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Abstract

In this paper, we construct an interregional trade model that has endogenous fertility rates in the manner of Helpman and Krugman (1985). The presented model shows that fertility rates in a large region become lower than those in a small region because of the agglomeration of manufacturing firms in the former. The agglomeration of firms in a region lowers the relative price of manufactured goods to child rearing costs, which raises the fertility rates.

We also find that a decline in transportation costs results in the agglomeration of manufacturing firms, which lowers fertility rates in both large and small regions. Finally, we extend our two-region model to a multi-region model and find that the number of manufacturing firms in larger regions is always greater than that in smaller regions, meaning that fertility rates in the former are always lower than those in the latter.

JEL Classification: J13, R10
Keywords: agglomeration, fertility rates, transportation costs, consumerism

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1 Introduction

Fertility rates have decreased in developed countries to the point that a low fertility rate is characteristic of a modern developed economy. For example, from 1960 to 2000, the total fertility rate decreased from 2.00 to 1.36 in Japan, from 2.72 to 1.64 in the UK, from 2.37 to 1.36 in Germany, from 2.73 to 1.88 in France, from 2.20 to 1.54 in Sweden, and from 3.64 to 2.06 in the US (Cabinet Office, Government of Japan (2004)).

Further, fertility rates in regions that have a high population density are lower than those in low population density regions. For example, in Japan, the prefectural-level total fertility rates in 2000 were only 1.07 in Tokyo, 1.28 in Kyoto, and 1.31 in Osaka, which are areas that have large populations. By comparison, the total fertility rates were 1.62 in Tottori, 1.65 in Fukushima, and 1.67 in Saga, which are areas that have low populations and low population densities (National Vital Statistics Report (2010)). We can also observe a similar trend in the US: in 2008, total fertility rates in populous states such as Massachusetts, New York, Pennsylvania, and Connecticut were less than 2, whereas those in the less densely populated states of Utah, Idaho, Alaska, and Texas were well above 2 (National Vital Statistics Report (2010)). [1]

The similar trend can be observed in the data which involves the difference of fertility rates across countries. Simon and Tamura (2009) find a strong negative relationship between population density and fertility rates in European countries and Canada, too. Figure 1 plots the relationship between fertility rates and population density in EU countries in 2010 based on data from Eurostat. The straight line in the figure is the regression line. This figure highlights that fertility rates are lower in the more density populated EU countries. Based on this trend, the present paper examines the driving forces behind these facts.[2]

Some researchers have stated that in modern industrialized countries, parents prefer to consume goods rather than bear children. For instance, Lutz (1996) points out that "consumerism" is the basis of the decline in fertility rates in modern developed countries:

Commentators often mention the increase in consumerism as a basic underlying cause for the recent fertility decline. The argument is that people would rather invest in pleasures for themselves than in children; they would rather buy a new car than have another child; they would rather spend their time watching TV than changing diapers. (p. 273)

In this paper, we propose a model in which "consumerism"—proxied by the agglomeration of manufacturing firms in a large region and a decline in transportation costs of manufactured goods—lowers fertility rates. We find that (i) fertility rates in the large region, which houses more manufacturing firms, become lower than those in the small region and (ii) a decline in transportation

[1] Sato (2007) points out that high land rents and congestion diseconomies in highly populated regions lower fertility rates.

[2] Generally, there is less international migration than interregional migration. The facts presented here show that fertility rates are higher in high population density regions irrespective of whether there is migration among regions.

costs results in the agglomeration of manufacturing firms in the large region and subsequently lowers the fertility rates in both the small and large regions. [3]

In the presented analysis, we assume that parents receive utility from both the consumption of differentiated goods and their number of children.[4] Parents allocate their fixed time to working or rearing children. Thus, there is a trade-off between nominal income and children. In our model, the agglomeration of manufacturing firms in a large region lowers the relative price of the differentiated goods in this region, since consumers can buy a variety of manufactured goods without incurring transportation costs. Thus, the agglomeration of manufacturing firms induces parents to extend their expenditure shares for differentiated goods. Through this mechanism, parents in the large region have fewer children, which explains the lower fertility rates seen in large regions compared with small ones.

In addition, the decline in transportation costs actually lowers fertility rates in all regions in our model. A decline in transportation costs lowers the relative price of differentiated goods in all regions, since consumers can purchase manufactured goods that are produced in other regions at lower prices, allowing parents to extend their expenditure shares for differentiated goods and resulting in a decrease in their number of children. Interregional transportation costs tend to be relatively low in developed countries because of the existence of transportation infrastructures such as highways, railroads, and airports as well as innovative transportation technology. Such a decline in transportation costs lowers fertility rates.

Theoretically, this paper presents a tractable interregional trade model in the mold of the models that have endogenous fertility rates proposed by Krugman (1980), Helpman and Krugman (1985), and Fujita, Krugman, and Venables (1999). These authors present Dixit–Stiglitz (1977)-type monopolistic competition models in which the interregional trade of differentiated goods incurs transportation costs and differentiated goods are produced by monopolistically competitive firms whose production functions are under increasing returns to scale. They show that manufacturing firms agglomerate in large regions, leading to a decline in transportation costs. The model proposed herein also suggests that manufacturing firms agglomerate in the large region, lowering fertility rates in this region.

