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Abstract

This study constructs a variety expansion growth model with public research spending, in which public researchers raise the productivity of private R&D. We show that the relationship between public research spending and the growth rate follows an inverted U-shape. This is because public research spending increases private R&D productivity, but crowds out labor input to private R&D. It is also shown that the welfare-maximizing level of public research spending is below the growth-maximizing level. With regards to tax policy, a zero-profit tax maximizes both growth and welfare. Finally, the study analyzes the stability of the steady state, showing that the equilibrium is indeterminate when the government's revenue source depends on asset income tax.

Keywords: Public expenditure, Endogenous growth, Innovation, Indeterminacy

JEL classification: E62, O41

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1 Introduction

It is widely recognized that productive government spending plays an important role in economic growth. Arrow and Kurz (1970) were the first to study this issue using growth models. They focused on infrastructure, such as highways, airports, railroads, and electrical facilities, and introduced the notion of productive public capital. However, in their model, growth is determined by exogenous factors. In terms of endogenous growth models, Barro (1990) presented the seminal model that includes productive government spending. He assumes that public services raise the productivity of private firms. Under this setting, the social rate of return on private capital becomes constant, and the long-run growth rate is determined endogenously. As a result, we have been able to investigate the relationship between productive government spending and the long-run growth rate. While Barro (1990) treats government spending as a flow variable, many studies have extended the Barro model by introducing the stock of public capital and examining the effects of public capital (see Futagami et al., 1993; Fisher and Turnovsky, 1998).[1,2]

Although many studies analyze the effects of productive government spending on economic growth, the Schumpeterian growth models developed by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992) focus mainly on the activities of private R&D firms motivated by monopoly profit. However, many empirical studies argue that the public research sector is an important factor in economic growth. For instance, Mansfield (1991) finds that 10% of the innovations in the United States between 1975 and 1985 could not have been developed, or could have developed with great delay, without academic research. Caloghirou et al. (2001) analyzed over 6,000 research joint ventures in 42 nations that received funding from the European Commission during the period 1983-1996. They found that 65% of research joint ventures involved one or more universities. Based on the Swedish Community Innovation Survey, Lööf and Broström (2008) found that university collaboration has a positive influence on the innovative activity of large manufacturing firms. On the other hand, few theoretical studies have considered public research policy from a macroeconomic perspective. Glomm and Ravikumar (1994) present a model in which the stock of technological knowledge depends upon public research, but the model does not allow for private R&D. In contrast, Park (1998) considers both

[1]Futagami et al. (1993) developed an endogenous growth model with productive public capital, and show different results to those of Barro (1990). Since their model includes two stock variables, it has transition dynamics. The growth-maximizing tax rate is the same as that of Barro (1990), but the welfare-maximizing tax rate is lower than that of Barro (1990). Fisher and Turnovsky (1998) suppose that public capital has a congestion effect. They show there is a trade-off between the degree of congestion and the substitutability between public and private capital in production.

[2]An extensive survey of the literature is provided by Irmen and Kuehnel (2009).

public and private research. Public research indirectly contributes to market production by influencing the knowledge accumulation of private R&D. However, the study is more concerned with open economy issues and international spillovers than public research policy. In order to analytically examine the growth and welfare effects of public research spending, this study incorporates public researchers who raise the productivity of private R&D in a variety expansion model, following the work of Grossman and Helpman (1991). The government expends public research spending and interest from government debt, but raises funds from taxes on asset income, consumption, and corporate profit.[3]

In this study, we obtained the following three main results. First, the relationship between public research spending and the growth rate follows an inverted U-shape. That is, growth-maximizing public research spending exists. The inverted U-shaped relationship occurs because public research spending raises the productivity of private R&D, but crowds out the labor input to private R&D. In addition, only the tax on profit has a negative effect on growth because an increase in the tax rate decreases monopoly profit, which is the rate of return for setting up new firms. Thus, a zero-profit tax maximizes growth and the government should finance expenditure by taxing asset income and consumption.

Second, the welfare-maximizing level of public research spending is lower than the growth-maximizing level. In this study, welfare is driven by households' consumption expenditure and the growth of differentiated goods (i.e., love of variety). While public research spending and the growth rate follow an inverted U-shape, households' consumption expenditure decreases with an increase in public research spending. This is because higher public research spending implies a heavier tax burden on households, so reducing their consumption expenditure. Therefore, there is a trade-off between households' consumption expenditure and the growth of differentiated goods. This trade-off leads to the welfare-maximizing level of public research spending being below the growth-maximizing level. With regards to tax policy, a zero-profit tax maximizes welfare because a decrease in profit tax increases growth, which increases welfare.

Third, an increase in asset income tax raises the possibility of indeterminacy, whereas an increase in government debt, consumption, and profit tax decreases the possibility of indeterminacy.[4] Some related studies have also pointed out the possibility that indeterminacy depends on fiscal policies (e.g., Guo and Harrison, 2008; Kamiguchi and Tamai, 2011). Guo and Harrison (2008) show that the presence of productive government spending and the distortion by

[3]Many theoretical studies have investigated the fiscal policy of productive public spending with debt financing. For example, see Bruce and Turnovsky (1999), Greiner and Semmler (2000), Ghosh and Mourmouras (2004), Futagami et al. (2008), Yakita (2008), Maebayashi et al. (2013), and Morimoto et al. (2013).

[4]In this study, *indeterminacy* means we cannot select a continuum of equilibrium trajectories that all converge on the same steady state.

taxes affect the possibility of indeterminacy. This implies that financing government spending using distortionary taxes might bring about equilibrium indeterminacy. Then, Kamiguchi and Tamai (2011) show that income tax financing has a greater influence on indeterminacy than the presence of productive government spending. It seems that the findings in these studies are similar to those of this study. However, these studies use real business cycle models, not an endogenous growth model, and so do not consider the effect of government debt. In fact, few studies examine the relationship between the debt policy rule and equilibrium indeterminacy. Futagami et al. (2008) shows that, in a closed economy model of endogenous growth with public services and a debt policy, the high growth steady state can be equilibrium indeterminate when the long-run debt-private capital ratio is sufficiently high. Morimoto et al. (2013) show that, in a small open economy model of endogenous growth with public capital and a debt policy, equilibrium indeterminacy arises when the long-run debt-GDP ratio is sufficiently high.[5] In contrast to these studies, this study shows that equilibrium indeterminacy arises when government debt is sufficiently small.

The rest of this paper is organized as follows. Section 2 establishes the model used in this study. Section 3 derives the equilibrium and dynamics of the economy. Section 4 examines the stability of the steady state. Section 5 analyzes the policy effect on economic growth and Section 6 investigates the policy effect on welfare. Finally, Section 7 concludes the paper.

2 Model

There is a unit continuum of identical households. Each household supplies one unit of inelastic labor supply. The factor market is perfectly competitive, while the goods market is monopolistically competitive, as explained below. The households have perfect foresight.

2.1 Households

The households maximize the following lifetime utility:

$$U_0 \equiv \int_0^\infty e^{-\rho t} \log C_t dt, \tag{1}$$

where C_t represents an instantaneous utility derived from the consumption of a composite good and $\rho > 0$ is a rate of time preference. C_t is given by

$$C_t = \left[\int_0^{N_t} c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

[5]Maebayashi et al. (2013) show that, in a closed economy model of endogenous growth with public capital and a debt policy, equilibrium indeterminacy never arises.

where $c_t(j)$ denotes the consumption of good j and N_t denotes the number of available varieties. We assume that $\varepsilon > 1$. ε is the elasticity of substitution between any two products. Denoting the expenditure of the households as $E_t = \int_0^{N_t} P_t(j)c_t(j)dj$, we obtain the demand function for good j as follows:

$$c_t(j) = \frac{P_t(j)^{-\varepsilon} E_t}{\int_0^{N_t} P_t(i)^{1-\varepsilon} di}, \tag{3}$$

where $P_t(j)$ is the price of good j, and $P_{D,t}$ is the price index, defined as

$$P_{D,t} = \left(\int_0^{N_t} P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}. \tag{4}$$

Then, the maximization problem for households is as follows:

$$\begin{aligned} &max && U_0 \\ &subject\ to && \dot{A}_t = (1 - t_A)r_t A_t + W_t - (1 + t_E)E_t, \end{aligned}$$

where A_t, r_t, and W_t represent the households' asset holding, rate of return on assets, and wage rate. The wage rate is the numeraire, and thus $W_t = 1$. The government taxes asset income at rate $t_A \in [0, 1)$ and consumption at rate $t_E \geq 0$. Substituting (3) into (2), we obtain the indirect sub-utility function as follows:

$$C_t = \frac{E_t}{P_{D,t}}. \tag{5}$$

The maximization subject to the intertemporal budget constraint yields the following Euler equation:

$$\frac{\dot{E}_t}{E_t} = (1 - t_A)r_t - \rho. \tag{6}$$

2.2 Firms

This subsection considers producer behavior. Producers undertake two distinct activities. They create blueprints for new varieties of differentiated goods, and manufacture the differentiated goods that have been created by R&D.

