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Intellectual Property Rights and Foreign Direct Investment:
A Welfare Analysis

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Abstract

This paper examines how intellectual property rights (IPR) protection affects innovation and foreign direct investment (FDI) using a North–South quality-ladder model incorporating the exogenous and costless imitation of technology and subsidy policies for both R&D and FDI. We show that for the interior steady state to be stable, either R&D or FDI subsidy rates must be positive. Our findings also indicate that strengthening IPR protection promotes both innovation and FDI. Moreover, a strengthening of IPR protection can also improve welfare if the initial IPR protection in the South is weak and the R&D subsidy rate is not too high.

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1 Introduction

Since the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPs) was signed in the Uruguay Round, developing countries that are members of the World Trade Organization (WTO) have been under pressure to adopt a set of minimum standards on intellectual property rights (IPR). To comply with these international agreements, some developing countries have recently strengthened their IPR protection. For example, according to the indexes in Park (2008), patent protection in Brazil, China, and India generally strengthened between 1990 and 2005 compared with before 1990.

This change toward strengthening IPR protection in developing countries is likely to have a great impact on innovation and foreign direct investment (FDI) in these countries for several reasons. For example, strengthening IPR protection in a developing country makes it difficult for local firms to copy products developed by other firms and decreases the risk of technology imitation in that country. Thus, strengthening IPR protection is likely to influence the decision of a firm with advanced technology on whether to transfer production to a developing country. In addition, a decrease in imitation changes the monopolistic rent that the inventor of a good can earn, which is likely to influence R&D activities by firms in developed countries.

The present paper theoretically investigates the impact of strengthening IPR protection in developing countries using a dynamic general equilibrium model with two countries: the North, where new technology is invented, and the South, where new technology cannot be invented but can be transferred from the North through FDI. From this analysis, we derive three results. First, strengthening IPR protection in the South increases the wage in the South relative to that in the North. Second, strengthening IPR protection increases innovation in the North and the flow of FDI from the North to the South in both the long and the short run. Third, strengthening IPR protection can improve the welfare of both Southern and Northern households if the initial IPR protection in the South is sufficiently weak and the R&D subsidy rate is not too high.

A number of theoretical studies on technology transfer have examined the influence of strengthening IPR protection using North–South dynamic general equilibrium models where the chosen channel for technology transfer is FDI. However, these studies are divided on the results. For example, two of the most important studies in this field, Lai (1998) and Glass and...
Saggi (2002), obtained contrasting results. Lai (1998), using a model of variety-expanding-type innovation, concluded that strengthening IPR protection promotes both innovation and FDI, whereas Glass and Saggi (2002), using a model of quality-improvement-type innovation, suggested the opposite. In related work, Glass and Wu (2007) (hereafter G-W) introduced costless imitation, as did Lai (1998), into a quality-improvement-type R&D model similar to that of Glass and Saggi (2002), and examined how increasing the probability of imitation affects innovation and FDI. Their results showed that strengthening IPR protection impedes both innovation and FDI. This finding lies contrary to Lai (1998) but is similar to that in Glass and Saggi (2002). By comparing the settings and results in these papers, G-W (2007) surmised that we could attribute the different results in Lai (1998) and Glass and Saggi (2002) to whether innovation is variety expanding or quality improving.

In this paper, we show that this presumption is not correct using a quality-ladder-type model. More specifically, the present paper extends G-W’s (2007) model by introducing selected industrial policies into the model, i.e., subsidies for R&D and FDI, in order to reexamine the effect of strengthening IPR protection. Our model also includes the case of “inefficient followers” from their paper as a particular case where both of these subsidies are zero. In terms of results, our model shows that the unique interior steady state is necessarily unstable if both subsidies are zero. Hence, there is no equilibrium path converging to the interior steady state in the case of zero subsidies. In that case, following a policy change, the economy must move toward a “corner-solution equilibrium” in which some endogenous variables are zero. This result implies that the conclusion on IPR protection in G-W’s (2007) inefficient followers’ case needs to be reexamined because we cannot apply comparative statics of the steady state to the evaluation of a policy change. To address this issue, we prove that the unique steady state can be stable and comparative statics are applicable if the subsidy rates are higher than some critical level. Our model shows that if the interior steady state is stable, strengthening IPR protection necessarily promotes both innovation and FDI. This central conclusion is the opposite of the result in G-W (2007) and the same as that of the variety-expanding-type model in Lai (1998). Thus, our result proposes a counterexample to G-W’s (2007) conjecture that whether strengthening IPR protection promotes innovation and FDI depends on the type of innovation.

As a more important topic, we also explore the welfare effects of strengthening IPR pro-
tection. Many earlier studies in this area, including Lai (1998), Glass and Saggi (2002), and G-W (2007), did not analyze the welfare effects because they focused on the effects on innovation and FDI. However, we cannot draw conclusions about the desirability of IPR policies based only on their effects on innovation and FDI. Our welfare analysis shows that strengthening IPR protection in the South entails simultaneous dynamic and static effects on welfare. The former dynamic effect arises from promoting innovation. An increase in innovation then enables households to consume higher-quality goods over time and increases their welfare. But strengthening IPR protection also accounts for static effects through changing the number of imitated goods and the income of households. As the imitator firms produce the imitated goods competitively, they sell at a lower price than the other goods produced monopolistically under the patents. Therefore, strengthening IPR protection may reduce welfare through decreasing imitation and the number of cheaper goods. In addition, it may affect welfare through changing the wages and the values of shares owned by households. To evaluate the desirability of IPR policies, we then need to compare the sizes of the dynamic effects with those of the static effects. In this paper, we show that the dynamic effects outweigh the static effects if the initial IPR protection is sufficiently weak and the rate of R&D subsidy is not too high. This implies that stronger protection of IPR can improve the welfare of the South and the North.

The remainder of the paper is structured as follows. Section 2 describes the model. In Section 3, we derive the equilibrium path of this model. In Sections 4 and 5, we show that strengthening IPR protection promotes both innovation and FDI. Section 6 shows that strengthening IPR protection can improve welfare. In Section 7, we discuss the welfare effects of subsidy policies and the welfare effects in the model where imitation and FDI are costly processes. Section 8 provides some concluding remarks.

2 The Model

Our model has the same basic structure as that of G-W (2007), which is a version of the North–South quality-ladder model developed by Grossman and Helpman (1991, Ch. 12). The main

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1Our model is based on the “fully endogenous” rather than the “semi-endogenous” theory of growth. The fully endogenous growth model has often been criticized because of the “problem” of scale effects. However, Ha and
difference between our model and G-W (2007) is the existence of two subsidies, one for R&D and the other for FDI. Consider an economy consisting of two countries, the North and the South, denoted by \(N\) and \(S\), respectively. The population size of country \(i \in \{N, S\}\) is constant and given by \(L_i\). Each agent supplies one unit of labor inelastically at each point in time \(t \in [0, \infty)\) at a wage of \(w_i(t)\). We choose Southern labor as the numeraire and normalize \(w_S(t)\) to one at any point in time \(t\). We let \(w(t)\) denote the relative wage of the North to the South: \(w(t) \equiv w_N(t)/w_S(t) = w_N(t)\).

In this economy, there is a continuum of goods, indexed by \(j \in [0, 1]\), that are produced in the North or the South. One unit of good output requires one unit of labor input. Each good is classified by a number of “generations” \(m = 0, 1, 2, \cdots\). We normalize the generation number of every good to be zero at time \(t = 0\). A one-step newer generation of good \(j\) becomes available if innovation takes place in industry \(j\) as a result of successful R&D efforts by a firm. We assume that different generations of a good have different “qualities”. The quality of good \(j\) of generation \(m\) is provided by \(q_m(j) = \lambda^m\).

2.1 Consumers

Consumers living in country \(i \in \{N, S\}\) have the following lifetime utility:

\[
U_i = \int_0^{\infty} e^{-\rho t} \log u_i(t) dt, \tag{1}
\]

where \(\rho\) is a common subjective discount rate and \(\log u_i(t)\) represents instantaneous utility at time \(t\). We specify the instantaneous utility function as:

\[
\log u_i(t) = \int_0^{1} \log \left( \sum_m q_m(j)x_{i,m}(j, t) \right) dj, \tag{2}
\]

where \(x_{i,m}(j, t)\) denotes consumption by consumers living in country \(i\) of generation \(m\) of good \(j\) at time \(t\). The representative consumer in country \(i \in \{N, S\}\) maximizes his or her lifetime utility.

Howitt (2007), for example, have argued that fully endogenous theory is more consistent than semi-endogenous theory with the long-run data. Further, Aghion and Howitt (2006, p. 98) stated that “... there is no evidence pointing to the absence of a scale effect at the world level or in small closed economies”. Because debate remains as to whether endogenous or semi-endogenous theory is more appropriate, we adopt an endogenous growth model. This generally has the benefit of a simpler dynamic structure than the semi-endogenous growth model, so we can obtain clearer results, particularly for welfare analysis.
utility (1) under the following budget constraint:
\[
\int_0^\infty e^{-\int_0^t r(s)ds} E_i(t)dt = A_i(0) + \int_0^\infty e^{-\int_0^t r(s)ds} w_i(t)dt - \int_0^\infty e^{-\int_0^t r(s)ds} T_i(t)dt,
\]
where \( r(t) \) is the interest rate that consumers in both countries face at time \( t \), \( A_i(0) \) is the initial asset holdings of a consumer in country \( i \), and \( T_i(t) \) is a lump-sum tax levied by the government of country \( i \). The term \( E_i(t) \) represents the flow of expenditure by a consumer in country \( i \in \{N, S\} \) at time \( t \), namely:
\[
E_i(t) = \int_0^1 \left[ \sum_m p_m(j, t)x_{i,m}(j, t) \right] dj,
\]
where \( p_m(j, t) \) is the price of generation \( m \) of good \( j \) at time \( t \).

We can solve this consumer's utility maximization problem in two stages. First, for each product, the consumer chooses the single generation \( \tilde{m}(j, t) \) that carries the lowest quality-adjusted price \( p_m(j, t)/q_m(j) \). This implies the following static demand function:
\[
x_{i,m}(j, t) = \begin{cases} 
E_i(t)/p_m(j, t) & \text{for } m = \tilde{m}(j, t), \\
0 & \text{otherwise.} 
\end{cases}
\] (3)
Second, intertemporal utility maximization requires that \( \dot{E}_i(t)/E_i(t) = r(t) - \rho \). Therefore, aggregate world expenditure, \( E(t) \equiv L_NE_N(t) + L_SE_S(t) \), also changes over time according to the following condition:
\[
\frac{\dot{E}(t)}{E(t)} = r(t) - \rho. \tag{4}
\]

2.2 Production

In this model, we can classify firms into two types: “leaders”, which are firms that developed the current latest generation of each good, and “followers”, which are firms other than the leaders. These firms are located in the North or the South. Regardless of location, a firm can freely sell its good in both countries without incurring any transportation costs or tariffs. We assume that only Northern firms have the ability to undertake R&D and bring about innovation. If a Northern firm succeeds in developing a newer generation of good \( j \), the firm can patent that generation of good \( j \) in the North and monopolistically sell the patented good until it is
imitated. Hereafter, we refer to the leader producing a latest-generation good in the North as a “Northern leader”.

A Northern leader can become a “multinational” firm by shifting production to the South through FDI. By becoming a multinational firm, the firm can employ cheaper Southern labor for production. However, a multinational firm faces the risk of imitation by Southern followers because IPR protection is not perfect in the South. Once imitation takes place in industry $j$, the process for making the current latest generation of good $j$ is completely revealed and perfect competition prevails in the industry until the next generation is developed. The imitated good is sold at $w_S(t) = 1$ in both countries and the multinational firm loses its monopoly rents. We assume that follower firms cannot copy a good produced in the North by the Northern leader.

As in Grossman and Helpman (1991), a monopolistic leader (which refers to a leader whose good has not yet been imitated) can maximize profits by pricing its good at the upper limit of the prices such that rival firms are unable to operate. As described in Section 2.3, we focus on the case where innovation takes place only in industries in which the current latest generation has already been copied. In this case, the strongest rivals of each monopolistic leader are the Southern followers that have the ability to produce the second-highest quality of each good, which could cut the price to the marginal cost, $w_S(t) = 1$. Because a monopolistic leader needs to set the lowest quality-adjusted price to eliminate rivals from its market, the optimal price for each Northern leader and multinational firm is the marginal cost of the rivals multiplied by the degree of advantage in quality: $p = \lambda \cdot w_S(t) = \lambda$. This price setting and the demand function imply that the sales of a monopolistic leader are $E(t)/\lambda$. Therefore, the flow of profits for a Northern leader is:

$$\pi_N(t) = (\lambda - w_N(t)) \frac{E(t)}{\lambda} = \left(1 - \frac{w(t)}{\lambda}\right) E(t). \tag{5}$$

Likewise, the flow of profits for a multinational firm is:

$$\pi_F(t) = (1 + s_F)(\lambda - w_S(t)) \frac{E(t)}{\lambda} = (1 + s_F) \left(1 - \frac{1}{\lambda}\right) E(t), \tag{6}$$

where $s_F \geq 0$ is the rate of FDI subsidy. That is, the Southern government pays each multinational firm $100 \times s_F$ percent of profits as “FDI subsidies”.\footnote{In reality, we can consider that the governments of some developing countries provide a “subsidy” in the form} Note that $s_F$ is assumed to be zero in G-W’s (2007) model.
2.3 R&D and FDI

We make the same two assumptions as G-W (2007) to focus on the case where no R&D is undertaken in industries where a Northern leader or a multinational firm produces a good. First, the labor input required for one unit of R&D by followers is sufficiently larger than that for the leader of the industry. This first assumption ensures that only leaders undertake R&D. G-W (2007) referred to this situation as the case of “inefficient followers” because the R&D productivity of followers is relatively low. Second, no leader has an incentive to further research its own good until the good has been imitated. Under these two assumptions, only the leader firm that made the previous innovation for the good undertakes R&D after the good has been imitated.

As in Grossman and Helpman (1991), we assume that the success or failure of R&D follows a Poisson process: specifically, if the leader firm of industry \( j \) devotes \( a_N t_N(t) dt \) units of Northern labor for a time interval of length \( dt \) to research on good \( j \), it succeeds in developing the next generation of good \( j \) with probability \( \nu_N(t) dt \). We refer to \( \nu_N(t) \) as “innovation intensity”. A leader firm whose good has been imitated chooses \( \nu_N(t) \) to maximize expected net gains from R&D. If a firm succeeds in innovation by R&D, it obtains \( v_N(t) \), which denotes the market value of a Northern leader. Meanwhile, R&D costs \( w_N(t)a_N \nu_N(t) dt \) for the wage payments. Thus, the expected net gains from R&D are \( [v_N(t) - (1 - s_R)w_N(t)a_N]\nu_N(t) dt \), where \( s_R \in [0, 1) \) denotes the rate of R&D subsidy. That is, the Northern government bears \( 100 \times s_R \) percent of the R&D cost as subsidies. Note that G-W’s (2007) model corresponds to the case of \( s_R = 0 \) in our model. Because R&D activities have to be of positive but finite size, the following zero-profit condition on R&D must be satisfied:

\[
v_N(t) = (1 - s_R)w(t)a_N. \tag{7}\]

Following G-W (2007), we assume that a Northern leader can become a multinational firm instantaneously without cost. Under this assumption, a Northern leader firm must be indifferent as to whether it becomes a multinational firm or continues production only in the North. Thus, of tax incentives, including partial exemption from corporate taxes, to the FDI of multinational firms. See, for example, UNCTAD (2001) for a broad survey of the tax incentives governments in developing countries use to promote FDI.

\(^3\)In equilibrium, this assumption is satisfied if the quality increment by innovation, \( \lambda \), is sufficiently large.
the following equality must hold at each point of time:

\[ v_N(t) = v_F(t), \] (8)

where \( v_F(t) \) is the market value of a multinational firm whose good has not yet been imitated.

Arbitrage between assets requires that a stock of a leader firm yields the same expected rate of return as the risk-free interest rate, \( r(t) \). Thus, in equilibrium, the following no-arbitrage condition between the risk-free asset and the stock of a Northern leader is satisfied:

\[ r(t) v_N(t) = \pi_N(t) + \dot{v}_N(t), \] (9)

where the right-hand side is the return from holding the stock of a Northern leader and is equal to the sum of dividends and capital gains. Meanwhile, the shareholders of a multinational firm face the risk of imitation. If imitation takes place in industry \( j \), the multinational firm in the industry loses its monopoly rents. Following Lai (1998) and G-W (2007), we assume that every multinational firm is equally exposed to the risk of imitation at an exogenous rate \( M(>0) \) that depends on IPR protection in the South. Thus, the shareholders of a multinational firm suffer a capital loss of amount \( v_F(t) \) at rate \( M \). This implies that the following no-arbitrage condition between the risk-free asset and the stock of a multinational firm is satisfied in equilibrium:

\[ r(t) v_F(t) = \pi_F(t) + \dot{v}_F(t) - M v_F(t), \] (10)

where the right-hand side is the return from holding multinational firm stock.

