# Discussion Papers In Economics And Business

Valuation of sequential R&D investment under technological, market, and rival preemption uncertainty

Michi NISHIHARA

Discussion Paper 14-13

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

# Valuation of sequential R&D investment under technological, market, and rival preemption uncertainty

Michi NISHIHARA

Discussion Paper 14-13

April 2014

Graduate School of Economics and
Osaka School of International Public Policy (OSIPP)
Osaka University, Toyonaka, Osaka 560-0043, JAPAN

# Valuation of sequential R&D investment under technological, market, and rival preemption uncertainty*

Michi NISHIHARA†

**Abstract**

We develop a real options model for evaluating and optimizing an R&D project. The model can capture key features of R&D, including research duration, growth opportunity, debt financing, and uncertainty of technological, demand market, and rival preemption. Nevertheless, it is computationally tractable and thus helps practitioners to evaluate various cases of R&D investment. Further, by analyzing the model with a wide range of parameter values, we unveil the interactions of key R&D features. The effect of duration on investment depends on whether there is a possibility of rival preemption. Higher uncertainty of research duration speeds up project inception in the presence of rival preemption. Higher uncertainty of technological success, combined with a growth opportunity embedded in the R&D project, accelerates investment. Debt financing can greatly decrease time lag between the first stage project and growth project. These results are consistent with the empirical evidence.

**JEL Classifications Code:** G31; G33; 032.

**Keywords:** R&D; technological uncertainty; investment lag; real options; compound option.

*This is the first version on 1 April, 2014.

†Corresponding Author. Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan, E-mail: nishihara@econ.osaka-u.ac.jp, Phone: 81-6-6850-5242, Fax: 81-6-6850-5277

# 1 Introduction

Research and development (R&D) investment does not only bring the progress of society via innovation and spillover effects but also is a key determinant of a firm's long-term growth. It is critical for a firm's management to accurately evaluate and execute an R&D project. This paper contributes both theory and practice by developing a tractable model for evaluating and optimizing R&D investment as well as unveiling interactions of several features associated with R&D investment.

The difficulties of R&D management lie in that three types of uncertainty are inevitably embedded in R&D projects. The first is technological uncertainty. Technological specifications, time schedule, and budget are planned before project initiation. However, in many cases, the outcome does not go successfully as planned (e.g. Raz, Shenhar, and Dvir (2002)). The risks of technological success, research duration, and investment costs are called technological uncertainty. The second is market uncertainty. This stands for uncertain cash flow, which a newly developed technology will generate. The dynamics of cash flow is not deterministic but affected by product-specific and macroeconomic shocks on demand in the product market. The third is a risk of rival preemption. For instance, if a competitor takes out a patent for a technology first, an R&D project concerning the technology will be aborted.

In practice, it is challenging to evaluate an R&D project and make a managerial decision of R&D involving these risks (e.g. Raz, Shenhar, and Dvir (2002)). Although the Net Present Value (NPV) method remains dominant for project valuation in the real world, there is an observed growing trend that the real options method[1] is adopted as a complement (e.g. Hartmann and Hassan (2006) and Baker, Dutta, and Saadi (2011)). Without doubt, the academic literature argues that the real options methodology adds value to project valuation involving high uncertainty and managerial flexibility, such as R&D projects. There have been a lot of case studies that apply the real options method to R&D project valuation (e.g., Perlitz, Peske, and Schrank (1999), Loch and Bode-Greuel (2001), Lee and Paxson (2001), and Cassimon, Backer, Engelenc, Wouwee, and Yordanovf (2011)).

Instead of examining a case study for a specific company or project, this paper develops a generic and tractable model of R&D investment so that one can analyze various cases of R&D investment with it. We extend the framework of American compound option as follows. Consider a firm that has the timing option to initiate an R&D project by paying a sunk cost. The project will take time to complete, and after completion, the developed technology will generate cash flow and a growth opportunity. When the firm exercises the growth option by paying a sunk cost, it will increase cash flow from the technology.

[1] The real options method is no longer a new concept because it has already been developed for more than thirty years. For details, we recommend a recent textbook Guthrie (2009) to both academicians and practitioners.

We consider this sort of compound option model, because growth opportunities, such as expanding production, starting a license business, and applying the new technology to other products, are frequently associated with R&D investment (e.g., Loch and Bode-Greuel (2001), Ho, Tjahjapranata, and Yap (2006), and Cassimon, Backer, Engelenc, Wouwee, and Yordanovf (2011)).

Furthermore, our model takes into consideration the three types of uncertainty, namely, technological, market, and rival preemption uncertainty. The level of technological success that influences cash flow, the lag between project inception and completion (henceforth research duration), and total costs are not deterministic but random, and the firm knows only their prior distributions. These technological risks are specific of R&D project valuation, but the existent real options models miss any of the three points. We do not propose any new setup with regard to market uncertainty. Instead, based on the standard literature, we assume that the dynamics of cash flow from the technology follows a stochastic process. We assume that the project value is potentially eliminated by rival preemption before project completion. The rival preemption occurs randomly, and the firm knows only its distribution. In other words, the firm does not possess the full information of its rival firms' R&D progress. We also extend the base model to a case with debt financing because an increasing number of papers, both theoretically and empirically, have pointed out the significance of financing sources.[2]

Before elaborating on the results, we differentiate our model from the most related and dominant models by Bar-Ilan and Strange (1998) and Weeds (2002). The model by Bar-Ilan and Strange (1998), like our model, can capture the effects of a growth opportunity and investment lag, but it does not include uncertainty of technological success, total costs, duration, and rival preemption. Weeds (2002) proposed a model with uncertain research duration and rival preemption, but the model does not include a growth opportunity and uncertainty of technological success and total costs. In addition, her assumptions of full information of rival firms and Poisson distributed duration are not very practical. We also note that the above models, unlike our model, do not include a case with debt financing. Thus, our model better helps evaluating an R&D project involving the three types of uncertainty compared to the previous models.

Our analysis of the model yields several empirical implications. First, we unveil interactions between research duration and rival preemption. In the absence of rival preemption, the firm tends to accelerate investment in the R&D project with longer duration because market demand at completion time is expected to be higher. This result is known in investment timing models with investment lags (e.g., Bar-Ilan and Strange (1996) and Bar-Ilan and Strange (1998)). Notably, we show that a slight possibility of rival preemption changes the duration effect. Longer duration increases the possibility of rival

[2] An incomplete list includes Mauer and Sarkar (2005), Sundaresan and Wang (2007), and Nishihara and Shibata (2013) in the real options studies, and Ho, Tjahjapranata, and Yap (2006), Brown, Fazzari, and Petersen (2009), and Hall and Lerner (2009) in the empirical studies of R&D.

preemption and decreases the project value. As this negative effect is much stronger than the positive effect, the effect of duration on investment becomes negative in the presence of rival preemption.

Next, we show that higher uncertainty of research duration and technological success drives the firm to launch the R&D project earlier. This leads to the lesson that uncertainty-investment sensitivity depends on the type of uncertainty. Indeed, the effect of market uncertainty on investment timing tends to be negative because higher market uncertainty increases the incentive for the firm to delay investment and receive additional information. On the other hand, technological uncertainty will be dissolved, not by waiting but by finishing the project. The R&D project value is likely to be convex with respect to research duration and levels of technological success. The former is caused primarily by the possibility of rival preemption, whereas the latter stems mainly from the growth option embedded in the R&D project. Because of this convexity, the higher uncertainty of research duration and technological success increases the project value, and hence, encourages investment in the R&D project. Our result is novel and can potentially account for empirical findings by Driver, Temple, and Urga (2006). They found that industries with high R&D intensity tend to indicate a positive effect of uncertainty on investment.

As for debt financing, we have empirical implications as follows. Access to debt financing for the growth project increases the project value and accelerates investment. This is straightforward and consistent with previous results in the literature (e.g., Hennessy (2004), Mauer and Sarkar (2005), and Sundaresan and Wang (2007)). More interestingly, the effects of debt financing on the first stage project are weaker than that on the growth project, which implies that the firm can reap the growth opportunity from the R&D project earlier by debt financing. A large number of empirical studies showed that large firms have advantages over small firms in profits from R&D projects (e.g., Ho, Tjahjapranata, and Yap (2006)). Our result may be related to the empirical finding because larger firms, due to less financing costs, are more likely to take the optimal capital structure.

The remainder of this paper is organized as follows. Section 2 illustrates the model setup and solutions. In Section 3, we exercise numerical analysis and provide empirical implications. In particular, we focus on interactions between research duration and rival preemption in Section 3.2, interactions between the growth option and uncertainty of technological success in Section 3.3, and effects of leverage in Section 3.4. Section 4 briefly summarizes the paper.

# 2 The model

## 2.1 Setup

We consider a firm that has an option to initiate an R&D project by paying the sunk cost $I_0$. The project will take $T_1$ years until completion. We call $T_1$ research duration. After completion, the firm will receive an instantaneous cash flow $A_1X(t)$ at time $t$.[3] Furthermore, we assume that the firm has a growth opportunity, such as expanding production, starting a license business, and applying the new technology to other products (e.g., Loch and Bode-Greuel (2001), Ho, Tjahjapranata, and Yap (2006), and Cassimon, Backer, Engelenc, Wouwee, and Yordanovf (2011)). After the firm exercises the growth option by paying the sunk cost $I_2$, it will receive an increased cash flow $A_1A_2X(t)$.