We assume that each differentiated good is produced by a single firm because the good is infinitely protected by a patent. We further assume that one unit of labor input produces one unit of a differentiated good. The firm manufacturing good j (firm j) maximizes its after-tax profit:

$$(1 - t_\pi)\Pi_t(j) = (1 - t_\pi)\big(P_t(j)X_t(j) - X_t(j)\big), \tag{7}$$

where $X_t(j)$ and $t_\pi \in [0, 1)$ represent the output for firm j and the tax rate on profit. Then firm j charges the following price:

$$P_t(j) = \frac{\varepsilon}{\varepsilon - 1} = P. \tag{8}$$

Therefore, all goods are priced equally. Pricing rules (8) and (3) yield $X_t(j) = X_t = (\varepsilon - 1)E_t/\varepsilon N_t$. Then, the per brand operating profits are as follows:

$$\Pi_t = \frac{E_t}{\varepsilon N_t}. \tag{9}$$

The no-arbitrage condition is given by

$$(1 - t_A)r_t = (1 - t_\pi)\frac{\Pi_t}{v_t} + \frac{\dot{v}_t}{v_t}, \tag{10}$$

where v_t denotes the value of a firm.[6]

Next, we consider the technology involved in developing a new good.[7] R&D firms create blueprints and expand the varieties of goods available for consumption. We assume that R&D activity needs labor input. Because the knowledge that has already been produced includes all that is needed for invention, greater knowledge means further invention. Since the knowledge is non-rival and non-excludable, an expansion in the number of varieties reduces the labor input. Furthermore, we incorporate public researchers into the model. Public research activities enhance the productivity of private R&D through a variety channel (for example, publications, scientific reports, conferences, research joint ventures, and university collaboration). We assume the following production function for R&D:

$$\dot{N}_t = f(G_t)N_t L_{R,t}, \tag{11}$$

where G_t and $L_{R,t}$ represent the number of public researchers and the amount of labor devoted to R&D. In addition, we postulate that f satisfies the following conditions:

$$f(0) > 0 \quad \mathit{and} \quad f'(G_t) > 0 \quad \mathit{and} \quad f''(G_t) < 0.$$

We assume that firms freely enter into the R&D race. Therefore, the free entry condition is given by

$$v_t = \frac{1}{f(G_t)N_t} \quad \Leftrightarrow \quad \dot{N}_t > 0. \tag{12}$$

Equation (12) shows that an increase in G_t decreases v_t. The reasoning is as follows. Greater R&D productivity creates an excess supply of blueprints. In a competitive market, the Walrasian adjustment mechanism reduces the patent value, v_t.

2.3 Government

The government taxes asset income, consumption, and corporate profit. We assume that the respective tax rates are held constant over time, and that the government maintains its debt,

[6]Note that profit tax can be translated as dividend tax.

[7]See Grossman and Helpman (1991) for more details of the R&D process.

$\bar{B}$, at the constant level. Therefore, the government finances public research spending and the interest on the debt. As a result, it satisfies the budget constraint

$$G_t + r_t \bar{B} = t_A r_t A_t + t_E E_t + t_\pi \Pi_t N_t. \tag{13}$$

Note that $\bar{B} = 0$ implies a balanced budget.[8,9]

3 Equilibrium

3.1 Dynamics

Labor is used for production, private R&D, and the employment of public researchers. The labor market equilibrium becomes

$$N_t X_t + L_{R,t} + G_t = 1. \tag{14}$$

From (12), the asset market equilibrium is as follows:

$$A_t = \bar{B} + N_t v_t = \bar{B} + \frac{1}{f(G_t)}. \tag{15}$$

From (9), (12), and (14), the no-arbitrage condition (10) becomes:

$$(1 - t_A) r_t = \left(1 - \frac{t_\pi}{\varepsilon}\right) f(G_t) E_t - f(G_t)(1 - G_t) - \frac{f'(G_t)}{f(G_t)} \dot{G}_t. \tag{16}$$

By using (9), (13), (15), and (16), we obtain

$$\frac{f'(G_t)}{f(G_t)} \dot{G}_t = \left(1 - \frac{t_\pi}{\varepsilon}\right) f(G_t) E_t - f(G_t)(1 - G_t) + \frac{1}{\Delta_t} \left\{ \left(t_E + \frac{t_\pi}{\varepsilon}\right) E_t - G_t \right\}, \tag{17}$$

where $\Delta_t \equiv t_A / \{(1 - t_A) f(G_t)\} - \bar{B}$. From (12), (16), and (17), Euler equation (6) becomes:

$$\frac{\dot{E}_t}{E_t} = \frac{1}{\Delta_t} \left\{ G_t - \left(t_E + \frac{t_\pi}{\varepsilon}\right) E_t \right\} - \rho. \tag{18}$$

Equations (17) and (18) formulate the autonomous dynamic system with respect to E_t and G_t.

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[8] Our main results do not change if the government uses a debt policy rule following that of Futagami et al. (2008). In that case, the budget constraint of the government is as follows:

$$G_t + r_t B_t = t_A r_t A_t + t_E E_t + t_\pi \Pi_t N_t + \dot{B}_t.$$

The government adjusts its debt according to the following rule:

$$\dot{B}_t = -\phi(B_t - \bar{B}),$$

where $\bar{B}$ and ϕ denote the target level of government debt and the adjustment coefficient of the rule, respectively.

[9] If $\bar{B} < 0$, the government lends to households and earns interest.

3.2 Steady state

In this section, we examine the steady state of the economy. Henceforth, we omit time index, t. Imposing $\dot{E} = \dot{G} = 0$ in (17) and (18), respectively, results in

$$\left(t_E + \frac{t_\pi}{\varepsilon}\right)E = G - \rho\Delta, \tag{19}$$

$$\rho = \left(1 - \frac{t_\pi}{\varepsilon}\right)f(G)E - f(G)(1 - G). \tag{20}$$

By eliminating E from equations (19) and (20), we have

$$\bar{B} = \left(\frac{t_A}{1 - t_A} + \frac{t_E + \frac{t_\pi}{\varepsilon}}{1 - \frac{t_\pi}{\varepsilon}}\right)\frac{1}{f(G)} + \frac{1}{\rho}\left(\frac{t_E + \frac{t_\pi}{\varepsilon}}{1 - \frac{t_\pi}{\varepsilon}} - \frac{1 + t_E}{1 - \frac{t_\pi}{\varepsilon}}G\right) \equiv \Lambda(G). \tag{21}$$

As shown in Figure 1, the intersection of the LHS and RHS of (21) determines the steady state value, G^*.[10] The steady state value E^* is obtained from (19), as follows:

$$E^* = \frac{1}{1 - \frac{t_\pi}{\varepsilon}}\left\{(1 - G^*) + \frac{\rho}{f(G^*)}\right\}. \tag{22}$$