2.4 Type of Industry

In this model, we classify every industry into the following three categories: (i) “type-N” industries where the Northern leader firm monopolistically produces the good; (ii) “type-F” industries where the multinational firm monopolistically produces the good; and (iii) “type-S” industries where Southern imitator firms produce the good under perfect competition. We represent the measure (number) of industries belonging to each category by \( n_N(t) \), \( n_F(t) \), and \( n_S(t) \), respectively. The sum of the measure of all industries is equal to one, so that \( n_N(t) + n_F(t) + n_S(t) = 1 \).

Hereafter, we focus only on the equilibrium such that all industries in the same category are symmetric. In this equilibrium, innovation intensity \( \iota_N(t) \) takes a common value in every
type-S industry. Because all innovation takes place in type-S industries, the measure of industries in which innovation takes place in a time interval of length $dt$ is given by $\iota(t)dt$, where $\iota(t) \equiv \iota_N(t)n_S(t)$ is the “aggregate rate of innovation” in the economy as a whole. These industries change the state from type-S to type-N in this time interval by successful innovation. Meanwhile, the measure of type-F industries in which imitation takes place is $Mn_F(t)dt$. These industries change the state from type-F to type-S in this time interval by successful imitation. Therefore, the value of $n_S(t)$ changes over time with the following equation of motion:

$$\dot{n}_S(t) = Mn_F(t) - \iota(t).$$

(11)

The values of $n_N(t)$ and $n_F(t)$ are determined at each point in time between 0 and $1 - n_S(t)$ depending on the Northern leader firms’ location choices.

2.5 Labor Market

Multinational and imitator firms employ Southern labor in the production of goods. Therefore, the labor market-clearing condition in the South is:

$$n_F(t)\frac{E(t)}{\lambda} + n_S(t)E(t) = L_S.$$  

(12)

Northern labor is devoted to R&D and production. Each leader firm whose good has been imitated undertakes R&D with intensity $\iota_N(t)$, so that the labor demand for R&D is given by $a_N\iota_N(t)n_S(t)$. Therefore, the labor market-clearing condition in the North is:

$$a_N\iota(t) + n_N(t)\frac{E(t)}{\lambda} = L_N.$$  

(13)

2.6 Government Budget Constraints

In this economy, the Northern and Southern governments subsidize R&D activities and multinational production, respectively. For simplicity, suppose that at each point in time each government runs a balanced budget where it finances its total subsidy payments with lump-sum taxes levied on each country’s consumers. To achieve a balanced budget, the following constraints must be satisfied in both countries: $s_F(1 - 1/\lambda)E(t)n_F(t) = L_ST_S(t)$ and $s_Rw(t)a_N\iota(t) = L_NT_N(t)$, where the left-hand sides are the total payments of subsidies by both governments.
and the right-hand sides are the tax revenues. The governments determine taxation \( T_i(t) \) such that these budget constraints are satisfied.

3 Market Equilibrium Path

In this section, we characterize the equilibrium path of the economy. The detailed derivation of the results in this section is given in Appendix A. In the following parts of the paper, we focus only on the interior equilibrium in which the aggregate rate of innovation is strictly positive and there are all types (type-N, type-F, and type-S) of industry at any time. Hereafter, let variables with an upper bar, e.g., \( \bar{E} \), denote the steady-state values of the corresponding variables.

On the equilibrium path of this model, some endogenous variables become constant by jumping to their steady-state values immediately at the initial time. In particular, \( E(t) \) and \( w(t) \) respectively take the following constant values for all \( t \) in the equilibrium:

\[
\bar{E} = \frac{a_N \lambda (1 - s_R) [\rho + M \lambda - \rho s_F (\lambda - 1)]}{(1 + s_F) (\lambda - 1)},
\]

\[
\bar{w} = \frac{\rho + M \lambda - \rho s_F (\lambda - 1)}{\rho + M}.
\]

We assume that \( \bar{w} \) is greater than 1 because \( s_F \) is sufficiently small such that \( s_F < M / \rho \). As the values of \( E(t) \) and \( w(t) \) are constant, \( r(t) \), \( v_N(t) \), and \( v_F(t) \) also become constant for all \( t \) on the equilibrium path. Equation (4) implies that \( r(t) = \rho \) for all \( t \).

In the steady state of the model, \( n_S(t) \), \( n_N(t) \), \( n_F(t) \), and \( \iota(t) \) also take constant values. The steady-state values of \( n_S(t) \), \( n_N(t) \), and \( n_F(t) \) are given by:

\[
\bar{n}_S = \frac{a_N M \lambda^2 L_S + \bar{E}^2 - \lambda (L_N + L_S) \bar{E}}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}] \bar{E}},
\]

\[
\bar{n}_F = \frac{\lambda (L_S + \lambda L_N - \bar{E})}{a_N M \lambda^2 - (\lambda - 1) \bar{E}},
\]

\[
\bar{n}_N = \frac{\lambda [a_N M \lambda (\bar{E} - L_S) - L_N \bar{E} (\lambda - 1)]}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}] \bar{E}}.
\]
These values must be positive if there is an interior steady state in the model. The aggregate rate of innovation \(\iota(t)\) is determined by the value of \(n_S(t)\) as follows:

\[
\iota(t) = \frac{1}{\alpha_N} \left\{ L_N + L_S - \frac{\bar{E}}{\lambda} [1 + (\lambda - 1)n_S(t)] \right\}.
\]  

(19)

However, \(n_S(t)\), \(n_N(t)\), \(n_F(t)\), and \(\iota(t)\) cannot jump to their steady-state values immediately at the initial time because \(n_S(t)\) is a state variable whose value is historically given, and \(n_N(t)\), \(n_F(t)\), \(\iota(t)\) are determined depending on the value of \(n_S(t)\). The equation of motion of \(n_S(t)\) is:

\[
\dot{n}_S(t) = \frac{a_N M \lambda^2 L_S + \bar{E}^2 - \lambda (L_N + L_S) \bar{E}}{a_N \lambda E} - \mu n_S(t),
\]

(20)

where \(\bar{E}\) is given by (14) and \(\mu \equiv M \lambda - (\lambda - 1) \bar{E}/(a_N \lambda) = M \lambda - (1 - s_R)[\rho + M \lambda - \rho s_F (\lambda - 1)]/(1 + s_F)\) is the speed of convergence to (divergence from) the steady state if \(\mu > 0\) (\(\mu < 0\)).

Note that this equation of motion is a linear function of \(n_S(t)\) and depends only on the values of \(n_S(t)\) itself and the exogenous variables. We solve the linear differential equation (20) as:

\[
n_S(t) = \begin{cases} 
    n_S + (n_S(0) - \bar{n}_S) e^{-\mu t} & \text{for } \mu \neq 0, \\
    n_S(0) + \frac{a_N M \lambda^2 L_S + \bar{E}^2 - \lambda (L_N + L_S) \bar{E}}{a_N \lambda E} t & \text{for } \mu = 0,
\end{cases}
\]

(21)

where \(n_S(0)\) denotes the initial value of \(n_S(t)\). Because \(n_S(t)\) is not jumpable, (21) implies that \(\mu\) must be strictly positive so that the steady state can be attained; otherwise, \(n_S(t)\) can never take the value of the interior steady state given by (16), except in the special case where \(n_S(0) = \bar{n}_S\). Likewise, \(n_F(t)\) and \(n_N(t)\) approach the steady-state values of (17) and (18) over time if and only if \(\mu > 0\) because the values of both variables depend on \(n_S(t)\). In this sense, the interior steady state of the model becomes unstable if \(\mu\) is negative. We summarize this result as follows.

**Proposition 1.** Suppose that there exists a unique steady state in which \(\bar{n}_S\), \(\bar{n}_F\), \(\bar{n}_N\), and \(\bar{\iota}\) are all positive. Then the economy approaches the steady state over time if and only if \(\mu\) is positive.

Proposition 1 shows that the key to the stability of the steady state is the sign of \(\mu\), being the coefficient of the equation of motion of \(n_S(t)\). Intuitively, we can interpret what determines the value of \(\mu\) and the stability of the steady state by equation (11). Other things being equal, an increase in \(n_S\) reduces the inflow into \(n_S\) (the first term in (11)) because it reduces the targets of imitation, \(n_F\), by the Southern labor constraint. This first effect is represented by
the first term in the definition of $\mu$. Meanwhile, other things being equal, an increase in $n_S$ also reduces the outflow from $n_S$ (the second term in (11)) because it reduces $\iota$.\footnote{This is shown in equation (19). Because a type-S industry employs more labor than a type-F industry, the labor constraint in the South requires that an increase in $n_S$ must reduce $n_S + n_F$ by sharply decreasing $n_F$. Thus, an increase in $n_S$ increases the measure of industries producing in the North, $n_N = 1 - (n_S + n_F)$, and thereby decreases the labor devoted to R&D in terms of the labor constraint in the North. This is why $\iota$ decreases with $n_S$.} This second effect is represented by the second term in the definition of $\mu$. If the former effect outweighs the latter, $\mu$ becomes positive. In this case, a change in $n_S$ decelerates over time and the economy asymptotically approaches the steady state. If the latter effect outweighs the former, $\mu$ becomes negative. In this case, a change in $n_S$ accelerates over time and the economy moves away from the steady state.

We hereafter assume that $\mu > 0$ so that the interior steady state is attainable.\footnote{If $\lambda$ is sufficiently large, this condition is not very restrictive. For instance, following Glass and Saggi (2002), we use $\rho = 0.05$, $\lambda = 4$, $L_N = 3$, and $L_S = 6$ as a numerical example. We further set $M = 0.037$, $a_N = 123.5$, and $s_F = 0.2$. In this case, the interior steady state exists and is stable if $0.220 < s_R < 0.487$, which does not appear a very narrow range.} Meanwhile, by the definition of $\mu$, we can immediately prove the following corollary.

**Corollary.** Suppose that the rates of the subsidies on R&D and FDI are equal to zero: $s_R = s_F = 0$. Then, the interior steady state of the model is unstable.

This corollary implies that the economy in the case of the inefficient followers in G-W (2007), which corresponds to the specific case of $s_R = s_F = 0$ in our model, does not tend to the interior steady state, except when it begins in the steady state by chance. In particular, when the government changes a policy, such as IPR protection, the economy without subsidies no longer moves toward the new interior steady state, even if it were originally in the interior steady state. Rather, it moves toward the “corner-solution equilibrium” such that the values of some endogenous variables are zero over time.\footnote{More specifically, if $n_S(0)$ is greater than $\bar{n}_S$, then $n_S(t)$ gradually increases and either innovation or FDI becomes zero at finite time. If $n_S(0)$ is smaller than $\bar{n}_S$, then $n_S(t)$ gradually decreases and either $n_S(t)$ or $n_N(t)$ becomes zero at finite time.} For this reason, if $s_R = s_F = 0$, a comparison of the interior steady states before and after a given policy change does not enable an accurate judgment on the effect of that policy. In this case, we should not evaluate the influence of a policy using analysis of the comparative statics. Thus, the conclusion on IPR protection...
policy in G-W’s (2007) inefficient followers’ case should be reexamined because it draws on comparative statics.

4 Long-Run Effects of Strengthening IPR Protection

In this section, we examine the influence of strengthening IPR protection on the steady-state values of the endogenous variables. Because we can interpret the strengthening of IPR protection in the South as a decrease in the imitation rate $M$, we carry out the comparative statics with respect to $M$.

4.1 The Effect on the Relative Wage

Using the comparative statics, we can find that strengthening IPR protection (a decrease in $M$) increases the wage in the South relative to the North, $1/w(t)$. Differentiating $\bar{w}$ given by (15) with respect to $M$, we have:

$$\frac{\partial \bar{w}}{\partial M} = \frac{(1 + s_F)(\lambda - 1)\rho}{(\rho + M)^2} > 0.$$  

The reason why stronger IPR protection increases the Southern relative wage is as follows. To start with, the no-arbitrage condition (10) implies that the market value of a multinational firm must equal the sum of the present value of its instantaneous profit flow, that is, $\bar{v}_F = \bar{\pi}_F/\frac{\rho}{\rho + M}$. This relation shows that, other things being equal, strengthening IPR protection (a decrease in $M$) increases $v_F$ directly. This is because the decrease in the risk of imitation enables a multinational firm to earn instantaneous profits $\pi_F$ for a longer time on average by extending the expected period of monopoly. The larger $v_F$ then provides the Northern leader firms with greater incentive to transfer production to the South. Therefore, other things being equal, the demand for Southern labor to produce a good would increase because more Northern leader firms would choose to convert to multinational firms. In equilibrium, the Northern (relative) wage $\bar{w}$ unambiguously falls to increase the incentive for production in the North. We summarize this result as follows.

Proposition 2. Strengthening IPR protection in the South increases the wage in the South relative to that in the North.
4.2 The Effect on Innovation

How then does strengthening IPR protection affect the “long-run” aggregate rate of innovation $\bar{\eta}$? G-W (2007) concluded that relaxing IPR protection (an increase in $M$) increases the aggregate rate of innovation in the case of inefficient followers. However, our model shows that this conclusion is not true in the parameter ranges where the interior steady state becomes stable. In fact, differentiating (19) with respect to $M$ and evaluating the steady-state value, we can prove the following proposition.

**Proposition 3.** *Strengthening IPR protection in the South increases the long-run aggregate rate of innovation.*

*Proof.* See Appendix B.

How does stronger IPR protection promote innovation in our model? Intuitively, it is useful to recall the Northern labor market-clearing condition (13). This condition requires that the sum of the labor inputs into R&D and production by the Northern leaders must be equal to the constant Northern labor supply. Accordingly, the lower the labor input into production by the Northern leaders, the more abundant the labor input into R&D. In our model, strengthening IPR protection decreases the labor input for production by Northern leaders through the following channels. The first channel is through a decrease in aggregate spending $\bar{E}$. By differentiating $\bar{E}$ given by (14) with respect to $M$, the effect of strengthening IPR protection (a decrease in $M$) on $\bar{E}$ proves to be negative as follows:

\[
\frac{\partial \bar{E}}{\partial M} = \frac{a_N \lambda^2 (1 - s_R)}{(1 + s_F)(\lambda - 1)} > 0.
\]

Because the decrease in aggregate spending $\bar{E}$ reduces the demand for goods produced by Northern leaders, each Northern leader decreases the labor input for production. The second channel is through a decrease in the measure of type-N industries. In our model, the measure of type-N industries $\bar{\eta}_N$ increases with the rate of imitation $M$; that is, strengthening IPR protection decreases the number of Northern patent holders that choose to operate in the North. By differentiating (18) with respect to $M$, we can verify this as follows:

\[
\frac{\partial \bar{n}_N}{\partial M} = \frac{1}{\mu} \left\{ (\lambda - 1)\bar{n}_F + \left[ \frac{M \lambda s}{E^2} + \frac{(\lambda - 1)\bar{n}_N}{a_N \lambda} \right] \frac{\partial \bar{E}}{\partial M} \right\} > 0. \tag{22}
\]
The fewer leaders operating in the North mean less use of Northern labor for production. Because both these effects decrease the labor input for production, strengthening IPR protection in the South increases the labor input for R&D and thereby promotes innovation.

Note that the conclusion of Proposition 3 is completely opposite to that of the inefficient followers’ case in G-W (2007). Why does the result of our model differ from that of G-W (2007)? The main reason is the stability of the steady state in the model. Recall that we restrict the parameter ranges so that the steady state becomes stable. As discussed in Section 3, at least \( \mu \) must be positive if the steady state is stable. Because \( \mu \) is positive, we find that \( \bar{n}_N \) increases with the rate of imitation \( M \) from (22). As a result, we can draw the conclusion that strengthening IPR protection unambiguously decreases the labor input for production in the North. However, in G-W’s (2007) model, the steady state is unstable and the value of \( \mu \) is negative. Therefore, from (22), \( \bar{n}_N \) must be decreasing with the rate of imitation \( M \). This means that the strengthening of IPR protection in their model necessarily increases the number of leader firms operating in the North (see also equation 29 in G-W, 2007). Because the sign of the abovementioned second channel lies opposite to that of our model, strengthening IPR protection can increase the labor input for production in the North. In fact, the negative effect of the latter channel outweighs the positive effect of the former, so that strengthening IPR protection decreases the labor input for R&D and the aggregate rate of innovation in G-W’s (2007) model.

The result of Proposition 3 is similar to that obtained with variety-expanding-type innovation models including an exogenous process of imitation in the South, such as in Lai (1998). However, our result is contrary to Mondal and Gupta (2008) who employed a variety-expanding-type innovation model including an endogenous process of imitation in the South, unlike Lai (1998) and the present paper. Mondal and Gupta (2008) showed that strengthening IPR protection impedes innovation.
4.3 The Effect on FDI

Strengthening IPR protection in the South increases the measure of type-F industries as well as the aggregate rate of innovation. By differentiating (17) with respect to $M$, we have:

$$\frac{\partial \bar{n}_F}{\partial M} = -\frac{1}{\mu} \left[ \lambda \bar{n}_F + \frac{(1 - \bar{n}_F) \lambda + \bar{n}_F \partial \bar{E}}{a_N \lambda} \right] < 0.$$  \hspace{1cm} (23)

We summarize this result as the following proposition.

