implies that the sign of $\frac{\partial \widehat{x}}{\partial s}$ is indeterminate.

Let

$$\Upsilon \equiv \frac{1}{s}\left\{\frac{b}{[1+\phi t(su+e)]^{\alpha\gamma}} + \frac{sw-b}{(1+\phi te)^{\alpha\gamma}}\right\}.$$

Differentiating (22) with respect to s leads to

$$\frac{\partial \overline{x}}{\partial s} = -\frac{1}{t}\frac{\partial \Upsilon}{\partial s},$$

where

$$\begin{aligned}
\frac{\partial \Upsilon}{\partial s} &= -\frac{b}{s^2}\left\{\frac{b}{[1+\phi t(su+e)]^{\alpha\gamma}}+\frac{sw-b}{[1+\phi te]^{\alpha\gamma}}\right\}+\frac{b}{s}\left\{\frac{-\alpha\gamma\phi t\frac{\delta(\delta+\lambda)}{(\delta+s\lambda)^2}}{[1+\phi t(su+e)]^{\alpha\gamma+1}}+\frac{\alpha\gamma\phi t\frac{\delta\lambda}{(\delta+s\lambda)^2}}{[1+\phi te]^{\alpha\gamma+1}}\right\} \\
&\quad +\frac{-\alpha\gamma w\phi t\frac{\delta\lambda}{(\delta+s\lambda)^2}}{[1+\phi te]^{\alpha\gamma+1}} \\
&= -\frac{b}{s^2}\left\{\frac{b}{[1+\phi t(su+e)]^{\alpha\gamma}}+\frac{sw-b}{[1+\phi te]^{\alpha\gamma}}\right\}-\frac{b}{s}\frac{\alpha\gamma\phi t\frac{\delta(\delta+\lambda)}{(\delta+s\lambda)^2}}{[1+\phi t(su+e)]^{\alpha\gamma+1}}-\frac{\alpha\gamma\phi t\frac{\delta\lambda}{(\delta+s\lambda)^2}}{[1+\phi te]^{\alpha\gamma+1}}\frac{sw-b}{s} \\
&< 0.
\end{aligned}$$

Therefore, $\frac{\partial \overline{x}}{\partial s}$ is positive.

Effect on $\Omega_w(x)$, $\Phi_w(x)$, and $g_w(x)$

Differentiating (19) with respect to s, yields

$$\frac{\partial \Omega_w(x)}{\partial s}=\frac{1}{\alpha}\left(\frac{w-tx}{w-t\overline{x}}\right)^{1/\alpha-1}\left[-\frac{(w-tx)}{(w-t\overline{x})^2}\frac{\partial\left(w-t\overline{x}\right)}{\partial s}\right],$$

where $\frac{\partial(w-t\overline{x})}{\partial s}=\frac{\partial \Upsilon}{\partial s}<0$. Therefore, we have $\frac{\partial \Omega_w(x)}{\partial s}>0$. In turn, $\frac{\partial \Phi_w(x)}{\partial s}$ and $\frac{\partial g_w(x)}{\partial s}$ are positive.

Effect on $\Omega_u(x)$, $\Phi_u(x)$, and $g_u(x)$

Differentiating (19) with respect to s, we get

$$\frac{\partial \Omega_u(x)}{\partial s}=\frac{\partial \Omega_0(x)}{\partial s}\left(\frac{b-stx}{b}\right)^{1/\alpha}-\frac{\Omega_0(x)}{\alpha}\left(\frac{b-stx}{b}\right)^{1/\alpha-1}\frac{tx}{b},$$

where the first term on the right-hand side is positive while the second term is negative. Therefore, the sign of $\frac{\partial \Omega_u(x)}{\partial s}$ is indeterminate. The impact on $\Phi_u(x)$ and $g_u(x)$ is also ambiguous.

Effect on W and U

Using (18), we get

$$\rho W = B+\frac{\delta}{\rho+\delta+s\lambda}\ln b-\frac{\alpha\delta}{\rho+\delta+s\lambda}\ln \Omega_u(0)+\frac{\rho+s\lambda}{\rho+\delta+s\lambda}\ln(w-t\overline{x}), \tag{A6}$$

$$\rho U = B+\frac{\rho+\delta}{\rho+\delta+s\lambda}\ln b-\frac{\alpha\left(\rho+\delta\right)}{\rho+\delta+s\lambda}\ln \Omega_u(0)+\frac{s\lambda}{\rho+\delta+s\lambda}\ln(w-t\overline{x}). \tag{A7}$$

The sign of $\frac{\partial W}{\partial s}$ and $\frac{\partial U}{\partial s}$ is indeterminate.