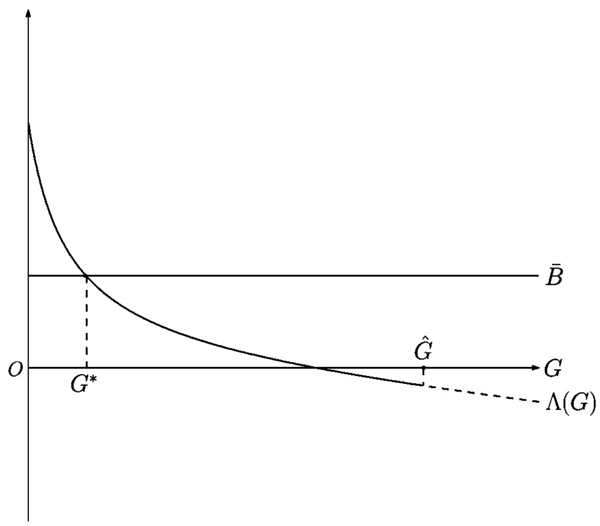

Figure 1

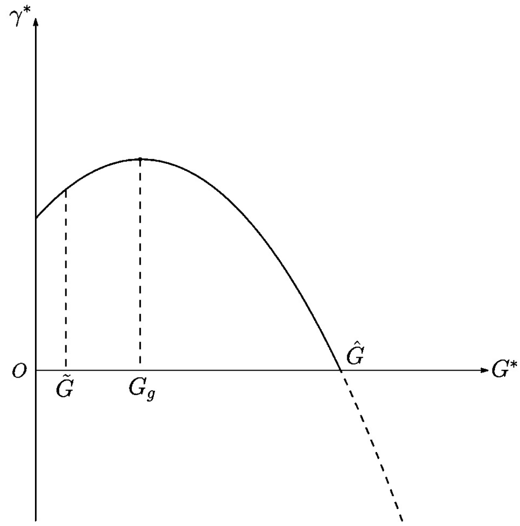

Figure 2

From (11), (14), and (22), the growth rate at the steady state is given by

$$\gamma^* \equiv \left(\frac{\dot{N}}{N}\right)^* = \frac{1}{\varepsilon - t_\pi}\Big\{(1 - t_\pi)f(G^*)(1 - G^*) - (\varepsilon - 1)\rho\Big\}. \tag{23}$$

Moreover, we assume a positive growth rate at $G^* = 0$, as follows:

$$\gamma^*|_{G^*=0} = \frac{1}{\varepsilon - t_\pi}\Big\{(1 - t_\pi)f(0) - (\varepsilon - 1)\rho\Big\} > 0. \tag{24}$$

[10] x^* represents the steady state value of variable x.

Differentiating γ^* with respect to G^*, we obtain

$$\frac{\partial \gamma^*}{\partial G^*} = \frac{1-t_\pi}{\varepsilon - t_\pi}\Big\{ f'(G^*)(1-G^*) - f(G^*) \Big\}. \tag{25}$$

From the assumption of f, $\partial\gamma^*/\partial G^*$ is decreasing in G^*.[11] Assuming that $f'(0) > f(0)$, there exists public research spending that satisfies $f'(G^*)(1-G^*) - f(G^*) = 0$. We define G^* as G_g, which yields the following relation:

$$\frac{\partial \gamma^*}{\partial G^*} \gtreqless 0 \;\; \Leftrightarrow \;\; G^* \lesseqgtr G_g. \tag{26}$$

Therefore, we can illustrate the graph of (23) as shown in Figure 2. As shown by (26), the relationship between the growth rate and G^* follows an inverted U-shape, and the growth-maximizing public research spending is G_g. From Figure 2, the growth rate becomes 0 if G^* is sufficiently large. In this case, R&D is not undertaken since a sufficiently large value of G^* makes labor input to private R&D zero. Here, $\hat{G}$ is defined as $\gamma^*|_{G^*=\hat{G}} = 0$. In Figure 1, we assume that the growth rate is positive in a balanced budget ($\bar{B} = 0$), that is, $G^*|_{\bar{B}=0} < \hat{G}$. Furthermore, to focus on the positive growth rate and $G^* \geq 0$, we impose the following condition:

$$\Lambda(\hat{G}) < \bar{B} \leq \Lambda(0).$$

These results are summarized in the following proposition:

Proposition 1

There is a positive growth steady state in this economy if $\Lambda(\hat{G}) < \bar{B} \leq \Lambda(0)$. In addition, if $f'(0) > f(0)$, there exists a positive growth-maximizing level of public research spending, G_g.

We next study the detailed relationship between the growth rate and public research spending at the steady state. From (11), the growth rate, $\gamma^* = f(G^*)L_R^*$, is determined by $f(G^*)$ and L_R^*. Differentiating γ^* with respect to G^* yields

$$\frac{\partial \gamma^*}{\partial G^*} = f'(G^*)L_R^* + f(G^*)\frac{\partial L_R^*}{\partial G^*}.$$

While the effect of the productivity of R&D is positive, $f'(G^*) > 0$, the effect of labor input, $\partial L_R^*/\partial G^*$, is ambiguous. From (14), labor input in private R&D at the steady state is given by $L_R^* = 1 - G^* - \frac{\varepsilon-1}{\varepsilon}E^*$. Differentiating L_R^* with respect to G^* yields

$$\frac{\partial L_R^*}{\partial G^*} = -1 - \frac{\varepsilon - 1}{\varepsilon}\frac{\partial E^*}{\partial G^*}. \tag{27}$$

[11] Differentiating $\partial\gamma^*/\partial G^*$ with respect to G^* yields

$$\frac{\partial^2 \gamma^*}{\partial G^{*2}} = \frac{1-t_\pi}{\varepsilon - t_\pi}\Big\{ f''(G^*)(1-G^*) - 2f'(G^*) \Big\} < 0.$$

The first term represents a direct crowding-out effect on private R&D labor input, and the second term represents the effect of labor demand on production, which depends on households' consumption expenditure. To more precisely investigate the effect of the second term, we differentiate E^* with respect to G^*, as follows:

$$\frac{\partial E^*}{\partial G^*} = \frac{1}{1-\frac{t_\pi}{\varepsilon}}\left\{-1-\rho\frac{f'(G^*)}{(f(G^*))^2}\right\} < 0. \tag{28}$$

Thus, E^* is decreasing in G^*. Since higher public research spending implies a heavier tax burden on households, an increase in public research spending reduces households' consumption expenditure. Then, we examine the value of G^* that crowds out private R&D labor input. From (27) and (28), we obtain

$$\frac{\partial L_R^*}{\partial G^*} = \frac{1}{\varepsilon - t_\pi}\left\{-(1-t_\pi)+(\varepsilon-1)\rho\frac{f'(G^*)}{(f(G^*))^2}\right\}.$$

From the assumption of f, $f'(G)/(f(G))^2$ is decreasing in G.[12] We define $\tilde{G}$ as $f'(\tilde{G})/(f(\tilde{G}))^2 = (1-t_\pi)/(\varepsilon-1)\rho$. Then we obtain the following relation:

$$\frac{\partial L_R^*}{\partial G^*} \gtreqless 0 \;\Leftrightarrow\; G^* \lesseqgtr \tilde{G}.$$

When $G^* < \tilde{G}$, the effect of labor demand on production exceeds the direct crowding-out effect; and $\partial L_R^*/\partial G^* > 0$ holds.[13] In contrast, when $G^* > \tilde{G}$, the direct crowding-out effect is sufficiently large; so $\partial L_R^*/\partial G^* < 0$ holds. Next, we compare G_g to $\tilde{G}$. Substituting $G^* = \tilde{G}$ into (25) yields

$$\left.\frac{\partial \gamma^*}{\partial G^*}\right|_{G^*=\tilde{G}} = f'(\tilde{G})L_R|_{G^*=\tilde{G}} > 0.$$

Therefore, from (26), we have $\tilde{G} < G_g$.

We summarize these results as follows. If $G^* < G_g$, the effect of an increase in R&D productivity exceeds the crowding-out effect, and thus an increase in G^* raises the growth rate. However, if $G^* > G_g$, the crowding-out effect exceeds the effect of an increase in R&D productivity. In this case, an increase in G^* decreases the growth rate. From these results, the growth rate and G^* follow an inverted U-shape.

Finally, we have to consider the condition of $G_g > 0$ that corresponds to $f'(0) > f(0)$. From the above discussion, if the productivity effect is larger than the crowding-out effect at $G^* = 0$, there exists a positive growth-maximizing level of public research spending. On the other hand, if the crowding-out effect is larger than the productivity effect at $G^* = 0$, public research spending crowds out private R&D input, that is, there is no need for public research spending.