Our model includes three types of uncertainty. First at all, we represent market uncertainty by the cash flow $X(t)$, which dynamically changes by project-specific and macroeconomic shocks on demand in the product market. Following the standard literature, we assume that $X(t)$ follows a geometric Brownian motion:

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t)\ (t > 0), \quad X(0) = x,$$

where $B(t)$ denotes the standard Brownian motion defined in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $\mu, \sigma(> 0)$ and $x(> 0)$ are constants. For convergence, we assume that $r > \mu$, where a positive constant $r$ is the discount rate. For the economic rationale behind these assumptions, refer to standard textbooks such as Dixit and Pindyck (1994) and Guthrie (2009).

Second, we represent technological uncertainty by a random vector $(A_1, T_1, I_1)$. Technical specifications, time schedule, and total costs, which are planned and estimated ex ante, may not be accomplished ex post (e.g., Raz, Shenhar, and Dvir (2002)). To deal with the technological risks, we consider the level of technological success, $A_1$, and research duration $T_1$, as random variables. The firm will observe the realized value of $A_1$ at completion time. At the same time, the firm will observe an extra cost $I_1$. For tractability, we assume that $(A_1, T_1, I_1)$ are independent of $X(t)$, which means that technological and market risks are irrelevant. There is less technological uncertainty for the growth project, and then, we assume that $A_2(> 1)$ and $I_2$ for the growth investment are positive constants.

The last uncertainty lies in rival preemption. The project value will be greatly destroyed when a competitor completes the similar product first, especially in competitions for patents and standardizations (e.g., Weeds (2002)). We assume that the firm does not have information of the R&D progress of rival firms. Instead, the firm knows that

[3] Some papers distinguish the lag between project inception and completion (the gestation lag) and project completion and commercial application (the application lag) (e.g., Pakes and Schankerman (1984)). For simplicity, we assume that the total lag is equal to $T_1$.

rival preemption may occur following an exponential distribution (Poisson arrival) with intensity $\lambda$, where $\lambda$ is a positive constant. Technically, until project completion, the cash flow $X(t)$ will be potentially killed at an instantaneous rate $\lambda \mathrm{d}t$.

The model differs from the compound real options model by Bar-Ilan and Strange (1998) in the sense that we incorporate technological uncertainty $(A_1, T_1, I_1)$ and rival preemption $\lambda \mathrm{d}t$.[4] Weeds (2002) also examined rival preemption and uncertainty of research duration. The model, however, does not include technological uncertainty $(A_1, I_1)$ and a growth opportunity.[5] The model adopts a game-theoretic framework with full information and assumes that research duration follows an exponential distribution. Although the setup is sufficient to provide economic implications, it seems almost impossible to apply the model to a real-world case. Actually, no firm wishes to inform its R&D progress to competitors, and no one plans Poisson distributed time schedule. In this paper, we model using a Poisson arrival not research duration, which can be estimated to a degree but a rival's R&D success, which will be an unexpected event.

## 2.2 Base problem

In this subsection, we provide technical instructions for how to evaluate and optimize the R&D project with the three types of uncertainty. We formulate the problem of finding the project value and the optimal policy as a two stage optimal stopping problem. Similar to Bar-Ilan and Strange (1998), we need to solve the problem backward. Suppose that the investment time $T^*$, the level of technological success, $A_1$, research duration $T_1$, and extra cost $I_1$ in the first stage project are all known. At time $s$, which is later than completion time $T^* + T_1$, the problem of finding the optimal investment time $T^{**}$ for the growth project is expressed as the following optimal stopping problem:

$$V_2(X(s), A_1) = \sup_{T^{**} \geq s} \mathbb{E}^{X(s)}[\int_s^{T^{**}} \underbrace{\mathrm{e}^{-r(t-s)}(1-\tau)A_1 X(t)}_{\text{cash flow before growth investment}} \mathrm{d}t + \int_{T^{**}}^{\infty} \underbrace{\mathrm{e}^{-r(t-s)}(1-\tau)A_1 A_2 X(t)}_{\text{cash flow after growth investment}} \mathrm{d}t - \mathrm{e}^{-r(T^{**}-s)} I_2], \tag{1}$$

where the investment time $T^{**}$ is optimized over all stopping times later than $s$. The notation $\mathbb{E}^{X(s)}[\cdot]$ denotes the expectation conditional on $X(s)$, and for later use we introduce the corporate tax rate $\tau$, which is a positive constant. The value function $V_2(X(s), A_1)$ in the problem (3) stands for the project value at time $s$ between completion of the first stage and initiation of the growth project.

[4]As a minor difference, Bar-Ilan and Strange (1998) does not consider profits before completion of the second stage. They consider duration of the second stage investment, but it is not essential because there is no investment after completion of the second stage.

[5]Nishihara and Ohyama (2008) extended Weeds (2002) to a case involving two alternative technologies, but the model, like Weeds (2002), does not include either technological uncertainty or a growth opportunity.

Because the firm's expected profits after the investment time $T^{**}$ are equal to

$$\mathbb{E}^{X(T^{**})}[\int_{T^{**}}^{\infty} \mathrm{e}^{-r(t-T^{**})}(1-\tau)A_1A_2X(t)\mathrm{d}t] = \frac{(1-\tau)A_1A_2}{r-\mu}X(T^{**}), \tag{2}$$

we can rewrite (3) as

$$V_2(X(s), A_1) = \underbrace{\frac{(1-\tau)A_1}{r-\mu}X(s)}_{\text{NPV}} + \underbrace{U_2(X(s), A_1)}_{\text{growth option value}}, \tag{3}$$

where $U_2(X(s), A_1)$ is the growth option value expressed by

$$\begin{aligned}
&U_2(X(s), A_1) \\
&= \sup_{T^{**}\geq s} \mathbb{E}^{X(s)}[\mathrm{e}^{-r(T^{**}-s)}\left(\frac{(1-\tau)A_1(A_2-1)}{r-\mu}X(T^{**}) - I_2\right)] \\
&= \begin{cases} \left(\dfrac{(1-\tau)A_1(A_2-1)x^{**}(A_1)}{r-\mu} - I_2\right)\left(\dfrac{X(s)}{x^{**}(A_1)}\right)^{\beta} & (X(s) < x^{**}(A_1)) \\[2ex] \dfrac{(1-\tau)A_1(A_2-1)X(s)}{r-\mu} - I_2 & (X(s) \geq x^{**}(A_1)). \end{cases}
\end{aligned} \tag{4}$$

The investment trigger $x^{**}(A_1)$ and the positive characteristic root $\beta$ are defined by

$$x^{**}(A_1) = \frac{\beta(r-\mu)I_2}{(\beta-1)(1-\tau)A_1(A_2-1)}. \tag{5}$$

and

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} (> 1), \tag{6}$$

respectively. For $A_1 = 0$ (failure of the first stage), we define $x^{**}(A_1) = \infty$ and $U_2(X(s)) = 0$. The firm's optimal policy, $T^{**} = \inf\{t \geq s \mid X(t) \geq x^{**}(A_1)\}$, is called the threshold policy, which means that the firm invests in the growth project as soon as the cash flow $X(t)$ hits the threshold $x^{**}(A_1)$. Note that if $X(T^* + T_1) \geq x^{**}(A_1)$ is satisfied, the firm proceeds to the second stage immediately after completion of the first stage.

Next, we turn to a problem of evaluating and optimizing the first stage investment. At this moment, the level of technological success, $A_1$, research duration $T_1$, and the cost $I_1$ are random variables, and the prior distributions of $(A_1, T_1, I_1)$ are known. At time $s(\geq 0)$, the project value is expressed as the value function of the optimal stopping problem as follows:

$$V_1(X(s)) = \sup_{T^*\geq s} \mathbb{E}^{X(s)}[\mathrm{e}^{-(r+\lambda)(T^*-s)}\left\{\mathrm{e}^{-(r+\lambda)T_1}V_2(X(T^*+T_1), A_1) - \mathrm{e}^{-rT_1}I_1 - I_0\right\}], \tag{7}$$

where $\lambda$ is the intensity of rival preemption and $V_2(X(T^*+T_1), A_1)$ is given by (3). In (7) the expectation is taken over all random variables $(X(T^*+T_1), A_1, T_1, I_1)$. The expected total costs are $I_0 + \mathbb{E}[\mathrm{e}^{-rT_1}I_1]$. As $(T_1, I_1)$ are independent of $X(t)$, we can remove $I_1$ by regarding the estimated total costs $I_0 + \mathbb{E}[\mathrm{e}^{-rT_1}I_1]$ as the initial cost $I_0$. From now on, without loss of generality, we assume that $I_1 = 0$.