Many studies have also presented models that have endogenous fertility rates. For example, Becker, Murphy, and Tamura (1991) present a model in which fertility is closely related to the accumulation of human capital. In their model, parents obtain utility not only from consumption but also from the quantity and quality of their children. Parents allocate a fixed amount of time to working, parenting, and educating their children. Hence, there is a quantity/quality trade-off for parents based on their optimum number of children and their qualities. Authors such as Galor and Weil (2000), Tamura (2002), Kalemli-Ozcan,

[3] In this paper, agents that engage in "consumerism" spend a larger share of their incomes (time) on consumption than they do on rearing children.

[4] Becker, Murphy, and Tamura (1991), Eckstein and Wolpin (1985), and Galor and Weil (1996, 2000), among others, assume that parents receive utility from their number of children.

Ryder, and Weil (2000), and Kalemli-Ozcan (2002, 2003) all follow Becker, Murphy, and Tamura (1991) by constructing models with a quantity/quality trade-off.

Other types of endogenous fertility rate models have also been proposed in the literature. Sato and Yamamoto (2005), for example, construct a model in which urbanization induces an agglomeration economy and congestion diseconomies and fertility rates decrease with urbanization. In their model, the agglomeration economy raises parents' incomes, which increases fertility rates owing to the income effect and reduces fertility rates owing to the substitution effect, whereas congestion diseconomies lower parents' incomes, which reduces fertility rates. Sato and Yamamoto (2005) show that the substitution effect and congestion diseconomies overcome the income effect of an agglomeration economy and urbanization, concluding that economic growth reduces fertility rates. Similar to the present paper, Sato (2007) also examines regional variations in fertility rates but overlooks the decline of fertility rates in developed countries, while Maruyama and Yamamoto (2010) focus on the relationship between fertility rates and economic growth, whereas this paper is interested in the regional variation of fertility rates and the agglomeration of manufacturing firms.

The remainder of this paper is organized as follows. Section 2 presents the model. In Section 3, we analyze the model and present the results. Section 4 extends our two-region model to a multi-region model and shows that manufacturing firms agglomerate in larger regions and that the fertility rates in these regions subsequently drop. Section 5 concludes.

2 The model

There are two regions, 1 and 2. Variables that refer to region 1 have the subscript 1 and those that refer to region 2 have the subscript 2. Each region is endowed with a fixed amount of labor, L_1 and L_2, respectively, while region 1 is larger than region 2: $L_1 > L_2$. [5] We assume that agents in both regions obtain utility from the consumption of homogeneous agricultural goods and differentiated manufactured goods as well as their number of children. Labor can be used to produce agricultural goods and differentiated manufactured goods, and/or to rear their children. While labor can be mobile between sectors in the same region, it cannot be mobile between different regions.

The utility function of the agent in region i ($i = 1, 2$) is given by

$$U_i = A_i + \frac{1}{\mu}\left[C_i^{\alpha} m_i^{1-\alpha}\right]^{\mu}, \tag{1}$$

[5] In our model, there is no interregional migration. As discussed in the Introduction, fertility rates are higher in high population density regions irrespective of whether there is migration among areas. Because our focus in this paper is to examine the mechanism behind this trend, we assume no interregional migration for analytical simplicity.

where

$$C_i = \left[\int_0^{n_i} x_i^i(j)^\rho dj + \int_0^{n_{i'}} x_i^{i'}(j')^\rho dj'\right]^{\frac{1}{\rho}}, \ 0 < \rho < 1, \ \rho > \alpha\mu, \ i, i' \in \{1, 2\}, \ i \neq i'. \tag{2}$$

Here, A_i is the consumption of agricultural goods in region i, C_i is the consumption of manufactured goods in region i, and μ is a positive parameter. $x_k^l(j)$ denotes the consumption of manufactured goods variety j in region k produced in region l. n_i is the number of varieties produced by a firm in region i. [6] Here, $\frac{1}{1-\rho}$ represents the elasticity of substitution among differentiated goods. We assume $\rho > \alpha\mu$ to ensure the concavity of preferences over $x_k^l(j)$. Following Becker (1965) and others, we assume that if parents have a child, they use time to rear him or her. Their budget constraint thus becomes

$$w_i(1 - \gamma m_i^\phi) = A_i + \int_0^{n_i} p_i^i(j)x_i(j)dj + \int_0^{n_{i'}} p_i^{i'}(j')x_i^{i'}(j')dj', \tag{3}$$

where $p_k^l(j)$ denotes the price of manufactured goods variety j in region k produced in region l and w_i denotes the wage rate in region i. γm_i^ϕ is the cost of rearing children. We assume that the per capita cost of rearing children decreases with the number of children and that $\mu(1-\alpha) < \phi < \frac{\mu(1-\alpha)}{1-\alpha\mu}$. The condition $\mu(1-\alpha) < \phi < \frac{\mu(1-\alpha)}{1-\alpha\mu}$ ensures that children are substitutional to differentiated goods. We take homogeneous agricultural goods as the numeraire.