**Proposition 4.** *Strengthening IPR protection in the South increases FDI in the long run.*

By using the equation of motion of $n_S(t)$, we obtain an interpretation of Proposition 4. From (11), the value of $\bar{n}_F$ is given by the ratio of the aggregate rate of innovation to the imitation rate, $\tilde{i}/M$, in the steady state. Because strengthening IPR protection decreases the imitation rate $M$ while increasing the aggregate rate of innovation $\bar{i}$, it necessarily increases the measure of type-F industries $\bar{n}_F$.

The result of Proposition 4 is also contrary to the conclusion of G-W’s (2007) inefficient followers’ case. G-W (2007) concluded that a decrease in $M$ necessarily decreases $\bar{n}_F$. However, our model shows that a decrease in $M$ necessarily increases $\bar{n}_F$ from (23) because $\mu$ is positive. The difference in the result of our model and that of G-W (2007) is also attributable to the difference in the stability of the steady states in the two models. Recall that under G-W’s (2007) setting, strengthening IPR protection decreases the aggregate rate of innovation because the value of $\mu$ is negative, unlike in our model. As $\bar{n}_F$ is equal to $\tilde{i}/M$ in both models, strengthening IPR protection influences $\bar{n}_F$ through two channels: the first is a direct effect through the decrease in the imitation rate, and the second is an indirect effect through the decrease in the aggregate rate of innovation. The former has a positive effect, whereas the latter has a negative effect on the value of $\bar{n}_F$ in their model. Because the latter indirect effect is sufficient to outweigh the former direct effect in their model, strengthening IPR protection decreases $\bar{n}_F$. Note that this conclusion in G-W (2007) crucially depends on the result that $\tilde{i}$ is increasing with $M$ under their setting. The reason why $\tilde{i}$ is increasing with $M$ is the instability of the steady state of their model. Thus, their result on the effect of a change in $M$ on $\bar{n}_F$ also arises from the lack of stability of the steady state.

The result of Proposition 4 accords with that in some of the literature. Lai (1998) and Gustafsson and Segerstrom (2011), who assumed variety-expanding type of innovation, also
concluded that strengthening IPR protection in the South increases technology transfer to the South within multinational firms. Dinopoulos and Segerstrom (2010) obtained the same result by using a quality-improvement-type innovation model that exhibits scale-invariant growth such as Segerstrom (1998), Li (2003), and Dinopoulos and Segerstrom (2007). However, there are a few exceptions, including Glass and Saggi (2002) and Mondal and Gupta (2008), both of which assumed an endogenous process of imitation. Contrary to our findings, they showed that strengthening IPR protection in the South impedes FDI.

5 Short-Run Effects of Strengthening IPR Protection

In Section 4, we concluded that strengthening IPR protection positively affects the aggregate rate of innovation and FDI in the long run by comparing the steady-state values before and after the policy change. However, it is also important to explore how strengthening IPR protection influences the aggregate rate of innovation and FDI on the transitional path to the new steady state. Unlike many existing studies in this literature, we can investigate this, so to speak, “short-run” effect of strengthening IPR protection because the transitional dynamics are explicitly analyzed in our model.  

Suppose that the economy is initially in the steady state and the Southern government strengthens IPR protection at the initial time. We define the short-run effects of strengthening IPR protection as the magnitudes of \( \frac{\partial \iota(t)}{\partial M} \) and \( \frac{\partial n_F(t)}{\partial M} \) for any \( t \in [0, \infty) \). Note that \( \frac{\partial \iota(t)}{\partial M} \) and \( \frac{\partial n_F(t)}{\partial M} \) are not necessarily equal to \( \frac{\partial \bar{\iota}}{\partial M} \) and \( \frac{\partial \bar{n}_F}{\partial M} \) because our model includes the transitional process to the steady state. If \( \frac{\partial \iota(t)}{\partial M} \) and \( \frac{\partial n_F(t)}{\partial M} \) are negative for all \( t \in [0, \infty) \), we can conclude that innovation and FDI are promoted at any point in time after strengthening IPR protection. On this short-run effect, we can prove the following proposition.

**Proposition 5.** Suppose that the economy is initially in the steady state. Then, strengthening

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7 Most North–South innovation models tend to have a complicated dynamic structure and so are unable to analytically examine the transition of the equilibrium paths. The exceptions are Helpman (1993), Arnold (2002) (which is based on Helpman (1993)), and Tanaka, Iwaisako, and Futagami (2007). These studies analyzed the transition of the equilibrium path by employing a relatively simple dynamic structure in their models.
IPR protection in the South increases innovation and FDI for all $t \in [0, \infty)$.

Proof. See Appendix C.

Proposition 5 shows that $\partial \iota(t)/\partial M < 0$ and $\partial n_F(t)/\partial M < 0$ for all $t$. It therefore implies that both innovation and FDI necessarily increase, even on the transitional path, after strengthening IPR protection. That is, strengthening IPR protection promotes innovation and FDI not only in the long run, but also in the short run. Proposition 5 shows that the result of the short-run analysis is the same as that of the long-run analysis. Thus, it reinforces our conclusion regarding the effect of IPR protection.

6 Welfare Analysis

In Sections 4 and 5, we showed that strengthening IPR protection enhances innovation and FDI in both the long and short run. However, we should judge a policy’s desirability by how it affects welfare. In this section, we examine the welfare effects of IPR protection with the results obtained in Section 5. By considering the short-run welfare effect, we can examine how a marginal increase in $M$, that is, relaxing IPR protection in the South, affects welfare.

To conduct the welfare analysis, we first consider the instantaneous utility function. Substituting (3) into (2), we obtain the utility level of a consumer in country $i$ at time $t$ as follows: $\log u_i(t) = (\log \lambda) \int_0^1 \tilde{m}(j, t) dj + \int_0^1 \log x_i(j, t) dj$, where $x_i(j, t)$ denotes the quantity demanded for the current latest generation of good $j$ at time $t$. The first term in this equation represents the welfare brought by the qualities of all goods consumed. Because $\tilde{m}(j, t)$ equals the current state-of-the-art generation number of good $j$ at time $t$, $\int_0^1 \tilde{m}(j, t) dj$ in the first term is equal to the aggregate number of innovations brought in the interval from time 0 to time $t$. Thus, the first term can be replaced with $(\log \lambda) \int_0^t \iota(\tau) d\tau$. This means that welfare positively depends on the aggregate rate of innovation, $\iota(t)$. The second term in the instantaneous utility function represents the welfare brought by the qualities of all goods consumed. Because $\tilde{m}(j, t)$ equals the current state-of-the-art generation number of good $j$ at time $t$, $\int_0^1 \tilde{m}(j, t) dj$ in the first term is equal to the aggregate number of innovations brought in the interval from time 0 to time $t$. Thus, the first term can be replaced with $(\log \lambda) \int_0^t \iota(\tau) d\tau$. This means that welfare positively depends on the aggregate rate of innovation, $\iota(t)$. The second term in the instantaneous utility function represents the welfare brought by the quantities of all goods consumed. Substituting (3) into the second term, we can rewrite the term as $\log E_i(t) - \int_0^1 \log p(j, t) dj$, which means that welfare depends positively on total spending, $E_i(t)$, and negatively on the prices of the state-of-the-art goods, $p(j, t)$. The price of type-S goods is equal to one because of
competition, whereas the price of the goods other than type-S is \( \lambda > 1 \) because the monopolistic leaders supply them. Therefore, the part of welfare that depends on the prices reduces to

\[
\int_0^1 \log p(j, t) dj = n_S(t) \log 1 + (1 - n_S(t)) \log \lambda = (1 - n_S(t)) \log \lambda,
\]

which depends on the measure of type-S industries. An increase in the measure of type-S industries lowers the prices of goods and improves the welfare of consumers. Considering these results, we can rewrite instantaneous utility as:

\[
\log u_i(t) = \left( \log \lambda \right) \int_0^t \nu(\tau) d\tau + \log E_i(t) - (1 - n_S(t)) \log \lambda.
\]

By differentiating the lifetime utility function with respect to \( M \), we obtain the change of welfare from the marginal increase in \( M \) as follows:

\[
\frac{\partial U_i}{\partial M} = \int_0^\infty e^{-\rho t} \left( \log \lambda \right) \frac{\partial}{\partial M} \left( \int_0^t \nu(\tau) d\tau \right) dt \quad + \quad \frac{1}{\rho E_i} \frac{\partial E_i}{\partial M} \left. \right|_{M} \quad + \quad \int_0^\infty e^{-\rho t} \left( \log \lambda \right) \frac{\partial n_S(t)}{\partial M} dt.
\]

Equation (24) shows that the total welfare effect of relaxing IPR protection (an increase in \( M \)) can be decomposed into the following three parts.

First, relaxing IPR protection impedes innovation and thus reduces welfare. As shown in Proposition 5, a marginal increase in the imitation rate necessarily reduces the aggregate rate of innovation for all \( t \). We refer to the welfare effect through this channel as the innovation-impeding effect. In the remainder of the analysis, we assume that the economy is initially in the steady state, namely \( n_S(0) = \bar{n}_S \). Under this assumption, we derive the innovation-impeding effect from the results in Appendix C as follows:

\[
\int_0^\infty e^{-\rho t} \left( \log \lambda \right) \frac{\partial}{\partial M} \left( \int_0^t \nu(\tau) d\tau \right) dt \quad = \quad \left( \log \lambda \right) \frac{\mu}{\rho^2(\rho + \mu)} \frac{\partial \bar{\bar{\nu}}}{\partial M} - \frac{1}{a_N \lambda \rho(\rho + \mu)} \left( 1 + (\lambda - 1)\bar{n}_S \right) \frac{\partial \bar{E}}{\partial M} \quad < 0,
\]

which is necessarily negative because an increase in \( M \) reduces \( \bar{\bar{\nu}} \) and increases \( \bar{E} \).

Second, relaxing IPR protection may affect nominal spending and welfare. We refer to this effect as the nominal-spending effect. Only these nominal-spending effects differ between the South and the North.
Finally, relaxing IPR protection affects the measure of type-S industries, $n_S(t)$. An increase in the measure of type-S industries where the price is lower improves welfare. We refer to this welfare effect as the competition effect. Using the results of the analysis of the short-run effect in Appendix C, we can compute the competition effect as follows:

$$ \int_0^\infty e^{-\rho t} (\log \lambda) \frac{\partial n_S(t)}{\partial M} dt = (\log \lambda) \frac{\mu}{\rho(\rho + \mu)} \frac{\partial \bar{n}_S}{\partial M}. $$

(26)

The sign of the competition effect is indeterminate because the sign of $\partial n_S/\partial M$ is ambiguous.

### 6.1 The Effect on Welfare in the South

We now examine whether strengthening IPR protection in the South can increase welfare in the South. In Sections 6.1 and 6.2, we focus on the case where Southern households initially possess no assets. Further, we assume no subsidy for FDI, $s_F = 0$, in order to make the results of the welfare analysis clearer. Based on these assumptions, the intertemporal budget constraint for a Southern consumer is reduced to $E_S = 1$, because the Southern wage is normalized to one and no tax is levied on Southern consumers. As the nominal spending in the South, $E_S$, is independent of $M$, the nominal-spending effect is absent in the South in this case. Therefore, strengthening IPR protection in the South improves the welfare of the South if the condition stated in the following proposition is satisfied.

**Proposition 6.** Suppose that the economy is initially in the steady state. Strengthening IPR protection in the South then increases the welfare of Southern consumers if $M > \rho s_R / [\lambda (1 - s_R)]$.

**Proof.** See Appendix D.

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8 In Appendix G, we briefly explore what happens when these assumptions are relaxed. If the Southern households have some assets and the FDI subsidy rate is positive, the nominal spending of a Southern household is not equal to one. This implies that the effect of relaxing IPR protection on the welfare of the South becomes more complex than is analyzed in Section 6.1 because it depends also on the nominal spending effect. Further, through the change in the value of the assets, a tax levied in the trading partner affects income and spending in both countries. Thus, the welfare effect of the North also becomes more complex than that analyzed in Section 6.2. However, our numerical analysis in Appendix G indicates that the results for welfare do not change if both the subsidy rate for FDI and the share of assets held by Southern households are sufficiently small.
The interpretation of the condition in Proposition 6 is somewhat complex. If relaxing IPR protection increases the measure of type-S industries, it increases welfare through reducing prices but decreases welfare by impeding innovation. One of the reasons why innovation decreases is that an increase in \( n_s \) reduces the labor input for R&D (see also footnote 4). The effect of reducing R&D through the increase in \( n_s \) intensifies as \( M \) is larger, \( s_R \) is smaller, and \( \lambda \) is larger. In addition, the innovation-impeding effect worsens if the subjective discount rate \( \rho \) is small. This is because the slowdown in quality improvement in the future is evaluated as a heavier loss if \( \rho \) is small. Thus, if the parameters satisfy the condition given in Proposition 6, the innovation-impeding effect becomes large enough to dominate the competition effect. Because the nominal spending effect is equal to zero given the assumptions, strengthening (relaxing) IPR protection improves (harms) welfare in the South.

Proposition 6 implies a desirable IPR policy in the South. Until now, earlier studies examined only whether strengthening IPR protection enhances innovation and FDI and not whether it could improve welfare. In contrast, we can evaluate the welfare effect of strengthening IPR protection analytically because of the tractable dynamic structure of the present model. The result of the analysis shows that strengthening IPR protection improves welfare in the South if \( M \) is sufficiently high and \( s_R \) is so low as to satisfy the inequality given in Proposition 6. In other words, we can conclude that strengthening IPR protection is a desirable policy change for the South as long as the initial protection of IPR is so weak in the South as to satisfy the condition.

\[9\] If relaxing IPR protection decreases the measure of type-S industries, it unambiguously decreases welfare in the South through both decreasing those goods with a lower price and impeding innovation.

\[10\] If IPR protection is initially strong and \( M \) is sufficiently low so as not to satisfy the inequality given in Proposition 6, the effect of strengthening IPR protection is indeterminate. In Appendix H, we provide a numerical analysis of the case where the condition of Proposition 6 is not satisfied. According to the analysis, further strengthening IPR under strong protection tends to worsen the welfare of the South if \( s_R \) is considerably large, and vice versa. Thus, the policy recommendation for the South may depend on the R&D subsidy policy of its trading partner, namely, the North. The authors appreciate the comment of an anonymous referee regarding this point.

\[11\] The condition given in Proposition 6 is not so restrictive. For example, if we set \( \rho = 0.05 \), \( \lambda = 4 \), and \( s_R = 0.3 \) as in footnote 5, the condition is satisfied when \( M > 0.0054 \).
6.2 The Effect on Welfare in the North

Next, we analyze the welfare effect in the North. Unlike the South, nominal spending in the North, \( E_N \), depends on \( M \). From \( E_S = 1 \) and \( \bar{E} = L_N E_N + L_S E_S \), the nominal spending of a Northern household is derived as \( E_N = (\bar{E} - L_S)/L_N \). An increase in \( M \) then increases aggregate expenditure \( \bar{E} \) but does not change \( E_S \), so it must increase the expenditure of a Northern household, \( E_N \). Thus, the total welfare effect of a change in IPR protection in the North is more complex than in the South. Nevertheless, by imposing a certain condition, we can show that strengthening IPR protection in the South improves welfare in the North as shown by the following proposition.

**Proposition 7.** Suppose that the economy is initially in the steady state. Strengthening IPR protection in the South increases the welfare of Northern consumers if 

\[
s_R < 1 - \left[ 1 + \frac{(\log \lambda) L_S}{\sigma_N \lambda (\rho + M \lambda)} \right] \frac{\lambda - 1}{\lambda - 1 + \log \lambda},
\]

*Proof.* See Appendix E.

If the condition provided in Proposition 7 is satisfied, the innovation-impeding effect becomes large enough to dominate the nominal spending effect and the competition effect.\(^{12}\) The condition tends to be satisfied if \( L_S \) is relatively small. This is because the nominal spending effect in the North is weak in that case.\(^{13}\) Meanwhile, the effect of \( \lambda \) on the condition is rather complex. First, an increase in \( \lambda \) intensifies the innovation-impeding effect because a quality improvement is highly valued by households in the case of a large \( \lambda \). Second, it also changes the innovation-impeding effect through influencing \( \bar{E} \). A change in \( \bar{E} \) affects the demand for goods and thereby the labor input for production. It therefore changes the allocation of labor resources between production and R&D, which affects the aggregate rate of innovation. Third, an increase in \( \lambda \) affects income and spending in the North through changing the wage and the

\(^{12}\) According to our numerical analysis, the condition given in Proposition 7 turns out to be comparatively restrictive. However, it does not mean that stronger IPR protection tends to decrease the welfare of Northern consumers because the condition is sufficient but not necessary. Even if the condition given in Proposition 7 is not satisfied, the welfare of the North may improve.