2.2 Wage

Effect on $\widehat{x}$ and $\overline{x}$

From (A5), it can be seen that w has no impact on $\widehat{x}$. Differentiating (22) yields

$$\frac{\partial \overline{x}}{\partial w}=\frac{1}{t}\left(1-\frac{1}{(1+\phi te)^{\alpha\gamma}}\right)$$

Therefore, we have $\frac{\partial \overline{x}}{\partial w} > 0$.

Effect on $\Omega_w(x)$, $\Phi_w(x)$ and $g_w(x)$

Differentiating (19) yields

$$\begin{aligned}\frac{\partial \Omega_w(x)}{\partial w} &= \frac{1}{\alpha}\left(\frac{w-tx}{w-t\overline{x}}\right)^{1/\alpha-1}\frac{1}{w-t\overline{x}}\left[1-\frac{\partial\left(w-t\overline{x}\right)}{\partial w}\frac{w-tx}{w-t\overline{x}}\right] \\ &= \frac{1}{\alpha}\left(\frac{w-tx}{w-t\overline{x}}\right)^{1/\alpha-1}\frac{1}{w-t\overline{x}}\left[1-\frac{1}{(1+\phi te)^{\alpha\gamma}}\frac{w-tx}{w-t\overline{x}}\right] \\ &> \frac{1}{\alpha}\left(\frac{w-tx}{w-t\overline{x}}\right)^{1/\alpha-1}\frac{1}{w-t\overline{x}}\left[1-\frac{1}{(1+\phi te)^{\alpha\gamma}}\frac{w-t\widehat{x}}{w-t\overline{x}}\right]\end{aligned}$$

for $x > \widehat{x}$. Plugging (A5) into the above inequality, we obtain $\frac{\partial \Omega_w(x)}{\partial w} > 0$. Then, the sign of $\frac{\partial \Phi_w(x)}{\partial w}$ and $\frac{\partial g_w(x)}{\partial w}$ is positive.

Effect on $\Omega_u(x)$, $\Phi_u(x)$ and $g_u(x)$

w has no impact on $\Omega_u(x)$, $\Phi_u(x)$, and $g_u(x)$.

Effect on W and U

Differentiating (A6) and (A7) yields

$$\rho\frac{\partial W}{\partial w} = \frac{\rho+s\lambda}{\rho+\delta+s\lambda}\frac{1}{w-t\overline{x}}\frac{\partial(w-t\overline{x})}{\partial w},$$

$$\rho\frac{\partial U}{\partial w} = \frac{s\lambda}{\rho+\delta+s\lambda}\frac{1}{w-t\overline{x}}\frac{\partial(w-t\overline{x})}{\partial w},$$

where $\frac{\partial(w-t\overline{x})}{\partial w} > 0$. Therefore, we have $\frac{\partial W}{\partial w} > 0$ and $\frac{\partial U}{\partial w} > 0$.

2.3 Unemployment benefit

Effect on $\widehat{x}$ and $\overline{x}$

From (A5), we see that b has a positive impact on $\widehat{x}$.

Differentiating (22) yields

$$\frac{\partial \overline{x}}{\partial b} = \frac{1}{st}\left\{\frac{1}{(1+\phi te)^{\alpha\gamma}} - \frac{1}{[1+\phi t(su+e)]^{\alpha\gamma}}\right\},$$

which implies $\frac{\partial \overline{x}}{\partial b} > 0$.

Effect on $\Omega_w(x)$, $\Phi_w(x)$ and $g_w(x)$

Differentiating (19) yields

$$\frac{\partial \Omega_w(x)}{\partial b} = -\frac{1}{\alpha}\left(\frac{w-tx}{w-t\overline{x}}\right)^{1/\alpha-1}\frac{w-tx}{(w-t\overline{x})^2}\frac{\partial\left(w-t\overline{x}\right)}{\partial b},$$

where

$$\frac{\partial\left(w-t\overline{x}\right)}{\partial b}=\frac{1}{s}\left\{\frac{1}{\left[1+\phi t(su+e)\right]^{\alpha\gamma}}-\frac{1}{(1+\phi te)^{\alpha\gamma}}\right\}<0.$$

Therefore, $\frac{\partial\Omega_w(x)}{\partial b}>0$. In turn, we have $\frac{\partial\Phi_w(x)}{\partial b}>0$, and $\frac{\partial g_w(x)}{\partial b}>0$.