Then, we can obtain the following demand functions:

$$m_i = \left(\frac{w_i\gamma\phi}{1-\alpha}\right)^{\frac{1-\alpha\mu}{B}} \alpha^{-\frac{\alpha\mu}{B}} P_i^{\frac{\alpha\mu}{B}}, \tag{4}$$

$$P_i = \left(\int_0^{n_1+n_2} p_i(j)^{\frac{\rho}{\rho-1}} dj\right)^{\frac{\rho-1}{\rho}}, \tag{5}$$

$$x_i^l(j) = \left(\frac{w_i\gamma\phi}{1-\alpha}\right)^{\frac{\mu(1-\alpha)}{B}} \alpha^{\frac{\mu(1-\alpha)-\phi}{B}} p_i^l(j)^{\frac{1}{\rho-1}} P_i^{\frac{X}{B(1-\rho)}}, \tag{6}$$

where P_i is the "price index" in region i, while $B = \mu(1-\alpha) - \phi(1-\alpha\mu) > 0$ and $X = \rho\mu(1-\alpha) - \phi(\rho - \alpha\mu) > 0$ because $\mu(1-\alpha) < \phi < \frac{\mu(1-\alpha)}{1-\alpha\mu}$.

Next, we describe the production structure of the agricultural sector. The agricultural goods market is perfectly competitive. We assume that in both regions, one unit of agricultural goods is produced with one unit of labor and that the interregional trade of homogeneous goods incurs no transportation costs. Therefore, the equilibrium wages in the two regions are both one: $w_1 = w_2 = 1$.

In the manufacturing sector, firms operate under Dixit–Stiglitz (1977)-type monopolistic competition. Each manufacturing firm produces differentiated

[6] The condition $\phi > \mu(1-\alpha)$ ensures the second-order condition of the consumer problem.

goods, and each variety is produced by one firm. To start production activities, a firm in region j is required to pay a fixed input requirement that comprises f units of labor. Moreover, a firm uses one unit of labor in its region as the marginal input to produce one unit of manufactured goods. Potential firms can freely enter production activities as long as the pure profits are positive and they can choose to locate in a region where profits are higher. Under this production structure, each manufacturing firm sets the following constant markup (mill) price:

$$p_1^1 = p_2^2 = \frac{1}{\rho}. \tag{7}$$

The interregional trade of manufactured goods incurs "iceberg"-type transportation costs. If a firm in one region sends one unit of its good to the other region, it must dispatch T units of the good. Hence, $T-1>0$ represents transportation costs. Thus, the price of imported manufactured goods in region i becomes $Tp_i^{i'}$ and $i \neq i'$. The price index in region i can therefore be written as

$$P_i = \frac{1}{\rho} \cdot (n_i + n_{i'}\tau)^{\frac{\rho-1}{\rho}}\ ,\ i, i' \in \{1,2\}\ ,\ i \neq i', \tag{8}$$

where $\tau \equiv T^{\frac{\rho}{\rho-1}}$ and τ represent the freeness of trade. $\tau = 0$ describes the case of autarky, whereas $\tau = 1$ implies free trade. From (6) and (7), the profits of firms in regions 1 and 2 can be expressed as follows:

$$\pi_i = (1-\rho)\rho^{\frac{\rho}{1-\rho}}\alpha^{\frac{\mu(1-\alpha)-\phi}{B}}(\frac{\gamma\phi}{1-\alpha})^{\frac{\mu(1-\alpha)}{B}}\left[L_i P_i^{\frac{X}{B(1-\rho)}} + \tau L_{i'} P_{i'}^{\frac{X}{B(1-\rho)}}\right] - f,\ i, i' \in \{1,2\}\,. \tag{9}$$

3 Equilibrium

In this section, we study the equilibrium that the manufacturing firms locate in both regions, that is

$$\pi_1 = \pi_2. \tag{10}$$

From (9) and (10), the relative price level is given by

$$\frac{P_1}{P_2} = (\frac{L_2}{L_1})^{\frac{B(1-\rho)}{X}}. \tag{11}$$

In this model, because region 1 is larger than region 2, the price level in region 1 is lower than that in region 2. From (4) and (11), we can thus obtain the relative fertility rates as follows:

$$\frac{m_1}{m_2} = (\frac{L_2}{L_1})^{\frac{\alpha\mu(1-\rho)}{X}} = l^{\frac{-\alpha\mu(1-\rho)}{X}}. \tag{12}$$

From $L_1 > L_2$, the fertility rates in region 1 are lower than those in region 2. To summarize the results of (11) and (12), we obtain the following proposition.

Proposition 1 *The larger region has a lower price level and lower fertility rates.*

We can explain this proposition intuitively. In the larger region, demand is higher and thus manufacturing firms agglomerate there. Thus, the price level in region 1 is lower than that in region 2 and larger regions consume more manufactured goods. In this model, because manufactured goods and bearing children are substitutes, parents that live in larger regions bear fewer children.