\(^{13}\) The nominal spending effect in the North is proportionate to \( 1/E_N \). Therefore, it decreases with the nominal spending of a Northern household, \( E_N \). As the nominal spending of a Southern household is fixed to one, the small population size in the South leads to large spending in the North for a given value of total spending in the world, \( \bar{E} \). In consequence, a small \( L_S \) implies that the nominal spending effect in the North is weak.
value of share holdings. Finally, it also intensifies the competition effect because $\lambda$ is equal to the ratio of the price of monopolistic goods to that of competitive goods. Because these effects simultaneously arise from the change in $\lambda$, how the welfare effect of IPR depends on $\lambda$ is ambiguous.

As shown in the proof, the condition in Proposition 7 is stricter than that in Proposition 6.\textsuperscript{14} Thus, when the inequality given in Proposition 7 holds, strengthening IPR protection in the South improves welfare in both the South and the North. This means that strengthening IPR protection in the South is a Pareto-improving policy as long as the initial protection of IPR is sufficiently weak in the South and the rate of R&D subsidy is so low as to satisfy the inequality in Proposition 7.

6.3 Comparison with the Literature

Although most of the studies in this field did not conduct a welfare analysis, there are some exceptions. For example, Helpman (1993), Grinols and Lin (2006), and Iwaisako et al. (2011) examined the welfare effect of IPR using a growth model. In the seminal study on IPR protection in the dynamic North–South model, Helpman (1993) conducted a welfare analysis using a variety-expanding innovation model. He showed that stronger IPR protection necessarily harms welfare in the South. Helpman (1993) constructed a model including the choice of multinationalization in addition to a model where the only mode of technology transfer is imitation. However, in his model, imitation does not affect multinationalization directly because he assumed a multinational firm equals a Northern firm in the risk of imitation. To correct this shortcoming, some additional research was subsequently undertaken.

In contrast, Grinols and Lin (2006) showed that stronger IPR protection may improve welfare in the South by introducing a group of goods consumed only in the South into Helpman’s

\textsuperscript{14} From (24) and the definition of $E(t)$, $\partial U_N / \partial M = (\partial U_S / \partial M) + (1 / \rho)(1 / E_N)(\partial E_N / \partial M)$ and $\partial E_N / \partial M = (1 / L_N)(\partial \bar{E} / \partial M) > 0$ if $E_S = 1$. Thus, there may be a case where stronger IPR protection is favorable for Southern consumers and unfavorable for Northern consumers. In this case, if the nominal spending effect to welfare in the North is sufficiently large (small) relative to the sum of the innovation-impeding effect and the competition effect, the magnitude of the decrease in the North’s welfare is larger (smaller) than the magnitude of increase in the South’s welfare. This is because $\partial U_S / \partial M$ is equal to the sum of the innovation-impeding effect and the competition effect if $E_S = 1$. 

– 23 –
(1993) imitation model. Although our results on the welfare of the South appear similar to those of Grinols and Lin (2006), the assumption concerning the channel of technology transfer differs. In brief, our model includes FDI, whereas Grinols and Lin (2006) focused on technology transfer through the direct imitation of Northern goods. In addition, we obtain the welfare results analytically, unlike Grinols and Lin (2006) who employed a numerical method.

Similarly to our conclusion, Iwaisako et al. (2011) also showed the possibility that strengthening patent protection in the South can improve welfare in the South by using a quality-improvement-type innovation model. However, the instrument of IPR protection examined in that analysis differs from the present paper. In general, IPR authorities can control the protection of patents using two instruments: patent length and patent breadth. Patent length refers to the duration for which a patentee can sell the patented product monopolistically, whereas patent breadth refers to the scope of the products that patentees can prevent firms without patents from producing and selling. Iwaisako et al. (2011) focused on the effects of broadening patent breadth. In the present paper, we assume that IPR authorities can control the probability of imitation, which is associated with the expected duration of IPR. Therefore, this paper focuses on the effects of extending patent length rather than patent breadth. Hence, our study and Iwaisako et al. (2011) complement each other in that the welfare effects of strengthening IPR protection in the South are examined more completely.\(^\text{15}\)

By using a model where R&D activities are undertaken in both the North and the South, Grossman and Lai (2004) also conducted a welfare analysis. They concluded that the level of patent protection that maximizes welfare in the South tends to be weaker than in the North. However, to obtain this result, they specified the utility function in quasilinear form and assumed no knowledge spillovers in the R&D process. In addition, they assumed the “obsolescence” of goods; that is, a newly developed good provides utility to consumers for only a finite length of time. These assumptions imply no growth of utility in their model. Therefore, we cannot simply compare their results with that from a growth model including the present paper.

\(^{15}\)Using a closed-economy framework, Iwaisako and Futagami (2003) and Palokangas (2011) both examined the welfare effects of patent length and breadth. The welfare effects of other patent instruments, for example, leading and lagging patent breadths, blocking patents, and the division rule governing profits between basic and applied researchers, have also been examined using a closed-economy framework by Li (2001), O’Donoghue and Zweimüller (2004), Chu (2009), and Chu and Furukawa (2012).
Furthermore, Lin (2010) conducted a welfare analysis in an extended North–South model composed of three countries. In his model, the middle country’s firms can imitate the goods invented in the North and thereafter shift production to the South through FDI. Lin (2010) also assumed that Southern local firms might imitate the goods where production is shifted to the South. However, the purpose of his paper was to evaluate not the welfare change of strengthening IPR protection in the South but rather that of tightening South-bound FDI in the middle country.

7 Discussion

7.1 The Welfare Effects of Subsidy Policies

Using the same method as in Section 6, we can also investigate the welfare effects of a change in the subsidy policies. Details of the analysis are given in Appendix I. We summarize the results as follows. First, a marginal increase in the R&D subsidy necessarily improves the welfare of the South. This is because the effect through promoting innovation necessarily outweighs the effect through changing the proportion of imitated goods. Second, a marginal increase in the R&D subsidy and the introduction of the FDI subsidy improve the welfare of the North if the condition given in Proposition 7 is satisfied. Finally, introduction of the FDI subsidy improves the welfare of the South if

\[
\frac{M\lambda \log \lambda}{\alpha N (\lambda - 1) \rho (\rho + \mu)(\rho + M \lambda)} - 1 > 0.
\]

The reason why the second and the third results are satisfied is also that the effect through promoting innovation is sufficiently large if the conditions are satisfied.

7.2 Costly Imitation and FDI

In previous sections, we assumed that imitation is costless. However, imitation of high-technology goods involves large costs in the real world. In addition, FDI also involves nonnegligible cost. For instance, affiliates in developing countries often conduct R&D for the absorption of parent-firm technology, as mentioned in Dinopoulos and Segerstrom (2010) and Gustafsson and Segerstrom (2011). In this section, we introduce a few results on the effects of strengthening IPR protection in the South in an extended model where imitation and FDI are a costly
process, as in Glass and Saggi (2002). The model is described in Appendix F and its analysis is in Appendix J.

The costly-imitation model shows that strengthening IPR protection (an increase in the imitation cost) tends to reduce innovation in the North. There are some changes from the basic model and thus we cannot strictly compare the results of the costly-imitation model with those of the costless-imitation model. Nonetheless, the result is opposite to that obtained in the costless-imitation model.\textsuperscript{16} Moreover, it is similar to the results in Mondal and Gupta (2008) using an expanding-variety model with a costly-imitation process. Thus, also in the costly-imitation models, the claim in G-W (2007) that whether strengthening IPR protection promotes innovation or not depends on whether innovation is of the variety-expansion or quality-improving type is not correct. This difference in results suggests that the effects of strengthening IPR protection may then depend on whether imitation is costless or costly.

The numerical results of the costly-imitation model show that the relation between the strength of IPR protection and the welfare of Southern consumers is ambiguous. Stronger IPR protection reduces welfare under a lower innovation cost, whereas it increases the welfare under higher innovation cost. When the innovation cost is lower, innovation is larger, and thus the negative welfare effect through impeding innovation also tends to be large enough to dominate the other positive welfare effects and vice versa.\textsuperscript{17}

8 Concluding Remarks

This paper examined the effects of strengthening IPR protection in developing countries using a simple North–South quality-ladder-type R&D model where FDI is determined endogenously. We find that strengthening IPR protection promotes innovation and FDI in both the long and

\textsuperscript{16} We surmise the reason for this result in the costly-imitation model is as follows. In the costly-imitation model, strengthening IPR protection increases the cost of imitation, and thus wastes Southern labor and crowds out FDI. The decrease in FDI then increases the production sectors in the North and consequently reduces the innovation.

\textsuperscript{17} In the costly-imitation model, we consider only the effects on the steady state and obtain the results only numerically because of the intractability of the model. To investigate the welfare effects under costly imitation more precisely, we would need to examine the short-run welfare effects as in the analysis of the costless-imitation model.
short run. This finding contrasts with the result in G-W (2007) but is identical to that in Lai (1998), who employed a variety-expansion-type North–South model. This result is thus important not only in the sense that it reverses the results of the comparative statics in G-W’s (2007) model, but also because it shows that the type of innovation, whether as a quality improvement or a variety expansion, does not play a key role in determining the effects of strengthening IPR protection on innovation and FDI. Accordingly, hereafter we must examine carefully which aspects determine the effects of IPR protection on innovation and FDI.

As a central issue, we analytically examined the welfare effect of strengthening IPR protection. We showed that the positive welfare effect through promoting innovation dominates the other effects if the initial protection of IPR in the South is sufficiently weak and the rate of R&D subsidy is not too high. Thus, strengthening IPR protection can increase welfare in both the South and the North. This is important because most related studies did not evaluate the policy of strengthening IPR protection from the viewpoint of welfare. Hence, the result of the present paper provides valuable information about desirable IPR policies in developing countries.

In the basic model, we assumed costless imitation for tractability. In practice, imitating products involving high-level technology can entail large costs. Thus, we need also to examine the welfare effects of strengthening IPR protection in the case where imitation is costly. By limiting the analysis to the steady state, Section 7.2 and Appendix J examined the effects numerically. This suggests it is necessary hereafter to examine not only the welfare effect in the steady state but also that in the short run, as in the analysis of the costless-imitation model. This is far beyond the scope of the present paper and thus remains as future work.

The present paper also assumes no cost of trade, such as in the form of tariffs. However, the presence of a trade cost would certainly affect the incentives for FDI and thereby change the effects of strengthening IPR protection on FDI and innovation. By extending the present model, we would also be able to analyze the effects of a change in trade cost as in Grieben (2005), Dinopoulos and Segerstrom (2007), Grieben and Şener (2009), and Dinopoulos and Unel (2011). It is also well worth examining how the presence of a trade cost would change the effects of strengthening IPR protection on FDI, innovation, and welfare.

Finally, as we examined the welfare effects of strengthening IPR in a model that exhibits
scale effects, one direction for future research would be to remove the scale effects from the model. However, if we extended the model in this direction, the dynamic structure of the model would be too complex to obtain clear analytical results, obliging us to rely on numerical analysis. Although this extension is beyond the scope of the present paper, it would be well worth conducting in the future.

Appendix A Derivation of the Equilibrium Path

In this appendix, we derive the equilibrium path of the economy. From (7) and (8), the following equality is satisfied in the equilibrium:

\[ v_N(t) = v_F(t) = (1 - s_R)w(t)a_N. \]  

(27)

Because this equality implies that \( \dot{v}_N(t) = \dot{v}_F(t) \) for all \( t \), substituting (9) and (10) into the relation leads to the following equality:

\[ \pi_F(t) - \pi_N(t) = Mv_F(t), \]  

(28)

where the left-hand side represents the “benefit” of becoming a multinational firm for a Northern leader firm, which can be measured by the increment in profits at each point in time, and the right-hand side represents the “cost” of becoming a multinational firm for a Northern leader firm, which corresponds to the risk of losing its monopolistic rents through imitation at each point in time. Substituting (5), (6), and (27) into (28), we have the following relation between \( E(t) \) and \( w(t) \):

\[ E(t) = \frac{a_N M \lambda(1 - s_R)w(t)}{s_F(\lambda - 1) + w(t) - 1}, \]  

(29)

which implies that:

\[ \frac{\dot{E}(t)}{E(t)} = \frac{s_F(\lambda - 1) - 1}{s_F(\lambda - 1) + w(t) - 1} \frac{\dot{w}(t)}{w(t)}. \]  

(30)

Noting that \( \dot{w}(t)/w(t) = \dot{v}_N(t)/v_N(t) \) from (7), we obtain the following equality by substituting (5), (7), and (29) into (9):

\[ \frac{\dot{w}(t)}{w(t)} = \frac{\dot{v}_N(t)}{v_N(t)} = r(t) = \frac{M(\lambda - w(t))}{s_F(\lambda - 1) + w(t) - 1}, \]  

(31)
where \( r(t) \) can be computed from (4), (30), and (31) as:

\[
r(t) = \frac{s_F(\lambda - 1) + w(t) - 1}{w(t)} \rho - \frac{[s_F(\lambda - 1) - 1]M(\lambda - w(t))}{w(t)[s_F(\lambda - 1) + w(t) - 1]}. \tag{32}
\]

Substituting (32) into (31), we can derive the equation of motion of \( w(t) \) as follows:

\[
\dot{w}(t) = (\rho + M)w(t) + \rho s_F(\lambda - 1) - (\rho + M \lambda).
\tag{33}
\]

The only endogenous variable in this equation is \( w(t) \), which is a jumpable variable. Thus, (33) implies that \( w(t) \) must jump to its steady-state value immediately; otherwise, \( w(t) \) would never reach the steady-state value because the coefficient of \( w(t) \) is strictly positive. From the differential equation, the steady-state value of \( w(t) \) is given by:

\[
\bar{w} = \frac{\rho + M \lambda - \rho s_F(\lambda - 1)}{\rho + M}.
\]

As \( w(t) \) takes a constant value at all times, we can show that \( E(t), r(t), v_N(t), \) and \( v_F(t) \) must also be constant over time. From (29), \( E(t) \) must immediately jump to the following steady-state value:

\[
\bar{E} = \frac{a_N \lambda (1 - s_R)[\rho + M \lambda - \rho s_F(\lambda - 1)]}{(1 + s_F)(\lambda - 1)}.
\]

Therefore, \( r(t) = \rho \) is satisfied for all \( t \) because \( \dot{E}(t)/E(t) = r(t) - \rho = 0 \) at any time. In addition, (27) shows that \( v_N(t) \) and \( v_F(t) \) must also take the following constant value of the steady state for all \( t \):

\[
\bar{v}_N = \bar{v}_F = \frac{(1 - s_R)a_N[\rho + M \lambda - \rho s_F(\lambda - 1)]}{\rho + M}.
\]

Meanwhile, we can show that the values of \( n_N(t), n_F(t), \) and \( \iota(t) \) are determined depending on the value of the only state variable in the model, \( n_S(t) \). By rewriting (12), we have:

\[
n_F(t) = \lambda \left( \frac{L_S}{E} - n_S(t) \right). \tag{34}
\]

Furthermore, substituting (34) into \( n_N(t) + n_F(t) + n_S(t) = 1 \) yields:

\[
n_N(t) = 1 - \frac{\lambda L_S}{E} + (\lambda - 1)n_S(t). \tag{35}
\]

Using (13) and (35), we can compute \( \iota(t) \) as follows:

\[
\iota(t) = \frac{1}{a_N} \left\{ L_N + L_S - \frac{\bar{E}}{\lambda} [1 + (\lambda - 1)n_S(t)] \right\}. \tag{36}
\]

\[\text{--- 29 ---}\]
By using the above results, we find that the motion of $n_S(t)$ in this model is determined by only the values of $n_S(t)$ itself and some exogenous variables. Substituting (34) and (36) into (11), we have the following equation of motion of $n_S(t)$:

$$
\dot{n}_S(t) = \frac{a_N M \lambda^2 L_S + \bar{E} - \lambda(L_N + L_S) \bar{E}}{a_N \lambda \bar{E}} - \left[M \lambda - \frac{(\lambda - 1) \bar{E}}{a_N \lambda}\right] n_S(t). \tag{37}
$$

Because $n_S(t)$ as well as the other variables must be constant in the steady state, the steady-state value of $n_S(t)$ can be computed from (37) as follows:

$$
\bar{n}_S = \frac{a_N M \lambda^2 L_S + \bar{E} - \lambda(L_N + L_S) \bar{E}}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}] \bar{E}}. \tag{38}
$$

Then, substituting (38) into (34) and (35) yields the steady-state values of $n_F(t)$ and $n_N(t)$:

$$
\bar{n}_F = \frac{\lambda[L_S + \lambda L_N - \bar{E}]}{a_N M \lambda^2 - (\lambda - 1) \bar{E}},
$$

$$
\bar{n}_N = \frac{\lambda[a_N M \lambda(\bar{E} - L_S) - L_N \bar{E}(\lambda - 1)]}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}] \bar{E}}.
$$

### Appendix B Proof of Proposition 3

In this appendix, we prove Proposition 3. It is sufficient for the proof to show that $\frac{\partial \bar{\iota}}{\partial M} < 0$.