Because the expected profits after the investment time $T^*$ are equal to

$$U_1(X(T^*)) = \mathbb{E}^{X(T^*)}[\mathrm{e}^{-(r+\lambda)T_1} V_2(X(T^* + T_1), A_1)], \tag{8}$$

we can reduce (7) to

$$V_1(X(s)) = \sup_{T^* \geq s} \mathbb{E}^{X(s)}[\mathrm{e}^{-(r+\lambda)(T^*-s)}(U_1(X(T^*)) - I_0)]. \tag{9}$$

Although we like to solve the optimal stopping problem (9), it is computationally hard. For, the payoff function in (9), $U_1(\cdot)$, is not analytically derived but numerically computed. Accordingly, it is impossible to check whether $U_1(\cdot)$ satisfies some regularity conditions, under which the optimal stopping time exists in a class of threshold policies.

We propose the following tractable method for computing the project value and optimal policy. In the first place, we restrict our attention within a class of threshold policies, which are of the form, $T^*_{x^*} = \inf\{t \geq 0 \mid X(t) \geq x^*\}$. This restriction is not very strong because $U_1(\cdot)$ is an increasing function. We now maximize the right-hand side of (9) by moving threshold $x^*$ rather than by moving stopping time $T^*$. At the initial time $s = 0$, the problem of finding the project value and the optimal policy is equal to[6]

$$\begin{aligned} V_1(x) &= \sup_{x^*} \mathbb{E}^x[\mathrm{e}^{-(r+\lambda)T^*_{x^*}}(U_1(X(T^*_{x^*})) - I_0)] \\ &= \sup_{x^* \geq x} \left(\frac{x}{x^*}\right)^{\tilde{\beta}} (U_1(x^*) - I_0), \end{aligned} \tag{10}$$

because we have $\mathbb{E}^x[\mathrm{e}^{-(r+\lambda)T^*_{x^*}}] = (x/x^*)^{\tilde{\beta}}$ and $X(T^*_{x^*}) = x^*$ for $x^* \geq x$. The characteristic root $\tilde{\beta}$ is defined by (6) replaced $r$ with $r + \lambda$.

Problem (10) is more tractable than problem (9) because it is an optimization problem of one-dimensional function. Nevertheless, in general, computing $U_1(\cdot)$ in (10) requires multiple integration, defined by (8), which makes problem (10) difficult to compute. When random variables $(A_1, T_1)$ have discrete distributions, (8) can be reduced to a single integral with respect to $X(t)$, which makes problem (10) computationally tractable. We suppose $N$ scenarios, in which $(A_1, T_1)$ take $(A_1^i, T_1^i)$ with probability $p^i$ for $i = 1, \ldots, N$. Then, problem (10) is reduced to

$$V_1(x) = \sup_{x^* \geq x} \left(\frac{x}{x^*}\right)^{\tilde{\beta}} \left(\sum_{i=1}^{N} p^i \mathbb{E}^{x^*}[\mathrm{e}^{-(r+\lambda)T_1^i} V_2(X(T_1^i), A_1^i)] - I_0\right), \tag{11}$$

because $(A_1, T_1)$ are independent of $X(t)$. Problem (11) is computable so that it can help practitioners to evaluate and optimize various cases of R&D investment. In section 3, we will conduct numerical analysis by solving problem (11).

[6]To avoid unnecessary disorder, we use the same notation $V_1(x)$ in (10) as that of (9). Strictly, the project value in (10) can be smaller than that of (9) because we restrict the firm's policies within the threshold policies.

## 2.3 Levered case

In this subsection, as a supplement, we extend the base model in Section 2.2 by allowing a firm to take the optimal capital structure in the growth project. In practice, an initial stage of an R&D project, involving quite high risks, is difficult to be financed by debt issuance (e.g., Brown, Fazzari, and Petersen (2009) and Hall and Lerner (2009)). After the success in the first stage, a firm is more likely to use debt financing for a growth project with less risks. For instance, Ho, Tjahjapranata, and Yap (2006) empirically showed that the firm size and financial leverage interactions influence growth opportunities from R&D investments. We explore the effects of the capital structure on R&D investment by supposing that the firm can optimally use debt financing for the growth project.

We build the levered setup on the dominant models by Fan and Sundaresan (2000), Goldstein, Ju, and Leland (2001), and Sundaresan and Wang (2007). Following Fan and Sundaresan (2000) and Sundaresan and Wang (2007), we assume that the firm will be able to avoid costly liquidation. Indeed, equity and debt holders negotiate and reduce coupon payments when the cash flow $X(t)$ is lower than a critical level. The firm's operation will return to normal when $X(t)$ restores beyond the critical level.

As in Section 2.2, we first consider the growth investment problem by supposing that the investment time $T_L^*$, the level of technological success, $A_1$, and research duration $T_1$ are all known. We denote the levered case by subscript $L$. The expected profits after the second investment time $T_L^{**}$ becomes

$$\frac{(1-\tau)A_1 A_2 \phi}{r-\mu} X(T_L^{**}) \tag{12}$$

in the levered case with the optimal capital structure. Note that (12) $= \phi\times$ (2). The multiplier $\phi$, which results from debt financing, is a positive constant defined by

$$\phi = 1 + \frac{\tau(1-\eta\alpha)}{(1-\tau(1-\eta))h}(>1), \tag{13}$$

where

$$h = \left[\frac{\beta(1-\gamma)}{\beta-\gamma}\right]^{-\frac{1}{\gamma}}(>1) \tag{14}$$

$$\gamma = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}(<0). \tag{15}$$

Constants $\alpha \in (0,1)$ and $\eta \in [0,1]$ denote the proportion of the firm value which is lost by liquidation and the bargaining power of equity holders in the renegotiation between equity and debt holders, respectively. For details in derivation of (12)–(15), refer to Fan and Sundaresan (2000) and Sundaresan and Wang (2007).

The growth option value, which corresponds to (4) in the unlevered case, becomes

$$
\begin{aligned}
&U_{2L}(X(s), A_1)\\
&= \sup_{T_L^{**} \geq s} \mathbb{E}^{X(s)}[\mathrm{e}^{-r(T_L^{**}-s)} \left( \frac{(1-\tau)A_1(A_2\phi - 1)}{r-\mu} X(T_L^{**}) - I_2 \right)]\\
&= \begin{cases} \left( \dfrac{(1-\tau)A_1(A_2\phi-1)x_L^{**}(A_1)}{r-\mu} - I_2 \right) \left( \dfrac{X(s)}{x_L^{**}(A_1)} \right)^{\beta} & (X(s) < x_L^{**}(A_1)) \\ \dfrac{(1-\tau)A_1(A_2\phi-1)X(s)}{r-\mu} - I_2 & (X(s) \geq x_L^{**}(A_1)), \end{cases}
\end{aligned} \tag{16}
$$

where the investment threshold $x_L^{**}(A_1)$ is defined by

$$
x_L^{**}(A_1) = \frac{\beta(r-\mu)I_2}{(\beta-1)(1-\tau)A_1(A_2\phi-1)}. \tag{17}
$$

Note that (17) $= (A_2 - 1)/(A_2\phi - 1) \times$ (5). Because of the optimal capital structure, the growth option becomes more valuable, and then, the growth investment takes place earlier. Because the first stage project is executed by all-equity financing, we can perform the same discussion as that of Section 2.2 for the first stage project by replacing $U_2(\cdot)$ with $U_{2L}(\cdot)$. In a similar fashion to (7)–(11), we can compute the project value and the optimal policy, and hence, we omit the description.

# 3 Numerical analysis and implications

## 3.1 Basic analysis

Our focus in this paper is not a case study of a specific R&D project. Instead, we show numerical results for a wide range of parameter values, demonstrating several properties of the project value and the optimal policy in a generic R&D project. We set the base parameter values as

$$
r = 0.08, \mu = 0.06, \sigma = 0.2, \tau = 0.15, \alpha = 0.3, \tag{18}
$$

following Leland (2004) and Sarkar (2008) based on the market data. There are several methods for estimating the market parameters and discount rate in a real options model (e.g., using the capital asset pricing model). For instance, Chapter 3 of Guthrie (2009) explains the details of standard calibration methods.

On the other hand, technological parameter values, such as levels of technological success, research duration, and investment costs, can be estimated by a project team. These values greatly differ over the project types and the industries. For example, in a project of developing a new drug, research duration is quite long, and the probability of technological success is quite low (Kellogg and Charnes (2000),Loch and Bode-Greuel (2001), Hartmann and Hassan (2006)). Considering the fact that the average duration is around 2 to 4 years in a majority of the literature (e.g., Pakes and Schankerman

(1984)), in the base case, we assume that $T_1$ takes a value in $\{2, 3, 4\}$ with probability 1/3. We consider three levels of technological success; $A_1 = 0$ (failure), $A_1 = 1$ (success as expected), and $A_1 = 2$ (success more than expected), and we assume that $A_1$ takes a value in $\{0, 1, 2\}$ with probability 1/3.