Effect on $\Omega_u(x)$, $\Phi_u\left(x\right)$ and $g_u(x)$

Differentiating (19) with respect to b yields

$$\frac{\partial\Omega_u(x)}{\partial b}=\frac{\Omega_u(0)}{\alpha}\left(\frac{b-stx}{b}\right)^{1/\alpha-1}\frac{stx}{b^2}.$$

Therefore, we have $\frac{\partial\Omega_u(x)}{\partial b}>0$. In turn, we have $\frac{\partial\Phi_u(x)}{\partial b}>0$ and $\frac{\partial g_u(x)}{\partial b}>0$.

Effect on W and U

Differentiating (A6) and (A7) with respect to b, we get

$$\rho\frac{\partial W}{\partial b}=\frac{\delta}{\rho+\delta+s\lambda}\frac{1}{b}+\frac{\rho+s\lambda}{\rho+\delta+s\lambda}\frac{1}{w-t\overline{x}}\frac{\partial\left(w-t\overline{x}\right)}{\partial b},$$

$$\rho\frac{\partial U}{\partial b}=\frac{\rho+\delta}{\rho+\delta+s\lambda}\frac{1}{b}+\frac{s\lambda}{\rho+\delta+s\lambda}\frac{1}{w-t\overline{x}}\frac{\partial\left(w-t\overline{x}\right)}{\partial b}.$$

Since $\frac{\partial(w-t\overline{x})}{\partial b}<0$, we conclude that the sign of $\frac{\partial W}{\partial b}$ and $\frac{\partial U}{\partial b}$ is indeterminate.

2.4 Commuting cost

Effect on $\widehat{x}$ and $\overline{x}$

Differentiating (A5) with respect to t leads to

$$\frac{\partial\widehat{x}}{\partial t}=-\frac{b}{st^2}\left[1-\left(\frac{1+\phi te}{1+\phi t(su+e)}\right)^{\alpha\gamma}\right]+\frac{\alpha\gamma b\phi u}{t\left[1+\phi t(su+e)\right]^2}\left(\frac{1+\phi te}{1+\phi t(su+e)}\right)^{\alpha\gamma-1}.$$

Therefore, the sign of $\frac{\partial\widehat{x}}{\partial t}$ is indeterminate.

Differentiating (22) with respect to t yields

$$\frac{\partial\overline{x}}{\partial t}=-\frac{\overline{x}}{t}-\frac{1}{t}\frac{\partial\Upsilon}{\partial t},$$

where $\frac{\partial\Upsilon}{\partial t}<0$. Therefore, the sign of $\frac{\partial\overline{x}}{\partial t}$ is indeterminate.

Effect on $\Omega_w(x)$, $\Phi_w(x)$, and $g_w(x)$

Differentiating (19) with respect to t yields

$$\frac{\partial\Omega_w(x)}{\partial t}=-\frac{1}{\alpha}\left(\frac{w-tx}{w-t\overline{x}}\right)^{1/\alpha-1}\frac{1}{w-t\overline{x}}\left[x+\frac{w-tx}{w-t\overline{x}}\frac{\partial\left(w-t\overline{x}\right)}{\partial t}\right].$$

Since $\frac{\partial(w-t\overline{x})}{\partial t}<0$, the sign of $\Omega_w(x)$ is indeterminate. In turn, the impact on $\Phi_w(x)$ and $g_w(x)$ is indeterminate.

Effect on $\Omega_u(x)$, $\Phi_u(x)$, and $g_u(x)$

Differentiating (19) with respect to t yields

$$\frac{\partial \Omega_u(x)}{\partial t} = \frac{\partial \Omega_0(x)}{\partial t}\left(\frac{b-stx}{b}\right)^{1/\alpha} - \frac{\Omega_u(0)}{\alpha}\left(\frac{b-stx}{b}\right)^{1/\alpha-1}\frac{sx}{b}.$$

Therefore, the sign of $\frac{\partial \Omega_u(x)}{\partial t}$ is indeterminate. In turn, the sign of $\frac{\partial \Phi_u(x)}{\partial t}$ and $\frac{\partial g_u(x)}{\partial t}$ is indeterminate.