By substituting (11) into (8), the relationship between n_1 and n_2 is given by

$$n_1 = \frac{l^{\frac{\rho B}{X}} - \tau}{1 - \tau l^{\frac{\rho B}{X}}} n_2, \tag{13}$$

where l denotes the relative population size, that is $l \equiv L_1/L_2 > 1$. From (13), the number of firms locating in region 1 is larger than the number of firms locating in region 2. [7] Then, firms agglomerate in the larger region. Eq. (13) shows that when $n_1 > 0$ and $n_2 > 0$, $\tau < l^{\frac{-\rho B}{X}} \equiv \bar{\tau}$. When $\tau > \bar{\tau}$, all manufacturing firms locate in region 1. From (13) and the free-entry condition of $\pi_1 = 0$, the number of manufacturing firms locating in region 2 is given by [8]

$$n_2 = \frac{1 - \tau l^{\frac{\rho B}{X}}}{1 - \tau} (1 + \tau)^{\frac{-\alpha\mu\phi(1-\rho)}{X}} \Psi^{-\frac{\rho B}{X}}, \tag{15}$$

where

$$\Psi = f(1-\rho)^{-1} \rho^{\frac{\alpha\mu\phi}{B}} \alpha^{\frac{\phi - \mu(1-\alpha)}{B}} \left(\frac{\gamma\phi}{1-\alpha}\right)^{\frac{-\mu(1-\alpha)}{B}} L_2^{-1}. \tag{16}$$

Then, by differentiating (15) and (13) with respect to τ, we can obtain the following equations:

$$\frac{\partial n_2}{\partial \tau} = -n_2 \left[\frac{l^{\frac{\rho B}{X}} - 1}{(1-\tau)(1 - \tau l^{\frac{\rho B}{X}})} + \frac{\alpha\mu\phi(1-\rho)}{(1+\tau)X} \right] \Psi^{-\frac{\rho B}{X}} < 0, \tag{17}$$

$$\frac{\partial n_1}{\partial \tau} = \frac{n_2}{(1-\tau^2)(1 - \tau l^{\frac{\rho B}{X}})X} F(\tau), \tag{18}$$

where

$$F(\tau) = (1+\tau)(l^{\frac{\rho B}{X}} - 1)X - \alpha\mu\phi(1-\tau)(1-\rho)(l^{\frac{\rho B}{X}} - \tau). \tag{19}$$

Thus, we can obtain the following proposition (see the Appendix for the proof).

Proposition 2 *Suppose $\tau < \bar{\tau}$ is satisfied.*

1) A decline in transportation costs decreases the number of manufacturing firms locating in region 2.

[7] By subtracting the denominator of (13) from the numerator of (13), we can show $\frac{n_1}{n_2} > 1$.

$$l^{\frac{\rho B}{X}} - \tau - 1 + \tau l^{\frac{\rho B}{X}} = (1+\tau)(l^{\frac{\rho B}{X}} - 1) > 0, \tag{14}$$

because $l > 1$.

[8] See the Appendix for the proof.

2) When $l > (\frac{X}{X-\alpha\mu\phi(1-\rho)})^{\frac{X}{\rho B}}$, a decline in transportation costs increases the number of manufacturing firms locating in region 1*. When* $1 < l < (\frac{X}{X-\alpha\mu\phi(1-\rho)})^{\frac{X}{\rho B}}$ *holds, a decline in transportation costs decreases the number of manufacturing firms locating in region* 1 *in* $0 < \tau < \tau^*$*, where* $F(\tau^*) = 0$*, while a decline in transportation costs increases the number of manufacturing firms locating in region* 1 *in* $\tau^* < \tau < \bar{\tau}$*.*

3) A decline in transportation costs decreases the total number of manufacturing firms.

In this model, in which we assume that region 1 is larger than region 2, the demand level of region 1 is larger than that of region 2, meaning that some manufacturing firms move their production plants to the larger region. The subsequent decline in transportation costs lowers the number of firms locating in region 2 and increases the number of manufacturing firms locating in region 1. By contrast, it could be considered that the decline in transportation costs facilitates exports to the manufacturing firms located in both regions. Consequently, the manufactured goods market becomes competitive and the profits generated by manufacturing firms drop. Therefore, the number of manufacturing firms locating in region 1 decreases. We also investigate how this decline in transportation costs affects the price indexes and fertility rates in both regions. Thus, we can obtain the following proposition (see the Appendix for the proof).

Proposition 3 *Suppose* $\tau < \bar{\tau}$ *is satisfied. A decline in transportation costs decreases the price indexes and fertility rates in both regions.*

Furthermore, such a decline in the interregional transportation costs of manufactured goods lowers the price levels in both regions. Because manufactured goods and bearing children are substitutes, a decrease in the price of manufactured goods reduces the number of children.

In the next step, we examine the case that $\tau > \bar{\tau}$. When $\tau > \bar{\tau}$, all manufacturing firms agglomerate in region 1 and $n_2 = 0$. Eqs. (**??**) and (4) show that in this case

$$\frac{m_1}{m_2} = \tau^{\frac{\alpha\mu(1-\rho)}{\rho X}}. \tag{20}$$

When $\tau > \bar{\tau}$, $\tau^{\frac{\alpha\mu(1-\rho)}{\rho X}} > (\frac{L_2}{L_1})^{\frac{\alpha\mu(1-\rho)}{X}} = l^{\frac{-\alpha\mu(1-\rho)}{X}}$. Thus, the full agglomeration of manufacturing firms (i.e., when all manufacturing firms agglomerate in the large region) reduces the difference in the fertility rates of the two regions.

Proposition 4 *When* $\tau > \bar{\tau}$*, the full agglomeration of manufacturing firms occurs, reducing the difference in fertility rates between the two regions with a decline in transportation costs.*

This proposition states that when transportation costs lower, full agglomeration is observed. If full agglomeration occurs, the difference in the fertility rates between the large and small regions begins to reduce. Moreover, another decline in transportation costs reduces the difference between these fertility rates further, since $\partial\tau^{\frac{\alpha\mu(1-\rho)}{\rho X}}/\tau > 0$.