Differentiating (19) with respect to $M$, we have:

$$
\frac{\partial \bar{\iota}}{\partial M} = -\frac{1}{a_N \lambda} \left\{ (\lambda - 1) \bar{E} \frac{\partial \bar{n}_S}{\partial M} + [1 + (\lambda - 1) \bar{n}_S] \frac{\partial \bar{E}}{\partial M} \right\}, \tag{39}
$$

where $\frac{\partial \bar{n}_S}{\partial M}$ can be computed from (16) as follows:

$$
\frac{\partial \bar{n}_S}{\partial M} = \frac{\bar{n}_F}{\mu} + \frac{1}{a_N \lambda \mu} \left[1 + (\lambda - 1) \bar{n}_S - \frac{a_N M \lambda^2 L_S}{\bar{E}^2} \right] \frac{\partial \bar{E}}{\partial M}. \tag{40}
$$

Substituting (40) into (39) yields:

$$
\frac{\partial \bar{\iota}}{\partial M} = -\frac{1}{a_N \lambda \mu} \left\{ (\lambda - 1) \bar{E} \bar{n}_F + \frac{(\lambda - 1) \bar{E}}{a_N \lambda} \left[1 + (\lambda - 1) \bar{n}_S - \frac{a_N M \lambda^2 L_S}{\bar{E}^2} \right] \frac{\partial \bar{E}}{\partial M} \right.
$$

$$
+ \frac{a_N M \lambda^2 - (\lambda - 1) \bar{E}}{a_N \lambda} [1 + (\lambda - 1) \bar{n}_S] \frac{\partial \bar{E}}{\partial M} \right\}
$$

$$
= -\frac{1}{a_N \lambda \mu} \left[ (\lambda - 1) \bar{E} \bar{n}_F + M \lambda \left( 1 - \frac{\lambda - 1}{\lambda} \bar{n}_F \right) \frac{\partial \bar{E}}{\partial M} \right]
$$

$$
< 0,
$$

– 30 –
where the second equality uses the Southern labor market-clearing condition, \( \bar{n}_S = \frac{L_S}{E} - \bar{n}_F \), and the last inequality uses \( 0 < \bar{n}_F < 1 \). Thus, we can confirm that stronger IPR protection increases the long-run aggregate rate of innovation \( \bar{\iota} \).

### Appendix C  Proof of Proposition 5

In this appendix, we prove Proposition 5. We first show that \( \frac{\partial \bar{\iota}(t)}{\partial M} < 0 \) for all \( t \in [0, \infty) \) if \( n_S(0) = \bar{n}_S \). Differentiating (19) with respect to \( M \), we have:

\[
\frac{\partial \bar{\iota}(t)}{\partial M} = -\frac{1}{a_N \lambda} \left\{ (\lambda - 1) \bar{E} \frac{\partial n_S(t)}{\partial M} + [1 + (\lambda - 1)n_S(t)] \frac{\partial \bar{E}}{\partial M} \right\} \quad (41)
\]

Note that \( \frac{\partial n_S(t)}{\partial M} \) is not necessarily equal to \( \frac{\partial \bar{n}_S}{\partial M} \). By differentiating (21) with respect to \( M \), we can compute \( \frac{\partial n_S(t)}{\partial M} \) as follows:

\[
\frac{\partial n_S(t)}{\partial M} = (1 - e^{-\mu t}) \frac{\partial \bar{n}_S}{\partial M} - te^{-\mu t} (n_S(0) - \bar{n}_S) \frac{\partial \mu}{\partial M} \quad (42)
\]

If the economy is initially in the steady state and \( n_S(0) = \bar{n}_S \), the second term in (42) is equal to zero. Thus, substituting (21), (39), and (42) into (41) and applying \( n_S(0) = \bar{n}_S \), we obtain the following relation:

\[
\left. \frac{\partial \bar{\iota}(t)}{\partial M} \right|_{n_S(0)=\bar{n}_S} = -\frac{1}{a_N \lambda} \left\{ (\lambda - 1) \bar{E} (1 - e^{-\mu t}) \frac{\partial \bar{n}_S}{\partial M} + [1 + (\lambda - 1)\bar{n}_S] \frac{\partial \bar{E}}{\partial M} \right\} \\
= (1 - e^{-\mu t}) \frac{\partial \bar{\iota}}{\partial M} - \frac{1}{a_N \lambda} [1 + (\lambda - 1)\bar{n}_S] e^{-\mu t} \frac{\partial \bar{E}}{\partial M} \\
< 0,
\]

where the inequality holds because \( \frac{\partial \bar{\iota}}{\partial M} < 0 \) and \( \frac{\partial \bar{E}}{\partial M} > 0 \).

Next, we prove that \( \frac{\partial n_F(t)}{\partial M} < 0 \) for all \( t \in [0, \infty) \) if \( n_S(0) = \bar{n}_S \). Differentiating (34) with respect to \( M \) and substituting (42) and \( n_S(0) = \bar{n}_S \) into the derivative, we have:

\[
\left. \frac{\partial n_F(t)}{\partial M} \right|_{n_S(0)=\bar{n}_S} = \lambda \left( -\frac{L_S \partial \bar{E}}{E^2} \frac{\partial \bar{n}_S}{\partial M} - \frac{\partial n_S(t)}{\partial M} \right) \\
= \lambda \left[ -\frac{L_S \partial \bar{E}}{E^2} \frac{\partial \bar{n}_S}{\partial M} - (1 - e^{-\mu t}) \frac{\partial \bar{n}_S}{\partial M} \right] \\
= (1 - e^{-\mu t}) \frac{\partial \bar{n}_F}{\partial M} - e^{-\mu t} \lambda L_S \frac{\partial \bar{E}}{\partial M} \\
< 0,
\]
where the third equality uses the relation that $\frac{\partial \bar{n}_F}{\partial M} = \lambda \left( -\frac{L_S}{E^2} \frac{\partial E}{\partial M} - \frac{\partial \bar{n}_S}{\partial M} \right)$ and the inequality uses $\frac{\partial \bar{n}_F}{\partial M} < 0$ and $\frac{\partial \bar{E}}{\partial M} > 0$.

Thus, we have been able to show that strengthening IPR protection increases innovation and FDI for all $t \in [0, \infty)$ if $n_S(0) = \bar{n}_S$.

### Appendix D Proof of Proposition 6

In this appendix, we prove that $\frac{\partial U_S}{\partial M} < 0$ holds when $M > \frac{\rho s R}{\lambda (1 - s R)}$. From (24) and $E_S = 1$, $\frac{\partial U_S}{\partial M}$ is equal to the sum of the innovation-impeding effect and the competition effect. The sign of the competition effect depends on that of $\frac{\partial \bar{n}_S}{\partial M}$.

If $\frac{\partial \bar{n}_S}{\partial M} \leq 0$, the competition effect is nonpositive. Because the innovation-impeding effect is negative, $\frac{\partial U_S}{\partial M} < 0$ necessarily holds if $\frac{\partial \bar{n}_S}{\partial M} \leq 0$.

If $\frac{\partial \bar{n}_S}{\partial M} > 0$, the competition effect is positive. Then, substituting (25), (26), and (39) into (24), we obtain a simpler expression of the total welfare effect as follows:

$$\frac{\partial U_S}{\partial M} = (\log \lambda) \left\{ - \frac{\mu}{\rho (\rho + \mu)} \left[ (\lambda - 1) \frac{E}{a_N \lambda \rho} - 1 \right] \frac{\partial \bar{n}_S}{\partial M} - \frac{1}{a_N \lambda \rho^2} [1 + (\lambda - 1) \bar{n}_S] \frac{\partial \bar{E}}{\partial M} \right\}.$$

From this relation, even if $\frac{\partial \bar{n}_S}{\partial M} > 0$, imposing $\frac{(\lambda - 1) E}{a_N \lambda \rho} > 1$ guarantees that the innovation-impeding effect outweighs the competition effect, namely $\frac{\partial U_S}{\partial M} < 0$. Using (14) and $s_F = 0$, $\frac{(\lambda - 1) E}{a_N \lambda \rho} > 1$ can be reduced to $M > \frac{\rho s R}{\lambda (1 - s R)}$.

### Appendix E Proof of Proposition 7

In this appendix, we prove Proposition 7. If the inequality in Proposition 7 holds, we obtain $\frac{\partial U_N}{\partial M} < 0$ from (14). Therefore, we prove that $\frac{\partial U_N}{\partial M} < 0$ if $\frac{(E - L_S)(\log \lambda)}{a_N \lambda (\rho + \mu)} > 1$.

Because $E_N = \frac{E - L_S}{L_N}$, the nominal spending effect in the North is given by $\frac{1}{\rho (E - L_S)} \frac{\partial \bar{E}}{\partial M}$. Substituting this equation, (25), and (26) into (24), the total welfare effect in the North is reduced to:

$$\frac{\partial U_N}{\partial M} = (\log \lambda) \left\{ \frac{\mu}{\rho^2 (\rho + \mu)} \frac{\partial \bar{E}}{\partial M} - \frac{1}{\rho (E - L_S) (\log \lambda)} \left[ \frac{(E - L_S)(\log \lambda)}{a_N \lambda (\rho + \mu)} - 1 \right] \frac{\partial \bar{E}}{\partial M} \right\}.$$

- 32 –
The sign of the competition effect depends on that of \( \frac{\partial \bar{n}_S}{\partial M} \).

If \( \frac{\partial \bar{n}_S}{\partial M} \leq 0 \), the competition effect is nonpositive. From \( \frac{(E-L_S)(\log \lambda)}{a_N \lambda (\rho + \mu)} > 1 \), the second term in the curly brackets on the right-hand side is negative. Thus, the total welfare effect is negative if \( \frac{\partial \bar{n}_S}{\partial M} \leq 0 \).

On the other hand, if \( \frac{\partial \bar{n}_S}{\partial M} > 0 \), the competition effect is positive. Then, from (39), we can rewrite the total welfare effect as follows:

\[
\frac{\partial U_N}{\partial M} = (\log \lambda) \left\{ -\frac{\mu}{\rho (\rho + \mu)} \left[ \frac{(\lambda - 1)\bar{E}}{a_N \lambda \rho} - 1 \right] \frac{\partial \bar{n}_S}{\partial M} - \frac{\mu}{a_N \lambda \rho^2 (\rho + \mu)} \left[ 1 + (\lambda - 1)\bar{n}_S \right] \frac{\partial \bar{E}}{\partial M} \\
- \frac{1}{\rho (E - L_S)(\log \lambda)} \left[ \frac{(E - L_S)(\log \lambda)}{a_N \lambda (\rho + \mu)} - 1 \right] \frac{\partial \bar{E}}{\partial M} - \frac{(\lambda - 1)\bar{n}_S}{a_N \lambda \rho (\rho + \mu)} \frac{\partial \bar{E}}{\partial M} \right\}.
\]

Because \( \lambda \) is greater than one, \( \lambda - 1 \) is greater than \( \log \lambda \) necessarily. This implies that \( \frac{(\lambda - 1)\bar{E}}{a_N \lambda \rho} > \frac{(E-L_S)(\log \lambda)}{a_N \lambda (\rho + \mu)} > 1 \). Therefore, the first and third terms in the curly brackets on the right-hand side are negative, and we can confirm \( \frac{\partial U_N}{\partial M} < 0 \) even if \( \frac{\partial \bar{n}_S}{\partial M} > 0 \).

### Appendix F  The Costly-Imitation Model

In this appendix, we present an extended model where imitation and FDI are costly. An analysis of the extended model is given in Appendix J. We describe only the parts that are changed in the extended model. For simplicity, we assume that the R&D and FDI subsidies are equal to zero, that is, \( s_R = s_F = 0 \).

Following Glass and Saggi (2002), we assume that multinational firms need \( \zeta (>1) \) units of Southern labor to produce one unit of goods, whereas Southern imitator firms need only one unit of Southern labor. In this case, the marginal cost for the multinational firms is \( \zeta \), although their optimal price is the same as in the original model, \( p_F(t) = \lambda \). Thus, the profits of a multinational firm change to \( \pi_F(t) = (1 - \frac{\zeta}{\lambda}) E(t) \). On the other hand, to maximize profits, Southern imitator firms charge a price so that the multinational firms cannot earn a positive profit; \( p_S(t) = \zeta \). Therefore, the profits of a Southern imitator become \( \pi_S(t) = (1 - \frac{\zeta}{\lambda}) E(t) \).

In this model, we assume that imitation activities require labor inputs in contrast with the original model. If a Southern follower firm devotes \( a_m M(t) dt \) units of Southern labor to copy the current latest generation of good \( j \) that the multinational firm produces, the firm succeeds in imitating the good with probability \( M(t) dt \). Under this setting, we can interpret stronger
IPR protection in the South as an increase in $a_m$. Given the value of $a_m$, a Southern follower firm attempting to copy a good chooses $M(t)$ optimally. Therefore, “imitation intensity” $M(t)$ is endogenously determined in this extended model, whereas the imitation rate is constant and exogenous in the original model. Letting $v_S(t)$ denote the value of a Southern imitator firm, the equilibrium condition of imitation is given by:

$$v_S(t) = a_m.$$  \hspace{1cm} (43)

The shareholders of a Southern imitator firm earn dividends $\pi_S(t)dt$ and capital gains $\dot{v}_S(t)dt$, and face a capital loss of amount $v_S(t)$ with probability $\iota_N(t)dt$ over a time interval $dt$. Therefore, the no-arbitrage condition between the shares of a Southern imitator firm and the risk-free asset is:

$$r(t)v_S(t) = \pi_S(t) + \dot{v}_S(t) - \iota_N(t)v_S(t).$$  \hspace{1cm} (44)

As another extension, we assume that FDI is also costly and requires labor inputs in this extended model, whereas we abstract the FDI cost in the original model. More concretely, we assume that the success or failure of FDI also follows a Poisson process: if a Northern leader devotes $a_F\iota_F(t)dt$ units of Southern labor to adaptation of technology, it succeeds in shifting production of its good to the South with probability $\iota_F(t)dt$. Under this assumption, the equilibrium condition of FDI changes from (8) to

$$v_N(t) + a_F = v_F(t).$$  \hspace{1cm} (45)

Next, we consider the labor market-equilibrium conditions. In contrast with the original model, a multinational firm needs $\zeta$ units of labor for the production of one unit of the good. This implies that the aggregate labor demand of the multinational firms changes to $n_F(t)(E(t)/\lambda)\zeta$. The aggregate labor demand of the imitator firms also changes to $n_S(t)E(t)/\zeta$ because the price set by the imitator firms is $\zeta$ in this model. In addition, Southern labor is devoted to both imitation activities and FDI. Therefore, the labor market-clearing condition in the South changes from (12) to

$$a_F\iota_F(t)n_N(t) + a_mM(t)n_F(t) + n_F(t)\frac{E(t)}{\lambda} \zeta + n_S(t)\frac{E(t)}{\zeta} = L_S.$$  \hspace{1cm} (46)

The labor market-clearing condition in the North is the same as (13).
Finally, we describe the law of motion governing the measures of the industries belonging to each category. In an infinitesimal time interval of length $dt$, leader firms succeed in developing a newer generation of $\iota_N(t)n_S(t)dt$ goods and transferring production of $\iota_F(t)n_N(t)dt$ type-N goods to the South. Moreover, the imitator firms succeed in copying $M(t)n_F(t)dt$ type-F goods in the same time interval. Therefore, the measure of the industries where the Southern imitator monopolistically produces the good changes over time with the following law of motion:

$$\dot{n}_S(t) = M(t)n_F(t) - \iota(t),$$

(47)

where $\iota(t) \equiv \iota_N(t)n_S(t)$. Meanwhile, the law of motion of $n_N(t)$ is given by:

$$\dot{n}_N(t) = \iota(t) - \iota_F(t)n_N(t).$$

(48)

Appendix G  Generalization of Initial Distribution of Assets and FDI Subsidy Rate

In Sections 6.1 and 6.2, we assumed no asset holding by the South and no FDI subsidy. In this appendix, we briefly discuss how the welfare results change if these assumptions are relaxed. To assess the consequences of the generalization, we draw on a numerical example because it is difficult to compute the welfare effect analytically.