Effect on W and U

Differentiating (A6) and (A7) with respect to t yields

$$\rho\frac{\partial W}{\partial t} = -\frac{\alpha\delta}{\rho+\delta+s\lambda}\frac{1}{\Omega_u(0)}\frac{\partial \Omega_u(0)}{\partial t} + \frac{\rho+s\lambda}{\rho+\delta+s\lambda}\frac{1}{w-t\overline{x}}\frac{\partial(w-t\overline{x})}{\partial t},$$

$$\rho\frac{\partial U}{\partial t} = -\frac{\alpha(\rho+\delta)}{\rho+\delta+s\lambda}\frac{1}{\Omega_u(0)}\frac{\partial \Omega_u(0)}{\partial t} + \frac{s\lambda}{\rho+\delta+s\lambda}\frac{1}{w-t\overline{x}}\frac{\partial(w-t\overline{x})}{\partial t},$$

where $\frac{\partial \Omega_u(0)}{\partial t} > 0$ and $\frac{\partial(w-t\overline{x})}{\partial t} < 0$. Therefore, we have $\frac{\partial W}{\partial t} < 0$ and $\frac{\partial U}{\partial t} < 0$.

2.5 Job destruction

Differentiating (A5) with respect to δ yields

$$\begin{aligned}\frac{\partial \widehat{x}}{\partial \delta} &= \left(\frac{1+\phi t\frac{s\lambda}{\delta+s\lambda}}{1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}}\right)^{\alpha\gamma-1}\frac{\alpha\gamma b\phi s\lambda}{(\delta+s\lambda)^2}\frac{1+\phi t}{\left(1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}\right)^2} \\ &> 0.\end{aligned}$$

Differentiating (22) with respect to δ yields

$$\frac{\partial \Omega_u(0)}{\partial \delta} < 0,\ \frac{\partial \overline{x}}{\partial \delta} < 0.$$

Differentiating (19) with respect to δ and using the above results yields, we have $\frac{\partial \Omega_w(x)}{\partial \delta} < 0$ and $\frac{\partial \Omega_u(x)}{\partial \delta} < 0$, which in turn imply $\frac{\partial \Phi_w(x)}{\partial \delta} < 0$, $\frac{\partial \Phi_u(x)}{\partial \delta} < 0$, $\frac{\partial g_w(x)}{\partial \delta} < 0$, and $\frac{\partial g_u(x)}{\partial \delta} < 0$.

Effect on W and U

The effects of δ on W **and** U are ambiguous.

2.6 Job asquisition

Plugging $e = \frac{s\lambda}{\delta+s\lambda}$ and $u = \frac{\delta}{\delta+s\lambda}$ into (A5) and differentiating it with respect to λ yields

$$\begin{aligned}\frac{\partial \widehat{x}}{\partial \lambda} &= -\left(\frac{1+\phi t\frac{s\lambda}{\delta+s\lambda}}{1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}}\right)^{-1}\frac{\alpha\gamma b\phi s\delta}{(\delta+s\lambda)^2}\frac{1+\phi t}{\left[1+\phi t\frac{s(\delta+\lambda)}{\delta+s\lambda}\right]^2} \\ &< 0.\end{aligned}$$

Differentiating (22) yields

$$\frac{\partial \Omega_u(0)}{\partial \lambda} > 0, \ \frac{\partial \overline{x}}{\partial \lambda} > 0.$$

Differentiating (19) with respect to δ and using the above results, we obtain $\frac{\partial \Omega_w(x)}{\partial \lambda} > 0$, $\frac{\partial \Omega_u(x)}{\partial \lambda} > 0$, which in turn imply $\frac{\partial \Phi_w(x)}{\partial \delta} > 0$, $\frac{\partial \Phi_u(x)}{\partial \delta} > 0$, $\frac{\partial g_w(x)}{\partial \lambda} > 0$, and $\frac{\partial g_u(x)}{\partial \lambda} > 0$.

Effect on W and U

The effects of λ on W **and** U are ambiguous.