[12] Differentiating $f'(G)/(f(G))^2$ with respect to G yields $\{f''(G)f(G) - 2(f'(G))^2\}/(f(G))^3 < 0$.

[13] Note that $\tilde{G}$ can be a negative value. In this case, there is no crowding-in effect.

4 Stability

We examine the local stability at the steady state. Approximating (17) and (18) linearly in the neighborhood of the steady states, we obtain

$$\begin{pmatrix} \dot{G} \\ \dot{E} \end{pmatrix} = \begin{pmatrix} J_{GG} & J_{GE} \\ J_{EG} & J_{EE} \end{pmatrix} \begin{pmatrix} G - G^* \\ E - E^* \end{pmatrix}.$$

Here, J_{ij} $(i, j = G, E)$ denotes entities in the Jacobian matrix of this system (see Appendix A for more detail):

$$\begin{aligned}
J_{GG} &= -\frac{f(G^*)}{f'(G^*)\Delta^*}\left\{1 + \rho\frac{t_A}{1-t_A}\frac{f'(G^*)}{(f(G^*))^2}\right\}, \\
J_{GE} &= \frac{f(G^*)}{f'(G^*)\Delta^*}\left\{\left(1 - \frac{t_\pi}{\varepsilon}\right)f(G^*)\Delta^* + \left(t_E + \frac{t_\pi}{\varepsilon}\right)\right\}, \\
J_{EG} &= \frac{E^*}{\Delta^*}\left\{1 + \rho\frac{t_A}{1-t_A}\frac{f'(G^*)}{(f(G^*))^2}\right\}, \\
J_{EE} &= -\left(t_E + \frac{t_\pi}{\varepsilon}\right)\frac{E^*}{\Delta^*},
\end{aligned}$$

where $\Delta^* = t_A/\{(1-t_A)f(G^*)\} - \bar{B}$.[14] The eigenvalues of the Jacobian matrix, J, are defined as λ_i $(i = 1, 2)$. Here, λ_1 and λ_2 are the roots of the characteristic equation, $\lambda^2 - (J_{GG} + J_{EE})\lambda + J_{GG}J_{EE} - J_{GE}J_{EG} = 0$. To check the stability, we investigate the sign of $detJ = J_{GG}J_{EE} - J_{GE}J_{EG}$ as follows:

$$detJ = -\frac{f(G^*)E^*}{f'(G^*)\Delta^*}\left[f(G^*) + \rho\frac{t_A}{1-t_A}\frac{f'(G^*)}{f(G^*)}\right].$$

Because the sign of the square bracket is positive, the sign of $detJ$ is determined by the sign of Δ^*. When $\Delta^* > 0$, the sign of $detJ$ is negative. We then have $\lambda_1 > 0$ and $\lambda_2 < 0$. In this case, the steady state is saddle-point stable. Since E and G are flow variables, the equilibrium is indeterminate.

On the other hand, when $\Delta^* < 0$, the sign of $detJ$ is positive. In this case, we must examine the sign of $trJ = J_{GG} + J_{EE}$, as follows:

$$trJ = -\frac{1}{\Delta^*}\left(1 - \frac{t_\pi}{\varepsilon}\right)\left[\frac{f(G^*)}{f'(G^*)} + \rho\frac{t_A}{1-t_A}\frac{1}{f(G^*)} + \left(t_E + \frac{t_\pi}{\varepsilon}\right)E^*\right].$$

Since $\Delta^* < 0$ implies $trJ > 0$, the eigenvalues of J have positive real parts. Thus, the steady state is unstable, that is, the equilibrium is determinate. From these results, we can state the following proposition:

[14] If $\Delta^* = 0$, the economy jumps to the steady state immediately. See Appendix B for more detail.

Proposition 2

The equilibrium is indeterminate if $\Delta^* > 0$*, while determinate if* $\Delta^* \leq 0$.

In this study, G^* is determined by the government's budget constraint, that is, G^* is affected by the fiscal variables. Therefore, from the definition of Δ^*, the fiscal variables influence the value of Δ^*. We next examine the relationship between Δ^* and the fiscal variables. To begin with, we investigate the effects of the fiscal variables on G^* through changes in the fiscal variables. As shown in Appendix C, we can state the following lemma:

Lemma

The effects of changes in policy variables $\bar{B}$ *and* t_i $(i = A, E, \pi)$ *are as follows:*

$$\frac{\partial G^*}{\partial \bar{B}} < 0, \ \ \frac{\partial G^*}{\partial t_i} > 0.$$

An increase in $\bar{B}$ raises the interest payment on debt, and G^* decreases. Meanwhile, an increase in t_i $(i = A, E, \pi)$ raises government revenue, and G^* increases. By using the Lemma, we examine the effects of Δ^* through changes in the fiscal variables. As shown in Appendix D, we obtain the following relationship:

$$\frac{\partial \Delta^*}{\partial \bar{B}} < 0, \ \ \frac{\partial \Delta^*}{\partial t_A} > 0, \ \ \frac{\partial \Delta^*}{\partial t_i} < 0. \quad (i = E, \pi)$$

From these results and Proposition 2, an increase in t_A increases the possibility of indeterminacy, whereas an increase in t_E, t_π and $\bar{B}$ reduces the possibility of indeterminacy. In addition, we consider the following extreme cases:

$$\begin{aligned}
\Delta^* &= \frac{t_A}{1 - t_A}\frac{1}{f(G^*)} - \bar{B} > 0, && \bar{B} \leq 0, \ \ t_A, t_E, t_\pi > 0, \\
\Delta^* &= \frac{1}{\rho}G^* > 0, && t_E = t_\pi = 0, \ \ \bar{B}, t_A > 0, \\
\Delta^* &= -\bar{B} < 0, && t_A = 0, \ \ \bar{B}, t_E, t_\pi > 0.
\end{aligned}$$

When $\bar{B} = t_A = 0$ is a special case, because $\Delta^* = 0$. In this case, we have to reconsider the equilibrium dynamics and stability. Appendix B shows the steady state is unstable, that is, the equilibrium is determinate. In summary, we can state the following corollary:

Corollary (Local Stability)

When t_A is sufficiently large and t_E, t_π, and $\bar{B}$ are sufficiently small, the equilibrium is indeterminate. On the other hand, when t_A is sufficiently small and t_E, t_π, and $\bar{B}$ are sufficiently large, the equilibrium is determinate.

To investigate the intuitive explanation of indeterminacy, suppose that the initial level of G is larger than the steady state level of public research spending, G^*, which increases the productivity of R&D.[15] From (12), the value of a firm, v, decreases. From (11) and (12), the no-arbitrage condition is given by

$$(1-t_A)r = (1-t_\pi)\frac{\Pi}{v} - f(G)L_R - \frac{f'(G)}{f(G)}\dot{G}.$$

In this case, the first term of the RHS of this condition increases. The second term changes slightly. This is because an increase in G reduces the labor input to R&D, L_R, and this partly offsets the increase in $f(G)$. If the government does not change the tax rates, $\dot{G} > 0$ cannot satisfy the budget constraint in the long run. Therefore, $\dot{G} < 0$ holds, and the third term of the RHS becomes positive. From these results, an increase in G raises the interest rate, r.[16]

Using this result, we provide an intuitive explanation of indeterminacy. Suppose that the economy is in a steady state and households expect the government to raise G. This expectation implies that households also expect an increase in r. Households then have a higher incentive to save. As a result, households' asset holding, A, increases and their expenditure, E, decreases. Under these circumstances, we can think of the government budget constraint as:

$$G = \underbrace{t_A r A - r\bar{B}}_{(1-t_A)r\Delta} + \left(t_E + \frac{t_\pi}{\varepsilon}\right)E. \tag{29}$$

From equation (29), when the government's interest payment exceeds the asset income tax revenue, $\Delta^* \leq 0$ holds.[17] This implies that the government's revenue source depends on consumption/profit tax financing. In this case, since t_A is sufficiently small, the effect of a reduction in E exceeds that of an increase in A, and the government then decreases G. Thus, the households' expectations are not self-fulfilling; that is, the equilibrium is determinate. On the other hand, $\Delta^* > 0$ implies that the asset income tax revenue is sufficient to finance the interest payment and the government's revenue source depends on asset income tax financing. In this case, since t_A is sufficiently large, an increase in A will raise G. In this case, households' expectations can be self-fulfilling and the equilibrium is indeterminate.