4 Multi-region case

In this section, we extend the presented model by assuming that there are $N > 2$ regions. Region i hosts an exogenously given mass of L_i consumers and $L_1 > L_2 > ... > L_N$ holds. In addition, the preferences of each region are identical and the utility function is given by (1). Then, although demand for manufactured goods and for bearing children is the same as that detailed in the previous section, the price index in region i is given by

$$P_i = \left[\sum_{k=1}^{N} \int_0^{n_k} p_i^k(j)^{\frac{\rho}{\rho-1}} dj\right]^{\frac{\rho-1}{\rho}}. \tag{21}$$

The production structures of the agricultural goods and manufactured goods sectors remain the same as before. Thus, the wage rate in each region becomes unity, because the interregional trade of agricultural goods incurs no costs and one unit of agricultural goods is produced with one unit of labor in each region. For the manufactured goods sector, the interregional transportation costs of manufactured goods remain the same for analytical simplicity. Therefore, the price of manufactured goods becomes $p_i^i = 1/\rho$ and $p_k^i = T/\rho$, while $i \neq k$. Hence, the profits of the manufacturing firms locating in region i are given by

$$\pi_i = (1-\rho)\rho^{\frac{\rho}{1-\rho}} \alpha^{\frac{\mu(1-\alpha)-\phi}{B}} (\frac{\gamma\phi}{1-\alpha})^{\frac{\mu(1-\alpha)}{B}} \left[(1-\tau)L_i P_i^{\frac{X}{B(1-\rho)}} + \tau \sum_{k=1}^{N} L_k P_k^{\frac{X}{B(1-\rho)}}\right] - f. \tag{22}$$

The price index in region i becomes

$$P_i = \frac{1}{\rho}\left[(1-\tau)n_i + \tau \sum_{k=1}^{N} n_k\right]^{\frac{\rho-1}{\rho}}. \tag{23}$$

Now, we examine the equilibria of the manufacturing firms located in each region. Thus, $\pi_1 = \pi_2 = ... = \pi_N$ holds. From (22) and (23), the relative price index between region i and region k is given by

$$\frac{P_i}{P_k} = (\frac{L_k}{L_i})^{\frac{B(1-\rho)}{X}}. \tag{24}$$

Thus, a larger region has a lower price index, and from $L_1 > L_2 > ... > L_N$, $P_1 < P_2 < ... < P_N$ holds. Because the larger region attracts more manufacturing firms, the price level in that region is lower. By using the above equation, we can obtain the relative fertility rates of regions i and k as follows:

$$\frac{m_i}{m_k} = (\frac{L_k}{L_i})^{\frac{\alpha\mu(1-\rho)}{X}}. \tag{25}$$

This equation shows that a larger region has lower fertility rates, and from $L_1 > L_2 > ... > L_N$, $m_1 < m_2 < ... < m_N$ holds. Because the price level in the larger region is lower, consumers in that region consume more manufactured goods compared with bearing children. Hence, by summarizing the results of the multi-region case, the following proposition can be obtained:

Proposition 5 *In the multi-region case, both the price level and fertility rates in the larger region reduce.*

This proposition states that fertility rates in large regions are lower than those in small regions, which is consistent with the findings of previous studies. Manufacturing firms agglomerate in large regions, which lowers their price indexes. Consequently, the relative price of working time, which provides agents with their nominal incomes, to bearing children is higher in large regions than it is small regions. Therefore, agents in large regions have fewer children than those in small regions. Hence, the number of firms in region i can be described as[9]

$$n_i = \frac{1}{1-\tau}\left(\frac{1-\tau+\tau N}{\eta}\right)^{\frac{B\rho}{X}}\left(\rho L_i^{\frac{B\rho}{X}} - \frac{\rho\tau}{1-\tau+\tau N}\sum_{k=1}^{N} L_k^{\frac{B\rho}{X}}\right). \qquad (26)$$

where $\eta \equiv f(1-\rho)^{-1}\rho^{\frac{\rho}{1-\rho}}\alpha^{\frac{\phi-\mu(1-\alpha)}{B}}(\frac{\gamma\phi}{1-\alpha})^{\frac{-\mu(1-\alpha)}{B}}$. The term $\frac{\rho\tau}{1-\tau+\tau N}$ is the increasing function of τ and becomes zero when $\tau = 0$. When $\tau = 1$, $\frac{\rho\tau}{1-\tau+\tau N} = \frac{\rho}{N}$. Then, in the regions where

$$L_i^{\frac{B\rho}{X}} - \frac{1}{N}\sum_{k=1}^{N} L_k^{\frac{B\rho}{X}} < 0$$

is satisfied, the transportation cost level τ_i^* satisfies $L_i^{\frac{B\rho}{X}} - \frac{\tau_i^*}{1-\tau_i^*+\tau_i^* N}\sum_{k=1}^{N} L_k^{\frac{B\rho}{X}} = 0$. Therefore, $n_i = 0$, when $\tau \geq \tau_i^*$. In addition, we can observe that $\tau_1^* < \tau_2^*$, when $L_1 < L_2$.. Thus, manufacturing firms disappear from smaller regions when transportation costs fall.