We first compute the nominal spending of households in the generalized case such that the Southern households have some assets and the FDI subsidy rate is positive. As discussed in Section 3 and Appendix A, $r(t) = \rho$ for all $t$ because $E(t)$ is constant on the equilibrium path. Thus, $E_i(t)$ also takes a constant value $E_i$ on the equilibrium path because of the intertemporal utility maximization. Considering these, the intertemporal budget constraint can be expressed as:

$$E_S = \rho A_S(0) + 1 - \rho \int_0^\infty e^{-\rho t}T_S(t)dt,$$

(49)

$$E_N = \rho A_N(0) + \bar{\omega} - \rho \int_0^\infty e^{-\rho t}T_N(t)dt.$$

(50)

By using the government budget constraints, (19), (21), and (34), the third terms of (49) and
(50) are given by:

\[\rho \int_{0}^{\infty} e^{-\rho t} T_{S}(t) dt = \rho \int_{0}^{\infty} e^{-\rho t} s_{F} \frac{(1 - 1/\lambda) \tilde{E}}{L_{S}} n_{F}(t) dt = \frac{\rho s_{F} (1 - 1/\lambda) \tilde{E}}{L_{S}} \int_{0}^{\infty} e^{-\rho t} \left( \frac{L_{S}}{\tilde{E}} - n_{S}(t) \right) dt = s_{F} (\lambda - 1) \tilde{E} \left[ \frac{L_{S}}{\tilde{E}} - \bar{n}_{S} - (n_{S}(0) - \bar{n}_{S}) \frac{\rho}{\rho + \mu} \right].\]

\[\rho \int_{0}^{\infty} e^{-\rho t} T_{N}(t) dt = \rho \int_{0}^{\infty} e^{-\rho t} \frac{s_{R} \tilde{w} a_{N}}{L_{N}} \bar{n}(t) dt = \frac{\rho s_{R} \tilde{w} a_{N}}{L_{N}} \int_{0}^{\infty} e^{-\rho t} \frac{1}{a_{N}} \left\{ L_{N} + L_{S} - \tilde{E} \left[ 1 + (\lambda - 1) n_{S}(t) \right] \right\} dt = s_{R} \tilde{w} \left[ L_{N} + L_{S} - \frac{\tilde{E}}{\lambda} - \frac{\tilde{E} (\lambda - 1) \tilde{E}}{\lambda} \bar{n}_{S} - (n_{S}(0) - \bar{n}_{S}) \frac{(\lambda - 1) \rho \tilde{E}}{\lambda (\rho + \mu)} \right].\]

The values of \(A_{i}(0)\) are computed as follows. Letting \(A(0) \equiv A_{N}(0) L_{N} + A_{S}(0) L_{S}\) denote the total initial asset holdings of the world, we obtain the following equation by summing up the spending of every household:

\[\tilde{E} = L_{S} E_{S} + L_{N} E_{N} = \rho A(0) + L_{S} + \tilde{w} L_{N} - \rho L_{N} \int_{0}^{\infty} e^{-\rho t} T_{S}(t) dt - \rho L_{N} \int_{0}^{\infty} e^{-\rho t} T_{N}(t) dt,\]

or equivalently,

\[A(0) = \frac{1}{\rho} \tilde{E} - \frac{1}{\rho} (L_{S} + \tilde{w} L_{N}) + L_{S} \int_{0}^{\infty} e^{-\rho t} T_{S}(t) dt + L_{N} \int_{0}^{\infty} e^{-\rho t} T_{N}(t) dt.\]

Because country \(i\)'s share of asset holdings must be given as an initial condition, we let \(\sigma \in [0, 1]\) denote the share of assets held by Southern households, that is, \(\sigma \equiv A_{S}(0) L_{S} / A(0)\). This means that \(A_{S}(0) = \sigma A(0) / L_{S}\) and \(A_{N}(0) = (1 - \sigma) A(0) / L_{N}\). Substituting these into (49) and (50), we have:

\[E_{S} = \frac{\sigma A(0)}{L_{S}} + 1 - \rho \int_{0}^{\infty} e^{-\rho t} T_{S}(t) dt = \frac{\sigma}{L_{S}} \left[ \tilde{E} - (L_{S} + \tilde{w} L_{N}) + \rho L_{S} \int_{0}^{\infty} e^{-\rho t} T_{S}(t) dt + \rho L_{N} \int_{0}^{\infty} e^{-\rho t} T_{N}(t) dt \right] + 1 - \rho \int_{0}^{\infty} e^{-\rho t} T_{S}(t) dt \]

\[= 1 - s_{F} (1 - \sigma) (\lambda - 1) + \frac{\tilde{E}}{L_{S}} - \frac{L_{S}}{L_{S}} - \frac{\sigma \tilde{w}}{L_{S}} \left[ (1 - s_{R}) L_{N} - s_{R} L_{S} + \frac{s_{R} \tilde{E}}{\lambda} \right] - \frac{(\lambda - 1) \tilde{E}}{L_{S}} \left[ \frac{\sigma s_{R} \tilde{w}}{\lambda} - s_{F} (1 - \sigma) \right] \left[ \bar{n}_{S} + (n_{S}(0) - \bar{n}_{S}) \frac{\rho}{\rho + \mu} \right].\]
\[
E_N = \frac{(1 - \sigma)A(0)}{L_N} + \bar{w} - \rho \int_0^\infty e^{-\rho t} T_N(t) dt \\
= \frac{1 - \sigma}{L_N} \left[ \bar{E} - (L_S + \bar{w}L_N) + \rho L_S \int_0^\infty e^{-\rho t} T_S(t) dt + \rho L_N \int_0^\infty e^{-\rho t} T_N(t) dt \right] \\
+ \bar{w} - \rho \int_0^\infty e^{-\rho t} T_N(t) dt \\
= s_F(1 - \sigma)(\lambda - 1) \frac{L_S}{L_N} + (1 - \sigma) \frac{\bar{E} - L_S}{L_N} + \frac{\sigma \bar{w}}{L_N} \left[ (1 - s_R)L_N - s_R L_S + \frac{s_R \bar{E}}{\lambda} \right] \\
+ \frac{(\lambda - 1)E}{\lambda} \left[ \frac{\sigma s_R \bar{w}}{\lambda} - s_F(1 - \sigma) \right] \left[ \bar{n}_S + (n_S(0) - \bar{n}_S) \frac{\rho}{\rho + \mu} \right].
\] (52)

Note that \(E_S = 1\) and \(E_N = (\bar{E} - L_S)/L_N\) are satisfied if \(\sigma = 0\) (households in the South have no asset) and \(s_F = 0\) (the FDI subsidy is zero), which is the same as analyzed in Sections 6.1 and 6.2. The derivatives of equations (51) and (52) with respect to \(M\) are:

\[
\frac{\partial E_S}{\partial M} \bigg|_{n_S(0) = \bar{n}_S} = \left\{ -\frac{\sigma}{L_S} \left[ (1 - s_R)L_N - s_R L_S + \frac{s_R \bar{E}}{\lambda} \right] - \frac{(\lambda - 1)\sigma s_R \bar{E} \bar{n}_S}{\lambda L_N} \right\} \frac{\partial \bar{w}}{\partial M} \\
+ \left\{ -\frac{\sigma}{L_S} \left[ \frac{s_R \bar{w}}{\lambda} - (1 - s_R) \frac{\partial \bar{E}}{\partial M} \right] \right\} \frac{\mu}{\rho + \mu} \frac{\partial \bar{n}_S}{\partial M},
\]

\[
\frac{\partial E_N}{\partial M} \bigg|_{n_S(0) = \bar{n}_S} = \left\{ -\frac{1 - \sigma}{L_N} \left[ (1 - s_R)L_N - s_R L_S + \frac{s_R \bar{E}}{\lambda} \right] + \frac{(\lambda - 1)\sigma s_R \bar{E} \bar{n}_S}{\lambda L_N} \right\} \frac{\partial \bar{w}}{\partial M} \\
+ \left\{ -\frac{1 - \sigma}{L_N} \left[ \frac{s_R \bar{w}}{\lambda} - \left(1 - s_R\right) \frac{\partial \bar{E}}{\partial M} \right] \right\} \frac{\mu}{\rho + \mu} \frac{\partial \bar{n}_S}{\partial M},
\]

Using these, we compute the nominal spending effect in the generalized case. The innovation-impeding effect and the competition effect are as in the text.

Figure 1 illustrates the welfare effects of relaxing IPR for each value of \(s_F\) and \(\sigma\). Following Glass and Saggi (2002), we set \(\rho = 0.05\), \(\lambda = 4\), \(L_N = 3\), and \(L_S = 6\) in Figure 1. We further set \(M = 0.037\), \(a_N = 123.5\), and \(s_R = 0.45\). Under these parameter values, it is required that \(0 \leq s_F \leq 0.24\) to ensure the existence and the stability of the steady state. The sign of the welfare effect is unchanged from the case of \(s_F = \sigma = 0\) if \(s_F\) and \(\sigma\) are sufficiently small. In this example, the effect on the welfare of the South \(\partial U_S/\partial M\) is negative for any \(s_F\) and \(\sigma\).
Figure 1: The welfare effect under positive FDI subsidy and positive asset holdings of the South.

Thus, strengthening IPR protection in this case increases the welfare of Southern consumers even if the FDI subsidy and the asset holdings of the South are positive. On the other hand, the sign of the effect on the welfare of the North $\partial U_N/\partial M$ depends on $s_F$ and $\sigma$. If the FDI subsidy rate and the asset holdings of the South are sufficiently large, $\partial U_N/\partial M$ can be positive: strengthening IPR protection in the South can harm the North. However, if they are sufficiently small, $\partial U_N/\partial M$ is negative, such that strengthening IPR protection in the South still benefits the North.

Appendix H Welfare Effects in the Case of Initially Strong IPR

In this appendix, we explore whether further strengthening IPR under strong protection is beneficial to the South. To do this, we examine the sign of $\frac{\partial U_S}{\partial M}$ in the case where the condition of Proposition 6 is not satisfied, by using two numerical examples. In the examples, we set $\lambda = 4, L_S = 6$, and $L_N = 3$ as in footnote 5, and set $a_N = 400$. Following the setting in Section 6.1, we assume that the FDI subsidy is zero and the Southern households have no asset. Figures 2 and 3 show the value of $\frac{\partial U_S}{\partial M}$ for each $M$ in the two cases. We set $\rho = 0.12$ and $s_R = 0.83$ in case 1 (Figure 2), while $\rho = 0.05$ and $s_R = 0.65$ in case 2 (Figure 3). To ensure the existence and the stability of the steady state, it is required that $0.0097 < M < 0.0196$ in case 1 and $0.009 < M < 0.0116$ in case 2. Note that both cases do not satisfy the condition provided in
Proposition 6; that is, IPR protection in the South is relatively strong.

According to the examples, the welfare effect of strengthening IPR is indeterminate if IPR is initially strong. Figure 2 shows that the sign of $\frac{\partial U_S}{\partial M}$ turns positive in case 1 if $M$ is sufficiently small. This implies that there is the optimal level of IPR protection inside the interval of $M$. However, Figure 3 shows that the sign of $\frac{\partial U_S}{\partial M}$ remains negative for any value of $M$ in case 2. Thus, unlike case 1, maximum IPR protection is beneficial to the South. Suppose, for example, that the initial imitation rate is $M = 0.01$. Then, the welfare effect of strengthening IPR protection differs between case 1 and case 2. In case 1, a further strengthening of IPR is unfavorable to the South, whereas in case 2, it is favorable to the South. Roughly speaking, it appears that further strengthening IPR under strong protection tends to worsen the welfare of the South if $s_R$ is considerably large.
Figure 2: Imitation rate and the effects of strengthening IPR on the welfare of the South: The case of $\rho = 0.12$ and $s_R = 0.83$. It is required that $0.0097 < M < 0.0196$ to ensure the existence and the stability of the steady state.

Figure 3: Imitation rate and the effects of strengthening IPR on the welfare of the South: The case of $\rho = 0.05$ and $s_R = 0.65$. It is required that $0.009 < M < 0.0116$ to ensure the existence and the stability of the steady state.
Appendix I  Welfare Effects of the Subsidy Policies

In this appendix, we show the welfare effects of the subsidy policies for R&D and FDI. Differentiating the lifetime utility function with respect to $s_R$ and $s_F$, we have:

$$\frac{\partial U_i}{\partial s_R} = \int_0^\infty e^{-\rho t}(\log \lambda) \frac{\partial \bar{E}}{\partial s_R} \left( \int_0^t \nu(\tau) d\tau \right) dt + \frac{1}{\rho} \frac{\partial E_i}{\partial s_R} - \int_0^\infty e^{-\rho t}(\log \lambda) \frac{\partial n_S(t)}{\partial s_R} dt. \quad (53)$$

$$\frac{\partial U_i}{\partial s_F} = \int_0^\infty e^{-\rho t}(\log \lambda) \frac{\partial \bar{E}}{\partial s_F} \left( \int_0^t \nu(\tau) d\tau \right) dt + \frac{1}{\rho} \frac{\partial E_i}{\partial s_F} + \int_0^\infty e^{-\rho t}(\log \lambda) \frac{\partial n_S(t)}{\partial s_F} dt. \quad (54)$$

An interpretation of the three terms of (53) and (54) is the same as that of (24). The first terms represent the positive welfare effect through promoting innovation. An increase in the subsidies stimulates R&D, so that it can improve welfare by speeding up the quality improvement.

The second terms represent the welfare effect through the effect on the nominal spending. An increase in the subsidies changes the profitability of R&D and FDI, which influences the firms’ decision on R&D and the location choices. Thus, it affects the spending of both countries through the change in the wages and the value of the shares of the leader firms. The third terms represent the welfare effect through changing the proportion of competitive goods. An increase in the subsidies changes the measure of type-S industries because it affects the aggregate rate of innovation, which is equal to the outflow from type-S industries, and FDI, which determines the inflow into type-S industries. It therefore influences welfare through the change in the proportion of cheaper goods.

To compute the value of each term, we first carry out comparative statics. Differentiating (14) with respect to $s_R$ and $s_F$, we have:

$$\frac{\partial \bar{E}}{\partial s_R} = -\frac{\bar{E}}{1 - s_R} < 0, \quad (55)$$

$$\frac{\partial \bar{E}}{\partial s_F} = -\frac{\lambda(M + \rho)\bar{E}}{(1 + s_F)(\rho + M\lambda - s_F(\lambda - 1)\bar{E})} < 0. \quad (56)$$

Using equation (16), we can compute $\frac{\partial n_S}{\partial s_R}$ and $\frac{\partial n_S}{\partial s_F}$ as follows:

$$\frac{\partial n_S}{\partial s_R} = \frac{1}{a_N \lambda \mu} \left[ 1 + (\lambda - 1)\bar{n}_S - \frac{a_N M \lambda^2 L_S}{E^2} \right] \frac{\partial \bar{E}}{\partial s_R}, \quad (57)$$

$$\frac{\partial n_S}{\partial s_F} = \frac{1}{a_N \lambda \mu} \left[ 1 + (\lambda - 1)\bar{n}_S - \frac{a_N M \lambda^2 L_S}{E^2} \right] \frac{\partial \bar{E}}{\partial s_F}. \quad (58)$$
which are analogous to (40). Therefore, we can compute the long-run effect of the subsidy policies on the aggregate rate of innovation:

\[
\frac{\partial \bar{\iota}}{\partial s_R} = -\frac{1}{a_N \lambda} \left\{ (\lambda - 1) \bar{E} \frac{\partial \bar{n}_S}{\partial s_R} + [1 + (\lambda - 1)\bar{n}_S] \frac{\partial \bar{E}}{\partial s_R} \right\}
\]

\[
= -\frac{1}{a_N \lambda \mu} \left\{ (\lambda - 1) \bar{E} \left[ 1 + (\lambda - 1)\bar{n}_S - \frac{a_N M \lambda^2 L_S}{E^2} \right] \frac{\partial \bar{E}}{\partial s_R} 
+ \frac{a_N M \lambda^2 - (\lambda - 1)\bar{E}}{a_N \lambda} [1 + (\lambda - 1)\bar{n}_S] \frac{\partial \bar{E}}{\partial s_R} \right\}
\]

\[
= -\frac{M}{a_N \mu} \left( 1 - \bar{n}_F + \frac{\bar{n}_F}{\lambda} \right) \frac{\partial \bar{E}}{\partial s_R}
> 0,
\]

(59)

\[
\frac{\partial \bar{\iota}}{\partial s_F} = -\frac{1}{a_N \lambda} \left\{ (\lambda - 1) \bar{E} \frac{\partial \bar{n}_S}{\partial s_F} + [1 + (\lambda - 1)\bar{n}_S] \frac{\partial \bar{E}}{\partial s_F} \right\}
\]

\[
= -\frac{1}{a_N \lambda \mu} \left\{ (\lambda - 1) \bar{E} \left[ 1 + (\lambda - 1)\bar{n}_S - \frac{a_N M \lambda^2 L_S}{E^2} \right] \frac{\partial \bar{E}}{\partial s_F} 
+ \frac{a_N M \lambda^2 - (\lambda - 1)\bar{E}}{a_N \lambda} [1 + (\lambda - 1)\bar{n}_S] \frac{\partial \bar{E}}{\partial s_F} \right\}
\]

\[
= -\frac{M}{a_N \mu} \left( 1 - \bar{n}_F + \frac{\bar{n}_F}{\lambda} \right) \frac{\partial \bar{E}}{\partial s_F}
> 0,
\]

(60)

where the second equalities in both equations use \( \mu \equiv M \lambda - (\lambda - 1)\bar{E}/(a_N \lambda) \) and the inequalities hold from \( 0 < \bar{n}_F < 1 \), (55), and (56). By differentiating (17) with respect to \( s_R \) and \( s_F \), we have:

\[
\frac{\partial \bar{n}_F}{\partial s_R} = -\frac{(1 - \bar{n}_F)\lambda + \bar{n}_F \frac{\partial \bar{E}}{\partial s_R}}{a_N \mu} > 0,
\]

and

\[
\frac{\partial \bar{n}_F}{\partial s_F} = -\frac{(1 - \bar{n}_F)\lambda + \bar{n}_F \frac{\partial \bar{E}}{\partial s_F}}{a_N \mu} > 0,
\]

both of which are similar to equation (23).