[15] Note that this increase in G is not caused by the fiscal policy change.

[16] Intuitively, when the price of a patent, v, decreases, the demand for the patent becomes higher than the risk-free asset. This raises the interest rate, r, because the no-arbitrage condition must hold.

[17] We can derive $t_A r A - r\bar{B} = (1-t_A)r\Delta$ from (15) and the definition of Δ.

5 Growth-Maximizing Policy

In this section, we investigate the policy maximizing the long-run growth rate. Equation (23) implies that only profit tax is a distorting tax, and that asset income/consumption tax can be equivalent to a lump-sum tax. We differentiate γ^* with respect to t_π, under given G^*, as follows

$$\left.\frac{\partial \gamma^*}{\partial t_\pi}\right|_{G^* given} = -\frac{\varepsilon - 1}{(\varepsilon - t_\pi)^2}\Big\{f(G^*)(1-G^*) + \rho\Big\} < 0.$$

Thus, a decrease in t_π raises the graph of the RHS of (23). From (25) and (26), the growth-maximizing public research spending is G_g, regardless of t_π. These relations are shown in Figure 3. By using these results, Proposition 1, and the Lemma, we can now state the following proposition representing the growth-maximizing policy.

Proposition 3

If $f'(0) > f(0)$, the policy mix of $t_\pi = 0$ and $\bar{B} = \Lambda(G_g)$ maximizes the long-run growth rate.

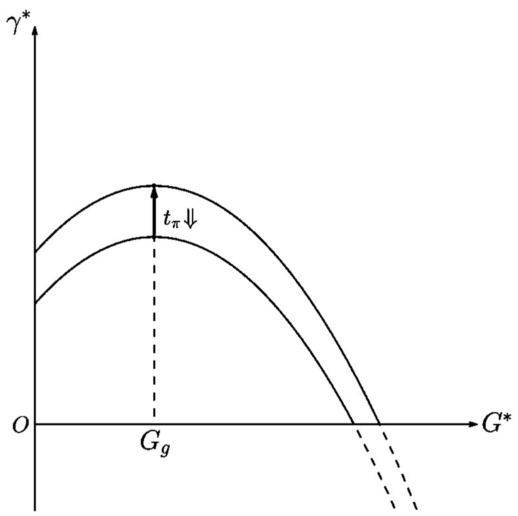

Figure 3

Proposition 3 states that the government should not levy corporate profits and finance its spending through asset income/consumption taxation. This is because profit tax has a negative growth effect by reducing the rate of return to setting up new firms. Furthermore,

$\Lambda(G_g)$ is increasing in t_A and t_E. This implies that larger government debt needs a larger asset income/consumption tax rate.

In the Barro model, when the government imposes tax only on household income, there exists a growth-maximizing government spending level. Meanwhile, when the government is able to use consumption or lump-sum tax, the growth rate is increasing in productive government spending. In other words, there is no growth-maximizing government spending level.[18] This implies that Proposition 3 is different to Barro's results. The reasoning is as follows. In the Barro model, when the government raises productive government spending, the production of the final output increases. Because the government transforms the final output into a productive input, an increase in the available resources can be devoted to further productive government spending. Thus, if the government can use a non-distortionary tax, the growth rate is increasing in government spending.[19] On the other hand, in this study, public research spending requires labor input. Since the total labor supply is constant over time, larger public research spending crowds out private R&D input, as discussed in Subsection 3.2. Therefore, there exists a growth-maximizing public research spending level, even if the government is able to use a non-distortionary tax.

6 Welfare-Maximizing Policy

In this section, we investigate the welfare level of the steady state. In the steady state, (2), (5), and (8) yield

$$\log C = \log E^* + \frac{1}{\varepsilon - 1}\gamma^* t + \log\Big(\frac{\varepsilon - 1}{\varepsilon}\Big) N_0^{\frac{1}{\varepsilon-1}}.$$

Without any loss of generality, we set $(\varepsilon - 1)N_0^{\frac{1}{\varepsilon-1}}/\varepsilon = 1$. If indeterminacy occurs, the effect of the welfare-maximizing policy is ambiguous. To focus on the determinate equilibrium, we impose the following condition using the definition of Δ^* and (D.1).

Condition : Determinate equilibrium

$$t_A \leq \frac{\bar{B}}{\frac{1}{f(G^*)} + \bar{B}} \quad or \quad t_E + \frac{t_\pi}{\varepsilon} \geq \frac{(1 + t_E)G^*}{\frac{\rho}{f(G^*)} + 1}.$$

[18] A detailed explanation is provided by Irmen and Kuehnel (2009).

[19] If the government's revenue source depends on only a distortionary income tax, an increase in productive government spending can decrease the after-tax private marginal product of private capital through a necessary increase in the distortionary income tax. Therefore, there exists a growth-maximizing government spending level.

Note that this condition implies that $\Delta^* \leq 0$. Since the economy jumps to the steady state, the welfare level at the steady state is calculated by

$$U^* = \frac{1}{\rho} \log E^* + \frac{1}{\rho^2(\varepsilon - 1)} \gamma^*. \tag{30}$$

Differentiating U^* with respect to G^*, we obtain

$$\frac{\partial U^*}{\partial G^*} = \frac{1}{\rho E^*} \frac{\partial E^*}{\partial G^*} + \frac{1}{\rho^2(\varepsilon - 1)} \frac{\partial \gamma^*}{\partial G^*}. \tag{31}$$

From the fact that the relationship between γ^* and G^* is an inverted U-shape (see Figure 2) and E^* is decreasing in G^* (see Equation (28)), we can depict equation (30) in (G^*, U^*) space, as shown in Figure 4. If the growth effect, $\partial \gamma^* / \partial G^*$, is sufficiently large, U^* and G^* follow an inverted U-shape (see Figure 4-1).[20] In contrast, if the growth effect is sufficiently small, U^* is decreasing in G^* (see Figure 4-2).

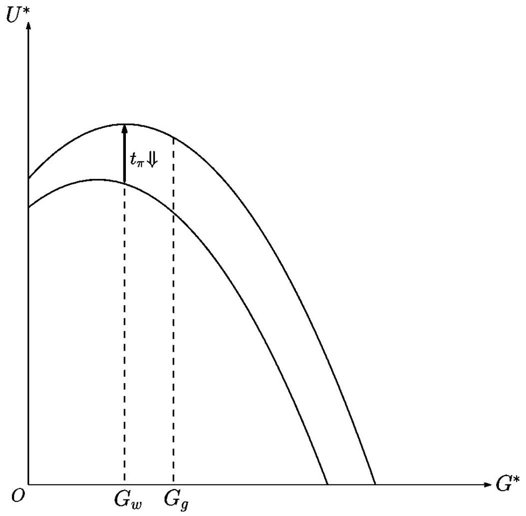

Figure 4-1 : the growth effect is large

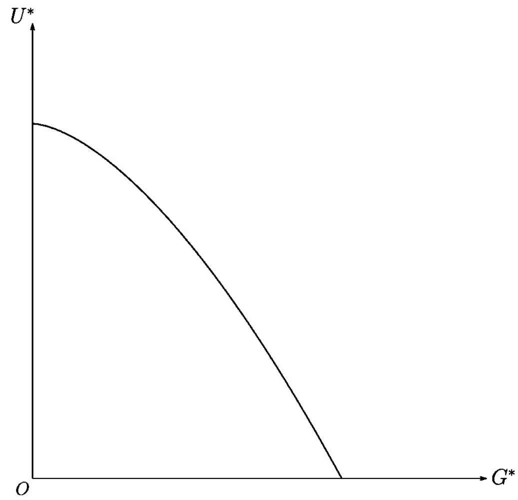

Figure 4-2 : the growth effect is small

We next consider the welfare-maximizing policy. From (22), (23), and (30), only the profit tax is a distorting tax and the asset income/consumption tax is equivalent to a lump-sum tax, as discussed in Section 5. Thus, differentiating U^* with respect to t_π, under given G^*, we obtain

$$\left.\frac{\partial U^*}{\partial t_\pi}\right|_{G^* given} = \frac{1}{\rho^2(\varepsilon - t_\pi)^2} \Phi(G^*),$$

where

$$\Phi(G) \equiv \rho(\varepsilon - 1 - t_\pi) - f(G)(1 - G).$$

Then, we investigate the sign of $\Phi(G^*)$ under $\dot{N} > 0$ (i.e., $0 \leq G^* < \hat{G}$). Appendix E shows that the sign of $\Phi(G^*)$ is negative when $0 \leq G^* < \hat{G}$. Thus, the sign of $\partial U^* / \partial t_\pi \big|_{G^* given}$ is

[20]We derive this condition later.

also negative. Analogous to the discussion of the growth-maximization policy, a decrease in the profit tax raises the graph of the RHS of (30) (see Figure 4-1) and the optimum can be attained by setting the profit tax to zero.