Following the standard literature of economics, we assume the convexity of the investment costs, i.e., $1/I_0 > (A_2 - 1)/I_2$. Note that if $1/I_0 \leq (A_2 - 1)/I_2$, the growth option is likely to be exercised immediately after the success of the first stage, which makes the analysis uninteresting. For expositional purposes, we set $I_0 = 10, A_2 = 2$, and $I_2 = 20$ in the base case. Then, $A_1(A_2 - 1)/I_2$ equals $1/20(< 1/I_0)$ and $1/10(= 1/I_0)$ for $A_1 = 1$ and $A_1 = 2$, respectively. This implies that the firm tends to postpone the growth project when the first stage results in the average success.

[Insert Table 1 and Figure 1 here.]

Table 1 presents the project values and investment triggers in the base case; the case with no growth option (we set $A_2 = 1$); the case in the presence of rival preemption (we set $\lambda = 0.2$); and the levered case (we set $\eta = 0.5$). We show the project value, $V_1(x)$, for $x = 0.2$.[7] Figure 1 shows the value functions $V_1(X(s))$ (see (11)) in the cases that correspond to Table 1.

First, we elaborate the result in the base case. The firm invests in the first stage project as soon as the cash flow $X(t)$ hits the investment threshold $x^* = 1.17$, and the R&D project will take $T_1 \in \{2, 3, 4\}$ years until completion. At completion time $T^* + T_1$, the firm will know the level of technological success $A_1 \in \{0, 1, 2\}$. If the R&D project fails ($A_1 = 0$), the firm will not receive any cash flow and will be finished with the project. Consider the case in which the first stage turns out to be successful. The firm will receive cash flow $A_1 X(t)$, and at the same time, it attains the growth opportunity. If $X(T^* + T_1)$ is larger than $x^{**}(1) = 2.46$, the firm will immediately invest in the growth project and receive increased cash flow $2A_1 X(t)$. According to our computation, due to $X(T^*) = 1.16 < 2.46$, the probability of this scenario is very low (it is approximately 0.03). If $X(T^* + T_1)$ takes a value in $[1.23, 2.46)$, the growth option will be immediately exercised only in the case of great success ($A_1 = 2$). In the case of average success ($A_1 = 1$), the firm postpones the growth project until $X(t)$ hits the investment trigger $x^{**}(1) = 2.46$. The probability that $X(T^* + T_1) \in [1.23, 2.46)$ is 0.53. The last scenario is that $X(T^* + T_1)$ is less than $x^{**}(2) = 1.23$. In this scenario, the firm delays the second stage investment until $X(t)$ hits either 1.23 or 2.46, depending on $A_1$. The probability of this scenario is 0.44.

[7] If one considers the expected waiting time until the project initiation, in the cases with no rival preemption it is more practical to set $x$ sufficiently close to the investment trigger $x^*$. However, for the purpose of comparing all cases with the same parameter values, we used $x = 0.2$. The results are qualitatively robust as long as $x$ is smaller than $x^*$ (see Figure 1).

We now briefly explain the other cases and compare the results with those of the base case. We can see from Figure 1 that a higher $X(s)$ increases the gap between the base case and the case with no growth option. This is because the value of the growth option increases with a higher $X(s)$. Because the project value becomes lower in the absence of growth opportunity, the investment trigger $x^* = 1.31$ is higher than that of the base case. The result is consistent with the stylized fact that growth opportunities increase a project value and encourages investment.

The third row in Table 1 shows the case with preemption. Table 1 and Figure 1 present the case of $\lambda = 0.2$, which means that the expected time up to rival preemption is 5 years, although we will show the results with varying levels of $\lambda$ later in this section. Figure 1 shows that the possibility of rival preemption greatly decreases the project value, especially for a low $X(s)$. Despite the decreased value, the firm tries to speed up investment because rival preemption can occur before completion of the first stage. Indeed, with $\lambda = 0.2$, the investment trigger $x^*$ decreases to 0.44 from 1.16. There is no possibility of rival preemption after completion of the first stage, and then we have the same investment triggers as those of the base case. Accordingly, the time lag between project completion of the first stage and the second investment is expected to be long.

We see from the last row in Table 1 that access to debt financing increases the project value and decreases the investment triggers. These leverage effects are consistent with previous results by Hennessy (2004), Mauer and Sarkar (2005), and Sundaresan and Wang (2007). In the levered case, we assume that the bargaining powers of equity and debt holders are equal, i.e., $\eta = 0.5$. The multiplier , caused by debt financing, is equal to 1.14. We will closely explore the effects of leverage in Section 3.4.

[Insert Figure 2 here.]

Figure 2 represents the project values $V_1(0.2)$ and investment triggers $x^*, x^{**}(1)$, and $x^{**}(2)$ with varying levels of $\lambda$. The other parameter values are the same as the base case. Note that second investment triggers $x^{**}(1)$ and $x^{**}(2)$ do not depend on $\lambda$. We see that $V_1(0.2)$ greatly decreases with $\lambda$. As expected from the expression (9), the graph of $V_1(0.2)$ shows convexity. In other words, for a lower $\lambda$, the sensitivity is greater. For instance, $V_1(0.2)$ decreases from 8.35 to 3.79 with a very small $\lambda = 0.05$. In contrast to the monotonic decrease in $V_1(0.2)$, the investment trigger $x^*$ decreases until $\lambda = 0.12$, and after this point, it increases. The possibility of rival preemption decreases the project value, but at the same time, it increases the firm's incentive to invest early and complete the technology before its rivals. The non-monotonicity in $x^*$ results from the trade-off. For a lower $\lambda$, the sensitivity of $x^*$ with respect to $\lambda$ is negative because the latter dominates the former. On the other hand, for a higher $\lambda$, the sensitivity is positive because the former dominates the latter.

[Insert Figure 3 here.]

Figure 3 plots the project values $V_1(0.2)$ and investment triggers $x^*, x^{**}(1)$, and $x^{**}(2)$ with varying levels of the cash flow volatility $\sigma$.[8] The other parameter values are set at the base case. Similar to Chapter 5.2.A of Dixit and Pindyck (1994), we can easily prove that $U_2(X(s), A_1)$ and $x^{**}(A_1)$ (see (4) and (5)) monotonically increase with $\sigma$. We can see from Figure 3 that $V_1(0.2)$ and $x^*$ also increase with $\sigma$. This result shows the robustness of the standard theory that higher market uncertainty increases an option value and delays the exercise of the option (e.g., Dixit and Pindyck (1994)) even if we incorporate several characteristics of R&D investment. We also examined the comparative statics for a wide range of parameter values, including positive $\lambda$. We found that the standard result is robust. Relatedly, Bar-Ilan and Strange (1996) and Bar-Ilan and Strange (1998) showed the similar effects, although their models do not include technological and rival uncertainty.[9]

So far, we have explained the results that were more or less known in the prior literature. We also found several findings that have not been addressed by previous studies. In the following subsections, we will elaborate key findings, logic behind the findings, and empirical implications.

### 3.2 Interactions between research duration and rival preemption

In this subsection, we reveal the effects of research duration combined with rival preemption. In the presence of rival preemption, longer duration increases the probability that other firms will complete the same technology first. Then, we can expect some interactions between research duration and rival preemption. Although Bar-Ilan and Strange (1996) and Bar-Ilan and Strange (1998) examined the effects of duration in the absence of rival preemption, to our knowledge, there are no papers that examine the mixed effects of duration and rival preemption. We summarize new findings below.

[Insert Figure 4 here.]

The upper and lower panels of Figure 4 show the project values $V_1(0.2)$ and investment triggers $x^*$, respectively, with varying levels of research duration $T_1$. In order to clarify the effects of $T_1$, we change levels of $T_1$ from 1 to 5, instead of taking $T_1$ as random variables.[10] In addition to the case with no rival preemption ($\lambda = 0$), the panels plot $V_1(0.2)$ and $x^*$ in

[8]For simplicity, we change $\sigma$, taking all other parameters, $r$ and $\mu$ as constants. This means that changes in $\sigma$ have only an idiosyncratic risk component. Most of the literature, including Dixit and Pindyck (1994), presents the comparative statics under this assumption, although some papers, including Wong (2007), examines the comparative statics assuming the relation between $\mu$ and $\sigma$.

[9]Bar-Ilan and Strange (1996) highlight that the standard volatility effect does not always hold for investment with an abandonment option in addition to investment lag.

[10]We have similar results for varying levels of $\mathbb{E}[T_1]$ even if we consider $T_1$ as a random variable.

the cases with rival preemption ($\lambda = 0.1, 0.2$, and 0.3) so that we can see the interactions of research duration and rival preemption. The other parameter values are set at the base case. The investment triggers $x^{**}(A_1)$ are independent of $T_1$ and $\lambda$ and remain unchanged from those of the base case in Table 1, and hence we omit plots of $x^{**}(A_1)$.

The upper panel of Figure 4 indicates the straightforward results that $V_1(0.2)$ monotonically decreases with $T_1$ and that the decrease is intensified with $\lambda$. That is, research duration is more crucial to the firm as rival preemption is expected to occur earlier. As expected from the expression (8), we find stronger convexity of a graph for a higher $\lambda$. Notably, the lower panel shows that the sensitivity of $x^*$ with respect to $T_1$ depends greatly on $\lambda$. In the absence of preemption $x^*$ monotonically decreases with $T_1$. The firm speeds up investment considering that $X(T^*)$ is expected to go up to $e^{\mu T_1} X(T^*)$ at completion time.[11] This result is well known in the previous researches such as Bar-Ilan and Strange (1998) and Bar-Ilan and Strange (1998).