Next, we analyze the dynamic effects of the change in the subsidies. Differentiating (19) with respect to \( s_R \) and \( s_F \) yields:

\[
\frac{\partial \bar{n}_S(t)}{\partial s_R} = (1 - e^{-\mu t}) \frac{\partial \bar{n}_S}{\partial s_R},
\]

(63)
\[ \frac{\partial n_S(t)}{\partial s_F} = (1 - e^{-\mu t}) \frac{\partial \bar{n}_S}{\partial s_F}. \] (64)

Thus, substituting these, (21), (59), and (60) into (61) and (62), we obtain the following relation:

\[
\frac{\partial \iota(t)}{\partial s_R} \bigg|_{n_S(0) = \bar{n}_S} = -\frac{1}{a_N \lambda} \left\{ (\lambda - 1) \bar{E} (1 - e^{-\mu t}) \frac{\partial \bar{n}_S}{\partial s_R} + [1 + (\lambda - 1) \bar{n}_S] \frac{\partial \bar{E}}{\partial s_R} \right\} \\
= (1 - e^{-\mu t}) \frac{\partial \iota}{\partial s_R} - \frac{1}{a_N \lambda} [1 + (\lambda - 1) \bar{n}_S] e^{-\mu t} \frac{\partial \bar{E}}{\partial s_R} \\
> 0,
\]

\[
\frac{\partial \iota(t)}{\partial s_F} \bigg|_{n_S(0) = \bar{n}_S} = -\frac{1}{a_N \lambda} \left\{ (\lambda - 1) \bar{E} (1 - e^{-\mu t}) \frac{\partial \bar{n}_S}{\partial s_F} + [1 + (\lambda - 1) \bar{n}_S] \frac{\partial \bar{E}}{\partial s_F} \right\} \\
= (1 - e^{-\mu t}) \frac{\partial \iota}{\partial s_F} - \frac{1}{a_N \lambda} [1 + (\lambda - 1) \bar{n}_S] e^{-\mu t} \frac{\partial \bar{E}}{\partial s_F} \\
> 0,
\]

where the inequalities hold from (55), (56), (59), and (60).

Using the above results, we can compute the welfare effects of the subsidy policies. First, the welfare effects through promoting innovation are:

\[
\int_0^\infty e^{-\rho t} (\log \lambda) \frac{\partial}{\partial s_R} \left( \int_0^t \iota(\tau) d\tau \right) dt \\
= (\log \lambda) \int_0^\infty e^{-\rho t} \int_0^t \left\{ (1 - e^{-\mu \tau}) \frac{\partial \iota}{\partial s_R} - \frac{1}{a_N \lambda} [1 + (\lambda - 1) \bar{n}_S] e^{-\mu \tau} \frac{\partial \bar{E}}{\partial s_R} \right\} d\tau dt \\
= (\log \lambda) \left\{ \frac{\mu}{\rho^2 (\rho + \mu)} \frac{\partial \iota}{\partial s_R} - \frac{1}{a_N \lambda \rho (\rho + \mu)} [1 + (\lambda - 1) \bar{n}_S] \frac{\partial \bar{E}}{\partial s_R} \right\} > 0, \quad (65)
\]

\[
\int_0^\infty e^{-\rho t} (\log \lambda) \frac{\partial}{\partial s_F} \left( \int_0^t \iota(\tau) d\tau \right) dt \\
= (\log \lambda) \int_0^\infty e^{-\rho t} \int_0^t \left\{ (1 - e^{-\mu \tau}) \frac{\partial \iota}{\partial s_F} - \frac{1}{a_N \lambda} [1 + (\lambda - 1) \bar{n}_S] e^{-\mu \tau} \frac{\partial \bar{E}}{\partial s_F} \right\} d\tau dt \\
= (\log \lambda) \left\{ \frac{\mu}{\rho^2 (\rho + \mu)} \frac{\partial \iota}{\partial s_F} - \frac{1}{a_N \lambda \rho (\rho + \mu)} [1 + (\lambda - 1) \bar{n}_S] \frac{\partial \bar{E}}{\partial s_F} \right\} > 0. \quad (66)
\]

Second, differentiating (51) and (52) with respect to \( s_R \) and \( s_F \), and applying \( n_S(0) = \bar{n}_S \), \( \sigma = 0 \), and \( s_F = 0 \), we get:

\[ \frac{\partial E_S}{\partial s_R} = 0, \]
\[ \frac{\partial E_N}{\partial s_R} = \frac{1}{L_N} \frac{\partial E}{\partial s_R}, \]
\[ \frac{\partial E_S}{\partial s_F} \bigg|_{s_F=0} = -(\lambda - 1) + \frac{(\lambda - 1) \bar{E}}{L_S} \bar{n}_S = -\frac{(\lambda - 1) \bar{E} \bar{n}_F}{\lambda L_S}, \]
\[ \frac{\partial E_N}{\partial s_F} \bigg|_{s_F=0} = (\lambda - 1) \frac{L_S}{L_N} - \frac{(\lambda - 1) \bar{E}}{L_N} \bar{n}_S + \frac{1}{L_N} \frac{\partial \bar{E}}{\partial s_F} = \frac{(\lambda - 1) \bar{E} \bar{n}_F}{\lambda L_N} + \frac{1}{L_N} \frac{\partial \bar{E}}{\partial s_F}. \]

Thus, in the case of \( \sigma = 0 \) and \( s_F = 0 \), the welfare effects through nominal spending are given by:

\[ \frac{1}{\rho} \frac{\partial E_S}{\partial s_R} = 0, \quad (67) \]
\[ \frac{1}{\rho} \frac{\partial E_N}{\partial s_R} - \frac{1}{\rho (\bar{E} - L_S)} \frac{\partial E}{\partial s_R}, \quad (68) \]
\[ \frac{1}{\rho} \frac{\partial E_S}{\partial s_F} = -\frac{(\lambda - 1) \bar{E} \bar{n}_F}{\rho \lambda L_S}, \quad (69) \]
\[ \frac{1}{\rho} \frac{\partial E_N}{\partial s_F} = \frac{1}{\rho (\bar{E} - L_S)} \left[ \frac{(\lambda - 1) \bar{E} \bar{n}_F}{\lambda} + \frac{\partial \bar{E}}{\partial s_F} \right]. \quad (70) \]

Third, from (63) and (64), the welfare effects through changing the proportion of competitive goods are:

\[ \int_0^\infty e^{-\rho t} (\log \lambda) \frac{\partial n_S(t)}{\partial s_R} dt = (\log \lambda) \frac{\mu}{\rho (\rho + \mu)} \frac{\partial \bar{n}_S}{\partial s_R}, \quad (71) \]
\[ \int_0^\infty e^{-\rho t} (\log \lambda) \frac{\partial n_S(t)}{\partial s_F} dt = (\log \lambda) \frac{\mu}{\rho (\rho + \mu)} \frac{\partial \bar{n}_S}{\partial s_F}. \quad (72) \]

Using equation (57), we obtain the sum of the welfare effects through promoting innovation and through changing the proportion of competitive goods:

\[ \int_0^\infty e^{-\rho t} (\log \lambda) \frac{\partial}{\partial s_R} \left( \int_0^t \nu(\tau) d\tau \right) dt + \int_0^\infty e^{-\rho t} (\log \lambda) \frac{\partial n_S(t)}{\partial s_R} dt \]
\[ = (\log \lambda) \left\{ \frac{\mu}{\rho^2 (\rho + \mu)} \frac{\partial \bar{n}_S}{\partial s_R} - \frac{1}{\alpha N \lambda \rho (\rho + \mu)} \left[ 1 + (\lambda - 1) \bar{n}_S \right] \frac{\partial \bar{E}}{\partial s_R} + \frac{\mu}{\rho (\rho + \mu)} \frac{\partial \bar{n}_S}{\partial s_R} \right\} \]
\[ = (\log \lambda) \left\{ \frac{\mu}{\rho^2 (\rho + \mu)} \frac{\partial \bar{n}_S}{\partial s_R} - \frac{1}{\rho (\rho + \mu)} \frac{M \lambda L_S}{E^2} \frac{\partial \bar{E}}{\partial s_R} \right\} \]
\[ > 0, \quad (73) \]

where the inequality holds from (55) and (59). Thus, from (53), (67), and (73), an increase in the R&D subsidy necessarily improves the welfare of the South.
Meanwhile, an increase in the R&D subsidy can improve the welfare of the North if the condition in Proposition 7 holds, proved as follows. Substituting (65), (68), and (71) into (53), we have:

\[ \frac{\partial U_N}{\partial s_R} = (\log \lambda) \left\{ \frac{\mu}{p^2(\rho + \mu)} \frac{\partial \bar{E}}{\partial s_R} - \frac{1}{\rho(E - L_S)(\log \lambda)} \left[ \frac{(E - L_S)(\log \lambda)}{a_N \lambda(\rho + \mu)} - 1 \right] \frac{\partial \bar{E}}{\partial s_R} - \frac{(\lambda - 1)\bar{n}_S}{a_N \lambda \rho(\rho + \mu)} \frac{\partial \bar{E}}{\partial s_R} + \frac{\mu}{\rho(\rho + \mu)} \frac{\partial \bar{n}_S}{\partial s_R} \right\} \cdot (74) \]

As discussed in Appendix E, \( \frac{(E - L_S)(\log \lambda)}{a_N \lambda(\rho + \mu)} > 1 \) is satisfied from (14) if the condition of Proposition 7 holds. Thus, (55), (59), and (74) imply that the total welfare effect of increasing the R&D subsidy is positive even if \( \frac{\partial \bar{n}_S}{\partial s_R} < 0 \).

Next, we analyze the welfare effect of a marginal increase in the FDI subsidy from zero. To explore the effect on the welfare of the North, we use similar logic to the proof of the effect of the R&D subsidy. More specifically, substituting (66), (70), and (72) into (54) yields:

\[ \frac{\partial U_N}{\partial s_F} = (\log \lambda) \left\{ \frac{\mu}{p^2(\rho + \mu)} \frac{\partial \bar{E}}{\partial s_F} - \frac{1}{\rho(E - L_S)(\log \lambda)} \left[ \frac{(E - L_S)(\log \lambda)}{a_N \lambda(\rho + \mu)} - 1 \right] \frac{\partial \bar{E}}{\partial s_F} - \frac{(\lambda - 1)\bar{n}_S}{a_N \lambda \rho(\rho + \mu)} \frac{\partial \bar{E}}{\partial s_F} + \frac{\mu}{\rho(\rho + \mu)} \frac{\partial \bar{n}_S}{\partial s_F} \right\} + \frac{1}{\rho(E - L_S)} \frac{(\lambda - 1)\bar{E}_F}{\lambda}. \cdot (75) \]

Thus, (56), (60), and (75) imply that the welfare effect of a marginal increase in the FDI subsidy from zero is positive if \( \frac{\partial \bar{n}_S}{\partial s_F} \geq 0 \) and the condition in Proposition 7 holds. Further, substituting (60) into (75), we have:

\[ \frac{\partial U_N}{\partial s_F} = (\log \lambda) \left\{ \frac{\mu}{p^2(\rho + \mu)} \frac{\partial \bar{E}}{\partial s_F} - \frac{1}{\rho(E - L_S)(\log \lambda)} \left[ \frac{(E - L_S)(\log \lambda)}{a_N \lambda(\rho + \mu)} - 1 \right] \frac{\partial \bar{E}}{\partial s_F} - \frac{(\lambda - 1)\bar{n}_S}{a_N \lambda \rho(\rho + \mu)} \frac{\partial \bar{E}}{\partial s_F} + \frac{1}{\rho(E - L_S)} \frac{(\lambda - 1)\bar{E}_F}{\lambda} \right\} \cdot (75) \]
Given the condition in Proposition 7, this equation shows that the total welfare effect of a marginal increase in the FDI subsidy from zero is positive, even if \( \frac{\partial\bar{n}}{\partial s_F} < 0 \). Thus, the introduction of the FDI subsidy improves the welfare of the North if the condition in Proposition 7 holds.

Finally, we analyze the effect of the FDI subsidy on the welfare of the South. Substituting (58), (66), (69), and (72) into (54), we obtain:

\[
\frac{\partial U_S}{\partial s_F} = (\log \lambda) \left[ \frac{\mu}{\rho^2(\rho + \mu)} \frac{\partial \bar{E}}{\partial s_F} - \frac{1}{\rho(\rho + \mu)} \frac{M\lambda L_S}{E^2} \frac{\partial \bar{E}}{\partial s_F} \right] - \frac{(\lambda - 1)\bar{E} \bar{n}_F}{\rho L_S}.
\]

Further, substituting (60) into (76) yields:

\[
\frac{\partial U_S}{\partial s_F} = -\frac{M(\log \lambda)(1 - \bar{n}_F)}{a_N \rho^2(\rho + \mu)} \frac{\partial \bar{E}}{\partial s_F} - \frac{M(\log \lambda)\bar{n}_F}{a_N \rho^2(\rho + \mu)} \frac{\partial \bar{E}}{\partial s_F} - \frac{M\lambda L_S(\log \lambda)}{\rho(\rho + \mu)E^2} \frac{\partial \bar{E}}{\partial s_F} - \frac{(\lambda - 1)\bar{E} \bar{n}_F}{\rho \lambda L_S},
\]

where the second equality uses (56) and \( s_F = 0 \). Because \( \frac{\partial \bar{E}}{\partial s_F} < 0 \) from (56), equation (77) implies that the welfare effect of a marginal increase in the FDI subsidy from zero is positive if

\[
\frac{M\lambda L_S(M + \rho)(\log \lambda)}{a_N(\lambda - 1)\rho(\rho + \mu)(\rho + M\lambda)} - 1 > 0.
\]

**Appendix J  Analysis of the Costly-Imitation Model**

In this appendix, we show the derivations of the equations that characterize the steady-state equilibrium of the costly-imitation model in detail, and examine the effects of strengthening IPR protection in the South. Because the model with costly imitation and FDI is more complex than the costless-imitation model, we focus only on the steady state. Hereafter, we let variables without time “\( t \)” denote the steady-state values of the variables.

**Steady State**

Using the Euler equation yields: \( r = \rho \). Combining the equilibrium condition of R&D and the no-arbitrage condition (9) yields:

\[
\rho w a_N = \left( \frac{1 - w}{\lambda} \right) E.
\]
Combining the equilibrium condition of imitation (43) and the no-arbitrage condition (44), we get:

$$\iota_N = \left(1 - \frac{1}{\zeta}\right) \frac{E}{a_m} - \rho. \quad (79)$$

From (7) and (45), the following equality is satisfied in the equilibrium: $$v_N(t) + a_F = v_F(t) = w(t)a_N$$. Because this equality implies that $$\dot{v}_N(t) = \dot{v}_F(t)$$ for all $$t$$, substituting (9) and (10) into the relation leads to the following equality: $$\pi_F(t) - \pi_N(t) = M(t)v_F(t) + r(t)a_F$$. Thus, in the steady state, we get:

$$M = \rho \left(\frac{1 - \frac{\zeta}{\lambda}}{1 - \frac{w}{\lambda}wa_N + a_F - 1}\right). \quad (80)$$

Using the laws of motion (47) and (48) yields:

$$Mn_F = \iota_F n_N = \iota_N n_S = \iota. \quad (81)$$

Finally, from (13), (46), and (81), the labor market-equilibrium conditions in the North and the South in the steady state are given by:

$$a_N \iota + n_N E \frac{1}{\lambda} = L_N, \quad (82)$$

$$(a_F + a_m) \iota + n_F E \frac{\zeta}{\lambda} + n_S E \frac{1}{\zeta} = L_S. \quad (83)$$

Hereafter, we describe the steady-state equilibrium using only two variables, $$\iota$$ and $$w$$. First, we derive $$n_S E$$ and $$n_F E$$ as functions of $$\iota$$ and $$w$$. From (78), we can express $$E$$ as a function of $$w$$ as follows:

$$E = \frac{\rho w a_N}{1 - \frac{w}{\lambda}}. \quad (84)$$

From (79) and (84), we get: $$\frac{\iota}{E} = \left(1 - \frac{1}{\iota}\right) \frac{1}{a_m} - \frac{1 - \frac{w}{\lambda}wa_N}{\iota a_N} \equiv \phi(w, a_m)$$. Because $$n_S E = (\frac{E}{\iota_N}) \iota$$ from (81), we obtain:

$$n_S E = \frac{\iota}{\phi(w, a_m)}. \quad (85)$$

Next, from (80) and (84), we get: $$\frac{M}{E} = \left(1 - \frac{\zeta}{\lambda}\right) \frac{1}{wa_N + a_F} - \frac{1 - \frac{w}{\lambda}wa_N}{\iota} \equiv m(w)$$. Because $$n_F E = (\frac{E}{M}) \iota$$ from (81), we have:

$$n_F E = \frac{\iota}{m(w)}. \quad (86)$$
Substituting (85) and (86) into (82) and (83), we obtain the following system of two equations in two unknowns \((\iota, w)\):

\[
\iota = \frac{\rho \omega N}{\lambda - w} - \frac{L_N}{\frac{1}{\lambda} \phi(w, a_m) + \frac{1}{m(w)}} - a_N.
\]

(87)

\[
\iota = \frac{L_S}{a_F + a_m + \frac{1}{\lambda} \frac{1}{m(w)} + \frac{1}{\lambda} \phi(w, a_m)}.
\]

(88)

We refer to the curves that satisfy (87) and (88) as the LN curve and the LS curve, respectively. These are both upward sloping, as shown in Figure 4. We assume that these curves intersect once.