Finally, we derive the welfare-maximizing public research spending level. By setting $t_\pi = 0$ and using (25), (28), and (31), we obtain

$$\frac{\partial U^*}{\partial G^*} = \frac{1}{\rho E^*}\left\{-1 - \rho\frac{f'(G^*)}{(f(G^*))^2}\right\} + \frac{1}{\rho^2\varepsilon(\varepsilon-1)}\Big\{f'(G^*)(1-G^*) - f(G^*)\Big\} \equiv \frac{1}{\rho}\Omega(G^*). \tag{32}$$

From Figure 4, when welfare-maximizing public research spending exists, the sign of $\partial U^*/\partial G^*$ at $G^* = 0$ becomes positive (i.e., the growth effect is sufficiently large at $G^* = 0$). Hence, the existence condition of positive welfare-maximizing public research spending becomes as follows:

$$\Omega(0) = \frac{1}{1+\frac{\rho}{f(0)}}\left\{-1 - \rho\frac{f'(0)}{(f(0))^2}\right\} + \frac{1}{\rho\varepsilon(\varepsilon-1)}\Big(f'(0) - f(0)\Big) > 0. \tag{33}$$

When ε and ρ are sufficiently small, and $f'(0)$ is sufficiently large, $\Omega(0) > 0$ holds (see Appendix F for more detail). Let us define G_w by $\Omega(G_w) = 0$. In this case, $\partial U^*/\partial G^* \gtreqless 0$ holds when $G^* \lesseqgtr G_w$ (see Figure 4-1), and thus G_w represents the welfare-maximizing public research spending. However, if $\Omega(0) \leq 0$, then $\partial U^*/\partial G^* < 0$ holds (see Figure 4-2). In this case, $G^* = 0$ maximizes the welfare. Furthermore, we can compare the welfare-maximizing level of public research spending with the growth-maximizing level. Substituting $G^* = G_g$ into (32) yields

$$\left.\frac{\partial U^*}{\partial G^*}\right|_{G^*=G_g} = \frac{1}{\rho E^*}\left\{-1 - \rho\frac{f'(G_g)}{(f(G_g))^2}\right\} < 0.$$

Therefore, we can see that the welfare-maximizing level of public research spending is lower than the growth-maximizing level. In summary, we can state the following proposition:

Proposition 4

If $\Omega(0) > 0$, the policy mix of $t_\pi = 0$ and $\bar{B} = \Lambda(G_w)$ maximizes welfare. In addition, the welfare-maximizing level of public research spending is below the growth-maximizing level.

In line with Proposition 3, Proposition 4 states that the government should not levy a tax on corporate profit and finance its spending through asset income/consumption taxation. From (30), welfare is driven by household consumption expenditure and the growth of differentiated goods. Thus, to maximize the growth effect, the tax rate on profit should be zero.

In the Barro model, when the government only imposes a tax on household income, the growth-maximizing policy is equivalent to the welfare-maximizing policy.[21] However, in this

[21] If the government can use consumption or lump-sum tax, welfare is increasing in government spending as well as the growth rate, as discussed in Section 5. That is, there is no welfare-maximizing government spending level.

study, the welfare-maximizing public research spending, G_w, is lower than the growth-maximizing public research spending, G_g. As mentioned above, higher public research spending decreases the household consumption expenditure. Therefore, there exists a trade-off between household consumption expenditure and the growth of differentiated goods. When the government increases public research spending to maximize the growth rate, this harms household consumption expenditure, and thus the welfare-maximizing public research spending level is below the growth-maximizing level.

7 Conclusion

In this study, we developed an R&D-based growth model to examine the effects of public research spending on private R&D activities. We find that a zero-profit tax maximizes growth and welfare at the steady state. Furthermore, the growth-maximizing public research spending level is above the welfare-maximizing level. We also find that the equilibrium is indeterminate when government debt, consumption, and profit tax are sufficiently small and asset income tax is sufficiently large.

Finally, this study is a first-generation R&D-based growth model that exhibits scale effects. For future research, it would be interesting to consider the non-scale growth model.[22]

Appendix

A. Derivation of the Jacobian matrix

From (17) and (18), we obtain

$$\dot{G} = \frac{f(G)}{f'(G)}\left[\Big(1-\frac{t_\pi}{\varepsilon}\Big)f(G)E - f(G)(1-G) + \frac{1}{\Delta}\left\{\Big(t_E+\frac{t_\pi}{\varepsilon}\Big)E - G\right\}\right],$$
$$\dot{E} = E\left[\frac{1}{\Delta}\left\{G-\Big(t_E+\frac{t_\pi}{\varepsilon}\Big)E\right\}-\rho\right].$$

Approximating these equations linearly in the neighborhood of the steady states, the following elements of the Jacobian matrix yield

$$J_{GG} = \frac{f(G^*)}{f'(G^*)}\left[\Big(1-\frac{t_\pi}{\varepsilon}\Big)f'(G^*)E^* - f'(G^*)(1-G^*) - f(G^*)\right] + \frac{f(G^*)}{f'(G^*)(\Delta^*)^2}\left[-\Delta^* + \left\{\Big(t_E+\frac{t_\pi}{\varepsilon}\Big)E^* - G^*\right\}\frac{t_A}{1-t_A}\frac{f'(G^*)}{(f(G^*))^2}\right],$$

[22]See Jones (1995) for a more detailed discussion of scale effects in R&D-based growth models.

$$J_{GE} = \frac{f(G^*)}{f'(G^*)}\left\{\left(1-\frac{t_\pi}{\varepsilon}\right)f(G^*) + \frac{1}{\Delta^*}\left(t_E + \frac{t_\pi}{\varepsilon}\right)\right\},$$

$$J_{EG} = \frac{E^*}{(\Delta^*)^2}\left[\Delta^* + \left\{G^* - \left(t_E + \frac{t_\pi}{\varepsilon}\right)E^*\right\}\frac{t_A}{1-t_A}\frac{f'(G^*)}{(f(G^*))^2}\right], and$$

$$J_{EE} = -\left(t_E + \frac{t_\pi}{\varepsilon}\right)\frac{E^*}{\Delta^*}.$$

From (20), we obtain

$$\left(1-\frac{t_\pi}{\varepsilon}\right)f'(G^*)E^* - f'(G^*)(1-G^*) - f(G^*) = 0. \tag{A.1}$$

By using (19) and (A.1), we can rewrite J_{ij} $(i, j = G, E)$ as follows:

$$\begin{aligned}
J_{GG} &= -\frac{f(G^*)}{f'(G^*)\Delta^*}\left\{1 + \rho\frac{t_A}{1-t_A}\frac{f'(G^*)}{(f(G^*))^2}\right\},\\
J_{GE} &= \frac{f(G^*)}{f'(G^*)\Delta^*}\left\{\left(1-\frac{t_\pi}{\varepsilon}\right)f(G^*)\Delta^* + \left(t_E + \frac{t_\pi}{\varepsilon}\right)\right\},\\
J_{EG} &= \frac{E^*}{\Delta^*}\left\{1 + \rho\frac{t_A}{1-t_A}\frac{f'(G^*)}{(f(G^*))^2}\right\},\\
J_{EE} &= -\left(t_E + \frac{t_\pi}{\varepsilon}\right)\frac{E^*}{\Delta^*}.
\end{aligned}$$