Proposition 6 *1) In those regions in which $L_i^{\frac{B\rho}{X}} - \frac{1}{N}\sum_{k=1}^{N} L_k^{\frac{B\rho}{X}} < 0$ is satisfied, $n_i = 0$, when $\tau \geq \tau_i^*$.*
2) $\tau_1^* < \tau_2^*$, when $L_1 < L_2$..

When one region has no firms, the relative fertility rates of region i, which has no manufacturing firms, to those of region k, which has manufacturing firms is[10]

$$\frac{m_i}{m_k} = \left(\frac{1-\tau+\tau N}{\tau}\frac{L_i^{\frac{B\rho}{X}}}{\sum_{k=1}^{N} L_k^{\frac{B\rho}{X}}}\right)^{\frac{(1-\rho)\alpha\mu}{\rho B}}. \qquad (27)$$

Therefore, $m_i/m_k > 1$ and $\partial(\ m_i/m_k)/\tau < 0$, since $\rho L_i^{\frac{B\rho}{X}} - \frac{\rho\tau}{1-\tau+\tau N}\sum_{k=1}^{N} L_k^{\frac{B\rho}{X}}$.

[9] We show the process with which we derive Eq. (26) in the Appendix.
[10] We show the process with which we derive Eq. (27) in the Appendix.

Proposition 7 *The fertility rates of (smaller) regions that have no manufacturing firms are larger than those of (larger) regions that have manufacturing firms. Moreover, the difference in the fertility rates of these two regions reduces with a decline in transportation costs.*

In summary, by using the multi-region case, we showed that (i) the number of manufacturing firms is larger in small regions than it is in large regions and (ii) fertility rates in small regions are higher than those in large regions. Furthermore, (iii) when transportation costs become low, small regions lose manufacturing firms. Finally, (iv) fertility rates in small regions are higher than they are in regions that have manufacturing firms.

5 Conclusion

In this paper, we presented an interregional trade model that has endogenous fertility rates in the manner of those of Krugman (1980), Helpman and Krugman (1985), and Fujita, Krugman, and Venables (1999). By using the presented model, we showed that manufacturing firms agglomerate in a large region, which lowers fertility rates in a large region compared with a small region. In addition, we found that a decline in transportation costs results in the agglomeration of manufacturing firms, which also lowers the fertility rates in both large and small regions. Moreover, by extending our two-region model to a multi-region one, we showed that the number of manufacturing firms in larger regions is always greater than that in the smaller regions, meaning that fertility rates in the larger region are always lower than those in the smaller region.

In our model, consumerism is an important determinant of fertility rates; however, many other candidates may influence fertility rates, too. In this regard, future research should aim to construct an interregional trade model with a quality/quantity trade-off in line with Becker, Murphy, and Tamura (1991). The second limitation of our model is that it is static. Constructing a dynamic interregional trade model is therefore another possible research extension for future studies.

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A The derivation of (15)

In this Appendix, we show the derivation of (15). From the free-entry condition, we can obtain the following equation:

$$(1-\rho)\rho^{\frac{\rho}{1-\rho}}\alpha^{\frac{\mu(1-\alpha)-\phi}{B}}(\frac{\gamma\phi}{1-\alpha})^{\frac{\mu(1-\alpha)}{B}}\left[L_1P_1^{\frac{X}{B(1-\rho)}}+\tau L_2P_2^{\frac{X}{B(1-\rho)}}\right]=f. \tag{28}$$

By using (8) and (13), we can rewrite the square brackets of (28) as follows:

$$\begin{aligned}\left[L_1P_1^{\frac{X}{B(1-\rho)}}+\tau L_2P_2^{\frac{X}{B(1-\rho)}}\right] &= L_2\rho^{\frac{-X}{B(1-\rho)}}\left[l(n_1+\tau n_2)^{\frac{-X}{\rho B}}+\tau(\tau n_1+n_2)^{\frac{-X}{\rho B}}\right]\\ &= L_2\left(\frac{(1-\tau^2)\rho n_2}{1-\tau l^{\frac{\rho B}{X}}}\right)^{-\frac{X}{\rho B}}(1+\tau).\end{aligned} \tag{29}$$

Then, by substituting (29) into (28), we can obtain the number of manufacturing firms locating in region 2.

B Proof of Proposition 2

B.1 Proof of (1) and (2) of Proposition 2

By differentiating (15) with respect to τ, we can obtain the following equation:

$$\begin{aligned}\Psi^{\frac{\rho B}{X}}\frac{\partial n_2}{\partial\tau} &= n_2\left[-\frac{l^{\frac{\rho B}{X}}}{1-\tau l^{\frac{\rho B}{X}}}+\frac{1}{1-\tau}-\frac{\alpha\mu\phi(1-\rho)}{(1+\tau)X}\right]\\ &= -n_2\left[\frac{l^{\frac{\rho B}{X}}-1}{(1-\tau)(1-\tau l^{\frac{\rho B}{X}})}+\frac{\alpha\mu\phi(1-\rho)}{(1+\tau)X}\right]<0,\end{aligned} \tag{30}$$

because $\tau<\bar{\tau}$. Then, we can differentiate (13) with respect to τ as follows:

$$\frac{\partial n_1}{\partial\tau}=\frac{l^{\frac{2\rho B}{X}}-1}{(1-\tau l^{\frac{\rho B}{X}})^2}n_2+\frac{l^{\frac{\rho B}{X}}-\tau}{1-\tau l^{\frac{\rho B}{X}}}\frac{\partial n_2}{\partial\tau}. \tag{31}$$