However, as seen from (8), a longer $T_1$ decreases the project value especially when $\lambda$ is high. This decrease in the project value leads to that $x^*$ monotonically increases with $T_1$ for $\lambda = 0.1, 0.2$, and 0.3 in the lower panel of Figure 4. According to our numerical analysis, with the slightest threat of rival preemption ($\lambda = 0.01$), the sensitivity of $x^*$ with respect to $T_1$ remains positive, while in the case with no rival preemption the sensitivity is negative. Thus, we conclude that in the presence of rival preemption, longer duration delays the firm's project initiation due to the decreased project value. This result is in sharp contrast with that of the case with no preemption.

Next, we explore the effects of uncertainty of research duration $T_1$. Actually, research duration can be extended from that of the initial schedule (e.g., Raz, Shenhar, and Dvir (2002)), although previous studies do not examine the effects. In order to clarify the effects of uncertainty of $T_1$, we consider random variables $T_1$ taking $1, 3$, and 5 with probability $w_T/2, 1 - w_T$, and $w_T/2$, respectively, and vary levels of weight $w_T$.[12] The variance of $T_1$ monotonically increases with $w_T$, while we maintain that $E[T_1] = 3$.

[Insert Figure 5 here.]

The upper and lower panels of Figure 5 show the project values $V_1(0.2)$ and investment triggers $x^*$, respectively, with varying levels of weight $w_T$. We exclude the graphs for $\lambda = 0$, which require very different scales, and we show the cases of $\lambda = 0.1, 0.2$, and 0.3 in the panels. The other parameter values are set at the base case. We can recognize from the panels that a higher $w_T$ enhances $V_1(0.2)$ and decreases $x^*$. The effects become stronger

[11] We do not consider the case of a negative $\mu$ because a negative growth rate is not practical for an R&D project.

[12] We have the same results for the base parameter value $T_1 \in \{2, 3, 4\}$, but in order to highlight the results, we presented the results for $T_1 \in \{1, 3, 5\}$ (of course, as the range of $T_1$ is wider, the effects are clearer.) For robustness, we also examined the comparative statics with respect to variance by changing the range of $T_1$ rather than $w_T$. The results are unchanged.

as $\lambda$ is higher. The reasoning is as follows. As explained with regard to the upper panel of Figure 4, $U_1(\cdot)$ is convex with respect to $T_1$ and the convexity is stronger with a higher $\lambda$. By Jensen's inequality and the convexity, $U_1(\cdot)$ increases with a higher $w_T$, leading to that a higher $w_T$ increases $V_1(0.2)$ and decreases $x^*$. The impact of $w_T$ is magnified by a higher $\lambda$.

In consequence, we have a key result that, combined with rival preemption, uncertainty of research duration plays a role in improving the project value and speeding up investment. Our result contrasts with the standard result that greater uncertainty tends to enhance the value of waiting and delaying investment (cf. Figure 3). Our result regards technological uncertainty that will not be dissolved by waiting, whereas the standard volatility effect is based on market uncertainty that will be dissolved by waiting. This difference causes the opposite effects of uncertainty. In an R&D project with high technological uncertainty, unlike in a case with only market uncertainty, a firm possesses incentive to dissolve technological uncertainty by accelerating investment. Our result is consistent with the empirical evidence in Driver, Temple, and Urga (2006). They observed positive effects of uncertainty on investment in industries with high R&D intensity.

### 3.3 Interactions between the growth option and uncertainty of technological success

In this subsection, we examine the effects of uncertainty of technological success $A_1$. Although the real options literature have stressed the effects of market uncertainty on investment, few papers have examined the effects of technological uncertainty on investment. Below, we reveal the effects of the variance of $A_1$ on project value and investment timing. We consider random variables $A_1$ taking $0, 1$, and $2$ with probability $w_A/2, 1-w_A$, and $w_A/2$, respectively, and vary levels of weight $w_A$. Note that the variance of $A_1$ monotonically increases with $w_A$ and $E[A_1] = 1$ is always satisfied.

[Insert Figure 6 here.]

The upper and lower panels of Figure 6 show the project values $V_1(0.2)$ and investment triggers $x^*$, respectively, with varying levels of $w_A$. In order to examine interactions between technological uncertainty and a growth opportunity, we plots the graphs for different sizes of growth opportunity, i.e., $A_2 = 1.5, 2, 2.5$, and $3$. The other parameter values are set at the base case. Note that in the case with no growth option ($A_2 = 1$), $V_1(0.2)$ and $x^*$ are 4.16 and 1.31, respectively, for all $w_A$. The second investment triggers $x^{**}(1)$ and $x^{**}(2)$ do not depend on $w_A$; indeed, we have $x^{**}(2) = 2.46, 1.23, 0.82$, and $0.62$ for $A_2 = 1.5, 2, 2.5$, and $3$, respectively. The investment triggers $x^{**}(1)$ are twice as large as $x^{**}(2)$.

In Figure 6, we recognize that $V_1(0.2)$ monotonically increases with $w_A$, while $x^*$ monotonically decreases with $w_A$. These effects of $w_A$ become greater as $A_2$ increases.

The intuition behind the results is as follows. The growth option value $U_2(X(s), A_1)$ is not negative but zero even if the first stage fails. On the other hand, $U_2(X(s), A_1)$ monotonically increases with $A_1(> 0)$. Because of this convexity of the growth option and Jensen's inequality, a higher $w_A$ increases $U_2(X(s), A_1)$. Considering the increased value of the growth option, the firm decreases the investment trigger $x^*$ for a higher $w_A$. Accordingly, the effects of uncertainty of $A_1$ become stronger as the size of the growth option, $A_2$, is larger.

We conclude that higher uncertainty of technological success improves the project value and encourages R&D investment by enhancing the growth option value. This argument aligns with that of compound option cases, in which greater uncertainty may speed up the exercise by increasing the embedded option value, although it contrasts with the standard effect of uncertainty in the real options literature. Furthermore, we highlight that the types of uncertainty determine the sensitivity of uncertainty on investment. Similar to uncertainty of research duration in Section 3.2, uncertainty of technological success will never be dissolved by waiting, and hence, the firm has no incentive to delay investment and obtain extra information. Our result is also consistent with Driver, Temple, and Urga (2006) who empirically showed that industries with high R&D intensity tend to have positive effects of uncertainty on investment.

On the other hand, because $x^{**}(1)$ and $x^{**}(2)$ are constants, a higher $w_A$ increases the lag between completion of the first stage and initiation of the second stage project for a fixed $A_1$. For instance, we take a look at the probability that the firm proceeds to the second stage immediately after completion of the first stage project. As explained in Section 3.1, in the base case ($w_A = 2/3$), the firm proceeds to the growth project immediately after the success of the first stage with probability 0.03, and it proceeds immediately to the growth project only in the case of great success with probability 0.53. For $w_A = 0.1$, due to the increased trigger $x^* = 1.26$, the probabilities go up to 0.06 and 0.6, respectively. Although one considers from this that a higher $w_A$ lengthens the waiting time, the interpretation is not correct in terms of the prior probabilities. Indeed, the prior probabilities that the firm invests in the second stage project right after completion of the first stage project are $0.03 \times 2/3 + 0.53 * 1/3 = 0.2$ for $w_A = 2/3$ and $0.06 \times 19/20 + 0.6 \times 1/20 = 0.09$ for $w_A = 0.1$. The ex ante expected waiting time also decreases with $w_A$. Therefore, we can state that greater uncertainty of technological success accelerates R&D investment.

## 3.4 Effects of leverage

Lastly, we explore the effects of the use of debt financing. Note that debt financing is available only for the second stage project with no risks of technological failure. We readily see from (16) and (17) that leverage increases the growth option value $U_{2L}(X(s))$ and decreases the second investment trigger $x_L^{**}(A_1)$. These correspond to the results in Sundaresan and Wang (2007). We now turn to the initial project value $V_1(0.2)$ and investment trigger $x^*$.

[Insert Figure 7 here.]

The upper and lower panels of Figure 7 show the project values $V_1(0.2)$ and investment triggers $x^*$, respectively, in both unlevered and levered cases with varying sizes of growth opportunity $A_2$. We take the size of growth opportunity $A_2$ as the horizontal axis because leverage influences the firm through the growth project. Naturally, the leverage effects increase with $A_2$. For comparison, in the lower panel, we present the second investment triggers $x^{**}(2)$ in addition to $x^*$, but we omit $x^{**}(1)$, which is twice as high as $x^{**}(2)$.

We can see from the upper panel of Figure 7 that $V_1(0.2)$ in the levered case is 1.16 to 1.17 times higher than that of the base case. The lower panel shows that $x^*$ and $x^{**}(2)$ in the levered case are lower than those of the base case. Thus, access of debt financing increases the project value and accelerates investment in the R&D project. This leverage effect is consistent with the prior literature (e.g., Hennessy (2004), Mauer and Sarkar (2005), and Sundaresan and Wang (2007)).