B. Dynamics and stability when $\Delta^* = 0$

From the definition of Δ^*, the case where $\Delta^* = 0$ implies that $\bar{B} > 0, t_A > 0$ or $\bar{B} = t_A = 0$. First, we examine the case where $\bar{B} > 0, t_A > 0$. We define G_Δ as $f(G_\Delta) = t_A/(1-t_A)\bar{B}$, that is, G_Δ satisfies $\Delta = 0$. When $G = G_\Delta$, (9) and (13) yield $E_\Delta \equiv G_\Delta/(t_E + \frac{t_\pi}{\varepsilon})$. Therefore, $\dot{G} = \dot{E} = 0$ holds, and the economy jumps to the steady state immediately. This result seems to imply that there are multiple steady states; G^* and G_Δ. However, G_Δ cannot hold the no-arbitrage condition unless $G^* = G_\Delta$. We rearrange equation (21) as follows:

$$\underbrace{\bar{B} - \frac{t_A}{1-t_A}\frac{1}{f(G)}}_{-\Delta} = \frac{t_E + \frac{t_\pi}{\varepsilon}}{1-\frac{t_\pi}{\varepsilon}}\frac{1}{f(G)} + \frac{1}{\rho}\left(\frac{t_E + \frac{t_\pi}{\varepsilon}}{1-\frac{t_\pi}{\varepsilon}} - \frac{1+t_E}{1-\frac{t_\pi}{\varepsilon}}G\right). \tag{B.1}$$

As shown in Figure B.1, the intersection of the LHS and RHS of (B.1) determines G^*. When $G^* \neq G_\Delta$, the following inequality holds:

$$\frac{t_E + \frac{t_\pi}{\varepsilon}}{1-\frac{t_\pi}{\varepsilon}}\frac{1}{f(G_\Delta)} + \frac{1}{\rho}\left(\frac{t_E + \frac{t_\pi}{\varepsilon}}{1-\frac{t_\pi}{\varepsilon}} - \frac{1+t_E}{1-\frac{t_\pi}{\varepsilon}}G_\Delta\right) > 0.$$

Thus, this inequality implies that the no-arbitrage condition cannot hold.

Second, we investigate the case where $\bar{B} = t_A = 0$. From (9) and (13), we obtain

$$G = \left(t_E + \frac{t_\pi}{\varepsilon}\right)E. \tag{B.2}$$

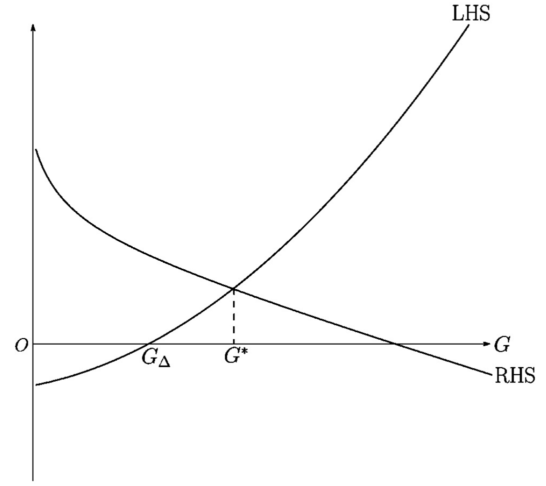

Figure B.1

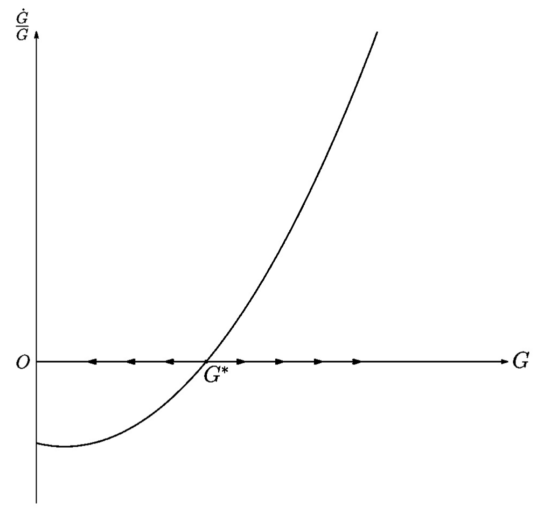

Figure B.2

Equations (6), (16), and (B.2) yield the following autonomous dynamic system, with respect to G:

$$\left(1+\frac{f'(G)G}{f(G)}\right)\frac{\dot{G}}{G}=\frac{1-\frac{t_\pi}{\varepsilon}}{t_E+\frac{t_\pi}{\varepsilon}}f(G)G-f(G)(1-G)-\rho\equiv\psi(G).$$

Note that $\psi(0)<0$ and $\psi(G)$ is increasing in G (or U-shaped). As shown in Figure B.2, the steady state is unstable, that is, the economy jumps to the steady state immediately.

C. Proof of Lemma

We examine the effects of the fiscal variables on G^* through changes in the fiscal variables. The effect of $\bar{B}$ can easily be examined by using (21). When $\bar{B}$ increases, the horizontal line goes up, as shown in Figure C.1. Thus, G^* decreases.

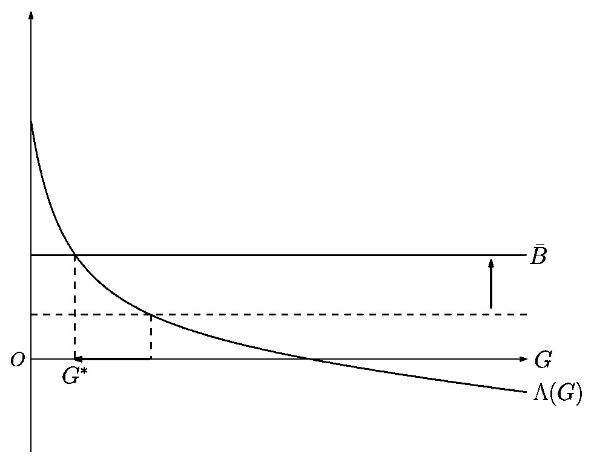

Figure C.1

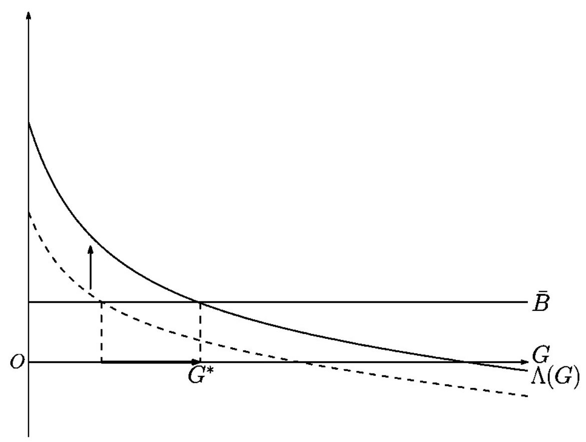

Figure C.2

Differentiating $\Lambda(G)$ with respect to t_A, t_E, and t_π, we obtain

$$\begin{aligned}
\left.\frac{\partial \Lambda}{\partial t_A}\right|_{G\ given} &= \frac{1}{(1-t_A)^2}\frac{1}{f(G)} > 0,\\
\left.\frac{\partial \Lambda}{\partial t_E}\right|_{G\ given} &= \frac{1}{1-\frac{t_\pi}{\varepsilon}}\left\{\frac{1}{f(G)} + \frac{1}{\rho}(L-G)\right\} > 0,\\
\left.\frac{\partial \Lambda}{\partial t_\pi}\right|_{G\ given} &= \frac{1+t_E}{\varepsilon\left(1-\frac{t_\pi}{\varepsilon}\right)^2}\left\{\frac{1}{f(G)} + \frac{1}{\rho}(L-G)\right\} > 0.
\end{aligned}$$

When the tax rates t_i $(i = A, E, \pi)$ increase, the graph of the RHS of (21) goes up (see Figure C.2). Therefore, G^* increases.