**Effects of Stronger IPR Protection on Innovation**

Next, we prove that an increase in \(a_m\), which is interpreted as strengthening IPR protection in the South, reduces the aggregate rate of innovation in the costly-imitation model. We let \(\Psi^N(w, a_m)\) and \(\Psi^S(w, a_m)\) denote the right-hand sides of (87) and (88). Totally differentiating (87) and (88), we get:

\[
A \begin{pmatrix} \frac{\partial \iota}{\partial a_m} \\ \frac{\partial \iota}{\partial w} \end{pmatrix} = \begin{pmatrix} \Psi^N_a \\ \Psi^S_a \end{pmatrix},
\]

(89)
where
\[
A = \begin{pmatrix}
1 - \Psi^N_w \\
1 - \Psi^S_w
\end{pmatrix},
\]

\[
\Psi^i_\psi = \frac{\partial \Psi^{i}(w, a_m)}{\partial a_m}, \quad \Psi^i_\lambda = \frac{\partial \Psi^{i}(w, a_m)}{\partial a_m} \quad \text{for } i = \{N, S\}. \quad \text{By differentiating } \Psi^N(w, a_m) \text{ and } \Psi^S(w, a_m) \text{ with respect to } w \text{ respectively, we obtain:}
\]

\[
\Psi^N_w = \frac{\partial \Psi^N(w, a_m)}{\partial w} = \frac{1}{\Lambda} \left( \frac{1}{\phi} + \frac{1}{m} \right) - a_N \left[ \frac{1}{\Lambda} \left( \frac{1}{\phi} + \frac{1}{m} \right) - a_N \right]^2 \Lambda \left( -\frac{1}{\phi^2} \phi_w - \frac{1}{m^2} m_w \right)
\]

\[
= \frac{E_\Lambda - L_N}{\left[ \frac{1}{\Lambda} \left( \frac{1}{\phi} + \frac{1}{m} \right) - a_N \right]^2 \Lambda \left( \frac{1}{\phi} \phi + \frac{1}{\phi} m \right)}
\]

\[
\Psi^S_w = \frac{\partial \Psi^S(w, a_m)}{\partial w} = -\frac{L_S}{a_m + a_F + \frac{1}{\Lambda} m + \frac{1}{\Lambda} \phi} \left( \frac{\phi}{\phi} \left( -\frac{1}{m^2} m_w \right) + \frac{1}{\phi} \left( -\frac{1}{\phi^2} \phi_w \right) \right)
\]

\[
= \frac{L_S}{a_m + a_F + \frac{1}{\Lambda} m + \frac{1}{\Lambda} \phi} \left( \frac{\phi}{\phi} \hat{m} + \frac{\phi}{\phi} \phi \right),
\]

where \( \phi_w = \frac{\partial \phi}{\partial w}, \quad m_w = \frac{\partial m}{\partial w}, \quad \hat{\phi} = \frac{\phi_w}{\phi^2} \) and \( \hat{m} = \frac{m_w}{m^2} \). By differentiating \( \Psi^N(w, a_m) \) and \( \Psi^S(w, a_m) \) with respect to \( a_m \) respectively, we obtain:

\[
\Psi^N_a = \frac{\partial \Psi^N(w, a_m)}{\partial a_m} = -\frac{E_\Lambda - L_N}{\left[ \frac{1}{\Lambda} \left( \frac{1}{\phi} + \frac{1}{m} \right) - a_N \right]^2 \Lambda \left( -\frac{1}{\phi^2} \phi_a \right)}
\]

\[
= -\frac{E_\Lambda - L_N}{\left[ \frac{1}{\Lambda} \left( \frac{1}{\phi} + \frac{1}{m} \right) - a_N \right]^2 \Lambda \hat{\phi}_a,}
\]

\[
\Psi^S_a = \frac{\partial \Psi^S(w, a_m)}{\partial a_m} = -\frac{L_S}{a_m + a_F + \frac{1}{\Lambda} m + \frac{1}{\Lambda} \phi} \left( \frac{1}{\phi} \phi_a \right)
\]

\[
= -\frac{L_S}{a_m + a_F + \frac{1}{\Lambda} m + \frac{1}{\Lambda} \phi} \left( \frac{1}{\phi} \hat{\phi}_a \right),
\]

where \( \phi_a = \frac{\partial \phi}{\partial a} \) and \( \hat{\phi}_a = \frac{-\phi_a}{\phi^2} \).

Suppose the LN and LS curves intersect only once. Then the LN curve intersects the LS curve from below, as in Figure 4. Thus \( \Psi^N_w > \Psi^S_w \), and \( \det A = -\Psi^S_w + \Psi^N_w > 0 \). Applying
Cremer’s rule to (89), we get:

\[
\frac{\partial \nu}{\partial a_m} = \frac{1}{\det A} \left( -\Psi_w^N \Psi_w^S + \Psi_w^S \Psi_w^N \right).
\]

Substituting \(\Psi^i_w\) and \(\Psi^a_w\) into \((-\Psi_w^N \Psi_w^S + \Psi_w^S \Psi_w^N)\) yields:

\[
-
\Psi_w^N \Psi_w^S + \Psi_w^S \Psi_w^N = \left[ \frac{E}{N} - L_N \right] \left( 1 + \frac{1}{m} \right)^2 \left[ a_n + a_F + \frac{1}{m} \right] \frac{1}{\lambda} F,
\]

where

\[
F \equiv \hat{\phi}_a \left( \frac{\xi}{\lambda} \hat{m} + \frac{1}{\xi} \hat{\phi} \right) - \left( 1 + \frac{1}{m} \hat{\phi}_a \right) \left[ \left( \hat{m} + \hat{\phi} \right) + \frac{1}{\Psi^N} \frac{\partial E}{\partial w} \right] = \hat{\nu}
\]

From the definitions of \(\phi(w, a_m)\) and \(m(w)\) above equations (85) and (86), \(\hat{\phi}_a \equiv -\frac{\phi_a}{\phi'} > 0\), \(\hat{\phi} \equiv \frac{\phi_w}{\phi'} > 0\) and \(\hat{m} \equiv m_m > 0\). In addition, from (84), \(\frac{\partial E}{\partial w} > 0\). Therefore, if \(\frac{\xi}{\lambda} - \frac{1}{\xi} < 0\), \(F < 0\) and thus \(\frac{\partial \nu}{\partial a_m} < 0\). This case is depicted in Figure 4.

**Welfare Analysis**

Finally, we conduct the welfare analysis in the steady state by using the results of the positive analysis. In the same manner as in Section 6, we can derive the instantaneous utility. Only the part of welfare that depends on the prices changes to

\[
\int_0^1 \log p(j, t) dj = n_S(t) \log \zeta + (1 - n_S(t)) \log \lambda = -n_S(t)(\log \lambda - \log \zeta) + \log \lambda.
\]

We can rewrite the instantaneous utility as:

\[
\log u_i(t) = (\log \lambda) \int_0^t u(\tau) d\tau + \log E_i(t) + n_S(t)(\log \lambda - \log \zeta) - \log \lambda.
\]

Therefore, we obtain the lifetime utility in the steady state as follows:

\[
U_i = \frac{1}{\rho} \left[ \frac{\log \lambda}{\rho} t + \log E_i + n_S(\log \lambda - \log \zeta) - \log \lambda \right].
\]

We assume that Southern households possess only the stocks of the imitation firms, which the Northern households cannot. This assumption implies that the budget constraint of the Southern households is

\[
\frac{E_S L_S}{\rho} = \frac{w_S L_S}{\rho} + v_S n_S.
\]

Using (43) and \(w_S = 1\), we can rewrite the budget
constraint as \( E_S = 1 + \rho \frac{a_m n_S}{L_S} \). By differentiating the lifetime utility function with respect to \( a_m \), we obtain the change of welfare by the marginal increase in \( a_m \) as follows:

\[
\frac{\partial U_S}{\partial a_m} = \frac{1}{\rho} \left[ \frac{\log \lambda}{\rho} \frac{\partial l}{\partial a_m} + \frac{\rho L_S}{1 + \frac{L_S}{L_S} a_m n_S} \left( n_S + a_m \frac{\partial n_S}{\partial a_m} \right) \right]
\]

\[
+ (\log \lambda - \log \zeta) \frac{\partial n_S}{\partial a_m}
\]

This equation shows that the total welfare effect of strengthening IPR protection (an increase in \( a_m \)) can be decomposed into the following three parts. First, strengthening IPR protection reduces aggregate innovation and thus reduces welfare. We refer to this welfare effect as the *innovation-impeding effect*. In contrast with the costless-imitation case, strengthening IPR protection through increasing the imitation cost reduces aggregate innovation, at least in the steady state. Second, strengthening IPR protection affects nominal spending and welfare. We refer to this as the *nominal-spending effect*. An increase in the imitation cost increases the stock value of an imitation firm; however, the sign of \( \frac{\partial n_S}{\partial a_m} \) is ambiguous and thus the sign of the nominal-spending effect is indeterminate. Finally, strengthening IPR protection affects the measure of type-S industries, \( n_S(t) \). An increase in the measure of type-S industries, where the price is lower because type-S goods are supplied by Southern firms with the lower marginal cost, increases welfare. We refer to this welfare effect as the *competition effect*. The sign of \( \frac{\partial n_S}{\partial a_m} \) is ambiguous and thus the sign of the competition effect is indeterminate.

Our numerical results indicate that strengthening IPR protection increases the measure of type-S industries, \( n_S \); therefore, both the nominal-spending effect and the competition effect are positive. The numerical results show that with higher innovation cost, strengthening IPR protection increases the welfare, whereas it reduces the welfare with lower innovation cost. For instance, if we set the parameter value of the innovation cost \( a_N \) at 123.5, which is the same as in the numerical examples of the text, the negative innovation-impeding effect outweighs those two positive effects, so that the welfare of the South increases with \( a_m \), as shown in the upper panels of Figure 5. However, if we set \( a_N \) at the lower value, 50, the negative innovation-impeding effect overwhelms those two positive effects, and thus the welfare of the
Figure 5: The effects of an increase in imitation cost

The horizontal axes represent $a_m$. The vertical axes represent aggregate innovation $\iota$, measure of imitated sectors $n_S$, and the utility of the Southern consumer $U_S$, respectively. The parameter values are $\lambda = 4$, $\zeta = 1.6$, $\rho = 0.05$, $L_N = 3$, $L_S = 6$, and $a_F = 5$. The upper three panels are the numerical results as $a_N = 123.5$, and the lower three panels are those as $a_N = 50$.

South decreases with $a_m$, as shown in the lower panels of Figure 5. We interpret the result for the welfare effect somewhat differently from that obtained in the costless-imitation model. In the basic model where imitation is costless, strengthening IPR protection in the South increases welfare if the positive welfare effect through promoting innovation outweighs the negative welfare effects. On the other hand, in the model where imitation is costly, strengthening IPR protection impedes innovation. However, if the other positive welfare effects overwhelm the negative effect on innovation, strengthening IPR protection in the South increases the welfare of Southern consumers.
Appendix K  Notes on some results in the text

In this appendix, calculations of some results in the text are explained in more detail.

Derivation of the result of (22)

In this note, we derive the result of (22). By differentiating (18) with respect to $M$, we have

$$\frac{\partial \bar{n}_N}{\partial M} = \frac{a_N \lambda^2 (\bar{E} - L_S) [a_N M \lambda^2 - (\lambda - 1) \bar{E}] [a_N M \lambda^2 (\bar{E} - L_S) - \lambda L_N \bar{E} (\lambda - 1)]}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}]^2 E^2} \left[ \frac{[a_N M \lambda^2 - (\lambda - 1) \bar{E}] [a_N M \lambda^2 (\bar{E} - L_S) - \lambda L_N \bar{E} (\lambda - 1)]}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}]^2 E^2} \partial \bar{E} \partial M \right]$$

$$= \frac{-a_N \lambda^2 (\bar{E} - L_S) (\lambda - 1) \bar{E}^2 + a_N \lambda^3 (\lambda - 1) L_N \bar{E}^2}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}]^2 E^2} \left[ \frac{a_N M \lambda^2 - (\lambda - 1) \bar{E}}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}]^2 E^2} \frac{a_N M \lambda^2 (\bar{E} - L_S) - \lambda L_N \bar{E} (\lambda - 1)}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}]^2 E^2} \partial \bar{E} \partial M \right]$$

$$= \frac{a_N \lambda^2 (\lambda - 1) [L_S + \lambda L_N - \bar{E}]}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}]^2 E^2} \left[ \frac{a_N M \lambda^2 L_S}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}]^2 E^2} + \frac{(\lambda - 1) \bar{E} [a_N M \lambda^2 (\bar{E} - L_S) - \lambda L_N \bar{E} (\lambda - 1)]}{[a_N M \lambda^2 - (\lambda - 1) \bar{E}]^2 E^2} \partial \bar{E} \partial M \right]$$

$$= \frac{(\lambda - 1) \bar{n}_F}{\mu} + \left[ \frac{M \lambda L_S}{\mu E^2} + \frac{(\lambda - 1) \bar{n}_N}{a_N \lambda \mu} \right] \frac{\partial \bar{E}}{\partial M}$$

$$= \frac{1}{\mu} \left\{ (\lambda - 1) \bar{n}_F + \left[ \frac{M \lambda L_S}{E^2} + \frac{(\lambda - 1) \bar{n}_N}{a_N \lambda} \right] \frac{\partial \bar{E}}{\partial M} \right\} > 0,$$

where the fourth equality uses (17), (18), and the definition of $\mu$, and the inequality uses (22).
Derivation of the result of (23)

In this note, we derive the result of (23). By differentiating (17) with respect to \( M \), we have

\[
\frac{\partial \bar{n}_F}{\partial M} = - \frac{\lambda \bar{E}}{a_N M^2 - (\lambda - 1)^2} + \left\{ \frac{\lambda}{a_N M^2 - (\lambda - 1)^2} \right\} \frac{\partial \bar{E}}{\partial M}
\]

where the second equality uses (17) and the definition of \( \mu \), and the inequality uses (22).

Derivation of the result of (25)

In this note, we derive the result of (25). Using \( \frac{\partial \tilde{t}(t)}{\partial M} \bigg|_{n_S(0)=\bar{n}_S} \) derived in Appendix C, we obtain

\[
\int_0^\infty e^{-\rho t} (\log \lambda) \frac{\partial}{\partial M} \left( \int_0^t \tilde{t}(\tau) d\tau \right) dt
\]

where the inequality uses (22) and the result of Proposition 3.
Derivation of the result of (40)

In this note, we derive the result of (40). By differentiating (16) with respect to $M$, we have

\[
\frac{\partial \bar{n}_S}{\partial M} = \frac{\partial \bar{n}_S}{\partial \bar{E}} \frac{E}{2} a_N \mu \frac{\lambda^2}{\lambda + L_N - \bar{E}}
\]

\[
+ \left\{ \frac{2\bar{E} - \lambda (L_N + L_S)}{\lambda} \frac{\lambda^2}{\lambda + L_N - \bar{E}} \left[ a_N \mu \frac{\lambda^2}{\lambda + L_N - \bar{E}} \right] \right\} \frac{\partial \bar{E}}{\partial M}
\]

\[
= \frac{\partial \bar{n}_S}{\partial \bar{E}} \frac{E}{2} a_N \mu \frac{\lambda^2}{\lambda + L_N - \bar{E}}
\]

\[
+ \left\{ \frac{2\bar{E} - \lambda (L_N + L_S)}{\lambda} \frac{\lambda^2}{\lambda + L_N - \bar{E}} \left[ a_N \mu \frac{\lambda^2}{\lambda + L_N - \bar{E}} \right] \right\} \frac{\partial \bar{E}}{\partial M}
\]

where the fourth equality uses (16), (17), and the definition of $\mu$. 

\[
= \frac{\bar{n}_F}{\mu} + \frac{1}{\lambda} \left[ \frac{\mu}{\lambda^2} \frac{\lambda^2}{\lambda + L_N - \bar{E}} \right] \frac{\partial \bar{E}}{\partial M}
\]
References