We now look at the lower panel of Figure 7 more closely. We find that the impact of leverage on $x^*$ is weaker than that on $x^{**}(2)$. This is probably because debt financing influences the growth investment directly, while it influences the first stage investment at second hand. As a result, leverage tends to decrease the waiting period between success of the first project and initiation of the growth project. Indeed, for $A_2 = 2$, we have $x^* = 1.05 > x^{**}(2) = 0.96$ in the levered case, while we have $x^* = 1.16 < x^{**}(2) = 1.23$ in the base case. Most of empirical studies stated that large firms have advantages over small firms in growth opportunities from R&D investment (e.g., Ho, Tjahjapranata, and Yap (2006)). Our result could be related to the evidence because larger firms that suffer less financing costs are more likely to approximate the optimal capital structure.

# 4 Conclusion

In this paper, we developed a real options model for evaluating and optimizing an R&D project. The model can capture the effects of various features of R&D investment such as research duration, growth opportunity, and technological, demand market, and rival preemption uncertainty; nevertheless, the model is computationally tractable so that it can help real-world decision-making process of R&D investment. Further, we presented numerical results for a wide range of parameter values and unveiled several interactions of the key features of R&D. Below, we summarize notable results.

In the absence of rival preemption, the firm speeds up investment in a project with longer research duration, but a slight possibility of rival preemption reverses this effect of duration. Indeed, longer duration drives the firm to delay investment because it increases the possibility of rival preemption and decreases the project value. As uncertainty of research duration and technological success of the R&D project is higher, the project value increases and the optimal investment time is earlier. A higher possibility of rival preemption intensifies the effects of uncertainty of duration because it strengthens the convexity of the project value with respect to duration. On the other hand, the effects of uncertainty of technological success are amplified by an increase in the growth option. Access to debt financing in the growth project increases the project value and accelerates investment, but the impact on the first stage is relatively weak. Our results can potentially account for several empirical findings.

# Acknowledgment

This work was supported by the Public Trust for Intellectual Property Research Support Fund and KAKENHI (23310103).

# References

Baker, H., S. Dutta, and S. Saadi, 2011, Management views on real options in capital budgeting, *Journal of Applied Finance* 21, 18–29.

Bar-Ilan, A., and W. Strange, 1996, Investment lags, *American Economic Review* 86, 610–622.

Bar-Ilan, A., and W. Strange, 1998, A model of sequential investment, *Journal of Economic Dynamics and Control* 22, 437–463.

Brown, J., S. Fazzari, and B. Petersen, 2009, Financing innovation and growth: Cash flow, external equity, and the 1990s R&D boom, *Journal of Finance* 64, 151–185.

Cassimon, D., M. Backer, P. Engelenc, M. Wouwee, and V. Yordanovf, 2011, Incorporating technical risk in compound real option models to value a pharmaceutical R&D licensing opportunity, *Research Policy* 40, 1200–1216.

Dixit, A., and R. Pindyck, 1994, *Investment Under Uncertainty* (Princeton University Press: Princeton).

Driver, C., P. Temple, and G. Urga, 2006, Real options – delay vs. pre-emption: Do industrial characteristics matter?, *International Journal of Industrial Organization* 26, 532–545.

Fan, H., and S. Sundaresan, 2000, Debt valuation, renegotiation, and optimal dividend policy, *Review of Financial Studies* 13, 1057–1099.

Goldstein, R., N. Ju, and H. Leland, 2001, An EBIT-based model of dynamic capital structure, *Journal of Business* 74, 483–512.

Guthrie, G., 2009, *Real Options in Theory and Practice* (Oxford University Press: Oxford).

Hall, B., and J. Lerner, 2009, The financing of R&D and innovation, NBER WORKING PAPER SERIES.

Hartmann, M., and A. Hassan, 2006, Application of real options analysis for pharmaceutical R&D project valuation?empirical results from a survey, *Research Policy* 35, 343–354.

Hennessy, C., 2004, Tobin's q, debt overhang, and investment, *Journal of Finance* 59, 1717–1742.

Ho, Y., M. Tjahjapranata, and C. Yap, 2006, Size, leverage, concentration, and R&D investment in generating growth opportunities, *Journal of Business* 79, 851–876.

Kellogg, D., and J. Charnes, 2000, Real-options valuation for a biotechnology company, *Financial Analyst Journal* 56, 76–84.

Lee, J., and A. Paxson, 2001, Valuation of R&D real American sequential exchange options, *R&D Management* 31, 191–201.

Leland, H., 2004, Predictions of default probabilities in structural models of debt, *Journal of Investment Management* 2, 5–20.

Loch, C., and K. Bode-Greuel, 2001, Evaluating growth options as sources of value for pharmaceutical research projects, *R&D Management* 31, 231–248.

Mauer, D., and S. Sarkar, 2005, Real options, agency conflicts, and optimal capital structure, *Journal of Banking and Finance* 29, 1405–1428.

Nishihara, M., and A. Ohyama, 2008, R&D competition in alternative technologies: A real options approach, *Journal of the Operations Research Society of Japan* 51, 55–80.

Nishihara, M., and T. Shibata, 2013, The effects of external financing costs on investment timing and sizing decisions, *Journal of Banking and Finance* 37, 1160–1175.

Pakes, A., and M. Schankerman, 1984, The rate of obsolescence of patents, research gestation lags, and the private rate of return to research resources, in Z. Griliches, ed.: *R&D, Patents, and Productivity* . pp. 73–88 (University of Chicago Press: Chicago).

Perlitz, M., T. Peske, and R. Schrank, 1999, Real options valuation: the new frontier in R&D project evaluation?, *R&D Management* 29, 255–269.

Raz, T., A. Shenhar, and D. Dvir, 2002, Risk management, project success, and technological uncertainty, *R&D Management* 32, 101–109.

Sarkar, S., 2008, Can tax convexity be ignored in corporate financing decisions?, *Journal of Banking and Finance* 32, 1310–1321.

Sundaresan, S., and N. Wang, 2007, Investment under uncertainty and strategic debt service, *American Economic Review, Papers & Proceedings* 97, 256–261.

Weeds, H., 2002, Strategic delay in a real options model of R&D competition, *Review of Economic Studies* 69, 729–747.

Wong, K., 2007, The effect of uncertainty on investment timing in a real options model, *Journal of Economic Dynamics and Control 31* 31, 2152–2167.

Table 1: Project values and investment triggers.

| | $V_1(0.2)$ | $x^*$ | $x^{**}(1)$ | $x^{**}(2)$ |
|---|---|---|---|---|
| Base | 8.35 | 1.16 | 2.46 | 1.23 |
| No growth | 4.16 | 1.31 | N/A | N/A |
| Preemption | 0.63 | 0.44 | 2.46 | 1.23 |
| Leverage | 9.77 | 1.05 | 1.93 | 0.96 |

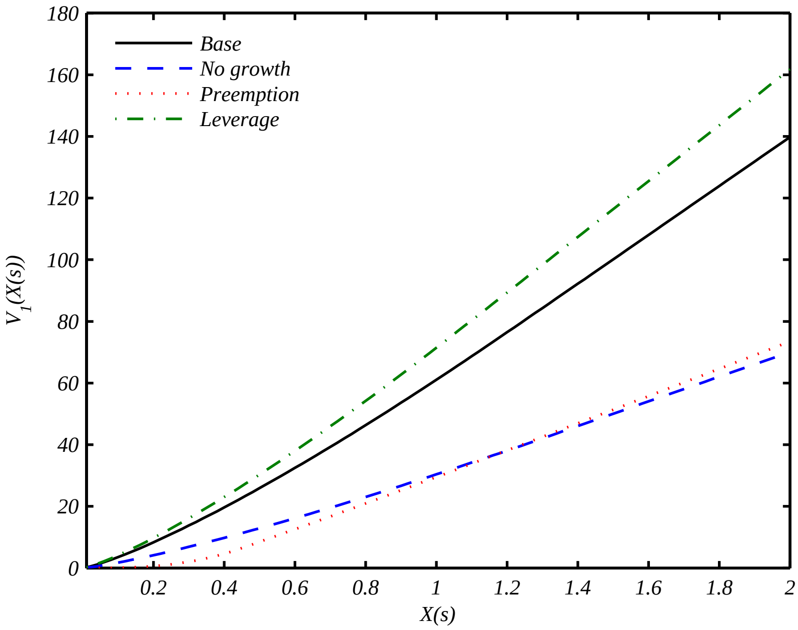


Figure 1: Value functions. The figure plots the project value functions $V_1(X(s))$ in the base case, the case with no growth option ($A_2 = 1$), the case with rival preemption ($\lambda = 0.2$), and the levered case ($\eta = 0.5$).