D. Effects of Δ^* through changes in fiscal variables

Differentiating Δ^* with respect to t_i $(i = E, \pi)$ yields

$$\frac{\partial \Delta^*}{\partial t_i} = -\frac{t_A}{1-t_A}\frac{f'(G^*)}{(f(G^*))^2}\frac{\partial G^*}{\partial t_i}. \quad (i = E, \pi)$$

By using the Lemma, we obtain $\partial\Delta^*/\partial t_i < 0$ $(i = E, \pi)$. To study the effect of $\bar{B}$ and t_A, we rewrite Δ^* using (22), as follows:

$$\Delta^* = \frac{1}{\rho(1-\frac{t_\pi}{\varepsilon})}\left[-\left(t_E + \frac{t_\pi}{\varepsilon}\right)\rho\frac{1}{f(G^*)} - \left(t_E + \frac{t_\pi}{\varepsilon}\right) + (1+t_E)G^*\right]. \tag{D.1}$$

Differentiating Δ^* with respect to $\bar{B}$ and t_A, we obtain

$$\begin{aligned}
\frac{\partial \Delta^*}{\partial \bar{B}} &= \frac{1}{\rho(1-\frac{t_\pi}{\varepsilon})}\left[\left(t_E + \frac{t_\pi}{\varepsilon}\right)\rho\frac{f'(G^*)}{(f(G^*))^2} + (1+t_E)\right]\frac{\partial G^*}{\partial \bar{B}},\\
\frac{\partial \Delta^*}{\partial t_A} &= \frac{1}{\rho(1-\frac{t_\pi}{\varepsilon})}\left[\left(t_E + \frac{t_\pi}{\varepsilon}\right)\rho\frac{f'(G^*)}{(f(G^*))^2} + (1+t_E)\right]\frac{\partial G^*}{\partial t_A}.
\end{aligned}$$

From the Lemma, $\partial\Delta^*/\partial\bar{B} < 0$ and $\partial\Delta^*/\partial t_A > 0$ both hold.

E. Sign of $\Phi(G^*)$

We investigate the sign of $\Phi(G^*)$ under $\dot{N} > 0$ (i.e., $0 \le G^* < \hat{G}$). Differentiating $\Phi(G^*)$ with respect to G^*, we obtain

$$\Phi'(G^*) = -f'(G^*)(1-G^*) + f(G^*) \lesseqgtr 0 \iff G^* \lesseqgtr G_g.$$

Therefore, $\Phi(G^*)$ and G^* follow a U-shape. Assumption (24) yields $f(0) > (\varepsilon-1)\rho/(1-t_\pi)$, and thus we obtain

$$\Phi(0) < -\rho t_\pi - \rho(\varepsilon-1)\frac{t_\pi}{1-t_\pi} < 0.$$

The definition of $\hat{G}$ yields

$$f(\hat{G})(1-\hat{G}) = \frac{(\varepsilon-1)\rho}{1-t_\pi}. \tag{E.1}$$

By using (E.1), we obtain

$$\Phi(\hat{G}) = -\frac{\rho t_\pi(\varepsilon - t_\pi)}{1-t_\pi} < 0.$$

Thus, the sign of $\Phi(G^*)$ is negative when $0 \leq G^* < \hat{G}$.

F. Condition of $\Omega(0) > 0$

From (33), the necessary condition for $\Omega(0) > 0$ is $f'(0) > f(0)$. In this case, $\partial\Omega(0)/\partial\varepsilon < 0$ apparently holds. Differentiating $\Omega(0)$ with respect to ρ and $f'(0)$ yields

$$\frac{\partial\Omega(0)}{\partial\rho} = -\left[\frac{1}{\big(f(0)+\rho\big)^2} + \frac{1}{\rho\varepsilon(\varepsilon-1)}\right]\Big(f'(0) - f(0)\Big) < 0,$$
$$\frac{\partial\Omega(0)}{\partial f'(0)} = \frac{-\rho^2\varepsilon(\varepsilon-1) + (f(0))^2 + \rho f(0)}{\rho\varepsilon(\varepsilon-1)\big\{(f(0))^2 + \rho f(0)\big\}}.$$

From the assumption in (24), we obtain $-\rho^2\varepsilon(\varepsilon-1)+(f(0))^2+\rho f(0) > 0$. Thus, $\partial\Omega(0)/\partial f'(0) > 0$ holds. From these results, when ε and ρ are sufficiently small and $f'(0)$ is sufficiently large, $\Omega(0) > 0$ holds.

References

[1] Aghion,P., Howitt,P., 1992. A model of growth through creative destruction. Econometrica 60, 323-351.

[2] Arrow,K.J., Kurz,M., 1970. Public investment, the rate of return and optimal fiscal policy. Johns Hopkins University Press, Baltimore, MD.

[3] Barro,R.J., 1990. Government spending in a simple model of endogenous growth. Journal of Political Economy 98, 103-125.

[4] Bruce,N., Turnovsky,S.J., 1999. Budget balance, welfare, and the growth rate: "Dynamic Scoring" of the long-run government budget. Journal of Money, Credit and Banking 31, 162—186.

[5] Caloghirou,Y., Tsakanikas,A., Vonortas,N.S., 2001. University-industry cooperation in the context of the European Framework Programmes. Journal of Technology Transfer 26, 153-161.

[6] Fisher,W.H., Turnovsky,S.J., 1998. Public investment, congestion, and private capital accumulation. Economic Journal 108, 399-413.

[7] Futagami,K., Iwaisako,T., Ohdoi,R., 2008. Debt policy rule, productive government spending, and multiple growth paths. Macroeconomic Dynamics 12, 445-462.

[8] Futagami,K., Morita,Y., Shibata,A., 1993. Dynamic analysis of an endogenous growth model with public capital. Scandinavian Journal of Economics 95, 607-625.

[9] Ghosh,S., Mourmouras,I.A., 2004. Endogenous growth, welfare and budgetary regimes. Journal of Macroeconomics 26, 623-635.

[10] Glomm,G., Ravikumar,B., 1994. Growth-inequality trade-offs in a model with public sector R&D. Canadian Journal of Economics 27, 484-493.

[11] Greiner,A., Semmler,W., 2000. Endogenous growth, government debt and budgetary regimes. Journal of Macroeconomics 22, 363-384.

[12] Grossman,G.M., Helpman,E., 1991. Innovation and growth in the global economy. MIT Press, Cambridge, MA.

[13] Guo,J.T., Harrison,S.G., 2008. Useful government spending and macroeconomic (in)stability under balanced-budget rules. Journal of Public Economic Theory 10, 383-398.

[14] Irmen,A., Kuehnel,J., 2009. Productive government expenditure and economic growth. Journal of Economic Surveys 23, 692-733.

[15] Jones,C.I., 1995. R&D-based models of economic growth. Journal of Political Economy 103, 759-784.

[16] Kamiguchi,A., Tamai,T., 2011. Can productive government spending be a source of equilibrium indeterminacy? Economic Modeling 28, 1335-1340.

[17] Lööf, H., Broström, A., 2008. Does knowledge diffusion between university and industry increase innovativeness? Journal of Technology Transfer 33, 73-90.

[18] Maebayashi,N., Hori,T., Futagami,K., 2013. Dynamic analysis of reductions in public debt in an endogenous growth model with public capital. Discussion Papers in Economics and Business 12-08-Rev, Osaka University.

[19] Mansfield,E., 1991. Academic research and industrial innovation. Research Policy 20, 1-12.

[20] Morimoto,K., Hori,T., Maebayashi,N., Futagami,K., 2013. Fiscal sustainability, macroeconomic stability, and welfare under fiscal discipline in a small open economy. Discussion Papers in Economics and Business 13-07, Osaka University.

[21] Park,W.G., 1998. A theoretical model of government research and growth. Journal of Behavior and Organization 34, 69-85.

[22] Romer,P.M., 1990. Endogenous technological change. Journal of Political Economy 98, 71-102.

[23] Yakita,A., 2008. Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. Journal of Public Economics 92, 897-914.