By substituting (30) into the above equation, we obtain

$$\begin{aligned}\frac{\partial n_1}{\partial\tau} &= n_2\left[\frac{(1-\tau)(l^{\frac{2\rho B}{X}}-1)-(l^{\frac{\rho B}{X}}-\tau)(l^{\frac{\rho B}{X}}-1)}{(1-\tau)(1-\tau l^{\frac{\rho B}{X}})^2}-\frac{\alpha\mu\phi(1-\rho)}{(1+\tau)X}\frac{l^{\frac{\rho B}{X}}-\tau}{1-\tau l^{\frac{\rho B}{X}}}\right]\\ &= \frac{n_2}{1-\tau l^{\frac{\rho B}{X}}}\left[\frac{l^{\frac{\rho B}{X}}-1}{1-\tau}-\frac{\alpha\mu\phi(1-\rho)(l^{\frac{\rho B}{X}}-\tau)}{(1+\tau)X}\right]\\ &= \frac{n_2}{(1-\tau^2)(1-\tau l^{\frac{\rho B}{X}})X}F(\tau),\end{aligned} \tag{32}$$

where

$$F(\tau)=(1+\tau)(l^{\frac{\rho B}{X}}-1)X-\alpha\mu\phi(1-\tau)(1-\rho)(l^{\frac{\rho B}{X}}-\tau). \tag{33}$$

From (32), the sign of $\partial n_1/\partial\tau$ depends on the sign of $F(\tau)$. The first derivative of $F(\tau)$ and the second derivative of $F(\tau)$ are given by

$$F'(\tau) = (l^{\frac{\rho B}{X}} - 1)X + \alpha\mu\phi(1-\rho)(l^{\frac{\rho B}{X}} + 1 - 2\tau) > 0, \tag{34}$$

$$F''(\tau) = -2\alpha\mu\phi(1-\rho) < 0. \tag{35}$$

Thus, $F(\tau)$ is a monotonically increasing function in $0 < \tau < \bar{\tau}$. The value of $F(\bar{\tau})$ is given by

$$F(\bar{\tau}) = (l^{\frac{\rho B}{X}} - l^{-\frac{\rho B}{X}})Q + \alpha\mu\phi(1-\rho)(1 - l^{-\frac{2\rho B}{X}}) > 0. \tag{36}$$

Therefore, because $F(\tau)$ is a monotonically increasing function and $F(\bar{\tau})$ is positive, when $F(0)$ is positive, $F(\tau) > 0$ holds in $0 < \tau < \bar{\tau}$. The value of $F(0)$ is given by

$$F(0) = (X - \alpha\mu\phi(1-\rho))l^{\frac{\rho B}{X}} - X. \tag{37}$$

When $l > (\frac{X}{X-\alpha\mu\phi(1-\rho)})^{\frac{X}{\rho B}}$ holds, $F(0)$ is positive and thus $\partial n_1/\partial\tau$ is also positive. When $l < (\frac{X}{X-\alpha\mu\phi(1-\rho)})^{\frac{X}{\rho B}}$ holds, $F(0)$ is negative. In this case, there exists τ^*, which satisfies $F(\tau^*) = 0$. Therefore, when $0 < \tau < \tau^*$, $F(\tau)$ is negative and $\partial n_1/\partial\tau$ is also negative. When $\tau^* < \tau < \bar{\tau}$, $F(\tau)$ is positive and $\partial n_1/\partial\tau$ is also positive.

B.2 Proof of (3) of Proposition 2

The total number of manufacturing firms is given by

$$n_1 + n_2 = (1+\tau)^{-\frac{\alpha\mu\phi(1-\rho)}{X}}(l^{\frac{\rho B}{X}})\Psi^{-\frac{\rho B}{X}}. \tag{38}$$

Then, by differentiating the above equation with respect to τ, we can obtain

$$\frac{\partial(n_1+n_2)}{\partial\tau} = -\frac{\alpha\mu\phi(1-\rho)}{X}(l^{\frac{\rho B}{X}} - 1)(1+\tau)^{-\frac{\alpha\mu\phi(1-\rho)+X}{X}}\Psi^{-\frac{\rho B}{X}} < 0. \tag{39}$$

Thus, a decline in transportation costs decreases the total number of manufacturing firms.

C Proof of Proposition 3

Neither (11) nor (12) depends on τ. Therefore, $\frac{\partial P_1}{\partial\tau} = \frac{\partial P_2}{\partial\tau}$ and $\frac{\partial m_1}{\partial\tau} = \frac{\partial m_2}{\partial\tau}$ hold. By substituting (13) into (8), the price index in region 1 is represented by

$$P_1 = \frac{l^{\frac{(\rho-1)B}{X}}}{\rho}\left[\frac{(1-\tau^2)n_2}{1-\tau l^{\frac{\rho B}{X}}}\right]^{\frac{\rho-1}{\rho}}. \tag{40}$$