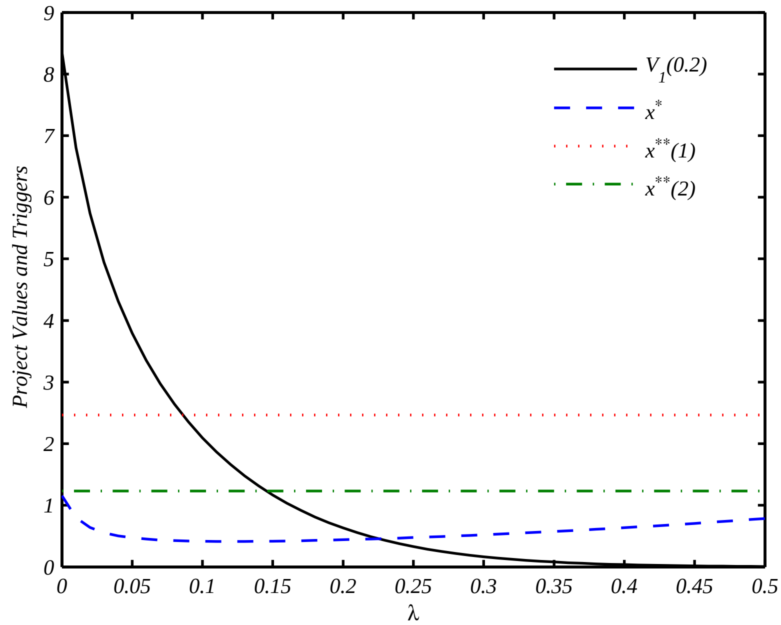


Figure 2: Effects of rival preemption. This figure plots the project values $V_1(0.2)$ and investment triggers $x^*, x^{**}(1)$, and $x^{**}(2)$ with varying levels of $\lambda$. The other parameter values are set at the base case.

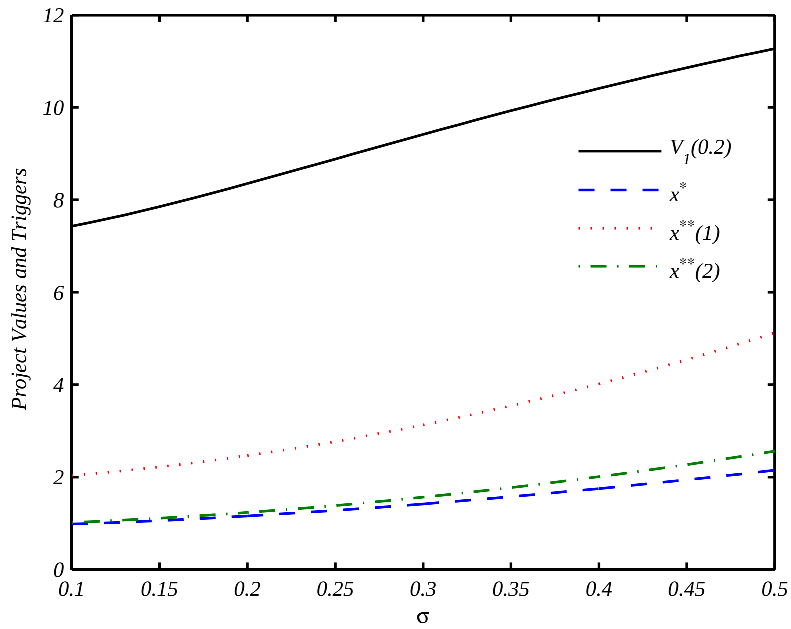


Figure 3: Effects of market uncertainty. This figure plots the project values $V_1(0.2)$ and investment triggers $x^*, x^{**}(1)$, and $x^{**}(2)$ with varying levels of market uncertainty $\sigma$. The other parameter values are set at the base case.

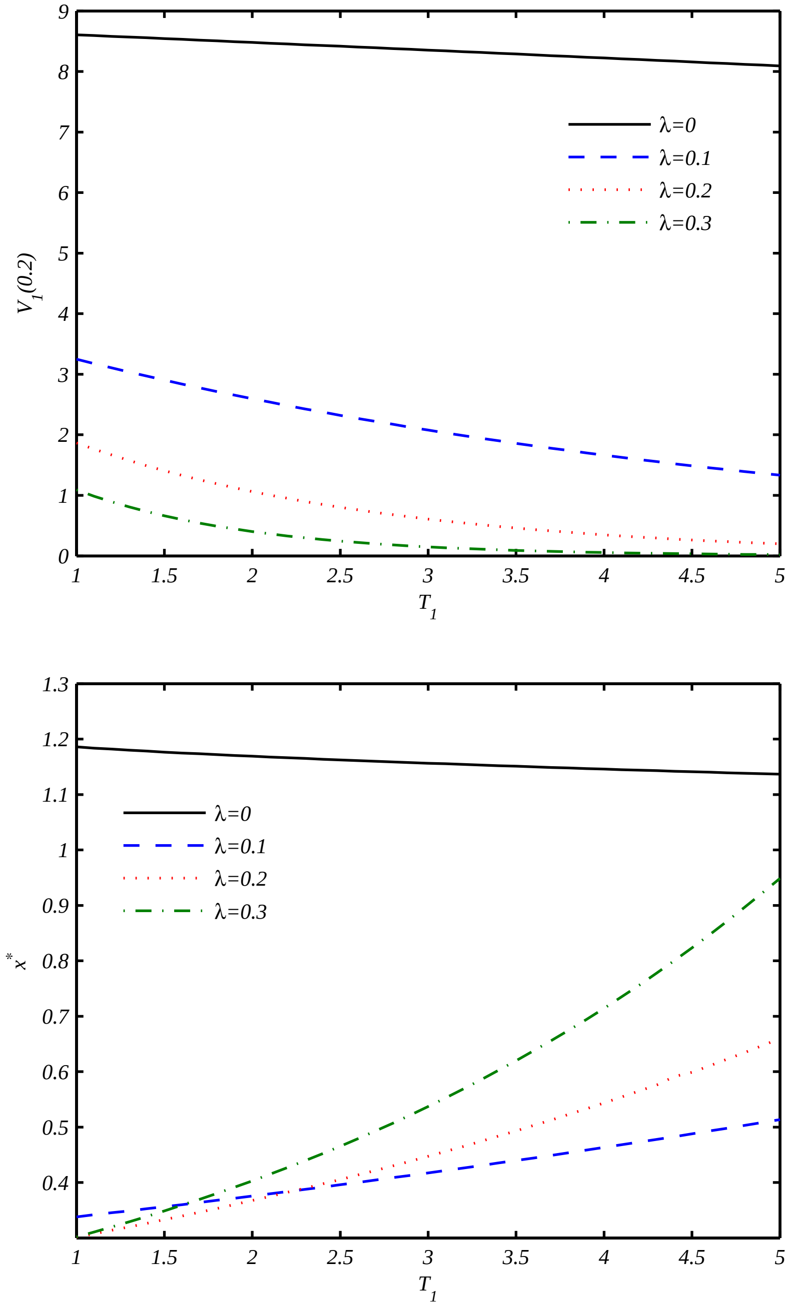


Figure 4: Effects of research duration. The upper and lower panels plot the project values $V_1(0.2)$ and investment triggers $x^*$ with varying levels of research duration $T_1$, respectively. The figure shows the cases of $\lambda = 0, 0.1, 0.2$, and $0.3$. The other parameter values are are set at the base case.

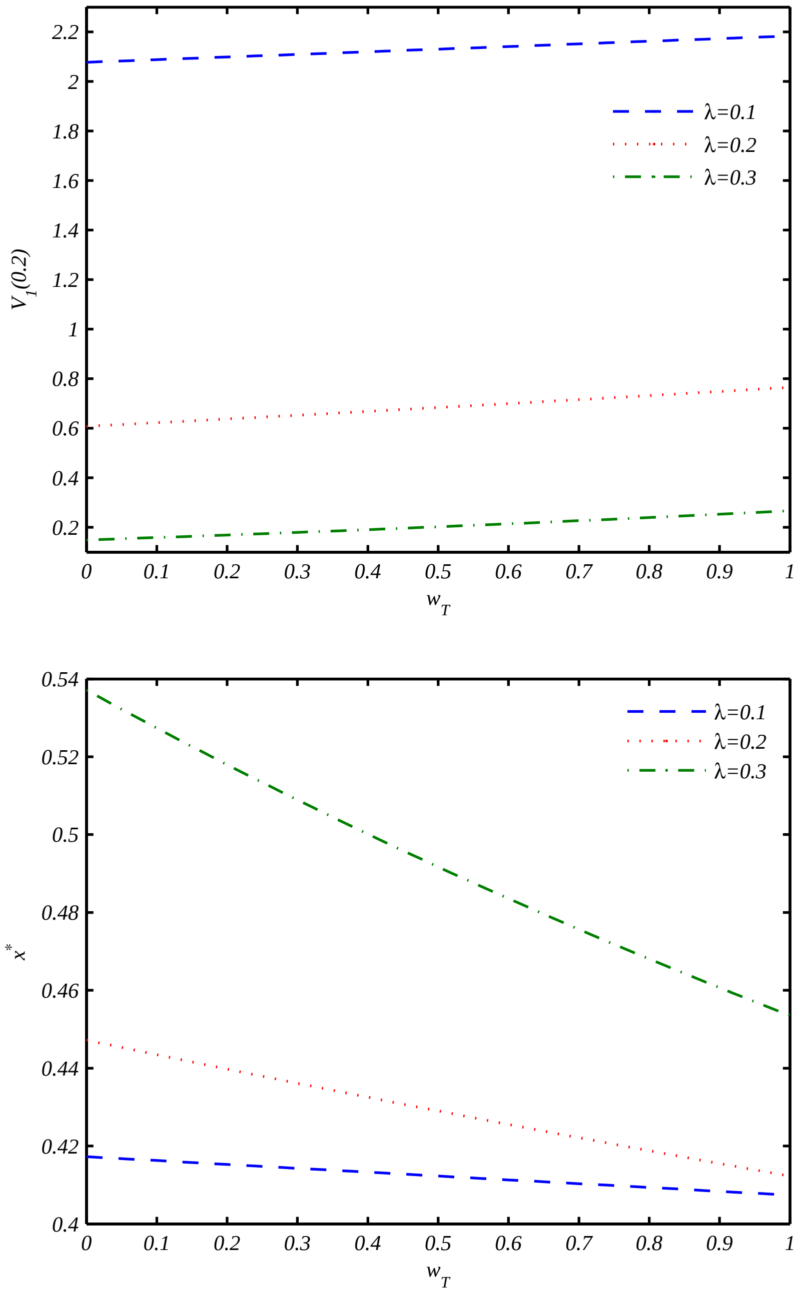


Figure 5: Effects of uncertainty of research duration. The upper and lower panels plot the project values $V_1(0.2)$ and investment triggers $x^*$ with varying levels of $w_T$, respectively. The figure shows the cases of $\lambda = 0, 0.1, 0.2$, and 0.3. The other parameter values are set at the base case.

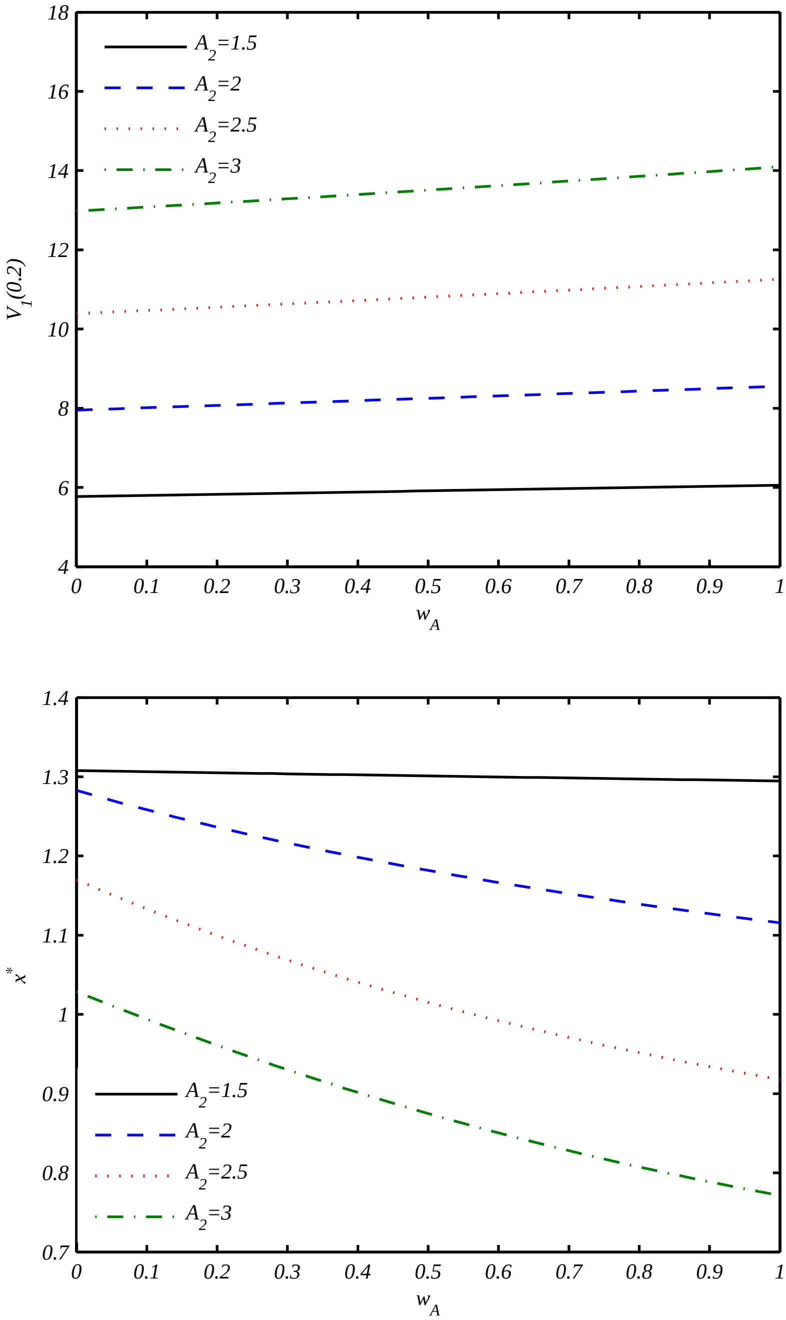


Figure 6: Effects of uncertainty of technological success. The upper and lower panels plot the project values $V_1(0.2)$ and investment triggers $x^*$ with varying levels of $w_A$, respectively. The figure shows the cases of $A_2 = 1.5, 2, 2.5,$ and 3. The other parameter values are set at the base case.

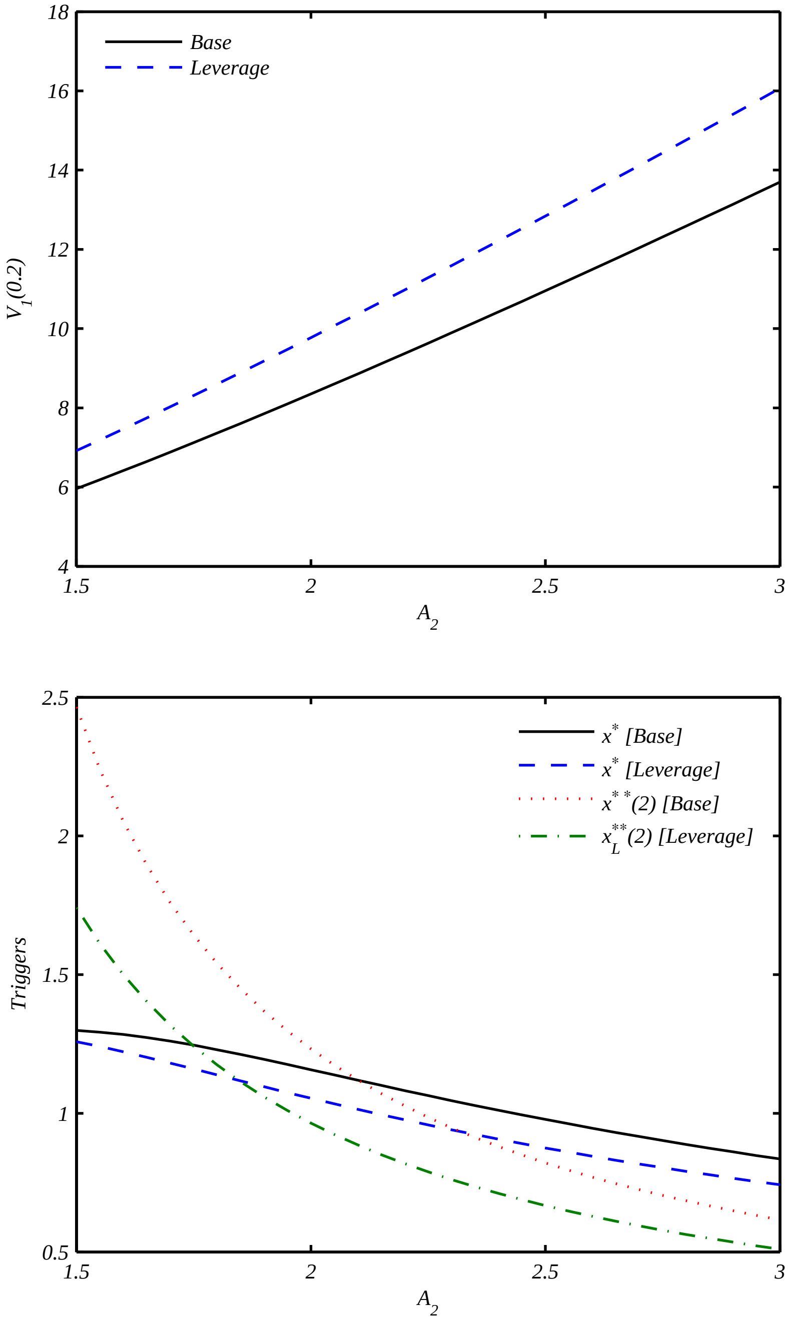


Figure 7: Effects of leverage. The upper and lower panels plot the project values $V_1(0.2)$ and investment triggers $x^*$ and $x^{**}(2)$, respectively, in both base and levered cases with varying sizes of growth opportunity $A_2$. The other parameter values are set at the base case.