Because $\rho < 1$, the sign of $\partial P_1/\partial\tau$ is not the same as the sign of the first derivative in the square brackets of (40). Then, in order to investigate the sign of $\partial P_1/\partial\tau$, we can obtain the following equation by differentiating the square brackets of (40) with respect to τ:

$$\frac{\partial}{\partial\tau}\left(\frac{(1-\tau^2)n_2}{1-\tau l^{\frac{\rho B}{X}}}\right)=\frac{l^{\frac{\rho B}{X}}(1+\tau^2)-2\tau}{(1-\tau l^{\frac{\rho B}{X}})^2}n_2+\frac{1-\tau^2}{1-\tau l^{\frac{\rho B}{X}}}\frac{\partial n_2}{\partial\tau}. \tag{41}$$

Then, by substituting (15) into the above equation, we can rewrite as follows:

$$\begin{aligned}\frac{\partial}{\partial\tau}\left(\frac{(1-\tau^2)n_2}{1-\tau l^{\frac{\rho B}{X}}}\right)&=n_2\left[\frac{l^{\frac{\rho B}{X}}(1+\tau^2)-2\tau}{(1-\tau l^{\frac{\rho B}{X}})^2}-\frac{(1+\tau)(l^{\frac{\rho B}{X}}-1)}{(1-\tau l^{\frac{\rho B}{X}})^2}-\frac{\alpha\mu\phi(1-\rho)(1-\tau)}{(1-\tau l^{\frac{\rho B}{X}})X}\right]\\&=n_2\left[\frac{-\tau(1-\tau)l^{\frac{\rho B}{X}}+1-\tau}{(1-\tau l^{\frac{\rho B}{X}})^2}-\frac{\alpha\mu\phi(1-\rho)(1-\tau)}{(1-\tau l^{\frac{\rho B}{X}})X}\right]\\&=\frac{n_2}{X}\frac{1-\tau}{1-\tau l^{\frac{\rho B}{X}}}\left[X-\alpha\mu\phi(1-\rho)\right] &(42)\\&=\frac{n_2}{X}\frac{1-\tau}{1-\tau l^{\frac{\rho B}{X}}}Q>0. &(43)\end{aligned}$$

Therefore, the sign of $\partial P_1/\partial\tau$ is negative and thereby a decline in transportation costs decreases the price levels in both regions. Next, we investigate how trade liberalization affects fertility rates. By differentiating (4) with respect to τ, we can obtain the following equation:

$$\frac{\partial m_1}{\partial\tau}=\left(\frac{\gamma\phi}{1-\alpha}\right)^{\frac{1-\alpha\mu}{B}}\alpha^{-\frac{\alpha\mu}{B}}\frac{\alpha\mu}{B}P_1^{\frac{\alpha\mu-B}{B}}\frac{\partial P_1}{\partial\tau}<0, \tag{44}$$

because $\partial P_1/\partial\tau<0$. Therefore, a decline in transportation costs decreases fertility rates in both regions.

D The derivations of (26) and (27)

In the multi-region case,

$$\pi_i=(1-\rho)\rho^{\frac{\rho}{1-\rho}}\alpha^{\frac{\mu(1-\alpha)-\phi}{B}}\left(\frac{\gamma\phi}{1-\alpha}\right)^{\frac{\mu(1-\alpha)}{B}}\left[(1-\tau)L_iP_i^{\frac{X}{B(1-\rho)}}+\tau\sum_{k=1}^{N}L_kP_k^{\frac{X}{B(1-\rho)}}\right]-f, \tag{45}$$

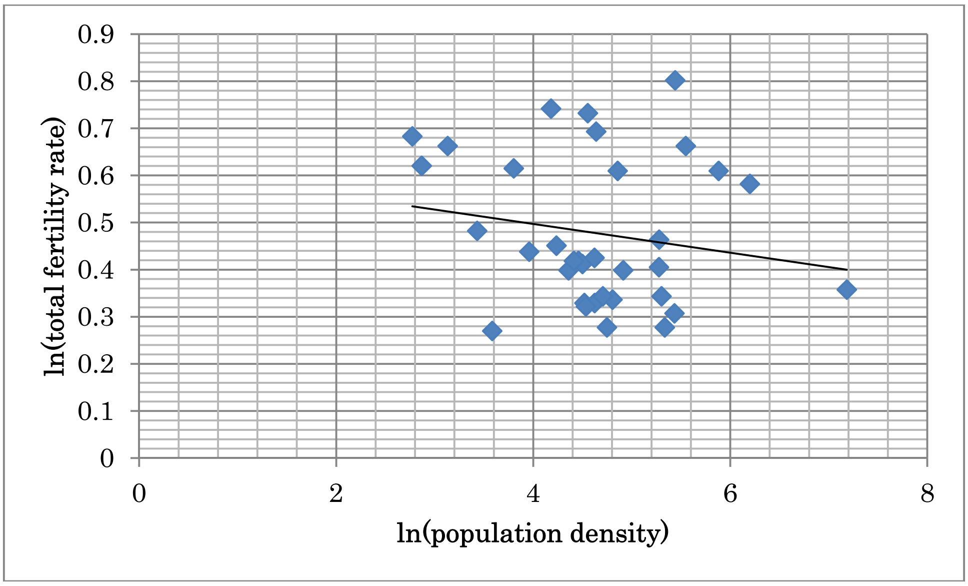

Figure 1: Population density and total fertility rate of EU countries for the year 2010.